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The Empirical Performance of Alternative Monetary and Liquidity Aggregates

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Abstract

This paper examines the empirical performance of alternatives to the monetary aggregates currently published by the Bank of Canada. The results show that real M1 and real M1α perform about equally well in providing leading information about real output at short horizons. However, on theoretical grounds, M1α is a more attractive aggregate, since it excludes federal government deposits held at trust companies (M1 excludes only such deposits held at banks). Also, the broad aggregates — M2+, M2δ (the sum of M2+, treasury bills, provincial savings bonds, and Canada Savings Bonds), M3β (the sum of M2δ, mutual funds, 1- to 3-year government bonds, mortgage-backed securities, foreign holders of Canadian dollar deposits, and foreign currency deposits of residents booked in Canada), and LLβ (the sum of M3β, bankers' acceptances, and commercial paper issued by non-financial corporations) — generally are the best in providing leading information about inflation at long horizons (one to two years) across the various models that were considered. Also, in the context of P* models of inflation, M1 performs well as a leading indicator of inflation with long leads. Based on the analyses carried out in this paper, the aggregates M1, M1α, M2+, M2δ, M3β and LLβ are found to be deserving of further attention.

Résumé

Dans cette étude, l'auteur examine la tenue sur le plan empirique de différents agrégats monétaires autres que ceux qui sont publiés à l'heure actuelle par la Banque du Canada. Il en ressort que M1 et M1α réels se comportent à peu près aussi bien l'un que l'autre en tant qu'indicateurs avancés de la production réelle sur de courts horizons. Toutefois, M1α est plus intéressant du point de vue théorique, puisqu'il n'englobe pas les dépôts du gouvernement fédéral tenus dans les sociétés de fiducie (M1 n'exclut que les dépôts du gouvernement fédéral qui sont tenus dans les banques). De même, dans les différents modèles analysés par l'auteur, ce sont les agrégats au sens large — M2+, M2δ (c.-à-d. M2+ plus les bons du Trésor, les obligations d'épargne provinciales et les obligations d'épargne du Canada), M3β (c.-à-d. M2δ plus les fonds mutuels, les obligations du gouvernement de un à trois ans, les titres hypothécaires, les dépôts en dollars canadiens détenus par des étrangers et les dépôts en monnaies étrangères comptabilisés au Canada au nom des résidents) et LLβ (c.-à-d. M3β plus les acceptations bancaires et le papier commercial émis par des sociétés non financières) — qui fournissent généralement les meilleurs renseignements sur l'évolution future de l'inflation sur de longs horizons (de un à deux ans). Par ailleurs, des tests effectués à l'aide de modèles P* de prévision de l'inflation montrent que M1 est un bon indicateur de l'évolution à long terme de l'inflation. Il ressort des analyses effectuées dans le cadre de cette étude que les agrégats M1, M1α, M2+, M2δ, M3β et LLβ méritent qu'on s'y intéresse davantage.

CONTENTS

1.0 Introduction and Summary

1.1 Introduction

The study of the monetary aggregates is of particular interest to monetary authorities since these aggregates can serve as information variables, or perhaps even transmission variables, in the conduct of monetary policy. The relationships between the aggregates and other macroeconomic variables such as nominal income, employment, interest rates and prices can also be used to forecast changes in these latter variables. In this light, central banks are interested in choosing aggregates that would act as useful indicators.

In recent years, financial innovations have fundamentally altered the characteristics of many monetary assets. These innovations have increased the liquidity of most of the deposit liabilities of the chartered banks and trust companies. Such developments seem to call for new ways of defining and measuring the monetary aggregates.

In this paper, a series of alternatives to the monetary aggregates currently defined and published by the Bank of Canada is developed. The alternative aggregates are based on an analysis of the various financial products offered by Canadian financial institutions. To assess the empirical superiority of the proposed monetary aggregates, each of the proposed aggregates was subjected to empirical analysis, using the current aggregates as benchmarks. The criteria used in examining the performance of the aggregates were

1. the information content and causality of the aggregates
2. the stability of the demand function for the aggregates
3. the performance of the aggregates in a P^* model of inflation, where inflation is expressed as a function of the output gap, money gap (deviation of money from its long-run demand) and term structure.

1.2 Summary

From the empirical exercises summarized in this paper, a number of conclusions can be drawn. With respect to providing leading information about real output at short horizons, it was observed that real M1 and real M1α perform about equally well; however, on theoretical grounds, M1α is a more attractive aggregate, since it excludes from M1 federal government deposits held at trust companies. (M1 excludes only such deposits held at banks.)

The study also finds that the broad aggregates generally are the best in providing leading information about inflation at long horizons, that is, one to two years, across the various models that were considered. Indeed, there is often not much to choose among the various aggregates in terms of minimizing root-mean-square error (RMSE) in the various models that were examined. This is not surprising. The various broad aggregates are correlated with one another, and depending on particular developments in the estimation period, one or another of these aggregates can dominate in terms of smallest forecast error, but they all generally tend to tell the same story. Nevertheless, M2+, M2δ and M3β appear to be the best of the broad aggregates. (See Table 1 for the definition of the new aggregates.) In addition, however, in the context of P* models, M1 performs well as a leading indicator of inflation with long leads.

Based on the analyses carried out in this paper, the aggregates M1, M1α, M2+, M2δ, M3β and LLβ, reported in Table 1, are found deserving of further attention. M1 and M1α are recommended because they represent measures of transactions money. An assessment of the performance of the aggregates in indicator models shows that M1 and M1α provide information about nominal and real income. Also, the money gaps of M1 and M1α were found to play dominant roles in P^* models of inflation.

M2+ and M2δ are recommended because they measure store-of-value money. On empirical grounds, M2+ was found to be the best contemporaneous indicator of inflation and nominal income. It also performs reasonably well in out-of-sample predictions of inflation. M2δ was observed to be one of the best aggregates for predicting the growth rates of the CPI and the CPI excluding food and energy (CPIXFE) in indicator models. In the context of the vector error-correction model (VECM), M2δ was found to be among the best aggregates for predicting the growth rates of the GDP deflator.

Table 1 : Recommended Monetary Aggregates

Mnemonic	Components
M1	Currency Outside Banks + Demand Deposits at Banks
M1α	M1 – Cash and Gross Demand and Notice Deposits held by Trust and Mortgage Loan Companies (TMLs) and Credit Unions – Receiver General's Deposits at TMLs
M2+	M2 + Deposits in Near-Banks + Personal Deposits at Government Savings Institutions + Money Market Mutual Funds (MMMFs) + Annuities of Life Insurance Companies
M2δ	M2+ + T-Bills +Provincial Savings Bonds (PSBs) + Canada Savings Bonds (CSBs)
M3β	M2δ + Mutual Funds + 1-3 yr. Govt. Bonds + Mortgage-Backed Securities + Foreign Holders of Canadian Dollar Deposits + Foreign Currency Deposits of Residents Booked in Canada
LLβ	M3β + Bankers' Acceptances + Commercial Paper Issued by Non-financial Corporations

M3β is recommended because it represents a broad and encompassing measure of money. Within the framework of indicator models, M3β is the best aggregate to predict inflation at short and long horizons. LLβ aggregate is recommended because empirical analyses show that it also performs well as a leading indicator of inflation with long leads.

The out-of-sample results in this paper must be accepted with caution, for the minimum RMSE might not be the best criterion by which to judge the forecasting performance of an aggregate. It was found that some of the aggregates that have relatively low root-mean-square forecast errors failed to capture the turning points of the path of inflation. For example, based on the RMSE, M3β is the best predictor of the one-quarter-ahead forecast of the CPI. However, graphs of the out-of-sample forecasts suggest that M2+ and M3α are about as good predictors of the one-quarter-ahead forecast for the CPI.

As noted above, the aim of this project was to provide an assessment of the usefulness of the alternative and conventional aggregates in a number of contexts. At this stage, the most promising avenue for further work is the examination of the performance of the six aggregates (M1, M1α, M2+, M2δ, M3β and LLβ) suggested above in distant early warning (DEW) inflation models, that is, models that provide better leading information of inflation with long leads, at least one to two years.

