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Gaining Credibility for Inflation Targets

by

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Abstract

In this paper, I consider a simple model in which agents learn about the inflation target of a central bank over time by observing the policy instrument or inflation outcomes. Measuring credibility as the distance between the perceived target and the actual target, an increase in credibility is beneficial to the central bank because it brings the policy consistent with attaining the inflation target closer to that required to attain the output target.

In this model, the crucial assumptions are that (i) the central bank knows what its target is, but lacks the means to credibly communicate it to agents, and (ii) observed changes in the policy instrument do not perfectly inform agents about the objective of the central bank. Optimal monetary policy therefore entails endogenizing the learning process of agents and solving the resultant "optimal-control" problem. I show that a linear approximation of the optimal-control problem is observationally equivalent to a "conservative central banker" in the sense of Rogoff (1985), results in most of the gains that are available from pursuing a higher-order approximation for reasonable degrees of initial credibility, and may actually be preferable if agents cannot determine the exact weights with which to update their view of the target. A conservative central banker is especially beneficial if society places a high weight on output deviations from target.

I then illustrate the impact of other factors on credibility formation, including choice of monetary policy instrument, transparency, and publishing forecasts.

JEL classification: E52
Bank classification: Credibility; Inflation targets

Résumé

L'étude porte sur un modèle simple où les agents économiques prennent connaissance de la cible d'inflation de la banque centrale au fil du temps en observant le comportement de l'instrument de politique monétaire ou l'évolution de l'inflation. La banque centrale a avantage à accroître sa crédibilité (définie comme une fonction de l'écart entre la cible perçue et la cible réelle), car l'atteinte de la cible d'inflation devient alors plus conciliable avec celle de la cible de production.

Le modèle auquel l'auteur fait appel repose sur deux hypothèses fondamentales : *a)* la banque centrale connaît sa cible d'inflation mais ne peut la communiquer de façon crédible aux agents; *b)* les variations observées de l'instrument de politique monétaire ne renseignent pas parfaitement les agents sur l'objectif de la banque centrale. L'optimisation de la politique monétaire exige donc l'endogénéisation du processus d'information des agents et la résolution du problème de

« contrôle optimal » qui en découle. L'auteur démontre qu'une approximation linéaire du problème de contrôle optimal donne lieu aux mêmes observations que le comportement d'une « banque centrale prudente » au sens de Rogoff (1985), qu'elle procure la plupart des avantages pouvant résulter d'une approximation d'ordre supérieur, compte tenu d'un degré raisonnable de crédibilité initiale, et qu'en fait, elle peut s'avérer préférable si les agents ne savent pas quel poids exact attribuer à chaque variable dans l'actualisation de leur perception de la cible. Une conduite prudente de la banque centrale est d'autant plus profitable que les agents accordent un poids élevé aux écarts de la production par rapport à la cible.

L'auteur termine son étude en illustrant l'incidence des trois facteurs suivants sur la crédibilité : le choix de l'instrument de politique monétaire, la transparence et la publication des prévisions.

Classification JEL : E52
Classification de la Banque : Crédibilité; Cibles en matière d'inflation

1. Introduction

In a survey of central bankers, Blinder (2000) found that there is widespread consensus on the desirability of obtaining credibility, but some disagreement as to exactly what credibility means in practical terms. There was also disagreement on how to build credibility, although having a history of honesty and independence figured prominently.

When the objective of monetary policy is stated clearly, the first of these disagreements is resolved: credibility can be measured simply as a function of the distance between the expectations of agents, or the perceived target, and the actual target, as is considered for inflation-targeting countries in Johnson (1998, 1997). This paper focuses on the second area of disagreement: how a central bank should go about building credibility.

To simplify the analysis, I consider this question in an explicit inflation-targeting environment. Even with an explicit target, stated objectives will not necessarily be believed, owing to incentives for the central bank to mislead agents, lack of confidence in the independence of the central bank to achieve its objectives, or doubts about the competence of the central bank. Otherwise, all central banks could enjoy instant credibility by stating their objectives. For most of the analysis, there will be no difference between the central bank and economic agents, except that (i) the central bank knows what its true target is, but lacks the means to credibly communicate it to agents, and (ii) observed changes in the policy instrument do not perfectly inform agents about the objective of the central bank.

In this paper, agents seek to learn about the objectives of the central bank by observing either movements in the policy instrument or economic outcomes. The faster they learn, the more quickly the central bank gains credibility. The central bank benefits from an increase in credibility in the linear-quadratic environment employed here, since it brings the policy consistent with attaining the inflation target closer to that required to attain the output target.

Optimal monetary policy in this framework entails endogenizing the learning process of agents and solving the resultant “optimal-control” problem. In general, this results in a reaction function for policy that is non-linear in the state variables of the model. I show that a linear approximation to the optimal-control problem is observationally equivalent to a “conservative central banker” in the sense of Rogoff (1985). Further, I find that an optimally conservative central banker retains most of the gains that are available from pursuing a higher-order approximation to the optimal-control problem for reasonable degrees of initial credibility, and this may actually be preferable if agents cannot determine the exact weights with which to update their view of the target. In contrast, true optimal control requires that both agents and the central bank understand and solve the optimal-control problem.

I then consider three different behaviours of the central bank that may be used to increase credibility in this framework: (i) choosing a policy instrument over which the central bank has a high degree of control; (ii) being transparent about the monetary policy framework, so that agents can infer the objective of monetary policy from observing the instrument, rather than economic outcomes; and (iii) publishing forecasts, so that agents base their updating of the target on the central bank's forecasts, rather than on forecasts from some other source. I find that in this framework, there may be large benefits from seeking to gain credibility for inflation targets by using one or more of these actions.

Section 2 summarizes the literature on monetary policy credibility. Section 3 outlines the model. Section 4 discusses the optimal-control problem, and section 5 other behaviours that may be used to enhance credibility. Section 6 offers some conclusions.

2. Related Literature

There is a widely held view that there are incentives for central banks to mislead agents as to their objectives. For example, Stein (1989) argued that the Federal Reserve Board cannot communicate its objectives credibly and precisely because it would benefit from manipulating expectations and pursuing a time-inconsistent policy. There is evidence that the misleading of agents has taken place. Thornton (1999) documents evidence from Federal Open Market Committee (FOMC) transcripts that the Federal Reserve started targeting the federal funds rate in 1982, although their official stated target was borrowed reserves until 1989. Therefore, in this paper, stated targets will not be believed unless the behaviour of the central bank is perceived to be consistent with them.

For this reason, it will take time for a central bank to gain credibility for any particular target. This is consistent with the experience of many countries. For example, Kaminsky and Leiderman (1998) show that high ex post real interest rates in Argentina, Israel, and Mexico after the commencement of disinflation programs were likely the result of a lack of credibility fuelling inflation expectations far above actual inflation. They use a multiple-regime model where agents rationally determine whether they think the economy is in a low-inflation or a high-inflation regime. Ricketts and Rose (1995) estimate similar models for each of the G-7 economies, and interpret the probability agents place on being in a low-inflation state as credibility. They find that credibility is difficult to gain, and easy to lose. Isard and Laxton (1998) consider a model calibrated to the Australian economy in which credibility is endogenous and the central bank undertakes experimentation when inflation is low, to learn more precisely the (unknown, time-varying) non-accelerating inflation rate of unemployment (NAIRU). They find that experimenting is costly in terms of credibility, although it may result in a slightly lower average rate of unemployment, at the expense of higher average inflation. Credibility is interpreted as the probability agents place on being in a low-inflation regime.

Closer to the methodology employed here, Tetlow and von zur Muehlen (2000) consider a simple model where agents must learn the parameters of the policy rule. They show that rationally updating agents will have difficulty learning the parameters of a policy rule using least squares if the rule conditions on many variables, potentially resulting in unstable outcomes. However, they will learn more quickly if the central bank restricts itself to only two or three parameters. They also consider the impact of a change in policy rule on a skeptical public, and the possibility of "actively teaching" agents about the change by exaggerating it in their reaction function to reduce transitional costs. Active teaching of the restricted type they consider is not optimal in their model.

In the model considered here, something akin to active teaching will be optimal if society places a high weight on minimizing output volatility, where active teaching may be interpreted as appointing a conservative central banker in the sense of Rogoff (1985)—that is, one who places a higher weight on inflation volatility (and therefore a lower weight on output volatility) than society.

In some models, assisting the learning of economic agents is not an unambiguously good thing. These models assume that while inflation itself is costly, the central bank benefits from inflation surprises. Therefore, it may be optimal for the central bank to be obscure about its objectives or, in the case of rational learning, use its policy to slow the learning process. For example, Vickers (1986) considers a world in which the preferences of a policy-maker are unknown, in that individuals do not know whether the policy-maker cares about inflation (is "dry") or unemployment (is "wet"). In his model, inflation is bad, but surprise inflation is good, since it drives unemployment below its natural level. He finds that incomplete information can result in better outcomes than complete information, since wet policy-makers will keep inflation low early in their tenure, to emulate the behaviour of dry policy-makers, resulting in lower average inflation.

Cukierman and Meltzer (1986) consider a central bank with time-varying preferences where, owing to imperfect information, agents only learn of changes in preferences with a lag. They find that when the central bank can choose the accuracy of monetary control, it will not always be optimal to choose the most effective control, since ambiguous control allows the policy-maker to generate positive inflation surprises in the future. The policy-maker will choose more ambiguous control the more uncertain their preferences. Similarly, Cripps (1991), using a variant of the Cukierman and Meltzer model with constant preferences, finds that it is optimal for the government to slow the rate of the public's learning by being less informative about its preferences.

One limitation of Cukierman and Meltzer (1986) and related papers is that the loss function of the central bank can be interpreted as being linear in output, so that the central bank would accept arbitrary increases in the variance of output to lower the variance of inflation. Also, they implicitly link increased transparency with improved control by the central bank. Faust and Svensson (2001) show that the former has the effect of ensuring that the central bank behaves in the same manner, irrespective of the level of credibility they enjoy. They extend the framework by considering a loss function that is quadratic in inflation and unemployment, in which transparency is linked to the ability of economic agents to deduce the intentions of the central bank from observables, rather than control by the central bank. Their goal is to formally assess the importance of dynamic credibility and transparency under persistently low inflation, when the central bank has a time-varying employment target. They simulate the learning process, and find that low credibility results in a more inflationary policy, but one that is less expansionary in the sense that inflation will be lower than expectations. They find that increased transparency of the central bank's intentions is generally desirable, although it makes the bank's reputation and credibility more sensitive to its actions, and can be costly.

Aspects of the behaviour of some central banks in recent years are not well captured by the current literature. First, many have (in the language of Vickers 1986) sought to be more dry than their predecessors. Second, some have replaced obfuscation with greater transparency as a means of gaining credibility for a new policy regime characterized by explicit inflation targeting.[1] Geraats's (2001) explanation for this is similar to the one explored here. She uses a two-period model to formalize why publishing forecasts and all the information used to create them might be desirable. She argues that central banks benefit from such increased transparency by establishing a reputation more quickly. Also, if they care about output variability, transparency allows them to respond to shocks at less cost to their reputation. In contrast, weak central banks are likely to prefer opaqueness.

