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## **Abstract**

This paper examines the ability of linear and nonlinear models to replicate features of real Canadian GDP. We evaluate the models using various business-cycle metrics. From the 9 data generating processes designed, none can completely accommodate every business-cycle metric under consideration. Richness and complexity do not guarantee a close match with Canadian data. Our findings for Canada are consistent with Piger and Morley's (2005) study of the United States data and confirms the contradiction of their results with those reported by Engel, Haugh, and Pagan (2005): nonlinear models do provide an improvement in matching business-cycle features. Lastly, the empirical results suggest that investigating the merits of forecast combination would be worthwhile.

*JEL classification: C32, E37*

*Bank classification: Business fluctuations and cycles; Econometric and statistical methods*

## **Résumé**

Les auteurs évaluent la capacité des modèles linéaires et non linéaires à reproduire les caractéristiques de l'évolution du PIB réel canadien en ayant recours à diverses mesures du cycle économique. Aucun des neuf processus générateurs de données qu'ils élaborent ne permet de recréer chacune des caractéristiques du cycle considérées. La richesse et la complexité du modèle ne garantissent pas une concordance parfaite avec les données canadiennes. Selon les résultats obtenus par les auteurs pour le Canada, les modèles non linéaires permettent de mieux reproduire les caractéristiques du cycle; ces résultats sont conformes à ceux de Piger et Morley (2005) fondés sur les données américaines et vont à l'encontre, comme les résultats de ces deux auteurs, de ceux présentés par Engel, Haugh et Pagan (2005). Enfin, les résultats empiriques donnent à penser qu'il vaudrait la peine d'examiner les avantages à tirer de la combinaison des prévisions.

*Classification JEL : C32, E37*

*Classification de la Banque : Cycles et fluctuations économiques; Méthodes économétriques et statistiques*

## 1 Introduction

In the view of early business-cycle writers, (e.g., Keynes 1936; and Mitchell 1927), the business cycle, or *classical cycle*, is characterized by long expansions in real aggregate economic activity, punctuated by sudden, infrequent, and violent contractions. These phenomenon summarize the so-called turning-point features of the business cycle with two distinct phases, namely recessions and expansions. Consequently, a central feature of the Burns and Mitchell (1946) business-cycle methodology is that the relation between expansions and recessions is asymmetric. Business-cycle asymmetry implies, among other things, that the mean duration of each phase is different: recessions are short-lived relative to expansions.

Neftçi (1984) and Hamilton (1989) propose models that include a regime-switching mechanism that explicitly addresses Burns and Mitchell’s idea of asymmetry in a time series. Their approach has influenced a large body of work by suggesting that univariate linear models are not “optimal” in describing the empirical process of macroeconomic time series such as real output or the unemployment rate. The literature, however, is not unanimous on whether nonlinear models are indeed admitted by the macroeconomic data (e.g., Hansen 1992; Garcia 1998) or on whether nonlinear models provide a better approximation of the business-cycle features of the data than do linear ones (Harding and Pagan 2002a).

For instance, Fisher (1925, p. 191) points out that “so-called business-cycles” likely result from pure randomness in output fluctuations, something which he describes as the “Monte Carlo cycle”; this is later associated by Neftçi (1984) with the presence of a “drift” in most macroeconomic time series. Harding and Pagan (2002a) find that non-linear effects are not important in explaining the U.S. business cycle, while Engel, Haugh, and Pagan (2005; EHP) point out that the benefits of nonlinear methods come at non-negligible costs: first, nonlinear methods are more cumbersome; second, the expansions suggested by these models tend to last longer than what is observed in the data. Morley and Piger (2006) find opposite results, however, when using a different set of nonlinear specifications.

The objective of this paper is to investigate the ability of a variety of models to reproduce selected features (e.g., amplitudes of business-cycle phases, frequency of recessions, higher-order moments, etc) of the Canadian data, including the aforementioned business-cycle asymmetry. To analyze this question, we borrow both from EHP and from Morley and Piger (2006) and extend their analysis by considering a simple multivariate model as well. We compare the ability of univariate linear and nonlinear models and a multivariate linear model to match Canadian data in a Monte Carlo exercise. We extend the range of metrics

and statistics considered to address some of Pérez Quirós's (2005, p.664) critiques of EHP.<sup>1</sup>

There are two key findings. First, from the nine data generating processes under investigation, none can completely accommodate our diversified set of metrics. Second, our simulation exercise shows that rich and complex functional forms do not guarantee a close match with data. Our findings for Canada are also in line with Morley and Piger's (2006) study for the United States and confirms their contradictions with the results reported by Engel, Haugh, and Pagan (2005): nonlinear models do provide an improvement in matching business-cycle features.

The implications of our results are twofold. On the one hand, our findings provide support to the notion of forecast combination because of model uncertainty. On the other hand, given that recessions occur very rarely in Canada, the results suggest that when developing models of Gross Domestic Product (GDP), we should also examine the model's ability to match selected business-cycle metrics such as the frequency of observing a recession, its average duration, its average depth, as well as the variability of output growth. For the formulation of monetary policy, where analyses based on stochastic simulations are one of the key inputs, evaluation of business-cycle metrics could enrich the understanding of the model and its implications for the real economy.

The rest of the paper is organized as follows. Section 2 describes some concepts relevant to business-cycle analysis and presents some results for Canada. Section 3 discusses how business-cycles features relate to a mixture of distributions, in particular to Markov-Switching models. Section 4 presents the results of a Monte Carlo exercise that compares the performance of each specification. Section 5 concludes with brief remarks.

## 2 Concepts and their Application to Canadian Data

### 2.1 The data

The data employed in our study are seasonally-adjusted market-prices Canadian GDP at (chained) 1997 dollars taken from Statistics Canada's national accounts. The log level series, denoted as  $Y_t$ , spans from 1961Q1 to 2005Q2. Because we are interested in asymmetries in GDP associated with the business cycle, the level series is detrended by log differencing:  $y_t = Y_t - Y_{t-1}$ . The top of Figure 1 plots real Canadian GDP ( $Y_t$ ), the middle plots its (annualized) growth rate ( $y_t$ ), while the bottom panel plots the change of the growth rate. Unless stated otherwise, our analysis of the stochastic properties of the Canadian data is

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<sup>1</sup>This paper does not, however, revisit an important ongoing debate: namely the statistical justification of nonlinear models by means of linearity tests or by means of out-of-sample forecast evaluations.

based on the log difference approximation to the quarterly growth rate, namely the basic  $(1 - L)$  filter, where  $L$  is the lag operator such that  $Ly_t = y_{t-1}$ .<sup>2</sup> As argued by Harding and Pagan (2002a, 2005), removing the trend, or a so-called “permanent component” of GDP, is not recommended when investigating business-cycle features. Not only may such a decomposition oddly reveal that GDP series may not exhibit a business-cycle—a situation that arises when detrended GDP is not serially correlated—, but the conclusion about the properties of the business cycle will also largely depend upon the decomposition method used (e.g., Hodrick-Prescott filter, Beveridge-Nelson).<sup>3</sup>

## 2.2 Dating the business cycle

In the spirit of Burns and Mitchell, the study of movements in the level of aggregate economic activity requires determining turning points between the phases of economic activity. We employ the version of Bry and Boshans’ (1971) monthly dating algorithm modified by Harding and Pagan (2002a) for use at the quarterly frequency. The algorithm, henceforth referred to as BBQ, requires that business-cycle phases last a minimum of two quarters, while a full cycle is required to last a minimum of five quarters.<sup>4</sup> Harding and Pagan (2002b) show that their modified algorithm performs well when compared to the National Bureau of Economic Research (NBER) business-cycle dates in the United States. Furthermore, they argue the algorithm is a transparent and simple method for dating the business cycle.