One of the principal ways that this research could be taken ahead is to construct DEW models that include unique, stable and well-specified long-run money-demand functions, that is, cointegrating relationships among money, prices, output and interest rates that satisfy economic theory with respect to both the elements of the cointegrating vector and the speed-of-adjustment parameters (the loadings). Hendry (1995) has recently estimated such relationships for M1. Thus, the next stage of this work will be to build on the P^* results by developing DEW models of inflation based on the output gap, Hendry's M1 gap and measures of the term structure of the interest rates.[1]

This paper aims at examining the empirical performance of the alternative aggregates. The paper is organized as follows. Section 2 describes and provides the rationale for the alternative aggregates. Section 3 describes the results of the Granger-causality tests, the information content and the predictability of the aggregates. Section 4 presents the results of the estimation of the demand-for-money functions. Section 5

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1. As noted in Section 5.1, the original version of the modified P^* model included an interest rate gap, which has been proxied by the term structure in earlier work. Recently, Mishkin (1990) and others have found the term structure to be a good predictor of inflation in the United States.

summarizes the results of the examination of the performance of the aggregates in a P^* model of inflation. Section 6 provides conclusions about the work.

2.0 Definition of the Proposed Monetary Aggregates

Construction of the alternative monetary aggregates is based on an analysis of the various financial products offered by Canadian financial institutions. Two questions were considered in defining the alternative monetary aggregates. First, do the components included in the alternative aggregates serve as transactions balances or as a medium of exchange? In other words, are the assets accepted in exchange for goods, services and other assets? The second question that was considered is whether the components are readily convertible into transactions balances, that is, whether economic agents view the asset as a highly liquid alternative to transactions balances.

On the basis of this approach, alternative monetary aggregates were proposed. The current monetary aggregates are presented in Table 2, and the proposed alternative aggregates (in decreasing order of liquidity) and the criteria used for construction are presented in Table 3. The data for the alternative aggregates are quarterly, seasonally adjusted, and are available from the second quarter of 1968 to the fourth quarter of 1992.

Table 2: Current Monetary Aggregates

Mnemonic	Components
M1	Currency Outside Banks + Demand Deposits at Banks
M2	M1 + Non-personal Notice Deposits + Personal Savings Deposits in the Banks
M3	M2 + Non-Personal Term Deposits + Foreign Currency Deposits of Residents Booked in Canada
M2+	M2 + Deposits in Near-Banks + Personal Deposits at Government Savings Institutions + Money Market Mutual Funds (MMMFs) + Annuities of Life Insurance Companies

Table 3 : Alternative Monetary Aggregates

Mnemonic	Components	Criteria for Aggregation
M1α	Currency + Low-Interest-Bearing Chequing Accounts (Personal and Non-personal)	Non-Interest-Bearing Assets
M1β	M1α + Interest-Bearing Chequing Accounts (Personal and Non-personal)	Transactions Balances
M2α	M1β + Non-Chequing Notice Deposits + CSBs + PSBs – (Registered Retirement Savings Plans + Registered Retirement Income Funds)	Assets with Short Maturity and Deterministic Prices
M2β	M2α + Money Market Mutual Funds + T-Bills	Assets held by Households with Short Maturity and Deterministic or Less Volatile Prices
M2γ	M2β + Fixed-Term Deposits (Personal and Non-personal) with More than 1 yr. to Maturity + Annuities – (Registered Retirement Savings Plans + Registered Retirement Income Funds)	Assets with Long Maturity and Deterministic or Less Volatile Prices
M2δ	M2γ + Registered Retirement Savings Plans + Registered Retirement Income Funds	Assets with Deterministic or Less Volatile Prices and Tax-Sheltered Status
M3α	M2δ + 1-3 yr. Govt. Bonds + Bond Funds + Equity Funds + Mortgage Funds + Mortgage-Backed Securities	Assets with Deterministic or Volatile Prices and Tax-Sheltered Status
M3β	M3α + Foreign Holders of Canadian Dollar Deposits + Foreign Currency Deposits of Residents Booked in Canada	Assets with Deterministic or Volatile Prices, Tax-Sheltered Status and Exchange Risk

M1α (see Table 3) is defined to capture all low-interest-bearing assets that are accepted in the economy for the discharge of debt. The aggregate also reflects the most liquid assets in the economy. M1β expands M1α to include all interest-bearing chequing accounts in the economy. Like M1α, M1β also measures the most liquid assets in the economy.

M2α is constructed so as to capture other savings instruments that can be readily converted into transaction balances and it captures all liquid assets in the economy. M2β measures all transactions balances and liquid assets with short maturity periods. M2γ is the sum of M2β and fixed-term deposits and annuities. These fixed-term deposits are not very liquid. Annuities have also been included in M2γ, since they are also fixed-term deposits that are held as stores of value. M2δ includes other assets that provide tax shelters to the holders. M2δ is defined as M2γ plus registered retirement savings plans (RRSPs) and registered retirement income funds (RRIFs).[2] In comparison to M2γ, M2δ is relatively illiquid because of the characteristics of the RRSPs and the RRIFs. M2δ is therefore classified as a set of transaction balances, liquid assets and non-liquid assets.

M3α is defined as M2δ plus the sum of 1- to 3-year government bonds, bond funds, equity funds, mortgage funds and mortgage-backed securities. The market value of each of the additional assets is very volatile and therefore cannot be predicted accurately. Furthermore, with the exception of 1- to 3-year government bonds and annuities, the assets face a default risk, as they are not insured. M3β adds to M3α foreign currency deposits of residents booked in Canada and foreign holders of Canadian dollar deposits. Although foreign currencies are easily convertible into Canadian funds, the fluctuations in exchange rates make it difficult to predict the market value of these deposits. The exchange rate risk therefore reduces the liquidity of these assets. Also the additional components to M3β are held for store-of-value reasons.

3.0 Estimation of the Indicator Models and Information Content

The results of the estimation of the indicator models are presented in this section. The exercise was carried out in the following steps. First, Granger-causality tests were conducted on each of the monetary aggregates with respect to selected macroeconomic variables. Second, based on the

2. To prevent double counting, other assets, such as money market mutual funds that are held as RRSPs and RRIFs, are excluded.

results of the Granger-causality test, indicator models of money and the goal variables were estimated. Third, the out-of-sample forecasts for the one-quarter-ahead, two-quarter-ahead, four-quarter-ahead, eight-quarter-ahead and twelve-quarter-ahead forecasts of the goal variables were examined. The forecasts were based on the predicted cumulative growth rates of the goal variables.[3]

3.1 Summary Statistics

Before embarking on the estimation of indicator models, I examined the correlation between the monetary aggregates and five macroeconomic variables (the three measures of inflation, plus nominal and real GDP). The results show that

- M2 is the best contemporaneous indicator of all measures of prices.
- M2+ is the best contemporaneous indicator of nominal GDP.
- Real M1 is the best contemporaneous indicator of real GDP.
- M2 is the best leading indicator of CPI over a one- to three-quarter horizon. Over a four- to seven-quarter horizon, M2+ is the best indicator, while M2γ is the best indicator for eight quarters out.
- M2 is the best leading indicator of CPIXFE over a one- to five-quarter horizon; for six to eight quarters out, M2+ is the best.
- M2 is the best leading indicator of the GDP deflator for one quarter ahead. M2+ is the best leading indicator over a two- to five-quarter horizon; for six to eight quarters out, M2γ is the best.
- M2+ is the best leading indicator of nominal GDP for one quarter ahead. M1 is the best leading indicator over a two- to four-quarter horizon. For a five- to eight-quarter horizon, M2γ is the best.
- Real M1α is the best leading indicator of real GDP over a one- to

3. I also examined the stationarity of the various aggregates along with other key macroeconomic variables. I found that, with the exception of M2, M3, M2+ and M2α, which are integrated of order 2, the monetary aggregates, velocities and selected goal variables (real and nominal GDP, CPI, CPIXFE and GDP deflator) are integrated of order 1.

two-quarter horizon. For three to five quarters out, the best is real M2α.

3.2 Granger-Causality Test

The Granger-causality test was conducted to determine the direction of causality between the one-quarter growth rates of GDP (real and nominal), CPI, CPIXFE (CPI excluding food and energy) and GDP deflator and the one-quarter growth rates of the proposed monetary aggregates. This test was conducted with a view to eliminating aggregates that were found not to cause a goal variable for our examination of indicator models.

From the Granger-causality tests, the following were observed:

- Each of the aggregates M1, M2, M2+, M3, M1α, M2δ, M3α and M3β was found to Granger-cause the CPI.
- Each of the aggregates M2, M2+, M3, M2δ, M3α and M3β was found to Granger-cause the CPI excluding food and energy (CPIXFE).
- Each of the aggregates real M1, real M1α, real M1β, real M2α and real M2β was found to Granger-cause real GDP.

3.3 Estimation of the Indicator Models and Information Content

The estimated indicator-models were constructed based on our results from the Granger-causality tests. Monetary aggregates which were found to have no direction of causality with the goal variables were excluded. For efficiency reasons, the regression equations for the monetary aggregates and the goal variables were jointly estimated.

