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Abstract

This paper tests between fads and bubbles using a new empirical strategy (based on switching-regression econometrics) for distinguishing between competing asset-pricing models. By extending the Blanchard and Watson (1982) model, we show how stochastic bubbles can lead to regime-switching in stock market returns. By incorporating state-dependent heteroscedasticity into the Cutler, Poterba, and Summers (1991) fads model, we show that it can also lead to regime-switching. Two main features of the bubbles model distinguish it from the fads model. First, the bubbles model implies that returns are drawn from two distinct regimes. Second, the bubbles model implies that deviations from fundamental price will help predict regime switches. Using U.S. data for 1926-89, we find evidence that is consistent with the fads model even when we allow for variation in expected dividend growth rates and expected discount rates. However, the restrictions that the fads model implies for a more general switching model are rejected. The rejections point in the direction of the bubbles model, although not all the implications of the bubbles model are supported by the data.

Résumé

Les auteurs de l’étude ont recours à une nouvelle approche empirique, fondée sur l’emploi de méthodes de régression avec changement de régime, en vue de différencier deux modèles d’évaluation des actifs, soit le modèle des bulles et le modèle des engouements. À l’aide d’une version élargie du modèle de Blanchard et Watson (1982), ils montrent comment la présence de bulles stochastiques peut provoquer un changement de régime de la courbe de rendement des valeurs boursières. Ils démontrent par ailleurs que, si on part de l’hypothèse que l’hétéroscédasticité varie selon l’état, le modèle de Cutler, Poterba et Summers (1991) relatif aux engouements peut également déboucher sur un changement de régime. Le modèle des bulles se distingue du modèle des engouements sur deux points importants. Premièrement, il repose sur l’hypothèse que les rendements sont tirés de deux régimes distincts. Deuxièmement, il postule que les écarts observés par rapport au prix fondamental aident à prévoir les changements de régime. Les résultats obtenus par les auteurs au moyen de données américaines couvrant la période 1926-1989 appuient le modèle des engouements même si on laisse varier les taux d’accroissement attendus des dividendes et les taux d’actualisation attendus. En revanche, les restrictions qu’il faut imposer à un modèle général de régression avec changement de régime pour qu’il se ramène au modèle des engouements sont rejetées. Ce rejet tend à accréditer le modèle des bulles, bien que les restrictions que ce dernier suppose ne soient pas toutes corroborées par les données.

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Introduction

In the last few years, there has been considerable academic interest in models of the stock market in which price deviates from fundamental price.[1] Two of the main alternatives are the fads model proposed by Summers (1986) and the stochastic bubbles model proposed by Blanchard and Watson (1982).[2] In this paper, we propose an empirical strategy for distinguishing between fads and bubbles. To the best of our knowledge, this is the first paper that attempts to do so. The paper may also be of broader interest because we show how to use switching-regression econometrics to distinguish between competing asset-pricing models.

In a world with fads, Cutler, Poterba, and Summers (1991) show that a measure of the deviation of actual price from fundamental price will predict returns. We extend the Cutler, Poterba, and Summers (1991) model to incorporate state-dependent heteroscedasticity similar to that studied by Schwert (1989). With this extension, fads are consistent with a simple form of regime-switching in stock market returns.

By extending the Blanchard and Watson (1982) model, we show how stochastic bubbles can also lead to regime-switching in stock market returns. The intuition is as follows. Stochastic bubbles may either survive or collapse. This implies that stock market returns come from two distinct regimes, one of which corresponds to surviving bubbles and the other to collapsing bubbles. In addition, if the probability of collapse depends on the size of the bubble, switches in regime will be predictable using a measure of the size of the bubble from the previous period. These considerations lead naturally to a switching-regression framework.

Like many recent studies, we find evidence that stock market returns are predictable, a result that is consistent with the existence of fads.[3] This is interesting and potentially

[1] A very partial list of these papers includes Blanchard (1979), De Long et al. (1989; 1991), Froot and Obstfeld (1991), Scharfstein and Stein (1990), Summers (1986), and Tirole (1982; 1985).

[2] These are the two main alternatives to models in which actual prices correspond to fundamental prices. See, for example, the recent survey by Bollerslev and Hodrick (1992).

[3] Some of the early papers that provided evidence of this predictability include Campbell and Shiller (1987; 1988) and Fama and French (1988). For further references see the recent survey by Bollerslev and Hodrick (1992).

important, since we allow for some degree of heteroscedasticity and account for predictable variation in the dividend growth rate and the discount rate due to fundamentals in some of our empirical specifications.

The form of regime-switching implied by the fads model imposes testable restrictions on a general switching regression. In our data, these restrictions are generally rejected. This result stems primarily from two aspects of the switching regressions. First, regime switches are predictable using a measure of apparent deviations from fundamentals. Second, expected returns (conditional on the size of the deviation in the previous period) are higher in the survival regime than in the collapse regime. The first aspect is quite pronounced and contributes strongly to the rejection of the fads-model restrictions. The second aspect is considerably weaker and would not, on its own, lead to the rejection of the fads-model restrictions in most of the specifications we consider.

A problem with testing between fads and bubbles models is that there is no consensus on the correct model of fundamentals. We therefore take an approach to measuring fundamentals (and thus the apparent deviations from fundamentals) that is not based on a particular asset-pricing model. To check the robustness of our results, we use three different techniques for measuring the apparent deviation from fundamentals, each of them based on successively less restrictive assumptions. For the most part, the qualitative results are similar, irrespective of the measure we use.

Our paper is organized as follows. Section I describes the fads model and shows how it is affected by the introduction of state-dependent heteroscedasticity. Section II introduces our extension of the Blanchard and Watson (1982) model and shows how it leads to a switching-regression specification for stock market returns. Section III discusses the estimation of the two models and shows how they imply different parameter restrictions on a general switching regression. Section IV discusses different techniques for measuring apparent deviations from fundamental price. Sections V and VI present empirical results for the fads and bubbles models, respectively. Section VII discusses the interpretation of the empirical results and suggests some possible directions for future research.

I. The Fads Model

Traditional models of efficient financial markets imply that stock prices are non-stationary and returns are not predictable. Summers (1986) points out that if there are fads in the stock market, we may observe long temporary price swings that can be modeled as a slowly decaying stationary component in prices. The decay over time in the transitory component will lead to mean reversion in stock prices.

The following model, used in Fama and French (1988) and Cutler, Poterba, and Summers (1991), can capture both the traditional model and the idea of fads:

$$p_t = p_t^* + e_t \quad \textbf{(1)}$$

$$p_t^* = p_{t-1}^* + \epsilon_t \quad \textbf{(2)}$$

where p_t is the log of the stock market price in period t, p_t^* is the non-stationary component of the log price, and ε_t is white noise. Under the traditional model, log prices are a random walk, $e_t = 0$ and returns (the difference in log prices) are white noise.[4] We can think of p^*_t as the fundamental price because it does not include a fads element. Under the fads model, prices have a stationary component:

$$e_t = \rho_u e_{t-1} + v_t \quad \textbf{(3)}$$

Thus the fads model can be characterized as a situation in which $\sigma_e^2>0$ and $\rho_e>0$, where σ_e^2 is the variance of e_t. The stationary component e_t in stock prices implies that returns will be predictable.[5]

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[4] Strictly speaking, efficiency in financial markets implies that log prices are a martingale, rather than a random walk. Our presentation of the traditional and fads models follows Cutler, Poterba, and Summers (1991). Later in this section we allow the variance of ε_t to vary over time; then in the traditional model (i.e., when $\sigma^2_e=0$) log prices are a martingale.

[5] The predictability of returns can also come from time variation in required returns in an efficient market. Fama (1991), Fama and French (1988), and Poterba and Summers (1988) discuss the question of interpretation in more detail and provide references to a variety of explanations. We return to this issue in Section IV when we discuss measuring apparent deviations from fundamental price.

Cutler, Poterba, and Summers (1991) consider a situation in which a proxy is available for the fundamental price. Because there is no universally accepted model of fundamentals, any such proxy is likely to be measured with error. We can model this using an errors-in-variables approach:

$$p_t^f = p_t^* + w_t \quad \textbf{(4)}$$

where p^f is the proxy and w is the measurement error (which is assumed to be serially uncorrelated). Cutler, Poterba, and Summers also suggest the following statistic λ as a way of reflecting the degree of measurement error:

$$\lambda \equiv \frac{\sigma_e^2}{[\sigma_e^2 + \sigma_w^2]} \quad \textbf{(5)}$$

If p^f is a perfect measure of fundamentals (so $\sigma^2_w=0$), then $\lambda=1$; if p^f measures p^* with error, then $\lambda<1$. In any case, since it is a ratio of variances it is always non-negative.

To see how fads lead to the predictability of returns based on lagged information, we can use equations (1)-(4) to express returns in terms of differences between the proxy for fundamentals and log price. This suggests regressions of the form:

$$p_{t+1} - p_t = \beta_0 + \beta_b(p_t - p_t^f) + \upsilon_{t+1} \quad \textbf{(6)}$$

One example of a proxy for fundamental price is the log of the real dividend. (This is the proxy that Cutler, Poterba, and Summers (1991) use in their empirical work on stock market returns.) Equation (6) is then a regression of returns on the lagged log dividend-price ratio. It is easy to show that (1)-(4) imply that:

$$\underset{T\to\infty}{plim}\ (\hat{\beta}_b) = -(1-\rho_e)\lambda \quad \textbf{(7)}$$

The fads model therefore implies that estimates of β_b should be negative.

