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## **A Dynamic Theory of Optimal Capital Structure and Executive Compensation\***

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### ABSTRACT

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We put forward a theory of the optimal capital structure of the firm based on Jensen's (1986) hypothesis that a firm's choice of capital structure is determined by a trade-off between agency costs and monitoring costs. We model this tradeoff dynamically. We assume that early on in the production process, outside investors face an information (**information not informational**) friction with respect to withdrawing funds from the firm that (**Anna said that not which**) dissipates over time. We assume that they also face an agency friction that (**Anna said that not which**) increases over time with respect to funds left inside the firm. The problem of determining the optimal capital structure of the firm as well as the optimal compensation of the manager is then a problem of choosing payments to outside investors and the manager at each stage of production to balance these two frictions.

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# 1. Introduction

We put forward a theory of the optimal capital structure of the firm and the optimal compensation of the firm's managers based on Jensen's (1986) hypothesis that a firm's choice of capital structure is determined by a trade-off between agency costs and monitoring costs. We model this trade-off dynamically by assuming that outside investors in a firm face different obstacles to recouping their investment at different times. Early on in the production process, outside investors face an information friction — the output of the firm is private information to the manager of the firm unless the outside investors pay a fixed cost to monitor the firm. With time, the output of the firm is revealed to outside investors and, hence, the information friction disappears. At this later stage in the production process however, outside investors face an agency friction — the firm's manager can divert resources not paid out to investors in the early phases of production towards perquisites that provide him with private benefits. The problem of determining the optimal capital structure of the firm as well as the optimal compensation of the manager is then a problem of choosing payments to outside investors and the manager at each stage of production to balance these two frictions.

Our theory is developed in an dynamic optimal contracting framework, and, as a result, our model yields predictions about the joint dynamics of a firm's capital structure and its executive compensation. The choice of compensation for the manager is shaped by the assumption that the manager is risk averse while the outside investors are risk neutral. Our theory has the following implications regarding optimal capital structure and executive compensation. Each period, the payouts from the firm can be divided into payments to the manager that consist of a non-contingent base pay and a performance component of pay based on the realized output of the firm, as well as two distinct payments to the outside investors

that resemble payments to debt and outside equity respectively. The debt-like payment to outside investors is made early in the period. It comes in the form of a fixed lump — the failure of which to pay leads to monitoring. The equity-like payment to outside investors comes in the form of a residual which depends upon the performance of the firm and is paid at the end of the period.

In our model, the fact that the manager receives some form of performance based pay is not motivated by the desire to induce the manager to exert greater effort or care in managing the firm. Instead, the performance based component of the manager's pay simply serves to induce the manager to forsake expenditures on perquisites for his own enjoyment. Hence, our model's predictions for whether it is optimal to have the performance component of the manager's compensation depend on the total market value of the firm or on some narrower measure of current performance such as current sales are determined entirely by our assumptions regarding the extent of the agency friction. If the manager is able to appropriate a broad measure of the firm's resources for his own benefit, then his performance bonus will be based on this broad measure. If the manager's ability to appropriate resources is more limited in scope, then his bonus pay will be based on this narrower measure of performance.<sup>1</sup>

Our theory also has implications for the relationship between the optimal financial structure of the firm and its optimal production plan. Our theory predicts that there is a wedge between the marginal product of capital in the firm and rental rate on capital that depends upon the expected monitoring costs associated with bankruptcy and the inefficient

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<sup>1</sup>Performance bonuses are typically conditioned on firm-specific factors and not industry or national factors. This form of agency friction is not inconsistent with that observation. An agency friction that arises from an information friction with costly effort is not, since anything that helps to signal the level of effort should be factored into the optimal contract.

risk-sharing between outside investors and the manager induced by the agency friction. Under certain parametric assumptions, we are able to compute the magnitude of this wedge between the marginal product of capital and its rental rate in terms of readily observed features of the firm's financial structure and its executive compensation.<sup>2</sup>

We also use our model to examine the role of financial hedging in the firm's optimal capital structure. In the data, firms are frequently seen to use financial instruments to hedge against both idiosyncratic and aggregate risks. According to standard theory, these financial hedges add no value. In our baseline model, financial hedging by the firm would actually be counter-productive. However, when we extend the model to allow for public signals of the productivity of the firm, we show that hedges can play a role in fine-tuning the efficient contract in terms of achieving the optimal trade-off between bankruptcy risk and the agency friction.

To extend our model to a dynamic setting, we assume that there is an information cycle in which outside investors first face an information friction and then face an agency friction. This cycle is repeated indefinitely. We associate this cycle with an accounting or capital budgeting cycle at the firm. We show the qualitative decomposition of the optimal contract into four payments — base pay and performance pay for the manager, and debt and equity-like payments to the outside investors — holds both in a static setting in which there is only one information cycle, and in a dynamic setting in which there are an arbitrary number of information cycles. Thus, repetition of our contracting problem does not change,

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<sup>2</sup>While all of the models that generate debt constraints as part of the contract generate a wedge between the inside return to capital and the outside cost of capital (e.g. Atkeson 1991, Hopenhayn and Clementi 2002, Albuquerque and Hopenhayn 2004, Bernake and Gertler 1989, and Charlstrom and Furest 1997), the advantage of our set-up is that it tightly ties this wedge to observable aspects of the contract.

qualitatively, the interpretation of our efficient contract as a theory of capital structure and executive compensation.