The information content of a monetary aggregate (M) relative to a goal variable (G) is defined as

$$I(G|M) = -0.5\log[(1-R_*^2)/(1-R^2)] \qquad (1)$$

where R_*^2 and R^2 are, respectively, the coefficients of determination for the following regression equations:

$$G = A^p(L)G + B^q(L)M + \nu \quad (2)$$

$$G = A^p(L)G + \xi \quad (3)$$

where ν and ξ are error terms and A^p and B^q are lag operators of length p and q, respectively. For a given level of R^2, equation (1) suggests that the measure of information content rises with an increase in R_*^2. Hence, the R_*^2 for the estimated indicator models was used to rank the information content of the monetary aggregates; the three best models are reported in Table 4.

Table 4: Three Best Indicator Models in Estimation, Based on $\bar{R}^2$

Rank	CPI	CPIXFE	GDP Deflator	Nominal GDP	Real GDP
1	M2 (0.6782)[a]	M2 (0.6111)	M2+ (0.6750)	M1 (0.6672)	RM1α[b] (0.3188)
2	M2+ (0.6641)	M2+ (0.6026)	M3 (0.6736)	M1α (0.6615)	RM1 (0.3064)
3	M1 (0.6538)	M3 (0.6003)	M2 (0.6721)	M2β (0.5963)	RM2α (0.2845)

a. $\bar{R}^2$ is recorded in parentheses.

b. Note that an "R "prefix indicates that money is in real terms, using the CPI.

The results, which support earlier works of the Bank of Canada, suggest that[4]

- M2 is the most informative monetary aggregate for CPI and CPIXFE.
- M2+ is the most informative monetary aggregate for the GDP deflator.
- M1 is the most informative monetary aggregate for nominal GDP.
- Real M1α is the most informative monetary aggregate for real GDP.

4. See Hostland, Poloz and Storer (1988) and Muller (1990).

3.4 Performance of the Indicator Models in Forecasting

To ascertain the reliability of the indicator models as sources of information for key macroeconomic variables, their performance in predicting the goal variables is examined. The predictive performance of the competing models was gauged by running recursive regressions over the sample period. This method involves estimating the model from 68Q1 to 81Q4 and making a *k*-quarter-ahead forecast, where k = 1, 2, 4, 8 and 12. The models are reestimated by extending the sample size by a quarter at a time and making a *k*-quarter-ahead forecast each time. The process continues until the data points are exhausted. The RMSE for each model is then calculated.

The estimated models were used to determine which of the alternative monetary aggregates is a useful leading indicator of macroeconomic variables, such as inflation and output (real and nominal), at distant horizons. Thus, the performance of the monetary aggregates in DEW models is examined in this section. Generally, the DEW model is of the form

$$G_t^k = \alpha + \sum_{i=0}^{p} \beta_i g_{t-k-i} + \sum_{j=0}^{q} \gamma_j m_{t-k-j} + \varepsilon \qquad (4)$$

where $G_t^k = (400/k)\log(G_t/G_{t-k})$ and k = 1, 2, 4 and 8. Also, *g* and *m* are the quarterly growth rates of the goal variables and the monetary aggregates, respectively. Using the various measures of money, equation (4) was used to examine the out-of-sample *k*-quarter growth rates of the goal variables.[5] The recursive method of regression was applied to equation (4) to obtain the RMSE for each of the indicator models. Note that the lag lengths of the indicator models were determined by the Akaike information

5. I stopped at the eight-quarter-ahead forecast because of the sample size.

criterion. The three best predictors of the goal variables are presented in Table 5.

Table 5: Three Best Aggregates in the Prediction of the Annualized Cumulative Growth Rates of the Goal Variables

Quarters	Rank	CPI	CPIXFE	GDP Deflator	Nominal GDP	Real GDP
1	1	M3β (1.8734)[a]	M3β (1.6754)	M3β (1.8472)	M1 (2.7229)	RM1α[b] (3.3233)
	2	M2 (1.9442)	M2 (1.7366)	M2+ (1.8989)	M1α (2.7761)	RM1 (3.3685)
	3	M2+ (1.9846)	M3 (1.7632)	M2 (1.9471)	M2β (2.9731)	RM2α (3.3815)
2	1	M2+ (1.9597)	M3β (1.5637)	M2+ (1.7153)	M2α (2.9683)	RM2α (2.9839)
	2	M3 (1.9720)	M3 (1.6697)	M3β (1.8972)	M2β (3.0816)	RM1α (3.0661)
	3	M2 (1.9978)	M2 (1.7246)	M3 (1.9582)	M1α (3.1036)	RM2β (3.0716)
4	1	M2+ (1.8377)	M3β (1.3654)	M2+ (2.2241)	M2α (3.5396)	RM2α (2.6505)
	2	M2 (1.9277)	M2 (1.4048)	M2 (2.3183)	M1 (3.6613)	RM2β (2.7612)
	3	M1 (1.9652)	M2+ (1.5280)	M3β (2.3423)	M1α (3.8089)	RM1α (2.8952)
8	1	M2+ (1.7031)	M2 (1.1987)	M3β (3.0238)	M2α (4.0194)	RM2α (2.1295)
	2	M2δ (1.7032)	M3β (1.3875)	M2+ (3.2876)	M2β (4.4093)	RM2β (2.3896)
	3	M3α (1.9344)	M2δ (1.5158)	M2 (3.3654)	M1α (4.4220)	RM1 (2.4448)

a. The RMSE is recorded in parentheses.

b. Note that an “R” prefix indicates that money is in real terms, using the CPI.

Based on the minimum RMSE and Table 5, the main results of the out-of-sample forecast of the annualized cumulative growth rates of goal variables are as follows:

- M3β is the best predictor of the one-quarter growth rate of the CPI; M2+ is the best predictor of the two-, four- and eight-quarter cumulative growth rates of the CPI.

- M3β is the best predictor of the one-, two- and four-quarter cumulative growth rates of CPIXFE; for the eight-quarter cumulative growth rates, M2 is the best predictor.

- M3β is the best predictor of the one- and eight-quarter cumulative growth rates of the GDP deflator, while M2+ is the best for the two- and four-quarter cumulative growth rates.

- M1 is the best predictor of the one-quarter growth rate of nominal GDP; M2α is the best predictor of the two-, four- and eight-quarter cumulative growth rates of nominal GDP.

- Real M1α is the best is the best predictor of the one-quarter growth rate of real GDP; real M2α is the best predictor of the two-, four- and eight-quarter cumulative growth rates of real GDP.

3.5 Performance of the Liquidity Aggregates

In this section the empirical performance of two liquidity aggregates are compared with those of the monetary aggregates. The liquidity aggregates are LL (M3 + bankers' acceptances + commercial paper issued by non-financial corporations) and LLβ (M3β + bankers' acceptances + commercial paper issued by non-financial corporations). The analyses were conducted as follows. First, the correlation between the liquidity aggregates and selected macroeconomic variables (the three measures of inflation, plus nominal and real GDP) were examined. Second, Granger-causality tests were conducted on each of the liquidity aggregates with respect to the macroeconomic variables. Third, the information content of the liquidity aggregates were assessed through estimation of indicator models based on LL and LLβ aggregates. Lastly, the out-of-sample forecast performance of indicator models based on the liquidity aggregates was examined and compared with the models based on the monetary aggregates.

The results of the correlation coefficients between the four-quarter growth rate of the LL and LLβ aggregates and the four-quarter growth rates of the selected macroeconomic variables show that

- LL and LLβ aggregates dominate M2 and M2+ as the best contemporaneous indicator of CPIXFE, the GDP deflator, and nominal income.
- Real M1 and M2 are maintained as the best contemporaneous indicators of real income and the CPI, respectively.
- With regard to the leading indicator properties (intertemporal correlations), the LL and LLβ aggregates do not perform better than any of the monetary aggregates considered in the previous section.

The results of the Granger-causality tests between the LL and LLβ aggregates and the selected macroeconomic variables show that

- The LL and LLβ aggregates independently Granger-cause all the three measures of prices.
- Also, the LL and LLβ aggregates do not independently Granger-cause both nominal and real GDP.

Comparing the information content of the LL and LLβ with that of the monetary aggregates, it was observed that

- the LLβ aggregate dominates M2+ as the most informative aggregate for the GDP deflator.
- With regard to the CPI, CPIXFE and GDP (nominal and real), the LL and LLβ aggregates are marginally less informative than any of the monetary aggregates.

