



BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2008-49

Financial Intermediation, Liquidity and Inflation

by Jonathan Chiu and Cesaire Meh

Bank of Canada Working Paper 2008-49

December 2008

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Acknowledgements

We have benefited from the comments and suggestions from Gabriele Camera, James Chapman, Ling Sun, Miguel Molico, Chris Waller, Randy Wright, and from seminar participants at the Bank of Canada, the Canadian Economic Association Meeting, the Conference on Computing in Economics and Finance in Paris, the Far Eastern Econometric Society Meeting, the Money, Banking, Payments and Finance Conference at the Federal Reserve Bank of Cleveland, the Philadelphia Workshop on Monetary and Macro Economics, and the Workshop on Money and Payments at the Federal Reserve Bank of New York.

Abstract

This paper develops a search-theoretic model to study the interaction between banking and monetary policy and how this interaction affects the allocation and welfare. Regarding how banking affects the welfare costs of inflation: First, we find that, with banking, inflation generates smaller welfare costs. Second, we show that, lowering inflation improves welfare not just by reducing consumption/production distortions, but also by avoiding intermediation costs. Therefore, understanding the nature of intermediation cost is critical for accurately assessing the welfare gain of lowering the inflation target. Regarding how monetary policy affects the welfare effects of banking: First, banking always improves efficiency of production, but the banking technology has to be efficient to improve welfare (especially in low inflation economy). Second, welfare effects of banking depend on monetary policy. For low inflation, banking is not active. For high inflation, banking is active and improves welfare. For moderate inflation, banking is active but reduces welfare. Owing to general equilibrium feedback, banking is supported in equilibrium even though welfare is higher without banking.

JEL classification: E40, E50

Bank classification: Monetary policy framework

Résumé

Les auteurs élaborent un modèle théorique de recherche afin d'étudier l'interaction entre l'activité bancaire et la politique monétaire, tout comme l'incidence de cette interaction sur l'allocation des ressources et le bien-être. L'activité bancaire est un facteur d'atténuation : c'est la première conclusion que les auteurs tirent de l'analyse du rôle de l'activité bancaire au regard des coûts de l'inflation sur le plan du bien-être. Ils montrent en second lieu qu'abaisser l'inflation accentue le bien-être, non seulement parce que cette action réduit les distorsions de la consommation et de la production, mais encore parce qu'elle permet d'éviter les coûts d'intermédiation. Saisir, par conséquent, la nature du coût d'intermédiation est capital si l'on veut évaluer au plus près le gain de bien-être attendu d'une cible d'inflation moins élevée. S'agissant de la façon dont la politique monétaire modifie les effets de l'activité bancaire sur le bien-être, les auteurs font deux constats. D'abord, si l'intermédiation bancaire améliore toujours l'efficacité productive, le bien-être n'augmente cependant qu'à condition que les banques disposent d'une technologie efficiente (surtout au sein d'une économie à basse inflation). Ensuite, les effets de l'activité bancaire sur le bien-être dépendent de la politique monétaire. Ainsi, l'intermédiation est limitée en régime de basse inflation mais s'accroît et favorise le bien-être quand l'inflation est forte; à un taux d'inflation modéré, elle est dynamique mais diminue le bien-être. L'enchaînement d'effets, révélé

à l'équilibre général, explique que l'activité bancaire soit soutenue à l'équilibre même si, sans elle, le bien-être progresserait.

Classification JEL : E40, E50

Classification de la Banque : Cadre de la politique monétaire

1 Introduction

Central banks care about the welfare effects of changing the inflation target. Existing monetary theories generally agree that inflation increases the opportunity cost of holding liquidity and thus distorts the allocation of resources which require liquidity in transaction. Many existing studies, however, abstract from financial intermediation, which plays an important role in the allocation of liquidity in modern economies. Therefore, these existing studies are not appropriate for investigating the interaction between financial intermediation and monetary policy and how this interaction affects the allocation and welfare.

To study these questions, we developed a micro-founded monetary model which endogenizes the roles of liquidity and financial intermediation. In particular, we explicitly model the micro-economic frictions that generate the roles of liquid assets (e.g. *money*) and financial intermediaries (e.g. *banking*) in facilitating the decentralized trading of production inputs. We use the model to study the relationship between money, banking, and social welfare: i) How banking affects the welfare costs of inflation, ii) How monetary policy affects the welfare effects of banking.

In this paper, we model the roles of competitive banks in channeling liquidity among entrepreneurs in the decentralized trading of production projects (denoted as “ideas”). As in Berentsen, Camera and Waller (2006), banks possess a record-keeping technology to keep track of financial records of borrowers and lenders and take deposits and make loans at a competitive interest rate. A key feature is that borrowing from banks may incur a fixed intermediation cost.

Let us summarize the main findings of the paper:

1. How does banking affect the welfare costs of inflation? First, we find that, with banking, inflation generates smaller welfare costs. Second, we show that, reducing inflation improves welfare not just by lowering the consumption/production distortions, but also by avoiding intermediation cost. Therefore, understanding the nature of intermediation cost is critical for accurately assessing the welfare gain of lowering the inflation

target.

2. How does monetary policy affect the welfare effects of banking? First, banking always improves efficiency of production, but the banking technology has to be efficient to improve welfare (especially in low inflation economy). Second, welfare effects of banking depend on monetary policy. For low inflation, banking is not active. For high inflation, banking is active and improves welfare. For moderate inflation, banking is active but reduces welfare. Owing to general equilibrium feedback, banking is supported in equilibrium even though welfare is higher without banking.

Let us briefly describe our model and give the basic intuition of our main findings. In this paper, banking is introduced to facilitate decentralized trading of production projects which are essentially intermediate inputs for production. In particular, we build on the setup in Silveira and Wright (2007) to study the roles of banking in the market for production projects (“ideas”) which are used as an input for production. Owing to the anonymity in the decentralized market for ideas, entrepreneurs need to bring liquidity (e.g., money) to this market to purchase ideas. Since innovators (i.e., sellers of ideas) have different random reservation prices with respect to their ideas, some entrepreneurs may end up with too much liquidity while others may end up with too little liquidity. The degree of this liquidity constraint depends on the real value of money, which in turn depends on the inflation rate. Inflation reduces the real value of money, and thus makes the liquidity constraint more binding. This problem can be resolved by having a financial intermediary (banks) which channels the funds from entrepreneurs with excess liquidity to those lacking liquidity. The welfare cost of inflation is lower when the banking sector is better developed because, by paying interest to deposit money, banking can reduce the opportunity cost of holding liquidity (i.e. inflation tax).

However, the use of banking involves fixed intermediation costs, in particular in enforcing the repayment from the borrowers. Naturally, costly banking is used when inflation is relatively high (liquidity problem is severe) and is not used when inflation is relatively low

(liquidity problem is mild). Therefore, in an economy with low inflation, the banking sector has to be more efficient (lower intermediation cost) for it to be welfare-improving.

An interesting finding is that, in an economy with moderate inflation, banking is used even though it is welfare-reducing. The intuition is that, when an entrepreneur chooses to borrow from a financial intermediary, he considers only his own net *private* gain from borrowing, ignoring the general equilibrium effect. However, borrowing will also lower his demand for money in the money market, and thus reduce the equilibrium value of money. A lower value of money is going to tighten other entrepreneurs' liquidity constraints, pushing more entrepreneurs to (costly) borrow. This will lead to welfare loss to society.

Apparently, by studying the market for ideas, our paper is closely related to Silveira and Wright (2007). Note that while we choose to study banking in the market for ideas, we do expect that the main findings of our paper can be generalized and applied to other decentralized trading. The way banking is modeled in this paper is related to Berentsen, Camera and Waller (2006). There are two key differences. First, Berentsen, Camera and Waller study environment in which enforcement of repayment by borrowers is either costless or infinitely costly. In our paper, there is perfect enforcement but is subject to a finite fixed cost. Second, the fractions of borrowers and lenders are fixed in their paper, but in our environment, it is endogenous and depends on the monetary policy. These differences generate some interesting new implications in our model. Another related paper is Bencivenga and Camera (2007) who also study the relationship between inflation and costly banking. We focus on the inefficiency of banking due to the competitive nature of the banking sector. This type of inefficiency is ruled out in their paper because a bank is modeled as an optimal contract among a coalition of agents. He, Huang, and Wright (2005) also study banking in the Lagos and Wright (2005) environment, but they focus on the safekeeping function of banking. Other related micro-founded models of money and banking include Andolfatto and Nosal (2003) and Sun (2007).

The road map is as follows. Section 2 describes the basic setup of the model. Section 3 considers an economy without banking. Section 4 and 5 then discuss economies with costless

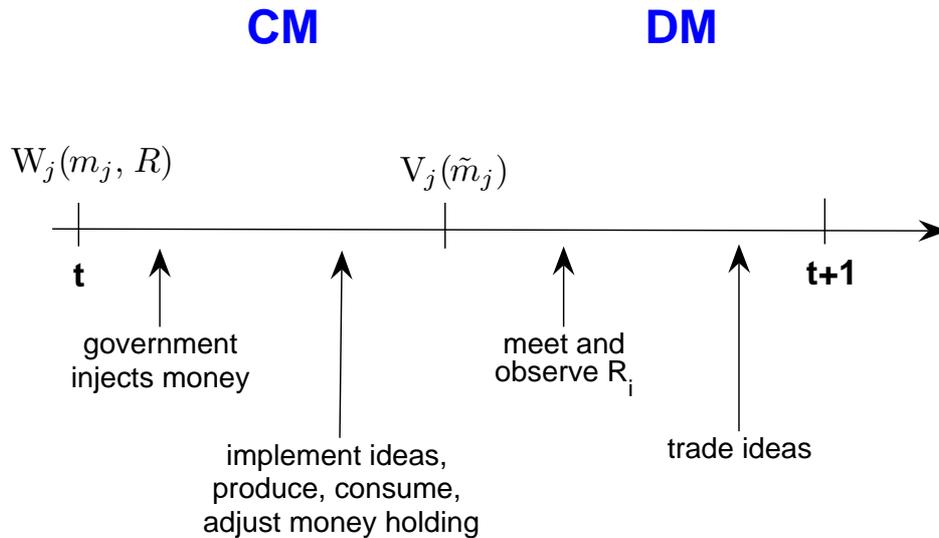


Figure 1: Timeline (No Banking)

and costly banking respectively. Section 6 considers various extensions. Section 7 concludes the paper.

2 Environment

Our paper builds on the framework of the market for ideas developed by Lagos and Wright (2005) and Silveira and Wright (2007) (SW) to study the roles of money and banking. Time is discrete and denoted $t = 0, 1, 2, \dots$. In this economy, there are two types of infinitely lived agents: measure one of innovators (who are good at coming up with ideas), and measure one of entrepreneurs (who are better at implementing ideas). There are two markets: centralized market, denoted CM, and a decentralized market, denoted DM. In this economy, there is an additional, perfectly divisible, and costlessly storable object which cannot be produced or consumed by any private individual, called *fiat money*.

The sequence of events is illustrated in Figure 1. Each period is divided into two sub-periods. In the first sub-period, agents implement ideas, perform production and consump-

tion and money holding adjustment in the CM. In the second sub-period, agents meet bilaterally in the DM and trade ideas which are implemented in the next CM. When the DM opens, each innovator comes up with a new idea that can be implemented in the following CM. By implementation, we mean that the idea will be used as an input in the production of the consumption good. The input value of an idea depends on who the implementor is. Entrepreneurs are good at implementing ideas. If an idea is implemented by an entrepreneur, it has an input value $I_e = 1$ and generates a return $R_e = \mathcal{R}(I_e)$. Innovators are not good at implementation and cannot realize the full values of ideas. If an idea is implemented by an innovator, it has a lower input value, $I_i \leq I_e = 1$, and thus yields a lower return $R_i = \mathcal{R}(I_i) \leq R_e$. Here, we assume that I_i is an i.i.d. random variable with a uniform (0,1) distribution, and its value is known when one enters the DM. If an innovator meets an entrepreneur in the DM, the former has an idea which can generate a return R_i for him, and can generate a return R_e for the latter.¹ When they meet, both entrepreneurs and innovators observe (R_i, R_e) . Because of the lack of information on trading history and the lack of commitment of entrepreneurs, money is required for the trading of ideas. The price at which an idea is traded is in terms of money, by which we mean some liquid assets the entrepreneur has on hand. The discount factor between one DM and the next CM is β . For simplicity, we will first consider the case in which an idea is both indivisible and rivalry. We discuss more general cases in Section 6.