The basic principles of the BBQ algorithm can be summarized by the following rules:

i) From the log level of real GDP ( $Y_t$ ), at time  $t$  a peak is defined as

$$Y_{t-2} - Y_t < 0, Y_{t-1} - Y_t < 0; Y_{t+1} - Y_t < 0, Y_{t+2} - Y_t < 0, \quad (1)$$

whereas a trough is said to occur if

$$Y_{t-2} - Y_t > 0, Y_{t-1} - Y_t > 0; Y_{t+1} - Y_t > 0, Y_{t+2} - Y_t > 0. \quad (2)$$

ii) Appropriate censoring is necessary to ensure that peaks and troughs alternate. If two consecutive peaks (troughs) are found, the peak (trough) with the higher (lower) value is selected.

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<sup>2</sup>This contrasts with Sichel (1993) and Knüppel (2004), among others, who detrend output using methods such as HP filtering, or some other linear filter.

<sup>3</sup>For excellent discussions on this issue, see Harding and Pagan (2002a, 2004) and Sichel (1993).

<sup>4</sup>Morley and Piger (2005) apply further modifications to the BBQ algorithm by selecting a turning-point threshold parameter to improve the signaling properties of the dating algorithm.

- iii) Additional censoring is necessary to ensure that each business-cycle phase has a minimal duration: the minimum duration for a full cycle is set to 5 quarters, and each phase is required to last a minimum of 2 quarters.

As a basis for assessing the accuracy of the BBQ algorithm using Canadian data, we compare the business-cycle chronology of the Economic Cycle Research Institute (ECRI) with that obtained from BBQ.<sup>5</sup> Because the ECRI follows the NBER methodology, it provides us with business-cycle dates that are consistent both with the existing literature on the U.S. and with Burns and Mitchell’s interpretation of business cycles.<sup>6</sup>

Table 1 compares the peak and trough dates from the ECRI with those from the BBQ algorithm. The BBQ algorithm provides turning points similar to the ECRI for the 1981-82 recession and the start of the 1990 recession. However, it differs from the ECRI chronology in two notable ways. First, the algorithm identifies the downturn seen in the first half of 1980 as a recession, while the ECRI does not; second, the length of the recession in the early 1990s is four quarters longer using the ECRI chronology rather than the BBQ chronology.

### 2.3 Selected business-cycle metrics

A number of metrics are available to analyze moments of GDP resulting from the business cycle. We employ metrics based on the level of GDP, the probability distribution function (pdf) of GDP growth, the average number of recessions, and the percentage of negative growth periods. The first set of metrics, based on the level of GDP, are the ones evaluated by Harding and Pagan (2002a); the second set stems from criticisms of Harding and Pagan (2005) by Perez Quirós (2005) and is based on the pdf (i.e., higher-order moments) of GDP growth and of the change in GDP growth. Finally, we compare the occurrence of recessions and of negative growth rates in GDP, metrics also analyzed by Morley and Piger (2006).

#### 2.3.1 Level metrics

While the BBQ algorithm provides a business-cycle chronology, it does not allow us to compare the relative performance of the different models in our simulation. We therefore follow Harding and Pagan (2002a) and EHP, and define a number of metrics that allow us to dissect the business cycle.

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<sup>5</sup>The ERCI applies the NBER’s dating methodology to a number of countries. Because there is no official agency in Canada responsible for dating the business cycle, the ERCI dates are chosen for consistency of methodology with the literature, which is mostly devoted to U.S. data.

<sup>6</sup>We are grateful to James Engel for providing us with his dating-algorithm Gauss code, which is available at <http://members.iinet.net.au/~james.engel>.

The first metrics of interest are the duration and amplitude of the business-cycle phases. Duration is defined to be the average length (in quarters) of a particular phase. Following notation close to Harding and Pagan (2002a) we denote  $D_i$  as the length of the  $i$ -th occurrence of the phase so that the mean durations are obtained from  $D^e = K^{-1} \sum_{i=1}^K D_i^e$  and  $D^c = K^{-1} \sum_{i=1}^K D_i^c$ , where  $K$  is the total number of occurrences of each phase; the superscripts  $e$  and  $c$  denote the phase: expansion and contraction, respectively.

The amplitude is defined as the percentage change from a business-cycle phase, and is expressed as the percentage increase (decrease) relative to the preceding trough (peak). We let  $A_i^n$  denote the amplitude of the  $i$ -th phase over the cycle and compute the average amplitude of both business-cycle phases in the same manner as the average durations.

The amplitude and duration of each phase can be used to approximate the cumulative change in output. This approximation, referred to as the triangle approximation in EHP, is defined as  $AD_i^n = (D_i^n \times A_i^n)/2$ , for  $n = c, e$ , and represents the cumulative change in output that would result if the economy evolved at a constant rate over a phase.

An alternative measure of cumulative output can be computed using a Riemann sum. This measure of actual movement in output during the  $i$ -th phase is defined as

$$C_i^n = \sum_{j=1}^{D_i} (Y_j - Y_{0,i}) - \frac{A_i^n}{2}, \quad \text{for } i = 1, \dots, K; \text{ and } n = c, e, \quad (3)$$

where  $\frac{A_i^n}{2}$  is the necessary adjustment that results from basing the approximation on a sum of rectangles, and where  $Y_{0,i}$  is the log level of GDP at the beginning of the  $i$ -th phase.

Finally, we compare the two measures of cumulated change in output within a phase to assess the degree of asymmetry present in the level of output. The metric of interest is thus the difference between the actual change and the constant-rate variation (i.e., the excess movement in output), which is defined as follows:

$$E_i^n = \frac{C_i^n - AD_i^n}{AD_i^n}, \quad \text{for } n = c, e, \quad (4)$$

with mean  $E^n = K^{-1} \sum_{i=1}^K E_i^n$ . Such excess metrics are another way to illustrate the shape of recessions and expansions.

EHP argue that a necessary condition for  $E$  to equal 0 is that the distribution of GDP growth must be symmetric about 0. This condition implies that symmetry and trend growth (in this case, the drift) in real output are important determinants of the degree of excess ( $E$ ). This is formalized by EHP (p. 654–55), who derive  $\Pr[y_t < 0 | y_{t-1} > 0]$  and show that it depends upon the intercept, the autoregressive coefficient(s), and on the ratio of long-run growth to the innovation variance.

EHP also report other metrics of interest such as the coefficients of variation of durations ( $CV_D$ ) and amplitudes ( $CV_A$ ). Again, these metrics are defined for both phases of the cycle. For durations, they are obtained from the following expression:

$$CV_D^n = \frac{\sqrt{\left(K^{-1} \sum_{i=1}^K (D_i^n - D^n)^2\right)}}{K^{-1} \sum_{i=1}^K D_i^n}, \quad \text{for } n = c, e, \quad (5)$$

while for amplitudes we have:

$$CV_A^n = \frac{\sqrt{\left(K^{-1} \sum_{i=1}^K (A_i^n - A^n)^2\right)}}{K^{-1} \sum_{i=1}^K A_i^n}, \quad \text{for } n = c, e. \quad (6)$$

It is important to note that, as EHP point out, metrics computed for the contraction phase must be interpreted with caution since they are calculated using a small sample.