In terms of the out-of-sample forecast of the annualized cumulative growth rates of the goal variables at various horizons, the following were observed:

- LLβ dominates M2+ as the best predictor of the two-quarter growth rate of CPI.

- LLβ dominates M3β as the best predictor of the one-, two- and four-quarter cumulative growth rates of CPIXFE.

- LLβ dominates M3β as the best predictor of the one-quarter growth rate of the GDP deflator.

The analyses carried out in this section show that the LLβ aggregate also performs equally as M3β in providing information for prices.

4.0 Demand-for-Money Functions

A great deal of research time has been spent by economists studying the quantity of money demanded by agents in a particular economy. In particular, the Bank of Canada has devoted much attention to this subject in a bid to ascertain whether any monetary aggregate could act as an intermediate target for monetary policy. Earlier research at the Bank, which found M1 to have a stable and predictable short-run demand function, led the Bank to target the growth rate of M1 from 1975 to 1981.

However, in the early 1980s, financial innovations fundamentally altered the characteristics of many monetary assets. These innovations, which increased the liquidity of most of the deposit liabilities of the chartered banks and trust companies, caused a shift out of M1-type accounts into new instruments. The demand for M1 became very unstable and unpredictable. The Bank therefore abandoned the targeting of M1 in 1981.

Recent research at the Bank has focussed on the demand for the broader monetary aggregates.[6] Until recently, these studies failed to find stable, long-run cointegrating relationships between the broad aggregates and real income, prices and interest rates, unless a dummy variable was included in the regression equation to account for the financial innovation in the early 1980s.

However, McPhail (1993) recently used the Johansen and Juselius (1990) methodology and found a stable long-run cointegration relationship between a broader aggregate (M2+ plus CSB and T-bill) and real income, interest rates and inflation. No dummy variable was required in McPhail's study, for the early 1980s, in order to find evidence of cointegration. The purpose of this section is to follow McPhail (1993) and investigate the existence of a stable long-run demand function for the alternative monetary aggregates using the Johansen and Juselius (1990) methodology.

4.1 The Johansen and Juselius Maximum-Likelihood Approach to Cointegration

Recent studies in the econometric literature have found certain drawbacks to the Engle and Granger (1987) residual-based tests for cointegration and have suggested other, new techniques. One of the techniques, which is now widely used in the literature, is the methodology of Johansen and Juselius (1990).

The Johansen and Juselius methodology can be illustrated by considering a vector autoregressive model of the form

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + \varepsilon_t \qquad (5)$$

where t = 1, …, T, Y_t is an N x 1 time-series vector and ε_t is a vector of independent Gaussian white noises with mean zero and finite covariance

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6. See McPhail (1993), Caramazza, Hostland and McPhail (1992) and Caramazza, Hostland and Poloz (1990). However, Hendry (1995) focusses on the long-run demand for M1.

matrix Ω_t. Johansen and Juselius then express equation (5) as a VECM model of the form

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \ldots + \Gamma_p \Delta Y_{t-p+1} + \Pi Y_{t-p} + \varepsilon_t \qquad (6)$$

where $\Gamma_i = -(I - A_1 - \ldots A_i), (i = 1, \ldots, p-1)$, and $\Pi = -(I - A_1 - \ldots A_p)$.

Now, if Y_t is a vector of I(1) variables, then the left-hand side and the first *p* elements of equation (6) are I(0) and the last element is a linear combination of I(1) variables. Johansen and Juselius use canonical correlation techniques to estimate all the distinct combinations of the levels of *Y* that are maximally correlated with the I(0) elements in equation (6). These combinations are the cointegration vectors.

Johansen and Juselius demonstrate that the matrix of coefficients in Π contains the essential information to determine the number of cointegration vectors between the variables in Y_t. The cointegration vectors would have the property that $\beta' Y_t$ is stationary, even though Y_t is nonstationary. Johansen and Juselius consider the test of the hypothesis that there are r (versus N) cointegration relationships. This test is formulated as the restriction:

$$H_o: \qquad \Pi = \alpha\beta' \qquad (7)$$

A step-by-step procedure for estimating β can be found in Johansen and Juselius (1990), Dickey, Jansen and Thorton (1991) and McPhail (1993).

4.2 Identifying the Cointegration Vectors

Besides the monetary aggregates, the other variables included in the system of equations to determine the cointegration vectors are real GDP, the GDP deflator and R90. With the exception of R90, the rest of the variables were converted into their logarithmic values. The basic Johansen and Juselius methodology is designed for variables that are integrated of order one. I therefore checked the order of integration for all the variables used in this study. The tests suggest that all the variables used in this research are generally integrated of order one.

In Table 6, I report the estimated cointegration vectors that have some of the characteristics of a money-demand function. For each of the cointegration vectors, in Table 6, the first term is the coefficient of money initialized to unity, the second term is income elasticity, the third term is semi-interest elasticity and the fourth is the semi-elasticity of inflation.

In summary, the Johansen and Juselius tests suggest the existence of cointegration relationships between the real values of the monetary aggregates (deflated by the GDP deflator), real GDP, R90 and inflation measured by the GDP deflator. The interpretation of these cointegration vectors as long-run money-demand functions is mixed.[7] In general, for the vectors which might be described as money-demand functions, I found the semi-interest elasticity for the narrower aggregates to be larger than those of the broader aggregates. I also found, in some cases, the semi-interest elasticities for the broader aggregates to be positive. I attribute the positive semi-interest elasticities to the absence of a measure of the own-rate of return for the aggregates in the VAR used to determine the cointegration vectors. As expected, I also found the income elasticities of the broader aggregates to be greater than the narrow aggregates. Also, the coefficients for inflation were positive for the narrower aggregates and negative for the broader aggregates.[8]

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7. This analysis focussed on whether there were cointegrating vectors that took on values that were consistent with money-demand functions, along the lines of past work conducted at the Bank and generally in the literature. However, this work did not address other criteria, such as determining unique cointegrating vectors in the cointegrating space, or ensuring that the speed-of-adjustment parameters in the dynamic equations (the loadings) in the Johansen-Juselius systems took on values that were consistently satisfying from a theoretical perspective. Both of these approaches are very time-consuming extensions. For an example of work that addresses these other issues in the context of examining the demand for M1, see Hendry (1995).

8. I also tested for homogeneity between real money balances and real income. In other words, I checked whether the long-run income elasticity is unity. Based on the likelihood-ratio statistic, the restriction on income is rejected for M2, M2+, M2δ and M3α.

Table 6: Estimated Money-Demand (Cointegration Vector) Functions

System	λ-max	Trace	Cointegration Vectors (β)	Loadings (α)			
				Money	LRY	R90	G1DEF
LRM1, LRY, R90, G1DEF	84.20[1] 48.66[1]	156.65[1] 72.44[1]	[1 -0.432 0.135 -0.019] [1 -0.341 0.018 -0.032]	-0.034 -0.023	-0.014 -0.010	-0.644 -1.571	-0.221 21.631
LRM2, LRY, R90, G1DEF	57.70[1]	93.86[1]	[1 -1.263 0.018 0.012]	-0.031	-0.053	-2.152	-6.691
LRM3, LRY, R90, G1DEF	49.42[1]	79.68[1]	[1 -1.061 0.204 -0.041]	-0.008	-0.009	-0.505	-0.145
LRM2+, LRY, R90, G1DEF	68.85[1]	105.48[1]	[1 -1.529 0.021 0.015]	-0.035	-0.045	-1.671	-6.550
LRM1α, LRY, R90, G1DEF	77.32[1] 49.63[1]	149.76[1] 72.44[1]	[1 -0.164 0.155 -0.019] [1 -0.165 0.020 -0.036]	-0.030 -0.028	-0.011 -0.006	-0.529 -1.480	-0.227 19.042
LRM1β, LRY, R90, G1DEF	34.74[1]	59.00[1]	[1 -1.365 0.131 -0.053]	0.002	-0.019	-1.695	1.020
LRM2α, LRY, R90, G1DEF	26.37[10]	52.04[1]	[1 -0.690 -0.068 0.049]	-0.012	0.019	2.154	-1.087
LRM2β, LRY, R90, G1DEF	28.48[5]	55.73[1]	[1 -0.703 -0.124 0.103]	-0.005	0.011	1.240	-0.745
LRM2γ, LRY, R90, G1DEF	57.66[1] 31.43[1] 17.50[5]	112.52[1] 54.85[1] 22.41[1]	[1 -1.395 0.009 0.016] [1 -0.914 -0.122 0.100] [1 -1.707 -0.002 -0.001]	-0.050 -0.000 -0.028	-0.033 0.011 0.035	-0.606 1.313 -6.116	-5.913 -1.748 2.209
LRM2δ, LRY, R90, G1DEF	66.91[1]	115.63[1]	[1 -1.547 0.009 0.015]	-0.056	-0.044	-1.363	-8.637
LRM3α, LRY, R90, G1DEF	66.69[1]	110.33[1]	[1 -1.623 0.013 0.018]	-0.041	-0.045	-1.521	-9.149
LRM3β, LRY, R90, G1DEF	55.77[1] 35.04[1] 16.64[5]	114.27[1] 58.50[1] 23.45[1]	[1 -0.902 -0.015 0.050] [1 -1.457 -0.110 0.061] [1 -1.831 -0.013 -0.017]	-0.019 -0.000 -0.030	-0.009 0.018 0.003	0.342 1.462 -2.146	-2.448 -1.495 3.673

