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**The Sale of Durable Goods by a Monopolist
in a Stochastic Environment**

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Abstract

This paper examines the sale of durable goods by a monopolist in a stochastic partial equilibrium setting. It analyzes the responses of prices and output to various types of shocks and notes the differences with non-durable goods and competitive markets. It shows that behavior in this model with constant marginal costs of production is in many respects similar to behavior under perfect competition with increasing marginal costs.

Résumé

L'auteur examine les ventes de biens durables en situation de monopole dans un cadre stochastique d'équilibre partiel. Il analyse les réactions des prix et de la production de ces biens en présence de divers types de chocs, en cherchant à établir si elles diffèrent de celles provoquées par ces chocs dans le cas des biens non durables et si elles varient selon que le marché est ou non concurrentiel. D'après le modèle qu'il utilise, le comportement que l'on observe lorsque les coûts marginaux de production sont constants serait similaire à plusieurs égards au comportement observé en régime de concurrence parfaite avec des coûts marginaux croissants.

Contents

1. Introduction	1
2. The model	4
3. The Linear Equilibrium	7
4. The Stationary Deterministic Case.....	11
5. Comparative Statics	12
5.1 Shift in Costs	13
5.2 Shifts in Demand	15
5.3 Shifts in the Interest Rate.....	17
6. Numerical Examples.....	18
7. Conclusion	21
Appendix	23
References	26
Tables	28
Figures	29

1 Introduction

It is safe to say that durable and non-durable goods respond differently to economic shocks. For instance, the output of durable goods is well known to be more variable, and their relative prices appear to be less procyclical (see Bils, 1987, Murphy et al., 1989, Rotemberg and Woodford, 1997).

Arguably, behaviour in the classic perfectly competitive context is well understood, as prices are identified with marginal costs. But market power in one form or another is prevalent in the economy, and in these circumstances, the pricing of durable goods involves further intertemporal considerations yet to be fully analyzed. In this paper, we study the sale of durable goods by a monopolist in a partial equilibrium setting where demand, the cost of production and the interest rate are stochastic. As a first approach, we assume that demand is linear and, except for monopoly power, there are ideal market conditions. For example, there is no private information and there is a perfect second hand market. We then characterize a simple equilibrium and use it to analyze the responses of output and prices to various types of shocks. The paper aims in particular to contrast the cases of durable versus non-durable goods and monopolistic versus competitive markets.

It turns out that, in the case of demand shocks, behaviour under monopolistic conditions with constant marginal costs of production is comparable to that under perfect competition with increasing marginal costs. For instance, following a temporary increase in demand, prices and output are higher at first, then lower, while output adjustments are gradual relative to the competitive case with constant marginal costs.

In the competitive case with increasing marginal costs, prices are higher at first because marginal costs are higher. Then, as the supply of durables in the market rises, output declines, hence costs and prices. In the monopolistic case, prices are higher at first because of intertemporal discrimination against buyers—faced with a higher demand, the firm asks high prices first. Consequently, the stock of durable goods in the market rises only gradually. Also, because profits are positive, the monopolist does not return the stock of durables immediately to its original level once it rises; rather, a higher output in any period induces a higher total supply in the future, hence lower future prices.

Thus, we show that, under elasticity-preserving shocks to demand,¹ the percentage deviation of output is smaller in the present model than under perfect competition with constant marginal costs of production, while the correlation between current and lagged output is significantly higher. The correlation between prices and current output is typically positive, but the correlation with lagged output is typically negative. More significantly, the correlation between prices and the level of demand (measured by the inverse slope of the demand curve) is smaller the more durable is the good, and eventually negative when movements in demand are serially correlated. Our results might therefore provide some explanation to the empirical findings that markups are less procyclical (or more countercyclical) for more durable goods. These results obtain in the absence of any structural or price adjustment costs.

Another feature that distinguishes the present model is that, because profits are

¹These amount to proportional shifts in demand, or equivalently, shifts that change only the slope of inverse demand (see section 5.2.b).

positive, changes in the interest rate affect both the opportunity cost of production as well as the opportunity gain of sales. If there are no costs of production, then an increase in the interest rate, which means a greater discount of the future, would lower current prices and increase current output. Thus, under shocks to the interest rate and assuming relatively small costs of production, the correlation between price and contemporaneous output is found to be negative.

The past literature examined the sale of durable goods by a monopolist under deterministic, if not stationary, conditions. Typically, it was concerned with the Coase conjecture for a monopolist who sells a non-depreciable good ([Fudenberg et al., 1985], [Gul et al., 1986]). Bond and Samuelson (1984) allowed depreciation. Conlisk et al. (1984) studied discount sales in a deterministic model where a new (but identical) cohort of consumers enters the market each period. Sobel [1984] looked at the same problem in an oligopolistic setting.

Domowitz et al. (1987) examined a two-period model in which the intercept of demand rises and then declines by the same amount, and Parker (1996) discussed the behavior of prices of durable goods when the distribution of utility flows of consumers has infinitely repeating cycles.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 derives a simple characterization of a (Markov subgame-perfect) equilibrium under linear demand. Section 4 reviews the stationary deterministic case. Section 5 carries out comparative static exercises and section 6 considers numerical examples. The paper closes with a few remarks.

2 The model

A monopolist sells durable goods to households who take prices as given and live for T periods, $1 \leq T \leq \infty$. At the beginning of each period, random shocks occur and become public knowledge before decisions are made; the monopolist then supplies a new quantity of goods while prices adjust (say through the intermediary of an auctioneer) to clear the market.²

Except for the seller's monopoly power, we assume ideal conditions for trade, e.g. there are no transaction costs or credit constraints, and the economy admits a perfect second hand market where goods can be resold at the prevailing price. It follows that in a partial equilibrium setting, the demand for the durable good's services at time t is a function of only the implicit rental price of the good

$$r_t \equiv p_t - \frac{(1 - \delta)}{1 + i_{t+1}} E_t p_{t+1}$$

where: p_s is the price of the durable good in period s ; δ is the rate at which the good depreciates ($0 \leq \delta \leq 1$);³ and i_{s+1} is the rate of return, known at s , on risk-free assets held from time s to $s + 1$.

Henceforth, we assume that the good is perfectly divisible and inverse demand is linear

$$r_t = r_t(D) = a_t - b_t D$$

where D is the total supply of durable goods and a_t , b_t follow exogenously given

²The presentation can be modified to let the firm set the price instead of the quantity.

³Non-durable as well as perfectly durable goods are therefore special cases.

stochastic processes.⁴

The firm is assumed to have a constant return to scale technology, the marginal cost c_t is stochastic and exogenously given. Also, $c_t > \frac{1}{1+i_{t+1}}E_t c_{t+1}$ for all t ,⁵ so the firm will have no incentives to keep inventories. Let

$$\bar{c}_t \equiv c_t - \frac{1-\delta}{1+i_{t+1}}E_t c_{t+1} (> 0)$$

be the opportunity cost of producing one unit at time t rather than $1-\delta$ units at time $t+1$.