Geraats (2001) examines credibility formation via an optimal-control problem, where the central bank endogenizes the credibility formation process when setting monetary policy. While this is conceptually appealing, it assumes a high degree of complexity on the part of economic agents

1. For example, see Perrier and Amano (2000) for a summary of steps taken by the Bank of Canada in recent years.

and the central bank. Both are assumed to understand the optimal-control problem, and take full account of it when they update their view of the target and set monetary policy, respectively. Given the complexity of the optimal-control problem, these are very strong assumptions for the day-to-day conduct of monetary policy.

Here, I extend the problem to the infinite horizon and show that a linear approximation to the optimal-control problem is observationally equivalent to period-by-period optimization with a conservative central banker. Further, I show that an appropriately conservative central banker extracts most of the gains available by solving the optimal-control problem.

3. A Simple Analytical Model

We will now construct a simple analytical model that will be used to investigate how a central bank should go about gaining credibility. Inflation is determined by a Phillips curve of the form

$$\pi_t = \pi_t^e + \beta(y_t - y^*) + \varepsilon_t, \tag{1}$$

where π_t and y_t are inflation and output, respectively, π_t^e is agents' expectations of inflation formed in period $t-1$, and ε_t is an inflation shock term. For the time being, we will assume that potential output (y^*) is known by all agents and is time-invariant. Monetary policy entails the setting of the monetary-policy instrument, which influences real output according to the relation

$$y_t = y^* - \gamma(r_t - r^* - \pi_t^e - \phi_t), \tag{2}$$

where r^* is the long-run equilibrium real interest rate, while ϕ_t is an independently and identically distributed random-noise term that reflects the mechanical operation of markets and obscures agents' observation of the desired level of the policy instrument, r_t. For example, in Canada, one may think of the current policy instrument as the Bank Rate,[2] which defines the interest rate at which the central bank is willing to make loans to major financial institutions to meet their daily settlement needs. This is generally adjusted in 25 basis-point increments. Under such a view, ϕ_t is the distance between the exact desired Bank Rate and the nearest 25 basis-point increment to which it is fixed.[3]

An alternative view is that r_t is the overnight rate, which is the rate at which major financial institutions borrow from and lend to each other to meet daily settlement requirements. This is a market in which the central bank also operates, largely to ensure that the overnight rate remains

2. The equivalent in the United States is the federal funds rate.
3. In the simulation exercises that follow, the variance of ϕ_t will be set consistent with this explanation.

within a 50 basis-point range, where the Bank Rate defines the upper bound. With this view, ϕ_t reflects shocks to the supply and demand of settlement balances that are not offset by the central bank.[4] Other sources of noise that could potentially increase the magnitude of ϕ_t many times over include any activity by the central bank in the market for purposes other than the pursuit of the inflation target (such as financial stability), agents and the central bank not sharing the same view as to the level of potential output[5] or the long-run equilibrium real interest rate, or the central bank measuring inflation expectations with error.[6] Conceptually, the impact of any of these would be to introduce more noise into observed interest rates.

The central bank seeks to minimize a loss function given by

$$L = E_0 \sum_{t=1}^{\infty} \rho^t L_t ,$$
$$L_t = (\pi_t - \pi^*)^2 + \omega(y_t - y^*)^2 . \tag{3}$$

where ρ is the discount rate and π^* is the inflation target, which is unknown to the public. $\omega = 0$ represents a central bank that cares only about inflation deviations from target, while for $\omega \rightarrow \infty$ the central bank cares only about deviations of output from potential.[7]

Monetary policy operates with a one-period lag, before the central bank learns of ε_t. If the central bank were to enjoy perfect credibility, that is $\pi_t^e = \pi^*$, it would set policy according to the rule

$$r_t = r^* + \pi^* . \tag{4}$$

The only source of loss to the central bank would then be random noise over which the central bank has no control, so there would be no distinction between monetary policy from minimizing the period-loss function and optimal policy, since there is no learning on the part of agents.

This paper focuses on the case where the central bank does not enjoy perfect credibility. The reaction function resulting from minimizing the period-loss function, treating credibility as exogenous, is given by

4. Bank of Canada (1998) provides a detailed discussion of the workings of monetary policy in Canada.
5. The estimate could be drawn from the same distribution, centred on the true value, so that such an assumption would not necessarily require the central bank to enjoy an informational advantage over economic agents. This will be explored further in section 5.3.
6. This case is addressed in Tarkka and Mayes (1999).
7. When we consider the role of publishing forecasts in section 5.3, the real target of the central bank (potential output) is unknown to both agents and the central bank. Further, it is directly linked to inflation performance in the model. In contrast, in Faust and Svensson (2001) the employment target is known by the central bank (but not agents) and has no links with inflation outcomes.

$$r_t = r^* + \pi_t^e + \left[\frac{1}{\gamma}\left(\frac{\beta}{\beta^2 + \omega}\right)\right](\pi_t^e - \pi^*) . \tag{5}$$

This reaction function is referred to elsewhere as the certainty-equivalent policy, since it is identical to the optimal policy in a world without uncertainty. Because the only sources of uncertainty to the central bank considered here are additive, the certainty-equivalent policy would be optimal if credibility were exogenous. It is used here to develop the model and as a benchmark against which to compare optimal policy later.

Agents do not know the inflation target of the central bank, but seek to learn about it over time. If there were no noise in the policy instrument ($\phi_t = 0$), agents could fully infer the target after only one period, since π^* is the only unknown in the reaction function.[8]

Output and inflation evolve in this model according to the following paths:

$$y_t = y^* - \frac{\beta}{\beta^2 + \omega}(\pi_t^e - \pi^*) - \gamma\phi_t , \tag{6}$$

$$\pi_t = \pi^* + \frac{\omega}{\beta^2 + \omega}(\pi_t^e - \pi^*) + \varepsilon_t - \beta\gamma\phi_t . \tag{7}$$

Even if the central bank publishes its objectives clearly, agents will not necessarily believe them, owing to the incentives for the central bank to deceive agents. To illustrate this point in a dynamic context, suppose that, as a result of particular shocks, the economy is in excess demand ($y_t > y^*$) and inflation lies below the target ($\pi_t < \pi^*$). One way for the monetary authority to achieve its target would be to convince agents that the target is higher than its true value, fuelling an increase in inflation expectations, while contracting the economy to close the output gap. Agents would then suffer losses as a result of lower-than-expected inflation and higher-than-expected real interest rates.

More generally, one may consider the lack of credibility to reflect a lack of belief that the central bank has the political independence to pursue its stated objective. In particular, suppose that agents suspected that the central bank wished to achieve higher output than was consistent with their inflation target, as in Barro and Gordon (1983). The central banks' true loss function would then take the form

$$L_t = (\pi_t - \pi^*)^2 + \omega(y_t - y^* - \alpha)^2 \tag{8}$$

8. This is the interpretation Geraats (2001) gives to transparency.

for some α, and policy would be observationally equivalent to that which would be observed if the loss function were of the form

$$L_t = \left(\pi_t - \pi^* - \frac{\omega\alpha}{\beta}\right)^2 + \omega(y_t - y^*)^2 . \tag{9}$$

If agents believe that the central bank is targeting a level of output above potential, that is exactly equivalent to seeking to target a rate of inflation above the stated target in this framework.

Agents are assumed to know the general form of the loss function, the central bank's reaction function, and the structural equations of the model, but not the inflation target, π^*. They form expectations based on the central bank seeking to minimize a loss function written in terms of the perceived value of this target,

$$L_t^P = (\pi_t - \pi_{t-1}^P)^2 + \omega(y_t - y^*)^2 . \tag{10}$$

Substituting this in (1) and (2) above, and taking expectations, yields the result

$$E_{t-1}(\pi_t) = \pi_t^e = \pi_{t-1}^P . \tag{11}$$

This is a rational expectation of inflation, taking the perceived loss function of the central bank as given. People expect inflation to be equal to the perceived target. This is not surprising since, from the perspective of the perceived loss function (10), there is no inconsistency between the output target and the perceived inflation target. The central bank's expectations of the output gap and the inflation gap are given by

$$E_{t-1}(y_t - y^*) = \frac{-\beta}{\beta^2 + \omega}(\pi_{t-1}^P - \pi^*) ,$$

$$E_{t-1}(\pi_t - \pi^*) = \frac{\omega}{\beta^2 + \omega}(\pi_{t-1}^P - \pi^*) . \tag{12}$$

Herein lies the advantage to the central bank in increasing their level of credibility. As long as there is less-than-perfect credibility, there is a trade-off between closing the output gap and closing the inflation gap. Under optimal discretionary monetary policy, these gaps are decreasing in the credibility of the central bank. Only under perfect credibility does the trade-off disappear, and closing the inflation gap becomes consistent with closing the output gap in expectation. That is, with perfect credibility in this model, the optimal monetary policy is identical irrespective of the value of ω in the loss function.

Note that in (12) above, the expected level of inflation of the central bank lies between the perceived target and the true target: owing to the impact of policy on output variability, the central bank seeks to only partially correct incorrect beliefs about inflation in any given period. This is consistent with Faust and Svensson (2001, page 389):

> A low credibility bank . . . will generate a larger (negative) inflation surprise from the public's perspective, leading to lower employment, and, in this sense, conducts a less expansionary policy. However, the low-credibility bank at the same time generates higher inflation than a high-credibility bank and, in this sense, conducts a more expansionary policy.

When monetary policy is conducted in a transparent manner, agents see the behaviour of real interest rates, and from them try to infer what the target of the central bank is. At the commencement of inflation targets, agents are assumed to have a prior estimate of the inflation target, π_0^P, whose distribution is characterized as

$$E(\pi_0^P - \pi^*)^2 = V_0 . \qquad (13)$$

Each period following the commencement of inflation targeting, agents optimally update this after observing the policy instrument, r_t. Each period provides a noisy observation of the inflation target given by

$$\hat{\pi}_t^* = \pi^* - \frac{\gamma(\beta^2 + \omega)}{\beta}\phi_t . \qquad (14)$$

If ϕ_t is independently and identically distributed, the most efficient estimate over t periods is obtained by averaging each period's observation, as

$$\bar{\pi}_t^* = \pi^* - \frac{\gamma(\beta^2 + \omega)}{\beta t}\sum_{i=1}^{t}\phi_i , \qquad (15)$$

$$E(\bar{\pi}_t^* - \pi^*)^2 = \frac{\gamma^2(\beta^2 + \omega)^2}{\beta^2 t}\sigma_\phi^2 . \qquad (16)$$

This is equivalent to Bayesian updating on π^*. Optimally combining $\bar{\pi}_t^*$ with π_0^P using Bayes Rule yields

$$\pi_t^P = \pi^* + \frac{V_0\left[\frac{-\gamma(\beta^2+\omega)}{\beta t}\right]\sum_{i=1}^{t}\phi_i + \frac{\gamma^2(\beta^2+\omega)^2}{\beta^2 t}\sigma_\phi^2(\pi_0^P - \pi^*)}{\frac{\gamma^2(\beta^2+\omega)^2}{\beta^2 t}\sigma_\phi^2 + V_0}, \tag{17}$$

with an expected variance of

$$E(\pi_t^P - \pi^*)^2 = \frac{\frac{\gamma^2(\beta^2+\omega)^2}{\beta^2 t}\sigma_\phi^2 V_0}{\frac{\gamma^2(\beta^2+\omega)^2}{\beta^2 t}\sigma_\phi^2 + V_0}, \tag{18}$$

where

$$\frac{dE(\pi_t^P - \pi^*)^2}{d\sigma_\phi^2} > 0\,,\ \frac{dE(\pi_t^P - \pi^*)^2}{dV_0} > 0\,,\ \frac{dE(\pi_t^P - \pi^*)^2}{d\omega} > 0\,. \tag{19}$$

That is, as the initial uncertainty of agents or the noise in the policy instrument increases, the perceived target lies further from the true target on average. The same holds true as ω increases, because interest rates are less influenced by the inflation target and more influenced by the output target, and so contain less information about the inflation target.