1: Significant at the 1% level
5: Significant at the 5% level
10: Significant at the 10% level

4.3 Estimating Dynamic Money-Demand Equations

Dynamic money-demand equations for all the monetary aggregates were estimated using the cointegration vectors estimated by Johansen and Juselius methodology as an error-correction model (ECM). Thus, the following regression equation was used in estimating the demand function for all the monetary aggregates:

$$\begin{aligned} A(L)\Delta Money_t = \; & A_0 + B(L)\Delta GDP_t + C(L)\Delta R90_t + D(L)\Delta infl_t \\ & + \gamma ECM_{t-1} + \varepsilon_t \end{aligned} \tag{8}$$

where Δ is the difference operator, the last term is an error term and money and GDP are in real terms. Within the above framework, the ECM term acts as a measure of disequilibrium in any period.[9] The results, reported in Table 7, show that with the exception of M1β, M2α, and M2β, the ECM term is very important in the dynamic money-demand equations for the monetary aggregates.

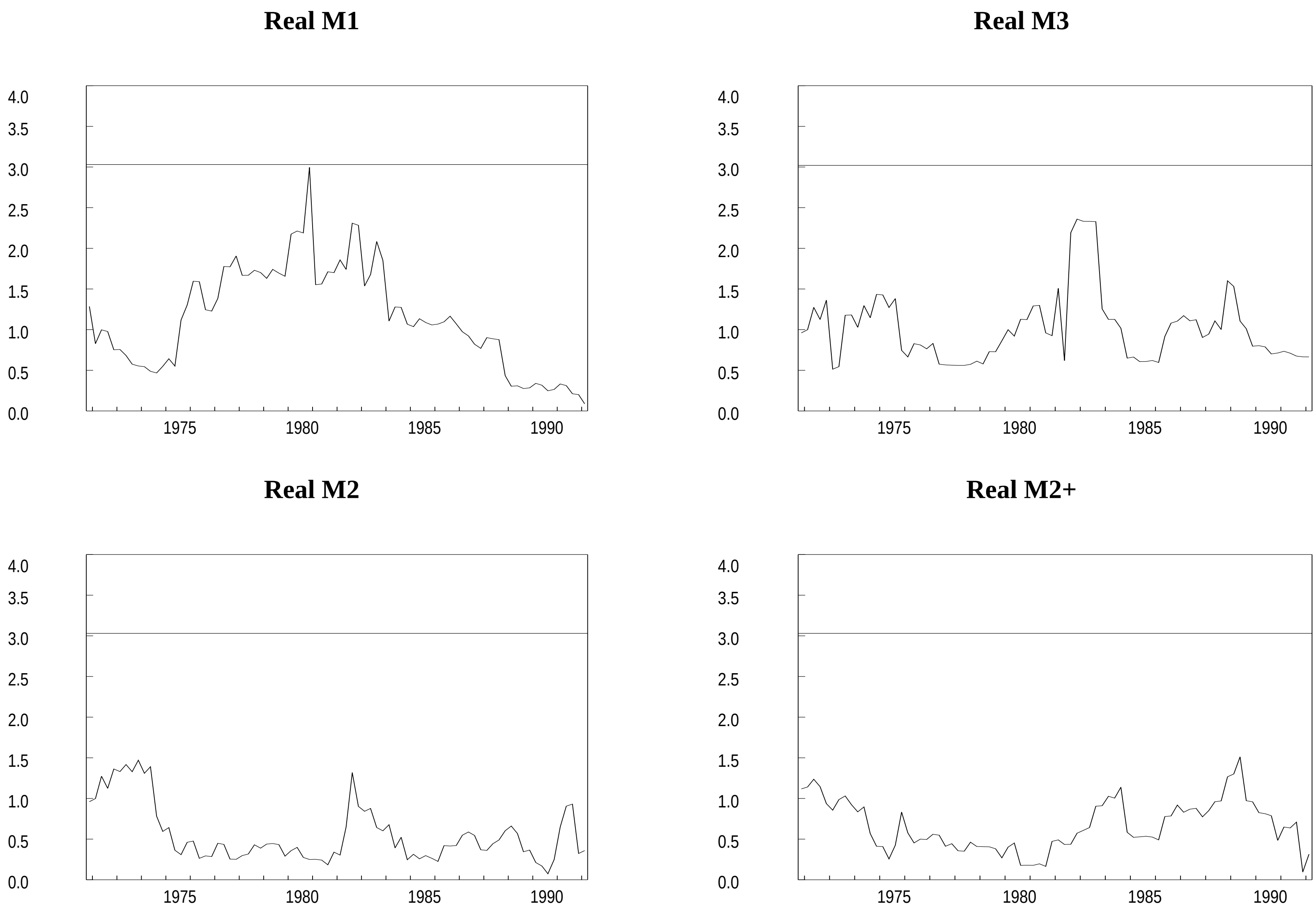
9. Note that in all cases only the cointegration vectors that were found to be statistically significant in the regression equation were included.

Table 7: Dynamic Money-Demand Equations

Money	Constant	Δ*Money*		Δ*LRY*		Δ*R90*		Δ*G1DEF*		ECM_{t-1}	$\bar{R}^2$	SEE.
		lag	coef	lag	coef	lag	coef x 10^{-3}	lag	coef x 10^{-3}			
M1	0.146 (4.78)[a]	3	0.104 (1.24)	0	0.176 (1.17)	1	-5.376 (4.66)	1	0.098 (0.19)	-0.024 (4.90)	0.544	0.012
M2	-0.085 (2.48)	1	0.533 (6.16)	4	0.135 (1.87)	1	-1.186 (2.11)	1	-1.978 (6.80)	-0.021 (2.58)	0.455	0.007
M3	0.006 (3.11)	1	0.470 (5.04)	4	0.203 (1.80)	4	-1.464 (1.64)	1	-1.210 (2.99)	-0.005 (2.83)	0.406	0.010
M2+	-0.098 (2.27)	1	0.609 (7.29)	4	0.120 (1.92)	4	-0.348 (0.70)	1	-1.738 (7.30)	-0.012 (2.85)	0.547	0.005
M1α	0.236 (5.12)	3	0.058 (0.67)	0	0.034 (0.19)	1	-4.354 (3.24)	1	-0.404 (0.67)	-0.024 (5.22)	0.465	0.015
M1β	-0.015 (0.46)	1	0.650 (9.62)	4	0.235 (1.48)	1	-7.546 (5.90)	1	-1.266 (2.03)	-0.003 (0.48)	0.606	0.015
M2α	0.003 (0.26)	1	0.507 (7.22)	2	0.324 (3.01)	0 1	-4.496 (5.24) -4.373 (4.73)	1	-2.069 (5.20)	-0.001 (0.13)	0.652	0.009
M2β	0.008 (1.07)	1	0.501 (6.67)	2	0.174 (1.43)	0 1	-3.218 (3.30) -4.137 (4.06)	1	-2.211 (4.92)	-0.002 (0.50)	0.564	0.011
M2γ	-0.188 (3.42)	1	0.316 (3.34)	4	0.116 (1.36)	0	-0.747 (1.13)	1	-1.727 (5.23)	-0.036 (3.53)	0.392	0.008
M2δ	-0.297 (3.33)	1	0.294 (3.12)	1	0.121 (1.30)	0	-0.995 (1.57)	1	-1.906 (6.23)	-0.042 (3.40)	0.416	0.007
M3α	-0.309 (3.62)	1	0.202 (2.07)	4	0.115 (1.47)	0	-1.204 (1.98)	1	-1.765 (5.65)	-0.039 (3.73)	0.358	0.007
M3β	0.029 (3.89)	1	0.388 (4.07)	4	0.084 (0.83)	0	2.552 (3.39)	1	-2.013 (5.14)	-0.014 (2.97)	0.401	0.009

