THE ROLE OF DEBT AND EQUITY FINANCING OVER THE BUSINESS CYCLE

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Abstract

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1 Introduction

The idea that changes in agency costs over the business cycle are important to understand the severity and persistence of business cycles has obtained a solid basis in economics. There are now several empirical and theoretical papers to support the basic idea. On the empirical side there is the classic paper by Gertler and Gilchrist (1994) that shows that sales of small firms are more sensitive to a monetary tightening than sales of large firms. On the theoretical side there are numerous papers that can generate the net-worth channel, that is, a deterioration in aggregate economic activity reduces firms' net worth, which aggravates the premium on external finance, which reduces lending and firm size, which in turn further slow down economic activity.

It is clear that incorporating agency problems into dynamic stochastic general equilibrium (DSGE) models great improve the empirical predictions of this class of models. But one shouldn't overlook that these models have some problems. First, in many models there is no default and in many models the default rate is procyclical. Both results are, of course, at odds with the data. Second, the theoretical models typically assume the owner and manager in the firm are the same agent and that the only type of external finance is debt finance. This means that in these models net worth only increases through retained earnings. The traditional view is that empirically this is not a bad assumption. Recent studies[1] document, however, that firms issue equity frequently and that equity issuance is an important financing source for increases in firm size. So although seasoned equity issues are indeed rare, the many other ways through which firms can issue equity are quantitatively very important. This means that the existing model is a bad characterization of many firms in the modern economy and especially a bad characterization of firms that are responsible for most of value added. The third deficiency of existing models is that a firm is simply an amount of capital and technology is such that the price of capital is equal to one (unit of consumption). Although, the introduction of adjustment costs allows for changes in the price of capital the change in the value of the firm is still pinned down by the technology to transform consumption into capital commodities.

In this paper we try to accomplish two things. In the first place we want to document the cyclical behavior of default rates and financing sources (debt, retained earnings, and net-equity issuance). Not surprisingly we show that

[1] Fama and French (2005) and Frank and Goyal (2005).

default rates are countercyclical. Also, we find that debt to asset ratios for small firms are clearly procyclical, whereas debt to asset ratios for large firms seem acyclical.

The second contribution of this paper is to construct a DSGE model with a firm problem that avoids the problems discussed above. The modifications we consider are the following.

- By allowing for diminishing returns to scale we generate the result that an increase in net worth decreases the default rate.

- We allow for a specification of net revenues that consists of two parts (e.g. revenues and costs) that both depend on the idiosyncratic shock but differently on aggregate productivity. With this specification it is easy to generate the result that increases in aggregate productivity decrease the default rate.

- We allow firms to issue equity. We model equity issuance in a simple way by assuming that there is a linear-quadratic cost of issuing equity. We then want to address the question when this will dampen or reinforce the net-worth channel. For example, it is possible that firms will choose to issue more equity when times are good to take advantage of the increased productivity and by doing so reinforce the net-worth channel. But it is also possible that firms will issue less equity during good times since the reduction in agency costs make it easier to use debt financing. Of course, it is a disadvantage to simply posit a linear-quadratic cost of issuing equity to capture the relevant trade-offs, but the plan is to calibrate this process so that the cyclical behavior of net-equity issuance is close to the observed behavior.

The organization of this paper is as follows. In the next section we discuss the cyclical behavior of default rates and document how the importance of alternative financing sources move over the business cycle. In Sections 3 and 4 we discuss in detail existing approaches and discuss the starting point of our framework. In Section 5 we show how relatively simple changes in the technology can overturn the undesirable results that increases in aggregate productivity lead to an increase in the default rate and increase in net worth have no effect on the default rate. In Section 6 we discuss the complete model with all the modifications and in Section 7 we give some preliminary results to document some of the features of the model.

2 Empirical observations

The goal of this section is twofold. First, using data for publicly traded firms document the business cycle dynamics of default rates. Second, analyze how asset growth is financed over the business cycle. The financing sources considered are debt issues, net equity issues and retained earnings.

2.1 Data

The balance sheet data is from Compustat. It consists of annual balance sheet and cash flow statement information from 1971 through 2004. Before 1971 both the coverage as well as the data availability is very incomplete in Compustat. The sample includes firms listed on the three U.S. exchanges (NYSE, AMEX and Nasdaq) and excludes firms not incorporated in the U.S.. Following standard practise, we exclude financial firms (SIC codes 6000-6999) and utilities (4900-4949). The balance sheet data are transformed to constant 2000 dollars using the GDP deflator.

The default rate is from Moody's and so only includes default on corporate bonds. The annual default rate is defined as the number of defaults during a year divided by the number of outstanding issuers at the beginning of the year. The denominator is adjusted to account for the number of rating withdrawals during the year. Defaults not only include bankruptcy but also missed interest payments and distressed exchange (workout).[2] This series does not include defaults on credit market instruments other than corporate bonds. Corporate bonds are the single most important credit market instrument for U.S. nonfarm, nonfinancial corporations. In particular, in 2003, it accounted for 1/3 of total liabilities outstanding and close to 3/5 of all credit market instruments.[3]

Figure 1 shows the annual default rate with NBER recessions represented by the shaded areas in the figure. The annual default rate series is available at the monthly frequency and covers the period between 1971 and 2004. The figure shows sharp increases during economic downturns. The figure

[2]In the period between 1995 and 2003, 77 per cent of defaults by U.S. nonfinancial corporations ended in bankruptcy (Moody's).

[3]Based on calculations using the *Flow of Funds Accounts*. Half of total liabilities are credit market instruments, defined as corporate bonds, bank loans, commercial paper and municipal securities. The other half includes mortgages, other loans, trade payable, tax payable, and other liabilities.

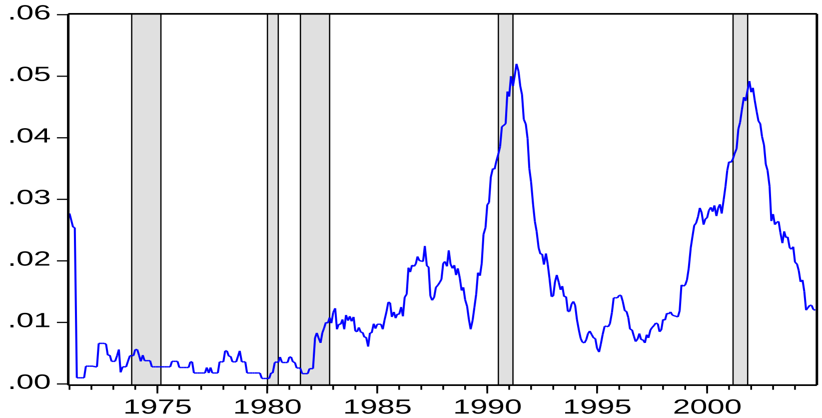
also shows that the behavior of the series in the first and the second half is quite different. First, in the second half of the series there are on average much more defaults. In particular, whereas from 1986 to 2004 the yearly default rate averaged 2.2 per cent, from 1970 to 1985 the yearly default rate was just 0.6 per cent. One possible explanation is that during the sample period, the use of corporate debt has become much more common, mainly due to deregulation and financial innovation[4] and that in the beginning of the sample corporate debt was mainly issued by firms with strong balance sheets that had a solid reputation. Moreover, the number of listed non-financial and non-utility firms in Compustat more than doubles during the period of 1971 through 2004. It is very well possible that the more recently listed companies have higher default rates. Our sample starts with 1530 firms in 1971 and ends with 3128 firms in 2004. Second, in the second half of the sample the increase in the default series clearly starts before the start of the official recession and decreases only gradually after the end of the recession. Given that defaults were quite low in the first half of the sample, it is perhaps not surprising that during this period defaults only increased noticeably when the economic downturn was most severe, that is, during the official recession. These observations are also true when we scale the default rate by the amount outstanding of corporate debt, to control for the size of the corporate debt market. Interestingly, the first peak in 1991 is now higher than the second peak in 2001 by a factor of two. This is not surprising since the 1991 recession was more severe than the last recession. Below we report our results for two non-overlapping sample periods, 1970:Q1–1985:Q4 and 1986:Q1–2004:Q4, in addition to the entire sample.

To document our stylized facts we further split the sample into two groups, small and large firms.[5] We follow Frank and Goyal (2003) and define small firms as having total assets below percentile 33 in each year. The average real book value of total assets for small firms is equal to 30 million (constant 2000) dollars, and the average number of employees is equal to 422

[4]Only in 1989 were commercial banks allowed to underwrite corporate bonds. Corporate equity underwriting was permitted for the first time in 1990. Before the Glass-Steagall Act of 1933 prohibited commercial banks from underwriting corporate securities and underwriting was restricted to top-tier investment banks. For more details see Gande, Puri, and Saunders (1999).

[5]This procedure is often done in this literature. Frank and Goyal (2003), Fama and French (2005) argue that to understand firms' capital structure decisions it is important to look at small and large public firms separately.

Figure 1: Default Rate on Corporate Bonds



Notes: The default rate series is from Moody's (mnemonic USMDDAIW in Datastream) and it is for all corporate bonds in the US. The plot shows the annual default rate, i.e., the number of defaults during a year divided by the number of outstanding issuers at the beginning of the year, adjusted by the number of rating withdrawals during the year.

workers. Large firms are defined as having total assets above percentile 67 in each year. The average size of a large firm corresponds to 4173 million (2000) dollars. In our sample large firms employ, on average, 2200 workers. The group of small firms has an average of 703 observations per year. The group of large firms is slightly bigger with an average of 940 observations per year.

For this sample the variables of interest are aggregate net debt issues, aggregate retained earnings and aggregate net equity issues as well as the aggregate increase in assets.[6] We construct the variables as in Fama and French (2005). Hence, net debt issues, ΔL, correspond to the change in liabilities (Compustat data item 181) between years t and $t-1$. Retained earnings, ΔRE, is the difference between Compustat's adjusted value of balance sheet retained earnings (36) between years t and $t-1$. The (book) measure of net equity issues, ΔSB, is the financing deficit minus the change in liabilities. The financing deficit is the difference between the change in total assets and retained earnings. The change in assets, ΔA, is then given by

$$\Delta A = \Delta L + \Delta RE + \Delta SB. \tag{1}$$

Fama and French (2005) argue that ΔSB is a noisy measure of stock issuance.[7] An alternative is the market measure of net equity issues, ΔSM, which is the change in the number of shares between years t and $t-1$ multiplied by the average stock price. The change in firm's net worth, ΔN, is equal to retained earnings plus book net equity issues. All the variables of interest are divided by total assets.

2.2 Stylized facts

Table 1 shows the cross-correlations between GDP and default rates for two sub-periods, 1971:Q1–1985:Q4 and 1986:Q1–2003:Q4 after both series have been detrended using the HP filter. The contemporaneous correlation between the cyclical components of these two variables is 0.74 in the second sub-period. It is negative but not significant in the first sub-period. For the

[6]The aggregate series are size-weighted averages of the firm's individual series.