The expected discounted loss function (3) at time 0 can now be computed as

$$L = \omega(\beta^2+\omega)V_0\left[\frac{1}{(\beta^2+\omega)^2} + \sum_{t=1}^{\infty}\rho^t\frac{\gamma^2\sigma_\phi^2}{\gamma^2(\beta^2+\omega)^2\sigma_\phi^2 + V_0\beta^2 t}\right] + \frac{1}{1-\rho}(\sigma_\varepsilon^2 + (\beta^2+\omega)\gamma^2\sigma_\phi^2) \tag{20}$$

where $\frac{dL}{d\sigma_\phi^2} > 0$, $\frac{dL}{dV_0} > 0$.

Credibility formation will now be illustrated with some simple simulations. For the purpose of the simulations, it will be assumed that the central bank is initially concerned only with an output target ($\omega = \infty$), and then institutes inflation targeting, coinciding with a change in the value of ω. In the pre-inflation targeting period, inflation shocks will be accommodated, and the central bank will set interest rates such that output is equal to potential, in expectation. Since there is no target to anchor expectations, inflation in such a world will follow a random walk, so that the best estimate of future inflation is current inflation.[9] Agents therefore use the last period's inflation rate as their expectation of inflation for each period. Simulating for 100 periods before the introduction of inflation targets will result in output gaps, and inflation gaps reflecting underlying uncertainties incorporated into the model, rather than initial conditions. For simplicity, I then assume that economic agents know that there has been a change in regime; they simply do not know what inflation target is being pursued.[10] Appendix A contains explanations for the parameter values used in the simulations.

Figure 1 shows credibility formation for different values of ω. The vertical axis is the level of credibility, where 0 corresponds with perfect credibility, while the horizontal axis starts with the first period of inflation targeting (period 101) on the left, through to period 200 on the right. As demonstrated above analytically, as the central bank cares more about output deviations from potential (that is, ω increases), it takes longer for the central bank to gain credibility. This is because movements in the policy instrument are less informative as to the inflation target. It is clear from these results that there may be a role for a conservative central bank to establish credibility for inflation targets, especially if society is concerned most about output (that is, ω is high).

4. Optimal Control and Conservative Central Bankers

In section 3 we developed a model of credibility formation where the central bank was assumed to treat credibility as exogenous, and therefore minimize the period-loss function in equation (3). While this is the optimal policy with perfect credibility, imperfect credibility drives a wedge between this discretionary policy and the optimal policy response. A possible remedy to this would be for the central bank to commit to following a policy rule of the form given in (4) above. If agents believed that monetary policy was going to be set according to this rule, the central bank would effectively enjoy perfect credibility independent of agents' views of the inflation target. However, such a commitment could not be verified by agents because of the presence of ϕ_t, so it would not be credible. In this model, the central bank cannot commit because it suffers from imperfect credibility, and if it enjoyed perfect credibility there would be no gains to commitment.

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9. This assumption is supported empirically. For example, Ricketts and Rose (1995) find that high inflation periods in the G-7 corresponded with non-stationarity in a Markov switching model, while Evans and Wachtel (1993) present evidence that U.S. inflation has been unstable over long periods of time.

10. Fuhrer and Hooker (1993) show that economic agents can take a very long time to learn of a change in regime if their learning methods do not allow for regime shifts to occur. I am assuming that agents have correctly inferred that a regime shift has taken place.

The optimal policy without commitment technology would entail solving an optimal-control problem. Since credibility is the only channel by which monetary policy today affects monetary policy in the future in this model, it is also the only state variable. The optimal control reaction function may therefore be written as

$$r_t = r^* + \pi_t^e + H(\pi_t^e - \pi^*), \tag{21}$$

for some function H. Note that this will coincide with the reaction function under perfect credibility given in (4) as $\pi_t^e \rightarrow \pi^*$.

Solving the optimal-control problem to determine the form of H is non-trivial, even in the simple model employed here, and even less so for any model with enough detail to capture realistic movements in macroeconomic variables. Suppose instead that the central bank were to linearize H, and choose policy optimally, contingent on this. The reaction function would then take the form

$$r_t = r^* + \pi_t^e + a_1(\pi_t^e - \pi^*) \tag{22}$$

for some a_1. It is easy to see by comparing equation (5) with equation (22) that the linearized optimal-control problem is equivalent to choosing a central banker with a preference parameter $\tilde{\omega}$ that satisfies $a_1 = (\beta / \gamma(\beta^2 + \tilde{\omega}))$ and treats credibility as exogenous. Therefore, choosing the degree of "conservatism" $\tilde{\omega}$ optimally is exactly equivalent to solving the linearized optimal-control problem. One aim of this paper is to investigate the optimal value of $\tilde{\omega}$, and compare outcomes based on it with those that would be obtained from higher-order approximations to the optimal-control problem.

Repeating the analysis in the previous section with a conservative central banker, the expected loss function (3) at time 0 can now be computed as

$$L = (\tilde{\omega}^2 + \omega\beta^2)V_0\left[\frac{1}{(\beta^2 + \tilde{\omega})^2} + \sum_{t=1}^{\infty}\rho^t\frac{\gamma^2\sigma_\phi^2}{\gamma^2(\beta^2 + \tilde{\omega})^2\sigma_\phi^2 + V_0\beta^2 t}\right] + \frac{1}{1-\rho}(\sigma_\varepsilon^2 + (\beta^2 + \omega)\gamma^2\sigma_\phi^2) \tag{23}$$

Differentiating with respect to $\tilde{\omega}$ and solving yields the following condition for the optimal degree of conservatism:

$$\tilde{\omega} = \omega - \left[\frac{\dfrac{1}{\beta^2} \displaystyle\sum_{t=1}^{\infty} \rho^t \left[\frac{\gamma^2 \sigma_\phi^2 \tilde{\omega} V_0 \beta^2 t}{(\gamma^2 (\beta^2 + \tilde{\omega})^2 \sigma_\phi^2 + V_0 \beta^2 t)^2} \right]}{\dfrac{1}{(\beta^2 + \tilde{\omega})^3} + \displaystyle\sum_{t=1}^{\infty} \rho^t \left[\frac{(\gamma^2 \sigma_\phi^2)^2 (\beta^2 + \tilde{\omega})}{(\gamma^2 (\beta^2 + \tilde{\omega})^2 \sigma_\phi^2 + V_0 \beta^2 t)^2} \right]} \right]. \tag{24}$$

Note that as long as $V_0 \neq 0$ and $\sigma_\phi^2 \neq 0$, $\tilde{\omega} = \omega$ if and only if $\tilde{\omega} = 0$. That is, the only time the optimal degree of conservatism is the same as that of society is when society cares only about inflation volatility. More generally, it can be shown that for $\omega > 0$, $0 < \tilde{\omega} < \omega$.[11]

Figure 2.1 illustrates these results with simulations for $\rho = 0.99$, for different values of ω. The vertical axis is the loss while the horizontal axis is $\tilde{\omega}$, the degree of conservatism of the central bank. The values of conservatism considered range from $\tilde{\omega} = \omega$ (the central bank is not conservative relative to society) to $\tilde{\omega} = 0$ (the central bank is completely conservative). For $\omega = 0.1$ or $\omega = 1$, there is little net benefit to a conservative central banker. However, as the weight on output volatility in the loss function increases, the costs of higher output deviations in the early periods are clearly more than offset by the benefits of smaller output deviations in later periods that result from gaining credibility faster via a more conservative central banker.

Figures 2.2 and 2.3 display the results for $\rho = 0.95$ and $\rho = 0.75$, respectively. Even with unrealistically high discount of the future, large gains result from appointing a conservative central bank to gain credibility for inflation targets, provided the weight on output volatility in society's loss function is sufficiently high.

We now consider how close installing an optimally conservative central banker comes to the truly optimal monetary policy, for reasonable degrees of initial credibility. One way to proceed is to consider higher polynomial functions in the state variable in equation (21). This serves to illustrate both the gains and the complications introduced by a non-linear reaction function. With the linear approximation, agents extract a signal on the inflation target given by equation (14), where the variance of that signal is a function of the variance of the central bank's control error, and is known by agents. Agents can therefore update their view of the target efficiently, making use of the optimal weights. In contrast, with a quadratic approximation to the optimal-control problem, agents are assumed to extract a signal of the target by taking the appropriate root of a quadratic function given by

11. See Appendix A2 for a proof.

$$a_1(\pi_t^e - \pi^*) + a_2(\pi_t^e - \pi^*)^2 + \phi_t = a_1(\pi_t^e - \hat{\pi}_t^*) + a_2(\pi_t^e - \hat{\pi}_t^*)^2, \tag{25}$$

where the left-hand side is the policy signal observed by agents. The signal is given by

$$\hat{\pi}_t^* = \pi_t^e - \frac{-a_1 + \sqrt{a_1^2 + 4a_2 r_t}}{2a_2}. \tag{26}$$

However, now the variance of this signal is a function of the true target, π^*, which is unknown by agents. This introduces noise into the updating process, since agents do not know the optimal weights to use, which will be costly to the central bank in expectation. Ignoring this issue will provide an upper bound on the gains to using a quadratic approximation over the linear approximation explored above. This bound can be calculated numerically for specific examples. The variance of the signal, ignoring the impact of the unknown target, can be computed as

$$V(\hat{\pi}_t^*) = \frac{1}{4a_2^2}\left[\frac{B+A}{2} - \left\{\frac{2(B^{1.5} - A^{1.5})}{3(B-A)}\right\}^2\right] \tag{27}$$

where $A = a_1^2 + 4a_2(a_1(\pi_t^e - \pi^*) + (a_2(\pi_t^e - \pi^*)^2)) - 2a_2\Phi$, $B = A + 4a_2\Phi$, and Φ is the range from which the uniformly distributed control error is drawn. For example, using a two-dimensional grid search over $[a_1, a_2]$ for the case of $\omega = 100$ with a discount rate of $\rho = 0.75$,[12] the results indicated that over 99 per cent of the expected gains from a quadratic approximation to the optimal-control problem over period-by-period discretionary policy can be gained with the linear approximation.[13,14]

To verify that this result is not unique to the quadratic approximation, the bound was also constructed using a cubic approximation.[15] Again, there was virtually no gain from pursuing a higher-order approximation over a linear approximation to the optimal-control problem.