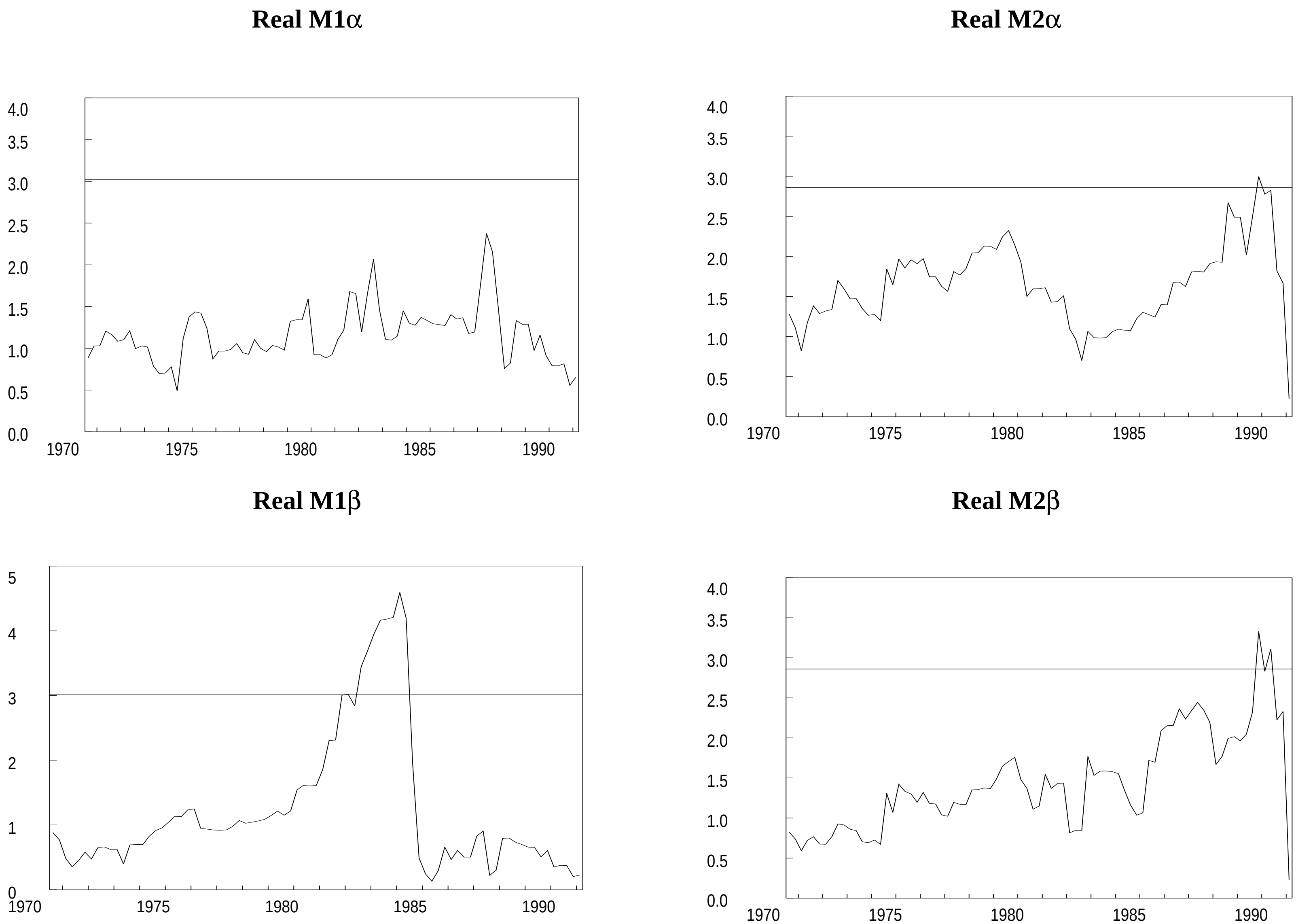
a. t-statistic in parentheses

Figure 1: Rolling Chow Tests for Current Monetary Aggregates



(1% critical value = 3.03)

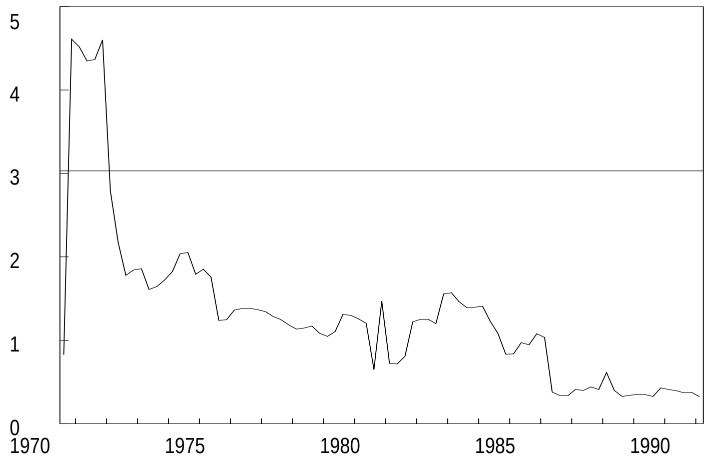
Figure 2: Rolling Chow Tests for Alternative Monetary Aggregates



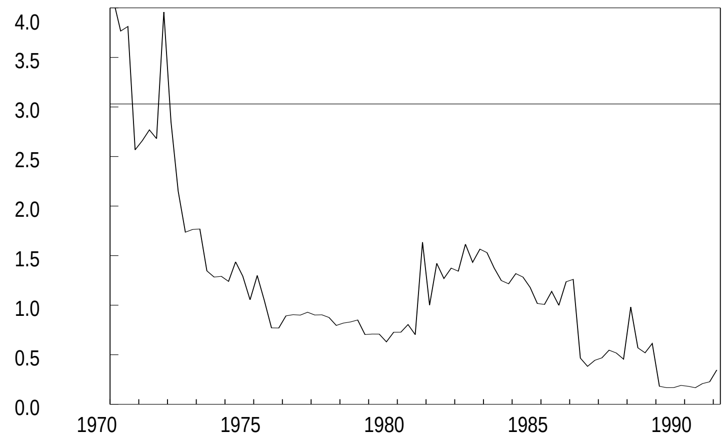
(1% critical value for M1α and M1β is 3.03, and for M2α and M2β is 2.86)

Figure 2 (Continued): Rolling Chow Tests for Alternative Monetary Aggregates

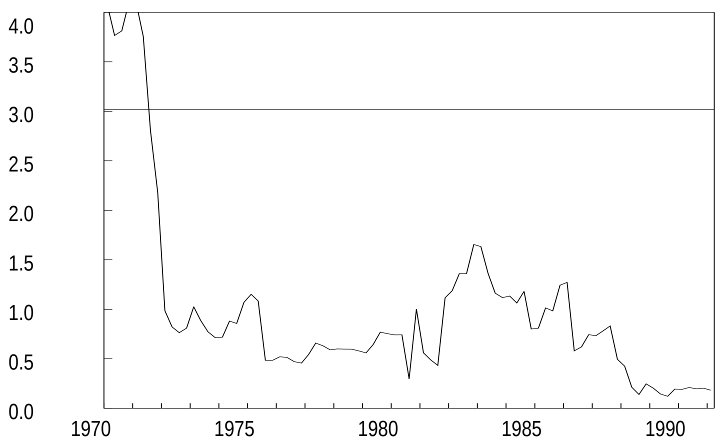
Real M2γ



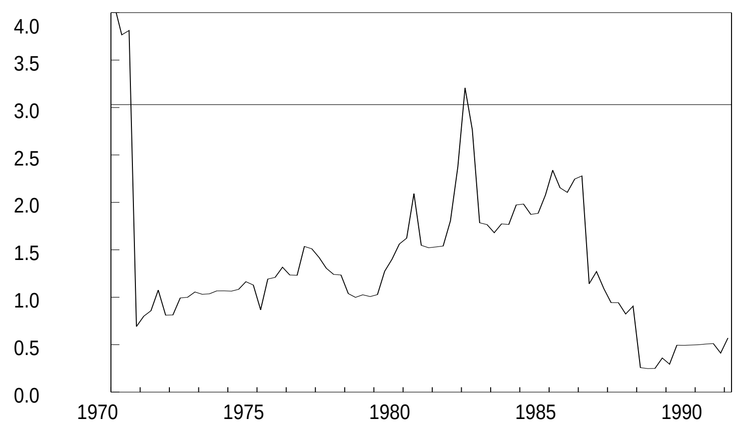
Real M3α



Real M2δ



Real M3β



(1% critical value is 3.03)

4.4 Stability of the Money-Demand Functions

A rolling Chow test is used to assess the stability of the dynamic money-demand functions for the monetary aggregates. Plots of the F-statistic for the various periods of the test are presented in Figures 1 and 2. Except for the dynamic equations for M2γ, M2δ, M3α and M3β, which were found to be slightly unstable at the beginning of the 1970s, the equation for M1β, which was seen to be unstable in the period from 1983 to 1985, and the equations for M2α and M2β, which were also observed to be slightly unstable in the early 1990s, the results suggest that the dynamic money-demand equations for the monetary aggregates (current and alternatives) are generally stable.