2.1 The CM

In the CM, agents implement ideas, produce, consume and adjust their money holding. We are going to consider a stationary environment. In a typical period, the utility of an agent is given by

$$U(X) - H,$$

¹Here, I_e being 1 is a normalization. Also, SW consider a more general case in which both innovators and entrepreneurs have random valuations.

where $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ denotes the utility of consuming $X \geq 0$ units of the consumption good, and $H \in \mathbb{R}_+$ denotes the labor effort on production. We assume that $U(\cdot)$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $U(0) = 0$, $U'(\bar{X}) = 1$ for some $\bar{X} > 0$. We now describe the implementation of ideas by individual $j = i, e$, where $j = i$ denotes an innovator and $j = e$ denotes an entrepreneur. Here, an idea is simply an intermediate input into production. The production technology of an individual j is

$$F(I_j, h),$$

where h is the employment of labor input, and $F : [0, 1] \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ denotes the production function. As a result, given real wage rate, w , in the CM, the return from implementing an idea with an input value I_j is given by

$$\mathcal{R}(I_j; w) = \max_h \left\{ F(I_j, h) - wh \right\}.$$

That is, the return is equal to the output net of the employment cost evaluated at the profit-maximizing level of employment. For simplicity, we first consider the case in which $F(I_j, h) = I_j + h$. Under this simplifying assumption, $R_j = \mathcal{R}(I_j) = I_j$ and $w = 1$.² Since $R_e = I_e \geq I_i = R_i$, it is efficient to have entrepreneurs implementing all the ideas. However, due to the trading frictions and the liquidity constraint in the decentralized market, ideas may not be allocated efficiently.

We now describe agents' money holding. Agents can hold any non-negative amount of money $\hat{m} \in \mathbb{R}_+$. The total money stock at the beginning of the CM is M . The gross growth rate is $\mu = \frac{M}{M_{-1}}$, where M_{-1} denotes the money stock in the previous period. Agents receive lump sum monetary transfers at the entrance of the CM. In what follows we express an agent's money holding as a fraction of the beginning of the period money supply: $\frac{\hat{m}}{M}$. Let m and \tilde{m} denote the normalized individual money holdings at the beginning of the CM and the DM respectively.

²We will discuss the general case in Section 6.

Let $\phi \in \mathbb{R}_+$ be the price of money balance in terms of the consumption good in the CM. We focus on stationary equilibrium in which the money growth rate, μ , is constant and the price of money is also constant over time. Let $W_j(m_j, R)$ be the value function for entrepreneurs ($j = e$) and innovators ($j = i$) entering the CM with m_j money holding and an idea with value R in hand. Then the budget constraint of agents in the CM is

$$R + H + \phi(m_j + \Delta m) \geq X + \phi\tilde{m}_j,$$

where \tilde{m}_j is money balance taken out of the CM, and Δm is the lump sum money transfer from the government. For $j = i, e$, the CM problem is

$$\begin{aligned} W_j(m_j, R) &= \max_{X, H, \tilde{m}_j \geq 0} U(X) - H + V_j(\tilde{m}_j) \\ \text{s.t. } X &= H + R + \phi(m_j - \tilde{m}_j + \Delta m) \end{aligned}$$

where $V_j(\tilde{m}_j)$ is the value function for entrepreneurs and innovators entering the DM with \tilde{m}_j , before meetings occur. From now on, we will assume that the utility function U is such that $H > 0$ even for the richest agents, so that we can focus on interior solution. Under this assumption, the budget constraint can be used to eliminate H in the objective function, simplifying the W_j to

$$\begin{aligned} W_j(m_j, R) &= \phi m_j + \phi \Delta m + R + \max_X \{U(X) - X\} + \max_{\tilde{m}_j} \{-\phi \tilde{m}_j + V_j(\tilde{m}_j)\} \quad (1) \\ &= W_j(0, 0) + \phi m_j + R, \end{aligned}$$

where $W_j(0, 0) = \phi \Delta m + U(\bar{X}) - \bar{X} + \max_{\tilde{m}_j} \{-\phi \tilde{m}_j + V_j(\tilde{m}_j)\}$. Therefore, $W_j(m_j, R)$ is linear in both m_j and R . We will use this result to derive the bargaining solution below.

2.2 The DM

When an entrepreneur and an innovator meet in the DM, the value of R_i is observed by both agents. Since $R_i \leq R_e = 1$, the entrepreneur can always implement the idea at least as good as the innovator. Efficiency requires that all ideas be implemented by the entrepreneurs. Owing to liquidity constraints in the market for ideas, this efficient allocation may not be supported. Let $p \in \mathbb{R}_+$ denote the money price they would agree if there were no issues of liquidity. The liquidity constraint requires that $\tilde{m}_e \geq p$. For simplicity, we assume that the price is determined by take-it-or-leave-it offers from the entrepreneur.³

3 Economy Without Banking

Innovator in DM

We first examine the case without banking. Consider an innovator bringing money holding \tilde{m}_i and idea R_i into the DM. If an innovator keeps her idea, her payoff is

$$\beta W_i\left(\frac{\tilde{m}_i}{\mu}, R_i\right) = \beta W_i(0, 0) + \phi \beta \frac{\tilde{m}_i}{\mu} + \beta R_i.$$

Here, the innovator does not spend her money balance and brings it forward to the next CM. The real value of this money balance (in terms of CM good) in the next CM is $\frac{\phi \tilde{m}_i}{\mu}$. Note that the next period money balance is re-scaled by the money growth rate because we normalize the money balance by the total stock of money. Also, we have made use of the result in (1) to evaluate the value in the next CM. If the innovator sells her idea at a price p , her payoff is

$$\beta W_i\left(\frac{\tilde{m}_i + p}{\mu}, 0\right) = \beta W_i(0, 0) + \phi \beta \frac{\tilde{m}_i + p}{\mu}.$$

Here, the innovator's real money balance in the next CM is increased by $\phi \frac{p}{\mu}$. Therefore,

³Silveira and Wright (2007) consider a more general case in which the price is determined by generalized Nash bargaining. Also, in their model, the innovator and the entrepreneur have an option to meet again in the next CM where the entrepreneur can raise more money. We abstract from these interesting extensions to focus on the effects of banking on the market for ideas in the simplest possible case.

the innovator has a reservation price $\bar{p}(R_i) = \frac{R_i\mu}{\phi}$ for an idea R_i . Apparently, the reservation price is increasing in R_i .⁴

Entrepreneur in DM

Consider an entrepreneur with money holding \tilde{m}_e meeting an innovator with idea R_i . The bargaining solution implies that, if $\tilde{m}_e \geq \bar{p}(R_i)$, then the entrepreneur can afford to buy the idea and get a payoff of $V_e^1(\tilde{m}_e, R_i)$ given by

$$V_e^1(\tilde{m}_e, R_i) = \beta W_e(0, 0) + \beta R_e + \beta \phi \frac{\tilde{m}_e - \bar{p}(R_i)}{\mu}. \quad (2)$$

Here, the entrepreneur obtains the idea and the real money balance in the next CM is reduced by $\phi \frac{p}{\mu}$. If $\tilde{m}_e < \bar{p}(R_i)$, the entrepreneur is liquidity constrained and cannot afford to purchase the idea and gets

$$V_e^0(\tilde{m}_e, R_i) = \beta W_e(0, 0) + \beta \phi \frac{\tilde{m}_e}{\mu}. \quad (3)$$

Whether the innovator trades or not, he gets $\beta(W_i(0, 0) + R_i) + \phi \beta \frac{\tilde{m}_i}{\mu}$: the innovator receives no trade surplus because she has no bargaining power.

Demand for money in CM

The value function of an innovator entering the DM is thus

$$\begin{aligned} V_i(\tilde{m}_i) &= \int_0^{R_e} \beta(W_i(0, 0) + R_i) dR_i + \phi \beta \frac{\tilde{m}_i}{\mu} \\ &= \beta W_i(0, 0) + \frac{\beta}{2} + \phi \beta \frac{\tilde{m}_i}{\mu} \end{aligned} \quad (4)$$

An innovator's optimal choice of money balance taken to the DM (i.e., \tilde{m}_i in (1)) is the

⁴For simplicity, we first consider the case in which lottery is not allowed. We will discuss the case where lottery is available in Section 6.

solution to $\max_{\tilde{m}_i} [-\phi\tilde{m}_i + V_i(\tilde{m}_i)]$ and is given by

$$\tilde{m}_i \begin{cases} = 0, & \text{if } \mu > \beta \\ \in [0, \infty), & \text{if } \mu = \beta \\ = +\infty, & \text{if } \mu < \beta \end{cases} \quad (5)$$

That is, an innovator chooses not to bring any money to the DM if the money growth rate is higher than β , indifferent between any amount of money if the money growth rate is equal to β , and to bring an infinite amount if the money growth rate is lower than β . We will focus on cases with $\mu \geq \beta$ and assume that when innovators are indifferent they choose $\tilde{m}_i = 0$. The intuition is that, since innovators do not spend money in the DM, they do not have incentives to bring any money to the DM if the opportunity cost is strictly positive (i.e., $\mu > \beta$). The value function of an entrepreneur entering the DM is

$$\begin{aligned} V_e(\tilde{m}_e) &= \int_0^{\tilde{m}_e \frac{\phi}{\mu}} V_e^1(\tilde{m}_e, R_i) dR_i + \int_{\tilde{m}_e \frac{\phi}{\mu}}^1 V_e^0(\tilde{m}_e, R_i) dR_i \\ &= \beta W_e(0, 0) + 2\beta\phi \frac{\tilde{m}_e}{\mu} - \frac{\beta(\phi\tilde{m}_e)^2}{2\mu^2} \end{aligned}$$

The two terms on the right hand side of the first equality capture the case when $\tilde{m}_e \geq \bar{p}(R_i)$ and $\tilde{m}_e \leq \bar{p}(R_i)$. The second equality is derived by using (2) and (3). An entrepreneur's optimal choice of money balance taken to the DM (i.e., \tilde{m}_e in (1)) is the solution to $\max_{\tilde{m}_e} [-\phi\tilde{m}_e + V_e(\tilde{m}_e)]$. This implies that, if $\tilde{m}_e > 0$, then

$$\tilde{m}_e = \frac{2\mu\beta - \mu^2}{\beta\phi}. \quad (6)$$

Equilibrium

The money market equilibrium in the CM requires

$$\tilde{m}_e + \tilde{m}_i = 1. \quad (7)$$

Denote the equilibrium price of money (with no banking) as ϕ^{NB} . Under the simplifying assumption that $\tilde{m}_i = 0$ for $\mu \geq \beta$, we define the equilibrium as follows.

Definition 1. *A stationary monetary equilibrium without banking is given by ϕ^{NB} satisfying (6) and (7) with $\phi^{NB} > 0$.*

Proposition 1. *(Existence of equilibrium without banking) For any $\mu \in [\beta, 2\beta]$, there exists a stationary monetary equilibrium without banking.*

If $\mu > \beta$, then $\tilde{m}_i = 0$ and $\tilde{m}_e = 1$. (6) then implies $\phi^{NB} = 2\mu - \mu^2/\beta$ which is non-negative for $\mu \leq 2\beta$.⁵ When $\mu \geq 2\beta$, money has no value and there is no monetary equilibrium (i.e., no ideas are traded).⁶ Let $\bar{R}_i^{NB} \in [0, 1]$ be the cut-off value of R_i such that an entrepreneur's liquidity constraint is just binding: $\tilde{m}_e = \bar{p}(R_i)$. In equilibrium, $\tilde{m}_e = 1$ and this cut-off is pinned down by the condition

$$\bar{R}_i^{NB} \mu = \phi^{NB}$$

The left-hand-side of the equation is the real reservation price of the marginal entrepreneur in terms of the *current* period consumption good while the right-hand side is the maximum real price an entrepreneur is able to pay (i.e., the real money balance $\phi^{NB}\tilde{m}_e = \phi^{NB}$).⁷ In Figure 2, the left-hand-side is represented by the upward sloping line and the right-hand-side is represented by the horizontal line.⁸ So the equilibrium amount of trade is given by:

⁵The upper bound being 2β is a result of the assumptions of $R_e = 1$ and $R_i \sim U(0, 1)$.