[7]Book net equity issues are a noisy measure of stock issues for three reasons: *pooling of interest mergers*, employee stock options and stock dividends. See Fama and French (2005) for details.

Table 1: Cross Correlations Between Default Rate and Real GDP

$corr(y_{t+i}, \Phi_{\bar{\omega}_t})$	-3	-2	-1	0	1	2	3
1971:Q1–2004:Q4	-0.25	-0.34	-0.38	-0.33	-0.26	-0.16	-0.08
s.e.	(0.14)	(0.14)	(0.13)	(0.13)	(0.14)	(0.15)	(0.15)
1971:Q1–1985:Q4	-0.20	-0.22	-0.21	-0.11	-0.02	0.07	0.10
s.e.	(0.17)	(0.16)	(0.16)	(0.16)	(0.17)	(0.17)	(0.17)
1986:Q1–2004:Q4	-0.46	-0.66	-0.77	-0.74	-0.63	-0.46	-0.27
s.e.	(0.15)	(0.11)	(0.08)	(0.08)	(0.11)	(0.15)	(0.18)

Notes: The default rate series is from Moody's (mnemonic USMDDAIW in Datastream) and it is for all corporate bonds in the US. The default rate series is HP filtered. Real GDP is logged and HP filtered. Standard errors are in parenthesis.

reasons highlighted above we take the second sub-period as a better proxy for the default rates of all types of debt issues. From this evidence we draw the first main conclusion:

Stylized fact #1: Default rates are countercyclical.

We will now turn to the balance sheet data. As pointed out by Fama and French (2005) and Frank and Goyal (2005) it is rare in this data set that a firm's net equity issuance is equal to (or close to) zero. Although seasoned offerings are rare there are many ways through which firms can issue equity (warrants, convertible debt, options, etc.). This leads to the second stylized fact:

Stylized fact #2: Non-zero equity issuance is a frequent event at the firm level.

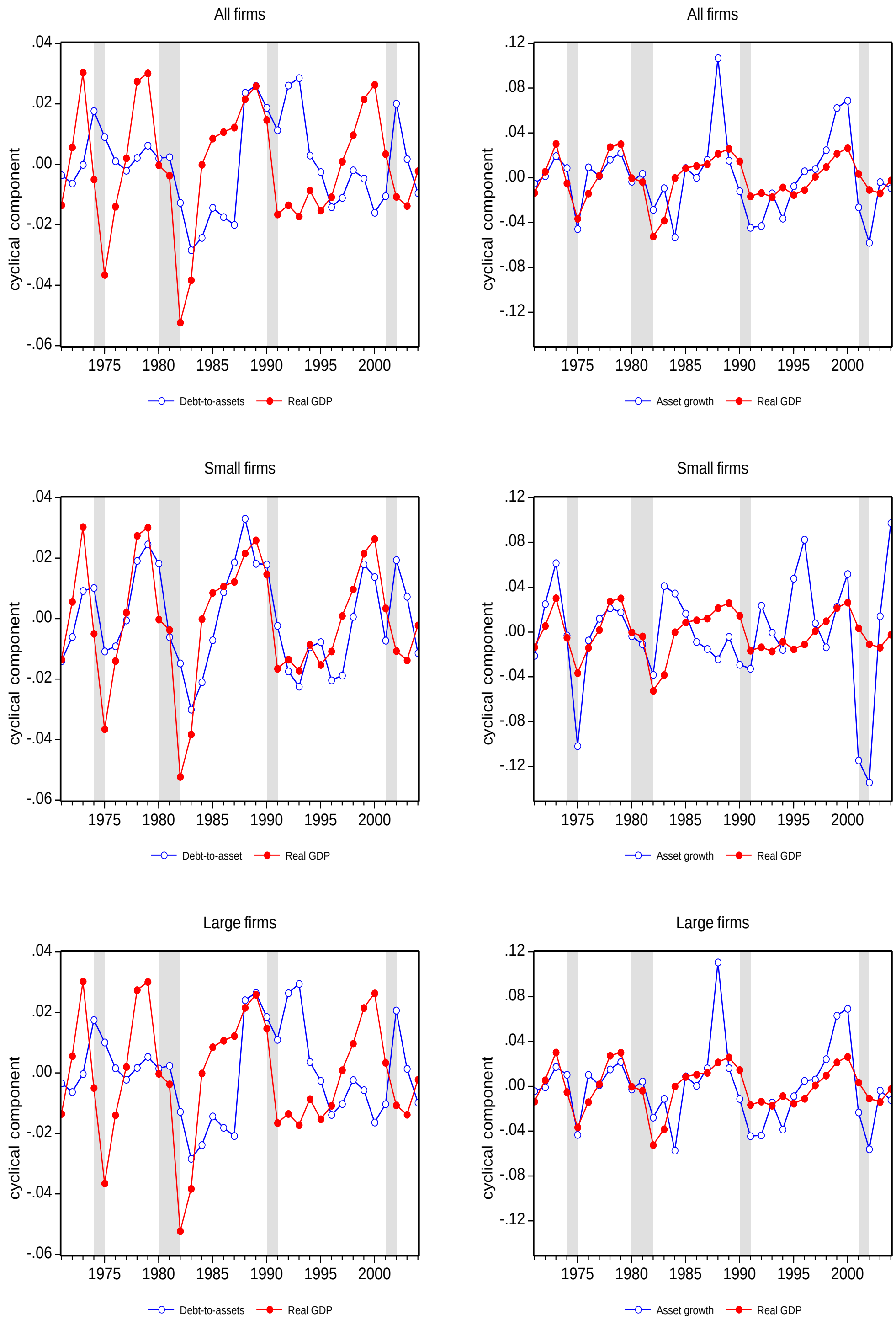
We now discuss the business cycle fluctuations of asset growth and debt-to-assets. Table 2 shows the cross-correlations between asset growth, debt-to-assets, accumulated-retained-earnings-to-assets, and accumulated-equity-issues-to-assets with real GDP after the series have been detrended using the HP filter. The contemporaneous correlation between the cyclical components

Table 2: Cross Correlations Between Asset Growth and Financing Sources with Real GDP

	All firms			Small firms			Large firms		
	-1	0	1	-1	0	1	-1	0	1
$corr(y_{t+i}, \Delta A_t/A_t)$	0.32	0.62	0.47	-0.31	0.24	0.25	0.34	0.61	0.46
	(0.18)	(0.14)	(0.17)	(0.15)	(0.18)	(0.17)	(0.18)	(0.15)	(0.18)
$corr(y_{t+i}, L_t/A_t)$	0.35	0.08	-0.24	0.74	0.69	0.26	0.33	0.07	-0.24
	(0.14)	(0.14)	(0.14)	(0.11)	(0.10)	(0.14)	(0.14)	(0.14)	(0.15)
$corr(y_{t+i}, RE_t/A_t)$	-0.15	0.12	0.26	0.19	0.19	0.00	-0.16	0.11	0.26
	(0.18)	(0.18)	(0.19)	(0.19)	(0.19)	(0.20)	(0.18)	(0.18)	(0.18)
$corr(y_{t+i}, SB_t/A_t)$	-0.34	-0.30	0.00	-0.30	-0.29	-0.04	-0.32	-0.29	0.01
	(0.22)	(0.21)	(0.23)	(0.19)	(0.19)	(0.20)	(0.22)	(0.22)	(0.23)

Notes: The sample includes U.S. firms on the Compustat files. Debt-to-assets, L/A, is liabilities (Compustat item 181) to total assets (6). Accumulated-Retained-Earnings-to-assets, RE/A, is retained earnings (36) to total assets. Finally, accumulated-equity- issues-to-assets is $(A - L - RE)/A$. All ratios are calculated as the aggregate value of the numerator divided by aggregate assets. Sample period is from 1971-2004. Asset growth, debt-to-assets, accumulated-retained-earnings-to-assets and accumulated-equity-issues-to-assets are HP filtered. Real GDP is logged and HP filtered.

Figure 2: Debt-to-Assets, Asset Growth, and Real GDP: Cyclical Components



Left panel: Cyclical components of debt-to-assets and real GDP for the entire sample, small and large firms, respectively. **Right panel:** Cyclical components of asset growth and real GDP for the entire sample, small and large firms, respectively.

of asset growth with real GDP is 0.6. Interestingly, the comovement for small firms is 0.24 and not statistically different from zero. This is not surprising if one takes into consideration that small firms are more likely to be growing even when the economy is not booming. On the other hand asset growth for large firms is strongly procyclical with a contemporaneous correlation of 0.60 with output. In addition, the correlation between debt-to-assets and real GDP is positive, but not statistically different from zero. Only for small firms is debt-to-assets strongly procyclical and moreover it lags output. For large firms debt-to-assets is acyclical. All these observations are consistent with the idea small firms face financing constraints. Otherwise we would expect to observe asset growth to be procyclical as we observe for large firms. In addition, we would also expect debt-to-assets to be acyclical, assuming no changes in taxes.

For the other financing sources the cross correlations are not statistically different from zero. Accumulated-retained-earnings-to assets is positively correlated with output and accumulated-equity-issues-to-assets is negatively correlated with output, but the comovements are not striking. Figure 2 displays the cyclical components of debt-to-assets, asset growth and real GDP. These plots show sharp decreases during economic downturns, specially for small firms. We summarize these observations as follows:

Stylized fact #3: Asset growth is procyclical for large firms. It is only slightly cyclical for small firms. Debt-to-assets is only procyclical for small firms and lags output.

We now consider the cyclical behavior of the three sources of financing: debt issues, $\Delta L/A$, retained earnings, $\Delta RE/A$, and equity issues, $\Delta SB/A$. We also look at the change in net worth, $\Delta N = \Delta RE + \Delta SB$. To illustrate the importance of firm size we again separate firms into two other groups— small and large firms— in addition to the total sample. Table 3 summarizes the contemporaneous correlation between the three sources of financing with asset growth and GDP . In particular, the comovement between asset growth and net debt issues is 0.9 for large firms and 0.7 for small firms. The table also plots the standard deviation of the change in each (scaled) finance source relative to the standard deviation of asset growth. Here we find a striking difference between small firms and large firms. The relative volatility of both retained earnings and equity issuance is higher for small firms than for large firms, whereas the opposite is true for debt finance. In particular, the standard deviation of $\Delta N/A$ is 70 per cent of the standard deviation of

$\Delta A/A$ for small firms, whereas for large firms this number is only 46 per cent.

The panels in Figure 3 plot asset growth for the corresponding firm category together with a financing source. The graphs illustrate the correlations results of Table 3. In particular, the left panels shows that net debt issues comove very closely with asset growth for all firm categories, but especially for large firms. Also, the importance for small firms of both retained earnings and net equity issuance is clearly visible.