4.5 Forecast of Selected Macroeconomic Variables

In this section, I use a representation of a VECM equivalent to equation (6) to access the performance of the monetary aggregates in forecasting real GDP and inflation (measured by the GDP deflator). The VECM is of the form

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \ldots + \Gamma_p \Delta Y_{t-p+1} + \alpha ECM_{t-p} + \varepsilon_t \qquad (9)$$

where

$$ECM_{t-p} = \hat{\beta}' Y_{t-p} \qquad (10)$$

and $\hat{\beta}$ is the statistically significant cointegration vector obtained by the Johansen and Juselius methodology. Note that the variables in the VECM are the same variables used in determining the cointegration vectors. The variables used in this paper are the monetary aggregates (deflated by the GDP deflator), real GDP, R90 and inflation measured by the GDP deflator.

The recursive regression method was used in computing the RMSE for the forecasts of real GDP and inflation. Three of the best predictors of the goal variables are presented in Table 8.

Table 8 : Three Best Aggregates in the Prediction of the Annualized Average Growth Rates of Real GDP and the GDP Deflator

Quarters	Rank	GDP Deflator	Real GDP
1	1	M1 (1.684)[a]	M2 (3.051)
	2	M1α (1.704)	M3α (3.133)
	3	M3α (1.774)	M1α (3.146)
2	1	M3α (1.630)	M2 (2.547)
	2	M2δ (1.638)	M1 (2.701)
	3	M2 (1.664)	M3α (2.709)
4	1	M2δ (1.591)	M1β (2.042)
	2	M3α (1.611)	M2 (2.116)
	3	M2 (1.712)	M2+ (2.182)
8	1	M2δ (1.549)	M1β (1.466)
	2	M3α (1.570)	M2+ (1.736)
	3	M2 (1.760)	M2 (1.818)
12	1	M3α (1.631)	M1β (1.137)
	2	M2δ (1.683)	M2+ (1.450)
	3	M2 (1.939)	M2 (1.622)

a. RMSE in parentheses.

Table 8 shows that real M1 is the best predictor of the one-quarter average growth rates of the GDP deflator. Also, real M3α is the best predictor of the two- and twelve-quarter average growth rates of the GDP deflator, while real M2δ is the best for the four- and eight-quarter average growth rates. I also find real M2 to be the best predictor of the one- and two-quarter average growth rates of real GDP, while real M1β is the best for the four-, eight- and twelve-quarter average growth rates.

5.0 Performance of the Aggregates in a P^* Model of Inflation

There is little disagreement among economists that, in the long run, the growth rate of an appropriately defined monetary aggregate is linked to the rate of inflation. Which aggregate is closely linked to prices? To what extent could one consider money to be exogenous? How does the price level adjust to a monetary shock? What is the impact of non-monetary factors, such as demand and supply shocks? These questions are part of the many important questions economists attempt to answer when it comes to establishing the link between money and prices.

In a bid to establish the link between money and prices, Hallman, Porter and Small (1989) have proposed the P^* model of inflation. The proponents of this model use econometric techniques to analyse the sources of inflation and find that a single variable, which they label as P^*, is the only relevant determinant of future inflation. The P^* is defined as the price level that is consistent with the current money supply and equilibrium in the goods and financial markets. The purpose of this section is to assess the performance of the alternative monetary aggregates in P^* models in forecasting inflation.

5.1 The Modified P^* Model of Inflation

The P^* model, proposed by Hallman, Porter and Small (1989), is based on the classic quantity theory of money. The model assumes that the velocity of money is stable and that the income elasticity of money demand is one. However, research at the Bank of Canada finds that, unlike in the

United States, the velocity of money in Canada is not stationary. Thus, the original version of the P^* model is not applicable to Canada. A modified version of the P^* model has therefore been proposed at the Bank. The modified version begins by postulating a long-run money-demand function, in other words the cointegrating vector for money and other economic variables. Let the long-run money demand be of the form

$$m_t - p_t = \varphi_0 + \varphi_1 y_t + \varphi_2 r_t + \varepsilon_t \tag{11}$$

where m_t, p_t, y_t and r_t are money, price level, real income and the interest rate (opportunity cost of holding money). With the exception of the interest rate, all the variables are expressed in their natural-logarithmic values.[10] From equation (11), the long-run equilibrium price level (p^*) that will correspond to the current level of money, the long-run interest rate, and potential output can be expressed as

$$p^* = m_t - \varphi_0 - \varphi_1 y_t^* - \varphi_2 r_t^* \tag{12}$$

From equations (11) and (12) the price gap is obtained as

$$p^* - p_t = \varphi_1 (y_t - y_t^*) + \varphi_2 (r_t - r_t^*) + \varepsilon_t \tag{13}$$

In other words, the price gap is defined as the sum of the output gap (weighted by the income elasticity, φ_1), the interest rate gap (weighted by the semi-interest elasticity, φ_2) and the money gap.[11]

Following Hallman, Porter and Small, the short-run dynamic equation for inflation is of the form

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10. The residual ε_t, the difference between actual money balances and long-run money demand, is referred to as the “money gap.”

11. In past work at the Bank, the interest rate gap has been proxied by the term structure. In our work, I dropped the term structure term because earlier research has found it to be statistically insignificant.

$$\pi_t = \Theta_0 + \Theta_1 ygap_{t-1} + \Theta_2 mgap_{t-1} + \sum_{i=1}^{2} \Phi_i \pi_{t-i} + \zeta_t \qquad (14)$$

where *ygap* and *mgap* are the output gap and the money gap, respectively.

5.2 Estimation of the P* Models of Inflation

In this section I estimate P* indicator models using past values of the one-quarter growth rate of money, the one-quarter growth rate of inflation, the money gap and the output gap. The models are of the form:

$$Gkp_t = \alpha_0 + \sum_{i=0}^{n} \alpha_{1i} G1p_{t-k-i} + \alpha_2 MGAP_{t-k} + \alpha_3 YGAP_{t-k} + \sum_{j=1}^{h} \alpha_{4j} G1m_{t-k} + \mu_t \qquad (15)$$

where *Gkp* is the *k*-quarter annualized growth rate of prices, *p* is price, *YGAP* is the output gap, and *MGAP* is the money gap. The equation is estimated for the twelve monetary aggregates across different horizons (*k* =1, 2, 4, and 8 quarters) and for the three measures of prices (GDP deflator, CPI, and CPIXFE).

The money gaps were created from the long-run cointegrating vectors estimated for the alternative aggregates, and the data on the output gap was taken from the Bank's Quarterly Projection Model (QPM).[12]

With the exception of the models reported in Table 9, the output-gap terms were found to be statistically significant positive coefficients. These

12. Section 4 reported results for cointegrating (money-demand) relationships for systems that included the GDP deflator as the measure of prices. The money gaps for the P^* equations for CPI and CPIXFE were obtained by reestimating the money-demand functions with systems that included CPI and CPIXFE.

results support the school of thought that argues that inflation is driven by the forces of aggregate demand.

Table 9 : Models with Statistically Insignificant Output-Gap (YGAP) Terms

Quarters	GDP Deflator	CPI	CPIXFE
1	M1β	M1	M1
		M2	M3
		M2+	M1α
		M1α	
2	None	M1	M1
		M1α	M2+
4	None	M1	None
		M1α	
8	M2	M1	None
	M2+	M2+	
		M1α	

The significance of the money gaps in the models is mixed. In Table 10, I present the statistically significant money-gap terms in the various inflation models. Also, in some of the models, the sign of the estimated coefficient of the money-gap terms is negative. A negative coefficient on the money-gap term is counterintuitive, because it suggests that a positive money gap will lead to a deflation.[13]

13. These results may suggest problems with the estimated long-run money-demand functions. That is, the cointegrating vectors for these aggregates may not properly represent money-demand relationships. (See footnote 6.)

