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**Michael Dolega**

Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
mdolega@bankofcanada.ca



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# Abstract

The author attempts to track Canadian labour productivity over the past four decades using a multivariate dynamic factor model that, in addition to the labour productivity series, includes aggregate compensation and consumption information. Productivity is assumed to switch between two regimes (the high-growth state and the low-growth state) with different trend growth rates according to a first-order Markov process. The author finds that labour productivity in Canada fell from the high-growth to the low-growth state towards the end of the 1970s, and that it has not yet reverted to the high-growth state. In particular, the model primarily attributes the resurgence of labour productivity growth in the late nineties to transitory effects.

*JEL classification: O4, O51, C32*
*Bank classification: Productivity*



# Résumé

L'auteur décrit l'évolution de la productivité du travail au Canada pendant les quatre dernières décennies au moyen d'un modèle multivarié à facteurs dynamiques alimenté par des données sur la productivité du travail ainsi que sur la rémunération et la consommation globales. Il fait l'hypothèse que la productivité oscille entre deux régimes que caractérisent des taux de croissance tendanciels différents (l'un élevé et l'autre faible), suivant un processus markovien d'ordre un. L'auteur constate que, vers la fin des années 1970, la productivité des travailleurs canadiens est passée d'un régime de croissance élevée à un régime de faible croissance, sans avoir renoué depuis avec son régime antérieur. Le modèle explique le gros de l'accélération de la croissance de la productivité du travail observée à la fin des années 1990 par des effets transitoires.

*Classification JEL : O4, O51, C32*
*Classification de la Banque : Productivité*

## 1 Introduction

The pickup in the growth of labour productivity[1] that occurred in the late nineties in both Canada and the United States was initially considered temporary by many analysts. Two important drivers of this acceleration were capital deepening – propelled by increased information and communication technology (ICT) investment – and multifactor productivity (MFP) gains in the ICT-producing and service industries. While the proponents of the “new economy” in the United States were vindicated by even stronger labour productivity figures after 2001, Canadian productivity growth slumped to mediocre rates.

The recent divergence of Canadian productivity growth rates from those of the United States was surprising, in view of the rather similar productivity growth records of the two economies over the past three decades, and also worrying, since slow labour productivity growth limits any improvement in living standards, arguably one of the most important goals of economic policy. Furthermore, the slowing of Canadian labour productivity growth increased the degree of uncertainty about estimates of trend productivity. Such increased uncertainty carries over to the estimation of the output gap, and, consequently, to the extent of inflationary pressures, a crucial ingredient in conducting monetary policy. In fact, some economists believe that the high level of U.S. inflation in the mid- to late seventies was a result of the U.S. Federal Reserve not being aware of a slowdown in the growth of trend labour productivity, which led to an overestimation of potential output and overly loose monetary policy.[2]

The primary purpose of this paper is to attempt to estimate the growth of trend labour productivity over the past forty years, and to draw inferences about its near-term developments. Particular attention is paid to determining whether the labour productivity revival experienced in the late nineties was in fact a pickup in the

1. Labour productivity is defined as real output per hour worked, unless otherwise stated.
2. See, for example, Orphanides (2003) and Cogley (2005).

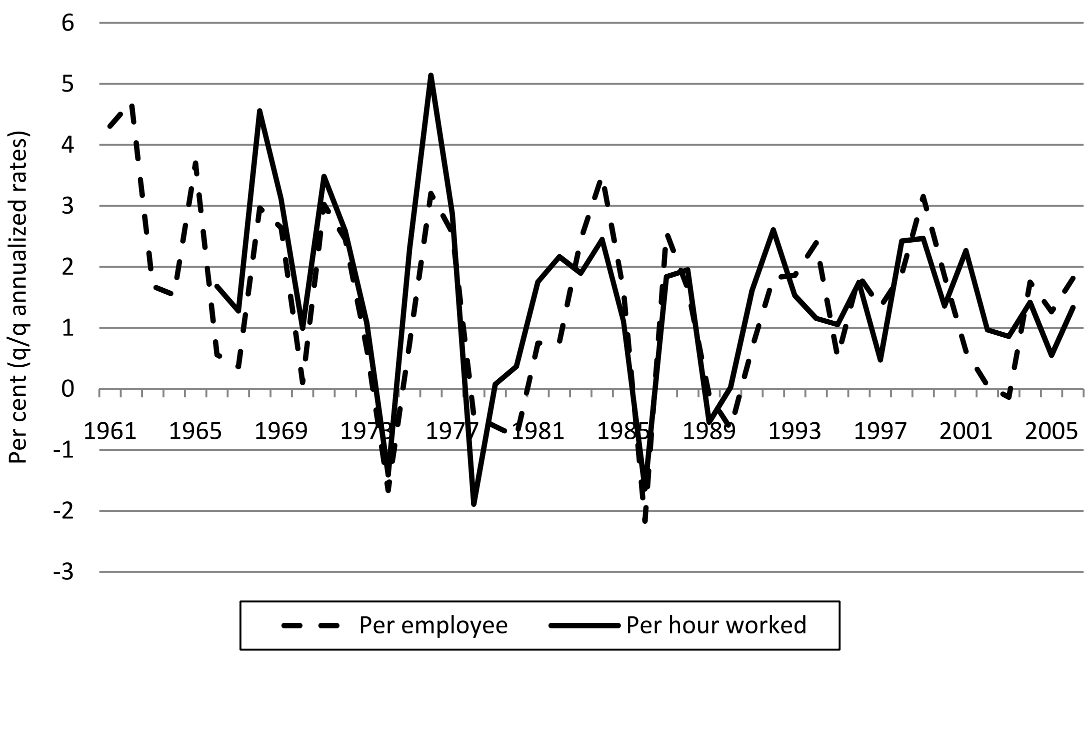
trend, or whether transitory factors instead played a major role in the accelerated growth experienced at that time. An innovation of this paper is that information on labour productivity, consumption, and labour income is used jointly to examine shifts in trend productivity growth.[3]

## 2 Background and Theory

### 2.1 History of productivity growth in Canada and the United States

Labour productivity in Canada has in the past four decades grown at an average annual rate of 1.8 per cent for the total economy and 2.0 per cent in the business sector. This masks considerable variability across time segments that may reflect cyclical developments or shifts in trend (Chart 1). For example, the years

**Chart 1**
**Labour Productivity Growth in Canada Measured by "Per Employee" and "Per Hour Worked"**




between 1962 and 1972 witnessed robust labour productivity growth, averaging over 3 per cent annually, and were followed by a slowdown to approximately

3. The approach follows Kahn and Rich (2006), who provide a similar analysis of U.S. data.

2.2 per cent per annum around the mid-1970s (Table 1). In the late 1970s, labour productivity growth suffered a further decrease to 1 per cent, which lasted for nearly two decades. Signs of recovery surfaced around 1997, but were interrupted in the early years of the following decade.

**Table 1**
**Average Annual Growth Rates of Labour Productivity in the Total Economy**

| | Canada | | United States |
|---|---|---|---|
| | Per employee | Per hour worked | Per hour worked |
| 1962-72 | 2.29 | 3.05 | 2.65 |
| 1973-77 | 1.53 | 2.23 | 1.56 |
| 1978-96 | 0.84 | 1.01 | 1.29 |
| 1997-2000 | 2.63 | 2.53 | 2.08 |
| 2001-05 | 0.73 | 1.07 | 2.60 |

Note: Canadian data are based on real output (at market prices), employment, and hours taken from Statistics Canada's Productivity Program Database. United States data are based on real output from the Bureau of Economic Analysis, and employment and hours are from the Bureau of Labor Statistics (2007).

The slowdown that occurred in the Canadian business sector in the 1970s is attributed primarily to a sharp decline in multifactor productivity[4] growth, which suffered an even larger setback in the 1980s when it turned negative.[5] Multifactor productivity growth picked up considerably to about 1.8 per cent between 1997 and 2000, and subsequently subsided to a healthy 0.8 per cent between 2001 and 2004. The latter period, however, witnessed a slowdown in labour quality improvement and a negative contribution of capital deepening (Statistics Canada 2002, 2005).

The recent productivity record in the United States differs considerably from the Canadian experience, with the 2001–05 growth in labour productivity accelerating to 3.2 per cent and that in multifactor productivity averaging 1.8 per cent, more than double its 1987–2000 average.[6]

4. Multifactor productivity (or total factor productivity) is defined as real output per combined inputs (capital and labour).

5. Harchaoui and Tarkhani (2005) indicate that, between the 1960s and 1970s, the average annual multifactor productivity growth rate fell from 2.1 per cent to 0.4 per cent. This rate fell further to -0 .1 per cent throughout the 1980s (see CANSIM Table 383-0016).