The middle and right panel of Figure 3 displays the cyclical behavior of retained earnings and net equity issues with asset growth. A graphical inspection of these plots confirms the result that the comovement between retained earnings and asset growth is more evident for small firms. We also observe a similar pattern between equity issues and asset growth.

Figure 3 shows a dramatic increase in asset growth and net debt issues for large firms that occurred in 1988. First, this was a year of unusual corporate restructuring activity, in particular in the form of leverage buyouts and stock repurchases. Second, a new accounting rule was introduced that required companies to consolidate the balance sheets of their wholly owned subsidiaries (Bernanke, Campbell, and Whited, 1990). This observation has a small effect on our results. For example, eliminating this observation from our sample changes the contemporaneous correlation between asset growth and net debt issues, retained earnings, and net equity issuance for large firms to 0.85, 0.59 and 0.40, respectively. These results lead us to the following observation:

Stylized fact #4: At the aggregate level we find that asset growth in large firms is mostly debt financed. Asset growth in small firms is driven by all three sources of financing, debt, retained earnings and equity issues.

3 External finance in the literature

The idea that agency problems cause a wedge between the expected rate that firms pay on outside financing and the opportunity cost of using retained earnings has been well established in the literature.[8] In the corporate finance literature there are now several papers that consider models in which there is a choice between debt and external equity (and both are subject to an

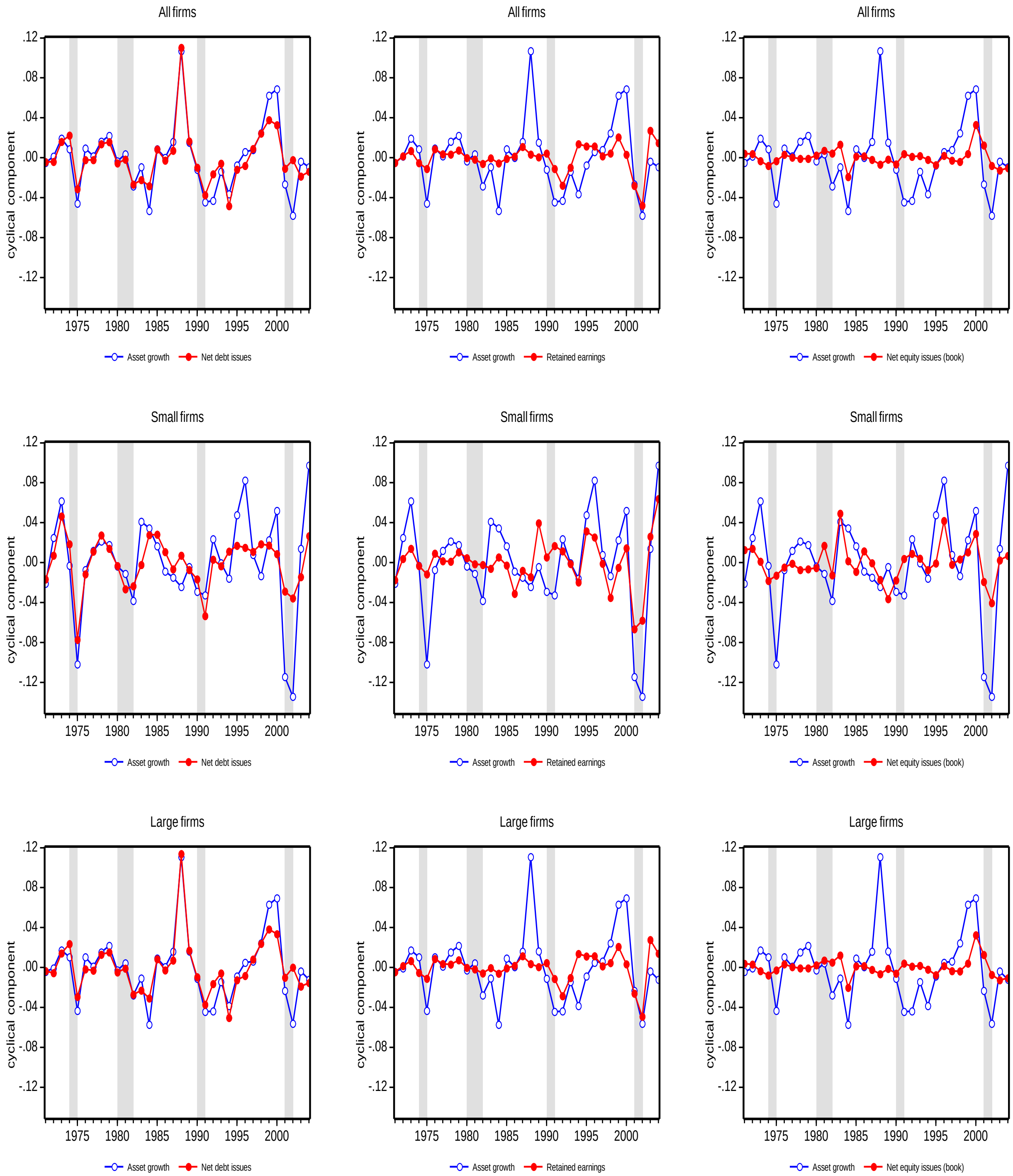
[8] Probably reference the Stiglitz handbook chapter for an early survey.

Table 3: Comovements and Relative Volatilities

	Asset growth rates		Real GDP	
	Relative Volatilities	Comovement	Relative Volatilities	Comovement
All firms				
$\frac{\Delta L}{A}$	0.81	0.89 (0.04)	1.39	0.63 (0.14)
$\frac{\Delta N}{A}$	0.46	0.61 (0.09)	0.80	0.25 (0.16)
$\frac{\Delta RE}{A}$	0.41	0.51 (0.11)	0.71	0.26 (0.17)
$\frac{\Delta SB}{A}$	0.26	0.27 (0.17)	0.44	0.02 (0.21)
$\frac{\Delta SM}{A}$	0.41	0.41 (0.19)	0.70	0.19 (0.24)
Small firms				
$\frac{\Delta L}{A}$	0.52	0.75 (0.09)	1.26	0.53 (0.14)
$\frac{\Delta N}{A}$	0.70	0.87 (0.05)	1.73	-0.05 (0.18)
$\frac{\Delta RE}{A}$	0.51	0.75 (0.09)	1.25	0.05 (0.20)
$\frac{\Delta SB}{A}$	0.38	0.62 (0.12)	0.92	-0.16 (0.16)
$\frac{\Delta SM}{A}$	0.52	0.69 (0.10)	1.28	-0.27 (0.18)
Large firms				
$\frac{\Delta L}{A}$	0.81	0.89 (0.04)	1.42	0.61 (0.14)
$\frac{\Delta N}{A}$	0.46	0.59 (0.09)	0.80	0.25 (0.17)
$\frac{\Delta RE}{A}$	0.41	0.49 (0.11)	0.71	0.27 (0.17)
$\frac{\Delta SB}{A}$	0.25	0.28 (0.17)	0.44	0.02 (0.22)
$\frac{\Delta SM}{A}$	0.41	0.41 (0.19)	0.70	0.19 (0.24)

Notes: See definitions of the variables in the text. Standard errors are in parenthesis.

Figure 3: Asset growth and Sources of Financing: Cyclical Components



Left panel: Cyclical components of asset growth and retained earnings. **Middle panel:** Cyclical components of asset growth and net equity issues at book values. **Right panel:** Cyclical components of asset growth and net equity issues at market values.

external finance premium).[9] Since GDP is produced mainly in firms that can issue equity[10] it is important to allow for external equity in macro models of the business cycle. Almost all DSGE models that incorporate agency problems in firm finance, however, restrict external finance to debt contracts. The first subsection discusses the alternative approaces used. Next, we discuss which agency problem is the basis of our framework and which modifications we introduce to improve key predictions of the model.

Take the money and run One possible problem that a lender may be concerned about is that the borrower takes the funds and either squanders the money or moves to a place on earth where he cannot be harassed by the lender. This is the agency problem considered in Kiyotaki and Moore (1997) (KM henceforth). In particular, KM assume that a borrower will always run away with any funds it could run away with. Consequently, lenders will require collateral and in particular the amount they lend has to be less than the value of the collateral. KM assume that there are two types of producers, farmers and gatherers, and both use the fixed factor (land), which is not only productive but can also serve as collateral. KM choose parameters such that the farmers borrow from the gatherers, are constrained in the amount they can borrow, and at the margin are more productive. Let r^l be the lending rate, k the amount of the fixed factor in the hands of the constrained agents, and q' the value of the collateral in the next period. Then the amount of funds borrowed, b has to satisfy the following equation.

$$(1 + r^l)b \leq q'k. \tag{2}$$

KM show that this framework generates two appealing predictions.

First, an increase in productivity leads to an increase in profits and, thus, an increase in net worth for constrained firms, which in turn makes it possible for the firm to borrow more. This leads to an increase in the scale of production by constrained firms and a further increase in the net worth in the next period. Consequently, a one-time increase in productivity can lead to a persistent increase in the demand of the fixed factor through the so-called net-worth channel. In general equilibrium, an increase in the use of the fixed factor by constrained agents must lead to a reduction in the use

[9] See for example Moyen (2004) and Hennessy and Whited (2005).

[10] According to the Economic Report of the President, 2003, corporations account for approximately 60 per cent of GDP.

by unconstrained agents. But because constrained agents are assumed to be more productive, this reallocation of resources does lead to an increase in aggregate productivity. To understand the effect of net worth on the amount the firm can borrow, consider the budget constraint of the borrower, which is given by

$$qk = n + b, \tag{3}$$

where n is the net worth of the agent. Suppose that the borrowing constraint is binding. Then it is easy to show that

$$\left(\frac{1 + r^l}{q'/q} - 1\right) b = n. \tag{4}$$

The term in brackets is the real rate of return, that is, the nominal rate of return adjusted for the increase in value of the collateral. The amount a firm can borrow is, thus, determined by the condition that the amount of net worth has to be equal to the real interest payments. This condition implies quite a high leverage ratio. KM also consider a modification of the model in which cultivating the fixed factor requires additional (uncollaterizable) investment. This reduces the leverage factor.

The second appealing prediction of the KM framework is that the value of collateral increases in response to a positive shock. An important component of net worth, n, is the value of the existing fixed factor, qk_{-1}. Consequently, an increase in the demand for this factor leads to an increase in the value of the firm's net worth and, thus, through the channel described above to a further increase in the demand of the fixed factor. Note that the value of collateral is not only pushed up by the current increase in the demand for the fixed factor, but—since it is a durable asset—also by the increased demand in subsequent periods. The framework, therefore, delivers an appealing within-period and intertemporal multiplier process.

