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## **Abstract**

This paper studies the capital accumulation and welfare implications of reducing capital income taxation in a general equilibrium economy with uninsurable investment risks. It has been shown that, with uninsurable investment risks, under-accumulation of capital may result compared to the complete markets economy. We show that reducing somewhat the capital income tax rate increases the capital stock and leads to a welfare gain. The complete elimination of the capital income tax, however, is not necessarily welfare improving.

*JEL classification: E21, E22, E62, G32, H24, H25*

*Bank classification: Economic models*

## **Résumé**

L'étude examine les conséquences, du point de vue du bien-être et de l'accumulation du capital, d'une réduction de l'impôt sur le revenu du capital dans le cadre d'un modèle d'équilibre général où les risques d'investissement ne sont pas assurables. Il a été démontré que la présence de tels risques peut provoquer une sous-accumulation du capital par rapport à ce que l'on observe dans une économie dotée de marchés complets. Les auteurs montrent qu'une réduction modérée du taux d'imposition du revenu du capital a pour effet d'accroître le stock de capital et entraîne une amélioration du bien-être. Cependant, l'élimination totale de l'impôt sur le revenu du capital ne s'accompagne pas nécessairement d'un gain de bien-être.

*Classification JEL : E21, E22, E62, G32, H24, H25*

*Classification de la Banque : Modèles économiques*

# 1 Introduction

The goal of this paper is to study the effects of changing the mix of capital income and labor income taxation in the presence of uninsurable investment risks on capital accumulation and welfare. A classic public finance result is that it is welfare-improving to set the capital income tax rate to zero in the long run. However, within a Bewley class of models with uninsurable earnings risks, Aiyagari (1995) argues that the capital income tax rate should be positive in the long-run. This happens because with uninsurable idiosyncratic earnings risks, agents save for precautionary reasons and this results in over-accumulation of capital. A positive capital tax rate thus reduces the over-accumulation and brings the capital stock closer to the level prevailing with complete markets.<sup>1</sup>

Besides Aiyagari (1995), there is a growing body of studies that use models with uninsurable idiosyncratic earnings risks to understand the consequences of different types of taxes (e.g., İmrohorođlu 1998; Domeji and Heathcote 2004; Conesa and Krueger 2006). Even though earnings risks are an important source of idiosyncratic uncertainty, several types of investment activities are also subject to uninsurable idiosyncratic risks. For instance, business owners invest a substantial fraction of wealth in their own businesses<sup>2</sup> and corporate managers hold a significant number of shares of the firm they manage.<sup>3</sup>

Angeletos (2003) and Meh and Quadrini (2006) show that with uninsurable investment risks such as those involved in entrepreneurial activities, the aggregate stock of capital could be smaller than in the environment with complete markets. That is, there could be under-accumulation of capital. The main mechanism that could lead to lower accumulation of capital can be explained as follows. With uninsurable investment risks, the risks in investment must be compensated by a risk premium, on top of the risk-free rate. This means that the marginal productivity of capital will be higher and the input of capital lower than in the complete-

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<sup>1</sup>İmrohorođlu (1998) quantitatively provides similar results in a model with overlapping generations.

<sup>2</sup>See Cagetti and DeNardi (2006), Gentry and Hubbard (2000), and Quadrini (1999).

<sup>3</sup>See Mikkelson, Partch and Shah (1997) and Himmelberg, Hubbard and Love (2000).

markets set-up. Then, if the equilibrium is characterized by under-accumulation of capital, Aiyagari's result in favor of positive capital income taxes, at least in the long-run, may be overturned. The objective of this paper is to investigate whether the reduction in capital income taxes in an environment with investment risks is welfare-improving.

To address this question, we construct a heterogeneous-agent general equilibrium model based on Meh and Quadrini (2006) with incomplete markets. Agents can invest in a risky technology, and save or borrow in risk-free bonds. There is a borrowing limit which depends on wage income and undepreciated capital. The risky technology exhibits decreasing returns to scale and agents supply labor inelastically. Taxes on capital income, labor income, and consumption are levied to finance exogenous and constant government consumption, which provides no utility to agents. In equilibrium, agents in the economy invest in risky capital as the expected rate of return is higher than the risk-free rate. There is a net-zero supply of risk-free bonds. Wealthier agents hold a positive amount of bonds to partially insure against investment risks and for better consumption smoothing. Poorer agents borrow in bonds to invest in the risky technology.

There is a trade-off when changing the mix of capital and labor income taxes. The severity of the trade-off depends on the agents' wealth. Capital income taxation is not desirable since it distorts the capital accumulation decision and results in lower aggregate capital, output and consumption. However, with uninsurable investment risks, where under-accumulation of capital already exists, the capital income tax lowers the capital stock even further below the complete-markets level. Thus, lowering the capital tax produces positive effects on the capital stock, output, and consumption relative to the economy with uninsurable labor income risks. Moreover, since the capital tax directly affects investment, the welfare of richer agents is affected more dramatically by capital income taxation. On the other hand, labor income taxation is not desirable even though labor supply is inelastic. The existence of borrowing constraints implies that agents save for precautionary reasons. Taxes on labor income hamper the agents' capacity to self-insure. In addition, the borrowing limit tightens as after-tax labor income falls, and this further

restricts agents' capacity to self-insure. Since poorer agents rely on labor income more heavily, a change in labor income tax affects these agents to a greater extent. Therefore, whether changing the mix of capital income taxes and labor income taxes is welfare-improving or not, is a quantitative question.

We examine the trade-off quantitatively by conducting tax revenue neutral experiments where we reduce the capital income tax rate and adjust simultaneously the labor income tax to keep government expenditure constant. We analyze both the long-run and short-run implications of capital income tax cuts.

Our key finding is that reducing somewhat capital income taxation increases capital accumulation and improves welfare. However, eliminating completely the capital income tax does not necessarily improve welfare when we account for the short-run costs associated with the transition.<sup>4</sup> Furthermore, even in the long-run the capital income tax that leads to the highest welfare is positive despite the under-accumulation of capital caused by uninsurable investment risks.

This finding can be understood by the following three observations. First, the relative importance of different sources of income matters. Since investment monotonically increases with wealth, risky investment is a larger source of income for richer agents while labor income is more important for poorer agents. As a result, when the tax burden shifts from investment income to labor income, rich agents are positively affected while poorer agents are negatively affected. Second, since labor is a safer source of income, the shift impedes agents' ability to self-insure. With uninsurable investment risks, accumulation of wealth is risky, especially for poorer agents who do not hold a positive amount of risk-free bonds. Hence less risky labor income acts as a partial insurance against risky investments. However,

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<sup>4</sup>The aggregate welfare gains/losses are computed as the proportional decrease in all agents' consumption in the pre-reform steady state so that a benevolent social planner, who weighs the utility of all agents equally, is indifferent between staying in the old regime or switching to the new regime with a lower capital income tax. Thus, although lowering the capital income tax eliminates part of the under-accumulation of capital, the "average" welfare may decrease. The welfare result depends on the balance of negative effects on poor agents and positive effects on rich agents.

as the labor tax increases, this effect is reduced. Third, when we account for the short-run effects, there is a general equilibrium effect which causes the interest rate to spike at the time of the capital tax cut before falling to a lower level in the post-reform steady state. The initial increase in the interest rate is beneficial for richer agents as they hold a positive stock of bonds. But for poorer agents, it represents an increase in the cost of financing investment as they are net borrowers.