6. Multifactor productivity data are defined as real value-added output per combined inputs for the non-farm private business sector provided by the Bureau of Labor Statistics. Data are as of the latest release date of 23 March 2007.

While trends in MFP are certainly worth investigating (French 2005), this paper concentrates on labour productivity, since multifactor productivity data are available only at an annual frequency and with a significant lag. This makes near real-time inference very difficult.

## 2.2 Literature review

While labour productivity was always an important theme of empirical research, interest in the topic surged at the end of the nineties. Increased attention to this issue seems to be related to the continued upsurge in the growth rate of labour productivity in the United States, and the subsequent debate on the causes and sustainability of such a rise. Early papers rebuke the concept of the "new economy" (Gordon 1998), or at least advise caution (Stiroh 1999). Hansen (2001), however, uses an iterative method based on Bai and Perron (1998), searching all possible break dates, and finds evidence of a break in 1997 for the durables manufacturing sector of the U.S. economy. The trend-and-cycle analysis of Roberts (2001), using a Kalman filter to extract a stochastic trend, also suggests that very little of the mid-nineties acceleration is attributed to cyclical factors. Gordon (2000) and Jorgenson, Ho, and Stiroh (2002) argue that productivity growth did in fact revive and is likely to remain intact for the immediate future. This is a view shared, amongst other authors, by Kahn and Rich (2006), who, utilizing a dynamic factor model with Markov switching, find that labour productivity growth in the United States fell to a low-growth regime in 1973 and rebounded to a high-growth regime in 1997.

In the Canadian context, the scope of research has been largely restrained to growth-accounting analyses, such as Sharpe (2004) and Macklem (2003). The exceptions are Robidoux and Wong (2003), who utilize an HP filter to perform their analysis, and van Norden (2005), who uses the Andrews (1993) mid-sample statistic (Sup $W^i$) and the Andrews (2003) end-of-sample statistic (Exp $W^i$): both papers find little evidence of change in trend labour productivity growth in the 1990s. Just as in Canada, the work done by authors in Europe has been primarily concerned with explaining why domestic productivity growth rates have not experienced the acceleration so well documented for the United States (see Benati 2006; Oulton and Srinivasan 2005; and Jimeno, Moral, and Saiz 2006).

## 2.3 Theory and methods

The analysis in this paper draws primarily on two empirical methodologies: state-space techniques, which are helpful for modelling common technology trends in multiple data series, and Markov-switching specifications, useful for modelling regime switching.

Data on the growth of labour productivity are often very volatile – especially at a quarterly frequency – making it difficult to draw definitive conclusions about its underlying trend. In order to make more confident inferences regarding this trend, we broaden the set of economic series to include – in addition to output – consumption and wage income (each is expressed relative to labour input). This approach is motivated by neoclassical growth theory, which states that these three series share a common trend (King, Plosser, and Rebelo 1988; Kahn and Rich 2006). This theoretical structure lends itself to the use of a dynamic factor model to describe co-movement across economic time series, with an unobserved factor used to capture the common technology-growth component. A state-space model is utilized to represent the observed variables as linear combinations of unobserved permanent (technology) and transitory (business cycle) state variables and error terms. State-space models are usually estimated by way of a Kalman filter, outlined in Kalman (1960). They were first introduced to economics by Harvey (1981) and quickly used in various applications afterwards. Stock and Watson's (1988) dynamic factor model of coincident economic indicators is a recent application of the state-space models.

Structural changes have been analyzed as far back as Chow's (1960) F-test. Quandt (1972) and Goldfeld and Quandt (1973) first proposed the notion that the probability of a switch depends on the regime in effect at that particular time. Markov switching was introduced along with a dynamic model in Hamilton's (1989) seminal paper, in order to deal with endogenous breaks. Estimating a model in state-space form with regime shifts posed serious difficulties until Kim's (1994) approximate maximum-likelihood procedure made it computationally feasible. This allowed a wide range of applications, including Kim and Murray's (2002) and Kim and Piger's (2002) papers, which adapt the dynamic factor model of Stock and Watson to a state-space representation with Markov switching. These more recent authors use this model to track GDP growth while allowing for Friedman-type "plucks." Finally, Kahn and Rich (2006) adapt Kim and Murray's model to analyze labour productivity for the United States. Their work serves as a foundation for this paper.

## 3 Model

The analysis in this paper uses the same econometric specification as Kahn and Rich (2006). Their specification, reviewed in what follows, is multivariate and contains four series: labour productivity, compensation per hour, consumption per hour, and detrended total hours worked. These series are assumed to be a linear sum of a deterministic linear trend, a common stochastic trend component, a business cycle transitory component, and an error term, or:

$$Q_{i,t} = D_i t + \gamma_i X_t + \lambda_i x_t + z_{i,t} \qquad \forall i \in \{1,2,3,4\},$$

where $Q_{i,t}$ is the log of the relevant quantity, $D_i$ is the growth rate of the deterministic trend (the unconditional mean growth rate), $X_t$ is the permanent stochastic component common to all the series, $x_t$ is the common transitory component, and $z_{i,t}$ are series-specific idiosyncratic shocks. The trend component comprises the sum of the first two terms: the deterministic trend and the stochastic trend.

The stochastic trend component allows for Hamilton-type (1989) switches of the mean growth rate from the unconditional mean $D_i$, and captures stochastic technological growth. The stochastic component is assumed to follow an autoregressive structure:

$$\phi(L)\Delta X_t = \mu(S_{1,t}) + \nu_t, \qquad \nu_t \sim N(0,1),$$

$$\mu(S_{1,t}) = \begin{cases} \mu_0, & if\ S_{1,t} = 0, \\ \mu_1, & if\ S_{1,t} = 1, \end{cases}$$

$$Pr[S_{1,t} = 0 | S_{1,t-1} = 0] = q_1,$$

$$Pr[S_{1,t} = 1 | S_{1,t-1} = 1] = p_1,$$

where the error is assumed to be normal with mean zero, variance is set to unity for reasons of identification of the model, and the weighted average of the two means ($\mu_0$ and $\mu_1$) equals zero.

The transitory component is taken to capture business cycles and includes Friedman's (1993) "plucks," which model asymmetries of the business cycle as follows:

$$\phi^*(L)x_t = \tau(S_{2,t}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0,1),$$

$$\tau(S_{1,t}) = \begin{cases} 0, & if\ S_{2,t} = 0, \\ \tau, & if\ S_{2,t} = 1, \end{cases}$$

$$Pr[S_{2,t} = 0 | S_{2,t-1} = 0] = q_2,$$

$$Pr[S_{2,t} = 1 | S_{2,t-1} = 1] = p_2,$$

where the $\tau$ term controls for asymmetries of the business cycle. Without asymmetries there is no need for the differentiated states, and setting $\tau = 0$ simplifies the model.

The idiosyncratic error specification is such that:

$$\psi(L)z_{i,t} = \eta_{i,t}, \qquad \eta_{i,t} \sim N(0, \sigma_i^2),$$

where the hours worked series ($i = 4$) follows an AR(2) process while the remainder ($i = 1,2,3$) follow an autoregressive process of degree one,[7] and where the roots of $\psi(L)$ are outside the unit circle.

## 4 Data

Data for the project apply to the total economy and come from the System of National Accounts produced by Statistics Canada. Expenditure-based gross domestic product at market prices, in 1997 chain-weighted dollars, is used as the output series, while real personal expenditures on consumer goods and services measure private consumption. The labour income measure used in the project is the sum of "wages, salaries, and supplementary labour income" and 65 per cent of "net income of non-farm unincorporated business including rent" at market prices. The measure of labour input is Statistics Canada's Labour Force Survey (LFS) estimate of total actual hours worked in all jobs.[8] All series are seasonally adjusted quarterly data between 1966Q1 and 2006Q1, inclusive, providing 160 observations.

The aforementioned data allow us to construct the needed series. Productivity is defined as a natural logarithm of output per hour worked, or $Q_1 = ln(Y/H)$; labour income per hour worked is $Q_2 = ln(W/H)$; private consumption per hour worked is $Q_3 = ln(C/H)$; and labour input is the detrended[9] log of hours worked, or $Q_4 = ln(\widetilde{H})$. Labour input is detrended since it is used only in controlling for business cycles.

