

Working Paper 98-2 / Document de travail 98-2

**International Borrowing, Specialization and Unemployment in a  
Small Open Economy**

by  
Patrick N. Osakwe  
and  
Shouyong Shi

Bank of Canada



Banque du Canada

ISSN 1192-5434  
ISBN 0-662-26635-8

Printed in Canada on recycled paper

Bank of Canada Working Paper 98-2

February 1998

**International Borrowing, Specialization and Unemployment  
in a Small, Open Economy**

Patrick N. Osakwe  
International Department, Bank of Canada†  
and  
Shouyong Shi  
Department of Economics, Queen's University

Correspondence to Patrick N. Osakwe, International Department,  
Bank of Canada, 234 Wellington Street, Ottawa, Ontario, K1A  
0G9, Canada. Phone: (613) 782-8596; Fax: (613) 782-7238; E-  
mail: [posakwe@bank-banque-canada.ca](mailto:posakwe@bank-banque-canada.ca).

†This paper represents the views of the authors and does not necessarily reflect those of the Bank of Canada or its staff. Any errors or omissions are those of the authors.

## **Acknowledgments**

The authors are grateful to Dan Bernhardt and Patrick Francois for helpful comments and the Social Sciences and Humanities Research Council of Canada for financial support. All errors are ours alone.

## **Abstract**

Empirical evidence suggests that the unemployment rate and the export/GNP ratio are positively correlated with external debt across developing countries. This paper develops a dynamic model that provides an explanation for the aforementioned relationships. The central idea of our paper is that international borrowing affects unemployment and specialization patterns by unevenly changing the risk-sharing structure - across sectors - between firms and workers. The economy produces a domestic good and an export good and faces uncertainty in its terms of trade. Unlike the domestic good, the production process for the export good lasts two periods and requires borrowing by firms in period one. To insure workers against income fluctuations, firms in the export sector find it optimal to offer an implicit contract through stable wages. This wage contract allows firms to lay off some workers in bad states of nature. An increase in international borrowing allows firms in the export sector to offer wage contracts to more workers thereby increasing the extent of specialization in the export good. As labour shifts from the domestic good sector into the more efficient export sector, a bad realization in the terms of trade results in higher unemployment. The paper shows conditions under which a state-contingent price subsidy will reduce the unemployment rate without (inefficiently) reducing the extent of specialization in the comparative advantage good.

## Résumé

Les résultats empiriques donnent à penser que, dans les pays en voie de développement, le taux de chômage et le ratio des exportations au PIB sont positivement corrélés à la dette extérieure. Les auteurs mettent au point un modèle dynamique qui explique cette relation. L'argument principal des auteurs est que les emprunts extérieurs agissent sur les profils du chômage et de la spécialisation en modifiant inégalement la structure du partage des risques (entre secteurs, entreprises et salariés). L'économie produit des biens pour la consommation intérieure et des biens pour l'exportation, et l'évolution des termes de l'échange est incertaine. Contrairement au cas des biens pour la consommation intérieure, le processus de production des biens pour l'exportation dure deux périodes et exige que les entreprises empruntent à la période un. Pour donner aux salariés l'assurance que leurs revenus ne fluctueront pas, les entreprises exportatrices préfèrent offrir un contrat implicite par l'entremise de salaires stables. Ce contrat leur permet de mettre des travailleurs à pied pendant les mauvaises périodes. Une hausse des emprunts à l'étranger amène les entreprises exportatrices à offrir des contrats salariaux à plus de travailleurs, augmentant ainsi la spécialisation dans le secteur d'exportation concerné. Au fur et à mesure que la main-d'oeuvre délaisse le secteur des biens pour la consommation intérieure pour le secteur de l'exportation, plus efficient, une détérioration des termes de l'échange donne lieu à des hausses de chômage plus importantes. Les auteurs exposent les conditions dans lesquelles une subvention éventuelle du côté des prix aura pour effet de réduire le taux de chômage sans réduire (de façon inefficace) la spécialisation dans le secteur du bien jouissant d'un avantage comparatif.

# Contents

I.	Introduction .....	1
2.	Structure of the Economy .....	3
	2.1 Agents and Goods .....	3
	2.2 Technology.....	5
	2.3 Timing .....	7
3.	Equilibrium .....	8
	3.1 Employment Contracts.....	8
	3.2 Equilibrium Definition.....	9
	3.3 Equilibrium Conditions .....	11
4.	Effects on Specialization and Unemployment.....	15
5.	Policy Implications .....	17
6.	Conclusion and Extension .....	21
	Appendix A .....	23
	Appendix B .....	24
	Appendix C .....	26
	References .....	28
	Tables 1-2.....	29
	Figures 1a-1b.....	31





# 1. Introduction

An interesting phenomenon of developing countries is that the unemployment rate and the relative importance of exports are positively correlated with the countries' external indebtedness. Table 1 lists the debt/GNP ratios, unemployment rates and export/GNP ratios for some Latin American and Asian developing countries. These numbers are more clearly depicted in Figures 1a and 1b. With Latin American countries alone (Figure 1a), there is a positive relation between the unemployment rate and the debt/GNP ratio across countries. This is accompanied by the positive relation between the export/GNP ratio and the debt/GNP ratio. Including some of the Asian developing countries does not change such a pattern of correlation (Figure 1b).<sup>1</sup>

The positive relation among the three variables represents a typical dilemma facing many developing countries and international development agencies. Decades of increasing foreign loans to developing countries seem to have produced mixed results. On the one hand, countries that have received the loans have become more specialized in sectors where the countries' comparative advantage lies. On the other hand, the increasing external debt has also produced persistently high unemployment rates. In order to find policies that attenuate the side effects of international borrowing, one must try to understand the mechanisms through which international borrowing affects unemployment and specialization in an open economy. This paper seeks to uncover one such mechanism.

We argue that international borrowing affects unemployment and specialization by unevenly changing the risk-sharing structure (across sectors) between firms and workers. To be more specific, we argue that a small open economy is subject to a considerable amount of fluctuation in its terms of trade. To insure workers against such income fluctuations, firms in the export sector find it optimal to offer an implicit insurance contract through stable wages. As is typical in the implicit contracts literature, this wage contract

---

<sup>1</sup> We realize that a much more rigorous statistical procedure is required in order to establish the relationships among the aforementioned variables. Due to data limitations, we were unable to conduct such a procedure. For statistics on exports of primary commodities as a percent of total exports see Table 2.

leads to (ex post) unemployment.<sup>2</sup> However, firms in the export sector cannot offer the wage contract to all workers in the economy, because they do not have sufficient internal funds to do so. Instead, only a fraction of the labour force is offered a wage contract by the export sector, while the rest is absorbed by the domestic good sector, which offers competitive wages. Unemployment is the fraction of agents offered a wage contract by the export sector but laid off after a bad realization in the terms of trade. Within this structure, an increase in international borrowing enables firms in the export sector to offer wage contracts to more workers. As labour shifts from the domestic good sector into the more efficient export sector, a bad realization in the terms of trade results in higher unemployment. This mechanism seems to be consistent with the positive relation among international borrowing, unemployment and specialization depicted in Figures 1a and 1b.

The idea that terms-of-trade fluctuations induce unemployment through implicit contracts is not new in the literature. For example, Fernandez (1992) examined the link between terms-of-trade uncertainty and unemployment. Deviating from earlier implicit contract models that focus on firm-specific technology shocks, Fernandez introduces aggregate uncertainty through terms-of-trade fluctuations. Our model is similar to Fernandez's model in the sense that it has aggregate uncertainty. However, her model is static and does not address the issue of specialization. Thus, it is not possible with that model to examine the relationship between international borrowing and specialization. Also different from Fernandez's model is the fact that the wage rates in our model are state independent.

The positive relationship between international borrowing and specialization in our model resembles that in Chang (1991), which shows that agents are likely to smooth consumption intertemporally by inefficiently diversifying their production if they do not have access to the equity market and are constrained in borrowing. Thus, increasing international borrowing enables agents to smooth risk more efficiently and induces specialization. A key feature of Chang's model is that labour is fully employed. Thus, it is not possible with that model to examine how international borrowing would affect specialization

---

<sup>2</sup> See, for example, Azariadis (1975), Bailey (1974), and the more recent contributions by Matusz (1985; 1986).

and unemployment simultaneously. Our paper seems a natural bridge between, and an extension to, the insightful analyses of Chang (1991) and Fernandez (1992).<sup>3</sup>

The model presented in this paper provides a framework that can be used to examine the efficacy of government policies aimed at reducing the unemployment rate in developing countries. In particular, the paper shows that, under certain conditions, a state-contingent price subsidy can reduce the unemployment rate in a small open economy without inefficiently reducing the extent of specialization in the country's comparative advantage goods.

The remainder of this paper is organized as follows. Section 2 describes the structure of the economy while section 3 characterizes the equilibrium of the model. The effect of international borrowing on specialization and unemployment is the subject matter of section 4. Section 5 deals with the implications of the results of our model for government policy, while section 6 provides concluding remarks and suggests ways in which the paper could be extended.