Our work is also related to other studies in the literature. A steady state welfare analysis of capital income taxation with labor income risks is conducted by İmrohoroğlu (1998). His quantitative result is in line with Aiyagari's in that a positive capital income tax rate is welfare-improving over zero. However, he has not considered investment risks. Meh (2005), Cagetti and DeNardi (2006), Kitao (2008), and Meh (2008) study the effects of taxation by explicitly considering entrepreneurship. Nevertheless, these studies do not focus on the under-accumulation that results from investment risks. Panousi (2008), using the framework of Angeletos (2003), also studies capital income taxation in the presence of investment risks. Yet, in Panousi's paper, there is no borrowing constraint and borrowing is subsidized. Therefore, contrary to our paper, she finds that capital income taxation quantitatively leads to an increase in capital accumulation. However, the two papers show that the removal of capital income taxation may lead to a welfare loss. Another strand of literature on optimal capital taxation includes Golosov, Kocherlakota and Tsyvinski (2003) and Albanesi and Sleet (2006), who argue for a positive optimal capital income tax in the presence of private information and earnings risks. Albanesi (2006) studies optimal taxation of entrepreneurial capital with private information.

The paper is organized as follow. In the next section, we describe the theoretical framework, characterize the agent's problem and define the general equilibrium. Sect. 3 conducts a quantitative analysis using parameterized versions of the model. Sect. 4 concludes.

## 2 The basic model

In this economy, there is a continuum of agents that maximize expected lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t U(c_t), \quad U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad (1)$$

where  $c_t$  is consumption at time  $t$  and  $\beta$  is the intertemporal discount factor. Agents are endowed with one unit of time per period supplied inelastically at the market wage rate  $w_t$ .

Each agent can run a risky technology that returns  $F(k_t, l_{t+1}, z_{t+1}) = [(z_{t+1}k_t)^\epsilon l_{t+1}^{1-\epsilon}]^\theta + z_{t+1}k_t$  in the next period with the inputs of capital  $k_t$  and labor  $l_{t+1}$ . This is the gross return inclusive of non-depreciated capital: the first term is production and the second is capital net of depreciation. The variable  $z_{t+1}$  is an idiosyncratic i.i.d. shock that is unknown when  $k_t$  is chosen but known when  $l_{t+1}$  is chosen. Note that the shock is assumed to affect the efficiency units of capital. For simplicity, we assume that the shock can take only two values,  $z_L$  and  $z_H$ , with  $z_L < z_H$ . The probability, denoted by  $p_i$ , with  $i = L, H$ , is strictly positive for both realizations of the shock. The parameter  $\theta$  is less than one and implies that the production function has decreasing returns to scale. The capital income share is given by  $\epsilon$ .

In addition to the risky investment, there are non-contingent assets that pay  $b$  units of output in the next period. Agents buy these assets from a financial intermediary. Because there are no aggregate shocks, by pooling many contracts the intermediary does not face any uncertainty. The assumption that financial markets are competitive then implies that the current value of these assets is  $\delta_t b$ , where  $\delta_t = 1/(1+r_t)$  is the market discount factor and  $r_t$  is the equilibrium riskless interest rate.

There is a government that finances its spending,  $G_t$ , by taxing consumption at the rate  $\tau_t^c$ , capital income at the rate  $\tau_t^k$ , and labor income at the rate  $\tau_t^w$ . In period  $t$ , taxable capital income is revenue net of depreciation, and it is given by  $[(z_t k_{t-1})^\epsilon l_t^{1-\epsilon}]^\theta - w_t l_t - (1 - z_t)k_{t-1}$ . The government operates under a balanced budget every period.

## 2.1 The agent's problem

Denote by  $a$  the agent's wealth or net worth before consumption. Given the sequence of prices and taxes, respectively  $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^\infty$  and  $T^t \equiv \{\tau_j^c, \tau_{j+1}^w, \tau_{j+1}^k\}_{j=t}^\infty$ , the optimization problem can be written as follows:

$$V_t(a) = \max_{c,k,b,l_i} \left\{ U(c) + \beta \sum_{i=L,H} V_{t+1}(a_i) p_i \right\} \quad (2)$$

subject to

$$a = (1 + \tau_t^c)c + k + \delta_t b, \quad (3)$$

$$a_i = (1 - \tau_{t+1}^w)w_{t+1} + b + \left[ [(z_i k)^\epsilon l_i^{1-\epsilon}]^\theta + z_i k - w_{t+1} l_{t+1} \right] - \tau_{t+1}^k \left[ [(z_i k)^\epsilon l_i^{1-\epsilon}]^\theta - w_{t+1} l_{t+1} - (1 - z_i k) \right] \quad \text{for } i = L, H, \quad (4)$$

$$b \geq -(1 - \tau_{t+1}^w)w_{t+1} - \left\{ (1 - \tau_{t+1}^k) \left[ [(z_L k)^\epsilon l_L^{1-\epsilon}]^\theta - w_{t+1} l_L \right] + \left[ 1 - (1 - \tau_{t+1}^k)(1 - z_L) \right] k \right\}. \quad (5)$$

This is the optimization problem not only in steady states, but for any deterministic sequence of prices and taxes. This motivates the time subscript  $t$  in the value function. Notice that  $z_i$ , with  $i \in \{L, H\}$ , denotes the next period's realization of the shock, which is unknown when the agent chooses the consumption and investment plan. The variable  $l_i$ , with  $i \in \{L, H\}$ , is the next period's labor demand after the realization of the shock  $z_i$ . Eq. (3) is the budget constraint and Eq. (4) is the law of motion for next period's net worth before consumption which is composed of the after-tax labor income, the return on bonds, and the gross capital income net of taxation. Eq. (5) refers to the borrowing constraint which is derived by rearranging Eq. (4) for  $a_L \geq 0$ . Specifically, each agent can borrow up to an amount equal to his labor income net of labor tax plus his lowest

realized after-tax capital income.<sup>5</sup>

At this stage, it is appropriate to discuss the effects of changing capital income taxation on the borrowing constraint. To do so, note first that our policy experiment is conducted in a revenue-neutral fashion. That is, we decrease the capital income tax rate  $\tau^k$  and raise simultaneously the labor income tax rate  $\tau^w$  (while leaving the consumption tax rate unchanged) to keep government expenditure constant at its benchmark value. The reduction of the capital income tax rate has two opposing effects on the borrowing constraint. On one hand, a decrease in the capital income tax rate relaxes the borrowing constraint of households because the after-tax capital income increases as observed in the second term of (5). A capital income tax cut also increases capital accumulation which relaxes the borrowing constraint since capital serves as a “collateral”, the third term of (5). On the other hand, because of the revenue-neutral experiment, the decrease in the capital income tax tightens the borrowing constraint since the after-tax labor income decreases, the first term of (5). The degree of tightness of the borrowing constraint will depend on the wealth distribution. The first effect is more likely to dominate for agents with higher wealth since they would make larger investments. The second effect is more likely to dominate for agents with low wealth who rely more on labor income.