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12. As the earlier results indicate, the qualitative results are robust across discount rates. This choice of discount rate substantially reduces the number of periods required to approximate the infinite-period discounted loss function. The choice of ω is to ensure that the optimal-control reaction function differs substantially from that obtained via period-by-period optimization.
13. Without conservatism, the expected discounted loss was 8.56×10^{-3}. The linear approximation resulted in a loss of 3.7814×10^{-3}, while the lower bound from the quadratic approximation was 3.7810×10^{-3}.
14. To calculate the actual gain associated with the quadratic approximation rather than the upper bound, the variance would need to be calculated substituting the perceived target (π_t^P) in place of the true target (π^*) when calculating A and B. Each period, with a new estimate of the true target, the weights used to update all previous periods would need to be revised.
15. See Appendix A3 for details.

The problem of inefficient Bayesian updating is a general problem for any non-linear reaction function. For reasonable initial credibility, a non-linear reaction function results in little gain over a linear reaction function, even ignoring inefficient Bayesian updating. Therefore, one may think of the linear approximation, which is observationally equivalent to the behaviour of a conservative central banker, as a feasible form of optimal control in this model.[16]

5. Other Factors to Enhance Credibility

This section uses the foregoing framework to discuss other factors that may be important in the credibility formation process.

5.1 Choice of monetary policy instrument

One other variable within the choice set of the central bank is the monetary policy instrument. In the case of Canada, this may be considered to be either the Bank Rate or the overnight rate. However, this has not always been the case. As recently as 1994, the 91-day treasury bill rate was used for this purpose. The important difference between such instruments for this study is the associated degree of control that the central bank exercises over the instrument.

As shown earlier,

$$\frac{dL}{d\sigma_{\phi}^{2}} > 0\,, \tag{28}$$

so choosing a policy instrument over which the central bank has a high degree of control is always optimal in this model. Not only does an increase in σ_{ϕ}^{2} directly impact on loss owing to reduced control of the central bank, it has a secondary effect via the increased time taken for agents to learn about the target.

Figure 3.1 illustrates the second of these costs, displaying simulation results of the credibility formation process for differing degrees of σ_{ϕ}^{2}. As the variance of the noise term in the policy instrument increases, it takes longer for the central bank to gain credibility. This is further compounded if the central bank places a large weight on output volatility.

More generally, with a conservative central banker (that is, $0 < \tilde{\omega} < \omega$),

16. If agents have a sufficiently diffuse prior on the monetary policy target, the linear approximation would impose significant costs relative to the bounds calculated numerically here for higher-order approximations. However, even then the gains may be small or negative if inefficient Bayesian updating were taken into account.

$$\frac{dL}{d\sigma_\phi^2} = \frac{\partial L}{\partial \sigma_\phi^2} + \frac{\partial L}{\partial \tilde{\omega}} \frac{d\tilde{\omega}}{d\sigma_\phi^2} = \frac{\partial L}{\partial \sigma_\phi^2} > 0 \tag{29}$$

by the envelope theorem and (26) above. Therefore, even with a conservative central banker, it is always optimal for an inflation-targeting central bank to minimize σ_ϕ^2.

It is interesting to ask in this context whether there is a trade-off between the choice of monetary policy instrument and conservativeness. That is, is a more or less conservative central bank preferred as the degree of instrument control increases (σ_ϕ^2 decreases)? A sufficient condition for a more conservative central bank to be optimal is

$$\sigma_\phi^2 > V_0 \frac{\beta^2 \sum_{t=1}^{\infty} \rho^t \left[\frac{t^2}{(\gamma^2(\beta^2 + \tilde{\omega})^2 \sigma_\phi^2 + V_0 \beta^2 t)^3} \right]}{\gamma^2(\beta^2 + \tilde{\omega})^2 \sum_{t=1}^{\infty} \rho^t \left[\frac{t}{(\gamma^2(\beta^2 + \tilde{\omega})^2 \sigma_\phi^2 + V_0 \beta^2 t)^3} \right]}. \tag{30}$$

That is, if the degree of instrument control is sufficiently low (σ_ϕ^2 large) relative to the uncertainty of agents (V_0), in the margin as that control improves, a more conservative central bank is optimal.[17] This is because the benefits from gaining credibility more rapidly with a more conservative central bank are sufficient to offset the costs of maintaining greater output volatility during the credibility formation process.

Similarly, a sufficient condition for a less-conservative central bank to be optimal is

$$\sigma_\phi^2 < \frac{V_0}{2} \frac{\beta^2 \sum_{t=1}^{\infty} \rho^t \left[\frac{t^2}{(\gamma^2(\beta^2 + \tilde{\omega})^2 \sigma_\phi^2 + V_0 \beta^2 t)^3} \right]}{\gamma^2(\beta^2 + \tilde{\omega})^2 \sum_{t=1}^{\infty} \rho^t \left[\frac{t}{(\gamma^2(\beta^2 + \tilde{\omega})^2 \sigma_\phi^2 + V_0 \beta^2 t)^3} \right]}. \tag{31}$$

If the degree of instrument control is sufficiently high (σ_ϕ^2 small) relative to the uncertainty of agents, in the margin as that control improves further, a less-conservative central bank is optimal. In this circumstance, with an optimally conservative central bank, there is a partial trade-off between transparency and degree of conservatism.

17. See Appendix A4 for a proof.

Combining these two results, all other things being equal, the optimally conservative central bank with either very poor or very good instrument control will display a similar degree of conservatism to society: in the former case because the instrument is so noisy as to preclude agents learning about the target over time, so that conservatism increases output variability about potential with little gain in future credibility, and in the latter case because the central bank gains credibility very rapidly irrespective of the degree of conservatism, so conservatism merely results in short-term costs in terms of increased output volatility. Between these two extremes, there is a gain to installing a conservative central banker.

5.2 Degree of transparency

In this paper, transparency means that agents have sufficient information to deduce the target of the central bank using movements in the policy instrument. The discussion to date has focused on a model in which economic agents are assumed to be highly sophisticated. They understand the framework in which monetary policy takes place, and as a result can infer from movements in the policy instrument what the central bank is trying to achieve. While steps have been taken by some central banks to be more open and transparent about the thinking behind movements in policy instruments, this is a recent phenomena. In many cases, monetary policy remains shrouded in secrecy.[18]

A necessary condition for transparency of this nature is that the central bank publish its internal view of the working of the economy, and, in particular, information pertaining to its view of the monetary transmission mechanism. Suppose, instead, that agents did not have sufficient knowledge of the monetary policy process to infer the inflation target from movements in the policy instrument, r_t. Agents might then seek to determine the objective of monetary policy by observing economic outcomes, on the presumption that observed inflation, on average, should equal the target.

Optimal Bayesian updating of the perceived target would take the following form:

$$\begin{aligned}
\pi_t^P &= \pi_{t-1}^P - V_{t-1}^P H^{-1}(\pi_t - \pi_{t-1}^P), \\
V_t^P &= V_{t-1}^P - (V_{t-1}^P)^2 H^{-1}, \\
H &= V_{t-1}^P + V(\pi_t), \\
V(\pi_t) &= \frac{\tilde{\omega}^2}{(\beta^2 + \tilde{\omega})^2} V_{t-1}^P + \beta^2\gamma^2\sigma_\phi^2 + \sigma_\varepsilon^2.
\end{aligned} \tag{32}$$

18. See Goodfriend (1986) for a discussion of secrecy at the Federal Reserve.

This version of the model cannot be solved analytically, so simulations are examined instead. The results, comparing a transparent central bank with a non-transparent central bank, are shown in Figure 3.2. Non-transparency unambiguously imposes a cost on society, since inflation contains more noise than interest rates, so that gaining credibility for the inflation targets takes longer. If society places a low weight on output stability the optimal degree of conservatism is increased (for example, $\omega = 1$), while if society places a high weight on output stability the optimal degree of conservatism is decreased (for example, $\omega = 100$).

5.3 Publishing central bank forecasts

In the foregoing analysis, there was no role for forecasts in the model. In reality, potential output is unknown, and monetary policy is based on the central bank's estimate of potential. That estimate may be thought of as serving the purpose of a forecast. In this model, measuring potential output with error is synonymous with measuring the equilibrium real interest rate with error, where the estimate of the equilibrium real interest rate ($\hat{r}^*$) will be related to the true level (r^*) according to the relation

$$\hat{r}^* = r^* - \frac{1}{\gamma}(\hat{y}^* - y^*), \tag{33}$$

where $\hat{y}^*$ is the central bank's estimate of potential output. Assuming that the error in forecasting potential is uncorrelated with other noise terms in the model, this will be used in the formulation of monetary policy via the following reaction function:

$$r_t = \hat{r}^* + \pi_t^e + \frac{1}{\gamma}\left[\frac{\beta}{\beta^2 + \omega}\right](\pi_t^e - \pi^*). \tag{34}$$

Suppose that agents know the estimate of potential output on which monetary policy is based. Results very similar to those already presented would then be obtained, revealing a role for conservatism in the gaining of credibility. However, if agents do not know this estimate of potential output, the policy instrument would be less informative as to the inflation target of the central bank. One sufficient (but not necessary) condition for this to be the case would be that the central bank has an informational advantage over economic agents. There is some evidence that this is the case, at least for the Federal Reserve. For example, Romer and Romer (2000) and Joutz and Stekler (2000) demonstrate that the Federal Reserve produces more accurate forecasts than commercial forecasters for a variety of variables and data sets. Romer and Romer argue that this is because the Federal Reserve commits more resources to forecasting than any single commercial forecaster. Others have argued that, because of their institutional nature, central banks should produce more accurate forecasts. Not only do they face less uncertainty as to their own future policy actions, but as Peek, Rosengren, and Tootell (1999) have shown, they typically have access to confidential bank supervisory data that contains information that is not available to agents in the economy, and yet is useful for forecasting.

Effectively, differing forecasts result in observed real interest rates being less informative to agents than to the inflation target. In particular, from the viewpoint of gaining credibility, the initial analysis holds, but with σ_{ϕ}^2 replaced by

$$\sigma_{\phi}^2 + \frac{E(\hat{y}_t^* - y'_t{}^*)^2}{\gamma^2}, \tag{35}$$

where $y'_t{}^*$ is the estimate of potential output that economic agents attribute to the central bank. The greater the error in measuring the central bank's forecast, the more costly the error is to society. To the extent that published forecasts are believed by agents to be those on which monetary policy is based, the lack of credibility imposed by this second term may be diminished.