## **2. Structure of the Economy**

### **2.1 Agents and Goods**

Consider a small open economy made up of  $Z$  risk-averse workers and a large number of risk-neutral firms. All agents live for two periods. There are two goods,  $X$  and  $Y$ , produced in sectors  $X$  and  $Y$  respectively. Allowing for two sectors is necessary for the discussion on specialization. Good  $X$  is a consumption good while good  $Y$  is an export good that is not consumed domestically. In other words, good  $Y$  is a cash crop. The assumption that good  $Y$  is a cash crop simplifies the indirect utility function, by making it independent of the relative price of the export good (see footnote 4), and facilitates the analysis of implicit contracts. The assumption is also supported by the observation that a developing country's consumption of its own export commodity is an insignificant proportion of total domestic output of the export good (see Chang 1991).

---

<sup>3</sup> There is also an extensive body of literature on international debt and trade. See Diwan (1990) and the excellent survey by Glick and Kharas (1986).

The size of industry Y is determined by the number of agents who have the specific skills for operating a firm in industry Y. We normalize this number to one. Also, let the size of industry X be one. The presence of specific skills in industry Y rules out complete specialization and ensures that pure profit (after deducting the payment to the specific skill) is zero in both industries. Since the number of firms in each industry is constant over time (in equilibrium), we measure the degree of the economy's specialization in industry Y by the fraction of the labour force offered contracts in sector Y.

An important fact of developing countries is that they face a considerable risk in their terms of trade. To capture this aggregate uncertainty, let good X be the numeraire and the relative price of good Y be  $P$ . Thus, the terms of trade uncertainty is represented by unanticipated changes in  $P$ . In order to focus on this source of uncertainty, we abstract from all other possible sources of uncertainty. Without loss of generality, we assume that there are only two states of nature,  $s \in \{g, b\}$ . The price of good Y in each of the two states is denoted  $P_g$  and  $P_b$ , with  $P_b < P_g$ . Call state  $g$  the good state and state  $b$  the bad state. Also,  $s$  is independently and identically distributed over time.

Workers have the same time-additive utility function with a discount factor  $\beta \in (0, 1)$ . As is typical in the implicit contract literature, it is convenient to work with the indirect utility function. Let a worker's indirect utility function in each period be  $U(I)$ , where  $I$  is the income in that period. The period utility function  $U$  is assumed to satisfy  $U' > 0$ ,  $U'' < 0$ ,  $U'(0) = \infty$  and  $U'(\infty) = 0$ .<sup>4</sup> Each worker is endowed with one indivisible unit of labor and cannot borrow. Firms, on the other hand, can borrow in the international financial market at the given world interest rate  $r$  up to a certain limit. For simplicity, we assume that the world interest rate is constant over time.

---

<sup>4</sup> Note that the indirect utility function depends only on income and not on the terms of trade. This is an outcome of the earlier assumption that the consumers in this economy do not consume good Y. To see this, suppose that the direct utility is  $U(c_x)$ , where  $c_x$  is the consumption of good  $x$ . The consumer's budget constraint is  $c_x \leq I$  and the maximization problem induces the indirect utility function  $U(I)$ .

## 2.2 Technology

To focus on employment, we assume that labour is the only input in the production of both goods X and Y. The production process for good X lasts one period and the technology is given by  $X_t = G_0 L_{xt}$ , where  $G_0$  is a constant and  $L_{xt}$  is effective labour employed in sector X at time  $t$ . The linear production function is necessary to ensure zero profit in sector X, since labor is the only input.

Unlike good X, the production process for good Y lasts two periods. In particular, for good Y to be produced, labour must be applied in both periods one and two, and output is realized only in period two. To understand the nature of the production process for good Y, suppose good Y is an agricultural good (e.g., cocoa or rubber): period one represents the time in which land tilling and planting operations are done, while period two captures the season in which the crop is harvested. Let  $L_{yst}$  be the labour input into the production of good Y in period  $t$  when the state is  $s$ . The production function of good Y is:

$$Y_1 = 0; \quad Y_{s2} = F(L_{ys'1}, L_{ys2}), \quad (2.1)$$

where  $Y_{s2}$  is the period-two output of good Y in state  $s$ . The production function  $F$  is increasing, concave in each argument and linearly homogeneous. In addition, we assume the following properties:

$$F(0, L) = F(L, 0) = F_2(0, L) = F_1(L, 0) = 0, \quad F_1(0, L) = F_2(L, 0) = \infty, \quad (2.2)$$

where  $F_1$  and  $F_2$  are the derivatives of the function  $F$  with respect to its two arguments. The first set of properties ensures that labour is indeed necessary in both periods for output of good Y to be positive. The second set of properties ensures that, if the amount borrowed is sufficiently small, a firm in sector Y will not choose default when the state is bad.<sup>5</sup> For the rest of the analyses, we assume that a firm cannot default on its debt, thereby abstracting from issues related to bankruptcy. Although default is an important aspect of

---

<sup>5</sup> See section 3.2 for more discussion.

the debt crisis, it is not central to our study of the relationship between borrowing, on the one hand, and specialization and unemployment on the other.

The two-period production structure for good Y captures in a simple way the firm's need to borrow: Since output is not generated in period one, firms must borrow in order to pay workers in the first period.<sup>6</sup> More generally interpreted, the assumed production structure of good Y captures the realistic notion that firms borrow in order to invest, provided that they do not have sufficient internal funds for such investment. Alternatively, a country may choose to borrow in the international financial market in order to smooth consumption intertemporally, as modelled in Chang (1991). Our assumption is more relevant for the current modelling because we focus on firms' choices, rather than on households' intertemporal consumption choices. Also, our approach seems more realistic given the fact that most highly indebted developing countries borrow to finance development projects rather than to smooth consumption levels.

The country is assumed to have a comparative advantage in the production of good Y. Since we do not explicitly model the world economy, standard notions of comparative advantage such as that determined by the country's relative factor endowment ratio cannot be used here. Instead, we follow Chang (1991) and define comparative advantage by the average value of product of labour in the two sectors. In particular, we require that the expected present value of the average product of labour in sector Y exceeds that in X for any labour input profile  $(L, L) > 0$ :<sup>7</sup>

$$E\left[P_s \frac{F(L, L)}{L}\right] > (2 + r)G_0 \quad \forall L > 0, \quad (2.3)$$

where E is the expectation operator.

---

<sup>6</sup> With the assumed properties on the function  $U$ , a worker's marginal utility of income in the first period is infinite when income in the first period is zero. Thus, workers will not agree to delay all wage payments to the end of period two.

<sup>7</sup> Note that this restriction does not depend on  $L$ , because the average product of labour,  $\frac{F(L, L)}{L}$ , is a constant independent of  $L$  when  $F$  has the assumed feature of linear homogeneity.

### 2.3. Timing

The information structure of the game is as follows.<sup>8</sup> In the first period, firms choose the sector in which to operate. The state of the world in the first period is then realized. Firms in sector Y offer an employment contract to a fraction of workers in the economy that specifies the wage rate in each state  $s$  and the associated employment probability, as described later. Firms in sector X hire workers in the competitive labour market. Workers choose which sector to work in and the labour market clears for the first period. However, since first-period contracts are offered after the state is revealed, no worker is laid off in the first period in any sector. Labour is then used in both sectors and output of good X is generated. Firms in sector X pay workers wages from their sales revenue, while firms in sector Y borrow in the international market to pay the first-period wage bill. The first period then ends.

At the beginning of period two, the labour market opens again and workers have the opportunity to switch between sectors. However, due to the fact that no new information has arrived since the last contract was signed, workers will stay in the sectors in which they worked in the first period. Then the labour market is closed. The second-period state of nature is realized. Firms in sector Y hire workers according to the employment contract. Some workers may be laid off in one of the states. Since the labour market is closed at this point, these workers are unemployed for the rest of the period. Firms in both sectors produce. Output is generated and the second-period wage is paid in both sectors. Firms in sector Y also repay the debt.

---

<sup>8</sup> We adopt this information structure because it greatly simplifies the exposition and provides a tractable framework that allows us to derive results analytically.

### 3. Equilibrium

#### 3.1. Employment Contracts

We first examine the contracts offered by firms in each sector. Denote an employment contract offered by a firm in sector  $i$  ( $=X, Y$ ) by  $M_i = (m_{i1}, m_{i2})$ , where  $m_{it} = (w_{ist}, e_{ist}, N_{it})_{s=g,b}$ . In this contract,  $w_{ist}$  is the wage rate in state  $s$  offered by the firm in period  $t$ ,  $e_{it}$  is the probability that a worker will be employed by the firm in state  $s$ , and  $N_i$  is the number of workers to whom the contract is offered. Effective employment in sector Y is  $L_{ist} = e_{ist}N_{it}$ . For sector X, firms simply offer the competitive wage that equals the marginal product of labour,  $G_0$ . Thus,  $w_{xst} = w_x \equiv G_0$  and  $e_{xst} = 1$  for all  $(s, t)$ . In this case, the level of employment in each firm is indeterminate. However, as usual, the level of employment in sector X as a whole is determined by the labour market clearing condition in conjunction with the demand for labour by sector Y. With this in mind, we will suppress the element  $N$  from the notation for the contract  $m_{xt}$  and simply denote  $N_{yt}$  as  $N$ . Then  $m_{xt} = (G_0, 1)$ .