The saving behavior of agents also depends on the initial wealth distribution.<sup>6</sup> In general, the risk-free bond  $b$  and the risky capital  $k$  increase with the initial wealth holdings of households. Specifically, agents with low wealth tend to borrow (i.e., negative  $b$ ) and hold a positive amount of risky capital  $k$  which is small on average for these agents. Wealthy households, on the other hand, tend to hold a positive amount of risk-free bonds  $b$  and positive risky capital  $k$ . The next period wealth,  $a_i$ , also depends on the realization of the shock. For all levels of initial wealth, the next period wealth is smaller the lower is the realization of the shock.

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<sup>5</sup>Notice that the borrowing constraint is imposed only in the case in which  $z = z_L$ . Indeed, if this constraint is satisfied for  $z = z_L$ , it is also satisfied for  $z = z_H$ .

<sup>6</sup>Note that in Panousi (2008), the portfolio of agents are independent of the initial wealth distribution. All agents of the same type make the same choice of  $k$  and  $b$ .

Because of the lack of complete insurance, this leads assets to be volatile. These results are consistent with Meh and Quadrini (2006).

Because the input of labor  $l_i$  is chosen after the observation of  $z_i$  with  $i = L, H$ , labor is determined by maximizing  $F(k, l_i, z_i) - w_{t+1}l_i$ . Therefore, the input of labor is fully determined by the input of capital, the shock and the wage rate by solving the following first order conditions with respect to labor input:

$$\theta(1 - \epsilon)(z_i k)^{\theta\epsilon} l_i^{\theta(1-\epsilon)-1} = w_{t+1}, \quad \text{for } i = L, H. \quad (6)$$

Using this first order condition, we find the optimal labor demand ( $l_i = l(k, w_{t+1}, z_i)$ ) as a function of the beginning-of-period  $t + 1$  capital  $k$ , the realized shock  $z_i$  at time  $t + 1$  and the wage rate  $w_{t+1}$ . Optimal labor demand is defined as follows:

$$l(k, w_{t+1}, z_i) = \left[ \frac{\theta(1 - \epsilon)(z_i k)^{\theta\epsilon}}{w_{t+1}} \right]^{1/(1-\theta(1-\epsilon))}. \quad (7)$$

The gross revenue net the cost of labor can then be written as:

$$R(w_{t+1}; k, z_i) = F(k, l(k, w_{t+1}, z_i), z_i) - w_{t+1}l(k, w_{t+1}, z_i)$$

The optimization problem (2) is a standard concave problem. We can then establish the following properties:

**Proposition 1** *Given the sequence of prices, there is a unique solution to problem (2), and the function  $V_t(a)$  is strictly increasing, concave, and differentiable at all  $t$ .*

**Proof 1** *It can be verified that the feasible set in problem (2) is convex and that the objective function is strictly concave. Therefore, if  $V_{t+1}$  is concave,  $V_t$  is strictly concave. Moving backward we can establish that  $\lim_{t \rightarrow -\infty} V_t$  is concave. Because the objective in problem (2) is strictly concave, the solution is unique. Standard arguments can be used to prove that the value function is differentiable. Q.E.D.*

Given Proposition 1, by substituting out  $c_t$  from the problem using (3), the solution to problem (2) is characterized by the following first order conditions for

$b_t$  and  $k_t$ , respectively:

$$\frac{U'(c_t)}{1 + \tau_t^c} = \beta(1 + r_t) E \left\{ \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \right\} + (1 + r_t)\lambda_t \quad (8)$$

$$\begin{aligned} \frac{U'(c_t)}{1 + \tau_t^c} = & \beta E \left\{ \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \cdot \left[ (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k \right] \right\} + \\ & \lambda_t \cdot \left[ (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z_L) + \tau_{t+1}^k \right], \end{aligned} \quad (9)$$

where  $\lambda_t$  is the Lagrange multiplier associated with the limited liability constraint (5). The multiplier is positive if the solution is binding.

Conditions (8) and (9) make clear that the after-tax expected return from the risky investment is always greater than the return from the risk-free asset — that is,  $1 + r_t < E \left\{ (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k \right\}$ . To see this, consider the case in which the limited liability constraint is not binding. Conditions (8) and (9) imply that:

$$\begin{aligned} (1 + r_t) \cdot E \left\{ \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \right\} = & E \left\{ (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k \right\} \cdot E \left\{ \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \right\} + \\ & \text{Cov} \left( (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k, \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \right). \end{aligned} \quad (10)$$

Rearranging Eq. (10) we obtain the following expression:

$$\begin{aligned} E \left\{ (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k \right\} = & 1 + r_t - \\ & \frac{\text{Cov} \left( (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k, \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \right)}{E \left\{ \frac{U'(c_{t+1})}{1 + \tau_{t+1}^c} \right\}} \end{aligned} \quad (11)$$

Because  $U'(c_{t+1})$  is in general negatively correlated with  $R_k(w_{t+1}; k, z)$ , the last term on the right-hand-side is negative, and therefore,  $1 + r_t < E \left\{ (1 - \tau_{t+1}^k) R_k(w_{t+1}; k, z) + \tau_{t+1}^k \right\}$ . The last term on the right-hand-side of Eq. (11) represents the risk premium that households receive from investing in the risky technology.

Let's compare this to the case in which  $z_L = z_H = z$  (no shocks) and in which there is no capital income tax. In this case, the covariance term in Eq.

(10) is zero, and the marginal returns from the two investments are equal — that is,  $1 + r_t = ER_k(w_{t+1}; k, z)$ . This environment is similar to the one studied in Aiyagari (1995). The only difference is that  $w_{t+1}$  is not deterministic in Aiyagari. However, even if  $w_{t+1}$  is stochastic at the individual level, the condition  $1 + r_t = ER_k(w_{t+1}; k, z)$  still holds. Because the interest rate is smaller than the intertemporal discount rate in the steady state equilibrium — that is,  $r < 1/\beta - 1$  —, the model with only earnings risks generates an over-accumulation of capital.<sup>7</sup>

With investment risks, the result that the interest rate is lower than the intertemporal discount rate still holds. However, the expected marginal return on capital is not necessarily smaller than the intertemporal discount rate for all agents. This may imply that in the aggregate economy, there is under-accumulation of capital relative to the complete markets level. We will show this result numerically in Sect. 3.