A supply schedule for the firm is a stochastic process $\{D_t(\cdot)\}_{t \geq 0}$ where, for each t , $D_t(\cdot)$ describes the total supply of goods at time t as a function of the present state of nature and the stock D_{t-1} of durables in the market at the end of period $t-1$.

A pricing schedule for the market is a stochastic process $\{p_t(\cdot)\}_{t \geq 0}$ where, for each t , $p_t(\cdot)$ describes the price that will prevail at time t as a function of the present state of nature and the current supply of durables.

The following notation will be useful: given $\{D_t(\cdot), p_t(\cdot)\}_{t \geq 0}$ and a stock D_{t-1} at the end of period $t-1$, let

$$(a) \quad D_{t-1}^*(D_{t-1}) \equiv D_{t-1}, \quad D_s^*(D_{t-1}) \equiv D_s[D_{s-1}^*(D_{t-1})] \quad (s \geq t)$$

$$(b) \quad p_t^*(D_{t-1}) \equiv p_t[D_t(D_{t-1})] \quad (s \geq t)$$

⁴Shifts in the slope b_t are perhaps more intuitive than shifts in the intercept a_t . The latter amount to constant shifts in demand irrespective of the rental price, whereas the former amount to proportional shifts, such as may arise from changes in the size of the population.

⁵This is not unreasonable if cost reductions are mainly driven by innovations rather than changes in factor prices.

Also let $\beta_{t,s}$ denote the discount rate (or alternatively, the degree of impatience) between t and s : $\beta_{t,t} = 1$; $\beta_{t,t+1} = \frac{1}{1+i_{t+1}}$; $\beta_{t,s+1} = \beta_{t,s}\beta_{s,s+1}$. So $\beta_{t,t+1}$ is known at t , but $\beta_{t,s}$ for $s > t + 1$ can be random.

By an **equilibrium** we shall mean a configuration $\{D_t(\cdot), p_t(\cdot)\}_{t \geq 0}$ of a supply schedule for the firm and a pricing schedule for the market such that for all t :⁶

1. Given the firm's supply schedule $\{D_t(\cdot)\}_{s \geq t}$, the pricing schedule $p_t(\cdot)$ clears the market, i.e. for every $D \geq 0$

$$p_t(D) = r_t(D) + \frac{1 - \delta}{1 + i_{t+1}} E_t[p_{t+1}^*(D)].$$

2. Given the market's pricing schedule $\{p_t(\cdot)\}_{t \geq 0}$, the firm's supply schedule $\{D_t(\cdot)\}_{s \geq t}$ is optimal for the firm, i.e., for every $D \geq 0$, the sequence $\{D_s^*(D)\}_{s \geq t}$ maximizes the firm's expected profits at time t

$$E_t \sum_{s=t}^T \beta_{t,s} [p_s(D_s) - c_s] [D_s - (1 - \delta)D_{s-1}]$$

over all possible sequences $\{D_s\}_{s \geq t}$ such that $D_{t-1} = D$ and $D_s \geq (1 - \delta)D_{s-1}$ for $s \geq t$.

The equilibrium concept we defined is essentially that of a Markovian subgame perfect equilibrium.⁷ It is "subgame perfect" in the sense that if $\{p_s(\cdot), D_s(\cdot)\}_{s \geq 0}$ is

⁶We further impose the transversality condition $\lim_{s \rightarrow \infty} \beta_{t,s} p_s [D_s^*(D)] = 0$ (uniformly over all D and all states of nature) to ensure that the price equals the discounted sum of present and future rental prices.

⁷It is possible to model, as in Gul et al., both the firm's actions as well as those of each individual buyer (under the provision that equilibrium strategies do not distinguish between past histories that

an equilibrium, then for all t and all D_{t-1} , $\{p_s(\cdot), D_s(\cdot)\}_{s \geq t}$ is an equilibrium over the subgame starting at time t with initial stock D_{t-1} .

3 The Linear Equilibrium

Suppose $\{p_s(\cdot), D_s(\cdot)\}_{s \geq 0}$ is an equilibrium with continuous functions $p_s(\cdot)$ and $D_s(\cdot)$.

We want now to derive first-order conditions on the equilibrium price and output.

One difficulty is that in certain instances the firm may choose not to produce new output, in which case the equilibrium path will have “corner points.” However, if the initial stock is not too large and variations in demand are not too wide, this problem does not arise. Specifically, letting D_t^c denote the quantity D such that $r_t[(1-\delta)D_t^c] = \bar{c}_t$, we assume that $D_t^c > 0$ and $(1-\delta)D_t^c < D_{t+1}^c$ at all t and all states of nature.⁸

Then it can be shown that $D_t(D) < D_t^c$ whenever $D < D_t^c$ and

$$D_t(D) > (1-\delta)D \text{ iff } D < D_t^c.^9$$

are equal almost everywhere). But, for our purposes, we have found no loss in aggregating the buyers' actions into a market pricing schedule. Our equilibrium concept corresponds to a Markovian subgame perfect equilibrium in the complete game.

⁸Equivalently, $a_t > \bar{c}_t$ and $\frac{(1-\delta)b_{t+1}}{b_t} \leq \frac{a_{t+1}-\bar{c}_{t+1}}{a_t-\bar{c}_t}$. If all parameters except b_t are constant, the condition becomes $(1-\delta)b_{t+1} \leq b_t$, i.e. demand must not drop by more than the depreciation rate δ per cent, which is obviously satisfied if demand is rising in time ($b_{t+1} \leq b_t$).

⁹See Srour, 1997a. Although the result is typical of this kind of problem (cf. Dixit and Pindyck), the proof is substantially complicated by the fact that the firm's payoffs depend on an infinite path of actions rather than a single one, as is usually assumed.

For all $s \geq 0$ and $D_{s-1} \geq 0$, let $\pi_s^*(D_{s-1})$ denote the supremum of the expected discounted sum at s of future cash flows when the stock at $s - 1$ is D_{s-1} . Then

$$\pi_s^*(D_{s-1}) = \sup\{\pi_s(D; D_{s-1}) : D \geq (1 - \delta)D_{s-1}\},$$

where

$$\pi_s(D; D_{s-1}) \equiv [p_s(D) - c_s][D - (1 - \delta)D_{s-1}] + \frac{1}{1 + i_{s+1}} E_s[\pi_{s+1}^*(D)].$$

Lemma 1 *Suppose $D_{t-1} < D_t^c$ and $p_{t+1}(\cdot)$ and $D_{t+1}(\cdot)$ are differentiable at all $D < D_{t+1}^c$. Then $D_t \equiv D_t(D_{t-1})$ satisfies the first-order condition*

$$r_t(D_t) - \bar{c}_t + p'_t(D_t)[D_t - (1 - \delta)D_{t-1}] = 0.$$

The first-order condition above can be interpreted as follows. The sale of an additional unit at time t raises revenues in that period by p_t and it reduces them by $(1 - \delta)p_{t+1}^*(D)$ at $t + 1$ (since the monopolist's market at $t + 1$ is $1 - \delta$ units smaller).¹⁰ So present discounted profits at t rise by the rental price $r_t(D)$ that unit earns from t to $t + 1$ less the opportunity cost \bar{c}_t of producing it at t rather than at $t + 1$. However, the higher supply at t also causes prices at t to drop by the amount $p'_t(D)$, which lowers profits by the amount $p'_t(D)[D - (1 - \delta)D_{t-1}]$. At the optimal choice, the two effects on profits must cancel each other out.