A simple extension of the fads model leads to regime-switching behaviour. Suppose that the variance of stock market returns varies over time, so that instead of being white noise, ε_t is heteroscedastistic. In particular, suppose that the heteroscedasticity is of the following form:

$$\begin{aligned} \varepsilon_{t+1} &\sim N(0,\sigma_S) \quad \textit{with probability } q \\ \varepsilon_{t+1} &\sim N(0,\sigma_C) \quad \textit{with probability } 1-q \end{aligned} \qquad \textbf{(8)}$$

with $\sigma_C > \sigma_S$, so that C refers to the high-variance state and S to the low-variance state.[6] Schwert (1989) has studied a similar form of heteroscedasticity.[7] To complete the model, we need a functional form for q which guarantees that it will be bounded between 0 and 1. We use the Logit form

$$q = \Phi(\beta_{q0}) \qquad \textbf{(9)}$$

where Φ is the logistic cumulative distribution function and β_{q0} is the mean of the logistic distribution function.

The fads model can be summarized in the following switching regression:

$$R_{S,t+1} = \beta_0 + \beta_b \cdot b_t + \varepsilon_{S,t+1} \qquad \textbf{(10)}$$

$$R_{C,t+1} = \beta_0 + \beta_b \cdot b_t + \varepsilon_{C,t+1} \qquad \textbf{(11)}$$

$$q = \Phi(\beta_{q0}) \qquad \textbf{(12)}$$

where b_t is the proportional deviation of actual stock market price from fundamental price. In Section III, we will show that this fads model represents a special case of a general switching regression. In the same section, we will discuss how to estimate and test the fads model.

II. Stochastic Bubbles and Regime-Switching

In this section, we begin by describing the Blanchard and Watson (1982) model of stochastic bubbles. By stochastic bubbles, we mean bubbles that may either survive or collapse in each period. The existence of stochastic bubbles implies that there are two regimes

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[6] This extends the Cutler, Poterba, and Summers (1991) fads model to allow for state-dependent heteroscedasticity.

[7] Schwert (1989) considers a Markov-switching model for variances. Equation (8) is slightly more restrictive; in effect, it constrains the rows of the probability transition matrix to be equal.

generating stock market returns, one where the bubble collapses and one where it survives. Rational investors take this into account when deciding whether or not to hold an asset. The period-to-period arbitrage condition allows us to impose some structure on asset returns in the surviving and collapsing regimes: in the surviving regime, returns should be sufficiently high to compensate the investor for the possibility that the bubble may collapse. Combined with the historical observation that larger overvaluations are more likely to collapse, this provides us with the essential elements for a regime-switching specification for stock market returns.

We begin by considering a simple asset-pricing model where risk-neutral investors choose between holding an asset that yields (1+r) and a risky stock. The period-to-period arbitrage condition for the stock is:

$$P_t = (1+r)^{-1} \cdot (E_t(P_{t+1}) + D_t) \tag{13}$$

where P_t and D_t are the stock's price and dividend at time t and E_t denotes the expectation conditional on information available at time *t*. One possible solution to this equation defines the fundamental price as

$$P_t^* \equiv \sum_{k=0}^{\infty} (1+r)^{-(k+1)} \cdot E_t(D_{t+k}) \tag{14}$$

All other prices are said to be "bubbly," with the size of the bubble defined as

$$B_t \equiv P_t - P_t^* \tag{15}$$

Since we have assumed that all asset prices, bubbly or not, follow (13), this implies that the bubble must satisfy the condition

$$E_t(B_{t+1}) = (1+r) \cdot B_t \tag{16}$$

Blanchard (1979) and Blanchard and Watson (1982) consider a particular stochastic solution to (16). They suppose there are two states of nature, one where the bubble survives (state S) and one where it collapses (state C). If the possibility of being in state S is some constant q and being in state C implies B_t=0, then (16) implies

$$E_t(B_{t+1} | S) = \frac{(1+r)}{q} \cdot B_t \tag{17}$$

The intuition here is that, if $B_t>0$, agents expect capital losses of B_t in state C, which must be balanced by expected capital gains in state S in order to earn the required rate of return on the bubble.

Historical accounts suggest that the probability of a bubble surviving decreases as the bubble grows. We therefore extend the Blanchard-Watson model by allowing the probability of survival q to depend on the proportionate size of the bubble

$$q \equiv q(b_t) \tag{18}$$

where $b_t \equiv B_t/P_t$ and

$$\frac{dq(b_t)}{d\ |b_t|} < 0 \tag{19}$$

Note the use of the absolute value of b_t, since the bubble may be positive or negative.[8]

While some notable market crashes have occurred in a single day, in other cases a collapse may occur over several months.[9] To model this, our second extension of the Blanchard and Watson model allows the expected value of the bubble, conditional on collapse, to be non-zero, thereby allowing for partial collapses. We assume that the expected size of a bubble in state C, which we define as u_tP_t, depends on the relative size of the bubble in the previous period, so that

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[8] There are theoretical arguments against negative bubbles that are often closely related to restrictions on the admissibility of non-fundamental solutions. On the admissibility of non-fundamental solutions, see, for example, Diba and Grossman (1988), Obstfeld and Rogoff (1983; 1986), and Tirole (1982; 1985).) Blanchard and Fischer (1989, 238) argue that "[These restrictions] often rely on an extreme form of rationality and are not, for this reason, altogether convincing. Often bubbles are ruled out because they imply, with a very small probability and very far in the future, some violation of rationality, such as non-negativity of prices or the bubbles becoming larger than the economy. It is conceivable that the probability may be so small, or the future so distant, that it is simply ignored by market participants." Moreover, recent work by Allen and Gorton (1991) and Leach (1991) has shown that restrictions on non-fundamental solutions are not robust to minor changes in assumptions, such as the introduction of heterogeneous agents, or allowing for more than two periods in an over-lapping generations model, or changing from discrete to continuous time. Our motivation for building the sort of model of speculative behaviour presented in this section is the same as Solow (1957, 323-324) in using an aggregate production function, which was controversial at the time: "Either this kind of [approach] appeals or it doesn't.... If it does, I think one can draw some ... useful conclusions from the results."

[9] The fall in the Tokyo stock exchange in the period following January 1990 is an example.

$$E_t[B_{t+1}|C] = u(b_t)\cdot P_t \qquad \textbf{(20)}$$

We further assume that u(˙) is a continuous and everywhere differentiable function and that:[10]

$$u(0) = 0 \qquad \textbf{(21)}$$

$$0 \le \frac{du(b_t)}{db_t} \le 1 \qquad \textbf{(22)}$$

(The differentiability assumption is made because we will be linearizing the model.) Assumptions (21) and (22) are added to ensure that a collapse means that the bubble is expected to shrink.[11] Imposing (16) then gives

$$E\left[B_{t+1}|S\right] = \frac{1+r}{q(b_t)}B_t - \frac{1-q(b_t)}{q(b_t)}u(b_t)\cdot P_t \qquad \textbf{(23)}$$

This shows that the expected value of the bubble in the surviving state is a decreasing function of the probability of survival $q(b_t)$. In other words, the greater the probability of collapse, the larger must be the gain on a positive bubble in the surviving state in order to compensate the investor for the possibility of collapse.

Note that when $q(b_t)\equiv q$, a constant, and $u(b_t)\equiv 0$, this model reduces to the Blanchard and Watson process.

It is straightforward to derive the expected excess returns R in each regime, where excess returns are the rate of return on the bubbly asset less the rate of return on the alternative asset:

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[10] As with assumptions on $q(b_t)$, the assumptions on $u(b_t)$ are not imposed on the data. Instead, they allow us to determine the expected signs and relative magnitudes of the parameters.

[11] To see this, draw a graph with $u(b_t)$ (which equals $E[B_{t+1}|C]/P_t$) on the vertical axis and b_t on the horizontal axis. The function $u(b_t)$ passes through the origin, since $u(0)=0$. The 45° line represents a situation where $E[B_{t+1}|C]/P_t = B_t/P_t$; i.e., where a "collapsing" bubble is the same size as the previous period's bubble. Since $0\le u'\le 1$, $u(b_t)$ always lies on or below the 45° line. Thus these assumptions ensure that a collapsing bubble is no larger than the bubble in the previous period.

$$E_t(R_{t+1} \,|\, S) \;=\; \frac{1-q(b_t)}{q(b_t)}\big[(1+r)b_t \;-\; u(b_t)\big] \tag{24}$$

$$E_t(R_{t+1} \,|\, C) \;=\; u(b_t) \;-\; (1+r)b_t \tag{25}$$

Noting that conditional expected excess returns are a function of b_t, we can take first-order Taylor series approximations of $E_t(R_{t+1}|S)$ and $E_t(R_{t+1}|C)$ with respect to b_t around some arbitrary value $\bar{b}$ to obtain:

$$E(R_{t+1} \,|\, S) \;=\; \beta_{SO} \;+\; \beta_{Sb} b_t \tag{26}$$

$$E(R_{t+1} \,|\, C) \;=\; \beta_{CO} \;+\; \beta_{Cb} b_t \tag{27}$$

where

$$\beta_{Sb} \;\equiv\; -\frac{1}{q(\bar{b})^2} \cdot \frac{dq(\bar{b})}{db_t} \cdot \big[(1+r)\cdot\bar{b} \;-\; u(\bar{b})\big] \;+\; \frac{1-q(\bar{b})}{q(\bar{b})} \cdot \left[1+r-\frac{du(\bar{b})}{db_t}\right] \tag{28}$$

$$\beta_{Cb} \;\equiv\; \left[\frac{du(\bar{b})}{db_t} - (1+r)\right] \tag{29}$$

Assuming $r \geq 0$, we can then prove that $\beta_{Sb} \geq 0$ and $\beta_{Cb} \leq 0$, and more generally that $\beta_{Sb} \geq \beta_{Cb}$.[12]

By dropping the expectations operator E_t in equations (26) and (27), we can rewrite them as

$$R_{S,t+1} \;=\; \beta_{SO} \;+\; \beta_{Sb} \cdot b_t \;+\; \varepsilon_{S,t+1} \tag{30}$$

$$R_{C,t+1} \;=\; \beta_{CO} \;+\; \beta_{Cb} \cdot b_t \;+\; \varepsilon_{C,t+1} \tag{31}$$

To complete the switching-regression model, we need a functional form for $q(b_t)$ that satisfies (19) and that guarantees that the resulting estimates of q will be bounded between 0 and 1. We use the Logit form

$$q \;=\; \Phi(\beta_{q0} \;+\; \beta_{qb} b_t^2) \tag{32}$$

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[12] The proof for β_{Cb} follows directly from (22). For β_{Sb}, we can use (22) and the fact that $1 \geq q(b_t) \geq 0$ to show that the second term in the expression is always non-negative. Equations (19), (20), and (22) together imply that the first term is also non-negative, so the sum of the two terms will be non-negative.

where Φ is the logistic cumulative distribution function.