## 2.2 Equilibrium

The solution of the agent's problem is a sequence of policy rules  $\{c_j(a), k_j(a), b_j(a)\}_{j=t}^{\infty}$ . Denote by  $M_t(a)$  the initial distribution of agents' assets. The general equilibrium can be defined as follows:

**Definition 1** *Given the initial distribution,  $M_t(a)$ , and a sequence of taxes and government expenditures,  $T^t \equiv \{\tau_j^c, \tau_{j+1}^k, G_j\}_{j=t}^{\infty}$ , a general equilibrium is defined by (i) a sequence of agents' policy functions,  $\{c_j(a), k_j(a), b_j(a)\}_{j=t}^{\infty}$ , and labor demand,  $l(k, w, z_i)$ ; (ii) a sequence of value functions,  $\{V_j(a)\}_{j=t}^{\infty}$ ; (iii) a sequence of prices,  $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^{\infty}$ ; (iv) a sequence of labor taxes  $\{\tau_{j+1}^w\}_{j=t}^{\infty}$ ; (v) a sequence of aggregate demands for labor,  $L(P^t) \equiv \{L_{j+1}(P^t)\}_{j=t}^{\infty}$ ; (vi) a sequence of aggregate capital,  $K(P^t, T^t) \equiv \{K_j(P^t, T^t)\}_{j=t}^{\infty}$ ; and (vii) a sequence of aggregate consumption,  $C(P^t, T^t) \equiv \{C_j(P^t, T^t)\}_{j=t}^{\infty}$ . These sequences must satisfy the following conditions: (i) the policy functions solve problem (2) at each point in time,*

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<sup>7</sup>This result, however, may not apply when the supply of labor is elastic. Pijoan-mas (2003) shows that precautionary savings could be negative in this case.

and  $\{V_j(a)\}_{j=t}^{\infty}$  are the associated value functions; (ii) the aggregate demands for labor, capital, and consumption are the aggregation of individual demands, and they satisfy  $L_{j+1}(P^t) = 1$  and  $(1 + \tau_j^c)C_j(P^t, T^t) + K_j(P^t, T^t) = \int a M_j(da)$ ; (iii) the sequence of labor taxes are such that the government budget balances in every period:  $\tau_j^c C_j(P^t, T^t) + \tau_j^w w_j + \tau_j^k (R(P^t, T^t) - K_j(P^t, T^t)) = G_j$ ; (iv) the distributions  $M_j(a)$ , for  $j > t$ , evolve according to the individual policies and the stochastic properties of the idiosyncratic shock.

**Complete markets economy:** One of the objectives in this paper is to compare the allocation obtained in a complete markets economy (i.e., with state-contingent contracts that are feasible and provide full insurance) with the allocations achieved in an incomplete markets economy where only non-state-contingent contracts are available. Since the availability of state-contingent contracts in a complete market allows the agent to fully insure against the investment risk, the first order conditions imply that  $(1 - \tau_{t+1}^k)ER_k(w_{t+1}; k_t, z_{t+1}) + \tau_{t+1}^k = 1 + r_t$ , where  $R_k$  is the derivative of gross revenue with respect to  $k$ . In the steady state, it must be that  $1 + r_t = 1/\beta$  for all  $t$ .

### 3 Numerical analysis

The goal of this section is to show numerically the macroeconomic and welfare implications of capital income taxation in the presence of uninsurable idiosyncratic investment risks. Although the analysis is not aimed at matching specific observations, it provides important information about the potential magnitude of these implications.

**Parameterization:** We assign the following parameter values. The period in the model is one year and the intertemporal discount factor is  $\beta = 0.95$ . The value of  $\beta$  is consistent with the values used in macroeconomic studies, although with incomplete markets the intertemporal discount rate is not equal to the interest rate. The risk aversion parameter is  $\sigma = 1.5$ .

Recall that we have assumed that the shock affects the efficiency units of capital. More specifically, if the investment at time  $t$  is  $k_t$ , the efficiency units of capital at the beginning of the next period (before choosing labor) is  $\tilde{k}_{t+1} = z_{t+1}k_t$ . The total resources returned by the risky technology is:

$$F(k_t, l_{t+1}, z_{t+1}) = \tilde{k}_{t+1} + (\tilde{k}_{t+1}^\epsilon l_{t+1}^{1-\epsilon})^\theta.$$

The first component is capital net depreciation, and the second component is production. After setting  $z_L = 0.5$  and  $z_H = 1.0$ , the probability of the low shock is chosen to have an expected depreciation rate of 8 percent, that is,  $p_L \cdot z_L + (1 - p_L) \cdot z_H = 0.92$ . This implies that, with 16 percent probability, capital depreciates by 50 percent, and, with 84 percent probability, there is no depreciation. The return-to-scale parameter is set to  $\theta = 0.95$ , and  $\epsilon = 0.35$ . This implies a labor income share of 60 percent.<sup>8</sup>

The last set of parameters are those characterizing the fiscal policy:  $\tau^k$ ,  $\tau^w$ ,  $\tau^c$ . Government spending,  $G$ , is set endogenously. To calibrate the three tax rates, we follow Mendoza, Razin and Tesar (1994) and İmrohoroğlu (1998). Using the average tax rates over the 1980s from the estimates of Mendoza et al. (1994), we set  $\tau^k = 0.40$ ,  $\tau^w = 0.20$ , and  $\tau^c = 0.055$ . The implied government expenditure-to-GDP ratio is 0.24. Table 1 reports the full set of parameter values for the baseline economy.

**Steady state properties:** Table 2 reports the steady state interest rate, aggregate capital stock, and concentration of wealth (as measured by the Gini index) in the baseline economy and in the complete markets economy. The table shows that the steady state capital stock is lower in the benchmark than in the complete markets economy.<sup>9</sup> A full analysis of the under-accumulation of capital can be

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<sup>8</sup>Given the choice of  $\beta = 0.95$ , the curvature of the production function,  $\theta = 0.95$ , implies that in equilibrium the total return from capital is between 5 and 10 percent, which is consistent with NIPA data for the U.S. economy once we consider all the proprietor's income as capital income.

<sup>9</sup>In the complete markets economy, the interest rate is equal to the intertemporal discount rate, and the stock of capital (normalized to 1) without taxation satisfies

Table 1: Parameter values for the baseline economy.

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Discount rate	$\beta$	0.95
Risk-aversion	$\sigma$	1.50
Return to scale	$\theta$	0.95
Capital income share	$\epsilon$	0.35
Lowest realization of shock	$z_L$	0.50
Highest realization of shock	$z_H$	1.00
Probability of $z_L$	$p_L$	0.16
Capital income tax rate	$\tau^k$	0.40
labor income tax rate	$\tau^w$	0.20
Consumption tax rate	$\tau^c$	0.055

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found in Meh and Quadrini (2006). The interest rate in the baseline economy is smaller than the intertemporal discount rate, which is the interest rate in the complete markets economy. This is not surprising given the results of Huggett (1993) and Aiyagari (1994). What differs here is that the aggregate stock of capital is smaller than in the complete markets economy. In other words, market incompleteness leads to under-accumulation of capital. This is a direct consequence of the fact that the accumulation of capital is risky, so agents require a risk premium.