The first-order condition reflects two facts. First, additional sales at time t essentially occur at the expense of sales at $t + 1$. This gives the seller an incentive to

¹⁰Of course, the higher supply at time t also induces a change in supply at $t + 1$, but this has only a second-order effect on the already optimized profits at $t + 1$.

postpone output in order to discriminate against buyers intertemporally. Therefore, the supply of goods should converge more gradually towards a steady state than it would, say, if behavior were governed by the equation

$$[p_t(D) - c_t] + p'_t(D)[D - (1 - \delta)D_{t-1}] = 0.^{11}$$

Second, the effects of present actions on future prices are reflected in present prices. Again, the supply of goods should converge more gradually towards a steady state than would be the case if the equation were

$$r_t(D) - \bar{c}_t + r'_t(D)[D - (1 - \delta)D_{t-1}] = 0.^{12}$$

The above suggests that the effects of a shock on output and prices will be more gradual and more persistent in the case of a monopolist seller of durable goods than they would be under perfect competition whereby

$$p_t(D) - c_t = 0 \text{ (or } r_t(D) - \bar{c}_t = 0), \quad D_t = \frac{a_t - \bar{c}_t}{b_t}$$

or if the goods were non-durable, whereby

$$r_t(D) - c_t + r'_t(D)D = 0.$$

The theorem below shows the existence and uniqueness of a “linear” equilibrium.¹³

¹¹This corresponds to the case where a new monopolist enters the market each period or the monopolist cares only about present cash flows.

¹²This corresponds to the case where buyers take future prices as fixed.

¹³Our presentation is related to that of Kahn (1986).

Lemma 2 Suppose $\{p_s(\cdot), D_s(\cdot)\}_{s \geq 0}$ is an equilibrium with continuous schedules such that, for all $D \leq D_{t+1}^c$, $D_{t+1}(D) = \gamma_{t+1} + \rho_{t+1}D$ and $p_{t+1}^*(D) = \mu_{t+1} - \nu_{t+1}D$. Then, for all $D \leq D_t^c$, $D_t(D) = \gamma_t + \rho_t D$ and $p_t^*(D) = \mu_t - \nu_t D$, where¹⁴

$$\begin{aligned}\gamma_t &= \frac{a_t - \bar{c}_t}{2b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}, \\ \rho_t &= \frac{b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}{2b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}(1-\delta), \\ \mu_t &= \bar{c}_t + \frac{b_t}{2b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}(a_t - \bar{c}_t) + \beta_{t,t+1}(1-\delta)\mu_{t+1}^e, \\ \nu_t &= \frac{[b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e]^2}{2b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}(1-\delta).\end{aligned}$$

Theorem 1 There is a unique equilibrium $\{D_t(\cdot), p_t(\cdot)\}$ with linear schedules.¹⁵

Sketch of Proof (A formal proof is provided in the appendix.) The claim in the finite horizon case follows immediately from the previous lemma. The finite horizon solutions can then be shown to converge to an equilibrium of the infinite horizon game which is the only one to satisfy the system of equations provided in lemma 1 (assuming some boundary conditions). \square

From now on, all statements will refer to the linear equilibrium.

¹⁴The superscript e refers to expectations conditional on information in the previous period, e.g.

$\nu_{t+1}^e = E_t \nu_{t+1}$.

¹⁵However, we do not know whether there are continuous non-linear equilibria. Gul et al. (1986) prove that this is not the case when the strategies are analytic in a neighborhood of 1.

4 The Stationary Deterministic Case

The stochastic processes of price and output take a particularly simple form if b_t and $\beta_{t,t+1}$ are constant. In that case, ν_t and ρ_t are also constant,

$$\nu_t = \nu \equiv \frac{[1 - \beta(1 - \delta)^2]^{-1/2} - 1}{\beta(1 - \delta)} b, \quad \rho_t = \rho \equiv \frac{1 - \delta}{1 + [1 - \beta(1 - \delta)^2]^{1/2}},$$

while γ_t and μ_t are linear in a_t and \bar{c}_t .

If a_t and c_t are constant as well, then so are γ_t and μ_t , and the equilibrium process is described by

$$D_{t+1} = \gamma + \rho D_t,$$

$$p_{t+1} = \mu - \nu D_t.$$

Let then \hat{D} , \hat{d} and \hat{p} be the steady-state stock, output and price respectively:

$$\hat{D} = \gamma + \rho \hat{D}, \quad \hat{d} = \delta \hat{D}, \quad \hat{p} = \mu - \nu \hat{D}.^{16}$$

An easy calculation shows

$$\hat{D} = k_1(\beta, \delta) \frac{a - \bar{c}}{b},$$

$$\hat{p} - c = k_2(\beta, \delta)(a - \bar{c})$$

where $k_1(\beta, \delta) = \frac{1}{1 + [1 - \beta(1 - \delta)^2]^{-1/2} \delta}$ and

$$k_2(\beta, \delta) = \frac{1}{1 - \beta(1 - \delta)} \frac{\delta}{\delta + [1 - \beta(1 - \delta)^2]^{1/2}}.$$

Thus, if for example the initial stock is smaller than the steady-state, then the supply of goods in the market increases towards \hat{D} at the rate of convergence $\frac{1}{\rho}$ (e.g., $\frac{\hat{D} - D_t}{\hat{D} - D_{t+1}} = \frac{1}{\rho}$), whereas output and price decrease at the same rate.

¹⁶Alternatively, $\hat{p} = r(\hat{D}) + \frac{1 - \delta}{1 + \rho} \hat{p}$.

Notice \hat{D} rises as b declines, a rises, c declines or β declines. Also, keeping \bar{c} constant across goods, the more durable the good, i.e. the smaller is δ , the larger the steady-state stock and the smaller the output.¹⁷ Heuristically, this is because the seller has an incentive to make new sales irrespective of the previous stock.

More importantly, as already hinted in the previous section, the more durable the good, the slower the rate of convergence: $\frac{\partial \rho}{\partial \delta} < 0$. Contrast this with the fact that (as long as the cost of providing one unit of the good's services, \bar{c} , is kept constant) the percentage variation of the steady-state stock, output or price differential ($\hat{p} - c$) due to a change of the parameters a, b or c is independent of δ and, with regard to the steady-state stock and output, is the same as under perfect competition.

5 Comparative Statics

It can be easily verified that a larger stock at the end of a period implies a larger stock and a lower price subsequently. Since output $d_t = D_t(D_{t-1}) - (1 - \delta)D_{t-1}$ and

$$\frac{\partial}{\partial D_{t-1}}[d_t] = \rho_t - (1 - \delta) < 0$$

output will also be smaller.