### 2.3.2 Higher-order moments

Business-cycle asymmetries may be present in the pdf of GDP growth and represent a complementary approach to that of Harding and Pagan (2002a) and EHP. They can be reflected in the pdf of growth rates in one of three ways.

First, a series may be deep. The notion of deepness, formally introduced by Sichel (1993), refers to the “relative depths of troughs and heights of peaks” (Sichel 1993). A process is said to be deep if the magnitude of the growth rates during expansions is smaller than the magnitude of growth rates during contractions. When a process is *deep*, it is negatively skewed, whereas a positively skewed process is referred to as *tall*. The series  $y$ , of mean  $\mu_y$ , is nondeep when the process is symmetric:

$$E \left[ (y - \mu_y)^3 \right] = 0.$$

Second, there is the notion of steepness. It refers to the “relative slope of expansions and contractions” (Sichel 1993). Processes defined to be negatively steep would enter recessions very rapidly, but recover slowly. A process is nonsteep when

$$E \left[ \Delta y^3 \right] = 0,$$

where  $\Delta = (1 - L)$ .

The last concept of interest, sharpness, is introduced by McQueen and Thorley (1993). A sharp series has the transition from contraction to expansion occurring more rapidly than

the transition from expansion to contraction, or vice versa. This feature results in the level series being more round at peaks (troughs) than at troughs (peaks). For an excellent technical discussion and graphical illustrations of these three concepts, see Clements and Krolzig (2003).

If business-cycle features such as asymmetry are present in the data, we would conclude that  $y_t$  is departing from normality. In the case where the marginal distribution of a process is a mixture of, say,  $M$  normal distributions, Timmermann (2000) and Clements and Krolzig (2003) explain how a state-dependent mean and variance will affect the realized density of the process. The view that the business cycle is characterized by sudden and violent recessions (e.g., Keynes 1936; Mitchell 1927), implies, among other things, that the empirical process is deep and (negatively) steep. In other words, GDP growth should exhibit negative skewness (i.e., contractions are short-lived but *violent*) and possibly some excess kurtosis. The latter could result from heteroskedasticity across phases, due to the ‘rare’ occurrence of large negative/positive growth rates.

Because the business cycle may affect the empirical distribution of GDP growth, it is instructive also to present metrics that examine how well models are able to match the business-cycle features that manifest themselves in the pdfs. In addition to metrics from Harding and Pagan (2002a), EHP, and Morley and Piger (2006), we also report metrics from the simulated series for the mean (Mean), the standard deviation (Std), as well as the third (Skew) and fourth (Kurt) central moments of GDP growth ( $y_t$ ) and its change ( $\Delta y_t$ ).

### 2.3.3 Negative-growth-rate metrics

We consider one final set of metrics that deal with negative innovations and recessions. These features are of primary importance when modelling the business cycle, but little attention is given to them in the literature.<sup>7</sup> To analyze the ability of our simulated models to capture recessions, we therefore report both the average number of recessions from each model as well as the percentage of quarters for which growth is negative.

## 2.4 Metrics of real Canadian GDP

Table 2 reports the metrics of Canadian GDP for the 1962Q1–2005Q2 period. The BBQ algorithm indicates that there have been three recessions in Canada over the last 44 years. The cycle is asymmetric: recessions last an average of 4 quarters while expansions last, on average, about 40 quarters. The amplitudes indicate that the level of output is on average 3

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<sup>7</sup>A notable exception is Morley and Piger (2005).

per cent lower after a recession and 40 per cent higher after an expansion. During recessions, the cumulative loss in output is 1 per cent smaller than a triangle approximation would imply, whereas the cumulative gains during expansions are (on average) 9.4 per cent higher than the triangle approximation. Figure 2 illustrates the excess observed in the data by plotting the gain curves for the expansion and recession phases. For both phases, the degree of excess seen in the data suggests that GDP exhibits some form of non-linearity.

The pdf of GDP growth provides conclusive evidence about the presence of non-linearities. While there is some positive skewness and kurtosis present in  $y_t$ , the null hypothesis of normality using the Jarque-Bera test is easily rejected (p-value: 0.01).<sup>8</sup> Similarly, the null hypothesis of the Jarque-Bera test is easily rejected for  $\Delta y_t$  (p-value: 0.00). This last series exhibits skewness and kurtosis, indicating that non-linearities may play an important role in defining its shape—particularly if the kurtosis is derived from heteroskedasticity between business-cycle phases (Clements and Krolzig 2003). The non-normality seen in the pdf of  $\Delta y_t$  implies that the pdf of  $y_t$  is non-normal as well.

We also briefly examine summary statistics and moments such as the median, the mean, standard deviation, skewness and kurtosis for  $y_t$  and for  $\Delta y_t$  (Table 3). Additionally, we evaluate the sensitivity of the estimates to the sample period by reporting estimates of the central moments for three subperiods.<sup>9</sup> For  $y_t$ , the mean and median are virtually equal over the full sample period, but discrepancies arise when subsamples are analyzed. While the full-sample estimate of the skewness coefficient indicates that output growth is a tall process, the process can either be deep or tall when looking at subperiods. Although we do not formally test the hypothesis that the mean has remained constant over time, the fact that it varies by roughly 1 percentage point over the two subsamples suggests that *structural* changes, in the spirit of Chow (1960) or Bai and Perron (1998), are probably not the dominant driver of the dynamics: because of the variability of the data, such a change seems small. The sign of skewness coefficient switches from positive to negative, ranging from 0.27 during the 1962Q2–1982Q2 period to -0.58 during the 1982Q3–2005Q2 period. DeLong and Summers (1984) examine U.S. output data and similarly found that the sign of skewness coefficient alternates.

The calculated skewness coefficient for  $\Delta y_t$  over the full sample provides strong evidence that output growth declines less rapidly when entering a recession than when recovering (i.e., a positively steep process). This assertion is supported for both subperiods. Over the last

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<sup>8</sup>The test is based on a whitened version—i.e., the residuals of an AR(1) for  $y_t$ .

<sup>9</sup>While the sample splitting was done in an *ad hoc* fashion, Chacra and Kichian (2004) have found a break in the mean of output growth in 1972Q4 and a break in the conditional variance in 1991Q2. The last subsample is chosen simply as to reflect the last ten years of observations.

ten years, however, the estimate of skewness coefficient for  $\Delta y_t$  is about zero. This likely results from the fact Canada did not experience a recession during this period. For this reason, this is not contradictory to the idea that output growth is a positively steep process. The tails of the distribution of  $\Delta y_t$  are also thick, suggesting that there is some information to be exploited by allowing for heteroskedasticity or for an intercept-switching mechanism.

Given our preliminary analysis, usual linear models will likely not provide a satisfactory approximation of the data generating process (DGP), although non-linear models could perform better.

### 3 Business-Cycle Asymmetry and Markov-Switching Models

#### 3.1 Mixtures of distributions

Burns and Mitchell's view of the business cycle implies that the mean growth rate differs whether the economy is in an expansion or in a recession. This concept is nicely formalized by Hamilton's (1989) Markov-Switching (MS) model of GDP. MS models are based on the principle of time-dependent mixtures of distributions.

Before further discussing the MS models, it is instructive to consider the following simple two-component mixture model for some process  $y = \{y_t\}_{t=1}^T$ , with mixing weight  $\pi$ :

$$p(y) = \pi f_1(y) + (1 - \pi)f_2(y), \quad \text{with } 0 < \pi < 1, \quad (7)$$

given some pdf of interest,  $f_1(\cdot)$  and  $f_2(\cdot)$ ,  $p(y)$  is thus a mixture pdf. A specification like (7) could easily be used to characterize a process where the dynamics of output growth are asymmetric in the sense that the process could depend upon whether the economy is contracting or expanding.