This is especially relevant at the present time, with discussion both inside and outside central banks regarding the possible emergence of a "new economy." The implication here is that it is important for the central bank to clearly articulate the view of the new economy on which monetary policy is based, even if there is a high degree of uncertainty about it. Otherwise, it runs the risk that agents may incorrectly infer that the inflation target has changed. From the perspective of credibility formation, articulating a view of the new economy is important, whether or not that view turns out to be correct.

6. Conclusions

In recent years, many central banks have moved to an explicit target for monetary policy, generally stated in terms of the inflation rate. This paper has analyzed how a central bank should go about gaining credibility for an inflation target. If the monetary authority does not have perfect control over its policy instrument, agents can only observe the target with noise, so it will take time for the monetary authority to gain credibility for its target. The monetary authority should then endogenize the credibility formation process, and solve the resulting optimal-control problem.

The solution to the optimal-control problem is not analytically tractable, but for the simple linear-quadratic environment employed here and reasonable initial credibility, a linear approximation to the optimal-control problem provides nearly all the gains attainable from higher-order approximations. Further, this is observationally equivalent to installing a conservative central banker in the sense of Rogoff (1985). Thus, at least up to a linear approximation, conservatism is observationally equivalent to optimal control for a monetary authority that is seeking to gain credibility. Further, to the extent that conservatism is an observable trait, these gains are possible without agents needing to solve the optimal control problem.

The potential gains to conservatism are increasing in the weight that society places on output deviations from potential. This counterintuitive result stems from the fact that the greater the weight on output volatility in the loss function of the central bank, the less information the policy instrument contains about the inflation target, so that credibility is more difficult to gain.

The credibility formation process may be further supplemented by: (i) choosing a policy instrument over which the central bank has a high degree of control; (ii) being transparent about the monetary policy framework, so that agents can infer the objective of monetary policy by observing the instrument rather than economic outcomes; and (iii) publishing forecasts, so that agents base their updating of the target on the central bank's forecasts, rather than forecasts from some other source.

This paper has implicitly highlighted the difference between a central bank seeking to gain credibility for a new target versus one that already enjoys a high level of credibility for an existing target. In the former case, there are large benefits to installing a conservative central banker, choosing a policy instrument over which the central bank has a high degree of control, acting in a transparent manner, and publishing forecasts. As credibility is gained, the benefits from each of these diminish, and in the limit disappear, as agents' expectations become more firmly anchored to the target.

References

Bank of Canada. 1998. "A Primer on the Implementation of Monetary Policy in the LVTS Environment." *Bank of Canada Review* (Autumn): 57–66.

Barro, R.J. and D.B. Gordon. 1983. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91(4): 589–610.

Blinder, A.S. 2000. "Central Bank Credibility: Why do We Care? How do We Build It?" *American Economic Review* 90(5): 1421–31.

Cripps, M. 1991. "Learning Rational Expectations in a Policy Game." *Journal of Economic Dynamics and Control* 15: 297–315.

Cukierman, A. 1992. "Central Bank Strategy, Credibility, and Independence: Theory and Evidence." Cambridge: MIT Press.

Cukierman, A. and A.H. Meltzer. 1986. "A Theory of Ambiguity, Credibility, and Inflation Under Discretion and Asymmetric Information." *Econometrica* 54(5): 1099–1128.

Duguay, P. 1994. "Empirical Evidence on the Strength of the Monetary Transmission Mechanism in Canada." *Journal of Monetary Economics* 33: 39–61.

Dupasquier, C. and N. Girouard. 1992. "Une nouvelle courbe de Phillips au Canada." Bank of Canada. Manuscript.

Evans, M. and P. Wachtel. 1993. "Inflation Regimes and the Sources of Inflation Uncertainty." *Journal of Money, Credit and Banking* 25(3) Part 2: 475–511.

Faust, J. and L.E.O. Svensson. 2001. "Transparency and Credibility: Monetary Policy with Unobservable Goals." *International Economic Review* 42(2): 369–97.

Fillion, J.-F. and A. Léonard. 1997. "La courbe de Phillips au Canada: un examen de quelques hypothèses." Bank of Canada Working Paper No. 97–3.

Fuhrer, J.C. and M.A. Hooker. 1993. "Learning about Monetary Regime Shifts in an Overlapping Wage Contract Model." *Journal of Economic Dynamics and Control* 17: 531–53.

Geraats, P.M. 2001. "Why Adopt Transparency? The Publication of Central Bank Forecasts." European Central Bank Working Paper No. 41.

Goodfriend, M. 1986. "Monetary Mystique: Secrecy and Central Banking." *Journal of Monetary Economics* 17: 63–92.

Isard, P. and D. Laxton. 1998. "Monetary Policy with NAIRU Uncertainty and Endogenous Credibility: Perspectives on Policy Rules and the Gains from Experimentation and Transparency." Reserve Bank of New Zealand Conference on Monetary Policy Under Uncertainty.

Johnson, D.R. 1997. "Expected Inflation in Canada 1988-1995: An Evaluation of Bank of Canada Credibility and the Effect of Inflation Targets." *Canadian Public Policy* 23: 233–58.

Johnson, D.R. 1998. "The Credibility of Monetary Policy: International Evidence Based on Surveys of Expected Inflation." In *Price Stability, Inflation Targets, and Monetary Policy,* 361–95. Proceedings of a conference held by the Bank of Canada, May 1997. Ottawa: Bank of Canada.

Joutz, F. and H.O. Stekler. 2000. “An Evaluation of the Predictions of the Federal Reserve.” *International Journal of Forecasting* 16: 17–38.

Kaminsky, G.L. and L. Leiderman. 1998. “High Real Interest Rates in the Aftermath of Disinflation: Is it a Lack of Credibility?” *Journal of Development Economics* 55: 191–214.

Peek, J., E.S. Rosengren, and G.M.B. Tootell. 1999. “Using Bank Supervisory Data to Improve Macroeconomic Forecasts.” *New England Economic Review* (September/October): 21–32.

Perrier, P. and R. Amano. 2000. “Credibility and Monetary Policy.” *Bank of Canada Review* (Spring): 11–17.

Ricketts, N. and D. Rose. 1995. “Inflation, Learning and Monetary Policy Regimes in the G-7 Economies.” Bank of Canada Working Paper No. 95–6.

Rogoff, K. 1985. “The Optimal Degree of Commitment to an Intermediate Monetary Target.” *Quarterly Journal of Economics* 100: 1169–89.

Romer, C.D. and D.H. Romer. 2000. “Federal Reserve Information and the Behavior of Interest Rates.” *American Economic Review* 90(3): 429–57.

Stein, J.C. 1989. “Cheap Talk and the Fed: A Theory of Imprecise Policy Announcements.” *American Economic Review* 79(1): 32–42.

Tarkka, J. and D. Mayes. 1999. “Forecasting and Uncertainty in Monetary Policy: The Value of Publishing Official Central Bank Forecasts.” Bank of Finland. Manuscript.

Tetlow, R.J. and P. von zur Muehlen. 2000. “Simplicity Versus Optimality: The Choice of Monetary Policy Rules when Agents must Learn.” *Journal of Economic Dynamics and Control* 25: 245–79.

Thornton, D.L. 1999. “The Fed’s Influence on the Federal Funds Rate: Is It Open Market or Open Mouth Operations?” Federal Reserve Bank of St. Louis. Manuscript.

Vickers, J. 1986. “Signalling in a Model of Monetary Policy with Incomplete Information.” *Oxford Economic Papers* 38: 443–55.