Assume a_t , b_t , c_t and $\beta_{t,t+1}$ are stochastically independent, and let a, b, c and β be selected values of these parameters to which we associate a steady-state.

¹⁷This was noted by Bond and Samuelson (1984).

5.1 Shifts in Costs

(a) A permanent and unexpected rise of the costs of production at time t decreases the rent $r_t(D) - \bar{c}_t$ that an additional sale can earn by an amount that is independent of the initial stock. Such a shock will therefore not affect the slopes of $D_s(\cdot)$ and $p_s^*(\cdot)$ (for $s \geq t$), e.g. $\frac{\partial}{\partial c_t} \rho_s = \frac{\partial}{\partial c_t} \nu_s = 0$. However, current and future output as well as the steady-state output will drop. For non-durable goods, output drops immediately to the new steady-state; for durable goods, starting from the old steady-state, output drops first below the new steady-state level—in the competitive case, the stock is brought immediately, and output a period later, to the new steady-state levels, whereas in the monopolistic case, the stock and output converge gradually to the new levels. Thus, the variation of output following a permanent shock to the costs of production is higher for durable than for non-durable goods. Prices of course rise, immediately under competition or for non-durable goods, and gradually under monopoly.

If the shock is temporary, current output falls by an amount that is even larger than when the shock is permanent, as the current opportunity cost of production increases by a larger amount in this case. So here too the variation in output is larger for durable goods. In the monopolistic case, both present and future stocks fall, hence present and future prices as well as future outputs rise. (Notice, the price-cost differential falls: $\frac{\partial}{\partial c_t} [p_t^*(D) - c_t] < 0$.)

(b) Expected higher costs at $t + 1$ cause production at t to be relatively cheaper, hence the output and stock at t will rise, $\frac{\partial D_t}{\partial c_{t+1}^e} > 0$. Both the higher supply at t and

the expected higher costs at $t + 1$ induce a lower expected output at $t + 1$, $\frac{\partial d_{t+1}^e}{\partial c_{t+1}^e} < 0$, and typically, a lower expected stock.¹⁸ Future stocks are therefore expected to fall and future prices to rise. Remarkably, the linear character of the equilibrium implies that current prices remain unchanged.

Theorem 2 *Suppose all parameters except costs are constant. Then expected changes in future costs have no effect on current prices.*

Proof. Let a be the quantity by which the supply would fall following a rise of the opportunity cost \bar{c}_t by 1 unit, so prices would increase by $-p'_t(D)a$. Since $\bar{c}_t \equiv c_t - \beta(1-\delta)c_{t+1}^e$, the effect of a unit increase in expected costs at $t + 1$ amounts to the combined effects of a decrease of the opportunity costs \bar{c}_t at t by $\beta(1-\delta)$ units and an increase of the opportunity costs \bar{c}_{t+1} at $t + 1$ by one unit. The former causes prices at time t to fall by $\beta(1-\delta)p'_t(D)a$ whereas the latter causes prices at $t + 1$ to increase by $-p'_{t+1}(D)a$. Thus prices at t change by $\beta(1-\delta)p'_t(D)a - \beta(1-\delta)p'_{t+1}(D)a = 0$.¹⁹

□

$$^{18} \frac{\partial}{\partial c_{t+1}^e} E_t(D_{t+1}^*) = -E_t \left[\frac{1}{2b_{t+1} + \beta_{t+1,t+2}(1-\delta)\nu_{t+2}^e} \right] + \rho_{t+1}^e \frac{\beta_{t,t+1}(1-\delta)}{2b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}.$$

Since $\rho_{t+1}^e < (1-\delta)$, it follows:

$$\frac{\partial}{\partial c_{t+1}^e} E_t D_{t+1} < 0 \text{ if } \beta_{t,t+1}(1-\delta)^2 < E_t \left[\frac{2b_t + \beta_{t,t+1}(1-\delta)\nu_{t+1}^e}{2b_{t+1} + \beta_{t+1,t+2}(1-\delta)\nu_{t+2}^e} \right],$$

which is the case if all parameters except c_t are constant.

¹⁹Alternatively, notice that v_t is independent of costs, and μ_t can be written in the form

$$\mu_t - \frac{\rho}{(1-\delta)}c_t = \left(1 - \frac{\rho}{(1-\delta)}\right)a + \beta(1-\delta)E_t \left[\mu_{t+1} - \frac{\rho}{(1-\delta)}c_{t+1} \right],$$

hence $\mu_t = \frac{1}{1-\beta(1-\delta)} \left(1 - \frac{\rho}{(1-\delta)}\right)a + \frac{\rho}{(1-\delta)}c_t$, which is independent of c_{t+1}^e .

5.2 Shifts in Demand

(a) Starting from a stationary deterministic setting whereby all parameters are constant (cf. section 4), it is not difficult to see that a permanent increase in demand due to either a permanent increase of the intercept or a permanent decline of the slope of demand causes an immediate increase of both output and price. (To illustrate, suppose the slope is halved. Let $p_t(\cdot)$ and $\hat{p}_t(\cdot)$ denote the equilibrium price schedules before and after the shift respectively. Then it is easy to see that for all D , $p_t^*(D) = \hat{p}_t^*(2D)$. It follows that $p_t^*(D) < \hat{p}_t^*(D)$ as claimed.)

(b) The outcome following temporary shocks to demand is a priori ambiguous. A rise in current output raises the stock in the market. Since the increase in demand is temporary, this implies lower future prices which offset the likely higher current rental prices. In light of certain empirical findings that markups of durable goods are countercyclical (see Bils, 1987, Murphy et al., 1989, Rotemberg and Woodford, 1997), it is of special interest to examine whether, under general conditions, prices and output move in opposite directions.

To investigate this question, it is first necessary to decide what type of demand shock to use as a benchmark. Clearly, as in the standard one-period scenario, arbitrary movements in prices and output can be obtained through shifts in the elasticity of demand. To see this more formally, write the first-order condition at $D_t \equiv D_t(D_{t-1})$ in the form

$$\frac{r_t(D_t) - \bar{c}_t}{p_t(D_t)} + \frac{1}{\epsilon_t} \frac{D_t - (1 - \delta)D_{t-1}}{D_t} = 0$$

where $\frac{1}{\epsilon_t} \equiv \frac{p_t'(D_t)D_t}{p_t(D_t)}$ is the inverse price-elasticity of demand. Then, a higher elasticity

of demand at D_t which leaves $r_t(D_t)$ and $p_t(D_t)$ unchanged (due for example to a rotation of the inverse demand curve $r_t(\cdot)$ around the point (D_t, r_t)) induces a larger supply by the firm and a lower price.