An important qualification with regards to the data needs to be noted. Productivity as defined in this project may substantially differ from the official headline

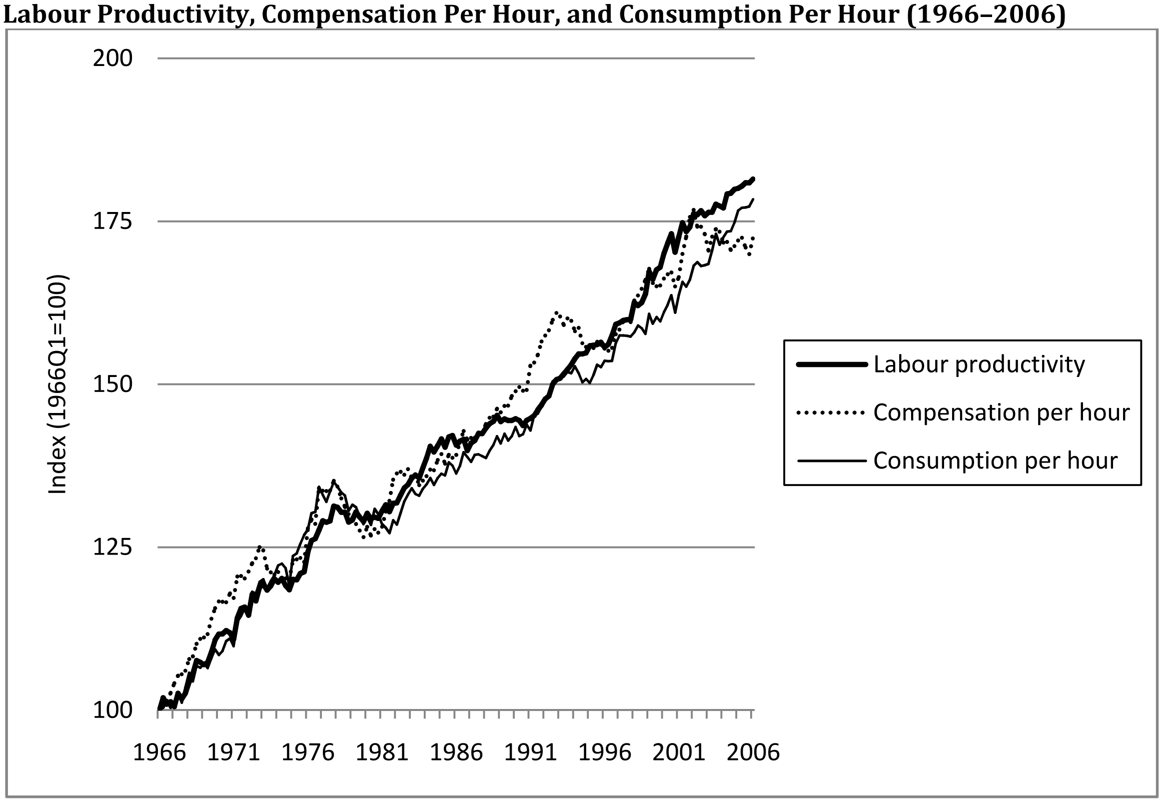
7. Autoregressive lag length is determined by extending the autoregressive process until further terms are no longer significant.
8. Since this series (v2685149) is not seasonally adjusted, an approximate adjustment is used by deriving the seasonal factors from the "actual total hours worked in main job" series (v4391505/v2685137).
9. The detrended variable is the residual of the regression of log-hours on a constant and a time variable.

productivity numbers published by Statistics Canada in their quarterly productivity report. There are two reasons for this divergence. First, the project output data are for the total economy, not the business sector only, as is highlighted in the productivity accounts. Second, while Statistics Canada's Productivity Program Database team builds the total hours worked figures mostly from LFS data, the two series are not identical, due to adjustments.

### 4.1 Summary statistics

The mean growth rate of labour productivity (as defined in the project) over the past four decades is 0.372 per cent quarter over quarter, or around 1.50 per cent annualized. Wages grew by an annualized rate of 1.37 per cent, while consumption grew by 1.45 per cent per year (Chart 2).

**Chart 2**
**Labour Productivity, Compensation Per Hour, and Consumption Per Hour (1966–2006)**



There is a significant amount of variance in the growth rates of all three series, especially in the wages and consumption data. This volatility makes it very difficult to determine clear patterns in the system, and is one of the motivating factors for using the model. The U.S. data for all three series, while slightly less volatile, have greater means, with annualized labour productivity growth averaging 1.93 per cent, or nearly half of a percentage point higher than the Canadian average (Table 2).

**Table 2**
**Summary Statistics for Project Data for Canada and the United States**

| | Canada | | | | United States | | | |
|---|---|---|---|---|---|---|---|---|
| | $\Delta ln(Y/H)$ | $\Delta ln(W/H)$ | $\Delta ln(C/H)$ | $ln(\widetilde{H})$ | $\Delta ln(Y/H)$ | $\Delta ln(W/H)$ | $\Delta ln(C/H)$ | $ln(\widetilde{H})$ |
| $\mu$ | 0.37 | 0.34 | 0.36 | - | 0.48 | 0.47 | 0.49 | - |
| $\sigma$ | 0.79 | 0.99 | 0.99 | 0.79 | 0.78 | 0.62 | 0.81 | 0.77 |
| $min$ | -1.7 | -2.3 | -2.2 | -1.9 | -1.4 | -0.97 | -1.4 | -3.6 |
| $max$ | 2.9 | 4.2 | 3.7 | 2.2 | 2.9 | 2.6 | 4.0 | 2.4 |

Note: Values are based on the sample period between 1966Q2 and 2006Q1 (1966Q2 and 2005Q3 for the United States).

## 4.2 Cointegration tests

Growth theory suggests that $Q_1$, $Q_2$, and $Q_3$ all share a common non-stationary trend capturing technological progress. Under the assumed Markov-switching model of technological growth, the trend is non-stationary, but not I(1).[10] Nevertheless, we believe that cointegration techniques may have some power to detect common-trend relationships and therefore conduct cointegration tests to provide some supporting evidence for the theoretical assumption of two cointegrating vectors (and one common trend). Two types of the Johansen (1991, 1995) test are used. The two variants[11] are Case 1, which includes an intercept only in the VAR, as well as Case 2*, which, in addition to the intercept in the VAR, includes a trend in the cointegrating equation. Both are appropriate for systems exhibiting an apparent trend in levels.

10. First-differences of an I(1) process are necessarily I(0), but regime switches result in potentially different means across regions. Thus, series are assumed to be integrated of order one within a regime.
11. The notation here follows Osterwald-Lenum (1992).

Tables 3 and 4 illustrate the maximum-eigenvalue statistics for the coincident data, one lag of productivity and wages per hour, and three lags of productivity and wages per hour. The maximum-eigenvalue statistic tests the null hypothesis that there are at most $(r)$ cointegrating vectors against the alternative of $(r+1)$ vectors.
The results of the tests are mixed and depend on whether lags of productivity and wages per hour are used, instead of coincident values in the cointegrating relationship. If the underlying trend was I(1), then results should not depend on whether lags or coincident values are used. However, in our set-up, with a Markov-switching model of technological growth, such lag dependence may provide additional information. In particular, the result that there are exactly two co-integrating vectors when lags of productivity and wages per hour are used, instead of coincident values, may suggest that trend movements in the productivity and wages per hour series lead trend movements in the consumption per hour series by one to three quarters.

**Table 3**
**Cointegration Test Results - Intercept in VAR (Case 1)**

| $H_0$: $(rank \leq r)$ | $H_1: (rank = rank + 1)$ $\lambda_{max}$ *statistic* | | | |
|---|---|---|---|---|
| $r$ | Coincident | One lag | Three lags | 90% c.v. |
| 0 | 137.1*** | 159.0*** | 145.1*** | 18.6 |
| 1 | 8.67 | 79.9*** | 72.2*** | 12.1 |
| 2 | 0.00075 | 0.086 | 0.023 | 2.7 |