⁶Note that the non-existence of monetary equilibrium for high money growth rates is related to the assumptions that ideas are indivisible and that lotteries are not allowed. Please see Section 6.

⁷Since the nominal price (in terms of a fraction of the current money stock) is $p = \frac{R_i \mu}{\phi}$, the real price (in terms of the current period goods) is given by $p\phi = R_i \mu$.

⁸One may interpret the figure as a supply-demand diagram which determines the equilibrium given a "demand curve" (ϕ) and a "supply curve" ($R_i \mu$).

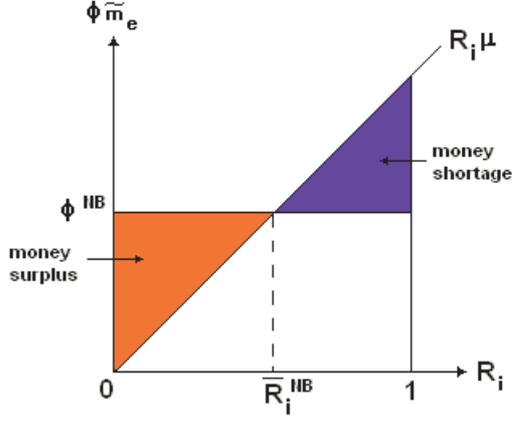


Figure 2: Potential gain from trading liquidity

$$\bar{R}_i^{NB} = \frac{\phi^{NB}}{\mu} = 2 - \frac{\mu}{\beta}$$

Note that there exists a unique stationary monetary equilibrium for $\mu \in [\beta, 2\beta]$ and that money growth always reduces trade. When $\mu = \beta$, the opportunity cost of holding money is zero. As a result, no entrepreneurs are liquidity constrained and all ideas are traded (i.e., $\bar{R}_i^{NB} = 1$).

Summarizing the findings in an economy without banking:

- At the Friedman's rule, all ideas are traded.
- When the inflation is moderate, a unique monetary equilibrium exists. Inflation reduces trades.
- When the inflation is high, there is no monetary equilibrium (in the absence of lotteries).

In Figure 2, an entrepreneur with $R_i \leq \bar{R}_i^{NB} = \frac{\phi^{NB}}{\mu}$ buys the idea at $p = R_i \mu$. After trade, these entrepreneurs still have money left over. The total real money surplus is $\frac{(\phi^{NB})^2}{2\mu}$. For the rest of the entrepreneurs, they are liquidity constrained and need extra funding to purchase the idea. The total money shortage is $(1 - \frac{\phi^{NB}}{\mu})^2 \mu / 2$. Therefore, there is a potential role for borrowing and lending between entrepreneurs whenever $\mu > \beta$.

4 Economy with Costless Banking

We now introduce intermediation into the economy and study the interaction between money and banking. Suppose in the DM there are competitive financial intermediaries (banks) taking deposits at an interest rate r^D and making loans at an interest rate r^L . As in Berentsen, Camera and Waller (2006), each bank has a record keeping technology allowing it to keep financial record of agents. Here, we assume that banks specialize in channeling funding across *entrepreneurs*.⁹ In particular, entrepreneurs can commit to repay the bank in the CM and banks can commit to repay depositors in the CM. Free entry implies zero profit for banks and thus $r^D = r^L = r$ for some $r \geq 0$. Figure 3 shows the sequence of events. In the DM, after meeting and observing the realization of R_i , an entrepreneur can choose to lend money to or borrow money from a bank before trading. In the next CM, deposits and loans will be repaid. In general, an entrepreneur meeting an innovator with low R_i has excess liquidity and would like to lend his surplus money holding to a bank after trade to earn interest income. An entrepreneur meeting an innovator with high R_i may find the surplus from trade smaller than the return from deposit and chooses instead to lend all his money holding to the bank. An entrepreneur with intermediate level of R_i is liquidity constrained and will choose to borrow from the bank to finance the trade. Figure 4 illustrates the flow of funds in the CM and DM. Anonymity of entrepreneurs in the market for ideas implies that money is still needed as a medium of exchange.

A competitive representative bank takes r^L and r^D as given, and chooses the amount of loans (l) and deposit (d) to maximize its profit (π):

$$\begin{aligned} \max_{l,d} \pi &= r^L l - r^D d, \\ \text{s.t.} \quad &d \geq l \end{aligned}$$

⁹Note that, in equilibrium, innovators do not have incentives to use banking even if they have access to banking. Also, we assume that banks cannot issue inside money in this economy. Equivalently, one can also allow banks to issue inside money but subject to an 100% reserve requirement.

Here, there is a feasibility constraint restricting that the amount of loans lent out has to be no more than the amount of deposits taken in. In equilibrium, it cannot be the case that $r^L > r^D$, otherwise banks will choose $l = d = +\infty$, implying $\pi = +\infty$. When $r^L < r^D$, banks choose $l = d = 0$ to earn $\pi = 0$. This cannot clear the loan market when entrepreneurs choose to save a positive amount. So whenever there is positive saving, we must have $r^L = r^D$. Banks' optimization problem then implies

$$\begin{cases} d = l & \text{if } r > 0 \\ d \geq l, & \text{if } r = 0 \end{cases} \quad (8)$$

In both cases, profits of the banks are zero.

Entrepreneur's decision in DM

In general, entrepreneurs may have to incur an additional fixed cost $\eta \geq 0$ to borrow from the bank. This section considers the case with costless banking ($\eta = 0$). We will consider the general case with costly banking ($\eta > 0$) in the next section.

After meeting in the DM, an entrepreneur with \tilde{m}_e and R_i chooses the amount of saving (lending if positive and borrowing if negative ($s \in \mathbb{R}$)), money brought to the next CM ($m_e \in \mathbb{R}_+$, as a fraction of *next* period money stock) and whether or not to buy the idea ($y \in \{0, 1\}$) to maximize the expected payoff:

$$\max_{s, y, m_e} \beta W_e(0, 0) + \beta y + \beta \phi (1 + r) \frac{s}{\mu} + \beta \phi m_e,$$

subject to

$$m_e \mu = \tilde{m}_e - y \frac{R_i \mu}{\phi} - s \geq 0.$$

The budget constraint says that the amount of money brought to next period is equal to the initial money holding minus the expenditure on purchasing idea and saving. We need to adjust the left-hand-side by the money growth rate because the two sides are normalized by money stocks in two different periods. Note that, by allowing borrowing (i.e. $s < 0$),

banking relaxes the liquidity constraint of entrepreneurs. Substituting this budget constraint into the objective function, we have

$$\max_{y, m_e} \beta W_e(0, 0) + \beta y R_e + \beta \frac{\phi}{\mu} (1+r)(\tilde{m}_e - \mu m_e - y \frac{R_i \mu}{\phi}) + \beta \phi m_e$$

Note that optimization implies $m_e = 0$ if $r > 0$ and $m_e \in \mathbb{R}_+$ if $r = 0$. Then, an entrepreneur's problem becomes

$$\beta W_e(0, 0) + \beta \frac{\phi}{\mu} (1+r)\tilde{m}_e + \beta \max\{R_e - (1+r)R_i, 0\}. \quad (9)$$

The last term captures an entrepreneur's comparison between the value of the idea ($R_e = 1$) and the opportunity cost (including interest) of buying the idea ($(1+r)R_i$). Therefore, the value function of an entrepreneur entering the DM is

$$V_e(\tilde{m}_e) = \int_0^1 \left[\beta W_e(0, 0) + \beta \frac{\phi}{\mu} (1+r)\tilde{m}_e + \beta \max\{R_e - (1+r)R_i, 0\} \right] dR_i$$

Denote the optimal saving of an entrepreneur with (m, R_i) by $s(\tilde{m}_e, R_i)$. (9) implies that the cut-off value $\bar{R}_i(r)$ that makes an entrepreneur indifferent between trading and no trading is given by

$$\bar{R}_i(r) = \frac{R_e}{1+r}$$

As a result, the value function of an entrepreneur entering the DM can be simplified to:

$$V_e(\tilde{m}_e) = \beta W_e(0, 0) + \beta \frac{\phi}{\mu} (1+r)\tilde{m}_e + \beta \int_0^{\frac{1}{1+r}} (R_e - (1+r)R_i) dR_i$$

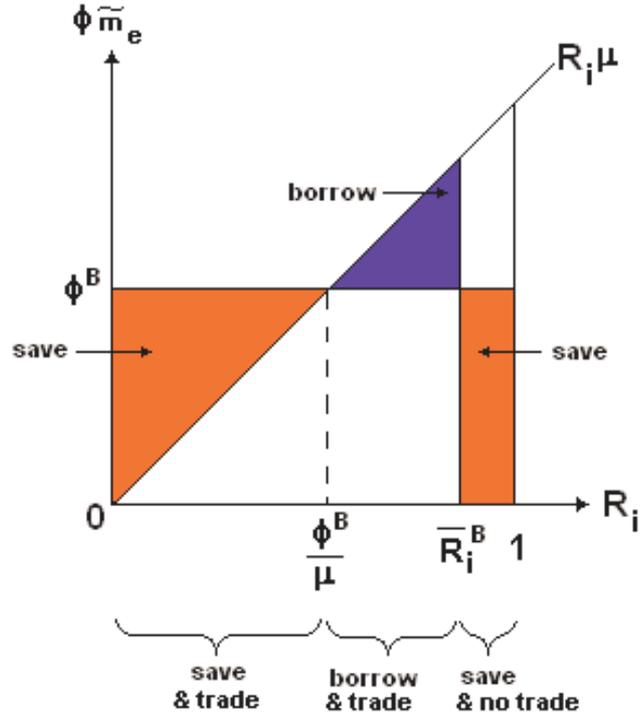


Figure 5: Saving and Borrowing

Therefore, $V'_e(\tilde{m}_e) = \beta \frac{\phi}{\mu} (1+r) > 0$ and thus the value function is linear. The market clearing condition in the CM requires that

$$\arg \max V_e(\tilde{m}_e) - \phi \tilde{m}_e.$$

So, in equilibrium, the optimal money demand is characterized by the first order condition of the above problem:

$$r = \frac{\mu}{\beta} - 1. \quad (10)$$

Basically, the use of banking relaxes entrepreneur's liquidity constraint in purchasing ideas in the DM. Therefore, when choosing the optimal amount of money brought to the DM, an entrepreneur simply looks at whether the real rate of return of money is higher

than the subjective discount rate (i.e., whether $\frac{\phi}{\mu}(1+r) - \frac{1}{\beta} > 0$) across two CM's. He will demand $\tilde{m}_e = 0$ when the real rate of return is lower than the subjective discount rate. He will demand $\tilde{m}_e = +\infty$ when the real rate of return higher than the subjective discount rate, and will demand any $\tilde{m}_e \in \mathbb{R}_+$ when the rate of return is equal to the subjective discount rate. To clear the money market in CM, the nominal interest rate, r , has to exactly compensate for the inflation and discounting.

Equilibrium

The cut-off value of idea is thus $\bar{R}_i^B = \bar{R}_i(r) = \frac{\beta}{\mu}$. Entrepreneurs' optimal choices of (y, s) as a function of R_i is illustrated by Figure 5¹⁰:

$$\begin{cases} y = 1, s = \tilde{m}_e - \frac{R_i\mu}{\phi} \geq 0 & , \text{ if } R_i \in [0, \frac{\phi}{\mu}] \\ y = 1, s = \tilde{m}_e - \frac{R_i\mu}{\phi} < 0 & , \text{ if } R_i \in (\frac{\phi}{\mu}, \bar{R}_i^B] \\ y = 0, s = \tilde{m}_e \geq 0 & , \text{ if } R_i \in (\bar{R}_i^B, 1] \end{cases} \quad (11)$$

As discussed earlier, the entrepreneurs with low and high R_i 's will save, and the entrepreneurs with medium R_i will borrow. Only the entrepreneurs with low and medium R_i 's will trade. The loan market clearing condition in the DM requires that the aggregate saving from the entrepreneurs is equal to the total deposit minus the total loans:

$$\int_0^1 s(\tilde{m}_e, R_i) dR_i = d - l.$$

Then condition (8) from the bank's optimization implies that

$$\begin{cases} \int_0^1 s(\tilde{m}_e, R_i) dR_i = 0 & , \text{ if } r > 0 \\ \int_0^1 s(\tilde{m}_e, R_i) dR_i \geq 0 & , \text{ if } r = 0 \end{cases}$$

Substituting in the saving functions from (11), we simplify the left-hand-side to $\tilde{m}_e - \frac{\beta^2}{2\mu\phi}$.