The employment contract offered by a firm in sector Y is more complicated. Since workers are risk-averse but firms are risk-neutral, it is optimal for firms to insure workers against the income risk generated by the terms of trade uncertainty. Loosely speaking, firms offer a relatively stable wage rate across states of nature. Typically, this implies that the wage rate exceeds the value of marginal product of labour when the state is bad but falls below the value of marginal product of labour when the state is good. Firms compensate for the loss in the bad state by the profit in the good state.

More specifically, let us specify the maximization problem of a firm in sector Y. Denote  $B$  as the upper bound on borrowing faced by a firm in sector Y after the resolution of uncertainty in period one. Also, let  $s'$  denote the state of nature in period one. The firm's expected profit in the two periods (excluding the implicit payment to the entrepreneur's skill) is:

$$\Pi_y = \sum_s q_s [P_{s2} F(L_{ys'1}, L_{ys2}) - w_{ys2} L_{ys2} - (1+r) w_{ys'1} L_{ys'1}] \quad (3.1)$$

where  $s$  is the second-period state and  $q_s$  is the probability of state  $s$  occurring in period



two. The firm's residual balance in the first period, after wage payments, does not appear in the expected profit equation because it is zero in the equilibrium that we consider below.

A firm in sector Y chooses  $(m_{y1}, m_{y2})$  to solve:

$$(PY) \quad \text{Max} \quad \Pi_y$$

subject to

$$\sum_s q_s [e_{ys2} U(w_{ys2}) + (1 - e_{ys2}) U_0] \geq \underline{U}_2 \quad (3.2)$$

$$U(w_{ys'1}) \geq \underline{U}_1 \quad (3.3)$$

$$B \geq w_{ys'1} L_{ys'1} \quad (3.4)$$

$$1 \geq e_{ys2} \quad \text{for } s = g, b. \quad (3.5)$$

The constraint (3.2) requires that the second-period contract offered by a firm in sector Y,  $m_{y2}$ , generate at least as much expected utility to a worker as the contract in sector X does. In this constraint,  $U_0 \equiv U(0)$  denotes the period utility when a worker has no labour income, and  $\underline{U}_2 \equiv U(G_0)$  denotes the utility derived from the contract  $m_{x2}$ . The constraint (3.2) must be imposed because workers can switch between sectors at the beginning of period two. For the same reason, the contract  $(m_{y1}, m_{y2})$  must also generate at least as much lifetime expected utility to a worker as does the combination of contracts  $(m_{x1}, m_{y2})$ . Therefore, the first-period contract offered by a firm in sector Y,  $m_{y1}$ , must be at least as good as the contract  $m_{x1}$ . This latter requirement is specified by (3.3), where  $\underline{U}_1 \equiv U(G_0)$ . Finally, the constraint (3.4) requires the amount of borrowing to cover the first-period wage bill and the constraint (3.5) requires the employment probability in period two to be at most one for all states.

### 3.2 Equilibrium Definition

The following is a definition of an equilibrium:

**Definition 3.1.** An equilibrium is a list of employment contracts  $(m_x, m_y)$  such that  
*(i)*  $m_{xt} = (G_0, 1)$  for all  $t$  and  $s$ ;

(ii) the employment contract  $m_y$  solves the problem (PY);

(iii) ex ante clearing of the labour market in period two, i.e.  $N_{x2} + N_{y2} = 1$ , and ex post clearing of the labour market in period one;

(iv)  $\Pi_y \geq 0$ .

The requirements (i) and (ii) in the above definition are self-explanatory. Condition (iii) requires full employment of labour after the resolution of uncertainty in period one, and every worker to be offered an employment contract before the realization of the state in period two. However, since firms in sector Y may lay off some workers after the realization of the state, there can be ex post unemployment in period two. The condition (iv) requires that the country be diversified in production: When (iv) is violated, no one would like to operate in industry Y. Thus, imposing (iv) is necessary for a non-trivial examination of how borrowing affects specialization.

To make the examination non-trivial, we further narrow our focus in the rest of this paper through the following restrictions. First, we assume that  $U(G_0) > U_0$ . Thus, being employed is always strictly better than being unemployed. Second, we are interested only in the case where the borrowing constraint (3.4) binds. This reflects our focus on economies that face binding restrictions on borrowing. Also, since there is no wedge between the borrowing and lending rates, firms cannot profit from borrowing an amount greater than what is required for the first-period wage bill. One can further argue that, since there is no information asymmetry in the model, lenders know the amount a firm needs to finance labor employment and would be reluctant to lend an amount greater than required since this would create a positive probability of the firm defaulting on the loan.

In the formulation of the problem (PY) and in the above definition, we have implicitly assumed that a firm in sector Y does not default on its debt. If the level of borrowing is sufficiently small, this can be viewed as an endogenous outcome in the current model, delivered by the properties of the production function  $F$ . Since the marginal product of labour in sector Y is infinite when labour input is near zero, firms can reduce employment probabilities sufficiently when the state is bad until total revenue just equals total production cost.

### 3.3. Equilibrium Conditions

For a firm in sector Y, the employment contract it offers to workers in the first period is identical to that offered by a firm in sector X. That is,  $m_{y1} = m_{x1} = (G_0, 1)$ . To see this, note that workers can switch sectors at the beginning of period two. That is, a worker can work for sector X in the first period and then work for sector Y in the second period. Thus, the contract  $(m_{y1}, m_{y2})$  must generate the same utility to the worker as the combination of contracts  $(m_{x1}, m_{y2})$ . Since utility is time separable, this implies that a worker must be indifferent between  $m_{y1}$  and  $m_{x1}$  in the first period. Therefore, equation (3.3) must bind in equilibrium, implying that  $w_{ys'1} = G_0$ . This piece of information and our focus on economies that face binding restrictions on borrowing imply that  $L_{ys'1} = \frac{B}{G_0}$ , with  $\frac{dL_{ys'1}}{dB} = \frac{1}{G_0} > 0$ . In other words, an increase in international borrowing increases effective labour employment in period one.

Having found the equilibrium values of  $w_{ys'1}$  and  $L_{ys'1}$ , firm Y's problem becomes that of choosing  $m_{y2} = (w_{ys2}, e_{ys2}, N_2)_{s=g,b}$  to maximize equation (3.1), subject to equations (3.2) and (3.5). Let  $\lambda$  be the Lagrange multiplier associated with (3.2) and  $q_s \gamma_s$  be the state-contingent Kuhn-Tucker multiplier associated with (3.5). The first-order conditions for this problem with respect to  $w_{ys2}$ ,  $e_{ys2}$  and  $N_2$  are as follows:

$$U'(w_{ys2}) = \frac{N_2}{\lambda} \quad (3.6)$$

$$P_{s2}F_2(L_{ys'1}, e_{ys2}N_2)N_2 - w_{ys2}N_2 + \lambda[U(w_{ys2}) - U_0] - \gamma_s = 0 \quad (3.7)$$

$$\sum_s q_s [P_{s2}F_2(L_{ys'1}, e_{ys2}N_2)e_{ys2} - w_{ys2}e_{ys2}] = 0. \quad (3.8)$$

Equation (3.6) requires the marginal utility of income, in period two, to be constant across all states of nature. This ensures that risk is distributed optimally in an environment where one agent is risk-averse. An implication of equation (3.6) is that the wage rate in period two is constant across all states of nature. This follows from the fact that the instantaneous utility function, and hence marginal utility, is independent of the relative

price of good Y which is the source of uncertainty in our model.<sup>9</sup> Equation (3.7) states that the cost of a marginal reduction in the probability of employment must exceed or equal the benefit. The optimal number of workers with whom the firm should contract is given by equation (3.8). The firm offers contracts until the expected benefit of a marginal increase in the number of contracted workers is equal to the expected cost.

In addition to (3.6), (3.7) and (3.8), we have:

$$\sum_s q_s [e_{ys2}U(w_{ys2}) + (1 - e_{ys2})U_0] - \underline{U}_2 = 0 \quad (3.9)$$

$$\gamma_s(1 - e_{ys2}) = 0, \quad \gamma_s \geq 0. \quad (3.10).$$

Equation (3.9) requires that the expected utility from working for firm Y be equal to the utility obtained by a worker who works for firm X. The Kuhn-Tucker conditions for  $e_{ys2}$  are represented by equation (3.10). From the first-order conditions, we derive the following result, familiar to the implicit contract literature, whose proof is presented in Appendix A.

**Lemma 3.1.** *In period two, there must be at least one state of nature in which there is full employment of labour.*

Lemma 3.1 is intuitive in the sense that if there is no state with full employment of labor in period two, the firm can do better by reducing the number of people offered contracts in period two (i.e.,  $N_2$ ). Since we are interested in the relationship between international borrowing and unemployment, we focus on a state in which there is unemployment. In general, the state that has unemployment may be either the good state or the bad state. However, it is reasonable to regard the bad state as the state which induces unemployment. Thus, we assume that  $e_{yb2} < 1$  throughout our discussion. From equation (3.10), this implies that  $\gamma_b = 0$ . Invoking lemma 3.1, we conclude that  $e_{yg2} = 1$  and  $\gamma_g \geq 0$ .