If we assume that an economy can be divided into two distinct phases, expansion and recession, and that we know the chronology of the phases, it is possible to construct an analytic expression for the mixing weight, as well as calculate the moments of the distributions in each phase. When the chronology is known, we can build a binary indicator function, say  $s_t$ , that discriminates between recession and expansion phases. If there are  $T = T_1 + T_2$  observations for the process  $y$ , with  $T_1$  observations associated with  $y$  being in a state of recession ( $s_t = 1$ ) and  $T_2$  observations associated with  $y$  being in a state of expansion ( $s_t = 2$ ), then  $\pi = T_1/T$ . Since  $\pi$  is observed, inference about chosen functional forms (i.e., the  $f_i(\cdot)$ s) can be calculated from known observations and is trivial in this context.

Over the whole sample, the average growth rate in Canada is about 3.5 per cent. The recession phase in Canada, which accounts for 15 of the last 174 quarters, has an average

growth rate of -1.8 per cent with a standard deviation of 2.7 percentage points. The growth rates observed during recessions are overwhelmingly negative, although some quarters are marked with slight positive growth rates. The expansion phase, which accounts for 159 of the 174 quarters, has an average growth rate of 4.0 per cent with a standard deviation of 3.1 percentage points.<sup>10</sup>

Based on the number of quarters for which each of the phases occurred in Canada, there is a 9 per cent probability (i.e.,  $\pi = 0.09$ ) that a quarter drawn at random is a recessionary quarter and a 91 per cent (i.e.,  $1 - \pi$ ) chance that the quarter will be an expansionary one. These probabilities can be interpreted as the probabilities that at time  $t$  a quarter will be drawn from either the expansionary or recessionary distributions. These probabilities can be used to form an unconditional distribution for GDP growth on using a mixture of two distributions. Figure 3 shows the (conditional) expansion and contraction densities. The realized process of Canadian GDP growth then has an unconditional density that is a weighted sum of the two underlying densities that describe the business cycle (Figure 4).

While mixture models provide a simple way to characterize the phenomenon of expansions and contractions, they do not address the issue of state persistence. In the following subsection, we consider the case of mixture models with Markov dependence.

### 3.2 Markov-Switching models

An important stylized fact about business cycles is that expansions last longer than recessions. An expansionary quarter is more likely to be followed by another quarter of positive growth, and vice versa. These features, however, cannot be parameterized under the simple mixture model described above. The probabilities of being in either states must therefore be serially correlated, or have Markov dependence. These are the so-called Markov-Switching models.

While it is trivial to analyze the mixture of distributions when a binary indicator function can be built using known business-cycle chronology, learning about mixtures is not trivial when either  $\pi$  or the  $f_i(\cdot)$ 's are unknown (see, e.g., Titterington, Smith, and Makov 1985; Hamilton 1989). MS models, as argued by Hamilton (1989, 2005), among others, are a natural way of combining a mixture of distributions using endogenously estimated mixing weights with Markov dependence. These models have been used to endogenously generate a binary indicator function using the probability of a particular phase occurring in a given quarter. MS models replicate U.S. business-cycle peak and trough dates from the NBER

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<sup>10</sup>In contrast, the United States has experienced 21 recessionary quarters during the same period, with a growth rate of -1.4 per cent during recessions, compared to 3.9 during expansions.

(Hamilton 1989; Chauvet and Hamilton 2005).

Consider the following MS-AR( $p$ ) model with switching means (MSM), as proposed by Hamilton (1989):

$$(y_t - \mu_{s_t}) = \sum_{j=1}^k \phi_j (y_{t-j} - \mu_{s_{t-j}}) + \varepsilon_t, \quad (8)$$

where  $k$  is the lag length. Although Hamilton (1989) treated the innovations as homoskedastic Gaussians, we generalize so that the variance of the innovations is state-dependent (MSMH), namely  $\varepsilon_t \sim i.i.d.N(0, \sigma_{s_t}^2)$ . The unobserved state,  $s_t$ , is the stochastic process governed by a discrete time, ergodic, first order autoregressive  $M$ -state Markov chain with transition probabilities, or mixing weights,  $\Pr [s_t = j | s_{t-1} = i] = p_{i,j}$  for  $\forall i, j \in \{1, \dots, M\}$ , and transition matrix,  $\mathbf{P}$  :

$$\mathbf{P} = \begin{pmatrix} p_{1,1} & \cdots & p_{M,1} \\ \vdots & \ddots & \vdots \\ p_{1,M} & \cdots & p_{M,M} \end{pmatrix}.$$

For more technical details, interested readers should consult Hamilton (1994, chap. 22). It is worth noting that the presence of business-cycle phenomena—e.g., sharpness, deepness, tallness, etc—depends upon the elements of  $\mathbf{P}$  and their implications on the higher-order moments of a series (Timmermann 2000; Clements and Krolzig 2003; and Knüppel 2004).

As with most of the existing literature—largely inspired by Hamilton (1989)—, we adopt the convention that the two-state model corresponds to

$$s_t = \begin{cases} 1, & \text{recession} \\ 2, & \text{expansion} \end{cases}.$$

While the two-state classification is a natural analogy to the traditional definition of business cycle phases, the possibility of a third state has been largely documented in the literature. For instance, Bodman and Crosby (2000) find that a three-regime model provides the most satisfactory specification for Canadian GDP growth. Similar results are also found for the GDP growth data of the United States (e.g., Sichel 1994; and Krolzig 1997). Because three-regime models allow for a richer description of the business cycle than two-regime models, we adopt the following taxonomy of phases:

$$s_t = \begin{cases} 1, & \text{low growth} \\ 2, & \text{medium growth} \\ 3, & \text{high growth} \end{cases}.$$

A natural alternative to the switching-mean specification is the switching-intercept (MSI) model:

$$y_t = \alpha_{s_t} + \sum_{j=1}^p \phi_j y_{t-j} + \varepsilon_t, \quad (9)$$

where the innovation process may be state-dependent. While similar, (8) and (9) differ on how the process evolves. Under general conditions, the two processes generate the same limiting value when the system is shocked. The propagation of the shocks over time, however, will differ. For (8), a shock causes a one-time jump whereas for (9) the process asymptotically approaches its limiting value.

Additional interesting generalizations of MS processes could also include regime-dependent autoregressive parameters, as discussed in Hansen (1992).

### 3.3 Types of Markov-Switching models

The assumption that the mixing weights are not *i.i.d.* processes, and that the intercept switches, rather than the mean, affects higher-order moments of the distribution of  $y_t$  and  $\Delta y_t$ , in particular the skewness and kurtosis (Krolzig 1997; Timmermann 2000; and Knüppel 2004). As a result, the form of asymmetry generated by the business cycle has important implications for selecting the form of MS model used.

A mean-switching model, which moves rapidly between the average growth-rates of the business-cycle phases, is more appropriate if the series rapidly enter and exit recessions. If, however, the series enter and exit recessions gradually, the intercept-switching model is more appropriate, since the realization of the process asymptotically approaches the mean of a given regime. For a discussion on the implied asymmetries of the two functional forms, see Knüppel (2004) and Clements and Krolzig (2003).

The nature of the asymmetry also has important implications for the number of regimes and the types of possible switching mechanisms. If the series is characterized as either deep or tall, then a two-regime MSM or MSI model will adequately capture all asymmetries resulting from the business cycle. If, on the other hand, the series exhibits steepness, then a third regime is required for the MSM models, whereas a two-state MSI model can admit steepness (Knüppel 2004).