¹⁰Here we assume that, when entrepreneurs are indifferent between saving and not saving (which happens when $r = 0$), they choose to save.

Imposing the money market clearing condition in the CM (i.e., $\tilde{m}_e = 1$), we have

$$\begin{cases} \phi = \frac{\beta^2}{2\mu} & , \text{ if } r > 0 \\ \phi \geq \frac{\beta^2}{2\mu} & , \text{ if } r = 0 \end{cases} \quad (12)$$

Definition 2. A stationary monetary equilibrium with costless banking is a pair (ϕ^B, r) satisfying (10), (12) with $\phi^B > 0$, $r \geq 0$.

When $\mu > \beta$, there exists a unique stationary monetary equilibrium with costless banking where $\phi^B = \frac{\beta^2}{2\mu}$, $r = \frac{\mu}{\beta} - 1 > 0$, $\tilde{m}_e = 1$, $\tilde{m}_i = 0$ and $\bar{R}_i^B = \frac{\beta}{\mu}$. A fraction $\bar{R}^B - \frac{\phi^B}{\mu} = \frac{\beta}{\mu}(1 - \frac{\beta}{2\mu})$ of entrepreneurs are borrowers and the rest are lenders. Since the interest rate in the loan market is positive, the excess supply of loans is zero.

When $\mu = \beta$, we have multiple equilibria: any $\phi^B \in [\frac{\beta}{2}, \infty)$, $r = 0$, $\tilde{m}_e = 1$, $\tilde{m}_i = 0$ and $\bar{R}_i^B = 1$. Fraction $\max\{1 - \frac{\phi^B}{\mu}, 0\}$ of entrepreneurs are borrowers and the rest are lenders. All these equilibria are equivalent in terms of real allocations and payoffs. They differ only in terms of the real value of money and the borrowing-lending decision in the DM. At the lower bound where $\phi^B = \frac{\beta}{2}$, half of the set of entrepreneurs are liquidity constrained and need to borrow. The excess supply of loans is zero. As the value of money (ϕ^B) goes up, fewer entrepreneurs are liquidity constrained and there are fewer borrowers. There is excess supply of loans in the loan market, but it is consistent with the interest rate being zero. For $\phi^B \geq \beta$, no entrepreneurs are liquidity constrained and there are no borrowers. Again, there is excess supply of loans. Also, at the Friedman rule, a banking equilibrium with $\phi^B = \beta$ is identical to an equilibrium without banking.

Inflation, Banking and Welfare

Note that the measure of trade ($\frac{\beta}{\mu}$) is decreasing in inflation. Maximum amount of trade ($\bar{R}_i^B = 1$) is achieved when $\mu = \beta$. Measuring the welfare by the average utility of all agents, we have the welfare for $k = NB, B$ given by:

$$\begin{aligned}
\bar{W}^k &= 2(U(\bar{X}) - \bar{X}) + \int_0^{\bar{R}_i^k} R_e dR_i + \int_{\bar{R}_i^k}^1 R_i dR_i \\
&= 2(U(\bar{X}) - \bar{X}) + \bar{R}_i^k + \frac{1}{2} - \frac{(\bar{R}_i^k)^2}{2}
\end{aligned}$$

The three terms on the right-hand-side of the first equality capture respectively the surplus in the CM, the value of ideas implemented by entrepreneurs and the value of ideas implemented by innovators. In particular, we have

$$\bar{W}^B = \bar{W}^{NB} + \int_{\bar{R}_i^{NB}}^{\bar{R}_i^{CB}} (R_e - R_i) dR_i,$$

where the second term captures the welfare gain from a better allocation of ideas. Now, we will compare the allocation with and without banking. Note that, without banking, the cut-off value, \bar{R}_i^{NB} , is pinned down by the money demand decision. In equilibrium, the first order condition (6) implies

$$\begin{aligned}
\frac{\phi}{\mu}(1 - \bar{R}_i^{NB}) &= \frac{\phi}{\mu}\left(\frac{\mu}{\beta} - 1\right) \\
\Rightarrow 1 - \bar{R}_i^{NB} &= \frac{\mu}{\beta} - 1
\end{aligned} \tag{13}$$

The left- and right-hand sides capture respectively the benefit and cost of bringing the marginal dollar to the DM. Bringing one extra dollar to the DM relaxes the liquidity constraint and allows $\frac{\phi}{\mu}$ more extra trades, each of which generates payoff $1 - \bar{R}_i^{NB}$ in terms of next period utility. At the same time, bringing one extra dollar incurs a (net) opportunity cost of $(\frac{\mu}{\beta} - 1)$ in terms of next period dollars. In terms of next period utility, the cost is $\frac{\phi}{\mu}(\frac{\mu}{\beta} - 1)$.

With banking, the cut-off value, \bar{R}_i^B , is pinned down by the borrowing decision. In equilibrium, condition (10) implies

$$1 - \bar{R}_i^B = \left(\frac{\mu}{\beta} - 1\right)\bar{R}_i^B \quad (14)$$

The left- and right-hand sides capture respectively the benefit and cost of borrowing for the marginal entrepreneur in the DM. Comparing the right-hand sides of (13) and (14), we can see that banking reduces entrepreneurs' cost of buying ideas: by lending out the money balance unused for trade, the excess balance is no longer subject to inflation and discounting.

Proposition 2. (*Inflation and welfare with costless banking*)

- (i) When $\mu = \beta$, $\bar{R}_i^B = \bar{R}_i^{NB} = 1$ and $\bar{W}^B = \bar{W}^{NB}$.
- (ii) When $\mu \in (\beta, 2\beta]$, $1 > \bar{R}_i^B > \bar{R}_i^{NB} > 0$, $\bar{W}^B > \bar{W}^{NB}$, $0 > \frac{d\bar{R}_i^B}{d\mu} > \frac{d\bar{R}_i^{NB}}{d\mu}$ and $0 > \frac{d\bar{W}^B}{d\mu} > \frac{d\bar{W}^{NB}}{d\mu}$.
- (iii) When $\mu > 2\beta$, $\bar{R}_i^B > \bar{R}_i^{NB} = 0$ and $\bar{W}^B > \bar{W}^{NB}$.

When $\mu = \beta$, all ideas are traded and welfare is maximized with or without banking. In this case, the existence of banking cannot improve welfare.

When $\mu \in (\beta, 2\beta]$, banking allows more ideas to be traded and thus implies higher welfare. The marginal effect of inflation is larger in magnitude when there is no banking for two reasons. First, the marginal effect of inflation on the number of trades is larger without banking (i.e., $|\frac{d\bar{R}_i^{NB}}{d\mu}| > |\frac{d\bar{R}_i^B}{d\mu}|$). Condition (13) suggests that, without banking, higher inflation raises the opportunity cost of holding money, and thus less ideas are traded (i.e., lower \bar{R}_i^{NB}). Condition (14) suggests that, with banking, the impact of inflation on \bar{R}_i^{NB} is smaller because unspent money holding can now be saved and thus does not subject to the inflation tax.

Second, the gain from the marginal trade is higher without banking ($1 - \bar{R}_i^{NB} > 1 - \bar{R}_i^B$). This is because the marginal value of trades is diminishing and because the number of trades is higher with banking. Therefore, inflation is less harmful in the presence of banking.

When $\mu > 2\beta$, monetary equilibrium does not exist without banking, but exists with costless banking. Without banking, the only reason to bring money to the DM is to buy

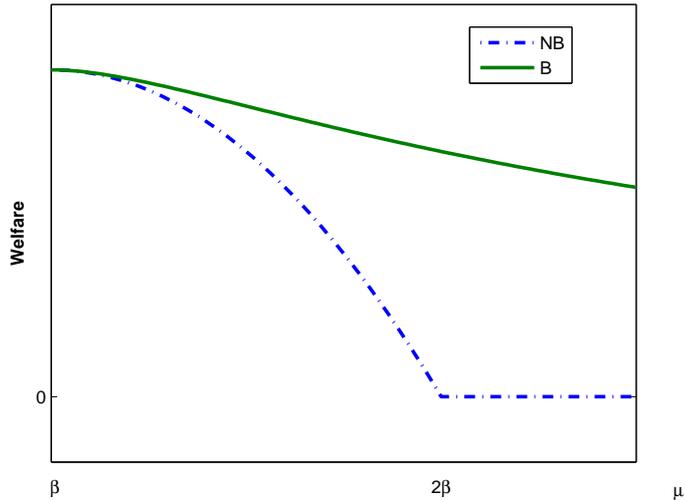


Figure 6: Welfare in No Banking and Costless Banking Equilibria

ideas. A very high inflation will make the cost of holding money so high so that there is no trades of ideas, implying zero value of money. With banking, there is an additional motive to bring money to the DM to lend to the banks. With high inflation, liquidity is relatively scarce in the DM (i.e., excess demand for loans if the price of money does not adjust). The scarcity of money in the DM induces entrepreneurs to demand more money in the CM, raising the price of money, ϕ . As a consequence of a higher ϕ , entrepreneurs' liquidity constraints in the DM are relaxed. So, when the money growth rate is higher than 2β , banking is needed to support a monetary equilibrium. (Figure 6)

Now, we compare the price of money in economies with banking (ϕ^B) and without banking (ϕ^{NB}). Mathematically, considering the price ϕ as a function of μ (i.e., $\phi^{NB}(\mu) = 2\mu - \frac{\mu^2}{\beta}$ and $\phi^B(\mu) = \frac{\beta^2}{2\mu}$), we have $\phi^{NB}(\beta) > \phi^B(\beta)$ and $\phi^B(2\beta) > \phi^{NB}(2\beta) = 0$. Since $\phi^B(\cdot)$ and $\phi^{NB}(\cdot)$ are strictly decreasing and continuous in μ , we have the following result:

Proposition 3. (*Value of money with costless banking*) *There exists a unique $\mu^* \in (\beta, 2\beta)$ such that $\phi^{NB}(\mu) \geq \phi^B(\mu)$ for $\mu \leq \mu^*$.*

To see the intuition why banking reduces ϕ when μ is low and increases ϕ when μ is high, let us consider the two cases illustrated in Figure 7. Start with an economy without banking. At the Friedman rule, every entrepreneur is liquidity unconstrained and has excess

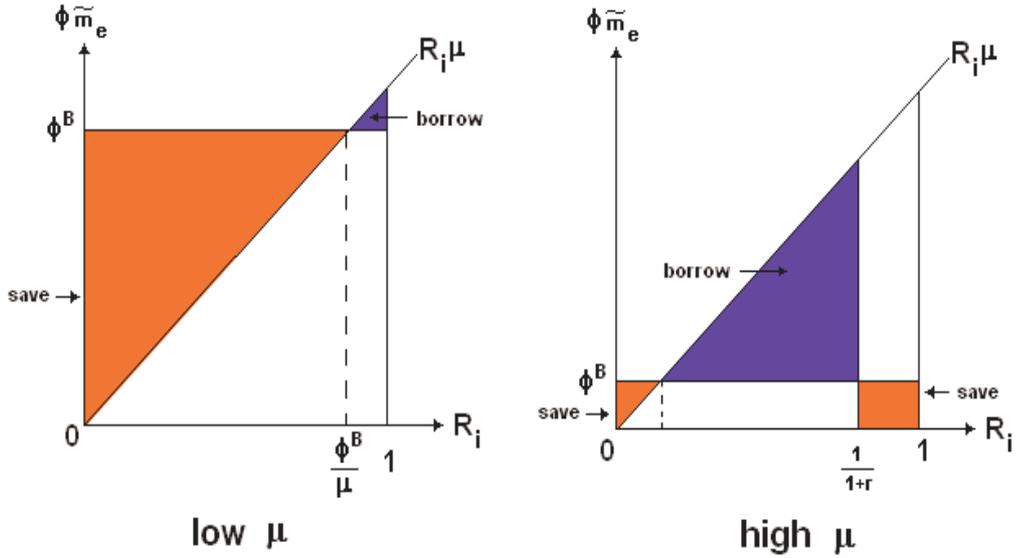


Figure 7: Borrowing and Saving in DM for different μ

liquidity after trades. Banking is not needed. Close to the Friedman rule (i.e., with low μ), the price of money is relatively high and most ideas are traded. Now, suppose we introduce banking into this economy. Let's first look at the *partial* equilibrium in the DM by keeping the original ϕ unchanged. As illustrated in the figure, there is excess supply of loans in the DM, driving the interest rate r to 0. Now, we consider the determination of ϕ in the general equilibrium. Anticipating $r = 0$ in the DM, entrepreneurs have lower incentive to demand money in the CM (because they can always borrow at $r = 0$ in the DM). As a result, the equilibrium price of money in the CM has to go down. Since the real money demand is $\phi\tilde{m} = \phi$, the use of banking comes with a lower demand for real money balances. In a sense, banking is a substitute for real money balances when the money growth rate is low.