Accordingly, we focus on temporary elasticity-preserving shifts in demand. These amount to proportional changes in demand or, equivalently, shifts that changes only the slope b_t of inverse demand.²⁰

It is easy to see that a lower slope, hence upward shift, in demand entails a higher output and higher rental prices at time t (hence higher stocks and lower prices in the future):

$$\frac{\partial \gamma_t}{\partial b_t} < 0, \quad \frac{\partial \rho_t}{\partial b_t} < 0, \quad \frac{\partial D_t(D_{t-1})}{\partial b_t} < 0, \quad \frac{\partial r_t(D_t)}{\partial b_t} < 0.$$

Let $D_t \equiv D_t(D_{t-1})$ and let \hat{D}_t be the quantity where the market price under the new demand, $\hat{p}_t(\hat{D}_t)$, equals the original price $p_t(D_t)$. It can be shown (see appendix) that the rise in demand leads the seller to increase the supply beyond \hat{D}_t and lower the price below $p_t(D_t)$ if and only if

$$\frac{D_{t-1}}{D_t} < \beta_{t,t+1} \frac{E_t p_{t+1}^*(D)}{\hat{p}'_t(\hat{D})} = \frac{\beta_{t,t+1} \nu_{t+1}^e}{\hat{b}_t + \beta_{t,t+1} (1 - \delta) \nu_{t+1}^e}.$$

This is the case if the initial stock D_{t-1} is small enough (in particular if $D_{t-1} = 0$), or the rise in demand is high enough, i.e. \hat{b}_t is small enough, provided its value

²⁰We define an elasticity-preserving shift in demand as one such that $\frac{\hat{p}'_t(\hat{D})\hat{D}}{\hat{p}_t(\hat{D})} = \frac{p'_t(D)D}{p_t(D)}$ whenever $\hat{p}_t(\hat{D}) = p_t(D)$ (where the hat superscript denotes quantities after the shift). The latter condition requires that the intercepts be constant: $\hat{p}_t(\hat{D}) - \hat{p}'_t(\hat{D})\hat{D} = p_t(D) - p'_t(D)D$. For a temporary shift, future demand is not affected. Therefore $\hat{p}_t(\cdot)$ and $p_t(\cdot)$ have the same intercept iff $\hat{r}_t(\cdot)$ and $r_t(\cdot)$ have the same intercept iff only the slope changes.

remains within the range allowed for the distribution of b_t . However, at a steady state, whereby $D_t = D_{t-1}$, and assuming a stationary process for b_t , it can be shown that the inequality fails, hence the current price will rise. (See the numerical example below for further detail.)

(c) The effects of changes in the intercept of demand a_t are derived in a similar fashion to those of changes in the cost: a rise of the intercept causes prices and output to rise in the current period, and subsequently to fall if the shock is temporary; an expected future rise causes current prices to rise, but output is unaffected.

5.3 Shifts in the Interest Rate

A higher interest rate (or lower degree of impatience) causes the present value of future goods, hence current price, to fall. By the same token, as can be seen directly from the first-order condition, current output rises (and future output declines) as the firm places a greater value on current versus future cash flows.²¹ However, this is true only for small enough costs of production. Otherwise, the higher opportunity cost of production \bar{c}_t can induce the firm to postpone some of its output to periods that are more favorable to borrowing.²²

²¹Of course these results describe behavior on the supply side only, since the interest rate does not affect demand in our model and there are no substitution effects between durables and non-durables. So our results do not conflict with findings in the literature that a rise in the interest rate is associated with a drop in sales [Mankiw, 1982].

²²More precisely, a one-time 1 per cent increase in the interest rate causes the return at $t + 1$ to an additional sale at t to change by an amount equal to $r_t(D) + r'_t(D)[D - (1 - \delta)D_{t-1}] - c_t$. For

6 Numerical Examples

Suppose now that the parameters a_t , b_t , c_t and $\beta_{t,t+1}$ follow a stationary process. Let a , b , c , and β be fixed values to which we associate a steady-state equilibrium. The system of equations in Lemma 2, log-linearized around the steady state, then yields a stationary solution²³ which approximates the original equilibrium and which can be used to analyze numerically the effects of shocks. We illustrate below our findings in the case of elasticity-preserving shocks to demand.²⁴

Tables 1 and 2 are obtained under the assumption that all parameters except b_t are constant: $a = 3$, $c = 0$, $\beta_{t,t+1} = .975$ (or $i_{t+1} = 2.56$ per cent);²⁵ and the stochastic process governing b_t is given by

$$\hat{b}_{t+1} = \omega \hat{b}_t + \epsilon_{t+1}, \quad \epsilon_t \text{ i.i.d., } \text{var}(\epsilon_t) = 1$$

where hats denote percentage deviations from steady state. Numbers shown in parentheses in the tables are calculated under the assumption of perfect competition. Figures 1 and 2 depict the impulse response of price and output (in percentage terms)

$c_t = 0$, this is positive; for c_t large enough, for example for $c_t > a$, it is negative.

²³cf. Blanchard and Kahn, 1988.

²⁴The findings in the cases of shocks to costs, the intercept of demand or the interest rate reveal no surprises and confirm our claims in section 5.

²⁵The assumption of zero costs is necessary to allow meaningful comparisons between goods with different degrees of durability. Alternatively, we can impose \bar{c} to be constant. Except for this provision, the particular values chosen for a , b and \bar{c} are inessential. The average value chosen for the interest rate is a compromise between opting for three-months or six-months time periods during which prices are assumed to be fixed. Again the results are robust.

to a 1 per cent decline in the slope (hence a rise in demand) for $\omega = 0$ and $\omega = 0.9$ respectively when $\delta = 0.025$. For the sake of comparison, we also show the response of output under perfect competition.

As suggested earlier, we find that a rise in demand causes an immediate increase of both price and output, followed by a drop if the shocks are i.i.d. ($\omega = 0$) or a gradual decline if the shocks are serially correlated ($\omega = 0.9$), and a subsequent adjustment back towards the steady state. In the case $\omega = 0.9$, prices fall below the mean while output is still high.

The correlation between prices and lagged output (typically one period lagged) is negative, whereas the correlation between price and contemporaneous output is positive. The latter does not seem to obey any particular relationship with δ (except that it is bell shaped); however, the correlation between price and the total supply is smaller the more durable is the good and eventually becomes negative. This reflects the fact that the more durable is the good, the longer will a given increase of the stock last and therefore the more likely will a high stock be contemporaneous with lower prices. More significantly, a similar relationship obtains between prices and the level of demand as represented by the inverse slope, and which arguably is a better proxy of aggregate output than the output of durable goods: the correlation between prices and the inverse slope is smaller the more durable is the good and eventually becomes negative when shocks are serially correlated.²⁶ Our results may therefore provide some explanation of empirical findings that prices and markups of

²⁶For $\delta = 0.025$, prices are negatively correlated with the level of demand if $\omega \geq 0.4$.

more durable goods are less procyclical.

The results also reproduce the well-known fact that output variance is higher for more durable goods. This is because the output in a stationary state is smaller for more durable goods; since the shock involves a given variation in demand for total supply, the percentage deviation of output will be higher for more durable goods. However, because the firm has an incentive to extract high prices first and because it takes into consideration the effect of higher output on future prices, it increases sales by less than would a perfectly competitive firm.²⁷ This, together with the firm's opportunity in each period to earn rent on additional sales irrespective of what has already been sold, causes the effects on output (as witnessed by the correlation between output and lagged output) to persist longer than they would under perfect competition.