**Table 4**
**Cointegration Test Results - Intercept in VAR, Trend in CE (Case 2*)**

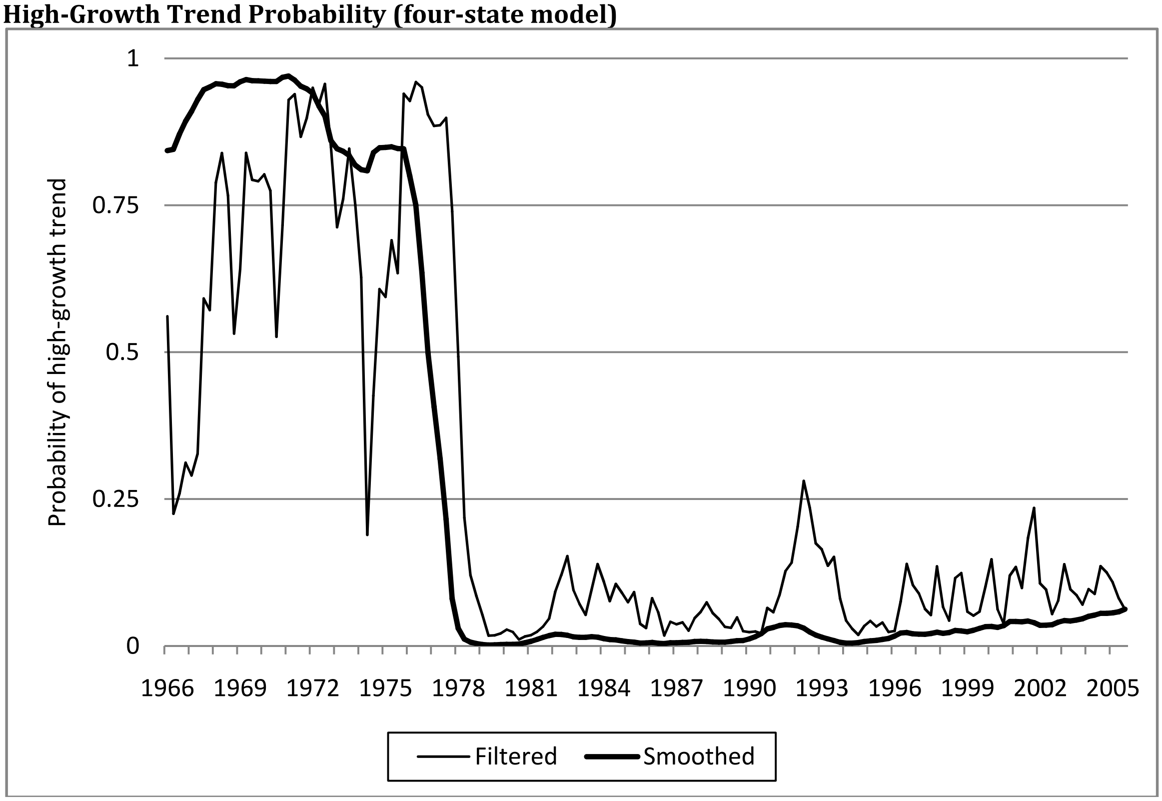
| $H_0$: $(rank \leq r)$ | $H_1: (rank = rank + 1)$ $\lambda_{max}$ *statistic* | | | |
|---|---|---|---|---|
| $r$ | Coincident | One lag | Three lags | 90% c.v. |
| 0 | 22.3* | 23.2** | 29.1*** | 23.11 |
| 1 | 12.0 | 16.9** | 17.1** | 16.85 |
| 2 | 0.25 | 1.2 | 0.66 | 10.49 |

Notes: ***, **, * indicate significance at the 5, 10, and 15 per cent level, respectively. Lag numbers range from coincident through one lag (productivity and wages per hour are lagged by one quarter) and three lags (productivity and wages per hour are lagged by three quarters).

## 5 Results

The most prominent conclusion drawn from the numerous variants of the model that were tested is that the trend in labour productivity growth in Canada has been in a low-growth state from the late 1970s through 2006Q1, growing at just above 1 per cent annually. Furthermore, the majority of variants indicate that the period from 1997 through 2001, a period characterized by stronger labour productivity growth in Canada, was subject to productivity shocks that were more transitory in nature than in the United States. While filtered probabilities (information up to and including point $t$) indicate up to a one-quarter chance of being in the high-growth state in the late nineties through 2002 (Chart 3), when we consider the full sample of smoothed probabilities (all information between 1966Q1 and 2006Q1), the probability decreases to below 5 per cent.

**Chart 3**
**High-Growth Trend Probability (four-state model)**



## 5.1 Trend state

The specification of the model is such that the labour productivity trend is at every point in time in one of two regimes: the low-growth state or the high-growth state. Estimates of the transition probabilities reveal that, in Canada, the low-growth state is slightly more persistent than the high-growth one. In other words, it is more probable that the economy will stay in the low-growth state if it was in that state last period, than it is that the economy will stay in the high-growth state if it was in that state last period. The estimates vary slightly across variants, but in most cases the low-growth transition probability, or $p_1$ , is about 1 percentage point higher than $q_1$, or the high-growth transition probability (Table 5).

**Table 5**
**Model Estimation Results**

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | Base model | Single lag model | Consumption data excluded | Wages data excluded | Productivity data only | United States data |
| $p$ | 0.984 | 0.979 | 0.350 | 0.985 | 0.971 | 0.982 |
| $q$ | 0.992 | 0.991 | 0.918 | 0.992 | 0.994 | 0.979 |
| $\mu_1$ | 0.55 | 0.49 | 2.56 | 0.81 | 0.29* | 0.50 |
| $\mu_0$ | -0.20 | -0.18 | -0.32 | -0.32 | -0.06* | -0.34 |
| $\gamma$ | 0.49 | 0.47 | 0.51 | 0.294 | 1.04 | 0.24 |
| $\lambda_1$ | 0.24 | 0.33 | 0.058* | 0.016* | 0.10 | -0.16 |
| $\lambda_2$ | 0.33 | 0.35 | -0.43 | | | 0.0029* |
| $\lambda_3$ | -0.78 | -0.73 | | -0.68 | | -0.48 |
| $\lambda_4$ | 0.35 | 0.30 | -0.082* | 0.56 | -0.50 | 0.55 |
| $\phi_1$ | -0.14* | 0.089* | -0.21 | -0.18* | -0.22 | 0.52 |
| $\phi_1^*$ | 0.64 | 0.47 | 1.47 | 0.91 | 1.71 | 1.44 |
| $\phi_2^*$ | 0.25 | 0.37 | -0.54 | 0.018* | -0.73 | -0.52 |
| $\psi_{1,1}$ | 0.87 | 0.96 | -0.14* | 0.89 | 0.43* | 0.91 |
| $\psi_{2,1}$ | 0.96 | 0.96 | -0.41* | | | 0.88 |
| $\psi_{3,1}$ | 0.76 | 0.82 | | -0.067* | | 0.070* |
| $\psi_{4,1}$ | 1.40 | 1.45 | 1.27 | 1.67 | 0.84 | 1.56 |
| $\psi_{4,2}$ | -0.43 | -0.48 | -0.32 | -0.69 | -0.17 | -0.61 |

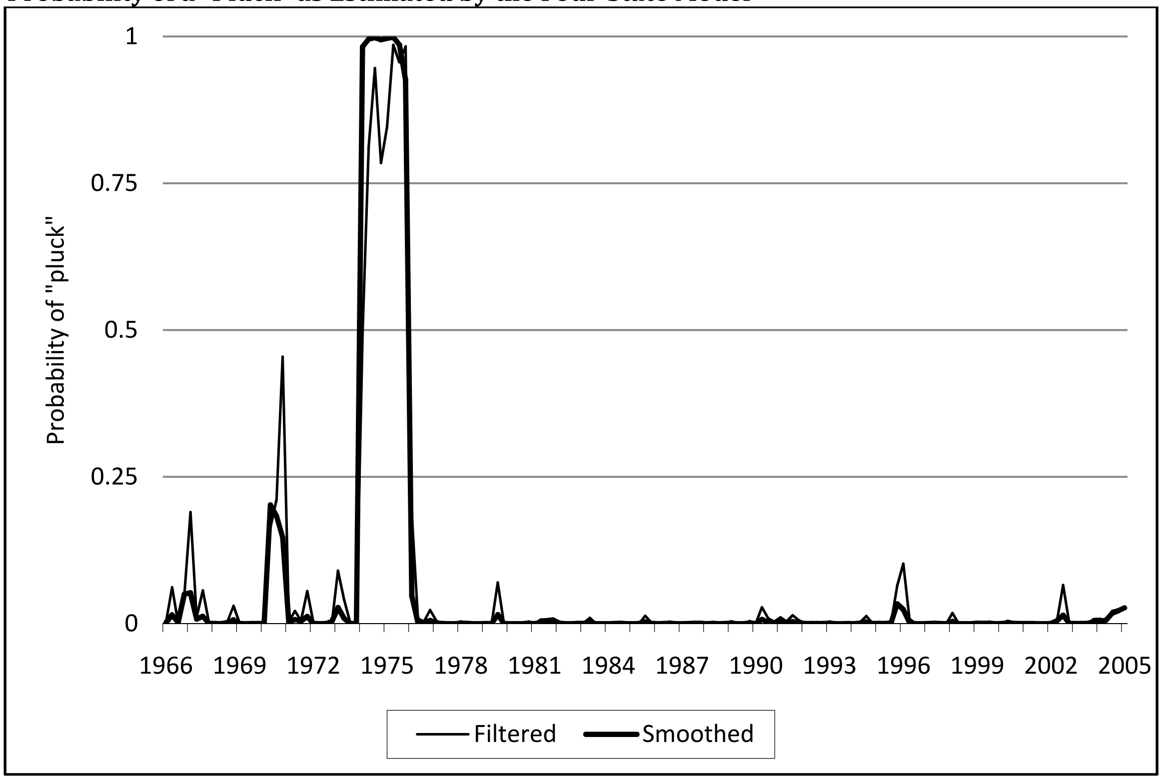
Note: All estimates are significant at the 10 per cent level, except those marked with an asterisk.