Figure 1: Credibility Formation

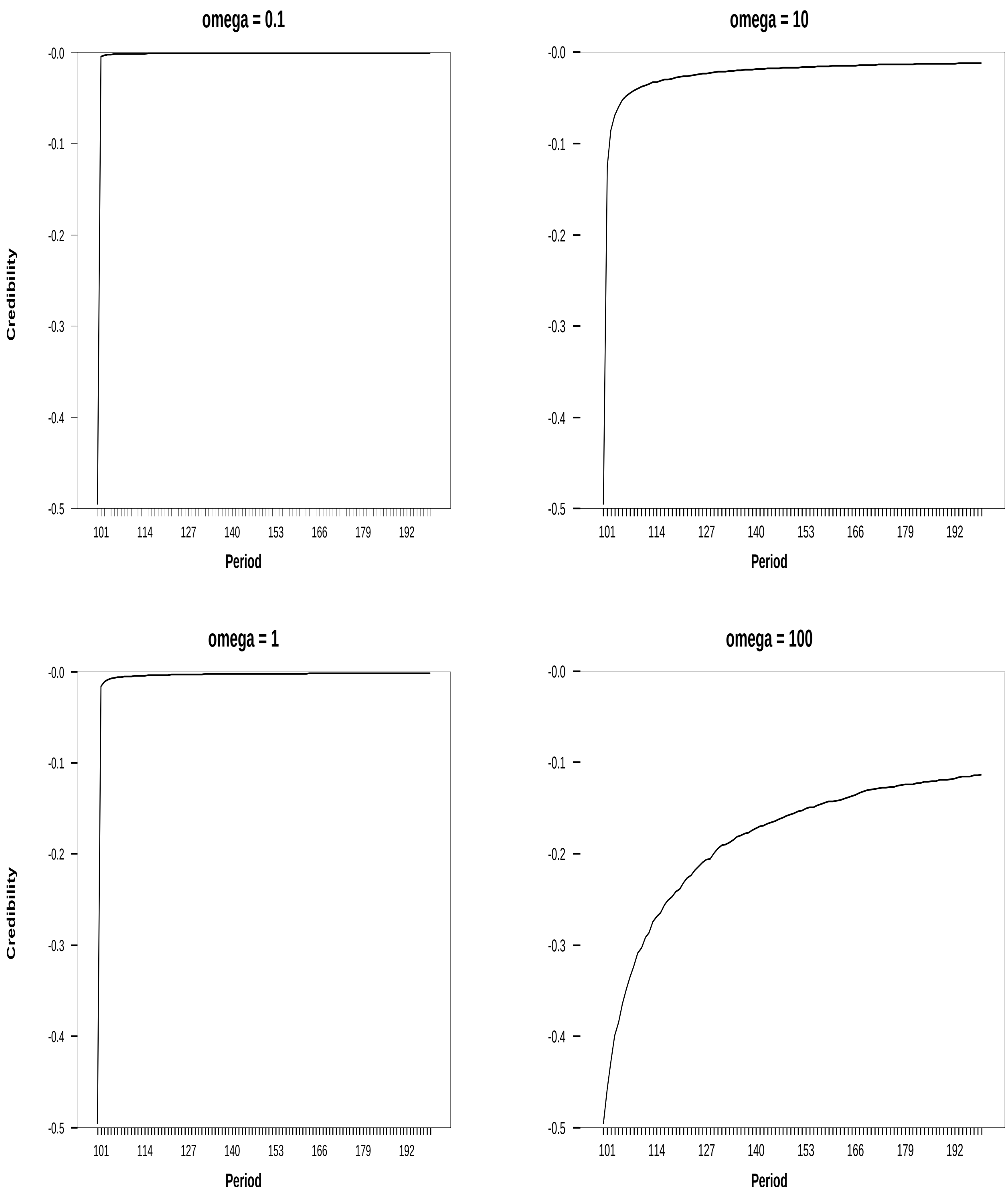


Figure 2.1: Loss with Conservative Monetary Authority

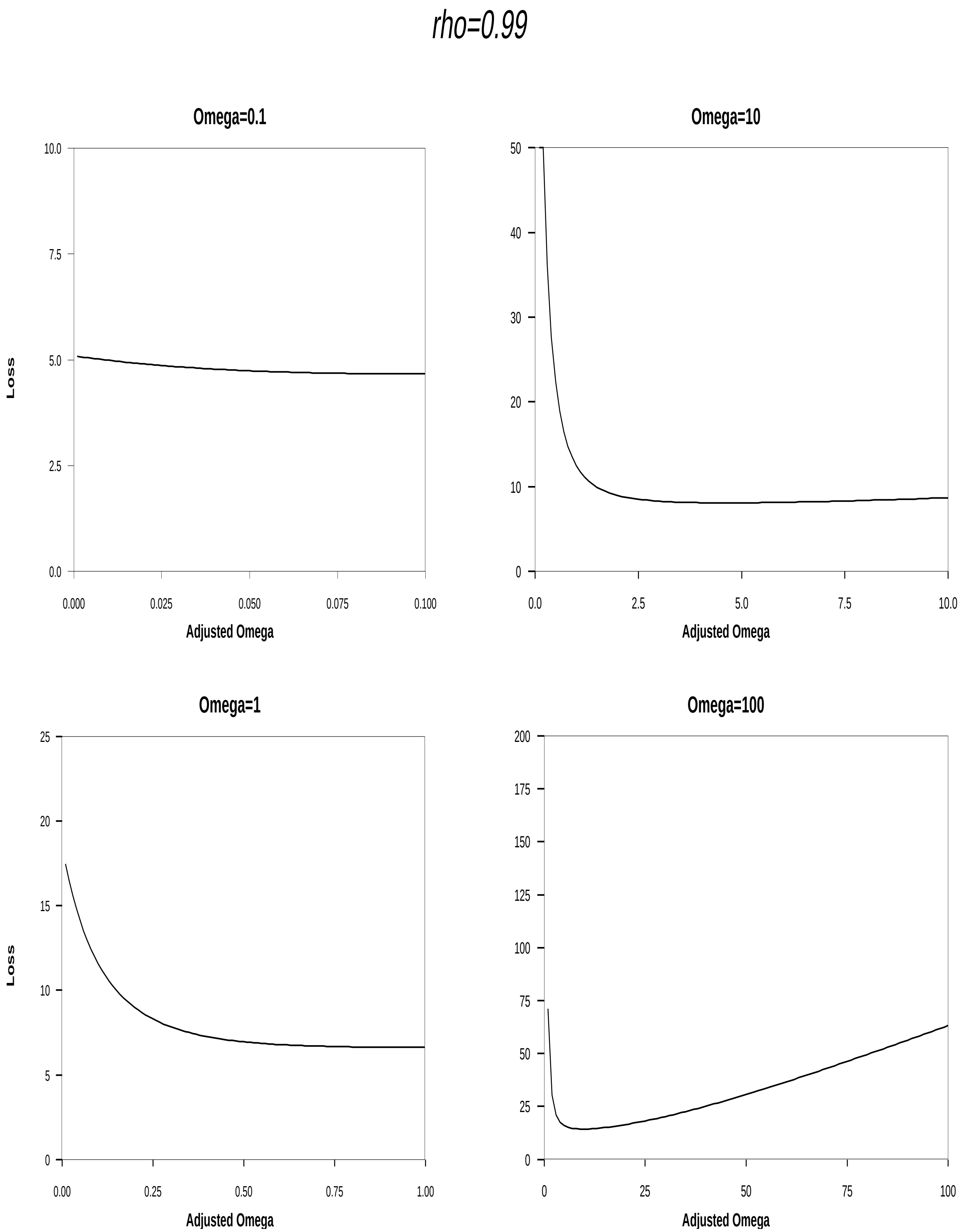


Figure 2.2: Loss with Conservative Monetary Authority

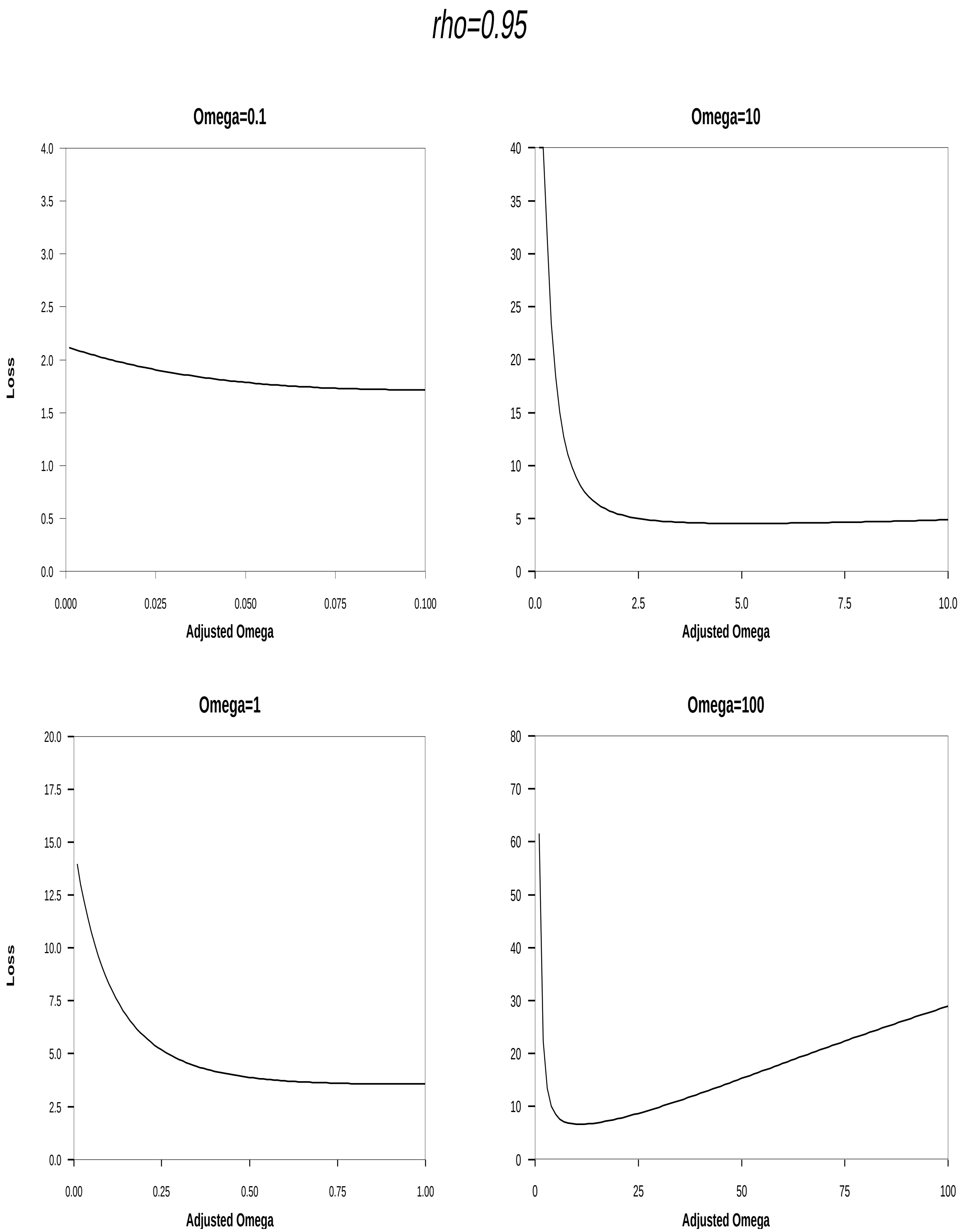


Figure 2.3: Loss with Conservative Monetary Authority

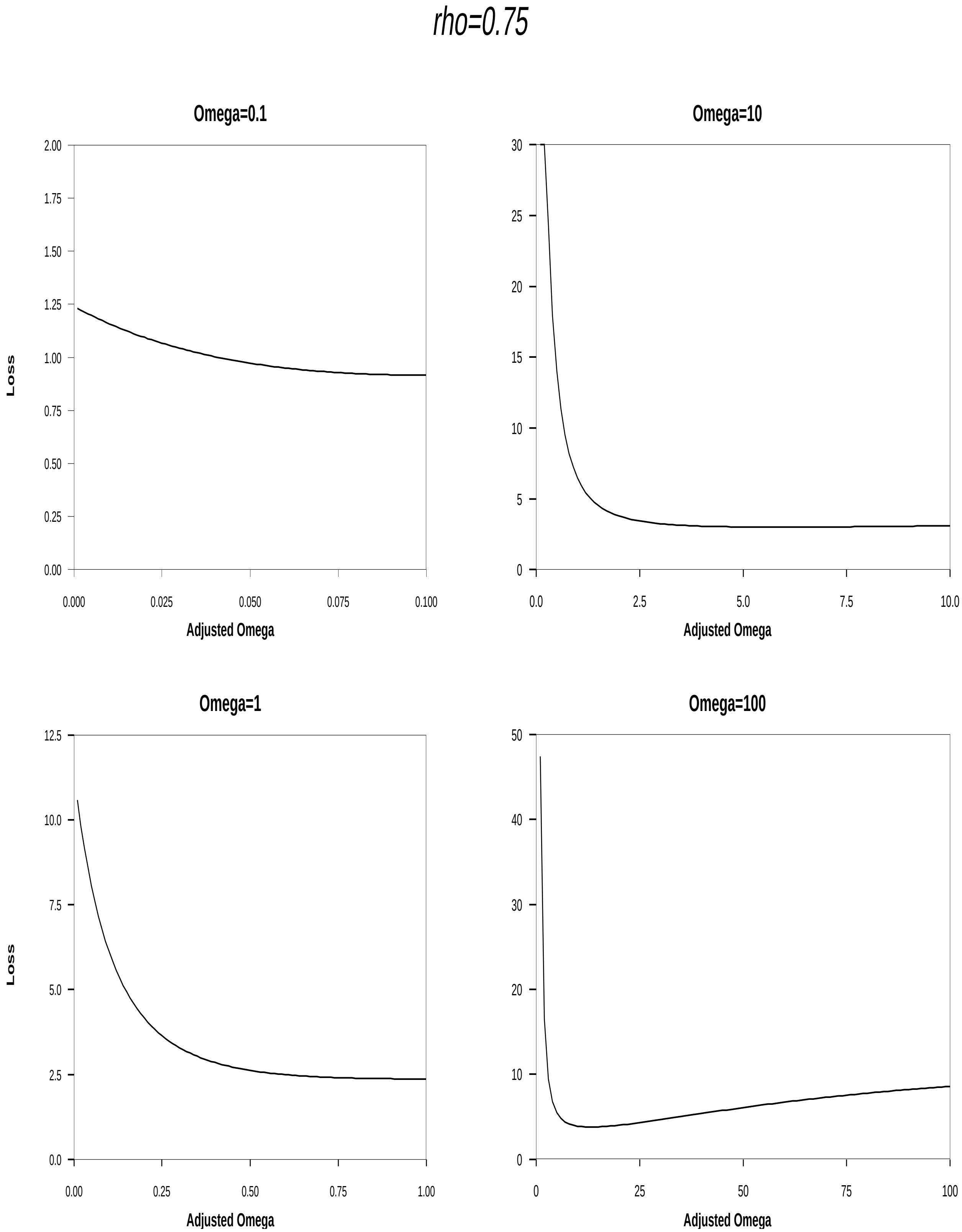


Figure 3.1: Credibility Formation

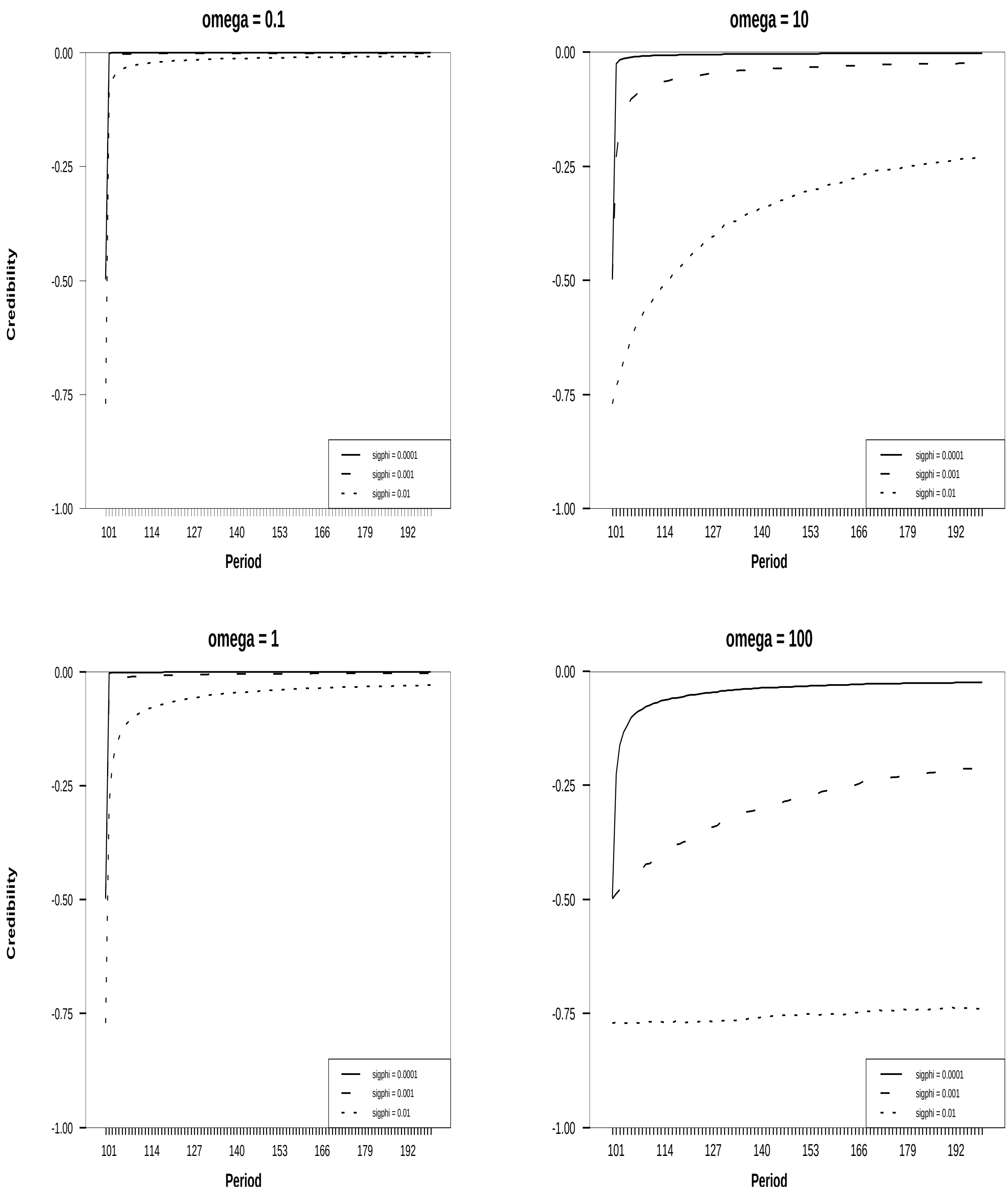
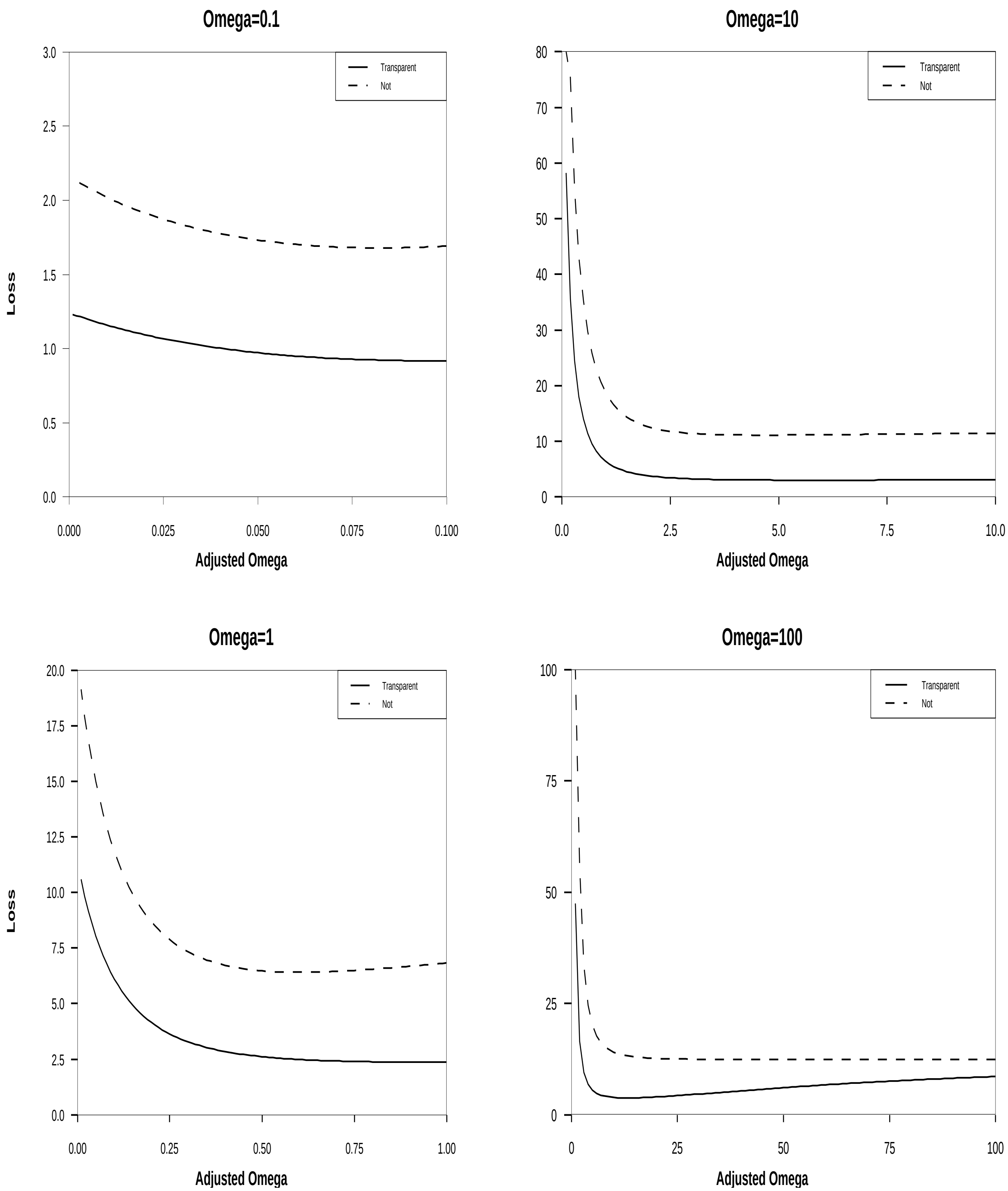


Figure 3.2: Transparent versus Non-Transparent Monetary Authority



Appendix A

A.1 Parameter values

The results obtained from the simulation exercises may in some cases be sensitive to the choice of parameter values. Here the values chosen are outlined, as well as the reasons for them. In general, parameter values are consistent with recent studies using Canadian data.

The loss function of the central bank is characterized by the following parameters: an inflation target of 2 per cent (that is, $\pi^* = 0.02$), and varying weights on the output gap in the loss function ($0 \le \omega < \infty$).

The standard deviation of inflation shock is $\sigma_\varepsilon = 0.006$, or 0.6 per cent on an annual basis. This is consistent with the total variability of inflation over the past 10 years.