## 4 Monte Carlo Experiment

In the previous sections, we discussed and examined business-cycle metrics and how certain models can accommodate these features. In this section, we report results of a Monte Carlo

experiment that evaluates the ability of selected DGPs to replicate the selected features of Canadian data, namely:

- i) Duration of phases ( $D^e, D^c$ );
- ii) Amplitude of phases ( $A^e, A^c$ );
- iii) Cumulative change in output ( $C^e, C^c$ );
- iv) Excess movement in phases ( $E^e, E^c$ );
- v) Coefficient of variation of duration of phases ( $CV_D^e, CV_D^c$ );
- vi) Coefficient of variation of amplitude of phases ( $CV_A^e, CV_A^c$ );
- vii) Number of recessions;
- viii) Frequency of negative growth rates; and
- ix) Mean (mean), Standard deviation (Std), skewness (Skew), and kurtosis (Kurt) on growth ( $y_t$ ) and on the change in growth ( $\Delta y_t$ ).

To conduct the experiment, the data are calibrated using the estimates from the models. In order to insure that the in-sample results of the different models are comparable, the starting date of  $y_t$  is trimmed by four quarters (the models analyzed have at most four lags). This leaves 173 observations for estimation, spanning from 1962Q2 to 2005Q2. The estimated parameters of each model are used to generate 1000 simulated level data, each of which spans 174 periods. The BBQ algorithm is applied to each series and the relevant metrics are calculated. The averages of each metric over the 1000 replications are reported. For each replication the initial value is set equal to the log of Canadian GDP in 1962Q1.

## 4.1 Design of data generating processes

### 4.1.1 Univariate models

EHP and Neftçi (1984) argue that a linear model with a drift to account for long run growth is sufficient to generate business cycles in GDP. While this is true, we note that in the general case where the process that generates  $y_t$  is an ARMA( $p, q$ ) function, say  $f(\cdot)$ , then  $y_t$  will be linear and Gaussian if all the arguments of  $f(\cdot)$  are linear and if its innovations are also Gaussians. This implies that linear structural and time-series models could be misspecified if

asymmetries or non-linearities exist. By definition, a linear model with Gaussian innovations will therefore fail to reproduce the skewness and kurtosis found in the pdf of  $\Delta y_t$ .

If a preferred linear model provides a good approximation to the series, then non-linearities are likely not a significant factor that needs to be addressed. To examine this question and to provide a benchmark that is comparable with the existing literature, AR( $p$ ) models are estimated for all combinations of  $p = 1, \dots, 4$ :

$$\phi(L)y_t = \alpha + \varepsilon_t, \quad (10)$$

where  $\phi(L)$  is the operator in the lag polynomial, with, for instance,  $\phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ ;  $\alpha$  is an intercept such that the mean,  $\mu$ , is defined as  $\alpha / (1 - \sum_{j=1}^p \phi_j)$ . The optimal lag is chosen using Akaike's information criterion (AIC). According to AIC, the optimal specification for (10) is a simple AR(1), for which we obtain the following estimates:

$$\begin{aligned} y_t &= 0.00605 + 0.3y_{t-1} + 0.00816\varepsilon_t \\ \varepsilon_t &\sim i.i.d. N(0, 1). \end{aligned} \quad (11)$$

Because it has been argued that long run growth is the only necessary component of simple linear models needed to generate a business cycle (Neftçi 1984), we also consider that GDP follows a random walk with drift (RWD):

$$\begin{aligned} y_t &= 0.008725 + 0.0086\varepsilon_t \\ \varepsilon_t &\sim i.i.d. N(0, 1). \end{aligned} \quad (12)$$

The differing business-cycle implications of the possible MS specifications, as well as model uncertainty arising from inconclusive signals from the pdf of  $y_t$  and  $\Delta y_t$ , leads us to consider a total of 6 different parameterization schemes to construct the MS DGP. Based on a preliminary analysis of the data, it appears that Canadian GDP growth over the last 44 years is a non-deep, positively steep series. Although this finding is not constant across all subdivisions of the data presented (during the 1962–82 and 1982–2004 subperiods it appears as though Canadian GDP growth may be respectively tall or deep as well as positively steep), the data are unambiguously positively steep. This pattern indicates that a two-regime model may not adequately capture the asymmetries of the data. Additionally, the kurtosis found in the pdf of  $\Delta y_t$  suggests that models that are heteroskedastic across regimes could be more appropriate.

To formally analyze the need for regime dependent variances and the need for a third regime, homoskedastic and heteroskedastic two- and three-regime MS-AR(1) models are

estimated. These models are specified with either an intercept- or a mean-switching mechanism.<sup>11</sup> The parameter estimates for these specifications are reported in Tables 4 and 5.

#### 4.1.2 Multivariate model

In their conclusion, EHP suggest that “The current generation of non-linear models has become extremely complicated, and one suspects that we have reached the limit concerning the ability of these models to generate realistic business cycles and that the introduction of multivariate models would be beneficial” (EHP, p. 661). We address this issue by simulating data from a multivariate model. The DGP is a simple vector autoregression of order  $p$  (VAR( $p$ )) of the Canadian economy. This model is inspired by Duguay’s (1994) IS-curve framework: it is approximated by relating real Canadian output growth to the slope of the yield curve<sup>12</sup>, real Canada-U.S. exchange rate<sup>13</sup>, and real U.S. output growth. To simulate data from this model, we use estimated parameters from a  $N$ -variable VAR model with lag order 2. Using matrix notation, this multivariate model can be written as follows:

$$\Psi(L)\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{u}_t,$$

where  $\mathbf{y}_t$  collects the four variables that enter the model.  $\Psi(L)$  is a matrix polynomial in the lag operator, such that  $\Psi(L) = 1 - \Psi_1 L - \dots - \Psi_p L^p$ , with  $p$  associated coefficient matrices  $\Psi$ . The  $N$ -dimensional vector  $\mathbf{u}_t$  collects the innovations and has variance-covariance matrix  $\Gamma = E[\mathbf{u}_t \mathbf{u}_t']$ ; all other correlations between period  $t$  and  $t - s$  are zero. Finally,  $\boldsymbol{\mu}$  is a vector of intercepts. One thousand replications of pseudo-artificial data are thus constructed using estimates of  $\Psi(L)$ ,  $\boldsymbol{\mu}$ , and  $\Gamma$ .

#### 4.2 Monte-Carlo evaluation results

Tables 6 and 7 report the calculated business-cycle metrics of our proposed DGPs. Items in bold are metrics for which the metrics calculated using actual data is less than the 10th percentile or more than the 90th percentile of the metrics calculated using simulated data. The RWD model generates business cycles with expansion durations compatible with the data, but the length of the recessions appear to be significantly too short with an average duration of only 2.5 quarters. The amplitude of expansions from the RWD model is on average a 35 per cent increase, which is similar to the 40 per cent gain seen in the data. The

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<sup>11</sup>Results for three-regime MSM models are not presented since we could not obtain economically sensible estimation results.

<sup>12</sup>Defined as the yield on the 90 day commercial paper minus the yield on the 10 year government bond.