However, the dynamic model with more than one information cycle does have a variety of additional implications with respect to capital structure and executive compensation. One feature that emerges is the distinction between temporary and persistent production shocks. Temporary positive production shocks raise current profits without changing the firm's future investment opportunities and lead the firm to reduce the future share of debt payments in the capital structure of the firm. Persistent positive production shocks result in an increase in the optimal size of the firm in the future, and if this increase is large enough, lead to an increase in the future share of debt payments. Increases in monitoring costs lead to a reduction in the share of payments going to debt and the probability of bankruptcy, while increases in agency costs lead to the reverse.

With respect to executive compensation we derive two results in a dynamic setting. The first of these results is that the compensation of incumbent managers is non-decreasing over time, regardless of the performance of the firm. The result that the pay of the incumbent manager is non-decreasing over time has implications for the dynamics of the firm's optimal capital structure since, *ceteris paribus*, the dynamics of executive compensation alter the terms of the trade-off between the information and agency frictions that outside investors face. In this manner, the dynamics of executive compensation in our model drive the dynamics of the optimal capital structure.

Finally when we augment our model to allow for observable shocks to managerial productivity, we find that incumbent managers are protected against the risk that they become unproductive with a "golden parachute" as a direct consequence of optimal risk sharing. We

show that the optimal retention strategy is to retain the manager if his productivity is above the threshold and fire him if he is below it. We show that the retention threshold is lower for managers who have been more productive in the past, and hence they are more likely to be retained. We interpret this result as a form of managerial entrenchment.

Our model delivers predictions for the division of payments from the firm between the manager, the owners of outside equity, and the owners of the firm's debt based on the trade-off of information and agency frictions. It is important to note that our dynamic model does not pin down the debt-equity ratio of the firm. This is because our model does not pin down the source of financing for ongoing investment in the firm. We present a simple example to demonstrate that, holding fixed the division of gross payments to debt and outside equity holders, the firm will have a different debt-equity ratio depending on whether debt is long term and ongoing investment is financed out of retained earnings or debt is short-term and ongoing investment is financed with new short-term debt. We conjecture that this failure of our model to pin down the debt-equity ratio of the firm in a dynamic setting may be a general feature of completely specified "trade-off" models of corporate finance.

This paper considers the optimal financial contract between outside investors and a manager in the presence of both information and agency frictions when there is the possibility of monitoring. It is therefore related to a wide range of prior research on each of these topics. The within period, or static, aspects of the information and monitoring aspect is similar to Townsend (1979), while the static aspects of the agency friction and the information friction are similar to Hart and Moore (1995), in that these frictions can rationalize a division of the firm's output into debt, and other payments.<sup>3</sup> However, unlike these prior papers, the

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<sup>3</sup>As in Jenson (1986) debt acts as a means of avoiding the agency friction associated with leaving the funds

inclusion of both frictions and monitoring, and the specific form of these friction leads to three different payment streams coming out of the firm, outside debt, outside equity, and managerial compensation.

Since we consider these frictions within a recursive environment, our paper is related to prior work on dynamic efficient contracting. However, unlike the literature on dynamic models of efficient financial contracting with information frictions, such as Atkeson (1991), Hopenhayn and Clementi (2002), Demarzo and Fishman (2004) or Wang (2004), our information friction is temporary since there is complete information revelation by the end of the period. As a result, while the costly state verification aspect of our model rationalizes outside debt, the dynamic aspects and the overall tractability of our model are similar to those of the dynamic enforcement constraint literature, such as Albuquerque and Hopenhayn (2004), and Cooley, Marimon and Quadrini (2004). In particular, it shares the feature that if the manager is retained, then his promised utility is weakly increasing over time, and these increases serve to reduce the extent of the enforcement friction.

In our model contracting is complete subject to explicit information and enforcement frictions. This is in contrast to a large literature that seeks to explain various aspects of the financial structure of firms as arising from incomplete contracting. Examples include Hart and Moore (1995,1997), as well as Aghion and Bolton (1992), which examines the efficient allocation of control rights, Dewatripont and Tirole (1994), in which outside investors choose their holdings of a debt as opposed to equity claims to generate the efficient decision with respect to the interference or not in the continuing operation of the firm, and Zweibel (1996), in which manager uses debt as a means of constraint their future investment choices to be

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in the firm and awaiting their payout as dividends.

more efficient.

## 2. Model

We begin by presenting a one period version of the model. We extend our results to a dynamic model in a later section.

There is a large number of risk neutral outside investors who are endowed with capital which they can rent in the market at rental rate  $r$ . There is also a large number of identical risk averse managers. These managers have an outside opportunity that offers them utility  $U_0$ . We assume that the managers have a utility function  $u(c)$  with  $u(0) = -\infty$ .

There is a production technology that transforms capital and the labor of a manager into output. The production process takes place over the course of three subperiods within the period. In the first subperiod, a manager is chosen to operate the production technology and capital  $K$  is installed. In the second subperiod, this production technology yields output  $y = \theta F(K)$  where  $\theta$  is a productivity shock that is idiosyncratic to this technology. In this subperiod, this productivity shock  $\theta$  and hence, output  $y$  as well, is private information to the manager. The set of possible shocks is an interval given by  $\Theta$  and the distribution of these shocks has c.d.f.  $P$  with density  $p$  and an expected value of one. We assume that there are diminishing returns to scale in the sense that  $F''(K) < 0$ .

In the second sub-period, the outside investors have the option of monitoring the output of the project to learn the realization of the shock  $\theta$  (and hence output  $y$  as well) at a cost of  $\gamma F(K)$  units of output. At the end of the second subperiod, the manager has the option of spending up to fraction  $\tau$  of whatever output of the firm that he has not paid out to the outside investors during this sub-period on perquisites that he alone enjoys. That

output that the manager does not spend on perquisites is productively reinvested in the firm. For simplicity, we assume that the gross return on this productive reinvestment in the firm between the second and third sub-periods is one.