III. Estimating and Testing the Fads and Bubbles Models

In this section, we show how both the fads and bubbles models nest within a general switching regression. Each model has implications for the parameters of the general switching regression. In some cases, these are zero restrictions; in other cases, they involve equalities or inequalities among the parameters in the three equations that make up the general switching regression. By testing these restrictions jointly and separately, we can make statements about the ways in which each model corresponds, or fails to correspond, to the data.

As shown in the discussion of the bubbles model, there may be a non-linear relationship between R_{t+1} and b_t that takes the form of state-dependency; i.e., the relationship between R_{t+1} and b_t exists but varies across states. If we knew with certainty which regime generated each observation of R_{t+1}, we could estimate these relationships using standard least-squares techniques. Given uncertainty about the classification of R_{t+1} into these regimes, however, standard estimation techniques will give biased and inconsistent estimates.[13] Nonetheless, consistent, efficient, asymptotically normal parameter estimates of such systems can still be obtained, provided that the equations are estimated simultaneously and that explicit account is taken of classification uncertainty.[14]

To understand the estimation procedure, suppose that in regime C

$$\boldsymbol{R}_{t+1} = \boldsymbol{R}_{t+1}^{C} = h_C(b_t) + \boldsymbol{e}_{t+1}^{C} \tag{33}$$

and that in regime S

$$\boldsymbol{R}_{t+1} = \boldsymbol{R}_{t+1}^{S} = h_S(b_t) + \boldsymbol{e}_{t+1}^{S} \tag{34}$$

This implies that we can write the probability density function of an observation conditional on it being generated by a given regime as

$$\phi_C(e_{t+1}^{C}) = \phi_C(\boldsymbol{R}_{t+1} - h_C(b_t)) \tag{35}$$

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[13] See Lee and Porter (1984) for a proof.

[14] See Goldfeld and Quandt (1976) and Kiefer (1978) for proofs.

and

$$\phi_S(e^S_{t+1}) = \phi_S(R_{t+1} - h_S(b_t)) \tag{36}$$

If we have no information on which regime generates each observation, we may denote the average probability that an observation comes from regime S as q. More generally, if we have a set of variables M_t that contain imperfect classifying information, we can write the probability that $R_{t+1} = R^S_{t+1}$ as $q(M_t)$. Therefore, the unconditional probability density function of each observation is

$$q(M_t) \cdot \phi_S(R_{t+1} - h_S(b_t)) + [1 - q(M_t)] \cdot \phi_C(R_{t+1} - h_C(b_t)) \tag{37}$$

and the likelihood function for a set of T observations

$$\prod_{t=1}^{T} [q(M_t) \cdot \phi_S(R_{t+1} - h_S(b_t)) + [1 - q(M_t)] \cdot \phi_C(R_{t+1} - h_C(b_t))] \tag{38}$$

Maximizing this likelihood function therefore estimates both (33) and (34) simultaneously with a set of parameters for $q(M_t)$, and it can be shown to lead to consistent and efficient estimates without the need for a priori knowledge on which observations correspond to a given regime.[15]

The general switching regression that encompasses the fads and bubbles models can be expressed as:

$$R_{S,t+1} = \beta_{SO} + \beta_{Sb} \cdot b_t + \varepsilon_{S,t+1} \tag{39}$$

$$R_{C,t+1} = \beta_{C0} + \beta_{Cb} \cdot b_t + \varepsilon_{C,t+1} \tag{40}$$

$$q = \Phi(\beta_{q0} + \beta_{qb} b_t^2) \tag{41}$$

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[15] For those familiar with Markov-switching models, it may be useful to note that this switching model has a related stochastic structure. A two-state Markov-switching model has two state-dependent probabilities: q(t) = Pr(S(t)=0 | S(t-1)=0) and p(t) = Pr(S(t)=1 | S(t-1)=1). The switching model presented here has one state-independent probability q(t) = Pr(S(t)=0). This is the special case of the Markov-switching model where q(t) = 1-p(t); i.e., the probability of today's state is independent of yesterday's state.

These three equations form a standard switching-regression model of the type described by Goldfeld and Quandt (1976) and Hartley (1978), and can be estimated by maximum-likelihood methods.

The fads model implies a number of restrictions on the general switching regression. First, according to the fads model, expected returns (conditional on b_t) should be the same in both regimes. This implies that β_{S0} should equal β_{C0} and that β_{Sb} should equal β_{Cb}. Second, as shown in Section I, the fads model implies that returns should be mean-reverting. This implies that the estimated value of β_b should be negative. Finally, according to the fads model, b_t should not influence which regime will occur in the subsequent period. This implies that β_{qb} should equal zero. The fads-model restrictions can be summarized as follows:

$$\beta_{S0} = \beta_{C0} = \beta_0 \quad \textbf{(42)}$$

$$\beta_{Sb} = \beta_{Cb} = \beta_b \quad \textbf{(43)}$$

$$\beta_{qb} = 0 \quad \textbf{(44)}$$

$$\beta_b < 0 \quad \textbf{(45)}$$

We can test the fads model in two basic ways. The first way is to impose the first three restrictions and compare the fit of the resulting regression with the fit of the unrestricted general switching regression. In Section V, we use a likelihood-ratio (LR) statistic to conduct this joint test.[16] The second way is to impose the first three restrictions (so as to estimate the fads model) and check whether β_b is negative.

The bubbles model also has implications for the general switching regression. First, according to the bubbles model, expected returns (conditional on b_t) should be greater in the states where the bubble survives than in the states where it collapses.[17] This implies that β_{S0}

[16] A number of authors have noted that while Lagrange Multiplier and Wald tests should be asymptotically equivalent to the LR tests, they sometimes give widely divergent results when applied to regime-switching models. For example, see Engle and Hamilton (1990). As the LR tests are thought to be the most reliable, we use these for most of the joint hypothesis tests we report.

[17] For $b_t < 0$, the expected returns in the surviving regime should be larger in absolute value (i.e., more negative) than those in the collapsing regime. Of course, the implications for the signs

will not necessarily equal β_{C0} and that β_{Sb} should be greater than β_{Cb}. Second, according to the bubbles model, collapses are more likely when bubbles are large in magnitude. This implies that β_{qb} should be positive. The implications of the bubbles model for the general switching regression can be summarized as follows:

$$\beta_{S0} \neq \beta_{C0} \qquad \textbf{(46)}$$

$$\beta_{Sb} > \beta_{Cb} \qquad \textbf{(47)}$$

$$\beta_{qb} > 0 \qquad \textbf{(48)}$$

There are two main ways in which we can test the bubbles model against the fads model. The first we have already discussed. The fads model implies certain restrictions on the general switching model. These restrictions are inconsistent with the bubbles model, so if we fail to reject them, we would conclude that there is no significant evidence for the bubbles model. The second sort of test involves the implications that are summarized in the previous three equations. If these parameter restrictions hold, it provides evidence in favour of the bubbles model.[18]

IV. Measuring Apparent Deviations from Fundamental Price

In this section, we describe three different approaches to measuring b_t, the proportional deviation of actual stock market price from fundamental price. To motivate the three approaches, consider the classic Gordon (1962) model:

$$P_t = \frac{D_t}{r-g} \qquad \textbf{(49)}$$

where g is the dividend growth rate. We can think of the fundamental price in the Gordon world as a function of current dividends, anticipated dividend growth, and anticipated interest

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of β_{Sb} and β_{Cb} are the same regardless of whether b_t is positive or negative.

[18] As always, other interpretations, which may involve neither bubbles nor fads, are possible. We are currently pursuing this point in a related paper that will examine whether switches in fundamentals (dividend growth and/or discount rates) can account for what we find in the data.

rates. Our first approach assumes a particular stochastic process for dividends and a constant interest rate. Under these assumptions, current dividends are a sufficient statistic for P_t* and thus b_t; in fact, P_t^* is a multiple of D_t. Our second approach allows for variable dividend growth rates, but maintains the assumption of a constant interest rate. Our third approach allows for variation over time in expectations of both dividend growth rates and interest rates.

A. Assuming the Dividend Process

We begin by assuming that stock market prices obey the following period-to-period arbitrage condition:

$$E_t(P_{t+1}) = (1+r)(P_t + D_t) \tag{50}$$

where r is the required rate of return, which we assume for the moment is constant. A number of previous studies have assumed that log dividends follow a random walk with drift:

$$d_t = \alpha + d_{t-1} + \epsilon_t \tag{51}$$

where d_t is the log of dividends.

Under these assumptions, it is straightforward to show that fundamental price is a multiple of current dividends:

$$P_t = \rho D_t \tag{52}$$

where:

$$\rho = \frac{1 + r}{e^{(\alpha + \sigma^2/2)} - 1} \tag{53}$$

This set of assumptions therefore provides one possible motivation for the measure of fundamental price used by Cutler, Poterba, and Summers (1991).

Under these assumptions, the proportional deviation of actual price from fundamental price is:

$$b_t^A = \frac{P_t - P_t^A}{P_t} = 1 - \frac{\rho D_t}{P_t} \tag{54}$$

We use the sample mean of the price dividend ratio to calculate ρ.