---

<sup>9</sup> In earlier implicit-contract models, production shocks are independently and identically distributed across firms and uncertainty is firm-specific, but not sector-specific. Therefore, there is no aggregate uncertainty and all aggregate variables are state independent. Our model is similar to Fernandez (1992) in the sense that there is aggregate uncertainty. However, unlike her model, the wage rate in our model is state independent.

To obtain a solution to firm Y's problem, note that equation (3.6) implies that  $w_{ys2} = w_{y2}$  for all  $s$  and that  $\lambda = \frac{N_2}{U'(w_{y2})}$ . Substituting for  $\lambda$  in equation (3.7), noting that  $e_{yb2} < 1$  and  $\gamma_b = 0$ , the first-order conditions can be reduced to a system of three equations in three variables ( $w_{y2}$ ,  $N_2$ ,  $e_{yb2}$ ).

$$P_{b2}F_2(L_{ys'1}, e_{yb2}N_2) - w_{y2} + \frac{[U(w_{y2}) - U_0]}{U'(w_{y2})} = 0 \quad (3.11)$$

$$q_g[P_{g2}F_2(L_{ys'1}, N_2) - w_{y2}] + q_b e_{yb2} [P_{b2}F_2(L_{ys'1}, e_{yb2}N_2) - w_{y2}] = 0 \quad (3.12)$$

$$q_g U(w_{y2}) + q_b [e_{yb2} U(w_{y2}) + (1 - e_{yb2}) U_0] - \underline{U}_2 = 0 \quad (3.13)$$

These equations determine a unique solution for  $w_{y2}$ ,  $N_2$ , and  $e_{yb2}$ .

To establish the relationship between international borrowing and specialization, we need to know: (i) how the employment probability in the bad state ( $e_{yb2}$ ) and the number of workers offered contracts in sector Y in period two ( $N_2$ ) respond to an increase in the second-period wage paid by firm Y ( $w_{y2}$ ); (ii) how the second-period wage paid by firm Y responds to an increase in first-period labour employment by firm Y ( $L_{ys'1}$ ). The procedure is as follows. Equation (3.13) implies:

$$e_{yb2} = \frac{\underline{U}_2 - q_g U(w_{y2}) - q_b U_0}{q_b [U(w_{y2}) - U_0]}. \quad (3.14).$$

This implies that  $e_{yb2} < 1$  if, and only if,  $U(w_{y2}) > \underline{U}_2$ . Invoking the property that  $U' > 0$ , we conclude that  $w_{y2} > G_0$ . In other words, the second-period wage paid by firm Y is greater than the wage paid by firm X. To obtain the relationship between the employment probability in the bad state and the second-period wage paid by firm Y, we differentiate equation (3.14) with respect to  $w_{y2}$ . Performing this differentiation yields:

$$\frac{\partial e_{yb2}}{\partial w_{y2}} = -\frac{(q_g + e_{yb2} q_b) U'(w_{y2})}{q_b [U(w_{y2}) - U_0]} < 0. \quad (3.15).$$

That is, with a higher wage offer by firm Y, in period two, workers must expect a higher chance of being unemployed in the bad state. We summarize these results in the following lemma.

**Lemma 3.2.**  $w_{y2} > G_0$  and  $\frac{\partial e_{yb2}}{\partial w_{y2}} < 0$ .

The next step is to show how the number of workers offered contracts by firm Y in period two responds to an increase in the second-period wage paid by firm Y. To derive this, rewrite equation (3.11) as:

$$F_2(L_{ys'1}, e_{yb2}N_2) = H(w_{y2}) \quad (3.16)$$

where

$$H(w_{y2}) \equiv \frac{1}{P_{b2}} \left[ w_{y2} - \frac{U(w_{y2}) - U_0}{U'(w_{y2})} \right]. \quad (3.17)$$

Clearly,  $\frac{\partial H}{\partial w_{y2}} < 0$ . Let the solution to equation (3.16) for  $e_{yb2}N_2$  be:

$$e_{yb2}N_2 = \phi(H(w_{y2}), L_{ys'1}). \quad (3.18).$$

The properties of  $F$  and  $H$  imply  $\phi_1 < 0$  and  $\phi_2 > 0$ . Using (3.13) to substitute for  $e_{yb2}$ , we obtain:

$$N_2 = \frac{1}{e_{yb2}(w_{y2})} \phi(H(w_{y2}), L_{ys'1}). \quad (3.19).$$

Differentiating the above equation with respect to  $w_{y2}$  yields:

$$\frac{\partial N_2}{\partial w_{y2}} = \frac{e_{yb2}\phi_1 \frac{\partial H}{\partial w_{y2}} - \phi \frac{\partial e_{yb2}}{\partial w_{y2}}}{(e_{yb2})^2}.$$

Since  $e_{yb2}$  is a decreasing function of  $w_{y2}$  (see lemma 3.2),  $N_2$  must be an increasing function of  $w_{y2}$ . This leads to the following lemma.

**Lemma 3.3.** *An increase in the second-period wage paid by firm Y increases the number of workers offered contracts by firm Y in period two (i.e.,  $\frac{\partial N_2}{\partial w_{y2}} > 0$ ).*

To show how the second-period wage paid by firm Y responds to an increase in first-period labour employment by firm Y requires manipulating equations (3.12), (3.15) and

(3.16) using the implicit function theorem and lemma 3.3. Since the algebraic manipulations involved in this procedure are lengthy and tedious, we relegate the proof to Appendix B and simply note that an increase in first-period labour employment by firm Y, and hence international borrowing, has no effect on the second-period wage paid by firm Y. We summarize this result in the following lemma.

**Lemma 3.4.**  $\frac{\partial w_{y2}}{\partial L_{ys'1}} = 0.$

The explanation for lemma 3.4 is as follows. An increase in first-period labour employment by firm Y has two opposing effects on the second-period wage paid by firm Y. The first, which we call the “complementarity effect,” arises from the fact that an increase in first-period labour employment increases the marginal product of period-two labour. This is a consequence of the assumption that first-period and second-period labour employment are complements. The second effect, which we call the “diminishing marginal productivity effect,” is negative because an increase in first-period labour employment tends to increase the marginal product of period-two labour thereby providing an incentive for firm Y to hire more labour in period two. As the firm hires more labour in period two, the marginal product of period-two labour decreases. In general, the ultimate effect of an increase in period-one labour employment on the second-period wage paid by firm Y depends on whether or not the “complementarity effect” dominates the “diminishing marginal productivity effect.” However, with a linearly homogeneous production function, the two effects cancel each other out.

#### 4. Effects on Specialization and Unemployment

Since there are  $Z$  workers in this economy and labour is the only input into the production of goods X and Y, the number of people offered contracts in sector Y captures the degree of specialization in the economy.

Clearly, borrowing increases specialization in good Y in period one. To understand this, note that period one is characterized by full employment of labour and that  $N_1$

$= L_{ys'1} = \frac{B}{G_0}$ . Therefore,  $\frac{dN_1}{dB} = \frac{dL_{ys'1}}{dB} = \frac{1}{G_0} > 0$ . For period two, recall that  $N_2 = N_2(w_{y2}, L_{ys'1})$ . Differentiating this equation with respect to  $B$  yields:

$$\frac{dN_2}{dB} = \frac{1}{G_0} \left[ \frac{\partial N_2}{\partial w_{y2}} \frac{\partial w_{y2}}{\partial L_{ys'1}} + \frac{\partial N_2}{\partial L_{ys'1}} \right]. \quad (4.1)$$

By lemma 3.4, the first expression in equation (4.1) is zero. From equation (3.19), we can show that  $\frac{\partial N_2}{\partial L_{ys'1}} = \frac{\phi_2}{e_{yb2}} > 0$ . Therefore,  $\frac{dN_2}{dB} > 0$ . This result is formalized in the following proposition.

**Proposition 4.1.** *An increase in international borrowing increases specialization in the comparative advantage good ( $Y$ ).*

An increase in borrowing enables firm  $Y$  to increase first-period labour employment. The increase in period-one labour employment has two effects on the number of workers offered contracts in sector  $Y$  in period two: the first is the direct effect and the second is the indirect effect through wage rates. The direct effect is captured by the second term in equation (4.1) and is positive because labour employment in periods one and two are complements. The indirect effect is represented by the first term in equation (4.1). It is zero in equilibrium because first-period labour employment does not affect the wage offered to workers by firm  $Y$  in period two.