In the third sub-period, the outside investors can freely observe both the output of the firm and the division of this output between spending on perks for the manager and productive reinvestment.

The contracting problem between the outside investors and the manager can be described as follows. A contract between these parties specifies a level of capital,  $K$ , to be hired in the first subperiod, a decision by the outside investors to monitor,  $m$ , and a payment from the manager to the outside investors in the second subperiod,  $v$ , and a payment from the outside investors to the manager in the third subperiod,  $x$ .

We assume that the outside investors can commit to a deterministic strategy for paying the cost to monitor the output of the project in the second sub-period as a function of the manager's announcement  $\hat{\theta}$  of the realization of the productivity shock  $\theta$ . Denote this strategy by  $m(\hat{\theta})$  and denote set of announced shocks for which monitoring takes place by  $M \subseteq \Theta$ .

The payments  $v$  from the manager to the outside investors in the second subperiod are contingent on the manager's announcement of the productivity shock  $\hat{\theta}$  as well as the outcome of the monitoring decision. Let  $v_0(\hat{\theta})$  denote the payment that the manager makes to the outside investors in the second sub-period as a function of the announcement  $\hat{\theta}$  in case monitoring does not take place, and let  $v_1(\hat{\theta}, \theta)$  denote the payment that the manager makes as a function both of the announcement  $\hat{\theta}$  and the true value of  $\theta$  in case monitoring does take place.

Finally, let  $x(\hat{\theta}, \theta)$  denote the payment from the outside investors to the manager in

the third subperiod as a function of the manager's report  $\hat{\theta}$  in the second sub-period and the realized production shock  $\theta$ . Note that it is not necessary for  $x$  to depend on the monitoring decision because the true value of  $\theta$  is revealed to outside investors in the third sub-period at zero cost.

For reasons of limited liability, we require

$$v_0(\hat{\theta}) \leq \hat{\theta}F(K), \quad v_1(\hat{\theta}, \theta) \leq \theta F(K), \quad \text{and} \quad x(\hat{\theta}, \theta) \geq 0. \quad (1)$$

We assume, without loss of generality, that  $x(\hat{\theta}, \theta)$  is chosen to ensure that the manager chooses not to take any perks for himself. This assumption implies a constraint on  $x(\hat{\theta}, \theta)$  that

$$u(x(\hat{\theta}, \theta)) \geq u(\tau(\theta F(K) - v_0(\hat{\theta}))) \text{ for all } \hat{\theta} \notin M, \text{ and} \quad (2)$$

$$u(x(\hat{\theta}, \theta)) \geq u(\tau(\theta F(K) - v_1(\hat{\theta}, \theta))) \text{ for all } \hat{\theta} \in M.$$

Given the terms of the contract,  $m$ ,  $v_0$ ,  $v_1$ , and  $x$ , the manager chooses a strategy for reporting  $\theta$  denoted  $\sigma(\theta)$ . We say that the report  $\sigma(\theta) = \hat{\theta}$  is *feasible* given  $v_0$  and  $\theta$  if either  $\hat{\theta} \in M$  or  $\hat{\theta} \notin M$  and  $v_0(\hat{\theta}) \leq \theta F(K)$ . Note that this definition requires that the manager has the resources to make the payment  $v_0(\sigma(\theta))$  in the event that he reports  $\sigma(\theta) = \hat{\theta} \notin M$ . We restrict the manager to choose a reporting strategies such that  $\sigma(\theta)$  is feasible given  $v_0$  for all  $\theta$ . We interpret this constraint as following from the assumption that there is an optimal contract in which the outside investors choose to monitor if the manager announces  $\hat{\theta} \notin M$  but then does not pay  $v_0(\hat{\theta})$  and that  $x(\hat{\theta}, \theta) = 0$  in this event.

We restrict attention to contracts in which the manager truthfully reports  $\theta$ . Hence, we impose the incentive constraint

$$u(x(\theta, \theta)) \geq u(x(\hat{\theta}, \theta)) \text{ for all } \theta \in \Theta \text{ and feasible } \hat{\theta} \text{ given } \theta \text{ and } v_0. \quad (3)$$

The manager's expected utility under the contract is given by the expectation of  $u(x(\theta, \theta))$ . Since managers have an outside opportunity that delivers them utility  $U_0$ , we require the individual rationality constraint

$$\int u(x(\theta, \theta))p(\theta)d\theta \geq U_0. \quad (4)$$

Note that this contracting problem is a partial equilibrium problem in the sense we assume that the outside investors have already purchased this production opportunity from the entrepreneur who created it and now they simply seek to design a contract with the manager that they hire on a competitive market to run this production opportunity. We do not model the costs that entrepreneurs pay to create these production opportunities nor the price that they receive when they sell a newly created production opportunity to outside investors.

### 3. Characterizing an efficient contract

In this section, we characterize a contract that maximizes the expected payoff to the outside investors

$$\int [(\theta - \gamma m(\theta))F(K) - x(\theta, \theta)] p(\theta)d\theta - rK \quad (5)$$

subject to the constraints (1), (2), (3), and (4) in the following two propositions. We refer to such a contract as an efficient contract.