These effects are particularly striking if the shocks apply directly to “purchases” of durable goods rather than stocks. That is, if the stochastic process governing b_t is described by

$$\hat{g}_{t+1} = \omega \hat{g}_t + \epsilon_{t+1}, \quad \epsilon_t \text{ i.i.d.}, \text{ var}(\epsilon_t) = 1$$

where $g_t \equiv b_t - (1 - \delta)b_{t-1}$. In that case, the demand for total supplies keep rising for a while following a shock before it declines.

Figure 3 depicts the impulse responses to a 1 per cent decline in g_t (corresponding to a 1 per cent increase in “purchases”) when $\delta = .025$ and $\omega = .9$. Under perfect

²⁷With higher serial correlation ω , the firm can increase its supply without fear of depressing prices. Thus, the percentage deviations of output and price rise with ω .

competition, the behaviour of output is identical to that of inverse g_t ,²⁸ specifically it rises immediately following the shock and then gradually returns to its original state. Under monopoly, output rises gradually to a peak before it declines, as the firm delays sales to preserve its market for times when demand and rental prices are higher.

7 Conclusion

We analyzed in this paper the sale of durable goods by a monopolist in a stochastic environment with linear demand and constant marginal costs of production. We found that behavior in the present model under demand shocks is in many respects similar to that in a perfectly competitive model with increasing marginal costs. For example, following a temporary increase in demand, prices are higher at first, then lower, while output adjustments are gradual relative to the competitive case with constant marginal costs. Furthermore, it was found that while the correlation between prices and current output is positive, the correlation between prices and the level of demand (measured by the inverse slope) is smaller the more durable is the good, and eventually negative when movements in demand are serially correlated. This raises the possibility that countercyclical markups for durable goods may obtain in a general equilibrium setting, where higher interest rates can also contribute to keep prices down during a boom (see Srour 1997b). These results are the consequence

²⁸The steady-state stock is $\frac{a-\bar{c}}{b}$; so output following the shock is $\frac{a-\bar{c}}{b} - (1-\delta)\frac{a-\bar{c}'}{b}$ where b' is the new slope and the percentage deviation of output is $-\hat{g}_t$.

of the intertemporal nature of the profit maximizing problem for durable goods in a monopolistic setting. The gradual adjustment of output and prices emerges in the absence of any structural or nominal adjustment costs.

Except for monopoly power, we assumed ideal market conditions. In particular there is no private information on the demand side and there is a perfect second hand market, so that the distribution of durable good holdings across consumers is irrelevant. Allowing weaker conditions could reveal challenging new directions of research.

Appendix

Proof of lemma 1

Recall

$$\pi_s(D_s; D_{s-1}) = [p_s(D_s) - c_s][D - (1 - \delta)D_{s-1}] + \beta_{s,s+1}E_s[\pi_{s+1}^*(D)].$$

The envelope theorem implies

$$\frac{\partial}{\partial D_t} \pi_{t+1}^*(D_t) = -(1 - \delta)[p_{t+1}^*(D_t) - c_{t+1}],^{29}$$

hence

$$\frac{\partial}{\partial D} \pi_t(D; D_{t-1}) = r_t(D) - \bar{c}_t + p'_t(D)[D - (1 - \delta)D_{t-1}].$$

By the remarks above, $D_{t-1} < D_t^c$ implies $(1 - \delta)D_{t-1} < D_t(D_{t-1}) < D_{t+1}^c$, so D_t is an interior point in the interval over which $\pi_t(D; D_{t-1})$ is maximized, hence the claim.

□

Proof of lemma 2

Let $D < D_t^c$. By lemma 1, $D_t \equiv D_t(D)$ satisfies the first order condition

$$r_t(D_t) - \bar{c}_t + p'_t(D_t)[D_t - (1 - \delta)D] = 0.$$

Since $p_t(D_t) = r_t(D_t) + \beta_{t,t+1}(1 - \delta)E_t p_{t+1}^*(D_t)$, it follows

$$a_t - b_t D_t - \bar{c}_t + [-b_t - \beta_{t,t+1}(1 - \delta)\nu_{t+1}^e][D_t - (1 - \delta)D] = 0.$$

²⁹ A rigorous proof can be found in Srour, 1997a.

Rearranging the elements yields an expression for $D_t \equiv D_t(D)$ as claimed, while the expression of $p_t^*(.)$ is derived from the identity

$$p_t^*(D) = r_t[D_t(D)] + \beta_{t,t+1}(1 - \delta)E_t p_{t+1}^*[D_t(D)].$$

Finally, the expressions of $D_t(.)$ and $p_t^*(.)$ must extend to D_t^c by left continuity. \square

Proof of theorem 1

The claim for finite-horizon games follows immediately from lemma 2. Let $\gamma_{t,T}$, $\rho_{t,T}$, $\mu_{t,T}$ and $\nu_{t,T}$ denote the parameters associated with the game that terminates at T with the modification that at T the cost is \bar{c}_T :

$$\gamma_{T,T} = \frac{a_T - \bar{c}_T}{2b_T}, \quad \rho_{T,T} = \frac{1}{2}(1 - \delta), \quad \mu_{T,T} = \frac{a_T + \bar{c}_T}{2}, \quad \nu_{T,T} = \frac{1}{2}(1 - \delta)b_T.$$

$$\frac{\partial \nu_t}{\partial \nu_{t+1}^e} = \beta_{t,t+1}(1 - \delta)^2 \frac{[b_t + \beta_{t,t+1}(1 - \delta)\nu_{t+1}^e][3b_t + \beta_{t,t+1}(1 - \delta)\nu_{t+1}^e]}{[2b_t + \beta_{t,t+1}(1 - \delta)\nu_{t+1}^e]^2}.$$

$$\text{Hence, } 0 < \frac{3}{4}\beta_{t,t+1}(1 - \delta)^2 < \frac{\partial \nu_t}{\partial \nu_{t+1}^e} < \beta_{t,t+1}(1 - \delta)^2$$

It follows $\nu_{T,T+1} > \nu_{T,T}$ since $\nu_{T+1,T+1}^e = \frac{1}{2}(1 - \delta)b_{T+1}^e > \nu_{T+1,T}^e \equiv 0$, and by induction $\nu_{t,T+1} > \nu_{t,T}$. The sequence $\nu_{t,T}$ therefore increases with T and converges (uniformly with respect to the states of nature): let $\bar{\nu}_t$ be the limit. Similarly, using the expressions of ρ_t and γ_t as functions of ν_{t+1}^e , it can be shown that $\rho_{t,T}$ increases and $\gamma_{t,T}$ decreases with T ; also both converge (uniformly): let $\bar{\rho}_t$ and $\bar{\gamma}_t$ be their limits.

The variation of $\mu_{t,T}$ with T is ambiguous. However,

$$|\mu_{t,T+1} - \mu_{t,T}| < \beta_{t,t+1}(1 - \delta) |\mu_{t+1,T+1}^e - \mu_{t+1,T}^e| + \frac{\beta_{t,t+1}(1 - \delta)}{4} |\nu_{t+1,T+1}^e - \nu_{t+1,T}^e| \frac{a_t - \bar{c}_t}{b_t}.$$

Hence, $\mu_{t,T}$ also converges uniformly, say to $\bar{\mu}_t$.