B. A VAR Projection on Dividend Changes and the Spread

Our second approach allows for variation over time in expected dividend growth and provides a way of capturing the information about future dividend growth contained in the information set available to market participants. This approach follows Campbell and Shiller (1987), who introduce a VAR approach to estimating and testing the present-value model of stock market prices. A simple algebraic manipulation allows us to use their methodology to construct estimates of b_t. Because the Campbell and Shiller (1987) approach is well known, our description of it is brief and focusses on the aspects that are directly relevant.

Consider the simple present value model of stock market prices:

$$P_t = E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i D_{t+i} \tag{55}$$

Define the innovation in stock price as:

$$\xi_t = P_t - E_{t-1} P_t \tag{56}$$

The present value model implies that the innovation can be expressed in terms of observable variables as:

$$\xi_t = P_t - (1+r)\,[P_{t-1} - D_{t-1}] \tag{57}$$

Note that this expression is equal to the excess return on stocks, multiplied by the stock market price.

If the present value model were true, then a linear function of current prices and dividends (the "spread") would be the optimal linear forecast of future dividend changes. Intuitively, this is because the current price reflects all available information, so innovations (i.e., excess returns) are unpredictable. Campbell and Shiller (1987) define the "spread" as the difference between price and a multiple of current dividends:

$$S_t \equiv P_t - \left(\frac{1+r}{r}\right) D_t \tag{58}$$

It is easy to show that S_t is the optimal linear forecast of S_t^*, where S_t^* is a weighted average of future dividend changes:

$$S_t = E_t S_t^* \tag{59}$$

where

$$S_t^* \equiv \frac{1+r}{r} \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \Delta D_{t+i} \tag{60}$$

A variety of studies, including Campbell and Shiller (1987), have found that excess returns are predictable. This predictability has two main interpretations. The first, which is emphasized by Fama and French (1988), for example, is that financial markets are rational and efficient and that discount rates vary over time in a predictable manner. The second, which is emphasized by Poterba and Summers (1988), for example, is that there may be some element of apparent irrationality in financial markets in the form of fads or bubbles. Under the first interpretation, we can obtain a better estimate of fundamental price by incorporating the sort of information contained in past dividends and prices that makes returns predictable. Under the second interpretation, by incorporating this additional information, we may bias our tests against finding either fads or bubbles because we have attributed to fundamental price some of the predictability that arises from fads or bubbles.

The additional information available from past dividend changes and stock market prices can be incorporated by estimating the following VAR representation for ΔD_t and S_t (where both variables have been demeaned):

$$\begin{bmatrix} \Delta D_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta D_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{61}$$

If b(L) were 0, then dividend changes would depend only on past dividend changes; if agents have additional information beyond the history of dividends that is reflected in past prices (and therefore past S), then S will have incremental explanatory power.

The equations (61) can be stacked into a first-order system:

$$\begin{bmatrix} \Delta D_t \\ . \\ . \\ \Delta D_{t-p+1} \\ S_t \\ . \\ . \\ S_{t-p+1} \end{bmatrix} = \begin{bmatrix} a_1 ... a_p & b_1 ... b_p \\ 1 & \\ \quad . & \\ \quad\; 1 & \\ c_1 ... c_p & d_1 ... d_p \\ & 1 \\ & \quad . \\ & \quad\; 1 \end{bmatrix} \begin{bmatrix} \Delta D_{t-1} \\ . \\ . \\ \Delta D_{t-p} \\ S_{t-1} \\ . \\ . \\ S_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ 0 \\ . \\ 0 \\ u_{2t} \\ 0 \\ . \\ 0 \end{bmatrix} \qquad \textbf{(62)}$$

where blank elements are zero. In companion matrix form, this can be rewritten as:

$$z_t = Az_{t-1} + v_t \qquad \textbf{(63)}$$

In our context, the value of such a VAR representation is that it allows us to form an optimal forecast of future dividend changes, conditional on a given information set H_t:

$$S_t^{/} \equiv E(S_t^* \mid H_t) = \theta e1^{/} \delta A(I - \delta A)^{-1} z_t \qquad \textbf{(64)}$$

where

$$\theta = \frac{1+r}{r}, \quad \delta = \frac{1}{1+r} \qquad \textbf{(65)}$$

and e1 is a row vector that picks out ΔD_t. Through a simple algebraic manipulation, we can transform the expression for the spread into an expression for fundamental price P^B_t, where P^B_t incorporates the optimal linear forecast of future dividend changes (S_t^*) based on past prices and dividends. Let:

$$P_t^B = S_t^{/} + \frac{1+r}{r} D_t \qquad \textbf{(66)}$$

Then we can define a second measure of deviations from fundamentals, namely:

$$b_t^B = \frac{P_t - P_t^B}{P_t} \qquad \textbf{(67)}$$

We set $r=1/[(\overline{P}/\overline{D}) - 1]$, where $\overline{X}$ denotes the sample average of X. This ensures that S_t has mean zero over our sample.

C. Incorporating Time Variation in Interest Rates

The VAR approach based on Campbell and Shiller (1987), which we have just outlined, is attractive, but it ignores a determinant of stock market prices that is frequently emphasized, namely variation over time in interest rates. To incorporate predictable variation in interest rates, as well as in dividend growth rates, we use the dividend-ratio model proposed by Campbell and Shiller (1988). Define the log of the stock market return as:

$$h_t \equiv \log(P_{t+1} + D_t) - \log(P_t) \tag{68}$$

This can be approximated quite closely as:

$$h_t \cong k + \delta_t - \kappa\delta_{t+1} + \Delta d_t \tag{69}$$

where $\delta_t \equiv d_{t-1} - p_t$, lower-case letters denote logs, k is a constant, and κ is a coefficient that emerges from the approximation.[19]

Equation (69) is a difference equation that can be solved forward subject to a terminal condition ($\lim_{i\to\infty} \kappa^i \delta_{t+i} = 0$) to obtain:

$$\delta_t \cong \sum_{j=0}^{\infty} \kappa^j (h_{t+j} - \Delta d_{t+j}) - \frac{k}{1-\kappa} \tag{70}$$

In other words, (70) says that the log dividend-price ratio can be expressed as a weighted sum of future returns h_{t+j} and dividend growth rates Δd_{t+j}. This equation contains economic content only if we put some conditions on the process for returns. In particular, Campbell and Shiller (1988) assume that expected stock market returns differ from the expected returns on some other asset by an amount that does not vary over time:

$$E_t h_t = E_t r_t + c \tag{71}$$

Taking expectations on both sides of (70) and substituting in (71), we obtain:

[19] We follow Campbell and Shiller (1988) in setting $\kappa=\exp(g-h)$, where h is the sample mean stock return and g is the sample mean dividend growth rate.

$$\delta_t \cong E_t \sum_{j=0}^{\infty} \kappa^j (r_{t+j} - \Delta d_{t+j}) + \frac{c - k}{1 - \kappa} \qquad \textbf{(72)}$$

Equation (72) can be thought of as a dynamic version of the Gordon model, in which both interest rates and dividend growth rates vary freely through time.

To construct an optimal linear forecast of the dividend-price ratio, we use a VAR of the following form:

$$\begin{bmatrix} \delta_t \\ \delta_{t-1} \\ r_{t-1} \\ r_{t-2} \\ \Delta d_{t-1} \\ \Delta d_{t-2} \end{bmatrix} = \begin{bmatrix} C_{111} & C_{211} & C_{112} & C_{212} & C_{113} & C_{213} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ C_{121} & C_{221} & C_{122} & C_{222} & C_{123} & C_{223} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ C_{131} & C_{231} & C_{132} & C_{232} & C_{133} & C_{233} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_{t-1} \\ \delta_{t-2} \\ r_{t-2} \\ r_{t-3} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ 0 \\ u_{2t} \\ 0 \\ u_{3t} \\ 0 \end{bmatrix} \qquad \textbf{(73)}$$

where all variables are expressed as deviations from their means. Using companion matrix notation, our forecast of the log dividend price ratio is:

$$\delta_t^C \equiv \sum_{j=0}^{\infty} \kappa^j e2' A^{j+1} z_t \qquad \textbf{(74)}$$

where e2 is a vector that picks out r_{t-1}-Δd_{t-1}. The resulting value of δ_t^C can be used to construct a third fundamental price and thus a third measure of b_t:

$$b_t^C = \frac{P_t - P_t^C}{P_t} = 1 - e^{-\delta_t^C} \frac{D_t}{P_t} \qquad \textbf{(75)}$$

V. Empirical Estimates of the Fads Model

In this section, we present estimates of the fads model derived in Section I as well as tests of the restrictions which the fads model implies for the general switching regression.

A. Assuming the Dividend Process (b^A)

Table I presents estimates of the fads model. We begin by focussing on the first column of the table, which presents parameter estimates based on b_t^A. A major implication of the fads model is that a regression of returns on the apparent deviation of actual prices from fundamental prices should yield a negative coefficient. As the first column shows, the point estimate of this coefficient (denoted β_b) is -0.013 and the t-statistic is -2.1, a result that is consistent with the fads model.[20] To get a sense of the economic significance of the point estimate, consider the value of b_t in September 1987 (one of the larger values in the sample and one of historical interest). With b_t=.29, the point estimate implies that excess returns will be lower by .37 per cent a month (5 per cent a year).