It is hard to believe, however, that a big concern for lenders in developed economies is that managers divert firm resources in a way that none is left for the lender. Indeed, a substantial fraction of business loans does not have collateral.[11] Moreover, the existence of collateralized loans doesn't mean that this agency problem is important. A collateralized loan is basically a loan that gives the lender priority when bankruptcy occurs and this is

[11] xxx

advantageous even if bankruptcy occurs for other reasons than diversion of assets.

Furthermore, KM rely on linear production functions and an infinite elasticity of intertemporal substitution. Cordoba and Ripoll (2004) show that if one relaxes those assumptions the effects are quantitatively important only for empirically implausible values. Moreover, if parameters are such that shocks lead to a substantial increase in leverage (and investment) then the corresponding increase in interest rates will be strong as well. This will substantially dampen the effect on net worth in the next period and thus reduce persistence.[12]

Limited contract enforceability The contracting environment in KM is very simple. Lending is assumed to occur through one-period lending contracts. Moreover, defaults never occur since loans are fully collateralized. An alternative approach followed in the literature is to design optimal contracts in which the lender commits to a sequence of state contingents capital allocations to the firm and payments to the entrepreneur. Albuquerque and Hopenhayn (2004) and Cooley, Marimon, and Quadrini (2004) use this approach and derive the optimal contract in an environment in which the borrower can divert the assets of the firm as in KM.

An attractive feature of these papers is that they can explain firm growth. In particular, the amount of capital assets under the control of the firm remains constant until the enforcement constraint is binding, that is, when the entrepreneur is tempted to divert the funds. If that happens then the amount of capital is increased and the temptation to divert funds is prevented by making the current relationship more attractive. In Cooley, Marimon, and Quadrini (2004) a positive aggregate shock represents a higher probability that a new project will have high productivity. The aggregate shock doesn't affect the productivity of existing projects. Consequently, entrepreneurs are more tempted to divert the capital of the existing relationship and start a new project. This means that for more firms the enforcement constraint will be binding and the amount of funds received from the lender will increase. The optimal contract, thus, has the attractive property that a productivity shock is magnified by the increased allocation of funds to firms.

[12]Kocherlakota (2000) makes a similar point but he doesn't allow for a reallocation of the fixed factor from productive to unproductive agents. As shown in the appendix, with no variation in use of the fixed factor the leverage effect of KM is eliminated.

Less appealing is that these additional funds are given because it is exactly during good times that entrepreneurs are more likely to divert capital and entrepreneurs will remain loyal to the existing contract only if it is made more attractive. Another undesirable feature is that firms—as in KM—never default. Also, Quadrini (2004) points out that state-contingent optimal contracts may not be renegotiation proof. That is, it may be possible that both the lender and the entrepreneur would prefer to renegotiate the contract. Quadrini (2004) designs the optimal contract that adds as an additional restriction that it is renegotiation proof. Interestingly, he shows that (randomized) liquidation may be part of that renegotiation proof contract in an environment in which liquidation would not be part of the optimal contract.[13]

Other moral hazard problems The assumption that the entrepreneur can divert the assets of the firm creates a particular (and quite extreme) moral hazard problem. Besides diverting the assets and leaving nothing to the creditor, there are other—less extreme—entrepreneurial actions that diminish the chance for the lender to get his money back. For example, the revenues could depend on the effort put in by the entrepreneur. Also, the amount of risk chosen by the manager may be hard to control by the lender. One would think that these are more plausible concerns than diverting assets. Although this agency problem has received considerable attention in the literature, there are few papers that have incorporated an unverifiable effort or investment choice into DSGE models. One exception is the paper by den Haan, Ramey, and Watson (2003) in which the outcome of the project is affected by the effort choice of the entrepreneur. The model in den Haan, Ramey, and Watson (2003) has besides this moral hazard problem also a matching friction, so that it takes time for a borrower to find a lender, and an inefficient allocation of funds across lenders. The latter friction implies that lenders who are in a relationship with a borrower may not have enough funds to write a sustainable contract, which stands in sharp contrast with the rest of the literature that assumes that lenders have unlimited access to funds. Unlimited access to funds may not always be an appropriate assumption. For example, bank equity regulation makes the ability of banks to extend loans dependent on the amount of bank equity, which is not easy to adjust.

[13]Quadrini (2004) also considers an environment in which the scrap value of the firm may exceed the joint value of the contract. In that environment liquidation can be part of the optimal contract as well.

Another example of a model that incorporates a more realistic moral hazard problem is the DSGE model in Covas (2004) who considers a model in which the manager of the firm chooses the risk of the project undertaken. An important feature of Covas (2004) is that the objectives of the owner and the manager do not necessarily coincide. This stands in sharp contrast with the rest of the papers in the macro literature in which the owner and the manager are typically the same agent.

Asymmetric information Uncertainty is an important source of agency problems. One possibility is that there is asymmetric information about the characteristics of the borrower when the contract is written. We are not aware of DSGE models that incorporate this type of agency problem. Given that the probability of drawing a "lemon" may very well increase during a downturn this may be an interesting approach. Another important reason why uncertainty matters is that the outcome of a project is random (even if the borrower puts in the best possible effort). The question arises whether this in itself leads to an increase in the external financing premium. In a classic paper, Townsend (1979) shows that if it is costly for the lender to observe the realization of the shock, then the optimal contract to deal with this agency problem is a debt contract. The relevant cost in the costly-state-verification (or CSV) framework is the cost the lender has to incur *to determine* the outcome of the realization. In the literature, observed bankruptcy costs are often used to calibrate the magnitude of these verification costs. But this is a bit odd, since the actual verification costs are likely to be much smaller than typical bankruptcy costs. Moreover, we are not aware of any evidence that suggests that the verification itself is very costly.

One-period debt contracts and uncertain outcomes Note, however, that the assumption that it is costly to determine how much resources are left in the firm is only needed for the result that the one-period contract is the optimal contract. If there are other reasons why state-contingent lending contracts are not feasible and lending occurs through one-period debt contracts (for example because of habits, convenience, or simplicity) then one can not only use this framework but one can also calibrate the model using observed bankruptcy costs. All equations, including those that determine what contract among all one-period debt contracts is the optimal one, are exactly the same as in the CSV framework. The only difference is that if verification costs are negligible then the model by itself does not imply that the one-period debt contract is the optimal contract.

Properties of the CSV framework An attractive feature of the optimally chosen one-period debt contract is that the amount lent increases with the net worth of the agent as well as the expected productivity of the firm. Consequently, as shown, for example, in Carlstrom and Fuerst (1997), this framework is helpful in propagating shocks through the so called net-worth channel. A positive shock increases net worth which increases the scale of operations, profits, and, thus, next period's net worth. This in turn leads to an increase in lending. A not so attractive feature is that defaults increase with aggregate productivity. That is, an increase in aggregate productivity leads to such an increase in the scale of the project that default happens more frequently. Moreover, the assumption of linear production technology implies that the subsequent increase in net worth has no effect on default rates. Bernanke, Gertler, and Gilchrist (1999) assume that the realization of the aggregate productivity is not yet know when the debt contract is determined. This means that an unexpectedly high aggregate productivity shocks results in a lower default rate. But they still have the property that an increase in the expected productivity leads to an expected increase in the default rate.[14]

We discuss the reasons for the procyclical default rate in this framework in more detail in Section 5.1. We also propose two modifications. The first modification is to replace the linear technology production function by a standard production function that satisfies decreasing marginal returns. With this production function, we find that increases in net worth have a negative effect on the default rate. The second modification adds an additional component to the production function. With this specification, it is easy to find parameters such that an increase in aggregate productivity reduces the default rate.

Bernanke, Gertler, and Gilchrist (1999) add adjustment costs of capital to reinforce the net-worth channel. With adjustment costs an increased desire to invest leads to an increase in the price of the capital. This means that a positive productivity shock increases the value of net-worth not only because of the increase in retained earnings but also because of the increase in the

[14] I think this is true but needs to be checked.

market value of the capital stock.[15] We find such a valuation effect very plausible and an attractive feature of the model to have. However, an increase in the price of capital because of the difficulty to quickly produce additional capital is likely to be only a part of the increased value of the firm during good times. Moreover, if productivity increases are not embodied in existing capital then the price of existing capital might even go down. In Cooley and Quadrini (2001) net worth of the firm represents the earnings potential of the firm. This is the approach we follow in this paper.

Predictions of the different frameworks The agency problems discussed above are quite different. Nevertheless,changes in net worth and productivity have similar effects on real activity, that is, these models all predict that an increase in productivity leads to an increase in real activity, which over time leads to an increase in net worth and a reduction in the external finance premium. There are important differences in the predictions of these different approaches as well. As discussed above, in some frameworks default doesn't occur and in some frameworks, the default rates are even procyclical. The frameworks also differ in the importance of changes in the risk free or discount rate. If a firm is nothing more than a stock of capital stock and produced commodities can be transformed into investment commodities (either one for one or at a slightly higher rate in the presence of adjustment costs), then there is no direct effect of the discount rate. However, if part of the firm are assets that are in fixed supply then the value of the firm is the discounted value of the earnings generated by this fixed asset and changes in the discount rate are then likely to have strong effects on the value of the firm.

Even across papers that use the same framework we find important differences in the choices needed to implement the problem. For example, in **?** the savings decision of the entrepreneur has an interior solution.[16] This means that entrepreneurs withdraw funds from the firm every period. Depending on how the entrepreneur changes his consumption level in response to a

[15]Note that with the linear production function typically assumed in this literature, this modification doesn't affect the result that changes in net worth have no impact on the default rate.

[16]That is, the savings decision affects the price of the commodity produced by the sector with the agency problem. In equilibrium, this price will adjust every period so that the Euler equation of the savings decision has an interior solution.

shock this can either dampen or reinforce the net worth channel.[17] Bernanke, Gertler, and Gilchrist (1999) on the other hand choose parameters such that entrepreneurs choose to keep all funds within the firm. Bernanke, Gertler, and Gilchrist (1999) prevent the net worth of firms to go to infinity by assuming that firms exogenously stop operating with some probability in which case the owners consume the net worth of the firm.

Making magnification possible in general equilibrium So far, the profession has not been successful in identifying external shocks that are large enough to explain the observed fluctuations in economic aggregates.[18] Therefore, the recent literature has tried to build DSGE models in which shocks are magnified and propagated and models with agency problems have been the main contender.[19]

It is important to realize though that in a general equilibrium this is not enough to magnify and/or propagate shocks. If firms increase the scale of their operation the question arises where the additional resources come from. One possibility is that entrepreneurs decrease their consumption and invest the additional funds in the firm.[20] In numerous papers, however, the high return on retained earnings forces the consumption level of the entrepreneur to be at a corner so that no further reductions in consumption are possible. Moreover, even if the entrepreneur increases the amount of retained earnings, this still wouldn't answer the question where the additional funds provided by the lender come from. Some papers simply assume that lenders have unlimited access to funds at an interest rate that is not affected by the shock.[21] This may be a plausible assumption when one studies the effect of a country-specific shock for a small country with excellent access to international funds. In this case the resources to finance the expansion come from abroad. Interest rates would also be constant if savers have a

[17] For example, in **?** a positive productivity shock does indeed lead to a sharp reduction in the amount of entrepreneurial consumption but is then followed by a moderate increase but this result may change if nonlinear utility functions for the entrepreneur are considered.