<sup>13</sup>The Canada-U.S. exchange rate is the noon-spot rate. Implicit price indices of chained real GDP were used to construct the real exchange rate.

average amplitude of the recessions, however, is not large enough and the RWD model fails to generate average excess growth similar to the data. The average number of recessions from the RWD model, at 3.9, is slightly above the actual, at 3.0, but it is nevertheless compatible with the actual data since it falls within the 10-to-90 per cent confidence interval. Unlike EHP, we find that the RWD model is able to replicate the variability ( $CV$ ) of the cycle reasonably well. On average, over 16 per cent of the simulated growth rates are negative, whereas they represent only about 13 per cent of the actual data, but this falls within the 10-to-90 per cent confidence interval. As expected, the RWD model is unable to match the higher order moments of  $\Delta y_t$ . Over all, our Monte Carlo results are in line with EHP: the RWD model can entertain a limited number of business-cycle metrics, but it fails significantly in a few aspects. First, it cannot, as expected, generate higher-order moments compatible with Canadian data. Second, the duration of recessions is too short. Third, the amplitude and the excess metrics over the contraction phases are not consistent with actual data.

For the AR(1) model, the results are very similar to the RWD model, with the notable exception that this model has an excess growth during expansions compatible with the data. While the AR(1) model creates an asymmetric pattern of expansions and contractions—because of the estimated drift term,  $\alpha$ , and serial correlation—, the duration of the simulated contractionary phases does not match the data. The expansionary phase has a duration that is markedly quite below the actual data (28 vs 40.25), but it nevertheless matches the data since the calculated metric is at the boundary of the 10-to-90 per cent confidence interval. The AR(1) model generates an average of 5.5 recessions, but again, the 10-to-90 per cent confidence interval encompasses the actual average duration of recessions.

The VAR(2) generates a number of business cycle metrics that are similar to the AR(1) and the RWD estimates, but performs worse than them in a number of areas. The expansion has a duration of 29.3, quite below the actual expansion length, but it nevertheless matches the actual data at the boundary of the 10-to-90 per cent confidence interval. The VAR(2), however, is less capable than the AR(1) to match the excess seen in the actual data, and, in fact generates average estimates of excess that have the opposite sign of the actual excess. As with the AR(1) and RWD models, the VAR(2) generates too many recessions and has too high a percentage of quarters with negative growth. Lastly, and again similar to the AR(1)'s DGP, the VAR is unable to match the higher order moments of  $\Delta y_t$ —again, this is to be expected given that the innovations are assumed to be Gaussians and that all the arguments of the pdf are linear. Overall, there is little improvement from moving to a richer, multivariate, linear model.

Table 7 reports the calculated business-cycle metrics of our nonlinear univariate DGPs.

The two-regime intercept-switching models provide an improvement by matching a few key business-cycle metrics such as phase duration and amplitude. The two-regime MSI and MSM models generate an asymmetric cycle for which the simulated averaged durations and amplitudes of both the expansion and contraction phases are compatible with the actual average. On the other hand, the three-regime MSI model generates expansion duration and amplitude compatible with actual duration and amplitude, but not for the contraction phase. Among all of our suggested DGPs, the MSIH2 perform best at reproducing the actual duration and amplitude of the Canadian data. Our MSI3 models are the only DGPs that are able to generate an amount of kurtosis that is compatible with the observed  $\Delta y_t$ , whereas none of DGPs can replicate the skewness that we see in this series.

Of the metrics reported in this paper, the most relevant for monetary policy are probably the variance of output, the duration of phases, the frequency of switching phase, depth of recessions, or the measures of kurtosis in  $y_t$  and  $\Delta y_t$ . These metrics examine the ability of a model to replicate certain important features of the data. They contribute, for instance, to minimizing policy makers' uncertainty regarding the true DGP. They also allow to improve our understanding about the recessions and expansions process; for instance: the frequency of recessions and the severity, or the strength of expansions. Finally, they help us understand movements between business cycle phases.

Using the variance of output growth as the decision criterion, there is no gain by using a multivariate or nonlinear modelling strategy.<sup>14</sup> In fact, both the RWD and the AR( $p$ ) are able to closely match this metric. Among the linear models, only the AR(1) is able to match business cycle phase durations and output variability. None of the linear models match the kurtosis measures, which suggest that they are unable to generate **extreme** events that are comparable to the actual data.

The only nonlinear model that provides a noticeable improvement over the linear models is the two-regime, mean-switching Markov-Switching model. It does as well, if not better on average, than the linear models at matching phase length duration and output variability, and provides a good average estimate for the kurtosis of  $y_t$ .

## 5 Conclusion

In this paper, we have discussed several important business-cycle concepts. We have investigated some properties of the Canadian business cycle and analyzed how various econometric specifications can accommodate business-cycle metrics and higher-order moments. In a

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<sup>14</sup>This is true for estimated models, but in the case where some parameters are calibrated, the resulting variance of output in a simulation exercise could turn out larger (smaller) than the actual data.

Monte Carlo exercise, we have compared the ability of a number of models (linear, nonlinear, univariate, and multivariate) to replicate the properties of actual Canadian data.

From this simulation exercise, a couple key findings emerge. First, from the nine data generating processes that we have designed, not a single one can completely accommodate our diversified set of metrics. Second, richness and complexity does not guarantee a close match with the actual data: even basic considerations such as the number of recessions or the frequency of observing negative growth rates can be markedly different from what has been observed in Canada since the 1960s.

With respect to the question of whether nonlinear or complex multivariate models can improve over simple models—e.g.,  $AR(p)$ —, we find that, while none of the models dominates over all the metrics, a two-regime Markov-Switching model, which lets the intercept and the variance switch, appears to match the data best among our nine models. In this respect, our findings for Canada are in line with Morley and Piger’s (2006) study of the United States data and confirms their contradictions with Engel, Haugh, and Pagan (2005): nonlinear models do provide an improvement in matching business-cycle features.

Our findings suggest that it would be worthwhile in future research to investigate the merits of forecast combination. A diversified portfolio of models could most probably reduce the overall uncertainty that surrounds the structure of the Canadian economy and how its statistical dynamics operate. As we have shown, each model tends to offer local advantages in capturing business-cycle features of interest while none can globally address all features of the data.

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**Table 1: Canadian Business-Cycle Dates**

Peak		Trough	
ECRI	BBQ	ECRI	BBQ
—	1980Q1	—	1980Q3
1981 Q1	1981Q2	1982Q4	1982Q4
1990 Q1	1990Q1	1992Q1	1991Q1

**Table 2: Business Cycle Metrics for Canadian GDP (1961Q1–2005Q2)**

Metric	
Duration (in quarters)	
expansion	4.0
recession	40.25
Amplitude	
expansion	-0.03
recession	0.4
Cumulative	
expansion	-0.08
recession	12.32
Excess	
expansion	1.07
recession	9.39
Coef. of variation of duration	
expansion	0.5
recession	0.76
Coef. of variation of amplitude	
expansion	-0.84
recession	0.81
Occurrence of negative growth	13.3
Number of recessions	3
Std. dev. of output growth	3.42
Skew. of output growth	0.15
Kurt. of output growth	3.33
Skew. of the change in growth	0.59
Kurt. of the change in growth	4.48

Notes: All metrics are defined on p. 13. Bold items denote metrics for which the value calculated using actual data is less (more) than the 10th (90th) percentile.