The result for β_b is consistent with the results of previous studies. As noted above, the regression reported in the first column of Table I is very similar to the Cutler, Poterba, and Summers (1991) test of the fads model. Their estimate of β_b is -0.01, which is close to our estimate.[21]

A number of authors have argued that the evidence for fads is weaker if heteroscedasticity is taken into account. For example, Kim, Nelson, and Startz (1991) argue that most of the evidence for mean reversion comes from the period before World War II, a period during which the volatility of stock market returns was unusually high.[22] We allow for heteroscedasticity by letting disturbances come from a high- and low-variance regime. As Table I shows, σ_C is about three times as large as σ_S, so heteroscedasticity appears to be a

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[20] We report t-ratios based on the inverse of the Hessian. The results that we report are asymptotically consistent and efficient under our assumptions. A number of recent studies (e.g., Kim, Nelson, and Startz (1991), McQueen (1992), Nelson and Kim (1992), and Richardson and Stock (1989)) have found that standard test statistics can be misleading in studies of stock market predictability. In this paper, we use more than 700 non-overlapping observations and account for conditional heteroscedasticity.

[21] Our results correspond most closely to the last row, first column of their Table 6. The sign of their estimate differs because they regress returns on fundamental price minus actual price and we regress returns on actual price minus fundamental price.

[22] See also McQueen (1992) and Nelson and Kim (1992) on heteroscedasticity.

genuine issue in the data.[23] Even after allowing for heteroscedasticity, there is still evidence that returns are predictable, since the results in column one of Table I show β_b to be significantly negative.

In Section III, we discuss two main types of tests of the fads model. The first imposes the fads-model restrictions on the general switching regression and then tests whether or not β_b is negative. However, this does not test between the fads and bubbles models. The second type of test examines whether the fads-model restrictions are valid; it is therefore a more useful test for distinguishing between fads and bubbles.

The fads-model restrictions on the general switching regression are shown in equations (42), (43), and (44). These restrictions reflect two fundamental differences between the fads model and the bubbles model. First, the bubbles model implies that returns are drawn from two distinct regimes, so that the intercept (β_{S0} and β_{C0}) and slope (β_{Sb} and β_{Cb}) coefficients may differ between regimes. Second, the bubbles model implies that the size of deviations from fundamental price influences the regime from which each stock market return is drawn, so that β_{qb} will not be equal to zero.

As shown in the lower portion of Table I, the likelihood-ratio test statistic for the joint test of the fads-model restrictions is 16.3. Since this test has a χ^2 distribution with three degrees of freedom under the null hypothesis of fads, the joint test strongly rejects the fads-model restrictions.

B. Alternative Measures of Deviations from Fundamental Price (b^B, b^C)

Constructing a measure of the fundamental stock market price is an inherently difficult and potentially contentious task. If we were attempting to test either the fads model or the bubbles model against the null hypothesis that stock market returns are driven exclusively by fundamentals, it would also be an extremely important task. When we are trying to distinguish between the fads model and the bubbles model, it is less clear that our choice of a model of fundamentals is crucial.

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[23] A formal test for the equality of σ_S and σ_C is difficult; see Hansen (1993) and Garcia (1992) for recent progress in this area.

Cutler, Poterba, and Summers (1991) argue that a noisy measure of fundamental price will tend to bias tests against finding evidence that non-fundamentals are important. Thus, if we use a noisy measure of fundamental price and still find evidence of either fads or bubbles, their view is that the noisiness of our measure of fundamentals strengthens the case that non-fundamentals matter. Others take a different view. For example, Cecchetti, Lam, and Mark (1990) argue that some of the evidence for fads can be explained by variations in the endowment process. In the present-value model outlined in Section II, this might be represented, for example, by variation in the expected growth rate of dividends.

Whether or not the measure of fundamentals makes a difference in testing between fads and bubbles is a question on which the previous literature is largely silent. Our approach is therefore eclectic and empirical: we consider various measures of fundamental price and examine whether they lead to different results.

The results in the first column of Table I assume that the expected dividend growth rate is constant. Our second measure of fundamental price allows for predictable variation in the dividend growth rate. In particular, it incorporates the information available in past dividend changes and stock market prices using a linear projection on this past information.[24]

We refer to the measure of deviation from fundamental price that incorporates predictable time variation in the dividend growth rate as b^B. The results for this measure are presented in column two of Table I. The point estimate of β_b is -0.012 with a t-statistic of 2.1, which is very close to the results using b^A. We can test the fads model against the bubbles model by determining whether the restrictions of the fads model on the general switching regression are valid. In the lower portion of Table I, the test statistic for the joint test of these restrictions is 10.8. As in the earlier results, this is a strong rejection of the fads-model restrictions.

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[24] If the existence of fads or bubbles leads returns to be predictable, then incorporating this predictability in the measure of fundamental price might bias tests against finding evidence for fads or bubbles. Whether incorporating this predictability into fundamental price would tend to bias the tests relatively more against fads or against bubbles is hard to say.

Fama (1991) and Fama and French (1988) argue that the predictability of returns is due to time variation in expected returns. One way to take this into account is to use excess returns as the dependent variable, as we do in this paper. This captures variations in the rate of return on the alternative asset. A second way to take this into account is to use predictable variation in discount rates in constructing the measure of fundamental price. This may help to capture predictable variation in the interest rate or the risk premium.

We refer to the measure of deviation from fundamental price that incorporates predictable variation in the discount rate (as well as in the dividend growth rate) as b^C. The results for this measure are presented in column three of Table I. The point estimate of β_b is -0.008 with a t-statistic of 2.5. The value of b_t^c is .51 in September 1987, so the point estimate of β_b implies that returns will be about .39 per cent lower in October (about 5 per cent lower at annualized rates). This is very close to the results using b^A.

The second type of test is whether the fads-model restrictions on the general switching regression are valid. In the lower portion of Table I, the test statistic for the joint test of these restrictions is 4.0. In contrast to the earlier results, the test fails to reject the fads-model restrictions.

We can briefly summarize the results for the full sample. The main focus of this paper is testing between fads and bubbles. We do this by testing the restrictions that the fads model imposes on the general switching regression. For two of our three measures of fundamental price, we find strong rejections of the fads-model restrictions. Our estimates of the fads model differ from previous research because we allow for state-dependent conditional heteroscedasticity; nevertheless, we still find evidence that returns are predictable, since our estimates of β_b are negative. The evidence that returns are predictable is also robust to allowing for variation in the dividend growth rate and the interest rate in constructing measures of fundamental price.

C. Subperiods

It is often suggested that much of the evidence for the predictability of returns comes from periods that include the 1929 crash and the Great Depression.[25] More generally, time-series econometric results are sometimes sensitive to the specific time period over which the estimation is done. In this subsection, we therefore present results for three subperiods — 1929-45, 1946-72, and 1973-89. The first includes the 1929 crash, the Great Depression, and World War II; the second includes the post-war boom up until the first major oil shock; and the third covers the most recent period, including the inflation of the 1970s and the 1987 crash.

The results for the period 1929-45 (Table II) are the most surprising. Once allowance is made for state-dependent heteroscedasticity (as our switching regression does), the evidence for fads is very weak. Contrary to the prediction of the fads model, the estimate of β_b is either positive or close to zero, depending on how we measure fundamental price. In all cases, the t-statistic on β_b is less than one in absolute value.

In the 1946-72 period, the estimates of β_b are all negative and all have t-statistics greater than two in absolute value as shown in Table III. However, when we test the fads-model restrictions, they are very strongly rejected regardless of which measure of fundamental price we use.

The results for the period 1973-89 provide the evidence most consistent with the fads model. As shown in Table IV, the estimates of β_b are all negative and all have t-statistics greater than two. Nevertheless, one of the three measures of fundamental price leads to a rejection of the fads-model restrictions.

The subperiod results suggest that the evidence for fads we find in the full sample does not derive primarily from the 1930s; the estimates of β_b are more consistent with the fads model during the post-war boom and the period since the 1973 OPEC shock. However, as in the full sample, the subperiod results frequently reject the restrictions implied by the fads model, suggesting that there is more in the data than fads.

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[25] See, for example, Kim, Nelson, and Startz (1991).

We discuss the interpretation of the empirical results for the fads model (and how they relate to the empirical results for the bubbles model) in the Conclusion.

VI. Empirical Estimates of the Bubbles Model

In this section, we present estimates of the bubbles model derived in Section I as well as tests of the restrictions implied by the bubbles model for the general switching regression. The bubbles model differs from the fads model in two main ways. First, the bubbles model implies that returns are drawn from two distinct regimes. In particular, we show in Section II that the bubbles model implies an inequality restriction on the slope coefficients in the surviving and collapsing regimes, namely that $\beta_{Sb} \geq \beta_{Cb}$. Moreover, since returns come from distinct regimes, there is no reason to expect the intercept coefficients (β_{S0} and β_{C0}) to be the same in the two regimes. Second, the bubbles model implies that deviations from fundamental price influence the regime from which the next period's stock market return is drawn. Specifically, the bubbles model in Section II suggests that a bubble is more likely to collapse when the actual price is far away from the fundamental price. This implies that β_{qb} will be positive.

A. Assuming the Dividend Process

As noted in the previous section, the fads-model restrictions are frequently rejected; the estimates of the bubbles model parameters in Table V help us to see why. We begin by focussing on the first column of the table, which presents parameter estimates based on b_t^A. The point estimate of β_{S0} is .0074 with a t-statistic of 4.1. If β_{Sb} were zero, this point estimate would translate into positive excess returns of 0.7 per cent a month (9 per cent a year) for a surviving bubble. The point estimate of β_{C0} is -.0294 with a t-statistic of -1.4. If β_{Cb} were zero, this point estimate would translate into negative excess returns of about 3 per cent a month (31 per cent a year). The point estimates of the intercept coefficients therefore imply substantial differences in expected returns between the surviving and collapsing regimes. The statistical significance of the difference is weaker; as the lower panel of the table shows, the marginal significance level of the restriction $\beta_{S0}=\beta_{C0}$ is about .08.