[18] See Cochrane (1994), for example.

[19] Alternatives are models with sun spots such as XXX or models with labor market frictions such as Andolfatto (XXX), Merz (XXX), and Den Haan, Ramey, and Watson (2000).

[20] But note that if the risk smoothing motive is strong enough, then entrepreneurs would like to increase their consumption.

[21] Papers that make this assumption are Kocherlakota (2000), Cooley, Marimon, and Quadrini (2004), xxx.

very high intertemporal elasticity of substitution. In this case the expansion would be financed by a reduction in (the growth of) aggregate consumption. Of course, this does lead to the counterfactual implication that consumption is countercyclical.

In this respect, the analysis of KM is quite interesting. Although KM assume that interest rates are constant, the key factor of production in their analysis is in fixed supply. Consequently, the expansion is financed by a *reallocation* of the factor from less productive to more productive agents. Unfortunately, Cordoba and Ripoll (2004) show that if one relaxes the assumptions of constant interest rates and constant marginal productivity that it is difficult to generate quantitatively interesting results in the KM framework. Although, Cordoba and Ripoll (2004) focus on the agency problem of KM, the message of their paper is likely to carry over to other environments. For example, if the borrowing constraints are relaxed and intermediaries are more willing to lend out funds, then the risk free rate may have to increase to attract these additional funds from depositors.

Note that in a framework such as CSV that has a procyclical default rate this problem is even worse since it means that more resources are lost due to the increased bankruptcy rate. In fact, in Carlstrom and Fuerst (1997, 1998) this effect is so strong that even in later periods, when the increase in entrepreneurial net worth has reduced agency problems, the output response is only marginally above the output response of the standard real business cycle model.

Consequently, agency costs by themselves may not be enough to generate a framework with quantitatively interesting magnification and propagation effects. In addition, one would need a mechanism through which *aggregate* resources increase. One mechanism is the increase in labor supply. Empirical estimates of the labor supply elasticities make clear that changes in the intensive margin are likely to be weak but changes in the extensive margin are empirically more plausible. den Haan, Ramey, and Watson (2000) build a labor market matching model into a general equilibrium framework and show that changes in the extensive margin are important in magnifying and propagation shocks.

4 Our framework

4.1 Summary

The starting point in this paper is the CSV framework. In particular, we assume the following:

- Default leads to the loss of resources.
- The idiosyncratic shock has not yet been realized when the lending contract is written.
- Lending occurs through one-period loans and borrowers are protected by limited liability. If the costs associated with default are verification costs then the one-period contract is the optimal type of contract. It is not clear to us, however, that the actual verification costs are very high and these are likely to be tiny relative to bankruptcy cost. Since there are other reasons (e.g., simplicity of habit) to motivate one-period debt contracts, we are perfectly happy simply assuming that this is one type of contract used by firms. Note that if one restricts the contract to be a one-period debt contract then all bankruptcy costs are relevant.
- The parameters of the one-period debt contract are chosen to maximize the expected profits of the entrepreneur subject to the break-even condition of the lender.
- We assume that consumption and investment commodities are produced with the same technology and the aggregate shock considered is a change in the productivity of this technology.[22]

Using this framework we modify the model in the following three ways.

[22] In Carlstrom and Fuerst (1997) the production of investment commodities occurs in firms that suffer from agency problems, whereas the production of consumption commodities is not subject to agency problems. A positive aggregate productivity shock increases the productivity of the technology to produce consumption commodities, but endogenously raises the prices at which entrepreneurs can sell the investment commodity. A consequence of not having the the endogenous price is that we cannot use the standard Euler equation to obtain an interior solution for the (aggregate) dividend choice.

- We consider a technology with diminishing marginal productivity of capital (instead of constant) and production incurs a fixed cost. The first modification generates a default rate that is decreasing with net worth, whereas the default rate does not depend on net worth when the technology is linear. The second modification implies that the default rate is decreasing with aggregate productivity, whereas it is increasing without fixed costs.

- We consider a model in which firm entry is restricted. Consequently, the net worth of the firm is affected by the discounted value of future earnings. This can also overturn the result that the default rate is procyclical. In Cooley and Quadrini (2001), the net worth of the firm also depends on expected future earnings, but quantitatively this doesn't play a big role in their setup. In particular, increases in the discounted value of future earnings are not strong enough to overturn the positive relationship between firm productivity and the default rate. More importantly, they do not consider aggregate uncertainty, so they cannot address the question how important these changes in discounted future earnings are for the business cycle dynamics of this type of model.[23]

- We allow the firm to attract outside equity. The standard approach is to assume that net worth can only increase through retained earnings. An important exception is Cooley and Quadrini (2001, 2005). In these papers, there is a linear cost of issuing equity, which means that firms will not pay dividends until the firm has reached a certain size. In Section 2 we pointed out that this is inconsistent with empirical evidence. We assume that the costs of issuing equity are continuous and strictly convex. With this approach firms issue equity much more frequently.

An important part of this paper is to modify the CSV framework so that it generates countercyclical default rates. We do this simply by considering an alternative production function. An alternative is proposed in Bernanke, Gertler, Gilchrist (1999). They assume that the contract is written before aggregate productivity is known. An increase in z means that there are more

[23]Cooley and Quadrini (2005) do consider aggregate shocks but in that paper agency problems are exogenously imposed (the amount of debt has to be less than the firm's equity). That is, there are no defaults and the value of the firm does not affect the external finance premium.

resources in the firm than expected and the default rate decreases. We find this solution not quite satisfying since an increase in the expected value of z still increases the default rate.

5 Generating a countercyclical default rate in a CSV framework

The standard implementation of the CSV framework has the counterfactual prediction that the default rate is countercyclical. There are two reasons for this prediction. First, keeping net worth constant, an increase in aggregate productivity leads to an increase in the default rate. Second, the increase in net worth that follows the increase in aggregate productivity has no effect on the default probability. In the first subsection we explain these predictions. In the second section we provide two modifications, namely diminishing marginal returns and the introduction of a fixed cost (or benefit), and show that with these modifications the framework can generate the desired responses of the default rate to changes in net worth and aggregate productivity.

5.1 Procyclical default rates in the standard CSV framework

In this section we describe the standard CSV model and we discuss why net worth has no effect on the default rate and why productivity has a positive effect.

5.1.1 Static CSV framework

Entrepreneurs have access to the following technology:

$$z\omega k, \tag{5}$$

where k stands for the amount of capital, z for the aggregate productivity shock (with $z > 0$), and ω for the idiosyncratic productivity shock (with $\omega \geq 0$). We assume that z is observed at the beginning of the period when the debt contract is written, but that ω is only observed at the end of the period. The entrepreneur's net worth is equal to n and he borrows $(i - n)$.

Borrowing occurs through one-period debt contracts.[24] That is, the borrower and lender agree on a debt amount, $(i-n)$, and a lending rate, r^l. The entrepreneur defaults if the resources in the firm are not enough to pay back the amount borrowed plus interest. That is, the entrepreneur defaults if ω is less than the default threshold, $\overline{\omega}$, where $\overline{\omega}$ satisfies

$$z\overline{\omega}k = (1+r^l)(i-n). \tag{6}$$

If the entrepreneur defaults then the lender gets

$$z\omega k - \mu z\omega k, \tag{7}$$

where μ represent bankruptcy costs, which are assumed to be a fraction of revenues.[25] Note that in an economy with $\mu > 0$ default is inefficient and would not happen if the first-best solution could be implemented. Let $\Phi(\omega)$ be the cdf of the idiosyncratic productivity shock. We can then write expected entrepreneurial income as

$$\int_{\overline{\omega}}^{\infty} z\omega k d\Phi(\omega) - (1-\Phi(\overline{\omega}))(1+r^l)(i-n). \tag{8}$$

The expected income of the lender is given by

$$(1-\Phi(\overline{\omega}))(1+r^l)(i-n) + \int_{-\infty}^{\overline{\omega}} z\omega k d\Phi(\omega) - \mu zk \int_{-\infty}^{\overline{\omega}} \omega d\Phi(\omega). \tag{9}$$

We follow standard practice and assume that the lender only cares about his expected income.[26] This requires that the lender is either risk neutral or has a well-diversified portfolio. Given values for n and z, the financial contract specifies a debt amount, $k-n$, and a lending rate, r^l, which then imply a value for $\overline{\omega}$. Alternatively, one can use (6) and define the financial contract as the pair $(k, \overline{\omega})$. If we use (6) then we can write the entrepreneur's expected income as

[24] If the default costs introduced below are the costs the lender pays to verify the realization of the idiosyncratic shock then we have the CSV framework of Townsend (1979) and the one-period debt contract is the optimal type of contract.

[25] In section 6 we consider alternatives.

[26] Note that z is known when the contract is written.

$$zkF(\overline{\omega}) \text{ with } F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega}))\overline{\omega} \tag{10}$$

and the lender's expected revenues as

$$zkG(\overline{\omega}) \text{ with } G(\overline{\omega}) = (1 - \Phi(\overline{\omega}))\overline{\omega} + (1 - \mu) \int_{-\infty}^{\overline{\omega}} \omega d\Phi(\omega) \tag{11}$$

We assume that the entrepreneur is risk neutral. For given values of n and z the values of $(k, \overline{\omega})$ are then chosen to maximize the expected entrepreneurial income subject to the constraint that the lender must break even. For simplicity we assume in this section that the cost of funds for the lender is equal to zero. The best one-period debt contract is, thus, given by the following maximization problem:

$$\begin{aligned} &\max_{k,\overline{\omega}} zkF(\overline{\omega}) \\ &\text{s.t.} \quad zkG(\overline{\omega}) = k - n \end{aligned} \tag{12}$$

The optimal values for k and $\overline{\omega}$ are given by

$$\frac{zF(\overline{\omega})}{1 - zG(\overline{\omega})} = -\frac{\partial F(\overline{\omega})/\partial\overline{\omega}}{\partial G(\overline{\omega})/\partial\overline{\omega}} \text{ and} \tag{13}$$

$$k = \frac{n}{1 - zG(\overline{\omega})}. \tag{14}$$

Below we will build intuition using the slopes of the iso-profit and zero-profit curves. The slope of the iso-profit curve is given by