Nominal interest rates at time zero are taken to be consistent with a long-run equilibrium real interest rate of 3 per cent and inflation expectations of 2 per cent: $r_0 = 0.05$; initial real output is the log of output in millions: $y_0 = 13.7$.

The impact of real interest rates on output is consistent with estimates obtained by Duguay (1994): $\gamma = 1.0$; and the slope of the Phillips curve is $\beta = 0.5$, which is consistent with a sacrifice ratio of 2 when inflation expectations are equal to lagged inflation.[1]

The economy is simulated for a finite number of horizons, but sufficient so that the total weight of all future periods is negligible. For example, with $\rho = 0.99$, a given output and inflation gap in period 1000 receives a weight of only 0.004 per cent of the weight that those same gaps would receive in period 1. With lower values of ρ, the weight afforded to future periods is even lower. Therefore, the economy is simulated for 1000 periods for $\rho = 0.99$, 200 periods for $\rho = 0.95$, and 50 periods for $\rho = 0.75$.

The simulations are for different values of $\tilde{\omega}$, over 1000 random draws of shocks on the economy. Using a sufficiently fine grid on $\tilde{\omega}$ allows conclusions to be drawn on the optimal conservatism of a central bank. Graphs are then produced of the average level of credibility, given by $-\left|\pi_t^P - \pi^*\right|$,[2] along with the value of the loss function, for different $\tilde{\omega}$.