Estimates of the trend productivity growth rate depend on the growth rate of the deterministic trend as well as a contribution from the stochastic trend. Estimates indicate that, in the high-growth state, the rate of growth of the stochastic component is roughly 0.44 per cent, whereas in the low-growth state it is -0.16 per cent. Both estimates are statistically significant, and statistically different from each other. The growth rate of the stochastic trend is assumed to follow an AR(1) process with the autoregressive term estimate being 0.14, which is not statistically different from zero. This suggests that the AR(1) component might not be necessary in this variant, but it is still included to facilitate comparisons with other variants and with the estimates for the United States. Establishing the contribution of the stochastic component to trend productivity growth requires that we consider the autoregressive term as well as the stochastic-trend factor loading coefficient. Taking account of these features, and adding in the growth rate of the deterministic component, the trend growth rate of productivity in the high-growth state is $D_1 + \gamma\left(\frac{\mu_1}{1-\phi_1}\right) = 0.36 + 0.49\left(\frac{0.55}{1+0.14}\right) = 0.60$ or 2.4 per cent annually, whereas in the low-growth state it is $D_1 + \gamma\left(\frac{\mu_0}{1-\phi_1}\right) = 0.27$ or 1.1 per cent annually. The results presented have a considerable level of robustness to specifications, with the remaining variants providing similar estimates of the relevant coefficients.

### 5.2 Transitory state

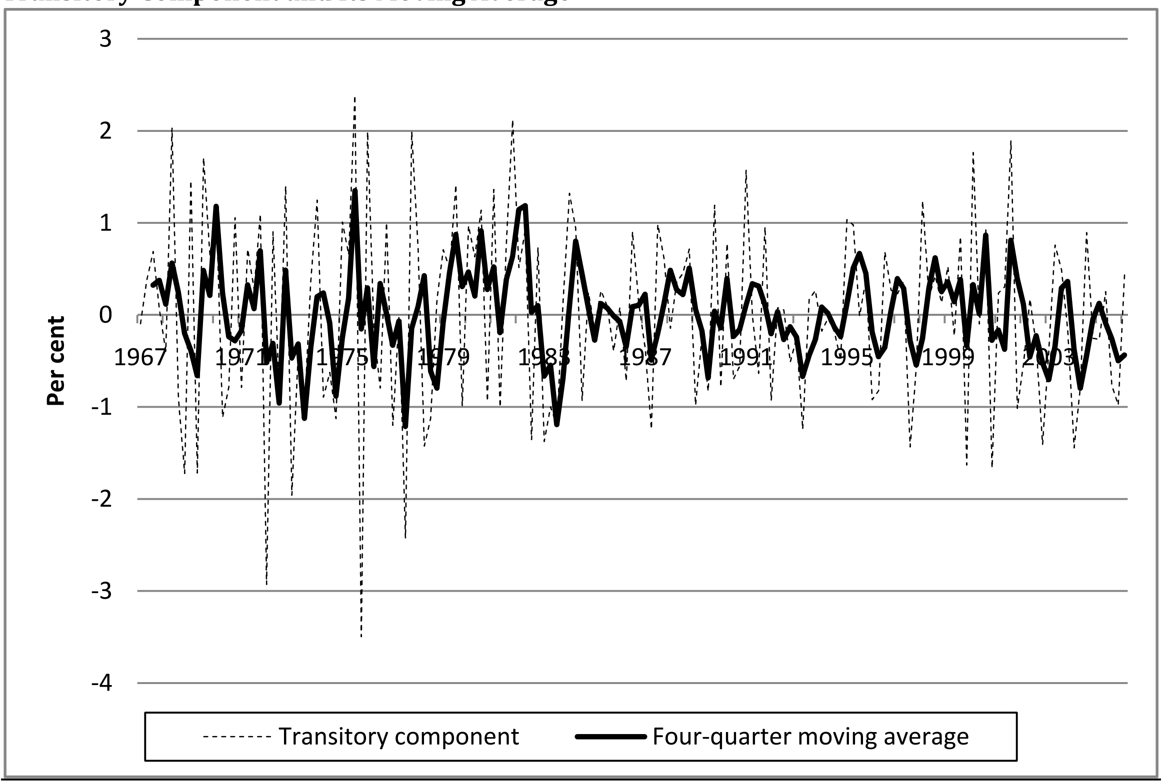
While many authors, including Beaudry and Koop (1993) and Sichel (1994), find evidence of a high-growth recovery phase while studying business cycles, Sichel indicates that a three-state Markov model is not especially useful in modelling output trends. Kim and Nelson (1999) and Kim and Murray (2002) recommend instead allowing for asymmetric behaviour in the transitory component, similar to Friedman's (1969, 1993) "plucking" model. This "pluck" is adopted by Kahn and Rich (2006) in modelling productivity growth. While labour productivity is considered procyclical by the majority of economists, the series does not seem to be subject to asymmetries. Estimating this model in the Canadian context yields very little evidence of business cycle asymmetries, with the only period exhibiting "plucking" behaviour being between 1974 and 1976 (Chart 4).

**Chart 4**
**Probability of a "Pluck" as Estimated by the Four-State Model**




Thus, it seems that, while useful in business cycle modelling, Friedman's "plucking model" falls short of being practical for modelling productivity. This observation eliminates the need to control for asymmetry and explicitly model separate transitory states. This reduces the model to a two-state model of trend component regimes only. Since the findings of both the four- and two-state models are very similar, we will concentrate on the two-regime model for the sake of simplicity and clarity. It is important to note that this model, while not controlling for asymmetry, still contains a transitory component that takes on only a single state. Hence, business cycle effects are still taken into consideration, only in a symmetric fashion. There is some evidence of recessionary effects in the mid-seventies and early eighties (Chart 5). The estimate of the transitory loading factor on the productivity, wages per hour, and hours worked series is around 0.3. The loading factor on the consumption per hour series is estimated to be around -0.7. This suggests, as Kahn and Rich (2006) point out, that consumption is less cyclical than the hours worked series.

**Chart 5**
**Transitory Component and Its Moving Average**




## 5.3 Impact of additional series

As mentioned earlier, growth theory implies that the labour productivity, compensation per hour, and consumption per hour series follow the same trend, or technological progress. In order to isolate the impacts of using the additional series on the growth of trend productivity, the compensation per hour and consumption per hour series are removed. The major benefit of using the supplementary series is the increased confidence of being in either of the two states, and a clearer shift from one regime to the other. Excluding wages per hour results in a slightly increased probability of being in the high-growth state around the mid-1970s, as well as towards the turn of the century (Chart 6). The estimation without consumption per hour decreases probabilities in both of the aforementioned time periods, with the smoothed probability of being in the high-growth state around 1999 rising to 30 per cent (over 80 per cent filtered), which is considerably higher than the full model suggests.

**Chart 6**
**Impact of Additional Series**


Using only the productivity and hours worked series reduces to about 50 per cent the probability of being in the high-growth state in the first 10 years of estimation. This is a dramatic decline, which suggests that, when using productivity data alone, we lack the evidence to conclude that a high-growth state prevailed in the decade following 1967. Removing the series has a large impact on estimates of transitory loading factors, with the productivity loading factor decreasing substantially, often not being statistically different from zero.

## 5.4 Permanent loading factor equality

The theoretical implication for the model of a single common trend in productivity, wages per hour, and consumption per hour is that the permanent loading factors are equal, or $\gamma_1 = \gamma_2 = \gamma_3$. In order to test this hypothesis, we estimate an unconstrained model , where the coefficients are allowed to vary across the three series, as well as the constrained model, where equality is ensured. In the constrained case, the permanent loading factor was estimated to be 0.48 with a standard error of 0.04. Removing the equality constraint yields estimates of $\gamma_1 = 0.37$, $\gamma_2 = 0.52$, and $\gamma_3 = 0.56$. The null hypothesis of equality across the three coefficients is rejected at the 5 per cent, but not at the 1 per cent, significance level.

## 5.5 When can we anticipate a switch if growth stays robust?

While the data up to and including 2006Q1 do not suggest a switch to the high-growth state, it is informative to probe how quickly the model would switch regimes if solid productivity growth was experienced in the future. For that purpose, the dataset is extended into the future assuming 0.6 per cent quarter-to-quarter (2.4 per cent annual) growth for the three cointegrated series, and assuming that the growth in hours worked is equal to the rate achieved in 2006Q1 (the last quarter of the dataset). Initial estimates suggest that strong growth lasting through 2008, or for at least 10 quarters, would be required to drive the probability of being in the high-growth regime past the 50 per cent mark (Chart 7).