The bubbles model in Section II implies that β_{Sb} should be greater than β_{Cb}. The point estimate of β_{Sb} is -.0136 with a t-statistic of 2.0. The point estimate of β_{Cb} is -.0554 with a t-statistic of 1.4. The inequality restriction implied by the bubbles model therefore holds but the point estimates are not sharp enough to reject the null hypothesis that $\beta_{Sb}=\beta_{Cb}$ at conventional significance levels. The economic significance of the coefficient estimates is quite different, however. For example, consider one of the larger values of b_t in our sample, namely the value of .26 in September 1929. Even at this relatively high value of b_t, the coefficient estimate implies a change in expected returns of only 35 basis points in the surviving regime. In the collapsing regime, the point estimate of β_{Cb} implies a change of 144 basis points.

The bubbles model in Section II suggests that bubbles are more likely to collapse when they comprise a large portion of the price of the stock. This implies that β_{qb} should be positive. The point estimate of β_{qb} is 1.71 with a t-statistic of 3.1.

The existence of distinct regimes in the bubbles model (and particularly the fact that the probability of a collapse depends on b_t) means that the relationship between returns and b_t can be highly non-linear. One way to illustrate this is to look at the probability that next period's return will be unusually high or low. In the fads model, expected returns next period depend on b_t; thus if b_t is large this period, there is an increased likelihood that next period's returns will be unusually low. In the bubbles model, a change in b_t has an effect through the slope coefficients (β_{Sb} and β_{Cb}), but there is an additional effect because b_t influences the probability that a given regime will occur. Mathematically, the probability of a return two standard deviations below the mean (a "crash") is:

$$Pr(R_{t+1}<x) \;=\; \Phi\left(\frac{x-\beta_{S0}-\beta_{Sb}b_t}{\sigma_S}\right)q(b_t) \;+\; \Phi\left(\frac{x-\beta_{C0}-\beta_{Cb}b_t}{\sigma_C}\right)(1-q(b_t)) \qquad \textbf{(76)}$$

The probability of an unusually high return ("rally") can be defined similarly.

If the non-linearity introduced by the bubbles model is empirically important, the magnitude of the fluctuations in the probability of a crash will be much larger in the bubbles model. Figure I graphs the probabilities of a crash generated by the fads and bubbles models. The scale of the fluctuations is very different. For example, from the beginning of the sample to September 1929, the probability of a crash generated by the fads model increases by only about two-tenths of a percentage point (from .022 to .024). Over the same time span, the probability of a crash generated by the bubbles model rises from about .03 to almost .10. Figure II plots the probability of a rally. In the months leading up to May 1932 (the lowest value of b_t in the sample), the probability of a rally generated by the fads model rises by about one percentage point (from .026 to .036). The probability of a rally generated by the bubbles model rises from about .01 to almost .09.

B. Alternative Measures of Deviations from Fundamental Price (b^B, b^C)

The results in the first column of Table V are based on the assumption that the expected dividend growth rate is constant. Our second measure of fundamental price allows for predictable variation in the dividend growth rate. The second column of Table V presents estimates of the bubbles model using b^B. The picture that emerges is broadly similar to the results in the first column of Table V. The point estimate of β_{S0} is positive, while that for β_{C0} is negative and much larger in magnitude. If β_{Sb} and β_{Cb} were zero, the point estimates of the intercept coefficients (β_{S0} and β_{C0}) would imply that annualized returns would be about 67 per cent higher in the surviving regime than in the collapsing regime. The marginal significance level for a test of the restriction $\beta_{S0}=\beta_{C0}$ is .08. The point estimate of β_{Cb} is negative and about six times smaller than the point estimate of β_{Sb}. Neither parameter is very precisely estimated, however, so the data fail to reject the restriction $\beta_{Sb}=\beta_{Cb}$. As predicted by the bubbles model in Section II, β_{qb} is positive with a t-statistic of about 2.5.

Our final measure of deviation from fundamental price is b^C, which incorporates predictable variation in the discount rate (as well as in the dividend growth rate) in the fundamental price. The results for b^C are presented in column three of Table V. The point estimates of the intercept and slope coefficients in the equations for the surviving and collapsing regimes are similar to those for b^A and b^B. The point estimate of β_{S0} is positive and relatively small in magnitude, while the point estimate of β_{C0} is negative and larger in magnitude. If β_{Sb} and β_{Cb} were zero, the point estimates of the intercept coefficients (β_{S0} and β_{C0}) imply that annualized returns would be about 45 per cent higher in the surviving regime than in the collapsing regime. The marginal significance level for the restriction $\beta_{S0}=\beta_{C0}$ is about .10. As with b^A and b^B, the point estimate of β_{Cb} is negative and smaller than β_{Sb} by about a factor of five. The point estimate of β_{qb} is positive; unlike the estimates of β_{qb} in columns one and two, however, the t-statistic is small.

Intuitively, the bubbles model suggests that: 1) there should be a regime in which the bubble survives and excess returns are positive to compensate the investor for the possibility that the bubble may collapse; 2) in the periods in which a positive bubble collapses, returns should be negative; and 3) the probability of a collapse should increase as the bubble grows larger. The empirical results for the full sample broadly correspond to this description of stochastic bubbles. There is one important exception: simple models of rational bubbles (and the derivations presented in Section II) imply that returns in the surviving regime should be higher when the bubble is larger. This conflicts with our finding that β_{Sb} is negative. There are theoretical reasons for interpreting this finding cautiously. In a broader theoretical model (which relaxes the assumption of risk neutrality and allows the intertemporal marginal rate of substitution to vary over time), we find that a bubbles model predicts that β_{Cb} will be negative and that β_{Sb} will be greater than β_{Cb}, but that no sign restriction can be placed on β_{Sb}.[26]

[26] See van Norden and Schaller (1992).

C. Subperiods

For the subperiod that includes the 1929 crash and the Great Depression, the results are broadly similar to those for the full sample. As shown in Table VI, the estimates of β_{S0} are greater than the estimates of β_{C0}, the estimates of β_{Sb} are considerably larger than the estimates of β_{Cb}, and the estimate of β_{qb} is positive. Unlike the full sample, for the period 1929-45, β_{Sb} is either positive or very close to zero.

For the period 1946-72, the estimate of β_{S0} is larger than the estimate for β_{C0}, as shown in Table VII. Unlike the full sample, this difference is highly significant; in fact, the difference between β_{S0} and β_{C0} is the main reason the fads model is so strongly rejected during the post-war boom subperiod. The difference in slope coefficients is smaller than in the full sample (and the direction is reversed) and β_{qb} is insignificantly different from zero.

The results for the period since the first OPEC shock are broadly similar to the full sample results. As shown in Table VIII, the estimate of β_{S0} is greater than the estimate of β_{C0}, the estimate of β_{Cb} is much smaller than the estimate of β_{Sb}, and the estimate of β_{qb} is positive (except when b^C is used).

VII. Conclusion

The objective of this paper is to test which model provides a better description of U.S. stock market returns: a fads model or a bubbles model. Our econometric approach to this question is novel. We show that both the fads model and the bubbles model can be represented as special cases of a general switching regression. This allows us to develop a new set of tests, involving the parameters of a general switching regression, to distinguish between fads and bubbles.

Previous authors have shown that the fads model implies that apparent deviations from fundamental price will help to predict stock market returns. Our estimates of the relevant coefficient (β_b) generally support this finding. When we test the restrictions that the fads model imposes on the general switching regression, however, we frequently reject these restrictions.

In our formulation, there are two key differences between the fads model and the bubbles model. The first arises from the fact that a fads model allows predictable variation in excess returns, while a bubbles model imposes the condition that expected returns are equal on the bubbly asset and the alternative asset. This condition of the bubbles model implies that returns must be high in the states where the bubble survives in order to compensate the investor for the low returns in the states where the bubble collapses.

The point estimates of the parameters that reflect this difference tend to point in the direction of the bubbles model. Because the regimes in the fads model differ only in their variances, the fads model implies that the slope and intercept coefficients in the regimes should be the same. The point estimates suggest economically important differences between the intercept coefficients. If the slope coefficients were zero, the intercept coefficients typically would imply that annualized returns in the surviving regime would be 40-70 per cent higher than in the collapsing regime.

The point estimates of the slope coefficients (β_{Sb} and β_{Cb}) are also quite different. The bubbles model implies that the coefficient in the collapsing regime should be negative and smaller than the coefficient in the surviving regime. The point estimate of the coefficient in the collapsing regime is always negative and typically about four to five times smaller than the coefficient in the surviving regime.

The second difference between the fads and bubbles models arises because the bubbles model in Section II suggests that a large bubble is more likely to collapse. In our formulation, this implies that the coefficient β_{qb} should be positive. Over the period 1928-89, this is what we find; moreover, the data strongly reject the hypothesis that β_{qb}=0 for two of our three measures of fundamental price.

Although the rejections of the fads model point in the direction of the bubbles model, the evidence is not definitive. First, the statistical significance of the differences in the intercept and slope coefficients between regimes is weak. Over the period 1928-89, the marginal significance level of the test for equal intercept coefficients is generally around .10. In the test for equal slope coefficients, the marginal significance level is even higher.

Second, intuition (and the derivations in Section II) suggest that expected returns should be an increasing function of the size of the bubble in the states where the bubble survives. In our data, the relevant coefficient (β_{Sb}) has the opposite sign. In Section VIB, we discuss theoretical reasons for not exaggerating the importance of this result for the bubbles model. Nonetheless, the result suggests that some caution is appropriate.

Taken together, our results suggest that there is more in the data than fads. The specific ways in which the data conflict with the fads model are frequently consistent with the bubbles model, but the evidence in favour of the bubbles model is not decisive.

We see a number of directions for further research. First, this paper shows how to use switching regression econometrics to distinguish between two specific asset-pricing models. In the future, economists may wish to think about how different asset-pricing models might lead to regime-switching behaviour. Differences between the models should suggest parametric tests based on the coefficient estimates from a general switching regression.