1. Recent estimates of the sacrifice ratio for Canada include 1.5 (Dupasquier and Girouard 1992), 1.7 (Duguay 1994), and 2.2 (Fillion and Léonard 1997).
2. This is consistent with the measure suggested in Cukierman (1992, page 176).

The noise term on interest rates, ϕ_t, is drawn from a uniform distribution centred on zero, where the range is set to be consistent with rounding introduced by 25 basis-point increments commonly observed in monetary policy, if interest rates are set equal to the nearest increment to the desired rate. That is, $\sigma_\phi = 7.217 \times 10^{-4}$, or approximately 7 basis points.

At the commencement of inflation targets, agents are assumed to believe that the inflation target is equal to the level of inflation in the previous, non-targeting period, and V_0 is equal to the variance across the 1000 simulations.

A.2 Conservative central banker

This section outlines a proof of $0 < \tilde{\omega} < \omega$ for $\omega > 0$. Recall that $\omega = 0$ iff $\tilde{\omega} = 0$. A sufficient condition is therefore that $\frac{d\omega}{d\tilde{\omega}} > 1$ for $\tilde{\omega} \geq 0$. That is, as ω increases from 0, $\tilde{\omega}$ also increases, but at a slower rate.

Differentiating (23), two sufficient conditions for $\frac{d\omega}{d\tilde{\omega}} > 1$ given $\tilde{\omega} \geq 0$ are $4\tilde{\omega}(\gamma^2\sigma_\phi^2\tilde{\omega}V_0\beta^2 t)([x_t - \gamma^2\sigma_\phi^2(\beta^2 + \tilde{\omega})] > 0)$ given $x_t = \gamma^2(\beta^2 + \tilde{\omega})^2\sigma_\phi^2 + V_0\beta^2 t$ and

$$\sum_{t=1}^{\infty}\rho^t\left[\frac{(\gamma^2\sigma_\phi^2)^2(\beta^2+\tilde{\omega})}{x_t^2}\right]\sum_{t=1}^{\infty}\rho^t\left(\left[\frac{x_t^2\gamma^2\sigma_\phi^2\tilde{\omega}V_0\beta^2 t - 4x_t(\gamma^2\sigma_\phi^2)^2\tilde{\omega}V_0\beta^2 t(\beta^2+\tilde{\omega})}{x_t^4}\right]\right.$$
$$\left.\geq \sum_{t=1}^{\infty}\rho^t\left[\frac{\gamma^2\sigma_\phi^2\tilde{\omega}V_0\beta^2 t}{x_t^2}\right]\sum_{t=1}^{\infty}\rho^t\left[\frac{x_t^2(\gamma^2\sigma_\phi^2)^2 - 4x_t(\gamma^2\sigma_\phi^2)^3(\beta^2+\tilde{\omega})^2}{x_t^4}\right]\right) \tag{36}$$

The first condition is always satisfied. Rearranging the second condition so that all summation terms have x_t^3 in the denominator, and cancelling common coefficients, leaves the following sufficient condition:

$$\left[\sum_{t=1}^{\infty}\rho^t\frac{t^2}{x_t^3}\right]\left[\sum_{t=1}^{\infty}\rho^t\frac{1}{x_t^3}\right] - \left[\sum_{t=1}^{\infty}\rho^t\frac{t}{x_t^3}\right]^2 \geq 0. \tag{37}$$

Defining $F(T) = \left[\sum_{t=1}^{T}\rho^t\frac{t^2}{x_t^3}\right]\left[\sum_{t=1}^{T}\rho^t\frac{1}{x_t^3}\right] - \left[\sum_{t=1}^{T}\rho^t\frac{t}{x_t^3}\right]^2$, it is possible to show that

$\lim_{T \to \infty} F(T) > 0$. First, $F(1) = 0$. Further, a sufficient condition for $F(T+1) > 0$ is $F(T) \geq 0$, as follows:

$$F(T+1) = F(T) + \rho^{T+1}\frac{(T+1)^2}{x_{T+1}^3}\left[\sum_{t=1}^{T}\rho^t\frac{1}{x_t^3}\right] + \rho^{T+1}\frac{1}{x_{T+1}^3}\left[\sum_{t=1}^{T}\rho^t\frac{t^2}{x_t^3}\right] \quad . \qquad (38)$$
$$-2\rho^{T+1}\frac{(T+1)}{x_{T+1}^3}\left[\sum_{t=1}^{T}\rho^t\frac{t}{x_t^3}\right]$$

Defining $A = \sqrt{\sum_{t=1}^{T}\rho^t\frac{t^2}{x_t^3}}$, $B = \sqrt{\sum_{t=1}^{T}\rho^t\frac{1}{x_t^3}}$, $F(T) \geq 0 \Rightarrow AB > \left[\sum_{t=1}^{T}\rho^t\frac{t}{x_t^3}\right]$.

A sufficient condition for $F(T+1) > 0$ is therefore $(T+1)^2A^2 + B^2 - 2(T+1)AB > 0$, or $[(T+1)A - B]^2 > 0$, which is always true. Therefore, for every $T > 1$, $F(T) > 0$, including the limiting case as $T \to \infty$.

Therefore, $0 < \tilde{\omega} < \omega$.

A.3 Cubic approximation

Defining the observed policy instrument as

$$o_t = a_1(\pi_t^e - \pi^*) + a_2(\pi_t^e - \pi^*)^2 + a_3(\pi_t^e - \pi^*)^3 + \phi_t, \qquad (39)$$

agents solve $o_t = a_1(\pi_t^e - \hat{\pi}^*) + a_2(\pi_t^e - \hat{\pi}^*)^2 + a_3(\pi_t^e - \hat{\pi}^*)^3$ for $\hat{\pi}^*$ to extract an estimate of the target each period. This may be rewritten as

$$x_t^3 + \frac{a_2}{a_3}x_t^2 + \frac{a_1}{a_3}x_t - \frac{o_t}{a_3} = 0, \qquad (40)$$

where $x_t = \pi_t^e - \pi^*$.

Defining $y_t = x_t + a_2/(3a_3)$, this may be rewritten as $y_t^3 + py_t + q$, where $p = a_1/a_3 - a_2^2/(3a_3^2)$, $q = -o_t/a_3 - a_2^3/(27a_3^3) - pa_2/(3a_3)$. Cardano’s formulas can then be applied. For the parameters considered here, there is only one real root given by

$$x = \sqrt[3]{-\frac{q}{2} + \frac{1}{2}\sqrt{\frac{4p^3 + 27q^2}{27}}} + \sqrt[3]{-\frac{q}{2} - \frac{1}{2}\sqrt{\frac{4p^3 + 27q^2}{27}}}. \qquad (41)$$

The signal of the target is then given by $\hat{\pi}_t^* = \pi_t^e - x + a_2/(3a_3)$. The lower bound of the variance of this signal (computed for known π^*) cannot be computed analytically. Instead, I use the variance of a quadratic approximation of the signal, as follows. The only source of variance in the signal is x because agents know π_t^e, so $V(\hat{\pi}_t^*) = V(x) = V(f(q))$. Since $f(q) \approx f(0) + qf'(0) + q^2 f''(0)/2$, $f(0) = f''(0) = 0$,

$$V(f(q)) \approx [f'(0)]^2 V(q) = V(q)/p^2 = V(\phi_t)/(a_3 p)^2. \tag{42}$$

A.4 Instrument choice and conservatism

Rewriting (24) as $\tilde{\omega} = \omega - F$ and differentiating with respect to $\tilde{\omega}$ for a constant ω yields

$$1 = -\frac{\partial}{\partial\tilde{\omega}}[F] - \frac{\partial}{\partial\sigma_\phi^2}[F]\frac{d\sigma_\phi^2}{d\tilde{\omega}}. \tag{43}$$

From section A1 above, $\frac{\partial}{\partial\tilde{\omega}}[F] > 0 \Rightarrow \frac{d\sigma_\phi^2}{d\tilde{\omega}} > 0$ iff $\frac{\partial}{\partial\sigma_\phi^2}[F] < 0$. The sign of this partial derivative,

following the cancellation of common factors and rearranging, will be the same as

$$\left(\sum_{t=1}^{\infty}\rho^t\left[\frac{x_t^3}{x_t^3}\right] + \sum_{t=1}^{\infty}\rho^t\left[\frac{x_t\gamma^2\sigma_\phi^2}{x_t^3}\right]\right)\left(\sum_{t=1}^{\infty}\rho^t\left[\frac{V_0^2\beta^4 t^2}{x_t^3}\right] - \sum_{t=1}^{\infty}\rho^t\left[\frac{\gamma^2\sigma_\phi^2(\beta^2+\tilde{\omega})^2 V_0\beta^2 t}{x_t^3}\right]\right)$$
$$-2\left(\sum_{t=1}^{\infty}\rho^t\left[\frac{x_t\gamma^2\sigma_\phi^2 V_0\beta^2 t}{x_t^3}\right]\sum_{t=1}^{\infty}\rho^t\left[\frac{\gamma^2\sigma_\phi^2(\beta^2+\tilde{\omega})^2 V_0\beta^2 t}{x_t^3}\right]\right) \tag{44}$$

If (28) is satisfied, then the first term is negative. If (29) is satisfied, a sufficient condition for the partial derivative to be positive is

$$\left(\sum_{t=1}^{\infty}\rho^t\left[\frac{x_t^3}{x_t^3}\right]\right)\left(\sum_{t=1}^{\infty}\rho^t\left[\frac{\gamma^2\sigma_\phi^2(\beta^2+\tilde{\omega})^2 V_0\beta^2 t}{x_t^3}\right]\right)$$
$$-2\left(\sum_{t=1}^{\infty}\rho^t\left[\frac{x_t\gamma^2\sigma_\phi^2 V_0\beta^2 t}{x_t^3}\right]\sum_{t=1}^{\infty}\rho^t\left[\frac{\gamma^2\sigma_\phi^2(\beta^2+\tilde{\omega})^2 V_0\beta^2 t}{x_t^3}\right]\right) > 0 \tag{45}$$

Multiplying out the x_t^3 in the numerator of the first term yields

$$\left(\sum_{t=1}^{\infty}\rho^{t}\left[\frac{x_t\gamma^2\sigma_{\phi}^2V_0\beta^2t}{x_t^3}\right]\sum_{t=1}^{\infty}\rho^{t}\left[\frac{\gamma^2\sigma_{\phi}^2(\beta^2+\tilde{\omega})^2V_0\beta^2t}{x_t^3}\right]\right) \tag{46}$$

$$+\left(\sum_{t=1}^{\infty}\rho^{t}\left[\frac{\left\{\gamma^2(\beta^2+\tilde{\omega})^2\sigma_{\phi}^2\right\}^3+\{V_0\beta^2t\}^3}{x_t^3}\right]\right)\left(\sum_{t=1}^{\infty}\rho^{t}\left[\frac{\gamma^2\sigma_{\phi}^2(\beta^2+\tilde{\omega})^2V_0\beta^2t}{x_t^3}\right]\right)$$

which is strictly positive.

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