In order to be almost certain of being in the high-growth state (i.e., high-growth-state filtered probability of approximately 95 per cent), we would have to witness about five years of strong growth (Chart 8). Both of these simulations are characterized by the two transition probabilities converging from, respectively, 0.984 and 0.992 for the low- and high-growth regimes in the original model, and 0.982 and 0.988 for the 10-quarter extension, to 0.983 for both in the 5-year extension.

**Chart 7**
**Estimation Period Extended: Ten Quarters of Robust Growth**

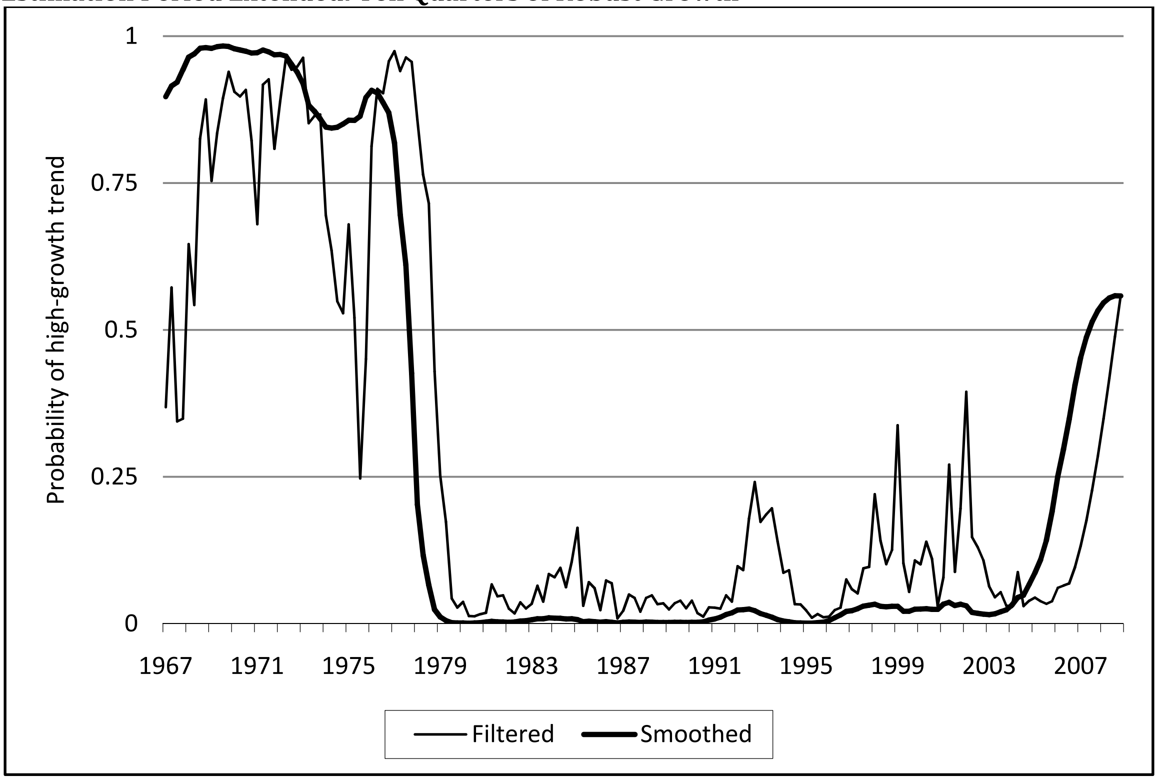
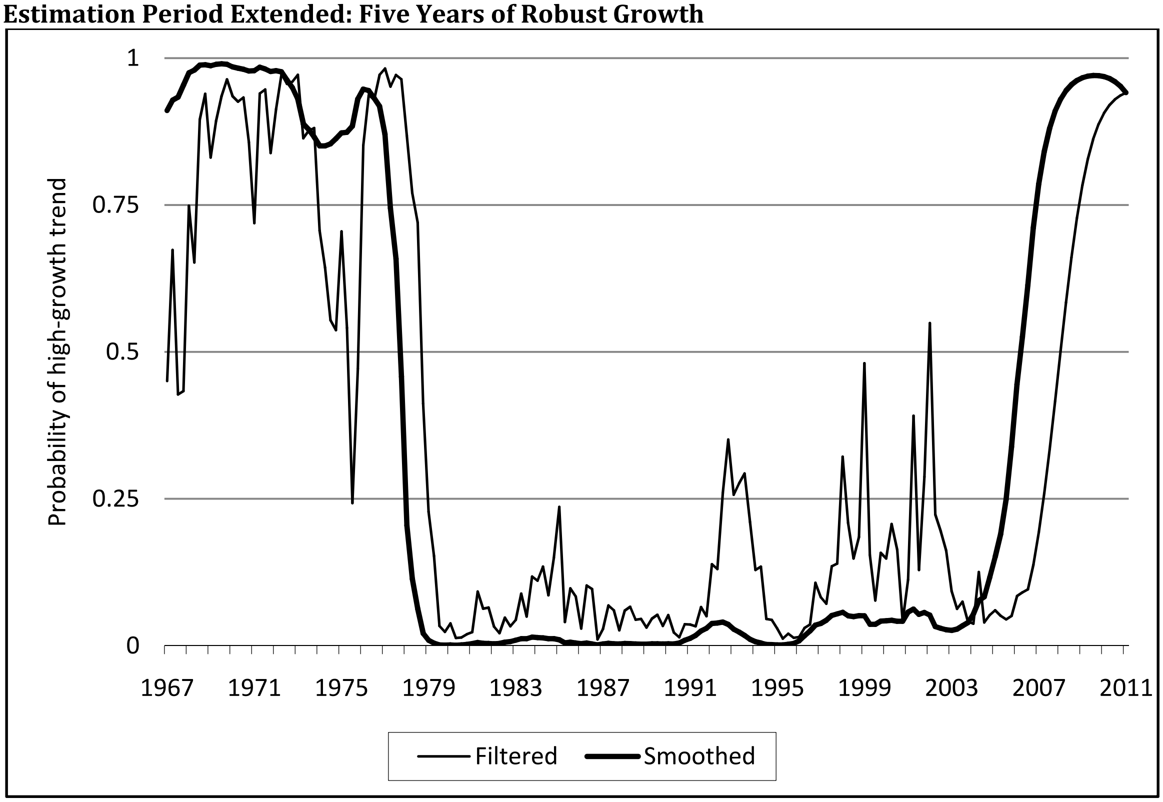


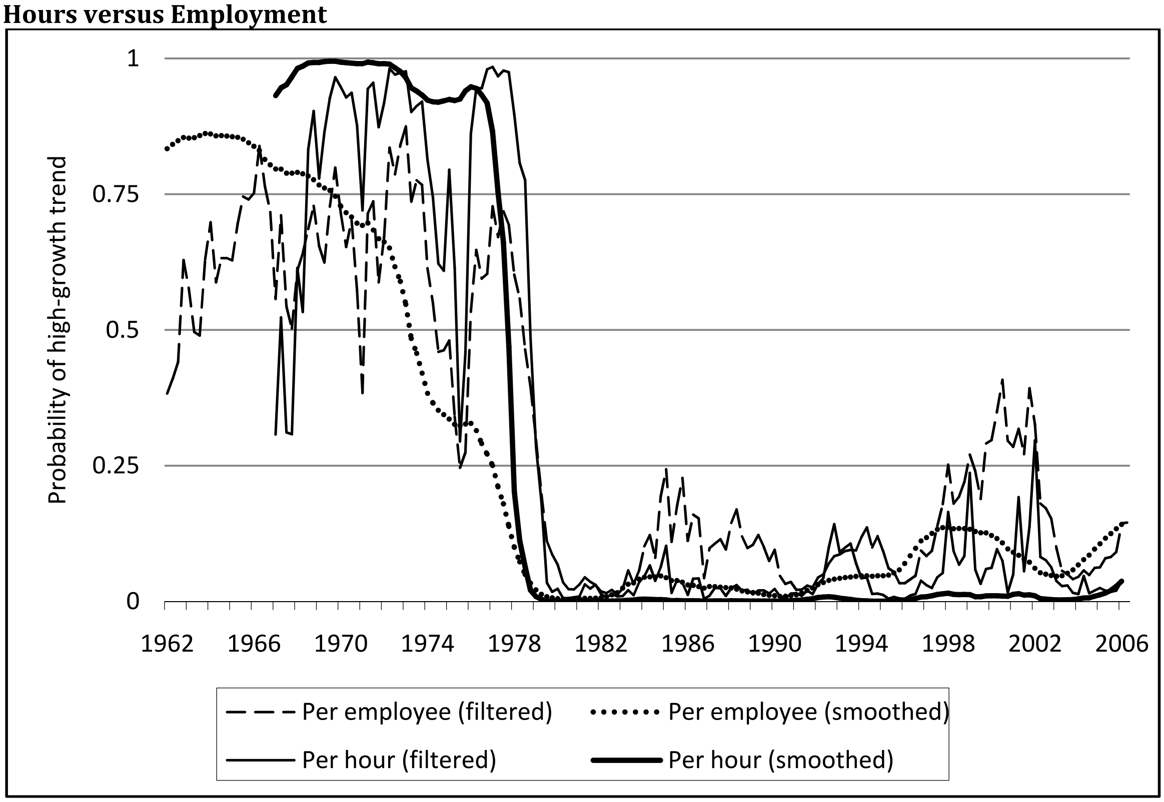

**Chart 8**
**Estimation Period Extended: Five Years of Robust Growth**



## 5.6 Per hour versus per employee models

While data for average hours worked are not available prior to 1966, a longer history of employment data is available. This allows for an extra twenty quarters of data beginning in 1961Q1. The "per employee" model allows us to check our baseline results, which make use of the total hours worked series, instead.