To establish the relationship between international borrowing and unemployment in this economy, note that there is full employment of labour in period one and that this economy's equilibrium exhibits unemployment only in period two. Therefore, the expected unemployment rate in this small open economy can be defined as:

$$u = \frac{q_b}{Z} (1 - e_{yb2}) N_2. \quad (4.2)$$

Differentiating this with respect to  $B$  yields:

$$\frac{du}{dB} = \frac{q_b}{Z} \left[ (1 - e_{yb2}) \frac{dN_2}{dB} - N_2 \frac{de_{yb2}}{dB} \right]. \quad (4.3)$$

From proposition 4.1, we know that  $\frac{dN_2}{dB} > 0$ . Using lemmas 3.2 and 3.4, we can show that  $\frac{de_{yb2}}{dB} = \frac{1}{G_0} \left( \frac{\partial e_{yb2}}{\partial w_{y2}} \frac{\partial w_{y2}}{\partial L_{ys'1}} \right) = 0$ . Therefore,  $\frac{du}{dB} > 0$ . This leads to the following proposition.



**Proposition 4.2.** *An increase in international borrowing results in an increase in the expected unemployment rate.*

To understand the intuition for proposition 4.2, note that international borrowing affects the expected unemployment rate through two channels. The first channel, represented by the first expression in equation (4.3), has to do with the notion that borrowing affects the extent to which the economy is specialized in the comparative advantage good (Y). In Proposition 4.1, we showed that international borrowing increases specialization in the comparative advantage good. Since unemployment is the fraction of agents offered contracts in sector Y but laid off after a bad realization in the terms of trade, the increase in specialization increases the expected unemployment rate. The second channel, captured by the second term in equation (4.3), is the effect of international borrowing on the employment probability in the bad state of nature. Clearly, this term increases (decreases) the expected unemployment rate if international borrowing decreases (increases) the probability of employment in the bad state. However, in the economy described above, international borrowing has no effect on the probability of employment in the bad state of nature which implies that the second term in equation (4.3) is zero. Therefore, international borrowing increases the expected unemployment rate in this economy because of its positive effect on the degree of specialization in good Y.

Proposition 4.2 is interesting because it is consistent with the positive correlation between international borrowing and unemployment rates depicted in Table 1 and Figures 1a and 1b. Our model, therefore, provides one reason why international borrowing and unemployment rates are positively correlated across developing countries.

## 5. Policy Implications

In this section, we examine the efficacy of government policies aimed at reducing the unemployment rate. In particular, we examine the effects of a state-contingent price subsidy to firm Y with a view to determining whether or not such a subsidy can reduce the unemployment rate without inefficiently reducing the extent of specialization in the

comparative advantage good.<sup>10</sup>

Consider a government policy that offers state-contingent price subsidies to firm Y in period two. Since the aim of the price subsidy is to reduce the unemployment rate and unemployment occurs only when there is a bad realization in the terms of trade ( $P$ ), we assume that the subsidy is given to firm Y only in bad states of nature. Let  $S_b$  denote the subsidy rate offered firm Y in period 2 when the bad state is realized. With a state-contingent price subsidy equations (3.11), (3.12) and (3.13) become:<sup>11</sup>

$$P_{b2}(1 + S_b)F_2(L_{ys'1}, e_{yb2}N_2) - w_{y2} + \frac{[U(w_{y2}) - U_0]}{U'(w_{y2})} = 0 \quad (5.1)$$

$$q_g [P_{g2}F_2(L_{ys'1}, N_2) - w_{y2}] + q_b e_{yb2} [P_{b2}(1 + S_b)F_2(L_{ys'1}, e_{yb2}N_2) - w_{y2}] = 0 \quad (5.2)$$

$$q_g U(w_{y2}) + q_b [e_{yb2}U(w_{y2}) + (1 - e_{yb2})U_0] - \underline{U}_2 = 0. \quad (5.3)$$

It can be shown, using equation 5.3, that the period-two employment probability in the bad state of nature ( $e_{yb2}$ ) is a decreasing function of the second-period wage paid by firm Y ( $w_{y2}$ ). To show the effect of a state-contingent price subsidy on specialization, we need to know how the second-period wage responds to an increase in a state-contingent price subsidy. In Appendix C, we show that  $\frac{\partial w_{y2}}{\partial S_b} < 0$ . That is, if the government offers firm Y a price subsidy in the bad state, firm Y will offer workers a lower wage in period two. The intuition behind this is that a state-contingent price subsidy increases the probability of employment in the bad state of nature and hence the expected utility workers derive from accepting a contract offer in sector Y. To ensure that the expected utility workers

---

<sup>10</sup> We also examined the case of a state-contingent wage subsidy. Since the qualitative results are similar to the price subsidy example presented in this paper, we will not discuss it here.

<sup>11</sup> In the framework developed in this paper, it does not matter whether the price subsidy is anticipated or not because the first-period labor employment is determined jointly by binding participation and borrowing constraints.

derive from a contract in sector Y is not more than their reservation utility level, profit-maximizing firms in sector Y respond by reducing the second-period wage (follows from equation 5.3).

Having established how the second-period wage responds to an increase in a state-contingent price subsidy, we can derive the relationship between a state-contingent price subsidy and specialization as follows. Recall, from Appendix C, that  $N_2 = N_2(S_b, w_{y2})$ . Differentiating this equation with respect to  $S_b$  gives:

$$\frac{dN_2}{dS_b} = \frac{\partial N_2}{\partial S_b} + \frac{\partial N_2}{\partial w_{y2}} \frac{\partial w_{y2}}{\partial S_b}. \quad (5.4)$$

Substituting for  $\frac{\partial w_{y2}}{\partial S_b}$  from equation (C.10) in the Appendix, we obtain:

$$\frac{dN_2}{dS_b} = \frac{\partial N_2}{\partial S_b} \left[ 1 - \frac{q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}}}{q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}} + q_b e_{yb2} P_{b2} (1 + S_b) \frac{\partial H^p}{\partial w_{y2}}} \right]. \quad (5.5)$$

After some manipulations, equation (5.5) yields:

$$\frac{dN_2}{dS_b} = \frac{q_b e_{yb2} P_{b2} (1 + S_b) \frac{\partial H^p}{\partial w_{y2}} \frac{\partial N_2}{\partial S_b}}{q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}} + q_b e_{yb2} P_{b2} (1 + S_b) \frac{\partial H^p}{\partial w_{y2}}}. \quad (5.6)$$

In Appendix C, we show that  $\frac{\partial N_2}{\partial S_b} > 0$ ;  $\frac{\partial H^p}{\partial w_{y2}} < 0$  and  $\frac{\partial N_2}{\partial w_{y2}} > 0$ . These derivatives and the property  $F_{22} < 0$  imply that  $\frac{dN_2}{dS_b} > 0$ . This result is formalized in the following proposition.

**Proposition 5.1.** *A state-contingent price subsidy to firm Y in period two increases specialization in the comparative advantage good.*

The explanation for proposition 5.1 is that a state-contingent price subsidy to firm Y in period two has two opposing effects on specialization. The direct effect is positive and is captured by the first term in equation 5.4. The second term in equation 5.4 is the indirect effect through the second-period wage rate. The indirect effect is negative. The net effect of a state-contingent price subsidy is positive because the positive direct effect swamps the indirect effect.

**Proposition 5.2.** *In general, a state-contingent price subsidy has an ambiguous effect on the expected unemployment rate in this economy. However, if the economy is not highly*

specialized in the comparative advantage good, then a state-contingent price subsidy will decrease the expected unemployment rate.

**Proof.** For the first part of the proposition, note that:

$$\frac{du}{dS_b} = \frac{q_b}{Z} \left[ (1 - e_{yb2}) \frac{dN_2}{dS_b} - N_2 \frac{de_{yb2}}{dS_b} \right]. \quad (5.7)$$

We have shown that  $e_{yb2}$  is a function of  $w_{y2}$ . Therefore,

$$\frac{de_{yb2}}{dS_b} = \frac{\partial e_{yb2}}{\partial w_{y2}} \frac{\partial w_{y2}}{\partial S_b} > 0.$$

Since both  $e_{yb2}$  and  $N_2$  are increasing functions of  $S_b$ , the sign of equation (5.7) is ambiguous in general. To prove the second part of the proposition, substitute for  $\frac{dN_2}{dS_b}$  and  $\frac{de_{yb2}}{dS_b}$  in equation (5.7). This yields:

$$\frac{du}{dS_b} = T \left[ (1 - e_{yb2}) q_b e_{yb2} P_{b2} (1 + S_b) \frac{\partial H^p}{\partial w_{y2}} + N_2 q_g P_{g2} F_{22} \frac{\partial e_{yb2}}{\partial w_{y2}} \right] \quad (5.8)$$

where,

$$T = \frac{q_b \frac{\partial N_2}{\partial S_b}}{Z \left[ q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}} + q_b e_{yb2} P_{b2} (1 + S_b) \frac{\partial H^p}{\partial w_{y2}} \right]} < 0.$$

After substituting for  $\frac{\partial H^p}{\partial w_{y2}}$  and  $\frac{\partial e_{yb2}}{\partial w_{y2}}$  and manipulating the resulting expression we obtain:

$$\frac{du}{dS_b} = T \left[ \frac{(1 - e_{yb2}) q_b e_{yb2} U'' (U - U_0)}{(U')^2} - \frac{N_2 q_g P_{g2} F_{22} (U_2 - U_0) U'}{q_b (U - U_0)^2} \right]. \quad (5.9)$$

Since  $T < 0$ ,  $\frac{du}{dS_b} < 0$  if and only if:

$$N_2 < \frac{(1 - e_{yb2}) q_b^2 e_{yb2} U'' (U - U_0)^3}{q_g P_{g2} (U_2 - U_0) F_{22} (U')^3}. \quad (5.10)$$

The intuition for proposition 5.2 is as follows. A state-contingent price subsidy has two opposing effects on the expected unemployment rate: It increases the degree of specialization of the economy in the comparative advantage good and this tends to increase the expected unemployment rate. However, it also increases the probability of employment in the bad state of nature thereby reducing the expected unemployment rate. Since the

two effects have opposite signs, the net effect of a state-contingent price subsidy on the expected unemployment rate is ambiguous in general. However, if  $N_2$  is sufficiently low initially, the employment probability effect will dominate the specialization effect resulting in a decrease in the expected unemployment rate.