Table 2 also shows that the Gini index for wealth is small relative to the data (see Quadrini and Ríos-Rull 1997).<sup>10</sup> This is because shocks are i.i.d. and because there are no other sources of heterogeneity. If we assume that only a sub-group of agents have access to the risky technology—as we will briefly do in the next section—the model will generate a much higher concentration of wealth.

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$ER_k(w; k, z) = 1/\beta$ .

<sup>10</sup>Notice that in the complete markets economy, the distribution of wealth is not determined: any distribution of wealth is a steady state equilibrium as long as aggregate wealth does not change. See Chatterjee (1994) for a proof of this result.

Table 2: Steady state interest rate, capital stock, and wealth inequality in the baseline economy and the complete markets economy.

	Interest rate (%)	Aggregate capital	Gini index (%)
Baseline economy	4.91	0.680	33.92
Complete markets economy	5.26	0.704	–

### 3.1 Quantitative analysis of capital income taxation

To study the desirability of capital income taxation in the presence of uninsurable investment risks, we conduct a revenue neutral experiment as in Lucas (1990). More specifically, we reduce the capital income tax rate and increase simultaneously the labor income tax rate (while leaving the consumption tax rate unchanged) to keep government expenditure constant at its benchmark value. Our benchmark economy has tax rates on capital income, labor income and consumption equal to 40%, 20%, and 5.5% respectively. We consider four scenarios: (i) the capital tax rate is cut in half to 20%; (ii) the capital income is reduced to 10%; (iii) the capital income tax is eliminated entirely; and (iv) capital accumulation is subsidized with a negative capital income tax.

Before presenting in detail the results, we briefly discuss the basic channel. The analysis of the consequences of lowering the capital income tax rate involves the quantitative assessment of two opposing effects on capital accumulation and boosts welfare. On the one hand, the reduction in capital taxation increases the return on investments in risky capital and thus encourages capital accumulation, output and welfare. The increase in capital accumulation also relaxes the borrowing constraint. On the other hand, the implied increase in the labor income tax rate required to keep government purchases constant inhibits the insurance benefits provided by the labor income. Since capital accumulation is risky in this economy, this discourages capital accumulation and reduces welfare. This second effect can be

seen as arising through the insurance role played by the capital income tax in our framework. This is because higher capital taxes allow for lower labor income taxes which provide partial insurance against entrepreneurial risks through higher after-tax labor income.

We explore the results as follows: first, we discuss the steady state aggregate and redistributive effects of capital income tax reforms. Second, we present the transition dynamics. Finally, we report the welfare effects of the capital tax reforms for two cases: (i) the post-reform steady state and (ii) where we account for the short-run transition between the two economies. We present the steady state welfare in order to compare our work to the literature (eg., Aiyagari 1995).

## 3.2 Steady state aggregate and distributional effects

Table 3 presents the steady state capital stock, interest rate, wage rate, and wealth Gini index for various reductions of the capital income tax rate. The first panel of the table presents the model economy with the baseline tax rates, and the second panel displays the results for several revenue-neutral combinations of capital and labor income tax rates (keeping the consumption tax rate constant). The table shows that reducing capital income taxation increases the capital stock substantially. For example, eliminating capital income taxation from 40% increases capital accumulation by about 33.1% in the long-run. Furthermore, cutting the capital income tax by half increases the steady state capital stock by almost 20%. Subsidizing capital accumulation through a negative capital income tax leads to the largest increase in capital accumulation. Specifically, moving the capital income tax from 40% to -10% boosts capital accumulation by 37.1%.

The table also shows that both the labor income tax and pre-tax wage rates increase monotonically as the capital income tax falls. The increase in the labor income tax is much larger than the rise in the wage rate. For example, eliminating capital income taxation causes the tax on labor income to increase by 56.6%, while generating a moderate increase of 9.56% in the wage rate. Because the labor income tax increases by a wider margin than the pre-tax wage rate, the after-tax

wage rate drops following the capital income tax reduction. Such a decrease in the after-tax wage rate tightens agents' borrowing limits and inhibits their ability to insure against investment risks. This effect highlights the desirability of the capital income tax since higher rates allow the government to keep the tax on labor income lower, *ceteris paribus*.

It is also evident from the table that the risk-free interest rate falls as the capital income tax decreases. For example, the elimination of capital taxation leads to a significant decrease in the interest rate from 4.91% to 3.77%. This can be explained by at least two forces. First, the implied increase in the labor income tax makes the borrowing limit tighter. When this happens, the risk-free rate has to fall to dissuade agents from accumulating large amounts of savings in safe bonds so that the bonds market can clear. Such a decline in the riskless interest rate facilitates more investment in the risky technology. Second, the reduction in capital income taxes increases the riskiness of the business investment (the return net of taxes is more volatile). This increases precautionary savings, which also contributes to the fall in the interest rate, for a given investment size.

The reduction of the capital income tax rate also increases wealth inequality. For instance, the Gini coefficient of wealth increases by almost 30%. This stems not only from the fact that more agents take on more risk by investing in risky capital but also from the fact that the borrowing limit has tightened.

### **3.3 Transition dynamics**

The steady state comparisons conducted above show that reducing capital income taxation has substantial long-run macroeconomic effects in an incomplete markets economy with uninsurable investment risks. We now assess the short-run implications of the tax reform by taking into account the transition to the new steady state. This experiment should be interpreted as a surprise permanent decrease in the capital income tax rate. In this section, we only present the results for the elimination of the capital income tax. The results for a reduction in the capital income tax to another value are qualitatively similar.

Table 3: Steady state interest rate, capital stock, and wealth inequality when the capital income tax is replaced by a higher labor income tax.

	$\tau^k$	$\tau^w$	Interest rate	Wage rate	Aggregate capital	Gini index of wealth
	(%)	(%)	(%)			(%)
Baseline	40.00	20.00	4.91	2.165	0.680	33.92
Experiments						
	20.00	25.82	4.48	2.293	0.813	39.71
	10.00	28.51	4.23	2.345	0.872	42.62
	0.00	31.32	3.77	2.372	0.905	44.04
	-10.00	34.09	3.19	2.394	0.932	46.81

Figure 1 plots the transition dynamics induced by a revenue neutral experiment where the capital income tax is eliminated. Note that the labor income tax rate is raised to keep government purchases constant every period  $t$ . As can be seen from the figure, the interest rate increases sharply after the policy change and then converges gradually to the new steady state level. The elimination of the capital income tax increases the demand for capital immediately. However, supply responds only gradually through capital accumulation. This explains the overshooting. As panel (c) shows, the aggregate capital stock converges to a higher level only gradually. As capital increases, the demand for labor also increases; to clear the labor market, the wage rate must rise (see panel b). Panel (e) also shows that the labor income tax initially jumps significantly from 20% to about 35% so as to make up for lost capital tax revenues, then gradually falls to its new steady state level of 31.32%. Although the pre-tax wage rate increases during the transition, the labor income tax increases to a much greater extent so that the after-tax wage decreases (see panel f). Because of the lack of an insurance market against investment risks and due to the presence of borrowing constraints, the fall in after-tax wage income inhibits agents' ability to protect themselves against risky investment. Panel (d) of the figure shows that the concentration of wealth,

measured by the Gini index, rises. This is because there is more risky capital accumulation and because the borrowing limit has fallen. The increase in the riskiness of business investments leads to more precautionary savings and this contributes to the increase in wealth concentration.