Clearly $\bar{\nu}_t, \bar{\rho}_t, \bar{\gamma}_t, \bar{\mu}_t$ satisfy the system of difference equations in lemma 2 and describe an equilibrium to the infinite-horizon game. Moreover, since $\frac{\partial \nu_t}{\partial \nu_{t+1}^c} < \beta_{t,t+1}(1 - \delta)^2$, there is a unique solution to the difference system such that $\lim_{T \rightarrow \infty} \beta_{t,T}(1 - \delta)^{2T} \nu_T = 0$ (which must hold under the transversality condition $\lim_{T \rightarrow \infty} \beta_{t,T} p_T^*(D) = 0$). \square

Proof of the inequality in section 5.2(b)

At D_t we have $\frac{\partial}{\partial D} \pi_t(D_t; D_{t-1}) = r_t(D_t) - \bar{c}_t + p'_t(D_t)[D_t - (1 - \delta)D_{t-1}] = 0$, and at \hat{D}_t , $\frac{\partial}{\partial D} \hat{\pi}_t(\hat{D}_t; D_{t-1}) = \hat{r}_t(\hat{D}_t) - \bar{c}_t + \hat{p}'_t(\hat{D}_t)[\hat{D}_t - (1 - \delta)D_{t-1}]$

Subtract the two previous expressions, making use of the fact, $\hat{r}_t(\hat{D}) - r_t(D) = -\beta_{t,t+1}(1 - \delta)E_t p_{t+1}^*(D)(\hat{D} - D)$, to deduce

$$\begin{aligned} \frac{\partial}{\partial D} \hat{\pi}_t(\hat{D}_t; D_{t-1}) &= -\beta_{t,t+1}(1 - \delta)E_t p_{t+1}^*(D)(\hat{D}_t - D_t) + [p'_t(D_t) - \hat{p}'_t(\hat{D}_t)](1 - \delta)D_{t-1} \\ &= (b_t - \hat{b}_t)(1 - \delta)D_t \left[\beta_{t,t+1} \frac{E_t p_{t+1}^*(D)}{\hat{p}'_t(\hat{D}_t)} - \frac{D_{t-1}}{D_t} \right] \end{aligned}$$

The claim follows.

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δ	$\text{sd}(\hat{d})$	$\text{sd}(\hat{D})$	$\text{sd}(\hat{p})$	$c(\hat{p}, \hat{d})$	$c(\hat{p}, \hat{d}_{-1})$	$c(\hat{d}, \hat{d}_{-1})$	$c(\hat{p}, \hat{D})$	$c(\hat{p}, -\hat{b})$
.025	9.17 (56)	.34(1)	.53	.48	-.36	-.11 (-.5)	-.62	.2
.1	3.67 (13)	.44(1)	.36	.67	-.49	-.17 (-.5)	-.27	.38
.5	1.43 (2.2)	.72(1)	.26	.82	-.65	-.2 (-.4)	.44	.66
.75	1.15 (1.9)	.86(1)	.15	.78	-.69	-.12 (-.24)	.6	.7
.95	1.02 ()	.97(1)	.03	.72	-.7	-.02 (-.01)	.69	.7
1	1	1(1)	0	0 (0)

Table 1: \hat{b}_t i.i.d., $\text{var}(\hat{b}_t) = 1$

δ	$\text{sd}(\hat{d})$	$\text{sd}(\hat{D})$	$\text{sd}(\hat{p})$	$c(\hat{p}, \hat{d})$	$c(\hat{p}, \hat{d}_{-1})$	$c(\hat{p}, \hat{D})$	$c(\hat{d}, \hat{d}_{-1})$	$c(\hat{p}, -\hat{b})$
.025	12.5	1.88	1.93	.37	.08	-.81	.68 (-.009)	-.45
.1	4.7	2	.82	.47	-.009	-.5	.62 (0)	-.19
.5	2.46	2.21	.28	.38	-.2	.08	.8 (.22)	.18
.75	2.34	2.26	.13	.28	-.22	.16	.87 (.5)	.21
.95	2.3	2.29	.026	.19	-.26	.17	.89 (.88)	.22
1	2.29	2.29	0

Table 2: $\hat{b}_{t+1} = 0.9\hat{b}_t + \epsilon_{t+1}$, ϵ_t i.i.d., $\text{var}(\epsilon_t) = 1$