PROPOSITION 1. *There is an efficient contract with the following properties: (i)  $v_1(\hat{\theta}, \theta) = \theta F(K)$  for all  $\hat{\theta} \in M$  and  $v_0(\hat{\theta}) = \theta^* F(K)$  for all  $\hat{\theta} \notin M$ , where  $\theta^* = \inf \{ \hat{\theta} | \hat{\theta} \notin M \}$ , (ii)  $M$  is an interval ranging from 0 to  $\theta^*$ , and (iii) for  $\hat{\theta} \neq \theta$ ,  $x(\hat{\theta}, \theta) = 0$  if  $\hat{\theta} \leq \theta^*$  and  $x(\hat{\theta}, \theta) = \tau(\theta - \theta^*)F(K)$  if  $\theta > \theta^*$ .*

**Proof:** To prove (i), we first consider the case of reports that lead to monitoring. For all  $\hat{\theta} \in M$ , setting  $v_1(\hat{\theta}, \theta) = \theta F(K)$  relaxes the constraint (2) as much as possible and has no effect on the objective (5) nor on any other constraint. Hence we can, without loss of generality, assume that  $v_1$  has this form.

Next, consider the case of reports that don't lead to monitoring. Let  $v_0^* = \inf \{ v_0(\hat{\theta}) | \hat{\theta} \notin M \}$ . Note that for all  $\theta < v_0^*/F(K)$ , it is not feasible for the manager to report  $\hat{\theta} \notin M$  since the manager would be unable to make the payment  $v_0(\hat{\theta}) \geq v_0^*$  in that case. This implies that  $v_0^* \leq \theta F(K)$  for all  $\theta \notin M$  since, otherwise, truthtelling would not be feasible. From (2) and (3), for all  $\theta \geq v_0^*/F(K)$ , we have that

$$x(\theta, \theta) \geq \sup_{\hat{\theta} \notin M} x(\hat{\theta}, \theta) \geq \tau(\theta F(K) - v_0^*), \quad (6)$$

since the manager can, with strategic reporting of  $\hat{\theta}$ , ensure that the payment that he makes to outside investors in the second subperiod is arbitrarily close to  $v_0^*$  and thus ensure that he consumes arbitrarily close to  $\tau(\theta F(K) - v_0^*)$  in perks. Since  $(\theta F(K) - v_0^*) \geq (\theta F(K) - v_0(\hat{\theta}))$  for all  $\hat{\theta} \notin M$ , we can set  $v_0(\hat{\theta})$  equal to a constant, here  $v_0^*$ , without affecting any of the

binding incentive constraints (2). Given a monitoring set  $M$ , setting the constant  $v_0^*$  as high as possible relaxes the constraint (2) as much as possible. Given this, constraint (1) implies that  $v_0^* = \theta^* F(K)$  with  $\theta^* \equiv \inf \left\{ \hat{\theta} \mid \hat{\theta} \notin M \right\}$  is an optimal choice.

To prove that  $M$  is an interval, note that if  $M$  were to contain some  $\tilde{\theta} > \theta^*$ , it would still be the case that, for that  $\tilde{\theta}$ ,  $x(\tilde{\theta}, \tilde{\theta}) \geq \tau (\tilde{\theta} - \theta^*) F(K)$  since the manager has the option of reporting any feasible  $\hat{\theta} \notin M$  that he chooses and hence ensuring himself of consumption arbitrarily close to  $\tau (\tilde{\theta} - \theta^*) F(K)$ . Hence, it is not possible to relax the constraint (2) for  $\theta > \theta^*$  any further by choosing to monitor for that report  $\tilde{\theta}$ . Since it is costly to monitor, an efficient contract must have  $\tilde{\theta} \notin M$  for almost all  $\tilde{\theta} > \theta^*$ . Thus, there is an efficient contract in which  $M$  is an interval ranging from 0 to  $\theta^*$ .

To finish the proof, observe that given the incentive constraint (3), we can, without loss of generality, set  $x(\hat{\theta}, \theta)$  for  $\hat{\theta} \neq \theta$  as low as possible to relax this incentive constraint as much as possible. Hence, it is efficient to set  $x(\hat{\theta}, \theta) = 0$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} \leq \theta^*$ , and it is feasible to do so since, in the event of monitoring, the outside investors take possession of the output of the firm and there are no resources for the manager to spend on perks for himself. From (2) and the results above, we set  $x(\hat{\theta}, \theta) = \tau(\theta - \theta^*)$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} > \theta^*$ . Here it is not possible to set the manager's compensation any lower. Note that by setting  $x(\hat{\theta}, \theta)$  in this way, we ensure that any choice of  $x(\theta, \theta)$  that satisfies nonnegativity for  $\theta \leq \theta^*$  and  $x(\theta, \theta) \geq \tau(\theta - \theta^*)F(K)$  for  $\theta > \theta^*$  also satisfies both the no perks condition (2) and is incentive compatible in that it satisfies (3). *Q.E.D.*

With these results, we can restate our optimal contracting problem much more simply as follows. The problem now is to choose a level of investment  $K$ , a monitoring set indexed by  $\theta^*$ , and payments to the manager  $w(\theta) = x(\theta, \theta)$  to maximize the expected payoff to the

outside investors:

$$\int (\theta F(K) - w(\theta)) p(\theta) d\theta - \gamma P(\theta^*) F(K) - rK, \quad (7)$$

subject to the constraints of individual rationality

$$\int u(w(\theta)) p(\theta) d\theta \geq U_0 \quad (8)$$

and a no-perks constraint

$$u(w(\theta)) \geq u(\tau(\theta - \theta^*) F(K)) \text{ for all } \theta \geq \theta^*. \quad (9)$$

With our assumption that managers' utility is unbounded below, we know that the limited liability constraint  $w(\theta) \geq 0$  is not binding.