### 3.4 Welfare effects

The previous two sections discussed the aggregate consequences in the steady state and during the transition. We now evaluate the welfare implications of lowering the capital income tax. As noted above, we report two sets of welfare results: one that compares only steady states and another that takes into account both the post-reform steady state and the short-run transition to this state.

For the first set of results, welfare effects are calculated as the aggregate additional consumption (proportionally distributed among agents) required to make all agents indifferent between living in the pre- and post-reform steady states. For the second set of welfare results, additional consumption is distributed to make all agents indifferent between remaining under the pre-reform tax system and undertaking a transition to the new steady state after the reform.

Let us formally define our welfare measure. Denote by  $V^{Initial}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Initial})$  the expected lifetime utility of an agent with net worth  $a$  that lives in the initial steady state of an incomplete market economy with capital income tax  $\tau^k = 0.40$ , labor income tax  $\tau^w = 0.20$  and consumption tax  $\tau^c = 0.055$ . The consumption in the initial steady state is given by  $c_t^{Initial}$ . The invariant distribution of agents over  $a$  in the initial steady state is denoted by  $M(a)$ . Define  $V^{New}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{New})$  to be the expected lifetime utility of an agent with net worth  $a$  following the revenue neutral tax reform. For the first set of results, at  $t = 0$ , the economy is already in the new steady state; for the second set,  $t = 0$  when the transition to the new steady state begins. The tax reform adjusts the labor income tax rate to keep government expenditure constant at its initial steady state value. The consumption gain from the policy shift, denoted by

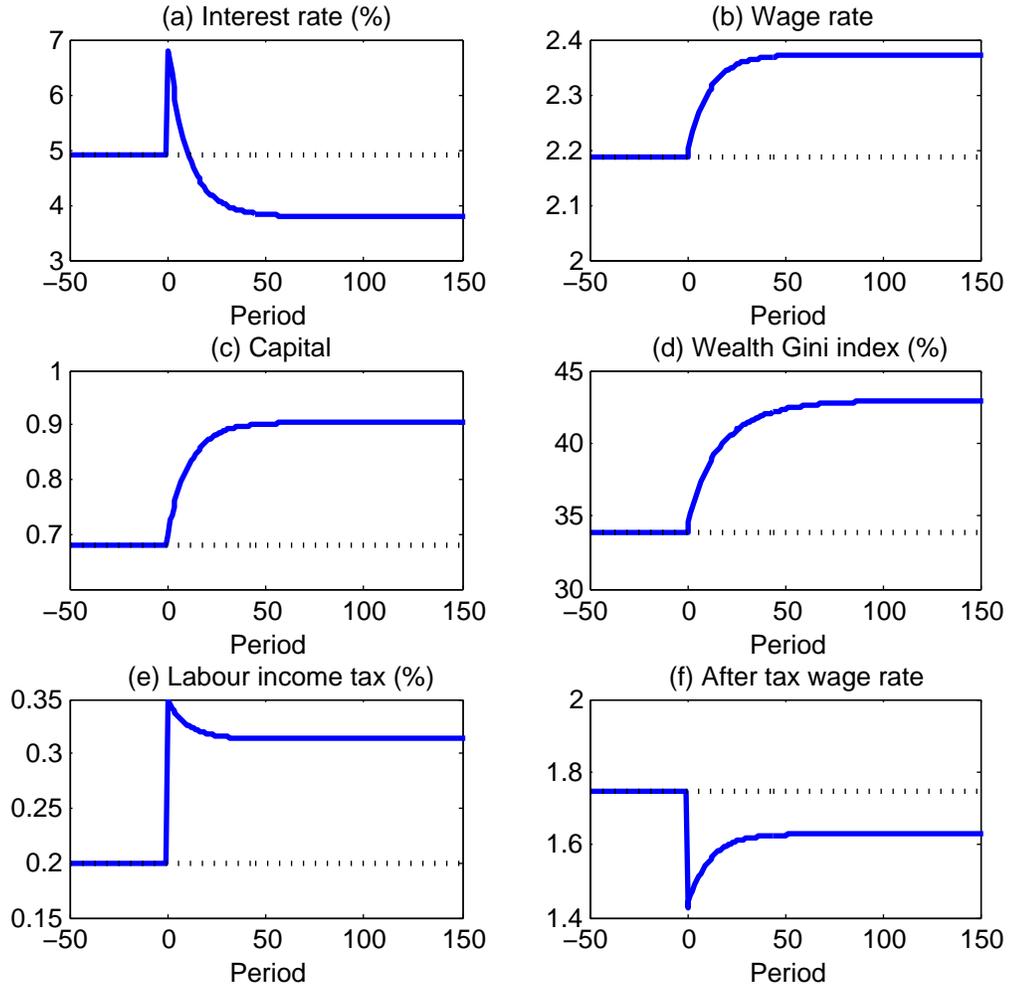


Figure 1: Transition to the new steady state with zero capital income tax when the labor income tax is raised to keep government purchases constant.

$g(a)$ , is determined by the following condition:

$$V^{New}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Initial} \cdot (1 + g(a))) = (1 + g(a))^{1-\sigma} \cdot V^{Initial}(a).$$

In other words, the consumption gain is determined by equalizing the lifetime utility achieved in the experiment with the lifetime utility obtained by increasing consumption in the initial steady state by  $c_t^{Initial}g(a)$  for all  $t$ .

The aggregate consumption gains (taking into account the transition to the new steady state) are given by:

$$\text{Gains} = \frac{\int_a c^{Initial}(1 + g(a))M(da)}{\int_a c^{Initial}M(da)} - 1,$$

where  $M(a)$  is the wealth distribution in the initial steady state.<sup>11</sup> The policy reform leads to a welfare gain if this measure is positive and to a loss otherwise. The welfare results are reported in Table 4.