Now, consider an economy without banking and μ is high. The equilibrium price of money is relatively low and most ideas are not traded. Now, suppose we introduce banking into this economy and again look at the *partial* equilibrium in the DM by keeping the original ϕ unchanged. As illustrated in the figure, there is excess demand for loans in the DM, driving up the interest rate r . But, in the general equilibrium, anticipating a high r in the DM, entrepreneurs have now higher incentive to demand money in the CM (because they do not want to borrow at a high rate and can always save at a high r in the DM). As a result, the

equilibrium price of money in the CM has to go up. In this case, banking is a complement for real money balances when the money growth rate is high.

Summarizing the findings for an economy with costless banking:

- At the Friedman's rule, banking is not used.
- Above the Friedman rule, banking is used and is welfare-improving.
- When the inflation rate is low, banking reduces the price of money. Banking and real money balances are substitutes.
- When the inflation rate is high, banking increases the price of money. Banking and real money balances are complements.
- When banking is used, inflation is less harmful.

5 Economy with Costly Banking

Entrepreneur's decision in DM

Suppose entrepreneurs have to incur a fixed effort/utility cost, η , to borrow but no cost to deposit. One can consider Section 3 as analyzing the case when this fixed cost is infinite, and Section 4 as analyzing the case when such cost is zero. Now, we consider the general case. One may interpret it as the borrower's cost of credibly committing to repay. An entrepreneur in the DM chooses saving (s), money brought to the CM (m_e) and whether or not to buy the idea ($y \in \{0, 1\}$):

$$\max_{s, y, m_e} \beta W_e(0, 0) + \beta y R_e + \beta \frac{\phi}{\mu} (1 + r) s - \iota(s) \eta + \beta \frac{\phi}{\mu} m_e$$

subject to $m_e \mu = \tilde{m}_e - y \frac{R_i \mu}{\phi} - s \geq 0$ and an indicator function

$$\iota(s) = \begin{cases} 1 & , \text{ if } s < 0 \\ 0 & , \text{ if } s \geq 0. \end{cases}$$

Again, the non-negativity constraint for m_e requires that $rm_e = 0$. Also, there is no reason to pay the fixed cost and borrow unless an entrepreneur is liquidity constrained. So, $\iota(s) = 0$ when $\tilde{m}_e - y \frac{R_i \mu}{\phi} \geq 0$ and thus the problem becomes

$$\begin{cases} \beta W_e(0, 0) + \beta \max\{1 + \frac{\phi}{\mu}(1+r)(\tilde{m}_e - \frac{R_i \mu}{\phi}), \frac{\phi}{\mu}(1+r)\tilde{m}_e\} & , \text{ if } \tilde{m}_e \geq \frac{R_i \mu}{\phi} \\ \beta W_e(0, 0) + \beta \max\{1 + \frac{\phi}{\mu}(1+r)(\tilde{m}_e - \frac{R_i \mu}{\phi}) - \frac{\eta}{\beta}, \frac{\phi}{\mu}(1+r)\tilde{m}_e\} & , \text{ if } \tilde{m}_e < \frac{R_i \mu}{\phi} \end{cases}$$

If $\tilde{m}_e \geq \bar{p}(R_i) = \frac{R_i \mu}{\phi}$, an entrepreneur is not liquidity constrained and will choose to save and trade if and only if

$$R_i \leq \bar{R}_1 \equiv \frac{1}{1+r}$$

If $\tilde{m}_e < p = \frac{R_i \mu}{\phi}$, an entrepreneur is liquidity constrained and will choose to borrow and trade if and only if

$$R_i \leq \bar{R}_2 \equiv \frac{1}{1+r} - \frac{\eta}{\beta(1+r)} \quad (15)$$

The optimal choice of an entrepreneur given any (\tilde{m}_e, R_i) pair is shown in Figure 8. Above the upward-sloping line $\phi\tilde{m}_e = R_i\mu$, entrepreneurs are not liquidity constrained. In this case, they choose to trade whenever $R_i \leq \bar{R}_1$. Below the upward-sloping line, entrepreneurs are liquidity constrained. In this case, they choose to trade whenever $R_i \leq \bar{R}_2$. Note that $\bar{R}_1 > \bar{R}_2$.

To solve for the equilibrium, we consider two different cases separately: $\phi\tilde{m}_e \leq \bar{R}_2\mu$ and $\phi\tilde{m}_e > \bar{R}_2\mu$. In the first case, the real value of money is so low that some liquidity constrained entrepreneurs want to borrow, implying $r \geq 0$. In the second case, the real value of money is so high that all entrepreneurs are not willing to borrow, implying $r = 0$.

Case 1: $\phi\tilde{m}_e \in [0, \bar{R}_2\mu]$

Note that an equilibrium with $r > 0$ can only exist when some entrepreneurs choose to

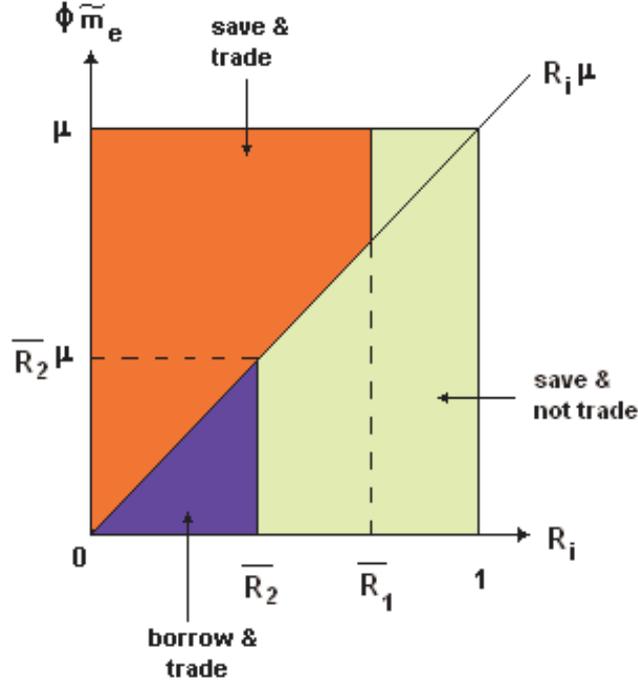


Figure 8: Optimal Choice of Entrepreneur

borrow (i.e., $s(1, R_i) < 0$ for some R_i). As Figure 8 suggests, this requires $\phi < \bar{R}_2 \mu$ (since $\tilde{m}_e = 1$ in equilibrium). Now, let us first characterize this equilibrium and then we can derive the condition for the existence of this equilibrium. In this equilibrium, an entrepreneur brings $\tilde{m}_e = 1$ to the DM and an entrepreneur chooses to trade and save if $R_i \in [0, \frac{\phi}{\mu}]$, chooses to trade and borrow if $R_i \in (\frac{\phi}{\mu}, \bar{R}_2]$ and chooses to save and not trade if $R_i \in (\bar{R}_2, 1]$. As shown in the Appendix, the value function over the relevant region (which is $[0, \frac{\bar{R}_2 \mu}{\phi}]$) is given by

$$V_e(\tilde{m}_e) = \beta(W_e(0,0) + R_e) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1 - \bar{R}_2) - \eta + \frac{\phi}{\mu}[\beta(1+r) + \eta]\tilde{m}_e$$

Therefore, $V_e'(\tilde{m}_e) = \frac{\phi}{\mu}[\beta(1+r) + \eta]$. The idea is that the marginal value of bringing an extra dollar to the DM consists of two components. The first part is the real return of money (i.e., $\beta\frac{\phi}{\mu}(1+r)$). The second part is that it helps to reduce the likelihood of being liquidity constrained and thus avoiding the expected fixed cost of borrowing a loan (i.e., $\frac{\phi}{\mu}\eta$). Again, V_e is linear in \tilde{m}_e in this region. In equilibrium, the money market clearing condition,

$\tilde{m}_e = 1$, implies that the first order condition with respect to the money demand in the CM has to be satisfied with equality (i.e., $V'_e(\tilde{m}_e) = \phi$). Therefore,

$$r = \frac{\mu - \eta}{\beta} - 1. \quad (16)$$

When (16) is satisfied, entrepreneurs are indifferent between any $\tilde{m}_e \in [0, \frac{\bar{R}_2 \mu}{\phi}]$. The equilibrium condition $\tilde{m}_e = 1$ then requires that

$$\phi \leq \bar{R}_2 \mu. \quad (17)$$

Let \bar{R}_i^{CB} denote the cut-off value of idea such that an entrepreneur is indifferent between trading or not. Condition (15) implies $\bar{R}_i^{CB} = \bar{R}_2 = \frac{\beta - \eta}{\mu - \eta}$.

As in condition (12), we can use the banks' optimal decision to derive equilibrium conditions regarding the excess supply of loans and the interest rate. The loan market clearing condition in the DM implies

$$\begin{cases} \tilde{m}_e - \frac{\mu(\beta - \eta)^2}{2\phi(\mu - \eta)^2} = 0 & , \text{ if } r > 0 \\ \tilde{m}_e - \frac{\mu(\beta - \eta)^2}{2\phi(\mu - \eta)^2} \geq 0 & , \text{ if } r = 0 \end{cases}$$

Imposing the money market clearing condition in the CM ($\tilde{m}_e = 1$), we have

$$\begin{cases} \phi = \frac{\mu(\beta - \eta)^2}{2(\mu - \eta)^2} & , \text{ if } r > 0 \\ \phi \geq \frac{\mu(\beta - \eta)^2}{2(\mu - \eta)^2} & , \text{ if } r = 0 \end{cases} \quad (18)$$

Equilibrium

Definition 3. A stationary monetary equilibrium with costly banking is a pair (ϕ^{CB}, r) satisfying (16), (17), (18) with $\phi^{CB} > 0$, $r \geq 0$.

We have the following result (derived in the Appendix):

Proposition 4. (*Existence of equilibrium with costly banking*) *If $\eta \leq \min\{\beta, \mu - \beta\}$, there exists an equilibrium with costly banking.*

When $\eta < \mu - \beta$, we have a unique equilibrium with $\phi^{CB} = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$, $r = \frac{\mu-\eta}{\beta} - 1 > 0$, and $\bar{R}_i^{CB} = \frac{\beta-\eta}{\mu-\eta}$. Fraction $\frac{(\beta-\eta)(2\mu-\beta-\eta)}{2(\mu-\eta)^2} > 0$ of entrepreneurs are borrowers.

When $\mu = \beta + \eta$, we have a continuum of equilibria with any $\phi^{CB} \in [\frac{(\beta+\eta)(\beta-\eta)^2}{2\beta^2}, \frac{(\beta-\eta)(\beta+\eta)}{\beta}]$, $r = 0$, and $\bar{R}_i^{CB} = 1 - \frac{\eta}{\beta}$. Corresponding to these equilibria, the equilibrium fraction of borrowers is $\max\{\bar{R}^{CB} - \frac{\phi^{CB}}{\beta}, 0\} \in [0, \frac{(\beta-\eta)(\beta+\eta)}{2\beta^2}]$. These equilibria are equivalent in terms of the allocation of ideas but are not payoff equivalent due to the fixed cost of borrowing. With the highest value of money (i.e., $\phi^{CB} = \frac{(\beta-\eta)(\beta+\eta)}{\beta}$), there are no borrowers and thus no fixed cost is incurred. As the value of money goes down, there are more and more borrowers and thus a higher total amount of fixed cost is incurred.

Now, we take a look at the conditions for the existence of an equilibrium with costly banking (i.e., $\eta \leq \min\{\beta, \mu - \beta\}$). Firstly, if $\beta < \eta$, the payoff of getting an idea ($\beta R_e = \beta$) is lower than the fixed cost (η), and thus no entrepreneurs want to borrow even when the price of the idea is zero. Secondly, note that the *net* real rate of return of buying money in the CM is

$$\begin{aligned} & \frac{\beta(1+r) + \eta}{\mu} - 1 \\ = & \frac{\beta r + (\beta + \eta - \mu)}{\mu}. \end{aligned}$$

Since $r \geq 0$, if $\eta > \mu - \beta$, then the net real rate of return is always positive, implying entrepreneurs would demand an infinite amount of money in the CM.