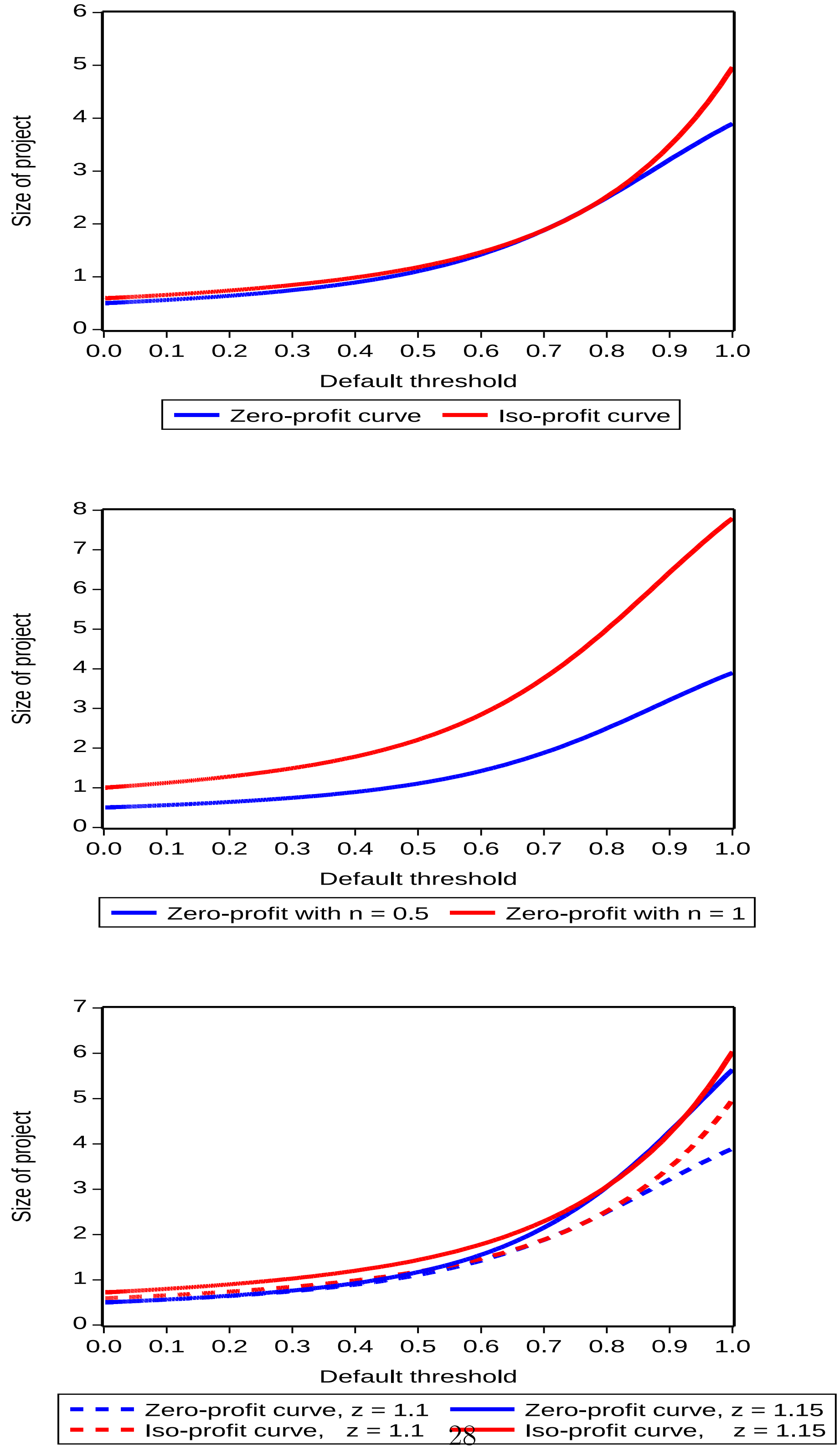
$$\frac{dk}{d\overline{\omega}} = -\frac{k\partial F(\overline{\omega})/\partial\overline{\omega}}{F(\overline{\omega})} \tag{15}$$

and the slope of the zero-profit curve is given by

$$\frac{dk}{d\overline{\omega}} = \frac{zk\partial G(\overline{\omega})/\partial\overline{\omega}}{1 - zG(\overline{\omega})}. \tag{16}$$

The top panel in Figure 4 shows the iso-profit curve for the entrepreneur and the zero-profit condition of the lender. Entrepreneurial profits increase

Figure 4: Static CSV Framework: Iso-Profit and Zero-Profit Curves



when the default rate decreases, that is, $\partial F(\overline{\omega})/\partial\overline{\omega} < 0$. A reduction of the default rate implies that the entrepreneur keeps the produced output for a wider range of realizations of ω (first term in 10). In addition, it reduces the interest payments (second term in 10), since according to (6) a lower value of $\overline{\omega}$ means that the interest payments must be less. A reduction in the default rate, thus, unambiguously increase entrepreneurial profits. This and the fact that entrepreneurial profits are increasing with the amount invested implies that the iso-profit curves are upward sloping.

A value of $\overline{\omega}$ equal to zero means that the entrepreneur never defaults, not even when he has no resources left. This means that the gross lending rate must be equal to zero. At that lending rate, the lender obviously only breaks even if he doesn't lend out anything at all. Consequently, k is equal to n when $\overline{\omega}$ is equal to zero. When bankruptcy costs are a fraction of revenues one assumption is needed to determine the shape of the zero-profit curve. In particular, we are interested in the case where the lender's profits increase as the default rate increases, that is $\partial G(\overline{\omega})/\partial\overline{\omega} > 0$. For this purpose we impose the same restriction as in Bernanke, Gertler, and Gilchrist (1999), which is a weak restriction on $\Phi(\omega)$, namely that XXX.

Changes in net worth In the standard framework discussed above, changes in net worth have no effect on the default rate. It only increases the amount invested. This result is graphically illustrated in the middle panel of Figure 4. It follows directly from the fact that equation (13) does not depend on k. The intuition is very simple. Because of the linearity, the slope of the iso-profit curves as well as the zero-profit curve are linear in k. That is for a given value of $\overline{\omega}$, the ratio of the slope of the iso-profit to the slope of the zero-profit line doesn't change. Since an increase in net worth is simply a parallel shift of the zero-profit line, this means that the change in net worth doesn't affect the optimal choice of $\overline{\omega}$. As discussed below, the linearity of the production function is essential.

Changes in aggregate productivity In the standard framework, an increase in aggregate productivity increases the default rate which is even less plausible than the effect of an increase of net worth on the default rate. It is easy to understand why the default rate increase with aggregate productivity. An increase in aggregate productivity implies that the zero-profit curve simply rotates upwards. That is, the zero-profit condition implies that an increase in $\overline{\omega}$ relative to k has become "cheaper". But an increase in z does not affect the slope of the iso-profit curves. Consequently, an increase in z not only means that the entrepreneur reaches a higher iso-profit curve, it also means that there is a movement along the iso-profit curve further increasing both k and $\overline{\omega}$. This is graphically illustrated in the bottom panel of Figure 4.

5.1.2 Modified CSV framework

In this section we will show that with two modifications to the CSV framework it is possible to achieve that both an increase in z and an increase in n result in a decrease of $\overline{\omega}$. The first modification is to replace the linear technology by a production function with decreasing marginal product of capital. The second modification adds an additional component to the production function that either represents a fixed costs or an additional source of revenue.

Suppose that technology is given by

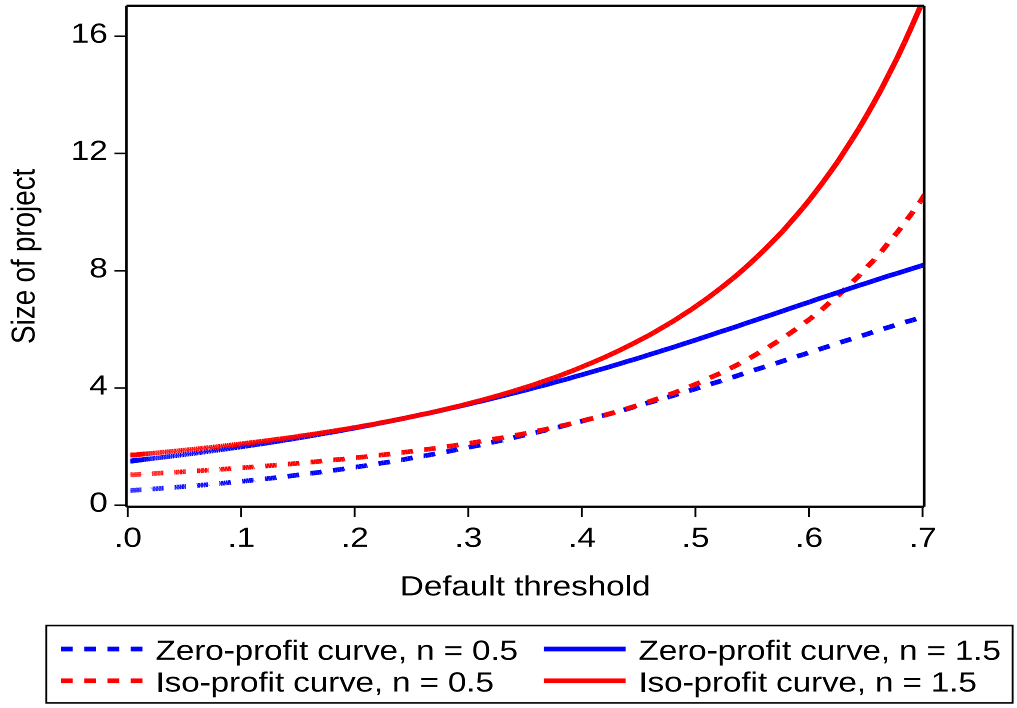
$$z\omega k^{\alpha} \tag{17}$$

and, thus, satisfies diminishing returns. It is easy to show that with this modification, an increase in net worth reduces the default rate. If the production function satisfies decreasing returns to scale then the slope of the iso-profit curve is still linear in k (for fixed value of $\overline{\omega}$). The slope of the zero-profit curve, however, is not and decreases relative to the slope of the iso-profit curve as k increases.[27] Consequently, when an increase in net worth pushes the zero-profit curve up, then the relative decrease in the slope of the zero-profit line pushes towards a lower $\overline{\omega}$ and a lower value of k. So this dampens the direct effect of net worth on k but one does obtain the desired effect on the default rate.

Now we consider specifications in which net revenues of the firm consist on two parts that depend differently on aggregate productivity. There are actually several specifications that work and in this section we illustrate two. In the first specification, the two parts of the production function both represent resources. In the second specification the first part represents a resource whereas the second represents a fixed cost.

[27]Basically because the cost of an one-unit increase in k is still equal to one but the benefits are decreasing with k.