**Table 3: Period Specific Summary Statistics**

Statistics	Median	Mean( $y$ )	Std( $y$ )	Skew( $y$ )	Kurt( $y$ )
<u>Sample: 1962Q2–2005Q2</u>					
$y_t$	3.47	3.46	11.71	0.15	3.33
$\Delta y_t$	-0.02	-0.03	16.53	0.58	4.43
<u>Sample: 1962Q2–1982Q2</u>					
$y_t$	3.66	4.05	15.21	0.27	2.66
$\Delta y_t$	-0.33	-0.16	25.91	0.53	3.25
<u>Sample: 1982Q3–2005Q2</u>					
$y_t$	3.39	2.94	8.2	-0.58	3.35
$\Delta y_t$	0.01	0.07	8.4	0.74	5.67
<u>Sample: 1994Q4–2005Q2</u>					
$y_t$	3.47	3.19	3.86	-0.35	2.37
$\Delta y_t$	0.09	-0.06	3.84	-0.02	2.76

Notes:  $y_t$  denotes the growth rate of real GDP, and  $\Delta y_t$  denotes its change.

**Table 4: Parameter Estimates for 2-Regime  
Markov-Switching Models**

Parameter	MSI	MSIH	MSM	MSMH
$p_{1,1}$	0.98	0.99	0.98	0.99
$p_{2,2}$	0.81	0.76	0.77	0.81
$\alpha_1$	3.38	3.23	3.91	3.68
$\alpha_2$	-1.78	-3.04	-2.14	-3.29
$\phi_1$	0.14	0.16	0.14	0.22
$\sigma_1$	2.98	3.03	2.97	3.13
$\sigma_2$	2.98	0.97	2.97	0.71

Notes: 1 refers to the expansionary state, and 2 to the contractionary state. The  $p$ 's are the transition probabilities defined in p. 11.

**Table 5: Parameter Estimates for 3-Regime  
Markov-Switching Intercept Models**

Parameter	MSI	MSIH
$p_{1,1}$	0.92	0.96
$p_{1,2}$	0.04	0.03
$p_{2,1}$	0.69	0.0
$p_{2,2}$	0.18	0.96
$p_{3,2}$	0.36	0.07
$p_{3,3}$	0.54	0.66
$\alpha_1$	2.46	3.39
$\alpha_2$	9.2	2.27
$\alpha_3$	-2.3	-2.73
$\phi_1$	0.26	0.28
$\sigma_1$	2.09	3.86
$\sigma_2$	2.09	1.66
$\sigma_3$	2.09	0.97

Notes: 1 refers to the high-growth state, 2 to the medium-growth state, and 3 to the low-growth state. The  $p$ 's are the transition probabilities defined in p. 11.

**Table 6: Business-Cycle Metrics for Linear Univariate and Multivariate Models**

Metric / Model	Actual	RWD	AR(1)	VAR(2)
Duration (in quarters)				
expansion	4.0	<b>2.47</b>	2.8	2.96
recession	40.25	37.68	28.0	<b>29.3</b>
Amplitude				
expansion	-0.03	<b>-0.01</b>	<b>-0.01</b>	-0.01
recession	0.4	0.35	0.03	<b>0.29</b>
Cumulative				
expansion	-0.08	<b>-0.02</b>	<b>-0.03</b>	<b>-0.03</b>
recession	12.32	11.39	7.1	<b>7.45</b>
Excess				
expansion	1.07	0.16	0.94	-0.55
recession	9.39	<b>0.37</b>	<b>0.04</b>	<b>-0.18</b>
Coef. of variation of duration				
expansion	0.5	0.23	0.35	0.37
recession	0.76	0.8	0.82	0.78
Coef. of variation of amplitude				
expansion	-0.84	-0.57	-0.64	-0.85
recession	0.81	0.8	0.85	0.8
Occurrence of negative growth	13.3	16.2	<b>16.7</b>	16.3
Number of recessions	3	3.9	<b>5.5</b>	5.2
Std. dev. of output growth	3.42	3.46	3.43	3.5
Skew. of output growth	0.15	0.0	0.01	0.0
Kurt. of output growth	3.33	2.97	2.95	2.96
Skew. of the change in growth	0.59	<b>0.0</b>	<b>0.01</b>	<b>0.0</b>
Kurt. of the change in growth	4.48	<b>2.97</b>	<b>2.97</b>	<b>2.95</b>

Notes: All metrics are defined on p. 13. Bold items denote metrics for which the value calculated using actual data is less (more) than the 10th (90th) percentile.

**Table 7: Business-Cycle Metrics for Markov-Switching Models**

Metric / Model	Actual	MSI2	MSIH2	MSI3	MSIH3	MSM2	MSMH2
Duration (in quarters)							
expansion	4.0	4.13	3.48	<b>8.49</b>	<b>16.96</b>	4.39	5.09
recession	40.25	33.97	37.83	<b>11.01</b>	26.66	41.67	46.74
Amplitude							
expansion	-0.03	-0.2	-0.02	<b>-0.06</b>	<b>-0.15</b>	-0.03	-0.05
recession	0.4	0.34	0.38	0.11	0.27	0.47	0.55
Cumulative							
expansion	-0.08	<b>-0.09</b>	<b>-0.08</b>	-0.45	-3.04	<b>-0.12</b>	<b>-0.26</b>
recession	12.32	10.24	12.35	<b>1.22</b>	<b>5.9</b>	16.53	20.96
Excess							
expansion	1.07	-0.01	-1.75	-4.37	-2.28	-0.29	-0.34
recession	9.39	<b>-0.31</b>	<b>-0.68</b>	5.08	<b>-4.16</b>	<b>0.34</b>	<b>0.33</b>
Coef. of variation of duration							
expansion	0.5	0.55	0.46	<b>0.76</b>	<b>0.09</b>	0.5	0.55
recession	0.76	0.84	0.8	<b>0.95</b>	0.76	0.81	0.79
Coef. of variation of amplitude							
expansion	-0.84	-0.82	-0.83	-0.83	-1.05	-0.71	-0.83
recession	0.81	0.87	0.82	0.89	0.76	0.83	0.79
Occurrence of negative growth	13.3	16.3	14.8	47.0	40.5	13.5	13.8
Number of recessions	3	4.3	3.8	<b>8.8</b>	3.9	3.3	2.8
Std. dev. of output growth	3.42	3.44	3.32	4.63	<b>4.32</b>	3.59	3.78
Skew. of output growth	0.15	<b>-0.26</b>	<b>-0.12</b>	<b>0.55</b>	<b>0.34</b>	<b>-0.43</b>	<b>-0.35</b>
Kurt. of output growth	3.33	3.15	2.86	2.94	2.65	3.47	2.98
Skew. of the change in growth	0.59	<b>0.0</b>	<b>0.01</b>	<b>0.18</b>	<b>0.0</b>	<b>0.0</b>	<b>0.01</b>
Kurt. of the change in growth	4.48	<b>3.01</b>	<b>3.02</b>	<b>4.17</b>	5.26	<b>3.14</b>	<b>3.22</b>

Notes: All metrics are defined on p. 13. Bold items denote metrics for which the value calculated using actual data is less (more) than the 10th (90th) percentile.