Given values of  $\theta^*$ ,  $K$ , and  $w(\theta)$  that solve this problem, the contract with  $M = \{\theta | \theta \leq \theta^*\}$ ,  $v_1(\hat{\theta}, \theta) = \theta F(K)$ ,  $v_0(\hat{\theta}) = \theta^* F(K)$ ,  $x(\theta, \theta) = w(\theta)$ ,  $x(\hat{\theta}, \theta) = 0$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} \leq \theta^*$ , and  $x(\hat{\theta}, \theta) = \tau(\theta - \theta^*) F(K)$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} > \theta^*$  is an efficient contract.

In the following two sections, we use the first order conditions of this simplified contracting problem to characterize the financial structure of this project and the relationship between financial structure and production efficiency. To that end, we consider the Lagrangian associated with the problem (7) given by

$$\max_{w(\theta), K, \theta^*, \lambda, \delta(\theta)} \int (\theta F(K) - w(\theta)) p(\theta) d\theta - \gamma P(\theta^*) F(K) - rK + \quad (10)$$

$$\lambda \left\{ \int u(w(\theta))p(\theta)d\theta - U_0 \right\} + \int \delta(\theta) \{u(w(\theta)) - u(\tau(\theta - \theta^*)F(K))\} p(\theta)d\theta.$$

#### 4. Debt, Equity and Executive Compensation

In this section, we interpret the characteristics of the efficient contract in terms of a contract compensating the manager that consists of a base level of pay and a performance bonus, together with a debt contract and an outside equity contract.

We begin by characterizing managerial compensation under this optimal contract in the following proposition.

**PROPOSITION 2.** *Under an optimal contract, the payments to the manager  $w(\theta)$  have the form  $w(\theta) = \bar{w}$  for  $\theta \leq \bar{\theta}$  and  $w(\theta) = \tau(\theta - \theta^*)F(K)$  for  $\theta > \bar{\theta}$ , where  $\bar{\theta}$  is the solution to  $\bar{w} = \tau(\bar{\theta} - \theta^*)F(K)$ .*

**Proof:** The first-order conditions of (10) with respect to  $w(\theta)$  are given by

$$1 = (\lambda + \delta(\theta)) u'(w(\theta)). \tag{11}$$

This first order condition implies that  $w(\theta)$  is constant for all values of  $\theta$  such that the constraint (9) does not bind ( $\delta(\theta) = 0$ ). We denote this constant by  $\bar{w}$ . Given  $\bar{w}$ , the constraint (9) binds for  $\theta > \bar{\theta}$  and does not bind for  $\theta < \bar{\theta}$ . Clearly,  $w(\theta) = \tau(\theta - \theta^*)F(K)$  when (9) binds. *Q.E.D.*

Note that for those values of  $\theta$  such that the constraint (9) is slack, the first order

condition (11) implies that

$$\lambda = \frac{1}{u'(\bar{w})}. \quad (12)$$

From proposition 2, we have that the payments made to the manager in the third sub-period are given by  $w(\theta) = \bar{w}$  in the event that  $\theta \leq \bar{\theta}$  and  $w(\theta) = \tau(\theta - \theta^*)F(K) = \tau(\theta - \bar{\theta})F(K) + \bar{w}$  in the event that  $\theta > \bar{\theta}$ . We interpret the payment  $\bar{w}$  as the manager's base pay and the additional payment to the manager of  $\tau(\theta - \bar{\theta})F(K)$  in the event that  $\theta > \bar{\theta}$  as the performance component of the manager's pay. For the outside investors, the value of this performance payment to the manager is given by

$$C = \left[ \int_{\bar{\theta}}^{\infty} \tau(\theta - \bar{\theta}) p(\theta) d\theta \right] F(K).$$

Note that the manager places a different value on these payments because he is risk averse. As we discuss below, this wedge between the valuation of this performance payment by the outside investors and the manager plays a role in determining the firm's optimal production plan.

To interpret the other payments under this optimal contract in terms of debt and equity, we must ensure that payments to outside investors after the initial investment in the first sub-period are non-negative so that they do not violate the limited liability constraint imposed on investors in corporations. To do so, we assume that the outside investors invest not only the capital  $K$ , but also the uncontingent portion of the manager's pay  $\bar{w}$  in the first sub-period. We associate the payments  $v_0$  or  $v_1$  made by the manager in the second

sub-period as the payments to debt holders. We associate the residual payments to outside investors as the payments to outside equity.

The payments made in the second sub-period are given by  $v_1(\theta, \theta) = \theta F(K)$  if  $\theta \leq \theta^*$  and  $v_0(\theta) = \theta^* F(K)$  if  $\theta > \theta^*$ . We interpret  $\theta^* F(K)$  as the face value of the project's debt. In the event that the realized value of the project exceeds the face value of the debt, the debt is paid. In the event that the realized value of the project is less than the face value of the debt, the project is bankrupt, monitored, and all remaining value is paid to the debt holders. If one assumes that the debt holders bear the cost of monitoring, the market value of the project's debt is given by

$$D = \left[ \int_0^{\theta^*} \theta p(\theta) d\theta + (1 - P(\theta^*))\theta^* - P(\theta^*)\gamma \right] F(K).$$