**Chart 9**
**Hours versus Employment**

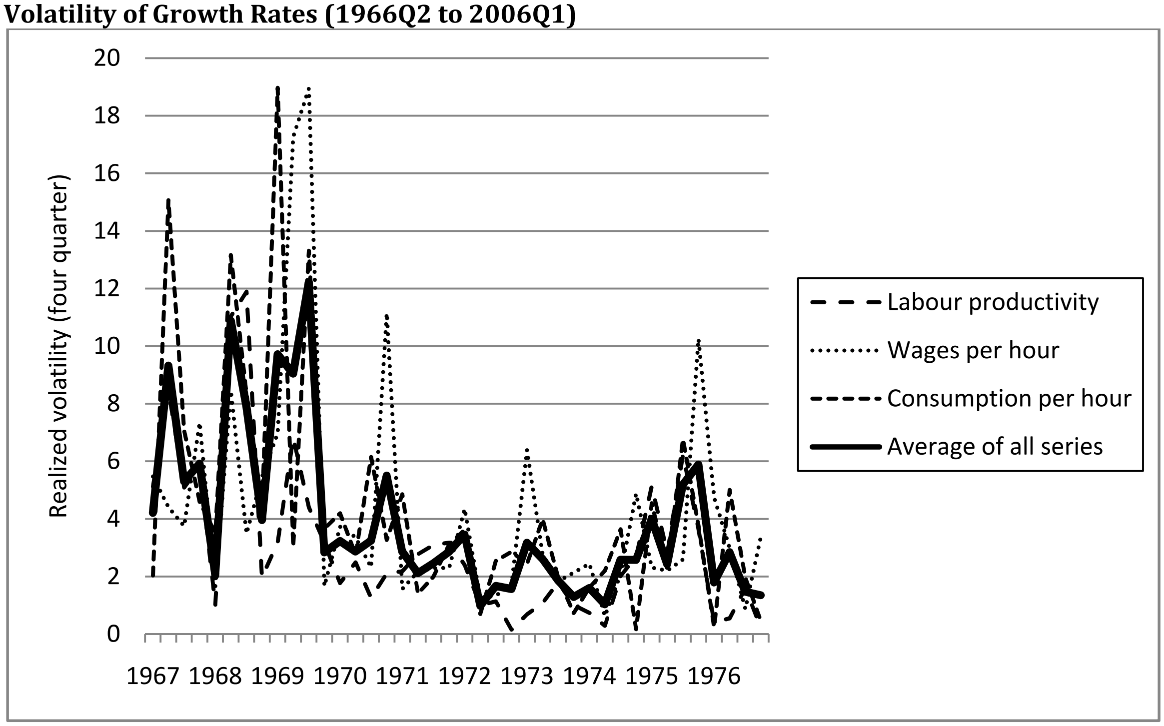



The "per employee" model differs significantly from the "per hour" model, especially prior to the mid-1980s, when average hours were continually falling. According to Harchaoui and Tarkhani (2005), hours worked by an average Canadian fell from 2,364 in 1947 to 1,986 in 1970, and are approximately 1,751 today. This results in the "per employee" productivity growth being well below the growth in output per hour primarily before 1983. In the period that followed, the changes in average hours did not experience a clear trend, resulting in both series moving in a similar manner. One notable exception is the increase in average hours worked over the past three years, which has caused output per employee to grow faster than actual labour productivity. As expected, the probability of being in the high-growth regime prior to 1980 is considerably lower for the "per employee" variant, with the switch to the low-growth regime taking place gradually, instead of rapidly, as in the hourly data variant (Chart 9). In the same vein, the probability of being in the high-growth regime post-1997 is higher in the "per employee" model, reaching a filtered probability of nearly 40 per cent around 1999.

### 5.7 Correcting for a decrease in volatility, post-1980

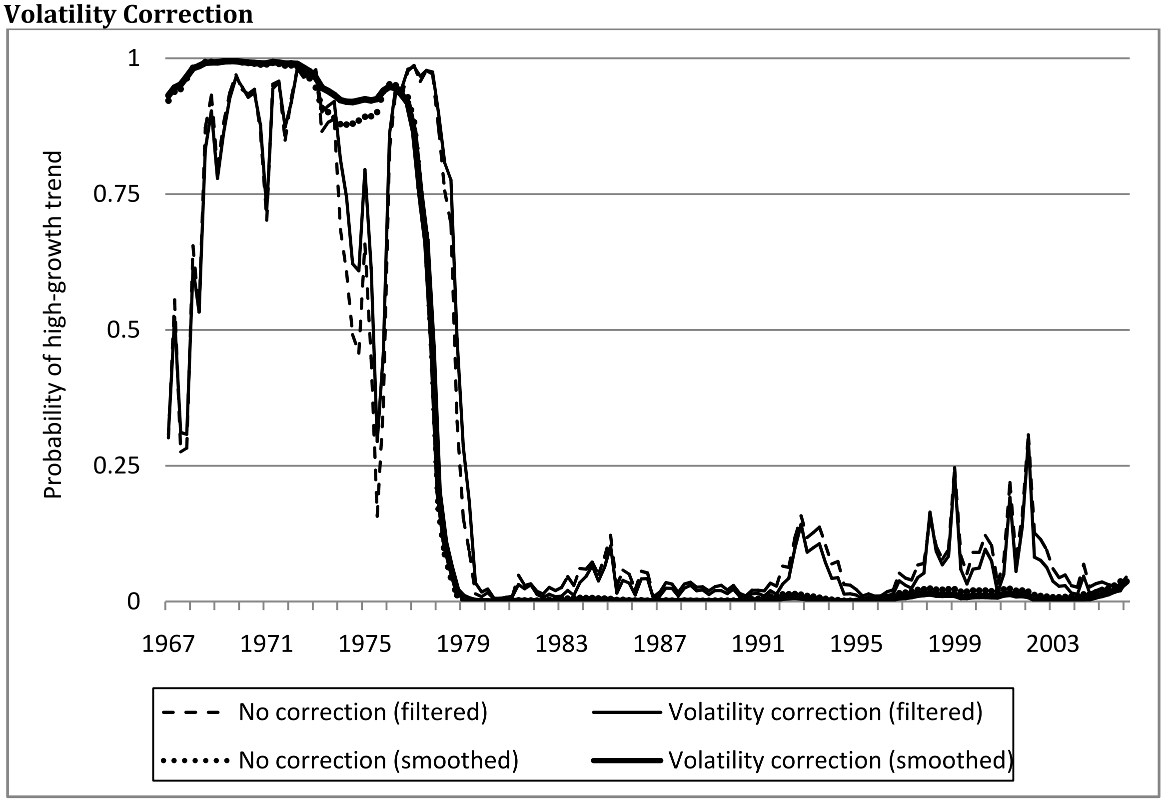
Since the early eighties, all series involved in the study began exhibiting decreasing volatility (Chart 10). Failing to account for this fact may lead to some inefficiencies in the estimated parameters. While there are many ways to approach this problem, a one-time decrease in volatility parameters for all series is used for simplicity.

**Chart 10**
**Volatility of Growth Rates (1966Q2 to 2006Q1)**



The main conclusions reached earlier remain intact, but correcting for volatility slightly increases the persistence of both regimes by about 0.2 percentage points. Furthermore, the correction results in estimates of the autoregressive coefficients of the error terms, particularly on the consumption per hour series, that are more in line with expectations given other series' estimates and the U.S. model. The filtered and smoothed probabilities alike do not differ very much, with the largest divergence being about 10 percentage points in the mid-1970s (Chart 11).