## 6. Conclusion and Extension

Empirical evidence suggests that the unemployment rate and the export-GNP ratio are positively correlated with external debt across developing countries. This paper formulates a two-sector, two-period model that simultaneously incorporates unemployment and a measure of specialization to illustrate one mechanism through which international borrowing affects specialization and unemployment in developing countries. We argue that international borrowing affects specialization and unemployment by unevenly changing the risk-sharing structure between firms and workers across sectors.

The economy faces uncertainty in its terms of trade (the relative price of the export good). To insure workers against income fluctuations, firms in the export sector find it optimal to offer an implicit contract through stable wages. This wage contract allows firms to lay off some workers in bad states of nature. Because the production process for the export good lasts two periods and its output is realized in period two, the only way in which firms in the export sector can finance labour employment in period one is through international borrowing. An increase in international borrowing, therefore, allows firms in the export sector to offer wage contracts to more workers, thereby increasing the extent of specialization in the export good. As labour shifts from the domestic good sector into the more efficient export sector, a bad realization in the terms of trade results in higher unemployment. This mechanism seems to be consistent with the positive relation among international borrowing, specialization and unemployment presented in Table 1 and Figures 1a and 1b.

We derive conditions under which a state-contingent price subsidy will be effective in reducing the unemployment rate without inefficiently reducing the extent of specialization in the comparative advantage good. An obvious extension of this research would be to

change the timing of the model in such a way that would require firms in the export sector to offer contracts to workers before the resolution of uncertainty in period one and to examine the robustness of our results to changes in the information structure. This alternative timing complicates the first-period contract significantly since the first-period wage paid by firm Y is no longer pinned down by a binding participation constraint. Our conjecture is that this alternative information structure may affect our results in a quantitative way, but will not affect the qualitative results derived in this paper.

## Appendix A

**Proof of Lemma 3.1.** We want to show that  $\gamma_g + \gamma_b > 0$ . Equation (3.7) can be rewritten as:

$$\gamma_s = P_{s2}F_2(L_{ys'1}, e_{ys2}N_2)N_2 - w_{ys2}N_2 + \lambda[U(w_{ys2}) - U_0]. \quad (A.1).$$

Thus

$$\sum_s q_s e_{ys2} \gamma_s = N_2 \sum_s q_s [P_{s2}F_2(L_{ys'1}, e_{ys2}N_2)e_{ys2} - w_{ys2}e_{ys2}] + \lambda \sum_s q_s [U(w_{ys2}) - U_0]e_{ys2}.$$

Using equation (3.8) in the above equation, we obtain

$$\sum_s q_s e_{ys2} \gamma_s = \lambda \sum_s q_s [U(w_{ys2}) - U_0]e_{ys2}. \quad (A.2).$$

From (3.10) we can show that

$$\sum_s e_{ys2} \gamma_s = \sum_s \gamma_s. \quad (A.3).$$

Therefore,

$$\sum_s q_s \gamma_s = \lambda \sum_s q_s [U(w_{ys2}) - U_0]e_{ys2}. \quad (A.4).$$

Since  $U(w_{ys2}) > U_0$  for all  $s$ ,  $\sum_s q_s [U(w_{ys2}) - U_0] > 0$  and hence  $\sum_s q_s \gamma_s > 0$ . Clearly, this requires  $\gamma_s > 0$  for at least one  $s$  (since  $\gamma_s \geq 0, \forall s$ ). Thus  $\gamma_g + \gamma_b > 0$ .

## Appendix B

**Proof of Lemma 3.4.** Using equation (3.16), we can rewrite equation (3.12) as:

$$-(q_b + q_g e_{yb2})w_{y2} + q_g P_{g2} F_2(L_{ys'1}, N_2) + q_b e_{yb2} P_{b2} H(w_{y2}) = 0. \quad (B.1)$$

Recall that equations (3.11) and (3.13) imply that  $e_{yb2}$  is a function of  $w_{y2}$  and that  $N_2$  depends on  $w_{y2}$  and  $L_{ys'1}$ . Therefore, equation (B.1) can be expressed as:

$$-(q_b + q_g e_{yb2}(w_{y2}))w_{y2} + q_g P_{g2} F_2(L_{ys'1}, N_2(w_{y2}, L_{ys'1})) + q_b e_{yb2}(w_{y2}) P_{b2} H(w_{y2}) = 0. \quad (B.2)$$

Rewrite equation (B.2) as:

$$J(w_{y2}, L_{ys'1}) = 0. \quad (B.3)$$

By the implicit function theorem,

$$\frac{dw_{y2}}{dL_{ys'1}} = -\frac{J_L}{J_w}. \quad (B.4)$$

From (B.2) and (B.3),

$$J_w = -(q_g + q_b e_{yb2}) + q_b (P_{b2} H - w_{y2}) \frac{\partial e_{yb2}}{\partial w_{y2}} + q_b e_{yb2} P_{b2} \frac{\partial H}{\partial w_{y2}} + q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}}. \quad (B.5)$$

Using equations (3.15) and (3.17) we can show that  $q_b (P_{b2} H - w_{y2}) \frac{\partial e_{yb2}}{\partial w_{y2}} = (q_g + q_b e_{yb2})$ .

Therefore,

$$J_w = q_b e_{yb2} P_{b2} \frac{\partial H}{\partial w_{y2}} + q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}} < 0. \quad (B.6)$$

Now consider  $J_L$ . From equations (B.2) and (B.3),

$$J_L = q_g P_{g2} [F_{21} + F_{22} \frac{\partial N_2}{\partial L_{ys'1}}] \quad (B.7)$$

where,

$$F_{21} \equiv \frac{\partial F_2(L_{ys'1}, N_2)}{\partial L_{ys'1}}$$

and

$$F_{22} \equiv \frac{\partial F_2(L_{ys'1}, N_2)}{\partial N_2}.$$



Let

$$F_{21}(e_{yb2}) \equiv \frac{\partial F_2(L_{ys'1}, e_{yb2}N_2)}{\partial L_{ys'1}}$$

and

$$F_{22}(e_{yb2}) \equiv \frac{\partial F_2(L_{ys'1}, e_{yb2}N_2)}{\partial N_2}.$$

Since  $F$  is linear homogeneous, it can be shown that

$$F_{21} = -\frac{N_2}{L_{ys'1}F_{22}} \quad (B.8)$$

$$F_{21}(e_{yb2}) = -\frac{e_{yb2}N_2}{L_{ys'1}F_{22}(e_{yb2})}. \quad (B.9).$$

To sign the derivative  $\frac{\partial N_2}{\partial L_{ys'1}}$ , differentiate equation (3.16) with respect to  $L_{ys'1}$ . This yields:

$$F_{21}(e_{yb2}) + e_{yb2}F_{22}(e_{yb2})\frac{\partial N_2}{\partial L_{ys'1}} = 0. \quad (B.10).$$

Solving equation (B.10) for  $\frac{\partial N_2}{\partial L_{ys'1}}$  and substituting for  $F_{21}(e_{yb2})$  using equation (B.9), we obtain

$$\frac{\partial N_2}{\partial L_{ys'1}} = \frac{N_2}{L_{ys'1}}. \quad (B.11).$$

Using equations (B.8) and (B.11) in equation (B.7) we can show that  $J_L = 0$ . Using this result we conclude that  $\frac{dw_{y2}}{dL_{ys'1}} = 0$ . This completes the proof.