Note that under the assumption that the debt holders bear the cost of monitoring, the value of  $D$  can be negative since it is net of the cost of monitoring. Alternatively, one may assume that the outside investors jointly contribute resources  $\gamma F(K)$  in addition to uncontingent payments  $K$  and  $\bar{w}$  in the first sub-period. Under this alternative assumption, the market value of the debt is given by

$$D = \left[ \int_0^{\theta^*} \theta p(\theta) d\theta + (1 - P(\theta^*))\theta^* + (1 - P(\theta^*))\gamma \right] F(K),$$

which is always positive.<sup>4</sup>

The residual payout from the project is associated with the payments to the outside

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<sup>4</sup>This alternative assumption can also help rationalize commitment to deterministic monitoring since the proceeds from monitoring are nonnegative even if  $\theta = 0$ , and are positive for  $\theta > 0$ .

equity holders. In the event of bankruptcy ( $\theta \leq \theta^*$ ), the outside equity holders receive no payment. In the event that  $\theta > \theta^*$  and  $\theta \leq \bar{\theta}$ , the outside equity holders receive payment  $(\theta - \theta^*) F(K)$ , which is the realized value of the project less the payment to the debt holders. (Recall that the base portion of the manager's pay,  $\bar{w}$ , was set aside in advance). In the event that  $\theta > \bar{\theta}$ , the outside equity holders receive  $(\theta - \theta^* - \tau (\theta - \bar{\theta})) F(K)$  which is equal to the realized value of the project less the payments to the debt holders and the payments to the manager on the performance portion of his compensation.

Note that in our model, the performance component of the manager's pay resembles an option on the value of the firm with strike price  $\bar{\theta}F(K)$ , or equivalently an option on the value of the equity of the firm with strike price  $(\bar{\theta} - \theta^*) F(K)$ . This result that the performance component of the manager's pay resembles an option on the firm is driven by our assumption that the agency friction applies to the entire value of the firm — that is, by our assumption that the manager can spend up to fraction  $\tau$  of all of the undisbursed output of the firm at the end of the second sub-period.

More generally, the measure of firm performance upon which the manager's performance pay is based is determined by the extent of the agency friction. To see this, consider a variant of our model in which firm output had two components: current cash flow  $\theta f(K)$  and undepreciated capital  $(1 - \delta)K$ . Assume that the manager is able to spend up to fraction  $\tau$  of undisbursed cash flow on perquisites, but that he cannot divert undepreciated capital for his own use. In this variant of the model, the constraint (2) on payments to the manager

would be modified to read

$$u(x(\hat{\theta}, \theta)) \geq u(\tau(\theta f(K) - v_0(\hat{\theta}))) \text{ for all } \hat{\theta} \notin M, \text{ and}$$

$$u(x(\hat{\theta}, \theta)) \geq u(\tau(\theta f(K) - v_1(\hat{\theta}, \theta))) \text{ for all } \hat{\theta} \in M,$$

and the limited liability constraint (1) would be modified to read

$$v_0(\hat{\theta}) \leq \hat{\theta}f(K) + (1 - \delta)K, \quad v_1(\hat{\theta}, \theta) \leq \theta f(K) + (1 - \delta)K, \text{ and } x(\hat{\theta}, \theta) \geq 0.$$

It is straightforward to show that the optimal contract in this variant of the model would break down into four payments as before, except in this case, the performance pay to the manager would be based on cash flow  $\theta f(K)$  and not on the value of the firm (which includes the value of undepreciated capital). It is also straightforward in this variant of the model to interpret the payments  $v$  backed by undepreciated capital  $(1 - \delta)K$  as payments to collateralized debt. It is worth noting, however, that our model does not have predictions regarding the optimal mix of collateralized and uncollateralized debt.

## 5. Capital Structure, Production and Monitoring

The standard result due to Modigliani and Miller (1958) is that in a frictionless world, the capital structure of a firm has no impact on its efficient production plan. Here we have assumed specific frictions that determine the optimal capital structure of the firm. In this section, we discuss the impact of these frictions of the firm's efficient production plan as characterized in Proposition 3 below. In particular, we show that under the optimal production plan, there is a wedge between the expected marginal product of capital within

the firm and the opportunity cost of capital. Our main result is that the magnitude of this wedge can be measured in terms of observable elements of the firm's capital structure and executive compensation.

We refer to an economy in which the monitoring cost  $\gamma = 0$  as a *frictionless environment*. In such an environment, the optimal contract specifies that the outside investors monitor the output of the project in the second sub-period for all values of  $\hat{\theta}$  and pay the manager constant compensation  $\bar{w}$  independent of the realized value of  $\theta$ . In this frictionless environment, the efficient capital stock satisfies  $F'(K) = r$ . In contrast, with financial frictions, there is a wedge between the marginal product of capital and its rental rate. We characterize this wedge in proposition 3.

**PROPOSITION 3.** *Assume that the support of  $\theta$  is unbounded above. Then, under the optimal contract,  $F'(K) > r$ , where*

$$\left\{ 1 - \gamma P(\theta^*) - \int_{\bar{\theta}}^{\infty} \left[ 1 - \frac{u'[\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \right] \tau(\theta - \theta^*) p(\theta) d\theta \right\} F'(K) = r. \quad (13)$$

**Proof:** The first order-condition of (10) with respect to  $K$  is given by

$$\left[ \int \{ \theta - \delta(\theta) u'(\tau(\theta - \theta^*)F(K)) \tau(\theta - \theta^*) \} p(\theta) d\theta - \gamma P(\theta^*) \right] F'(K) = r. \quad (14)$$

From (11) and (12), we have that

$$\delta(\theta) = \frac{1}{u'(w(\theta))} - \frac{1}{u'(\bar{w})}.$$

With this result and the assumption that the expectation of  $\theta$  is 1, this first order condition for capital simplifies to (13). Note  $\bar{w}$  is finite as long as the manager's reservation utility is finite. Hence the assumption that the support of  $\theta$  is unbounded above implies that  $P(\bar{\theta}) < 1$ .