**Steady state:** The first column of Table 4 presents the steady state welfare consequences of lowering capital income taxation. This first column shows that despite the fact that the elimination of capital income taxation or the subsidization of capital significantly increases capital accumulation, steady state welfare is higher with a positive capital income tax of 10%. Moving to a capital income tax of 10% generates a welfare gain of 3.71% of aggregate consumption while eliminating capital income taxation yields a welfare gain of 3.47%. In spite of the fact that uninsurable investment risks lead to an under-accumulation of capital, a zero capital income tax or a capital subsidy does not produce a higher steady state

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<sup>11</sup>The steady state aggregate consumption gain is given by  $\left(\frac{V^{new}}{V^{Initial}}\right)^{1/(1-\sigma)} - 1$ , where  $V^{new}$  is the aggregate welfare in the new steady state and computed with the invariant wealth distribution in the new steady state, and  $V^{Initial}$  is the aggregate welfare in the initial steady state and is obtained using the initial steady state invariant wealth distribution.

welfare than a positive capital tax. In other words, Aiyagari’s result in favor of a positive capital income tax in the long-run is maintained even in the presence of an under-accumulation of capital. The positive capital income tax is also in line with the findings of Golosov et al. (2003) and Albanesi and Sleet (2006) in an endogenous incomplete markets economy.

Table 4: Welfare effects, comparing steady states alone and taking transition dynamics into account, following the reduction of the capital income tax rate.

	Welfare, steady states only (%)	Welfare, including transition (%)	Percentage of agents in favor of the reform (%)
$\tau^k = 0.20$	3.12	0.766	86.10
$\tau^k = 0.10$	3.71	0.447	71.35
$\tau^k = 0.00$	3.47	-0.092	53.29
$\tau^k = -0.10$	2.69	-0.838	31.03

**Transition:** The second column of Table 4 displays the welfare results when we take into account the costs or gains associated with the transition. There are three points to highlight. First, as in the steady state case, the second column of the table shows that reducing capital income taxation to 10% and 20% improves welfare. However, the welfare gain is much smaller than that reported for the steady states. Specifically, the welfare gain is now 0.77% of aggregate consumption compared to the steady state when capital income tax is reduced to 20%. This is because of the short-run costs associated with the transition. Note also that when we account for the transition, a capital income tax rate of 20% generates a higher welfare gain than a capital income tax rate of 10%. Put differently, in contrast to the steady state results, a 10% capital income tax no longer leads to the highest welfare level when transition effects are taken into account.

Next, in contrast to the steady state results, the second column of the table shows that eliminating capital income taxation leads to a welfare loss and not a

welfare gain. The welfare loss is, however, small. For example, the elimination of the capital income tax, followed by an increase in the labor income tax to balance the government budget, generates a welfare loss of -0.092% of aggregate consumption. This is because the resulting increase in the labor income tax diminishes the after-tax wage and thus hinders the ability of agents to insure against investment risks. Moreover, the initial dramatic increase in the interest rate negatively affects financially constrained agents since their borrowing costs increase, which in turn reduces welfare.

Finally, the welfare gains/losses are not uniformly distributed. The last column of Table 4 shows the fraction of agents with positive consumption gains (winners) after the tax reform—taking into account the transition. It can be seen that a larger fraction of agents are in favor of a partial reduction in capital income taxation than a full elimination of capital income taxation or a capital subsidy. For further illustration of this point, we present graphically the results for the complete elimination of capital income taxation. The left panel of Figure 2 plots the welfare gains/losses as a function of the initial wealth of agents. It is interesting to note that there are large gains for (initially) wealthier agents and losses for (initially) low-wealth agents. The right panel plots the initial and final distribution of agents over assets. This highlights the relative importance of poorer agents (who lose in the transition) and wealthier agents (who are the main winners).

The distribution of the welfare gains/losses can be explained by at least two observations. First, after the elimination of the capital income tax, the aggregate demand for capital increases. Because supply responds slowly, the interest rate increases (see the first panel of Figure 1). The increase in the interest rate is beneficial for wealth holders — that is, richer agents. For poorer agents, the increase in the interest rate represents an increase in the cost of financing because they are net borrowers. Second, the large increase in the labor income tax impedes agents' ability to insure themselves against business failure, and this is especially harmful for less wealthy agents.

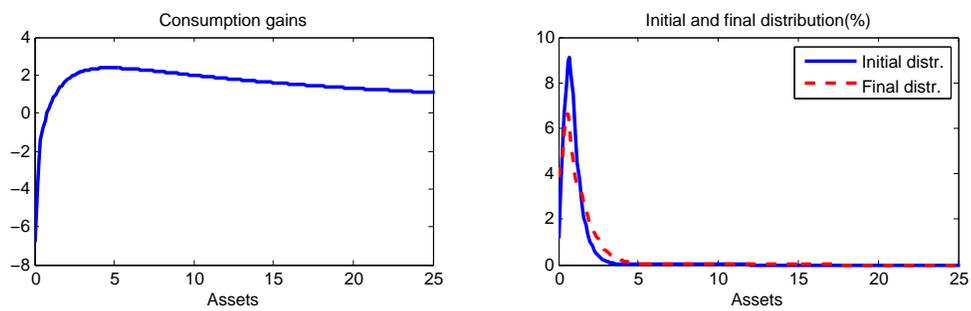


Figure 2: Distribution of the welfare gains following the elimination of capital income taxation, by household asset holdings.

**Discussion:** This analysis shows that moving to a moderate positive capital income tax leads to the highest steady state welfare gain even though uninsurable investment risks lead to under-accumulation of capital. This is an interesting result given the findings of Aiyagari (1995) and İmrohoroğlu (1998), who find that a positive capital income tax is necessary in the long-run because capital income taxation mitigates the over-accumulation of capital in light of uninsurable earnings risks. In our environment, where there is an under-accumulation of capital, one would have expected capital accumulation to be subsidized or at least subject to a zero capital income tax. The results in Table 4 speak instead to the contrary — that is, a positive capital income tax is welfare-improving despite the under-accumulation of capital.

When we account for the short-run costs associated with the transition, the result is even more pronounced. The best improvement in welfare is achieved with a positive tax rate that is even higher than the rate preferred in the steady state analysis. Moreover, a zero capital income tax is shown to lead to a small welfare loss.

This finding is due to the fact that the decrease in capital income taxation leads to an increase in the labor income tax, which impedes the ability of households to insure themselves against investment risks. When the transition effects are considered, the welfare result is also explained by the initial increase in the riskless interest rate, which increases the borrowing cost of financially constrained (low-wealth) agents.

**Sensitivity analysis: higher consumption taxation** Many papers find significant welfare gains from switching to consumption taxation rather than labor income taxation (see for example, Auerbach and Kotlikoff 1987; İmrohoroğlu 1998). In complete markets with inelastic labor supply, consumption taxation and labor taxation are equivalent. With uninsurable idiosyncratic risks, a switch to a consumption tax appears to be desirable a priori. First, the reform involves the same reduction in distortions, and therefore the aggregate effects of the welfare gain should be comparable. Second, the distributional losses should be smaller since

the strong correlation between wealth and consumption means that switching to a consumption tax involves a modest redistribution of the tax burden.

This economic intuition is confirmed for the case in which the capital income tax is eliminated in Table 5. The table illustrates that cutting capital income taxation increases the aggregate capital stock substantially. Eliminating capital income taxation increases the steady state capital stock by 33.68%. This result is consistent with the large body of literature on consumption taxation.