Case 2: $\phi \tilde{m}_e \in (\bar{R}_2 \mu, \infty)$

As shown in Figure 8, there is no borrowing in this equilibrium, implying $r = 0$, and accordingly $\bar{R}_1 = 1$. The equilibrium allocation is exactly the same as an equilibrium without banking: entrepreneurs with $R_i \leq \frac{\phi}{\mu}$ will trade, others will save in the bank at a zero interest

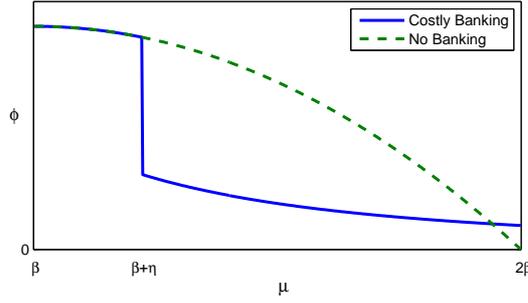


Figure 9: Inflation and Price of Money

rate. In this case, the equilibrium price of money is ϕ^{NB} which is derived in section 3. No entrepreneur has an incentive to borrow at a zero rate when the surplus from trade cannot cover the fixed cost of borrowing for the entrepreneur drawing $R_i = \frac{\phi^{NB}}{\mu}$. As shown in the Appendix, this equilibrium exists when

$$(\eta + \beta - \mu)(\beta - \eta) > 0 \quad (19)$$

Inflation, Banking and Welfare

Proposition 5. (*Banking and Trading*) *If $\eta \leq \min\{\beta, \mu - \beta\}$, then $\bar{R}_i^{NB} \leq \bar{R}_i^{CB} \leq \bar{R}_i^B$.*

Less ideas are traded with costly banking than with costless banking. More ideas are traded in an equilibrium with banking than in an equilibrium without banking. As shown in the last section, costly banking is a substitute for real money balances when the inflation rate is low, and is a complement when the inflation rate is high.¹¹ As shown in Figure 5:

Proposition 6. (*Value of money with costly banking*) *There exists a unique $\mu^* \in (\beta + \eta, 2\beta)$ such that $\phi^{NB}(\mu) \gtrless \phi^{CB}(\mu)$ for $\mu \lesseqgtr \mu^*$.*

¹¹Apparently, $\phi^B > \phi^{CB}$ because $\frac{d\phi^{CB}}{d\eta} < 0$. Considering the price ϕ as a function of μ (i.e., $\phi^{NB}(\mu) = 2\mu - \frac{\mu^2}{\beta}$ and $\phi^{CB}(\mu) = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$), we have $\phi^{NB}(\beta + \eta) > \phi^B(\beta + \eta)$ and $\phi^B(2\beta) > \phi^{NB}(2\beta) = 0$. Since $\phi^{CB}(\cdot)$ and $\phi^{NB}(\cdot)$ are strictly decreasing and continuous in μ .

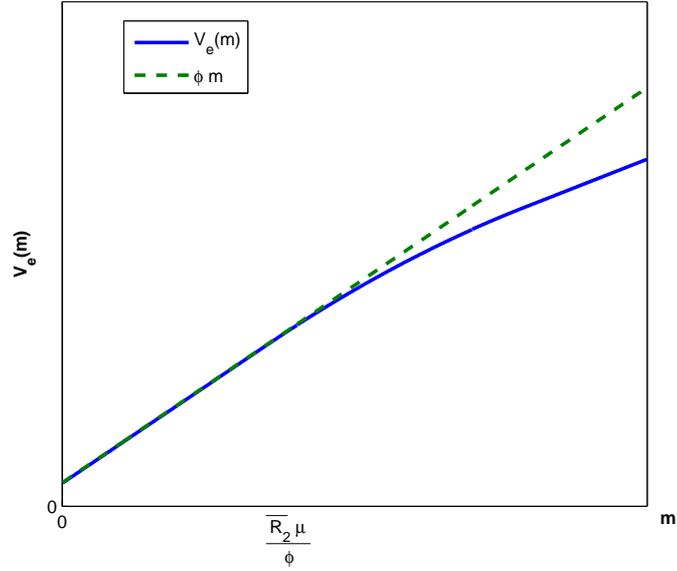


Figure 10: Value Function of an Entrepreneur in DM

While banking can increase trades, it also incurs the fixed cost. We can measure the welfare by the average utility of entrepreneurs and innovators. As before, when there is no banking, the welfare is

$$\begin{aligned}
 \bar{W}^{NB}(\mu) &= 2(U(\bar{X}) - \bar{X}) + \int_0^{\bar{R}_i^k} R_e dR_i + \int_{\bar{R}_i^k}^1 R_i dR_i \\
 &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1 - \bar{R}_i^{NB})^2}{2}
 \end{aligned}$$

When there is costly banking (i.e., $\mu \geq \eta + \beta$), the welfare is

$$\begin{aligned}
 \bar{W}^{CB}(\mu) &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1 - \bar{R}_i^{CB})^2}{2} - \eta \left[\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu} \right] \\
 &= \bar{W}^{NB} + \int_{\bar{R}_i^{NB}}^{\bar{R}_i^{CB}} (R_e - R_i) dR_i - \eta \left[\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu} \right].
 \end{aligned}$$

Therefore, the effects of banking on welfare can be decomposed into two parts: welfare gain from a better allocation of ideas and the welfare cost due to the fixed cost which is the product of fixed cost (η) and the number of borrowers ($\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu}$).

Now, we compare the steady state welfare between economies with different money growth rates, μ . We have shown that, for $\mu \in [\beta, \beta + \eta)$, banking is not viable and thus the welfare is given by \bar{W}^{NB} . As discussed above, when $\mu = \beta + \eta$, there is a continuum of banking equilibria with different welfare levels. All these equilibria support the same amounts of trades ($\bar{R}^{CB} = \bar{R}^{NB}$) but incur different amounts of fixed cost. There is a “high welfare equilibrium” associated with a high value of money and zero fixed cost. There is also a continuum of “low welfare equilibria” associated with lower values of money and positive amounts of fixed cost incurred. The lowest welfare level among these “low welfare equilibria” is given by $\bar{W}^{CB}(\beta + \eta)$. It is easy to show that $\bar{W}^{CB}(\beta + \eta) < \bar{W}^{NB}(\beta + \eta)$. By the continuity of \bar{W}^{NB} and \bar{W}^{CB} , for sufficiently small $\Delta > 0$, we still have $\bar{W}^{CB}(\beta + \eta + \Delta) < \bar{W}^{NB}(\beta + \eta + \Delta)$. Therefore, for moderate inflation, even though banking is used, it does not improve welfare. An economy without banking can achieve a higher welfare.

The welfare ranking is reversed when the inflation rate is high. In particular, when $\mu = 2\beta$, $\bar{R}_i^{NB} = 0$ and $\bar{W}^{NB}(2\beta) = 2(U(\bar{X}) - \bar{X}) + \frac{1}{2}$. The welfare level in a banking equilibrium is¹²

¹²Specifically, the welfare level in an economy with costly banking is

$$\begin{aligned}
\bar{W}^{CB}(2\beta) &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1 - \bar{R}_i^{CB})^2}{2} - \eta(\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu}) \\
&= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta - \eta)^2} - \eta \frac{(\beta - \eta)(2\beta - \eta)}{2(2\beta - \eta)^2} \\
&> 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta - \eta)^2} - \frac{(\beta - \eta)(2\beta - \eta)}{2(2\beta - \eta)^2} \\
&= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta - \eta)^2} - \frac{3\beta^2 - 4\beta\eta + \eta^2}{2(2\beta - \eta)^2} \\
&= 2(U(\bar{X}) - \bar{X}) + \frac{1}{2} \\
&= \bar{W}^{CB}(2\beta).
\end{aligned}$$

$$\begin{aligned}\bar{W}^{CB}(2\beta) &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1 - \bar{R}_i^{CB})^2}{2} - \eta\left(\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu}\right) \\ &> \bar{W}^{CB}(2\beta).\end{aligned}$$

Therefore, we have the following result:

Proposition 7. (*Inflation and Welfare*) For any $\eta \in (0, \min\{\beta, \mu - \beta\})$, there exists $\bar{\Delta}_1, \bar{\Delta}_2 > 0$ such that

- (i) $\bar{W}^{CB} < \bar{W}^{NB}$ for any $\mu \in [\beta + \eta, \beta + \eta + \bar{\Delta}_1]$,
- (ii) $\bar{W}^{CB} > \bar{W}^{NB}$, for any $\mu \in [2\beta - \bar{\Delta}_2, +\infty]$.

This is illustrated in Figure 11. The idea is that banking has both positive and negative effects on welfare. On the negative side, use of banking incurs a fixed cost which is a deadweight loss to society. On the positive side, banking improves the allocation of ideas. Note that the improvement of welfare depends on the inflation rate. When the inflation rate is low, most of the ideas are efficiently allocated even without banking, thus the gain from trading those remaining ideas is small. In this case, the welfare improvement from better allocation of ideas is outweighed by the deadweight loss. When the inflation rate is high, most of the ideas are not traded without banking. In this case, the welfare improvement from better allocation of ideas outweighs the deadweight loss.

Why is banking used even though it is welfare-reducing? This is because, when an entrepreneur decides on whether to borrow or not, he takes into account only his own private cost and benefit. He chooses to borrow whenever the net private gain from borrowing is larger than the fixed cost, ignoring the general equilibrium feedback effect. In particular, a borrower neglects that his borrowing in the DM reduces his demand for money in the CM. As a result, the price of money in the CM goes down. In equilibrium, a drop in the price of money will tighten other entrepreneurs' liquidity constraints, pushing more entrepreneurs to costly borrow from banks. So an individual's choice to borrow from a bank can lead to welfare loss to society.

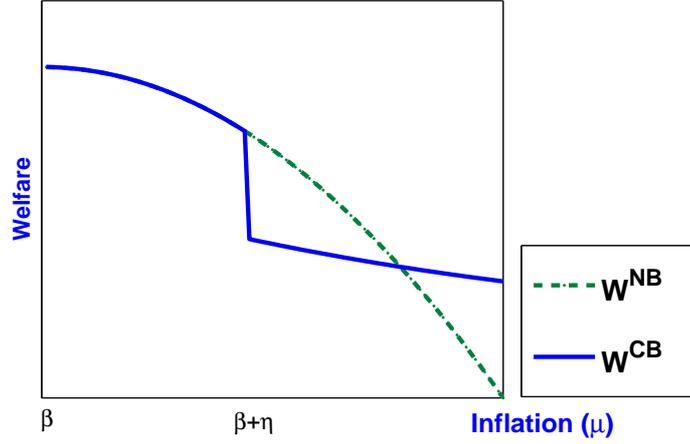


Figure 11: Welfare, Inflation and Banking

Now, we study the welfare effect of changing the fixed cost.