Thus, since

$$0 < 1 - \frac{u'[\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} < 1 \quad \forall \theta > \bar{\theta},$$

we have that

$$1 - \gamma P(\theta^*) - \int_{\bar{\theta}}^{\infty} \left[ 1 - \frac{u'[\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \right] \tau(\theta - \theta^*) p(\theta) d\theta < 1.$$

Hence,  $F'(K) > r$ . *Q.E.D.*

Note that the assumption that the support of  $\theta$  is unbounded above is sufficient to ensure that  $F'(K) > r$ , but it is not necessary. What is necessary is that either  $P(\theta^*) > 0$  or  $P(\bar{\theta}) < 1$ , so that either there is monitoring or there is not perfect risk sharing.

From (13), one can see that there are two parts to this wedge between  $F'(K)$  and  $r$ . The first part is the expected loss due to monitoring given by  $\gamma P(\theta^*)$ . This loss is a cost of debt since the monitoring that debt requires in the event of bankruptcy results in a loss of output. The second part of the wedge is the loss due to inefficient risk-sharing between the outside investors and the manager that arises as a result of the performance **based** component of the manager's compensation. Specifically, this is the loss due to the fact that the risk averse manager places a lower valuation on the state contingent component of his compensation

than the outside investors do, and this loss is represented by the term

$$\int_{\bar{\theta}}^{\infty} \left[ 1 - \frac{u' [\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \right] \tau(\theta - \theta^*)p(\theta)d\theta.$$

To interpret this term, observe that the outside investors value the manager's option at  $\int_{\bar{\theta}}^{\infty} \tau(\theta - \theta^*)p(\theta)d\theta$ . In contrast, the payment of  $\tau(\theta - \theta^*)$  to the manager in the event that productivity  $\theta > \bar{\theta}$  is realized raises the manager's utility by  $u' [\tau(\theta - \theta^*)F(K)] \tau(\theta - \theta^*)$  and hence relaxes the promise keeping constraint by an amount that is worth only

$$\frac{u' [\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \tau(\theta - \theta^*)$$

to the outside investors.

**Log Preference Example:** If we assume that  $u(c) = \log(c)$ , then the first order condition for capital (13) reduces to

$$\left\{ 1 - \gamma P(\theta^*) - \int_{\bar{\theta}}^{\infty} \tau(\theta - \bar{\theta})p(\theta)d\theta \right\} F'(K) = r. \quad (15)$$

Hence, in this case the wedge between the marginal product of capital and the rental rate on capital is given by one minus the sum of the fraction of expected output ( $F(K)$ ) devoted to the monitoring cost and the fraction of expected output paid to the manager as the option portion of his compensation. Thus, in this example, it is a simple matter to link our model's implications for the impact of financial frictions on efficient production plans to observables.

We turn next to the determination of the optimal extent of monitoring. The first order-condition of (10) with respect to  $\theta^*$  can be written as follows once we substitute for

$\delta(\theta)$  as we did in the capital condition (13),

$$\tau \int_{\bar{\theta}}^{\infty} \left( 1 - \frac{u'[\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \right) p(\theta) d\theta = \gamma p(\theta^*)F(K). \quad (16)$$

This condition implies that under the efficient contract,  $\theta^*$  is determined by a trade-off between the marginal cost of monitoring as captured by the right hand side of the above expression, and the marginal impact of monitoring on the cost of distorting the manager's consumption, as captured by the left hand side of the above expression.

## 6. Risk Hedges and Capital Structure

Financial hedges are contracts that firms enter into in order to insure themselves against certain (typically) exogenous events. We define financial hedges as contracts between the firm and outside investors that require the outside investors, in some states of nature, to pay additional funds into the firm after capital investments have already been financed.

Why do we see firms using financial hedges — both insurance against idiosyncratic risk and options and futures to hedge aggregate risks? In the literature, it has been argued that these financial hedges can be used to avoid risks which can lead to bankruptcy (Smith and Stoltz 1985) or to reduce the risk associated with stochastic cash-flows when external funds are more costly than internal funds (Froot, Scharfstein and Stein 1993).<sup>5</sup>

We do not find support for these arguments in our model. While our optimal contracting problem is sufficiently general to allow the firm to hedge risks, our results indicate that there is an efficient contract without such hedges. We find that financial hedges do not

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<sup>5</sup>Acharya and Bisin (2005) have recently argued the hedges can be use to reduce the incentive of risk averse managers to skew investment choices towards projects with aggregate risk that they can more readily offset in their private portfolios than idiosyncratic risk.

add value in our basic model despite the fact that the risk that the firm faces in our model, as indexed by  $\theta$ , can be interpreted to include the types of losses that firms typically insure against, and despite the fact that we have shown that this leads to a wedge between the internal and the external rate of return.

We do find, however, that if we augment our model to include an informative public signal of the firm's profitability in the second sub-period, then there is an efficient contract that is implemented with debt, outside equity, and a financial hedge contingent on this public signal. In this sense, our augmented model provides a novel theory of financial hedging. In this theory, firms include financial hedges as part of their capital structure to implement an optimal monitoring strategy contingent on publicly observed information.