Figure 5: Modified CSV Framework: Decreasing Returns to Scale



Multiple-project specification If production is given by (17) then an increase in z increases the slope of the zero-profit curve but doesn't change the slope of the iso-profit curves. For this production function expected firm income, $i(\overline{\omega}, k; z)$ is given by

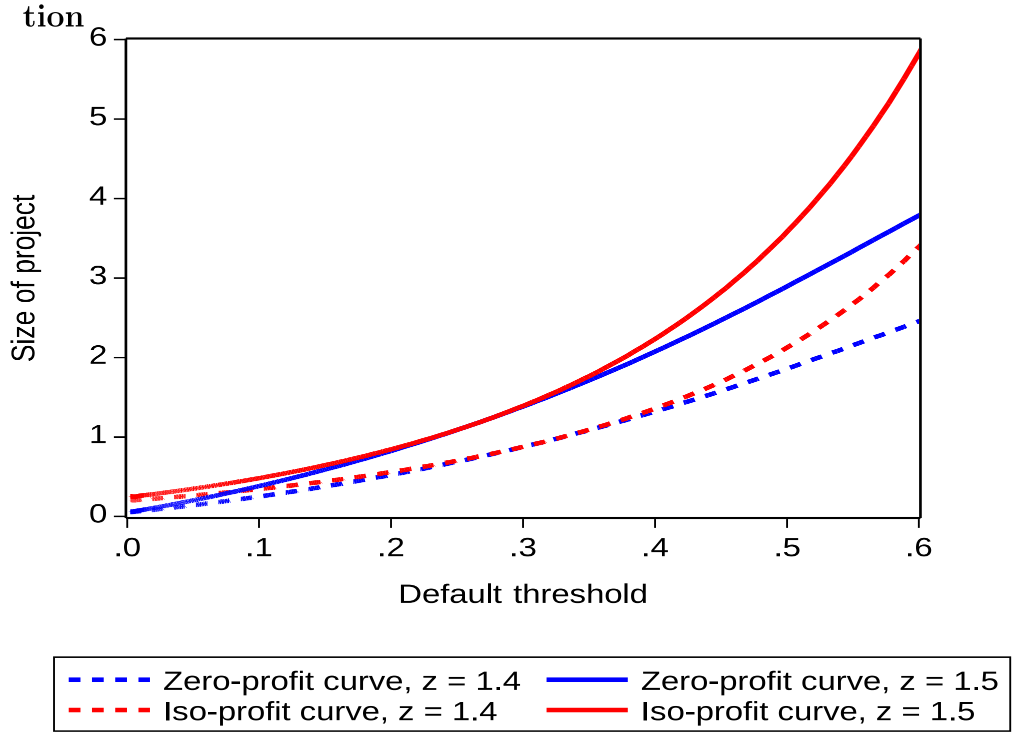
$$i(\overline{\omega}, k; z) = z\overline{\omega}k^{\alpha}F(\overline{\omega}) \tag{18}$$

with $F(\overline{\omega})$ given by (10). The reason why the slope of the iso-profit condition doesn't vary with z is that both $\partial i(\cdot)/\partial\overline{\omega}$ and $\partial i(\cdot)/\partial k$ depend linearly on z. Consequently, the value of z drops out of the expression for the slope of the iso-profit curve. One example of a production function for which that is not the case are the following.

$$z\omega k^{\alpha} + a_0 z^{a_1}\omega \text{ with } a_1 > 1 \tag{19}$$

The idea here is that the firm's net revenues consist of different components that (i) depend differently on z ($a_1 \neq 1$) and (ii) depend differently on ω and/or k. The specification used here is just one of several that can accomplish this. One nice feature of this specification is that the combination of $\omega = \overline{\omega}$ and $k = n$ is still on the zero-profit curve and an increase in z simply

Figure 6: Modified CSV Framework: Multiple-Project Specification



rotates the zero-profit curve upwards. But the increase in z also increases the slope of the iso-profit curves and it is easy to find values for a_0 and a_1 such that the default rate decreases. Figure 6 documents the effect of an increase in z on the default rate and the capital lend when a_0 is equal to 0.1 and a_1 is equal to 8.

Fixed-cost specification In the second specification, output net of the fixed cost is given by

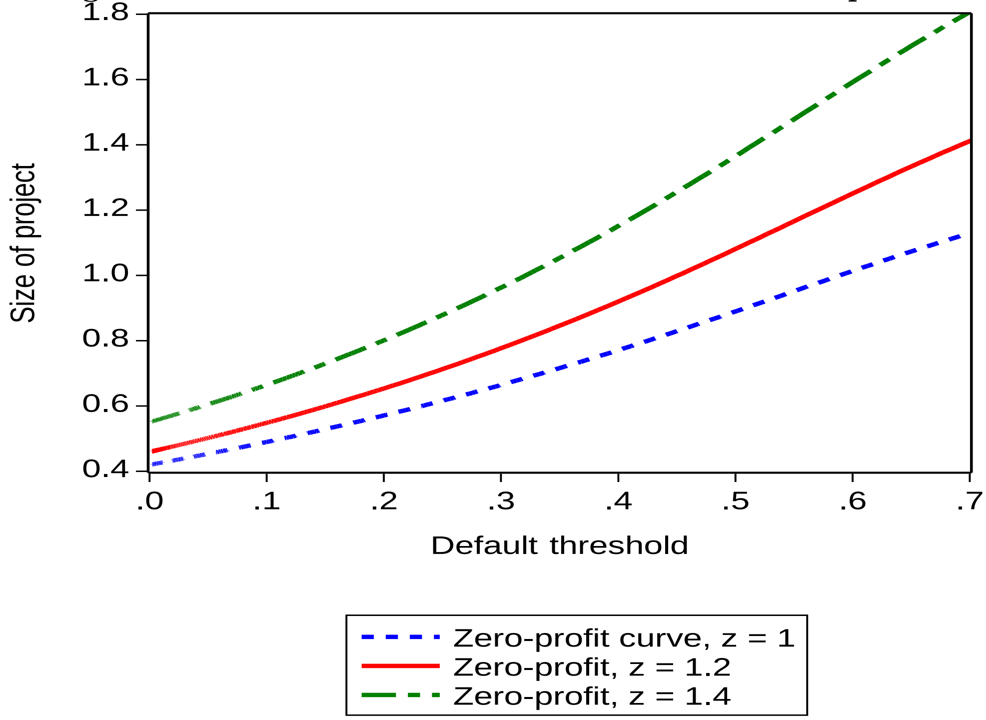
$$\begin{gathered} z\omega k^{\alpha} - c(z) \\ \text{with } c(z) = c_0 - c_1 z^{c_2}. \end{gathered} \tag{20}$$

Something here citing Ramey on the empirical relevance of fixed costs. It is easy to show that the expression for the expected income of the lender hasn't changed and the zero-profit condition for the lender is now given by

$$zk^{\alpha}G(\overline{\omega}) - c(z) = k - n \tag{21}$$

and the expression for $G(\overline{\omega})$ remains unchanged. That is, relative to the case with $c(\cdot)$ equal to zero, the zero-profit curve shifts down. An increase in z

Figure 7: Zero-Profit Curves and the Fixed-Cost Specification



simply shifts up the zero-profit curve and, thus, has the same effect as an increase in n. That is, in the presence of diminishing returns an increase in z would decrease the bankruptcy probability. Although, this specification is computationally very easy and quickly delivers the result one has to be be aware that now corner solutions are possible. The reason is that at $\overline{\omega} = 0$ there may not be a value for k bigger than n such that bank profits are zero. That is, because of the fixed cost, the bank would require $\overline{\omega} > 0$ even if it sets k equal to n, that is, even if it doesn't lend anything at all. The contract with $k = n$ would be an insurance contract. For large values of ω the firm pays the bank a fixed sum and when ω is so low that the fixed costs are not covered the bank will take the loss. Figure 7 plots zero-profit curves for different values of z and documents the different starting points of the zero-profit curve.

In the specification an increase in aggregate productivity lowered the fixed costs but did not affect the marginal product of capital. The results will go through if one uses a more standard specification, i.e., $z\omega k^{\alpha} - c(z)$, as long as the reduction of $\overline{\omega}$ caused by the parallel shift in the zero-profit curve dominates the reduction in $\overline{\omega}$ caused by the increase in the slope of the zero-profit curve.

6 Complete Model

This section describes the full dynamic model. It consists of the following: (i) a unit mass of firms; (ii) a large number of competitive banks that provide one-period loans to firms; and (iii) a representative household which owns the firms.

6.1 Environment

Firm produce goods in this economy and are owned by the representative household (shareholders). Firms can borrow funds from a bank, and at the same time households also provide funds to the firm at an additional cost. Operating profits of the firm are given by

$$\pi(z, k) = z\omega f(k) - c(z), \tag{22}$$

where z stands for aggregate productivity (with $z > 0$), ω for the i.i.d. idiosyncratic productivity shock (with $\omega \geqslant 0$), and k for the firm's capital stock. The term $c(z)$ is a fixed cost contingent on the aggregate state. The capital stock depreciates at rate δ. Further, assume that $f(\cdot)$ is continuously differentiable, strictly increasing, strictly concave with $f(0) = 0$, and satisfying the Inada conditions.

The timing assumptions are as follows. The debt contract is written at the beginning of the period at which point the value of z is known. The value of the idiosyncratic shock, ω, is observed at the end of the period. We assume that at that point next period's value of the aggregate shock, z', is known as well. The firm then makes the default decision. If the firm continues then the shareholders decide whether to liquidate the firm (take all resources out of the firm), pay out any dividends, or add additional equity to the firm.

6.2 Debt choice

This economy is populated by a large number of banks that lend funds to firms in a competitive market. Consequently, banks make zero expected profits on each loan they make to a firm. We assume that banks have unlimited access to funds. Below we describe the environment under two possible scenarios. Under the first scenario, firms defaults if the resources in the firm are not enough to pay back the amount borrowed plus interest. This ignores future earnings potential, or continuation value, as an asset of the firm. This assumption would be appropriate if adding equity to a firm in distress is prohibitively costly. Under the second scenario the firm defaults if, after repayment of the debt, the continuation value of the firm is negative. It is then possible that a firm that doesn't have enough resources to repay the debt will not default because the future earnings potential of the firm is attractive enough to acquire additional equity (and cover the costs of obtaining additional equity).