The table also shows that eliminating capital income taxation and replacing it with a higher tax on consumption reduces welfare.<sup>12</sup> The welfare loss from removing the capital income tax is -0.532% of aggregate consumption, while switching to a labor income tax is -0.092%. The finding that eliminating capital income taxation and replacing it with a consumption tax leads to a welfare loss is consistent with Domeji and Heathcote (2004). The difference is that we focus on uninsurable investment risks instead of earnings risks.

Table 5: Welfare and aggregate effects of reducing capital income taxation and compensating it by a consumption tax.

	$\tau^k$	$\tau^c$	Interest rate	Aggregate capital	Gini index of wealth	Welfare including transition
	(%)	(%)	(%)			(%)
Baseline	40.00	5.50	4.91	0.680	33.92	–
Experiments						
	20.00	11.56	4.54	0.813	39.51	0.440
	0.00	17.35	3.9961	0.909	42.52	-0.532

**Extension: Only a fraction of agents has access to the risky technology**

One possible interpretation of the risky investment is that it captures the risk associated with entrepreneurial activities. We can then assume that the agents

<sup>12</sup>In the case of the steady state welfare analysis, switching to consumption taxation leads a large welfare gain.

investing in the risky technology are the ones engaged in entrepreneurial activities. In line with this interpretation we assume that 10 percent of agents are in the position to invest in the risky technology.<sup>13</sup> We will refer to these agents as “entrepreneurs” and to the others as “workers”.

Entrepreneurs solve the same problem we have studied earlier. Workers, instead, solve a simpler problem. Because they face no risk, the consumption path can be easily determined using the Euler equation, the budget constraint, and the law of motion for wealth. That is:

$$\begin{aligned} U'(c_t) &\leq \beta(1 + r_t)U'(c_{t+1}) \\ a_t &= c_t + \delta_t b_t \\ a_{t+1} &= w_{t+1}(1 - \tau_{t+1}^w) + b_t \end{aligned}$$

The Euler equation is satisfied with the inequality sign if  $a_{t+1} = 0$ . That is, if the borrowing limit is binding. In the steady state the interest rate is lower than the intertemporal discount rate and the liability constraint binds, that is,  $a_t = 0$  for all  $t$ . The level of consumption is then equal to  $c_t = \delta w(1 - \tau^w)$ , where  $\delta$  and  $w$  are constant in a steady state. Table 6 reports some steady state statistics as well as the results from the tax reform.

The basic qualitative results do not change by assuming that only a fraction of the population has access to the risky investment. In particular, a decrease in the capital income tax leads to an increase in the aggregate stock of capital. The most notable change is the increase in the Gini index in the baseline. This is because only a small fraction of agents (the entrepreneurs) save. Although the model is stylized, this shows how entrepreneurial activities can generate a much higher concentration of wealth. This point is also made in Quadrini (2000) and Cagetti and DeNardi (2002).

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<sup>13</sup>See Quadrini (1999) for a documentation of the share of entrepreneurs in the population.

Table 6: Effects of reducing capital income taxation and compensating it by a labor tax when 10 percent of the population has access to the risky investment.

	$\tau^k$	$\tau^l$	Interest rate	Aggregate capital	Gini index of wealth
	(%)	(%)	(%)		(%)
Baseline	40.00	20.00	4.00	0.666	94.77
Experiments					
	20.00	25.45	2.79	0.791	94.89
	0.00	30.81	1.25	0.884	95.10

## 4 Conclusion

This paper investigates the effects of changing the mix of capital income and labor income taxation on capital accumulation and welfare in a general equilibrium model with uninsurable investment risks. The analysis is conducted focusing both on long-run and short-run effects.

There is a trade-off between capital income and labor income taxation. While a capital income tax discourages investments in the presence of uninsurable investment risks, a labor income tax reduces the insurance effect of labor income against those risks. The extent of this trade-off depends on the wealth of agents. Richer agents prefer a reduction in the capital income tax, whereas poorer agents prefer a reduction in the labor income tax.

As the capital income tax rate is reduced, the capital stock increases monotonically. However, welfare effects are not monotonic. When the capital income tax rate is reduced to a moderately positive level, there is a welfare gain both in the long-run and along the transition path to the new steady state. This implies that the welfare gains by richer agents dominate the loss incurred by poorer agents. As the tax rate is lowered further to zero, the welfare gain is reduced in the long run and it even leads to a small loss if we consider the short run effects along the transition path. Therefore, as the tax rate is lowered, the welfare losses among

poorer agents become larger relative to the gains among richer agents. As a result, despite the under-accumulation of capital, a moderate positive tax rate on capital income is still welfare-improving over zero capital tax rate or a capital subsidy.

Although this paper analyzes capital and labor income taxation and welfare with uninsurable investment risks, we have not fully addressed the optimal tax rates via Ramsey's solution. It is a natural next step. In addition, the model assumes that market incompleteness is exogenous. In another type of environment where the use of private insurance is endogenous, a provision of public insurance via a progressive income tax system can affect the degree to which private insurance is used. Since in the present paper the insurance effects played by labor income are important, an extension to include the interaction between public and private insurance is another topic for future research.

## Appendix: Computation of the equilibrium

**Steady state:** We start the procedure by guessing the steady state interest and wage rates. Given the prices, we solve problem (2) on a grid of points for the asset holdings  $a$  using value function iteration. After guessing the next period values of  $V(a)$  at each grid point, we approximate this function with a quadratic polynomial. Given the next period's value function, problem (2) is solved at each grid point using a maximizing routine that do not requires smoothness of the value function. We use the Fortran routine BCPOL.

Once the iteration on the value function has converged, we use the agents' policy rules to find the invariant distribution of agents over  $a$ . Starting from an initial distribution we iterate until convergence. After aggregating using the invariant distribution, we verify the clearing conditions in the capital and labor markets. Based on these conditions, we update the prices and restart the procedure until all markets (labor and capital) clear.

**New steady state:** The numerical procedure is similar to the procedure used to solve for the steady state of the baseline economy based on value function iteration. Because we conduct a revenue neutral experiment, we also guess the labor income tax rate needed to keep government expenditure constant at its benchmark value.

**Transition equilibrium:** To compute the transition from the initial steady state to the final steady state, we start the procedure by guessing sequences of prices,  $r$  and  $w$ , and labor income tax,  $\tau^w$ , for a certain number of periods. The number of periods is sufficiently long for the economy to get close to the new steady state equilibrium. Given the guessed sequences of prices and tax rates, we solve the agents' problem backwards at each grid point starting from the final transition period. In the final period, the economy should have converged to the new steady state, and therefore, we already know the solution. Once we have solved for all transition periods, we start from the initial period and compute the market clearing conditions and check that the government budget is balanced. We

then update the guessed sequences and continue until all markets clear and the government budget is balanced in all transition periods.

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