Figure 1: **Logarithm of Real Canadian GDP (top), Growth (middle), and Change in Growth (bottom)**

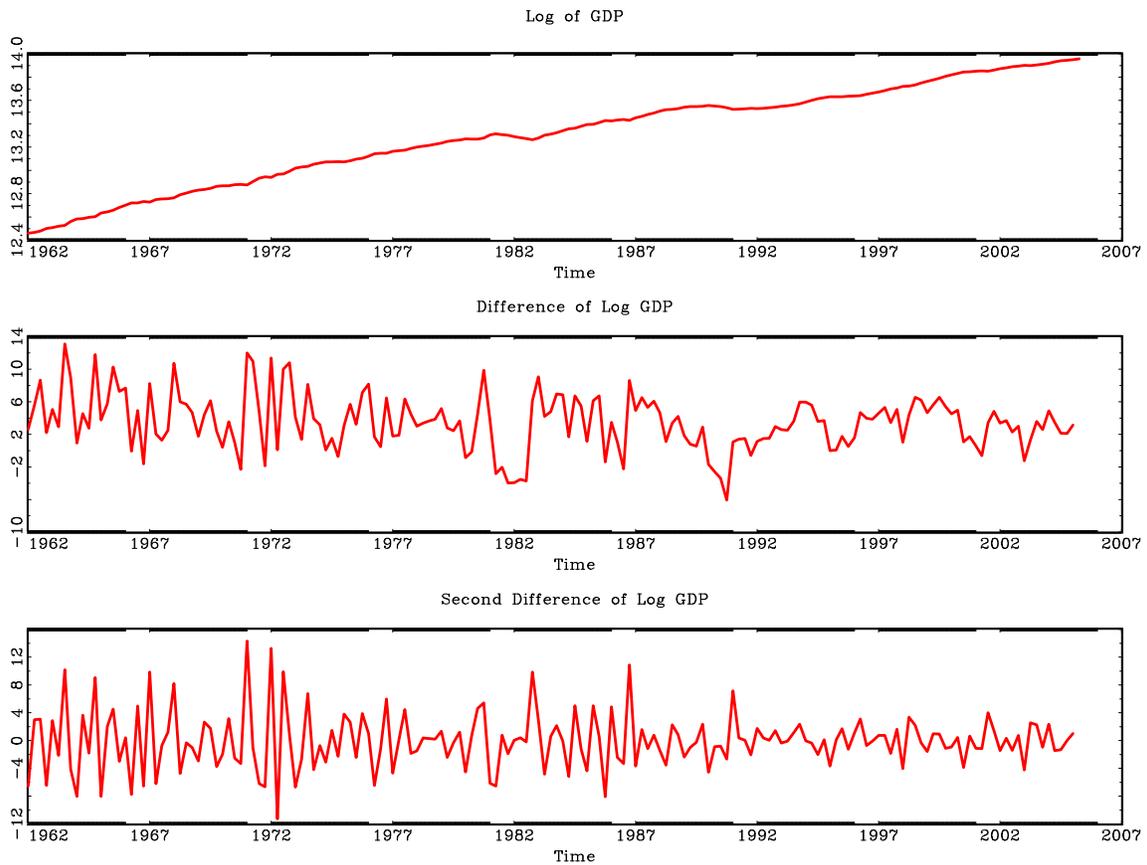


Figure 2: Gain Curve of Expansions

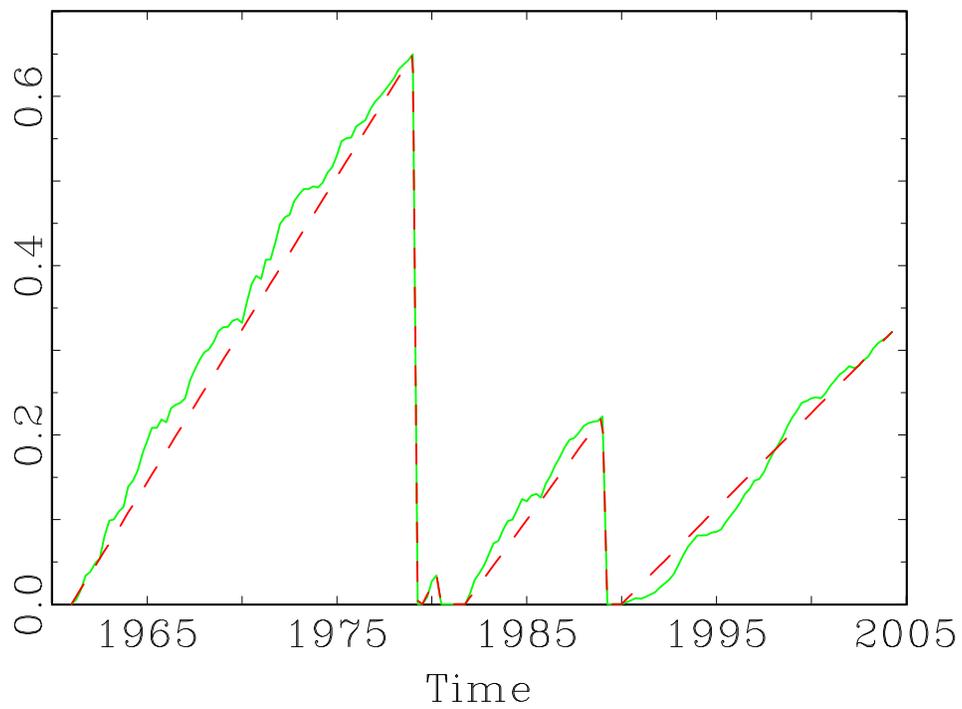


Figure 3: **Conditional Densities: Expansion vs. Contraction**

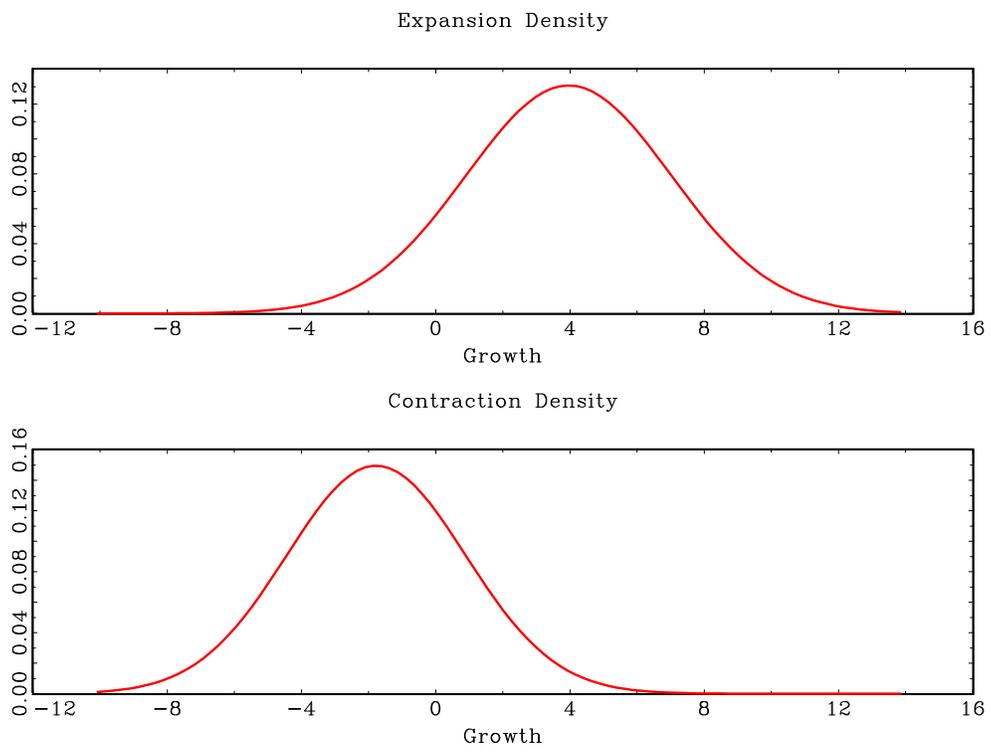


Figure 4: **Mixture of Densities** (Notes: the solid line is the unconditional density of Canadian GDP growth while long and short dashed lines are probability weighted conditional expansion and contraction densities, respectively. The mixing weight,  $\pi$ , is described in Section 3.1.)