Proposition 8. (*Fixed Cost and Welfare*) For any $\mu \in (\beta, 2\beta)$, there exists $\tilde{\Delta}_1, \tilde{\Delta}_2 > 0$ such that

- (i) $\bar{W}^{CB} > \bar{W}^{NB}$, for any $\eta \in [0, \tilde{\Delta}_1]$,
- (ii) $\bar{W}^{CB} < \bar{W}^{NB}$, for any $\eta \in [\mu - \beta - \tilde{\Delta}_2, \mu - \beta]$.
- (iii) For $\mu = 1$, there exists an $\bar{\eta} \in (0, 1 - \beta)$ such that,

$$\begin{cases} \bar{W}^{CB} > \bar{W}^{NB} & \text{if } \eta < \bar{\eta} \\ \bar{W}^{CB} = \bar{W}^{NB} & \text{if } \eta = \bar{\eta} \\ \bar{W}^{CB} < \bar{W}^{NB} & \text{if } \eta > \bar{\eta} \end{cases}$$

Proposition 2 implies that, when the fixed cost is zero, banking can always improve welfare. By continuity, banking can improve welfare for small fixed costs. Moreover, for a fixed cost sufficiently large relative to the inflation rate, the deadweight loss of banking outweighs the gain from a better allocation of ideas. Figure 12 plots various \bar{W}^{CB} for different sizes of the inflation rate. When $\mu = \beta$, the welfare is independent of the fixed cost, because banking is never used. For $\mu > \beta$, banking is used whenever $\eta < \mu - \beta$. Below this threshold, welfare is decreasing in the fixed cost. Above this threshold, banking is not used. Note that,

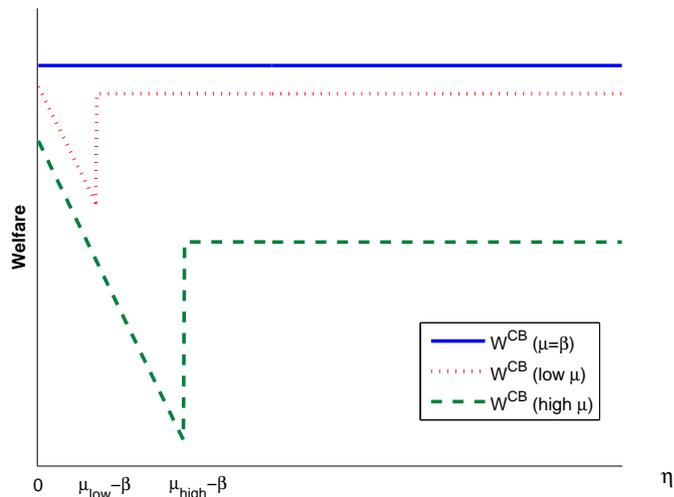


Figure 12: Effect of Fixed Cost on Welfare

due to the general equilibrium feedback mentioned above, the welfare can be higher when the fixed cost goes up. The implication is that the range of fixed costs that can support banking is increasing in the inflation rate.

Figure 13 shows the distribution of outcomes for different combinations of the fixed cost and money growth rate. In Figure 13, we can see that banking is used only when two conditions are satisfied: (i) $\eta < \beta$ (indicated by the vertical line) and (ii) $\eta < \mu - \beta$ (indicated by the upward-sloping straight line). Also, the curve indicating $\bar{W}^{CB} = \bar{W}^{NB}$ is concave because, when there is no banking, the marginal distortion of inflation is increasing in money growth rate. It is interesting to compare this result with Bencivenga and Camera (2007). In their model, banking potentially can also reduce welfare, but this suboptimal outcome cannot be supported in equilibrium because banking is modeled as an optimal contract.

Even when banking is costly, inflation is less harmful whenever banking is used:

Proposition 9. (*Banking and Welfare Cost of Inflation*) For any $\eta \in (0, \min\{\beta, \mu - \beta\})$, $|\frac{d}{d\mu}\bar{W}^{CB}| < |\frac{d}{d\mu}\bar{W}^{NB}|$.

One implication of this finding is that, measuring the welfare gain of reducing inflation by extrapolating observable data from the high inflation region with banking (far from the Friedman's rule) to the unobservable low inflation region without banking (close to the

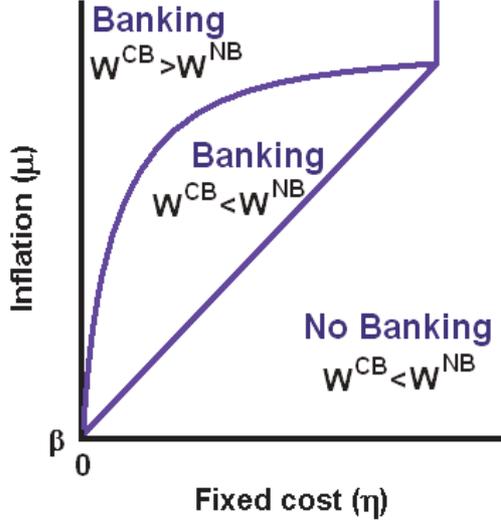


Figure 13: Distribution of Equilibrium

Friedman's rule), we may underestimate the actual welfare gain of following the Friedman's rule because this approach ignores the intermediation cost involved in using banking to solve liquidity problem. Mathematically, denote $\bar{W}^{CB}(\mu)$ as the level of welfare in an economy with costly banking. The actual welfare gain of moving from an inflation rate μ to the Friedman's rule is $G(\mu) = \bar{W}^{CB}(\beta) - \bar{W}^{CB}(\mu)$. A first order approximation of this welfare gain is $\hat{G}(\mu) = \frac{d\bar{W}^{CB}(\mu)}{d\mu}(\beta - \mu)$. We can show that

Proposition 10. (*Underestimation of Welfare Gain*) For any μ, β , there exists an $\bar{\eta}(\mu, \beta)$ such that $G(\mu) > \hat{G}(\mu)$ for all $\eta \in (\bar{\eta}, \beta)$.

The idea is that banking is used and the fixed intermediation cost is incurred only when the inflation rate is sufficiently high. As a result, measuring the welfare change by extrapolating from the high inflation region (where banking is used) to the Friedman's rule (at which banking is not used) does not take into account the potential saving of the fixed costs as the money growth rate drops to β .

Summarizing the findings for an economy with costly banking:

- Banking improves the allocation of ideas.
- When the inflation is low, banking is not used.

- When the inflation is high, banking is used and is welfare-improving. Banking increases the price of money and the real money demand.
- When the inflation is moderate, banking is used but is welfare-reducing. Banking reduces the price of money and the real money demand.
- When banking is used, inflation is less harmful.

6 Extensions

This section discusses the robustness of our findings when some of the simplifying assumptions are relaxed. First, we relax the assumption that “ideas” are pure private goods. In particular, we assume that, after trading an idea, the entrepreneur receives the implementation value R_e , while the innovator retains a fraction λ of her implementation value (i.e., λR_i), with $\lambda \in (0, 1)$.¹³ We can show that, there exists a $\bar{\lambda}$ such that for all $\lambda \leq \bar{\lambda}$, all our findings still hold true.¹⁴

Another simplifying assumption of our model is that ideas are indivisible and agents are not allowed to use lotteries to convexify their bargaining problem. Now, we consider the case in which it is feasible for agents to use lotteries. In particular, an offer from an entrepreneur to an innovator consists of a pair (p, θ) , where p is the price paid by the entrepreneur and $\theta \in [0, 1]$ is the probability of transferring the idea. While analytical solution is not generally feasible, one can show numerically that all the main findings still hold true. The only important difference from the benchmark case is that, in an economy without banking, a monetary equilibrium always exists for any monetary growth rate.¹⁵

One interesting extension is to consider a general non-separable production function. In

¹³When $\lambda = 0$, it is the benchmark case when ideas are pure private goods. When $\lambda = 1$, ideas are pure public goods. In this case, innovators will sell all their ideas at a zero price. Efficiency is always achieved. Therefore, we will focus on the interesting case with $\lambda \in (0, 1)$.

¹⁴In particular, banking can improve the allocation of ideas. When μ is low, banking is not used. When μ is moderate, banking is used but is welfare-reducing. When μ is high, banking is used and is welfare-improving. Also, banking is needed to support a monetary equilibrium when μ is high.

¹⁵The analysis and implications are the same if we assume that ideas are divisible instead of allowing for lotteries.

the Appendix, we discuss the case in which $F(I, h) = If(h)$ with $f' > 0$, and $f'' < 0$. In this case, the equilibrium prices (r^*, w^*, ϕ^*) and the fraction of ideas traded, I^* , are jointly determined. For example, an improved allocation of ideas in the DM will lead to a higher labor demand, which then tends to drive up the wage rate in the CM. A high wage rate will then affect the entrepreneurs' implementation returns relative to innovators' returns, and thus has feedback effects on the allocation of ideas in the DM.

7 Conclusion

This paper develops a search-theoretical model to study how money and banking interact to affect allocation and welfare. We highlight that banking and monetary models need to be studied together for properly assessing the welfare effect of banking and the welfare cost of inflation. An interesting implication of our model is that, due to general equilibrium feedback, banking can exist in equilibrium even when it is welfare-reducing. Moreover, the non-linear welfare effect of inflation implies that measuring welfare cost of inflation by extrapolating historical data may underestimate the actual cost.

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8 Appendix

Proof of Proposition 4

We first derive the value function V_e by evaluating its value over three regions: $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$, $(\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$ and $(\frac{\bar{R}_1\mu}{\phi}, \infty)$:

(1) For $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$:

$$\begin{aligned}
V_e^1(m) &= \int_0^1 V_e(m, R_i) dR_i \\
&= \beta W_e(0, 0) + \beta \int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) dR_i + \beta \int_{\frac{\phi}{\mu}m}^{\bar{R}_2} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) - \frac{\eta}{\beta} dR_i \\
&\quad + \beta \int_{\bar{R}_2}^1 \frac{\phi}{\mu}(1+r)m dR_i \\
&= \beta W_e(0, 0) + \beta \int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) dR_i + \beta \int_{\frac{\phi}{\mu}m}^{\bar{R}_2} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) - \frac{\eta}{\beta} dR_i \\
&\quad + \beta \int_{\bar{R}_2}^1 1 + \frac{\phi}{\mu}(1+r)(m - \frac{\bar{R}_2\mu}{\phi}) - \frac{\eta}{\beta} dR_i \\
&= \beta(W_e(0, 0) + 1 + \frac{\phi}{\mu}(1+r)m) - \beta \int_0^{\bar{R}_2} (1+r)R_i dR_i - \beta \int_{\bar{R}_2}^1 (1+r)\bar{R}_2 dR_i - \int_{\frac{\phi}{\mu}m}^1 \eta dR_i \\
&= W_e(0, 0) + \beta - \beta \int_0^{\bar{R}_2} (1+r)R_i dR_i - \beta \int_{\bar{R}_2}^1 (1+r)\bar{R}_2 dR_i - \eta(1 - \frac{\phi}{\mu}m) + \beta \frac{\phi}{\mu}(1+r)m \\
&= \beta W_e(0, 0) + \beta - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1 - \bar{R}_2) - \eta + [\beta \frac{\phi}{\mu}(1+r) + \eta \frac{\phi}{\mu}]m
\end{aligned}$$

(2) For $m \in (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$:

$$\begin{aligned}
V_e^2(m) &= \int_0^1 V_e(m, R_i) dR_i \\
&= \beta W_e(0, 0) + \beta \int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) dR_i + \beta \int_{\frac{\phi}{\mu}m}^1 \frac{\phi}{\mu}(1+r)m dR_i \\
&= \beta W_e(0, 0) + \beta \frac{\phi}{\mu}(1+r)m + \beta \int_0^{\frac{\phi}{\mu}m} 1 - (1+r)R_i dR_i \\
&= \beta W_e(0, 0) + \beta \frac{\phi}{\mu}(1+r)m + \beta \frac{\phi}{\mu}m - \beta(1+r) \frac{(\frac{\phi}{\mu}m)^2}{2}
\end{aligned}$$

(3) For $m \in (\frac{\bar{R}_1\mu}{\phi}, \infty)$:

$$\begin{aligned}
V_e^3(m) &= \int_0^1 V_e(m, R_i) dR_i \\
&= \beta W_e(0, 0) + \beta \int_0^{\bar{R}_1} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i}{\phi}) dR_i + \beta \int_{\bar{R}_1}^1 \frac{\phi}{\mu}(1+r)m dR_i \\
&= \beta W_e(0, 0) + \beta \int_0^{\bar{R}_1} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i}{\phi}) dR_i + \beta \int_{\bar{R}_1}^1 1 + \frac{\phi}{\mu}(1+r)(m - \frac{\bar{R}_1}{\phi}) dR_i \\
&= \beta W_e(0, 0) + \beta + \beta \frac{\phi}{\mu}(1+r)m - \beta(1+r) \int_0^{\bar{R}_1} R_i dR_i - \beta(1+r)\bar{R}_1(1 - \bar{R}_1) \\
&= \beta W_e(0, 0) + \beta + \beta \frac{\phi}{\mu}(1+r)m - \beta(1+r) \frac{\bar{R}_1^2}{2} - \beta(1+r)\bar{R}_1(1 - \bar{R}_1)
\end{aligned}$$

We now show that, in the proposed equilibrium, the optimal money holding stays in the interval $[0, \frac{\bar{R}_2\mu}{\phi}]$. In particular, we will show:

- (1) $V_e^1(m) - \phi m = k$ for $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$ where k is a positive constant
- (2) $V_e^2(m) - \phi m < k$ for $m \in (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$
- (3) $V_e^3(m) - \phi m < k$ for $m \in (\frac{\bar{R}_1\mu}{\phi}, \infty)$

First, consider the equilibrium with $r > 0$. Using the result that $\bar{R}_1 = \frac{\beta}{\mu-\eta}$, $\bar{R}_2 = \frac{\beta-\eta}{\mu-\eta}$, $\phi = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$, and $1+r = \frac{\mu-\eta}{\beta}$, we can simplify the V_e derived above to get the followings:

$$\begin{aligned}
(1) \quad &V_e^1(m) - \phi m \\
&= \beta(W_e(0, 0) + 1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1 - \bar{R}_2) - \eta + [\beta\frac{\phi}{\mu}(1+r) + \eta\frac{\phi}{\mu}]m - \phi m \\
&= \beta(W_e(0, 0) + 1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1 - \bar{R}_2) - \eta + \phi m - \phi m \\
&= \beta(W_e(0, 0) + 1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1 - \bar{R}_2) - \eta \\
&= \beta W_e(0, 0) + \beta - \frac{\beta^2 + \eta^2 - 2\beta\eta}{2(\mu - \eta)} - \frac{(\beta - \eta)(\mu - \beta)}{\mu - \eta} - \eta \\
&= \beta W_e(0, 0) + \frac{(\beta - \eta)^2}{2(\mu - \eta)} = k > 0
\end{aligned}$$

(2) Similarly, we get $V_e^2(m) - \phi m = \beta W_e(0, 0) + \frac{(\beta-\eta)^2}{2(\mu-\eta)^2} \left[(\beta - \eta)m - \frac{(\beta-\eta)^2 m^2}{4(\mu-\eta)} \right]$. We can show that $V_e^2 - \phi m$ is strictly concave and attains its maximum at $m = \frac{2(\mu-\eta)}{(\beta-\eta)}$ (which is the lower bound of region 2), with the maximum equals to k . Therefore, $V_e^2(m) - \phi m < k$ for all $m \in (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$.

(3) First, note that $V_e^3(m) - \phi m$ is linear and strictly decreasing with $\frac{d}{dm}[V_e^3(m) - \phi m] = -\frac{\phi}{\mu}\eta < 0$. So, for any $m \in (\frac{\bar{R}_1\mu}{\phi}, \infty)$, $V_e^3(m) - \phi m$ is lower than $V_e^3(\frac{\bar{R}_1\mu}{\phi}) - \phi(\frac{\bar{R}_1\mu}{\phi}) = W_e(0, 0) + \frac{-2\beta\eta + \beta^2}{2(\mu - \eta)}$ which is lower than k if $\frac{\eta^2}{2(\mu - \eta)} > 0$.

Now, we consider the equilibrium with $r = 0$. (16) implies that $\mu = \beta + \eta$. We can follow the same analysis as above to show that $V_e^2(m) - \phi m$ is maximized at $m = \frac{\bar{R}_2\mu}{\phi}$ which is equal to $V_e^1(m) - \phi m$ for any $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$. Also, $r = 0$ implies $\bar{R}_1 = 1$, so the third region vanishes.

So, we have proved that $\arg \max_m V_e(m) - \phi m \in [0, \frac{\bar{R}_2\mu}{\phi}]$, indeed in equilibrium an entrepreneur is indifferent between any m in this interval. Now we need to check that this is not an empty set, that is $\bar{R}_2 \geq 0$ which requires $\eta \leq \beta$. Finally, $r \geq 0$ requires $\eta \leq \mu - \beta$.

Proof of Condition (19)

We want to show that $\arg \max_m V_e(m) - \phi m > \frac{\bar{R}_2\mu}{\phi}$ when $r = 0$ and condition (19) is satisfied. First, it is easy to show that, when $r = 0$, $V_e^2(m) - \phi m$ in the previous proof attains its global maximum at $m = \frac{\bar{R}_2(\beta + \eta)}{\phi}$. Then, to show that choosing $m \leq \frac{\bar{R}_2\mu}{\phi}$ is not optimal (where V_e^1 is the corresponding value function), note that $V_e^2(\frac{\bar{R}_2\mu}{\phi}) - \phi\frac{\bar{R}_2\mu}{\phi} = V_e^1(m) - \phi m$ for all $m \leq \frac{\bar{R}_2\mu}{\phi}$. Therefore, we just need to show that $\frac{\bar{R}_2(\beta + \eta)}{\phi} > \frac{\bar{R}_2\mu}{\phi}$ which is equivalent to (19).

Proof of Proposition 4

Firstly, consider the case with $r > 0$. Condition (16) implies that $\mu - \eta - \beta > 0$. Condition (18) then implies $\phi^{CB} = \frac{\mu(\beta - \eta)^2}{2(\mu - \eta)^2}$. Then condition (17) is satisfied if and only if

Proof of Proposition 5

$\bar{R}^B = \frac{\beta}{\mu} \geq \bar{R}^{CB} = \frac{\beta - \eta}{\mu - \eta}$ is obvious. Also, $\bar{R}^{CB} = \frac{\beta - \eta}{\mu - \eta} \geq \bar{R}^{NB} = 2 - \frac{\mu}{\beta}$ if $(\mu - \beta)(\beta - \mu + \eta) \leq 0$.

$$\begin{aligned}
\phi^{CB} &\leq \bar{R}_2\mu \\
\Leftrightarrow \frac{\mu(\beta - \eta)^2}{2(\mu - \eta)^2} &\leq \frac{\beta - \eta}{\mu - \eta}\mu \\
\Leftrightarrow (\beta - \eta)^2 &\leq 2(\beta - \eta)(\mu - \eta) \\
\Leftrightarrow (\beta - \eta)(\beta + \eta - 2\mu) &\leq 0 \\
\Leftrightarrow \eta &\leq \beta.
\end{aligned}$$

Now, consider the case with $r = 0$. Condition (16) implies that $\mu - \eta - \beta = 0$. Condition (18) then implies $\phi^{CB} \geq \frac{\mu(\beta - \eta)^2}{2(\mu - \eta)^2}$. Then condition (17) is satisfied if and only if

$$\begin{aligned}
\phi^{CB} &\leq \bar{R}_2 \mu \\
\Leftrightarrow \phi^{CB} &\leq \frac{(\beta - \eta)(\beta + \eta)}{\beta} \\
\Rightarrow \eta &\leq \beta, \text{ if } \phi^{CB} \geq 0
\end{aligned}$$

Proof of Proposition 8

First, $\bar{W}^{NB}(\eta) < \bar{W}^{CB}(\eta)$ for $\eta = 0$.

Second, $\bar{W}^{NB}(\eta) > \bar{W}^{CB}(\eta)$ for $\eta = \mu - \beta$

Finally, for $\mu = 1$, $\text{sign}(\bar{W}^{CB} - \bar{W}^{NB}) = \text{sign}(D(\eta))$ where $D(\eta) = (1 - \beta)^2((1 - \eta)^2 - \beta^2) - \eta(\beta - \eta)\beta^2(2 - \eta - \beta)$. From above, we know already that $D(0) > 0$ and $D(1 - \beta) < 0$. Also, we can show that $\frac{dD}{d\eta}(1 - \beta) < 0$ and $\frac{d^2D}{d\eta^2} > 0$, implying that $\frac{dD}{d\eta} < 0$ for $\eta \in (0, 1 - \beta)$. Therefore, there exists a cutoff $\bar{\eta}$ such that $\bar{W}^{NB} = \bar{W}^{CB}$.

Proof of Proposition 10

For any $\eta \in (0, \min\{\beta, \mu - \beta\})$, $|\frac{d}{d\mu} \bar{W}^{CB}| < |\frac{d}{d\mu} \bar{W}^{NB}|$.

The welfare effect of inflation when there is costly banking is given by

$$\begin{aligned}
&\frac{d}{d\mu} \bar{W}^{CB} \\
&= -\frac{d}{d\mu} \left[\frac{(1 - \bar{R}_i^{CB})^2}{2} \right] - \frac{d}{d\mu} \left[\eta \left(\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu} \right) \right] \\
&= \frac{d\bar{R}_i^{CB}}{d\mu} (1 - \bar{R}_i^{CB}) - \frac{d}{d\mu} \left[\eta \frac{(\beta - \eta)(2\mu - \beta - \eta)}{2(\mu - \eta)^2} \right] \\
&= \frac{d}{d\mu} \left[\frac{\beta - \eta}{\mu - \eta} \right] \left(1 - \frac{\beta - \eta}{\mu - \eta} \right) - \eta(\beta - \eta) \left[\frac{(\mu - \eta) - (2\mu - \beta - \eta)}{(\mu - \eta)^3} \right] \\
&= -\frac{(\beta - \eta)(\mu - \beta)}{(\mu - \eta)^3} - \eta(\beta - \eta) \left[\frac{(\mu - \eta) - (2\mu - \beta - \eta)}{(\mu - \eta)^3} \right] \\
&= -\frac{(1 - \eta)(\beta - \eta)(\mu - \beta)}{(\mu - \eta)^3}
\end{aligned}$$

The welfare effect of inflation when there is no banking is given by

$$\begin{aligned}
&\frac{d}{d\mu} \bar{W}^{NB} \\
&= -\frac{d}{d\mu} \left[\frac{(1 - \bar{R}_i^{NB})^2}{2} \right] \\
&= \frac{d\bar{R}_i^{NB}}{d\mu} (1 - \bar{R}_i^{NB}) \\
&= \frac{d}{d\mu} \left[2 - \frac{\mu}{\beta} \right] (1 - \bar{R}_i^{NB}) \\
&= -\frac{\mu - \beta}{\beta^2}
\end{aligned}$$

So the welfare effect of inflation is smaller with banking when

$$|\frac{d}{d\mu} W^{CB}| < |\frac{d}{d\mu} \bar{W}^{NB}|$$

$$\begin{aligned}
&\Leftrightarrow \frac{(1-\eta)(\beta-\eta)(\mu-\beta)}{(\mu-\eta)^3} < \frac{\mu-\beta}{\beta^2} \\
&\Leftrightarrow \frac{(1-\eta)(\beta-\eta)}{(\mu-\eta)^3} < \frac{1}{\beta^2}, \text{ since } \mu - \beta > 0 \\
&\Leftrightarrow (1-\eta)(\beta-\eta) < \frac{(\mu-\eta)^2}{\beta^2}(\mu-\eta), \text{ since } \mu - \eta > \beta
\end{aligned}$$

which is true since

$$(1-\eta)(\beta-\eta) < \beta < \frac{(\mu-\eta)^2}{\beta^2}(\mu-\eta)$$

General Production Function

Suppose $F(I, h) = If(h)$ with $f' > 0$ and $f'' < 0$. Also, we assume that the utility of agents is given by $X - D(Y)$, with $D' > 0$ and $D'' > 0$. The labor demand is then given by the F.O.C. $f_h(I, h) = w$, which implies a labor demand function $h(I, w)$. The labor supply is characterized by $D'(H) = w$, which implies a labor supply function $H(w)$. We first consider the equilibrium with banking. Here, the equilibrium prices (ϕ^*, w^*) and allocation of ideas I^* are jointly determined:

(i) Money market clearing condition:

$$\frac{\mu}{\beta} = \frac{\pi(1, w^*) - \phi^*/\mu}{f(h^*)},$$

where $h^* = h(I^*, w^*)$ and $\pi(I, w^*) = If(h(I, w^*)) - h(I, w^*)$.

(ii) The fraction of ideas traded:

$$\frac{\phi^*}{\mu} = \pi(I^*, w^*).$$

(iii) The labor market clearing condition:

$$(D')^{-1}(w^*) = \int_{I^*}^1 h(I, w^*) dI + h(1, w^*)I^*.$$

Similarly, in an equilibrium with costless banking, the prices (ϕ^*, w^*, r^*) and allocation I^* are given by:

(i) Money market clearing condition:

$$r^* = \frac{\mu}{\beta} - 1.$$

(ii) The fraction of ideas traded:

$$1 + r^* = \frac{\pi(1, w^*)}{\pi(I^*, w^*)}.$$

(iii) The labor market clearing condition:

$$(D')^{-1}(w^*) = \int_{I^*}^1 h(I, w^*) dI + h(1, w^*) I^*.$$

(iv) The banking sector equilibrium:

$$\frac{\phi^*}{\mu} = \int_0^{I^*} \pi(I, w^*) dI.$$