Figure 1

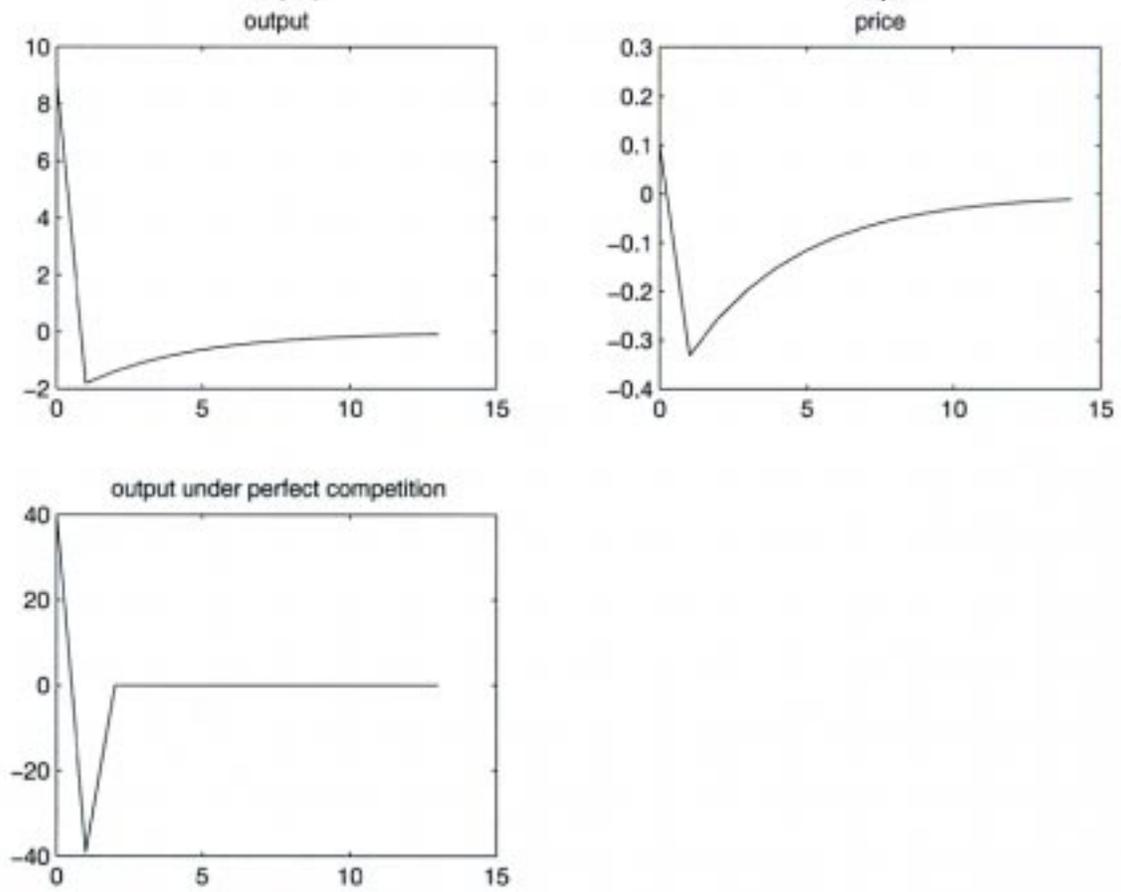


Figure 2

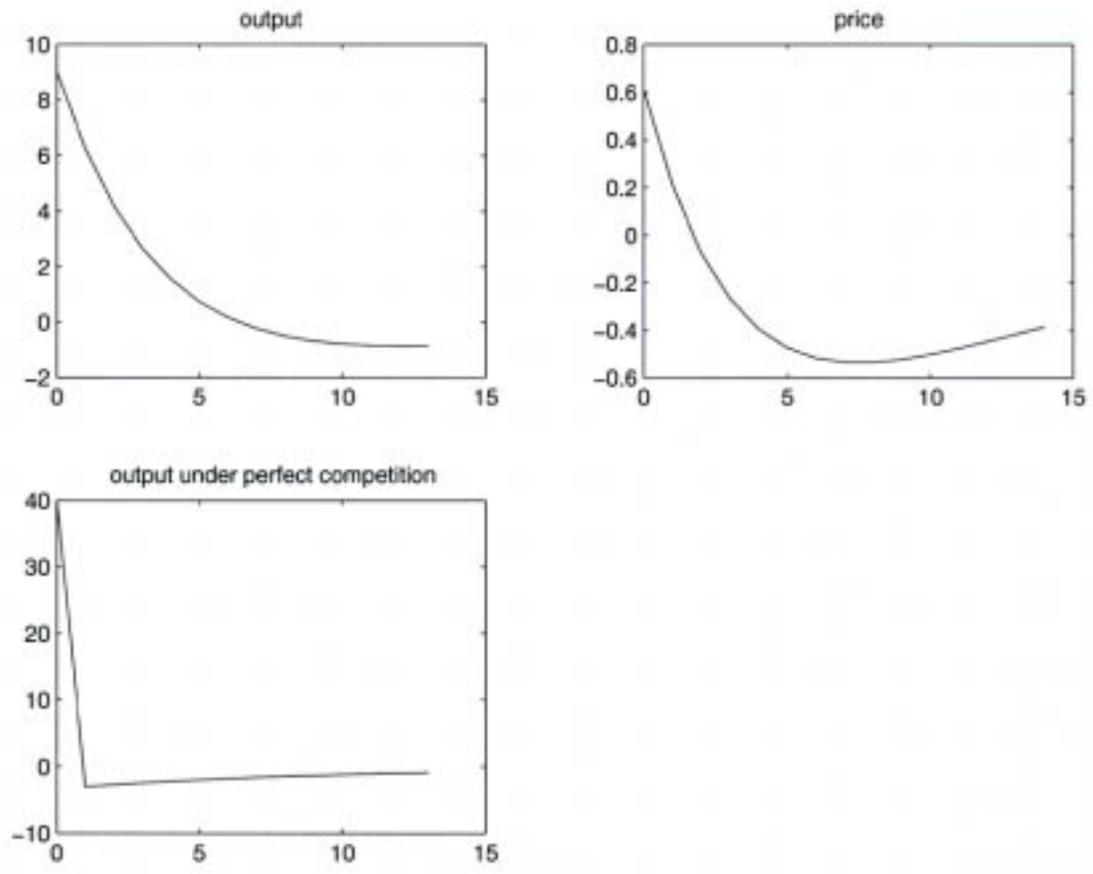
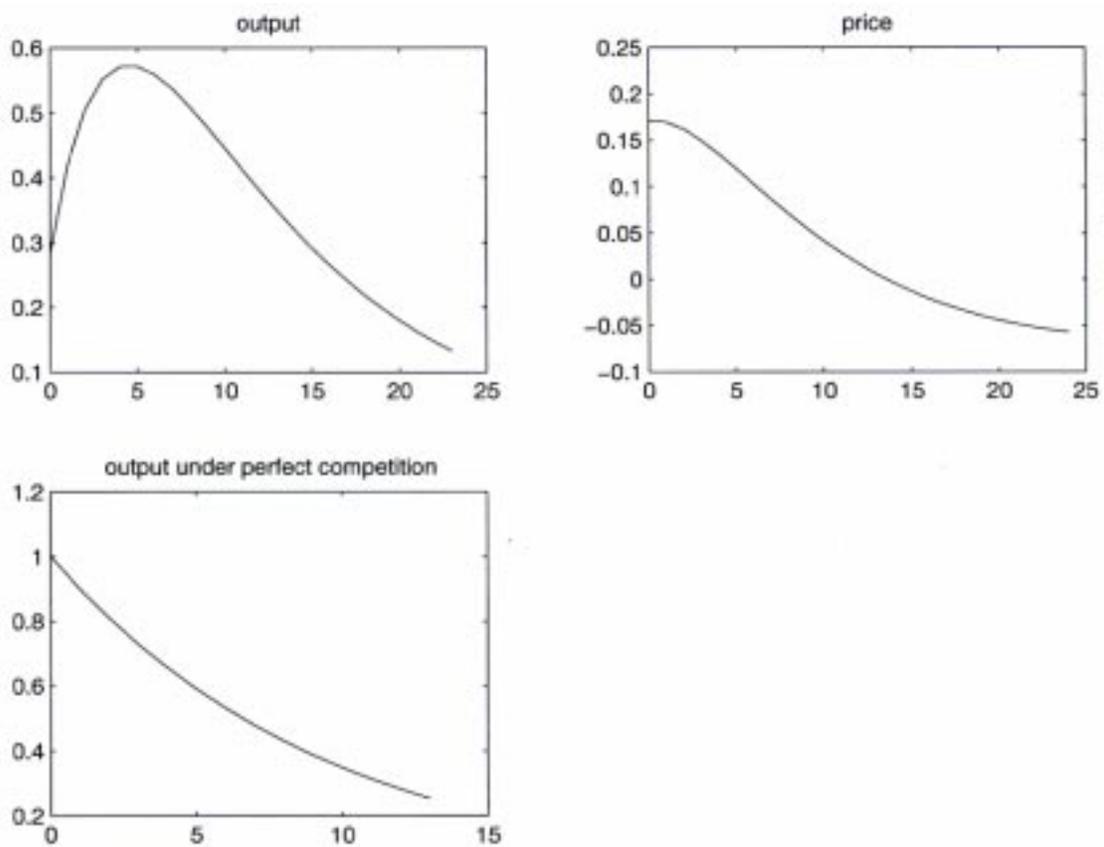


Figure 3



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