Second, we examine aggregate stock market returns for the U.S. It would be interesting to see if the patterns we find in U.S. aggregate data carry over to other countries and other assets, as well as whether they appear in the returns of individual firms or particular portfolios.[27]

Third, the focus in this paper is on models in which asset prices do not correspond to fundamental price. It is possible that fundamentals determine asset prices and that the regime-switching behaviour we find is due to changes in fundamentals. For example, Bollerslev and Hodrick (1992) and Cecchetti, Lam, and Mark (1990) have proposed stochastic processes for dividends that seem to account for some of the predictability of returns. Whether such processes for fundamentals can account for the sort of regime-switching we find is an interesting question and one that we are currently pursuing.

Fourth, apparent anomalies in asset markets may arise because of strong assumptions about the constraints faced by agents. For example, Brock and LeBaron (1990) show that in a production-economy asset-pricing model, the introduction of finance constraints on firms can

[27] Van Norden and Schaller (1993) offer some evidence on regime-switching in Canadian stock market returns, and van Norden (1996) examines regime-switching in foreign exchange. Neither of these papers tests a fads model against a bubbles model.

accentuate mean reversion.[28] Whether finance or other constraints can explain our results is another interesting question for future research.

[28] There is some empirical evidence to support this idea; see Jog and Schaller (1993). Woodford (1989) shows that imperfect financial intermediation can lead to non-linear (or even chaotic) dynamics.

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Table I
The Fads Model: Full Sample (1929-89)

Coefficients	b_t^A	b_t^B	b_t^C
β_0	0.0067 (4.015)	0.0047 (2.691)	0.0051 (2.988)
β_b	-0.0126 (-2.079)	-0.0117 (-2.122)	-0.0077 (-2.547)
β_{q0}	-1.1983 (-7.511)	-1.1951 (-7.469)	-1.1747 (-7.354)
σ_S	0.0392 (21.198)	0.0393 (20.919)	0.0390 (20.535)
σ_C	0.1278 (8.526)	0.1289 (8.460)	0.1274 (8.574)
Joint Test	16.32 (0.0010)	10.82 (0.0127)	4.02 (0.2598)

The coefficients are estimated from a switching regression of the form represented by equations (10)-(12). The figures in parentheses below the point estimates are t-statistics calculated using the inverse of the Hessian. The headings b_t^A, b_t^B, and b_t^C refer to three different measures of apparent deviations from fundamental price that are described more fully in Subsections A, B, and C of Section IV, respectively. The Joint test takes the general switching regression represented by equations (39)-(41) as the null hypothesis and the coefficient restrictions in equations (42)-(44) as the alternative hypothesis; the likelihood-ratio statistic is distributed χ^2 with three degrees of freedom. The marginal significance levels (p-values) are in parentheses below the Joint test statistics. The switching regression is estimated using the maxlik procedure in GAUSS with a combination of BFGS and BHHH algorithms. For b_t^A, the sample period is 1927-89, because lagged data are not required for the VARs (which are used to construct b_t^B and b_t^C).

Table II
The Fads Model: 1929-45

Coefficients	b_t^A	b_t^B	b_t^C
β_0	0.0085 (1.763)	0.0043 (0.730)	0.0007 (0.090)
β_b	0.0056 (0.405)	-0.0003 (-0.017)	-0.0067 (-0.697)
β_{q0}	-0.7387 (-2.427)	-0.7342 (-2.426)	-0.7508 (-2.301)
σ_S	0.0482 (7.643)	0.0513 (9.034)	0.0517 (7.730)
σ_C	0.1490 (5.993)	0.1538 (5.509)	0.1540 (5.611)
Joint Test	12.65 (0.0055)	5.60 (0.1329)	4.02 (0.2596)

See the notes to Table I. For b_t^A, the sample period is 1927-45, because lagged data are not required for the VARs (which are used to construct b_t^B and b_t^C).

Table III
The Fads Model: 1946-72

Coefficients	b_t^A	b_t^B	b_t^C
β_0	0.0084 (3.742)	0.0066 (3.065)	0.0057 (2.562)
β_b	-0.0180 (-2.462)	-0.0153 (-2.380)	-0.0092 (-2.480)
β_{q0}	-0.2525 (-0.044)	-0.2925 (-0.057)	-0.1803 (-0.025)
σ_S	0.0338 (2.607)	0.0338 (2.787)	0.0340 (2.209)
σ_C	0.0405 (2.108)	0.0408 (2.139)	0.0398 (1.931)
Joint Test	18.79 (0.0003)	19.09 (0.0003)	17.56 (0.0005)

See the notes to Table I.

Table IV
The Fads Model: 1973-89

Coefficients	b_t^A	b_t^B	b_t^C
β_0	0.0044 (1.396)	-0.0010 (-0.296)	0.0246 (2.897)
β_b	-0.0493 (-2.333)	-0.0424 (-2.364)	-0.0711 (-2.781)
β_{q0}	-0.7714 (-1.531)	-0.7795 (-1.573)	-0.6324 (-1.617)
σ_S	0.0360 (8.098)	0.0360 (8.240)	0.0337 (8.983)
σ_C	0.0759 (4.666)	0.0763 (4.681)	0.0741 (5.727)
Joint Test	4.04 (0.2570)	8.38 (0.0388)	1.41 (0.7031)

See the notes to Table I.

Table V
The Bubbles Model: Full Sample (1929-89)

Coefficients	b_t^A	b_t^B	b_t^C
β_{S0}	0.0074 (4.087)	0.0053 (2.818)	0.0059 (3.228)
β_{Sb}	-0.0136 (-1.976)	-0.0117 (-1.926)	-0.0071 (-2.239)
β_{C0}	-0.0294 (-1.402)	-0.0386 (-1.572)	-0.0253 (-1.349)
β_{Cb}	-0.0554 (-1.361)	-0.0757 (-1.553)	-0.0397 (-1.374)
β_{q0}	-1.7660 (-6.181)	-1.6593 (-5.872)	-1.3227 (-4.815)
β_{qb}	1.7132 (3.071)	1.2068 (2.459)	0.2396 (0.800)
σ_S	0.0406 (19.645)	0.0407 (19.812)	0.0394 (18.165)
σ_C	0.1338 (7.463)	0.1340 (7.475)	0.1271 (7.896)
Tests:			
$\beta_{S0}=\beta_{C0}$	3.0284 (0.0818)	3.1647 (0.0753)	2.7031 (0.1002)
$\beta_{Sb}=\beta_{Cb}$	0.9870 (0.3205)	1.6602 (0.1976)	1.2256 (0.2683)
$\beta_{S0}=\beta_{C0}$ and $\beta_{Sb}=\beta_{Cb}$	3.3819 (0.1844)	3.6452 (0.1616)	3.3148 (0.1906)

The coefficients are estimated from a switching regression of the form represented by equations (30)-(32). The figures in parentheses below the point estimates are t-statistics calculated using the inverse of the Hessian. The headings b_t^A, b_t^B, and b_t^C refer to measures of apparent deviations from fundamental price; see Subsections A, B, and C of Section IV, respectively. The first two tests are based on Wald statistics using the inverse of the Hessian and are asymptotically distributed χ^2 with one degree of freedom. The third test is based on a likelihood-ratio statistic and is asymptotically distributed χ^2 with two degrees of freedom. Marginal significance levels are listed below the test statistics. The estimation uses the maxlik procedure in GAUSS with a combination of BFGS and BHHH algorithms.

Table VI
The Bubbles Model: 1929-45

Coefficients	b_t^A	b_t^B	b_t^C
β_{S0}	0.0110 (1.775)	0.0048 (0.675)	0.0065 (0.372)
β_{Sb}	0.0128 (0.685)	0.0037 (0.217)	-0.0002 (-0.012)
β_{C0}	-0.0427 (-1.297)	-0.0543 (-1.068)	-0.0677 (-1.263)
β_{Cb}	-0.0741 (-1.361)	-0.1046 (-1.232)	-0.0965 (-1.374)
β_{q0}	-1.3388 (-2.773)	-1.3917 (-2.701)	-0.8444 (-0.682)
β_{qb}	1.8821 (2.341)	1.1490 (1.664)	0.1985 (0.239)
σ_S	0.0497 (6.712)	0.0561 (9.065)	0.0510 (2.951)
σ_C	0.1471 (5.231)	0.1676 (4.918)	0.1462 (2.820)
Tests:			
$\beta_{S0}=\beta_{C0}$	2.6087 (0.1063)	1.3002 (0.2542)	2.5374 (0.1112)
$\beta_{Sb}=\beta_{Cb}$	2.1674 (0.1410)	1.5069 (0.2196)	2.1955 (0.1384)
$\beta_{S0}=\beta_{C0}$ and $\beta_{Sb}=\beta_{Cb}$	2.6476 (0.2661)	1.5829 (0.4532)	2.7798 (0.2491)

See the notes to Table V. For b_t^A, the sample period is 1927-45, because lagged data are not required for the VARs (which are used to construct b_t^B and b_t^C).

Table VII
The Bubbles Model: 1946-72

Coefficients	b_t^A	b_t^B	b_t^C
β_{S0}	0.0324 (6.849)	0.0304 (7.146)	0.0296 (7.164)
β_{Sb}	-0.0268 (-2.852)	-0.0237 (-2.824)	-0.0142 (-2.115)
β_{C0}	-0.0013 (-0.300)	-0.0030 (-0.722)	-0.0036 (-0.881)
β_{Cb}	-0.0149 (-1.439)	-0.0132 (-1.264)	-0.0104 (-1.782)
β_{q0}	0.6634 (1.500)	0.5406 (1.351)	0.4315 (1.120)
β_{qb}	-0.3029 (-0.218)	0.1632 (0.151)	0.4462 (0.628)
σ_S	0.0161 (4.784)	0.0161 (4.840)	0.0161 (4.998)
σ_C	0.0382 (17.657)	0.0380 (18.467)	0.0379 (19.116)
Tests:			
$\beta_{S0}=\beta_{C0}$	32.1920 (0.0000)	36.9500 (0.0000)	39.5250 (0.0000)
$\beta_{Sb}=\beta_{Cb}$	0.6562 (0.4179)	0.6187 (0.4315)	0.2066 (0.6494)
$\beta_{S0}=\beta_{C0}$ and $\beta_{Sb}=\beta_{Cb}$	9.0357 (0.0109)	6.5806 (0.0372)	12.7790 (0.0017)

See the notes to Table V.