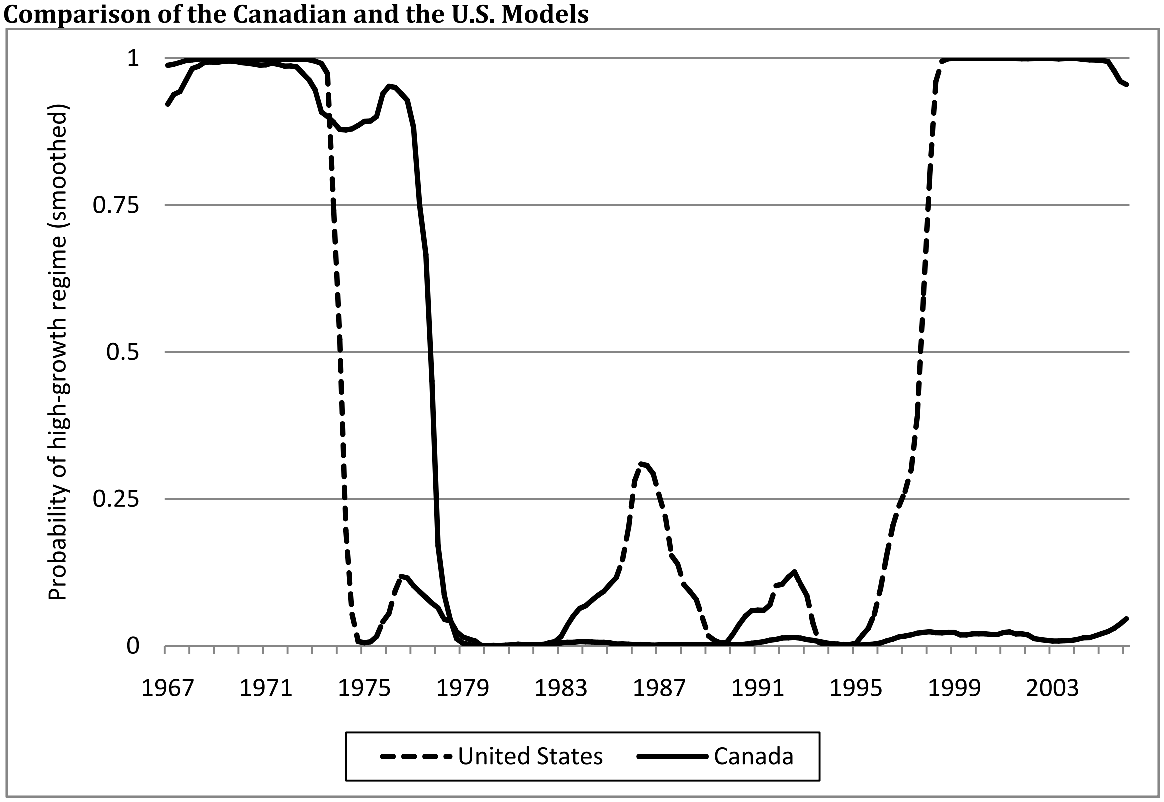
**Chart 11**
**Volatility Correction**




## 5.8 Comparing the Canadian and U.S. models

The most striking difference between the Canadian and the U.S. models is illustrated in Chart 12. Data for the United States before 1966 are removed to ensure comparability. While in Canada the switch from a high to a low regime took place around 1979,[12] in the United States this switch occurred about five years earlier. Furthermore, in the United States there was a clear switch back to the high-growth regime in 1997, whereas in Canada there is no indication of a regime change, at least not in the smoothed probabilities. The U.S. smoothed probabilities are much more clearly defined in the four-state model, although we present the two-state model for ease of comparison.

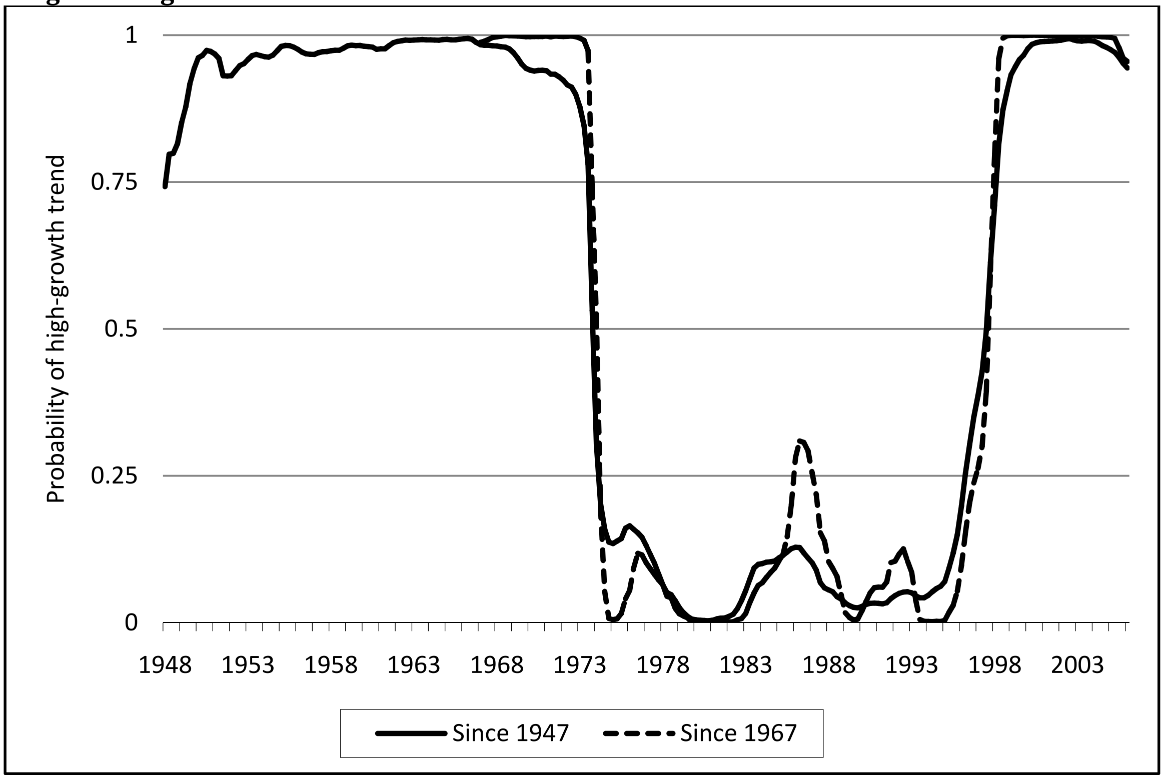
**Chart 12**
**Comparison of the Canadian and the U.S. Models**




The original United States dataset goes back farther in history by nearly twenty years. These additional data, while helpful in the estimation, do not drastically change the results. The persistence of the high-growth regime rises, but the timing of the regime changes is approximately identical in both models (Chart 13).

12. There is a slight indication of lower growth around 1973, but that does not materialize until the end of the seventies.

**Chart 13**
**Length of Original U.S. Dataset**




## 6 Conclusion

The model used in this paper indicates that Canadian labour productivity switched regimes from a high-growth trend to a low-growth trend towards the end of the 1970s, with very little evidence of a shift back to the high-growth regime by 2006Q1. While filtered probabilities indicate a potential switch around the turn of the century, they never reach levels near 50 per cent. Considering the entire period as the information set removes these signals altogether.

Estimating the model is not free of difficulties, with some variants being considerably sensitive to the specification of the transitory regime, the autoregressive structure, and, very often, the initial values. Furthermore, to assume that Canadian labour productivity takes on two separate trend regimes that switch between each other rather abruptly is clearly a simplification that is not flawless.

Data for the United States, for example, provide clearer signals with higher confidence of being in any particular regime, and rapid shifts between states.[13] Nonetheless, the message from the estimation, although not necessarily positive, is clear and helpful when considering how the trend in Canadian labour productivity is currently evolving.

A number of improvements can be made to the model. First, while relatively lengthy, the dataset may not contain a sufficient number of high-growth quarters. The years between 1966 and 1978, while experiencing high growth on average, were subject to brief periods of weakness. Extending the dataset forward when the data become available may allow the model to provide a clearer message.[14] Second, explicitly modelling the volatility of some components may significantly improve the estimation results, since it is clear that the productivity series in particular has undergone a structural change, decreasing in volatility over the years. Lastly, it could very well be the case that the two-regime model is not the best fit for an analysis of Canadian labour productivity. Allowing for three regimes may in fact be more appropriate, although with the current dataset it is very difficult to obtain acceptable estimates. Alternatively, the trend rate of productivity growth may evolve in a more continuous fashion.

13. This is especially true for the four-state model, where U.S. data seem to perform far better than Canadian data.

14. A clearer conclusion may also emerge if data prior to 1966 become available, although no such data release is anticipated.

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