## Appendix C

**Proof of Lemma 5.1** From equation (5.3), we can re-establish our earlier result that  $e_{yb2}$  is a decreasing function of  $w_{y2}$ . Rewrite equation (5.1) as:

$$F_2(L_{ys'1}, e_{yb2}N_2) = H^P(S_b, w_{y2}) \quad (C.1)$$

where,

$$H^P(S_b, w_{y2}) \equiv \frac{1}{P_{b2}(1+S_b)} \left[ w_{y2} - \frac{U(w_{y2}) - U_0}{U'(w_{y2})} \right]. \quad (C.2).$$

Clearly,

$$\frac{\partial H^P}{\partial w_{y2}} = \frac{1}{P_{b2}(1+S_b)} \left[ \frac{U''(w_{y2})[U(w_{y2}) - U_0]}{(U'(w_{y2}))^2} \right] < 0 \quad (C.3)$$

$$\frac{\partial H^P}{\partial S_b} = -\frac{1}{P_{b2}(1+S_b)^2} \left[ w_{y2} - \frac{U(w_{y2}) - U_0}{U'(w_{y2})} \right] < 0. \quad (C.4).$$

Let the solution to equation (C.1) for  $e_{yb2}N_2$  be:

$$e_{yb2}N_2 = \phi^P(H^P(S_b, w_{y2}), L_{ys'1}), \quad \phi_1^P < 0, \quad \phi_2^P > 0. \quad (C.5)$$

Solving equation (C.5) for  $N_2$ , noting that equation (5.3) implies that  $e_{yb2}$  is a function of  $w_{y2}$ , we obtain:

$$N_2 = \frac{1}{e_{yb2}(w_{y2})} \phi^P(H^P(S_b, w_{y2}), L_{ys'1}). \quad (C.6).$$

Differentiating the above equation with respect to  $w_{y2}$  and  $S_b$  yields:

$$\frac{\partial N_2}{\partial w_{y2}} = \frac{e_{yb2}\phi_1^P \frac{\partial H^P}{\partial w_{y2}} - \phi^P \frac{\partial e_{yb2}}{\partial w_{y2}}}{(e_{yb2})^2} > 0 \quad (C.7)$$

$$\frac{\partial N_2}{\partial S_b} = \frac{1}{e_{yb2}} \phi_1^P \frac{\partial H^P}{\partial S_b} > 0. \quad (C.8).$$

For the last part of the proof note that, using equation (C.1), equation (5.2) can be rewritten as:

$$-(q_b + q_g e_{yb2})w_{y2} + q_g P_{g2} F_2(L_{ys'1}, N_2) + q_b e_{yb2} P_{b2} (1 + S_b) H^P(S_b, w_{y2}) = 0. \quad (C.9).$$

We have shown that  $e_{yb2}$  is a function of  $w_{y2}$  and that  $N_2$  depends on  $S_b$  and  $w_{y2}$ . Using this piece of information and the implicit function theorem in equation (C.9) we can show that:

$$\frac{\partial w_{y2}}{\partial S_b} = -\frac{q_g P_{g2} F_{22} \frac{\partial N_2}{\partial S_b}}{q_g P_{g2} F_{22} \frac{\partial N_2}{\partial w_{y2}} + q_b e_{yb2} P_{b2} (1 + S_b) \frac{\partial H^p}{\partial w_{y2}}} < 0. \quad (C.10).$$

## References

- Azariadis, C. (1975), "Implicit Contracts and Underemployment Equilibria," *Journal of Political Economy*, 83, pp. 1183-1202.
- Bailey, M., (1974), "Wages and Employment under Uncertain demand," *Review of Economic Studies*, 41, pp. 37-50.
- Chang, P.H.K., (1991), "Export Diversification and International Debt under Terms of Trade Uncertainty: an Intertemporal Approach," *Journal of Development Economics*, 36, pp. 259-279.
- Cole, H. et. al. (1995), "Default, Settlement, and Signalling: Lending Resumption in a Reputational Model of Sovereign Debt," *International Economic Review*, 36, pp. 365-385.
- Diwan, I., (1990), "Linking Trade and External Debt Strategies," *Journal of International Economics*, 29, pp. 293-310.
- Fernandez, R., (1992), "Terms-of-Trade Uncertainty, Incomplete Markets, and Unemployment," *International Economic Review*, 33, pp. 881-894
- Glick, R. and H. Kharas (1986), "The Costs and Benefits of Foreign Borrowing: A survey of Multi-Period Models," *Journal of Development Studies*, pp. 279-299.
- ILO (1995), *The Yearbook of Labour Statistics*, International Labour Organization.
- Matusaz, S., (1985) "The Heckscher-Ohlin-Samuelson Model with Implicit Contracts," *Quarterly Journal of Economics*, pp. 1313-1329.
- , (1986), "Implicit Contracts, Unemployment, and International Trade," *The Economic Journal*, 96, pp. 307-322.
- Sachs, J., (1984), "Theoretical Issues in International Borrowing," *Princeton Studies in International Finance*.
- UNCTAD (1995), *Commodity Yearbook*, United Nations Conference on Trade and Development
- World Bank (1994), *World Debt Tables*, Washington D.C., World Bank.

**Table 1: Debt/GNP Ratio, Unemployment and Export/GNP Ratio**

	Debt/GNP Ratio (%) (Average 1985-89)	Unemployment Rate (Average 1985-89)	Export/GNP Ratio (%) (Ave. 1985-89)
<u>America</u>			
Bolivia	138.3	19.3	20.1
Brazil	40.1	3.2	10.9
Jamaica	177.3	21.1	66.3
Panama	123.5	13.4	120.6
Uruguay	65.7	9.0	27.6
Argentina	62.1	5.6	11.1
Nicaragua	443	5.6	17.1
Venezuela	65.7	10	26.9
Costa Rica	108.8	5.6	36.8
Chile	109.3	8.1	35.5
Paraguay	60.6	5.5	28.3
Mexico	65.6	3.8	19.5
<u>Asia</u>			
China	9.5	2.1	11.8
Cyprus	-	3.1	-
Hong Kong	-	2.0	-
Singapore	-	4.2	-
Thailand	40.8	3.5	32.8
Korea(Rep)	33	3.2	39.5
Pakistan	43.9	3.3	18.7
Philippines	84.3	7.7	29.1

**Source:** *Unemployment figures were computed using data obtained from The Yearbook of Labour Statistics, published by the ILO, 1995. Export/GNP and Debt/GNP ratios were calculated using data published in World Debt Tables (1993-94).*

**Table 2: Exports of Primary Commodities as a Percent of Total Exports (1980 and 1993)**

	<b>A (Excluding Fuels)</b>		<b>B (Including Fuels)</b>	
	<u>1980</u>	<u>1993</u>	<u>1980</u>	<u>1993</u>
<u>America</u>				
Bolivia	92	68.1	99	82.3
Brazil	59.2	38.8	61	40.4
Jamaica	91.7	76.7	93.6	77.6
Panama	66.9	74.8	89.8	76.6
Uruguay	53.5	47.7	53.5	47.8
Argentina	72.8	58.6	76.3	68.1
Nicaragua	83.8	86.6	86.1	87.6
Venezuela	4.2	7.9	95.2	93.1
Costa Rica	66.8	55.4	67.5	55.9
Chile	86.2	82.3	87.4	82.5
Paraguay	92.1	83	92.1	83.2
Mexico	22	10.7	88.2	24.6
<u>Asia</u>				
China	23.2	14.6	47.5	19.1
Cyprus	38.5	39.6	43.8	40.7
Hong Kong	5.8	4.1	5.9	4.3
Singapore	12.9	7	41.7	19.3
Thailand	70.3	26.3	70.3	27.4
Korea(Rep)	9.1	3.8	9.4	6
Pakistan	38	16	45.1	17
Philippines	63.6	21.7	64.2	23.8
<b>Source:</b> <i>UNCTAD Commodity Yearbook, 1995.</i>				

Figure 1a: Specialization and unemployment (excluding Asian countries)