Table VIII
The Bubbles Model: 1973-89

Coefficients	b_t^A	b_t^B	b_t^C
β_{S0}	0.0036 (0.913)	0.0014 (0.406)	0.0231 (2.095)
β_{Sb}	-0.0229 (-0.879)	-0.0245 (-1.051)	-0.0630 (-1.991)
β_{C0}	0.0025 (0.146)	-0.0636 (-1.337)	-0.0399 (1.145)
β_{Cb}	-0.1898 (-1.730)	-0.2431 (-1.765)	-0.1454 (-1.247)
β_{q0}	-1.1542 (-1.685)	-2.4383 (-3.148)	-0.3120 (-0.472)
β_{qb}	1.9339 (0.769)	6.2322 (2.092)	-1.3777 (-0.791)
σ_S	0.0368 (7.934)	0.0395 (13.825)	0.0345 (9.010)
σ_C	0.0749 (5.381)	0.0721 (5.553)	0.0758 (5.702)
Tests:			
$\beta_{S0}=\beta_{C0}$	0.0034 (0.9535)	1.8828 (0.1700)	0.1747 (0.6760)
$\beta_{Sb}=\beta_{Cb}$	2.1251 (0.1449)	2.5809 (0.1082)	0.3984 (0.5279)
$\beta_{S0}=\beta_{C0}$ and $\beta_{Sb}=\beta_{Cb}$	3.1280 (0.2093)	4.9634 (0.0836)	0.6708 (0.7151)

See the notes to Table V.

Figure 1
Probability of a Crash

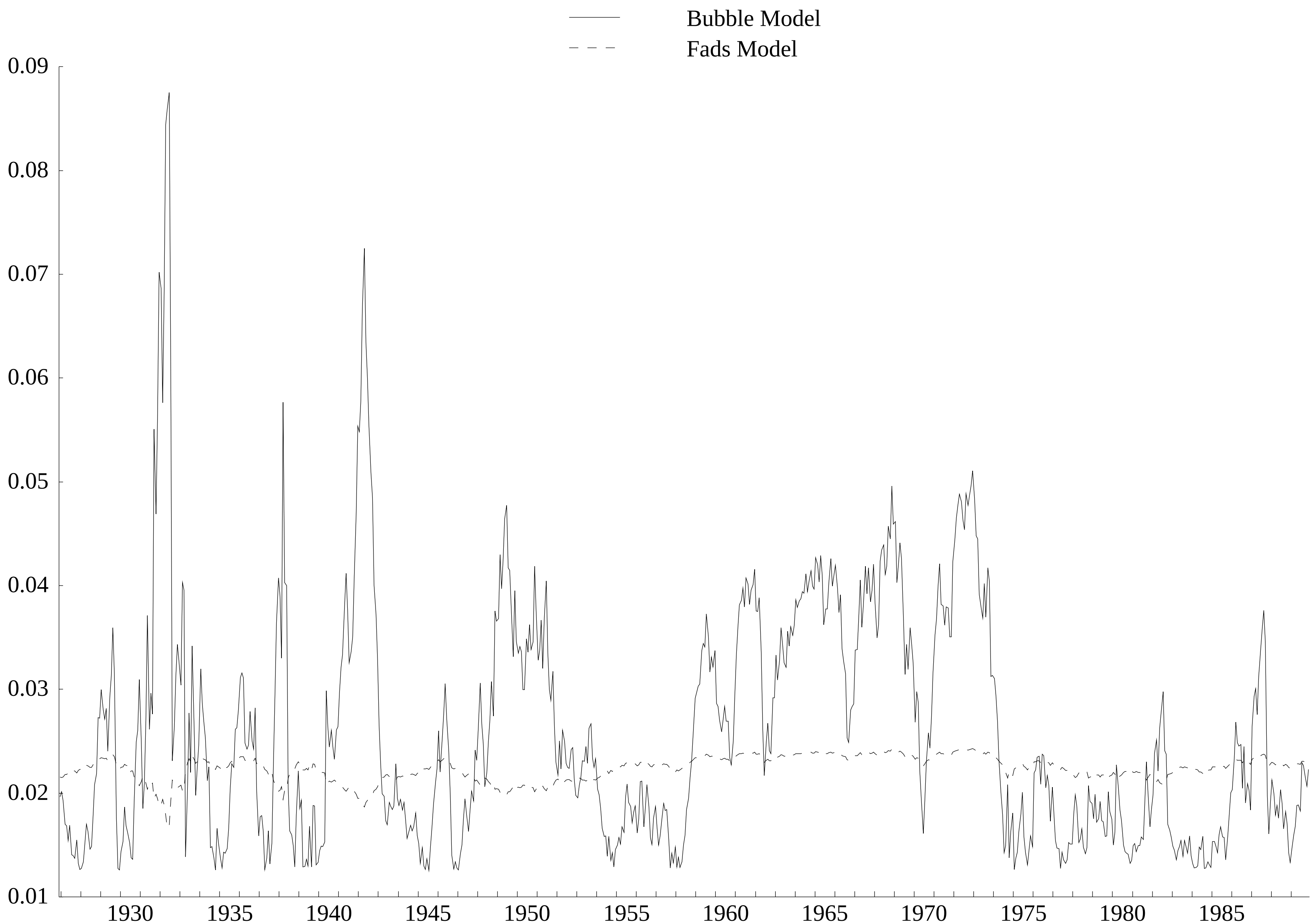
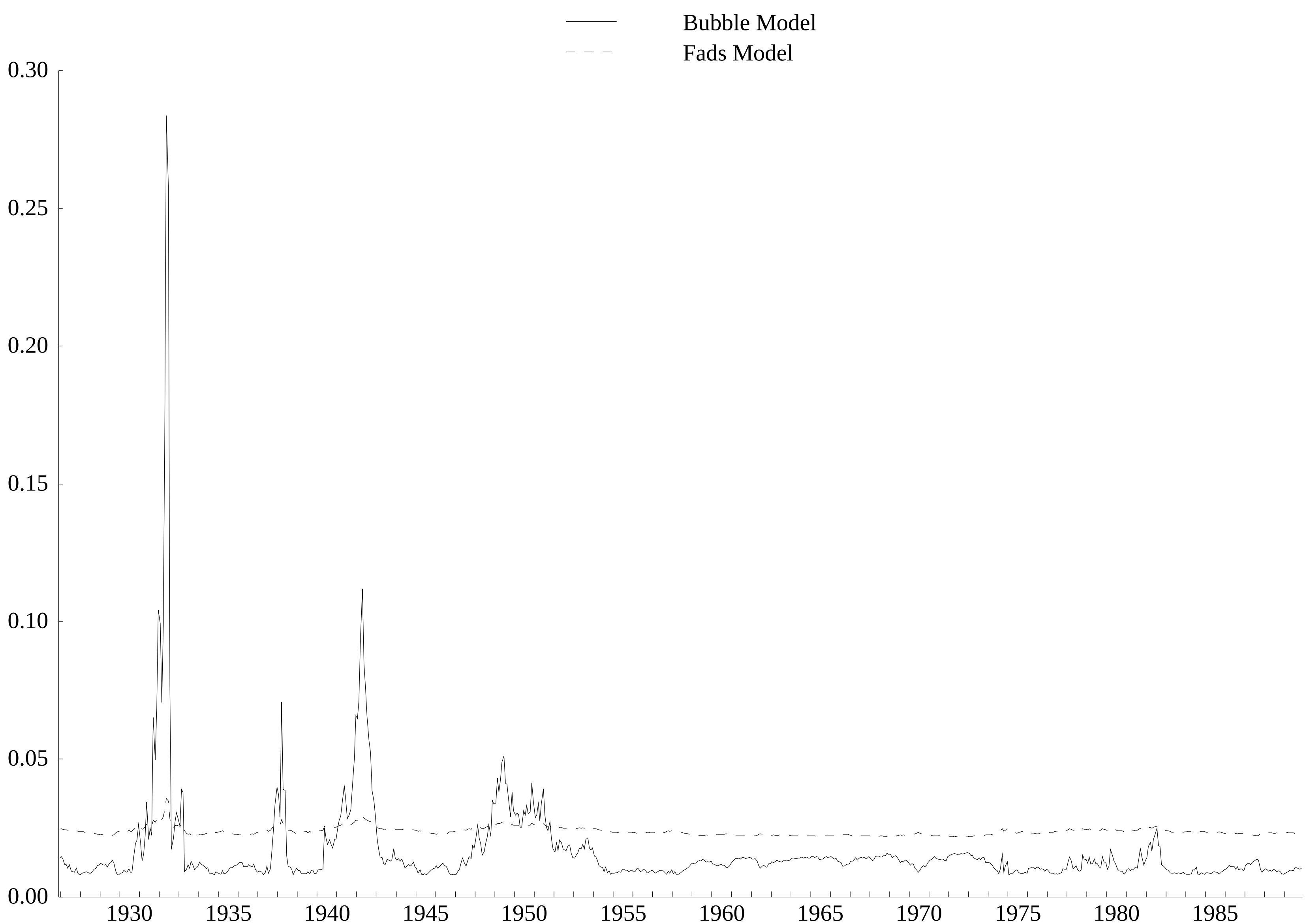


Figure 2
Probability of a Rally



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