Table 10 : Models with Statistically Significant Money-Gap (MGAP) Terms[a]

Quarters	GDP Deflator	CPI	CPIXFE
	None	M1	M1
1		M1α	M2
			M2+
			M3
1			M1α
			M1β*
	M3*	M1	M1
	M1β*	M1α	M2
		M2β*	M2+
2			M3
			M1α
			M1β
			M2γ
			M2δ
			M3β
	M2*	M1	M1
	M2+*	M1α	M2+
4	M3*	M2β*	M1α
	M1β*	M2γ	M2γ
			M2δ
			M3β

Table 10 (Continued): Models with Statistically Significant Money-Gap (MGAP) Terms[a]

Quarters	GDP Deflator	CPI	CPIXFE
8	M2*	M1	M1
	M2+*	M2*	M2β*
	M1α*	M3*	M2γ
	M1β*	M1α	M2δ
	M2α	M1β*	M3β
	M2β	M2β*	
	M3α*		

a. The asterisks represent models in which the estimated coefficient of the MGAP term is negative.

Table 10 suggests that in all the models for explaining the various quarter growth rates of the GDP deflator, only the MGAP terms for M2α and M2β (eight-quarter growth rate models) have significant positive coefficients. Although the estimated coefficients of the MGAP terms in the rest of the GDP deflator models are significant, the signs are negative; these results are contradictory to what economic theory suggests.

The results also show that for the CPI models, the estimated coefficients of the MGAP terms that are significant and positive are: M1 and M1α (all the *k*-quarter growth rate models) and M2γ (four-quarter growth rate models). Notably as well, the output gap is not significant in any of the M1- or M1α-based models for the CPI.

In the case of CPIXFE, the estimated coefficients of the MGAP terms that are significant and positive are M1 (all the *k*-quarter growth rate models), M2 and M3 (one- and two-quarter growth rate models), M2+ and M1α (one-, two- and four-quarter growth rate models), M2γ, M2δ and M3β (two-, four- and eight-quarter growth rate models).

Based on $\overline{R}^2$, Table 11 shows the three best inflation-indicator models for the three measures of inflation. Models with coefficients of MGAP that are either insignificant or significant and negative are reported with asterisks.

From the results, I observe that M3 provides the best in-sample fit for one- and two-quarter growth rates of the GDP deflator. For the four- and eight-quarter growth rates of the deflator, the best in-sample fit is obtained with M1β. However, these results are unsatisfactory and must be interpreted with caution, because the estimated coefficients of the MGAP terms are either insignificant or have negative signs.

With regard to CPI, M1α provides the best in-sample fit for the one- and two-quarter growth rates. Also, for the four- and eight-quarter growth rates of the CPI, M1 is the best.

In the case of CPIXFE, M1 provides the best in-sample fit for the one- and four-quarter growth rates. The aggregates that give the best in-sample fit for the two- and eight-quarter growth rates are M1α and M2+, respectively. But again, M1 performs practically as well as these aggregates.

Table 11 : Three Best Estimation Models, Based on $\bar{R}^{2a,b}$

Quarters	Rank	GDP Deflator	CPI	CPIXFE
1	1	M3* (0.6923)	M1α (0.7171)	M1 (0.6906)
	2	M2* (0.6629)	M1 (0.7160)	M1α (0.6790)
	3	M2+* (0.6584)	M2* (0.6867)	M3 (0.6600)
2	1	M3* (0.7526)	M1α (0.7399)	M1α (0.7242)
	2	M1* (0.7233)	M1 (0.7325)	M1 (0.7223)
	3	M2+* (0.7211)	M2* (0.7151)	M2+ (0.6937)

Table 11 (Continued): Three Best Estimation Models, Based on $\bar{R}^{2a,b}$

Quarters	Rank	GDP Deflator	CPI	CPIXFE
4	1	M1β* (0.7176)	M1 (0.7540)	M1 (0.7424)
	2	M3* (0.7152)	M1α (0.7329)	M1α (0.7211)
	3	M1* (0.7098)	M2β* (0.7027)	M2+ (0.6917)
8	1	M1β* (0.6691)	M1 (0.6739)	M2+ (0.6283)
	2	M3* (0.6561)	M3* (0.6397)	M1 (0.6209)
	3	M2* (0.6311)	M1α (0.6338)	M2β* (0.6157)

a. The asterisks represent models in which the estimated coefficient of the MGAP term is either insignificant or significant and negative.

b. $\bar{R}^2$ is recorded in parentheses.

The most striking results here are that, for all the *k*-quarter growth rates of the CPI and CPIXFE, M1 or M1α is either the best or second-best aggregate in fitting the data well. As well, in all of the M1- or M1α-based models of the CPI, the output gap is not significant at any horizon. For the GDP deflator, M1 is one of the best aggregates in fitting the two- and four-quarter growth rates.

5.3 Forecast of the Cumulative Growth Rates of the Three Measures of Inflation

This section of the paper examines the performance of the P* indicator models in predicting the three measures of inflation. The predictive performance of the competing models was assessed by the RMSE criterion. The three best predictors of the three measures of inflation are presented in Table 12.

Table 12: Three Best Models in the Prediction of the k-quarter Annualized Cumulative Growth Rate of Prices, Based on the RMSE

Quarters	Rank	GDP[a] Deflator	CPI	CPIXFE
1	1	M2* (2.0508)[b]	M3β* (1.9029)	M1 (1.6955)
	2	M3* (2.0571)	M1 (1.9615)	M3β* (1.7200)
	3	M2+* (2.0981)	M2β* (1.9670)	M1α (1.7746)
2	1	M2γ* (2.0393)	M3β* (1.7363)	M3β (1.4608)
	2	M2* (2.0531)	M2β* (1.7825)	M2α* (1.5258)
	3	M3β* (2.0744)	M2α* (1.7969)	M2γ (1.6151)
4	1	M1* (2.3630)	M2β* (1.7859)	M2α* (1.3424)
	2	M2* (2.3804)	M1 (1.8517)	M1 (1.3854)
	3	M1α* (2.3827)	M1α (1.9257)	M1α (1.4727)
8	1	M2* (2.4078)	M2β* (1.3964)	M2β* (1.3146)
	2	M1 (2.5152)	M1β* (1.7863)	M1 (1.3233)
	3	M1α (2.5507)	M1 (1.9931)	M1α (1.4112)

a. The asterisks represent models in which the estimated coefficient of the MGAP term is either insignificant or significant and negative.

b. The RMSE is recorded in parentheses.

I find from the results that, contrary to earlier evidence in Bank studies that broader aggregates, such as M2+, are the best in predicting inflation, M1 and M1α are among the best aggregates for predicting inflation at a longer horizon. Although the broader aggregates, such M2, M2β and M3β, also perform well in predicting inflation, the estimated coefficients of the MGAP terms are either insignificant or significant and negative.

Table 12 shows that M2 is the best predictor of the one- and eight-quarter growth rates of the GDP deflator. The best predictor of the two- and four-quarter growth rates of the deflator are M2γ and M1, respectively.

In the case of the CPI, M3β is the best predictor of the one- and two-quarter growth rates. For the four- and eight-quarter growth rates, M2β is the best predictor.

With regard to CPIXFE, no aggregate stands out. I find the best predictors of the one-, two-, four- and eight-quarter growth rates are M1, M3β, M2α and M2β, respectively.

6.0 Conclusions

Based on the empirical analyses carried out in this paper, the aggregates M1, M1α, M2+, M2δ, M3β and LLβ are found to be deserving of further attention. M1 and M1α are recommended because they provide information about nominal and real income. Also, the money gaps of M1 and M1α were found to play dominant roles in P^* models of inflation. M2+ was found to be the best contemporaneous indicator of inflation and nominal income. M2δ was observed to be one of the best aggregates for predicting the growth rates of the CPI and the CPI excluding food and energy (CPIXFE) in indicator models and one of the best for predicting the growth rates of the GDP deflator in a VECM. M3β is recommended because it is the best aggregate in predicting inflation at short and long horizons. The LLβ aggregate was also observed to provide leading information for inflation.

At this stage, the most promising avenue for further work is the examination of the performance of the recommended aggregates in DEW inflation models, that is, in models that provide better leading information of inflation with long leads, at least one to two years.

One of the principal ways that this research could be taken ahead is to construct DEW models that include unique, stable and well-specified long-run money-demand functions, that is, cointegrating relationships among money, prices, output and interest rates that satisfy economic theory with respect to both the elements of the cointegrating vector and the speed-of-adjustment parameters (the loadings). Thus, the next stage of this work will be to build on the P^* results by developing DEW models of inflation based on the output gap, deviation of money from its long-run path and measures of the term structure of the interest rates.

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