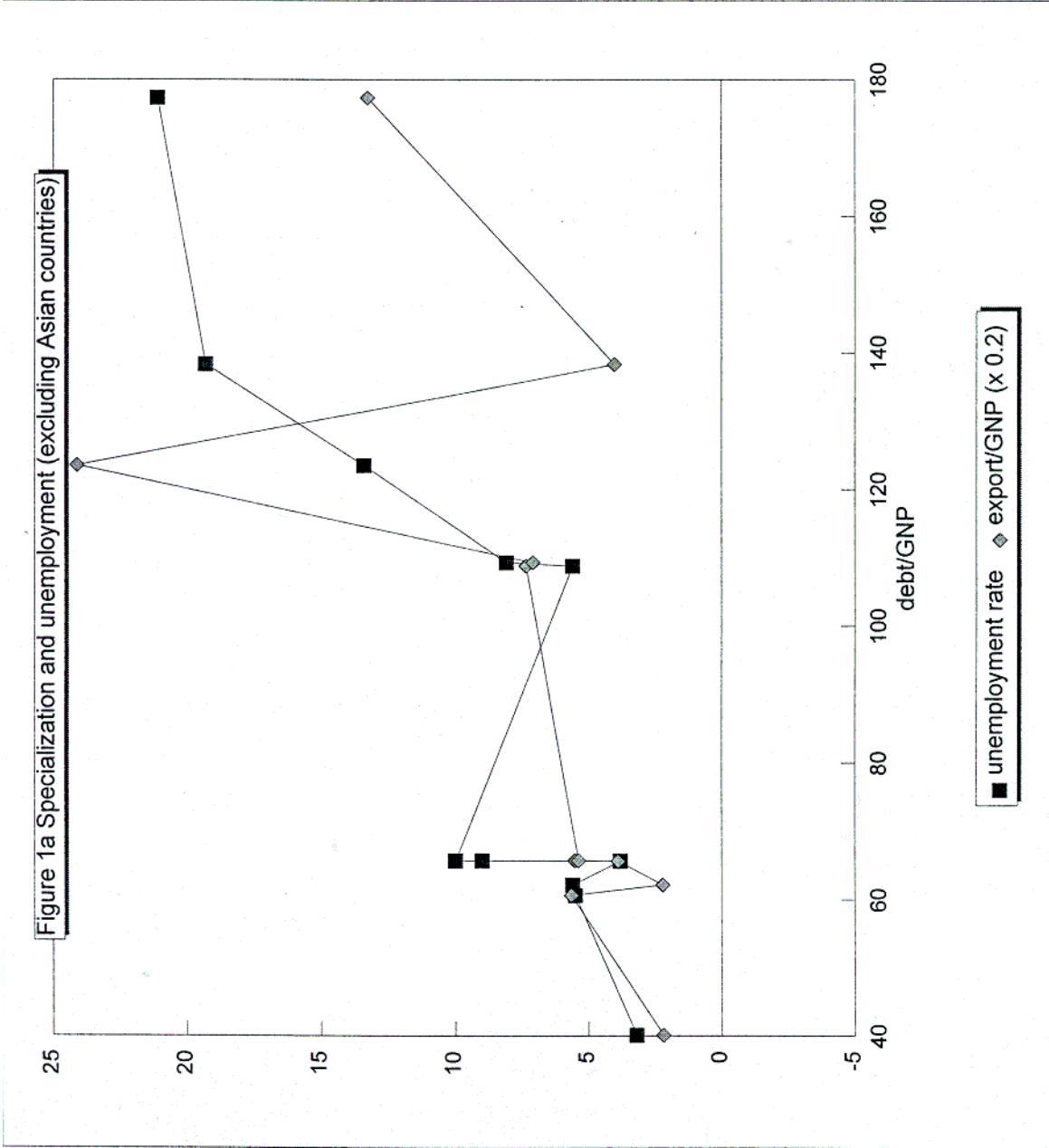
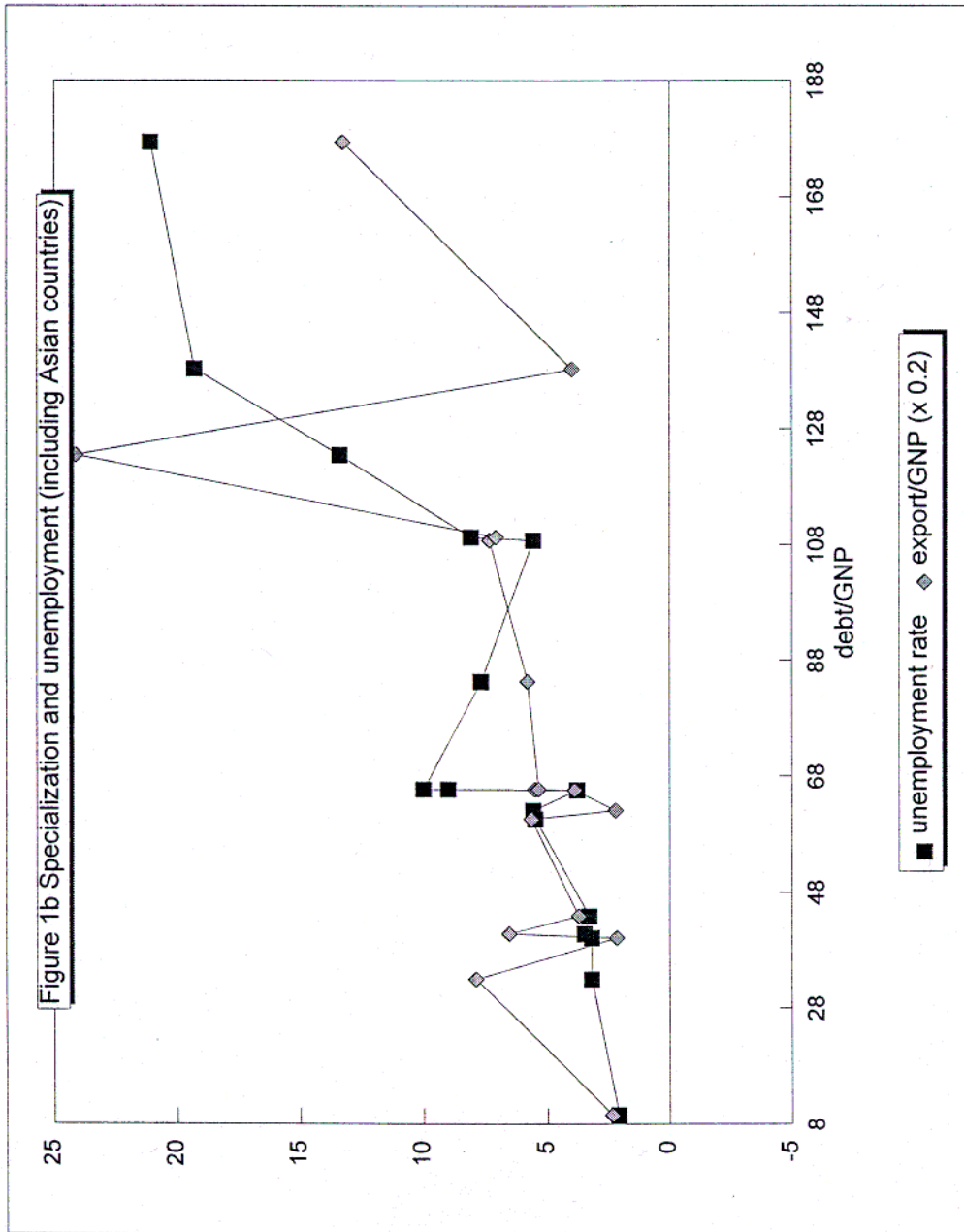


Figure 1b: Specialization and unemployment (including Asian countries)





## Bank of Canada Working Papers

### 1998

- 98-1 Food Aid Delivery, Food Security & Aggregate Welfare in a Small Open Economy: Theory & Evidence Patrick Osakwe
- 98-2 International Borrowing, Specialization and Unemployment in a Small, Open Economy Patrick Osakwe and Shouyong Shi

### 1997

- 97-1 Reconsidering Cointegration in International Finance: Three Case Studies of Size Distortion in Finite Samples M.-J. Godbout and S. van Norden
- 97-2 Fads or Bubbles? H. Schaller and S. van Norden
- 97-3 La courbe de Phillips au Canada: un examen de quelques hypothèses J.-F. Fillion and A. Léonard
- 97-4 The Liquidity Trap: Evidence from Japan I. Weberpals
- 97-5 A Comparison of Alternative Methodologies for Estimating Potential Output and the Output Gap C. Dupasquier, A. Guay and P. St-Amant
- 97-6 Lagging Productivity Growth in the Service Sector: Mismeasurement, Mismanagement or Misinformation? D. Maclean
- 97-7 Monetary Shocks in the G-6 Countries: Is There a Puzzle? B. S. C. Fung and M. Kasumovich
- 97-8 Implementation of Monetary Policy in a Regime with Zero Reserve Requirements K. Clinton
- 97-9 Mesures du taux d'inflation tendenciel T. Lafèche
- 97-10 The Structure of Interest Rates in Canada: Information Content about Medium-Term Inflation J. Day and R. Lange
- 97-11 A Band-Aid Solution to Inflation Targeting R. Amano, R. Black, and M. Kasumovich
- 97-12 A Micro Approach to the Issue of Hysteresis in Unemployment: Evidence from the 1988–1990 Labour Market Activity Survey G. Wilkinson
- 97-13 What Does Downward Nominal-Wage Rigidity Imply for Monetary Policy? S. Hogan
- 97-14 Menu Costs, Relative Prices, and Inflation: Evidence for Canada R. Amano and R. T. Macklem
- 97-15 The Effects of Budget Rules on Fiscal Performance and Macroeconomic Stabilization J. Millar
- 97-16 Canadian Policy Analysis Model: CPAM R. Black and D. Rose
- 97-17 Les marchés du travail régionaux : une comparaison entre le Canada et les États-Unis M. Lefebvre
- 97-18 Canadian Short-Term Interest Rates and the BAX Futures Market D. G. Watt
- 97-19 Modelling the Behaviour of U.S. Inventories: A Cointegration-Euler Approach I. Claus
- 97-20 A Measure of Underlying Inflation in the United States I. Claus

Earlier papers not listed here are also available.

Single copies of Bank of Canada papers may be obtained from  
Publications Distribution, Bank of Canada, 234 Wellington Street Ottawa, Ontario K1A 0G9  
E-mail: [publications@bank-banque-canada.ca](mailto:publications@bank-banque-canada.ca) WWW: <http://www.bank-banque-canada.ca/>  
FTP: [ftp.bank-banque-canada.ca](ftp://ftp.bank-banque-canada.ca) (login: anonymous, to subdirectory  
/pub/publications/working.papers/)