6.2.1 Without Valuation Effects

At the beginning of the period the firm borrows $(k-n)$ from one of the banks, where n is the firm's net worth. The firm agrees to repay $(1+r^b)(k-n)$ at the end of the period. Although we allow the firm to issue equity we assume that the costs are prohibitively expensive when the firm is in distress. The firm then defaults if ω is less than the default threshold, $\bar{\omega}$, where $\bar{\omega}$ satisfies

$$z\bar{\omega}k^{\alpha} + (1-\delta)k - c(z) = (1+r_b)(k-n). \tag{23}$$

If the firm defaults the bank gets

$$z\omega k^{\alpha} + (1-\delta)k - c(z) - \mu(1+r_b)(k-n), \tag{24}$$

where μ represent bankruptcy costs, which are assumed to be a fraction of the contracted repayment value of the debt. Let $\Phi(\omega)$ be the cumulative distribution function of the idiosyncratic shock. We can write the expected income of the bank for each loan it issues as

$$\int_0^{\bar{\omega}} [z\omega k^{\alpha} + (1-\delta)k - c(z)]\, d\Phi(\omega) + [1-(1+\mu)\Phi(\bar{\omega})](1+r_b)(k-n). \tag{25}$$

As in the static case, the financial contract is given by the $(k, \bar{\omega})$ pair that maximizes the firm's value subject to the lender being indifferent between loaning the funds or retaining them.

$$\begin{aligned} &\max_{k,\bar{\omega}} \quad \int_{\bar{\omega}}^{\infty} E[v(z',x)|z]d\Phi \\ &\text{s.t.} \quad x = z\omega k^{\alpha} + (1-\delta)k - c(z) - (1+r_b)(k-n), \\ &\quad zk^{\alpha}G(\bar{\omega}) + (1-\mu\Phi(\bar{\omega}))[(1-\delta)k - c(z)] \geqslant (k-n). \end{aligned} \tag{26}$$

where

$$G(\bar{\omega}) = [1 - (1+\mu)\Phi(\bar{\omega})]\bar{\omega} + \int_0^{\bar{\omega}} \omega \, d\Phi. \tag{27}$$

Note that we assume that the opportunity cost for the bank is equal to zero. The function $v(z', x)$ represent the value of the firm at the end of the period when next period's aggregate shock is equal to z' and resources within the firm are equal to x. This continuation value takes into account that at the end of the period the firm could liquidate the firm ($v(z', x) = x$), issue dividends ($n' < x$), or add additional equity ($n' > x$).

Furthermore, the participation constraint of the firm must hold as well. In particular we need that the maximized value in expression (26) to be greater or equal to $E[v(z', n)|z]$. This condition will always be satisfied below. When the lending contract is designed the firm has already decided that it isn't worth liquidating the firm. Because of the fixed cost, however, it may still be better for the firm to produce without a bank contract and set k equal to n. Thus, we have to check the participation constraint that the maximized value in expression (26) to be greater or equal to $E[v(z', n)|z]$. We never found this constraint to be violated.

Further, from the financial contract we can simplify expression (26) using the definition of the default threshold given in (23):

$$x = z(\omega - \bar{\omega})k^{\alpha}. \tag{28}$$

6.2.2 With Valuation Effects

We now consider the case where $v(0, z') > 0$. Obviously, this would mean that it is worthwhile for the firm to add equity, which would require that issuing equity for a distressed firm is not prohibitively expensive. The maximum amount of equity an entrepreneur is willing to add to the firm is given by the amount $-\overline{x}(z') > 0$, where $\overline{x}(z')$ satisfies

$$v(\overline{x}(z'), z') = 0. \tag{29}$$

The firm will now default when $\omega < \overline{\omega}$, where $\overline{\omega}$ satisfies

$$z\overline{\omega}k^{\alpha} + (1-\delta)k - c(z) - \overline{x}(z') = (1 + r_b)(k - n). \tag{30}$$

We assume that when $\omega < \overline{\omega}$ and the firm, thus, cannot raise enough equity to pay back the debt that part of the bankruptcy costs are that the value of the firm is equal to zero.[28] In case default occurs the bank then receives the cash in the firm and pays the bankruptcy cost

$$z\omega k^{\alpha} + (1-\delta)k - c(z) - \mu(1+r_b)(k-n). \tag{31}$$

We can then re-write the bank's expected revenues as

$$E\left[\begin{array}{c} \int_0^{\overline{\omega}(z')}[z\omega k^{\alpha} + (1-\delta)k - c(z)]\, d\Phi(\omega) \\ +[1-(1+\mu)\Phi(\overline{\omega}(z'))](1+r_b)(k-n) \end{array}\middle|\ z\right] \tag{32}$$

Note that with valuation effects on the default decision the bank makes zero-expected profits ex-ante but not ex-post, depending on the realization of next period's aggregate productivity. Hence, the profits of banks, Π could be either positive or negative. Given values of n and z, the financial contract specifies a debt amount $k-n$, and a lending rate r_b, such that:

$$\max_{k,\bar{\omega}} \quad E\left[\int_{\overline{\omega}(z')}^{\infty} v(z',x)d\,\Phi(\omega)\middle|\, z\right] \tag{33}$$

$$\text{s.t.} \quad x = zk^{\alpha}(\omega - \overline{\omega}(z')) + \overline{x}(z'), \tag{34}$$

$$\text{and} \quad (32). \tag{35}$$

6.3 Equity choice

In addition to debt financing the firm can also attract outside equity. To consider a more realistic and interesting equilibrium in which both debt and equity co-exist, we assume firms face a linear-quadratic cost of issuing equity. This cost function captures both the cost of underwriting fees and possible adverse selection problems in equity markets. More specifically, the cost of external equity is defined as:

$$\lambda = \lambda(e; z') \geq 0, \tag{36}$$

where e is new equity issued. When $e < 0$ then the firm issues dividends and we assume that in that case $\lambda(\cdot)$ is equal to zero. Furthermore, we assume that $\lambda(0; z') = \lambda'(0; z')$ and when $e > 0$ that $\lambda(e; z') > 0$ and $\lambda'(e; z') > 0$. We allow for the possibility that the cost of issuing equity is decreasing with

[28]This means that when $\omega < \overline{\omega}$ there is no point for the bank to lower the debt obligation so that additional equity can be attracted.

z'. The idea would be that an increase in z' increases the bargaining position of existing shareholders and makes it easier to attract new equity. Recall that we assume that the firm observes next period productivity, z', before it decides how much equity to issue in the current period.

Recall that $v(z', x)$ is the optimal value of a firm before issuing equity when next period's productivity is z' and with cash-on-hand equal to x. The firm's optimization problem can be specified in terms of the following dynamic programming problem:

$$v(z', x) = \max_{e,n'} -e - \lambda(e, z') + w(z', n') \tag{37}$$

$$\text{s.t.} \quad -e + n' = x, \tag{38}$$

where $w(z', n')$ is the firm's optimal value after the equity decision is made. Note that the firm can set $-e = x$, that is $n' = 0$. In this case the firm is liquidated and $v(z', x) = x$. We focus on the case where $v(z', 0) > 0$ and it is worthwhile to issue equity even when there is no net worth in the firm. Then liquidation never happens. The reason is that if a firm issues equity the slope of $w(\cdot)$ must be strictly greater than one (to cover the issuance cost). As n' increases the slope of $w(\cdot)$ decreases but never gets below one,[29] consequently we always have that $w(z', n') > n'$.

This value function is defined as follows:

$$w(z', n') = \frac{1}{1+r} E\left[\int_{\bar{\omega}'}^{\infty} v(z'', x')\, d\Phi(\omega)|z'\right]. \tag{39}$$

Finally, we define the firm's next period cash-on-hand, x', as follows:

$$x' = \begin{cases} 0 & \text{if } \omega < \bar{\omega}', \\ z'\omega' k'^{\alpha} + (1-\delta)k' - c(z') - (1+r_b')(k' - n') & \text{if } \omega \geqslant \bar{\omega}'. \end{cases} \tag{40}$$

Bank loans in our model are within-period loans and the opportunity costs is equal to zero. The discount rate is strictly bigger than zero. If instead we would assume that the opportunity cost for banks is equal to r, then firms would never pay out dividends. Because of diminishing returns there would be a maximum level of capital but firms would keep funds inside the firm even if net worth would exceed that level. Firms would simply invest the excess funds at rate r which is the same as what it would earn outside

[29] The slope will be equal to one when it starts paying out dividends.

the relationship, but keeping funds inside the firm serves as a buffer against drops in net worth (and avoiding paying issuance costs). Alternatively, we could assume that the opportunity costs for the banks is equal to r but that funds deposited by the firm at the bank earn less than r.

6.4 Household problem

In this economy there is a representative household which receives an endowment w and uses it to purchase consumption and shares of firms. Households maximize the expected lifetime utility derived from consumption:

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t, \tag{41}$$

where $0 < \beta = 1/(1+r) < 1$ is the discount factor. The household income comes from two sources: exogenous income and the dividends of firms. The household's budget constraint can be written as:

$$\begin{aligned} c + \int s'(z',x)p(z',x)\, d\Gamma(z',x) = w &+ \int s(z',x)p(z',x)\, d\,\Gamma(z',x) \\ &+ \int s'(z',x)d(z',x)\, d\Gamma(z',x) + \Pi, \end{aligned} \tag{42}$$

where $s(z',x)$ stands for the fraction of shares owned by the household, $p(z',x)$ for the price of the share and $d(z',x)$ for the dividend. The profits of banks are denoted by Π. As in Gomes (2001) we assume the payment of dividends occurs right after the shares of the firm are bought.

Taking first-order conditions w.r.t. the accumulation of shares we get:

$$p(z',x) = d(z',x) + \beta E[p(z'',x')|z']. \tag{43}$$

where $d(z',x) = -e(z',x) - \lambda(e,z')$.

Proposition 1 *In equilibrium* $p(z',x) = v(z',x)$.

7 Results

A Appendix

A.1 Kocherlakota (2000)

Kocherlakota (2000) argues that the collateral approach cannot deliver large effects for reasonable parameter values. Here we want to point out that this is true in the framework he considers but the framework of Kocherlakota is quite different from the KM framework. In particular, Kocherlakota assumes that the constrained agents are the only ones that hold the fixed factor. Consequently, there is no reallocation of the fixed factor from less productive to more productive agents as in KM. Whereas in KM the amount of the fixed factor held by constrained agents is the center piece of the analysis, in Kocherlakota it is the investment in the uncollaterizable investment. But this investment is not subject to the leverage effect discussed in the main text. Moreover, it is also not affected by changes in the current value of land. To see why consider the borrowing constraint (assumed to be binding) and the budget constraint used by Kocherlakota.

$$(1+r^l)b = q'k \tag{44}$$

$$qk + x = n + b \tag{45}$$

where x is the amount of uncollaterizable investment. In the Kocherlakota framework farmers use all the land so that k is constant and equal to the given supply of the aggregate factor. Suppose the aggregate supply is equal to 1. The solution for x is then given by

$$x = profits - (1+r^l)b_{-1} + q'(1+r'). \tag{46}$$

Note that the leverage effect has disappeared. The direct effect of a change in net worth on investment is equal to one whereas in KM it is huge. Also, note that the current-period land price has disappeared from this equation. In KM an increase in q increases n which has such a huge effect on b (because of the leverage effect) that this effect dominates the downward effect on b caused by the decrease in q'/q (keeping q' constant).

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