The result that financial hedges do not add value in our basic model emerges because the debt and equity contracts have been optimally chosen to offset the enforcement and incentive problems the outside investors face with respect to the manager. We see, in particular, that in the second sub-period, the outside investors want to extract from the firm as large a payment as is possible given the choice of monitoring. Additional funds paid into the firm at this point would only exacerbate the agency friction as modelled by the no-perks constraint (9). Nor is there an incentive for the outside investors to contribute further resources in the third sub-period. The outside investors are simply the residual claimant on the firm.

Here we show that when we augment the model to include an informative public signal of the firm's profitability in the second sub-period, the efficient financial contract including the second sub-period payment and the monitoring decision is contingent on this signal. We show that there is an interpretation of the efficient contract as a combination of simple debt and equity contracts along with a financial hedge contract. However, the purpose of the

hedge is not be to remove or reduce the risk of bankruptcy with simple debt contracts, but rather to fine tune it to allow the monitoring associated with bankruptcy to be undertaken in the optimal state-contingent fashion.

Consider a version of our model in which, in the second sub-period, outside investors observe a public signal  $\phi$  that is correlated with  $\theta$ . One might interpret this signal as a publicly verifiable event, such as a fire or flood in the case of firm-specific risks or a change in exchange rates or commodity prices in terms of aggregate risks. We retain the assumption that the realization of  $\theta$  remains the private information of the manager, but we now allow the outside investors to make the terms of the contract,  $M$ ,  $v_0, v_1$ , and  $x$  contingent on  $\phi$ . (Note that  $K$  cannot be contingent on  $\phi$  because it is chosen in the first sub-period before  $\phi$  is realized.) Assume that the unconditional distribution of this signal has density  $g(\phi)$  and the distribution of  $\theta$  conditional on  $\phi$  has density  $p(\theta|\phi)$ . It is straightforward to show, in this modified environment, that our simplified contracting problem can be written as one of choosing a monitoring strategy as indexed by  $\theta^*(\phi)$  and pay for the manager  $w(\theta, \phi)$  to maximize

$$\int \int (\theta F(K) - w(\theta, \phi)) p(\theta|\phi) g(\phi) d\theta d\phi - \gamma \int P(\theta^*(\phi)|\phi) g(\phi) d\phi F(K) - rK$$

subject to a constraint on the manager's reservation utility

$$\int \int u(w(\theta, \phi)) p(\theta|\phi) g(\phi) d\theta d\phi = U_0$$

and the no-perks constraint

$$u(w(\theta, \phi)) \geq u(\tau(\theta - \theta^*(\phi))F(K)) \text{ for all } \theta, \phi.$$

The optimal compensation for the manager again has base pay  $\bar{w}$  (independent of  $\phi$ ) when the no-perks constraint does not bind. Otherwise, this pay is given by  $w(\theta, \phi) = \tau(\theta - \theta^*(\phi))F(K)$ . The optimal monitoring strategy derived from this problem is governed by a first order condition similar to (16) given by

$$\tau \int_{\bar{\theta}(\phi)}^{\infty} \left( 1 - \frac{u'[\tau(\theta - \theta^*(\phi))F(K)]}{u'(\bar{w})} \right) p(\theta|\phi) d\theta = \gamma p(\theta^*(\phi)|\phi), \quad (17)$$

where  $\bar{\theta}(\phi) = \bar{w}/\tau + \theta^*(\phi)$ . Payments in the second sub-period are then given by  $v_0(\hat{\theta}, \phi) = \theta^*(\phi)F(K)$  for  $\hat{\theta} \geq \theta^*(\phi)$  in the event that there is no monitoring and  $v_1(\hat{\theta}, \theta, \phi) = \theta F(K)$  for  $\hat{\theta} < \theta^*(\phi)$ .

From (17), we see that the optimal monitoring strategy and the payment  $v_0(\hat{\theta}, \phi)$  in the second sub-period depend on  $\phi$  in a non-trivial manner. This optimal monitoring strategy and set of payments in the second sub-period can be implemented with debt that is uncontingent on the public signal  $\phi$  and a hedge against fluctuations in  $\phi$  as follows. Designate an uncontingent payment  $\theta^D F(K)$  to be the principal and interest due on the debt. Designate the difference  $(\theta^*(\phi) - \theta^D) F(K)$  as the payment due from the firm to the outside investors on the financial hedge. With this implementation of the optimal contract, the manager will be able to pay both the hedge and the debt if and only if  $\theta \geq \theta^*(\phi)$ . Outside investors contribute additional funds into the firm whenever a  $\phi$  is realized such that  $\theta^*(\phi) < \theta^D$ . These additional

funds will be paid immediately to the holders of the debt. This hedge can be interpreted, then, as a hedge taken on to reduce the costs of financial distress associated with bankruptcy. (Note that there are realizations of  $\theta$  such that it is not possible for the firm to pay both its uncontingent debt and its financial hedge. For such values of  $\theta$ , we interpret the firm as defaulting on both its debt and its hedge contracts.)

Note that the hedge component of the optimal financial contract is contingent only on the publicly observed variable  $\phi$  (outside of the possibility that the firm might be monitored and default on its hedge payment). In this sense, the financial hedges in our model are similar the use of formal insurance contracts against firm-specific risks or financial futures, forwards and options on aggregate risks such as currency or commodity prices.

Under certain regularity conditions, we can derive implications for how the optimal hedge payment  $(\theta^*(\phi) - \theta^D) F(K)$  varies with  $\phi$ . Consider, for example, a firm for which  $\theta$  is normally distributed with mean  $\phi$  and a constant variance. Assume as well that the optimal financial structure has  $\theta^*(\phi)$  below  $\phi$  (the mean of  $\theta$ ). Under these assumptions,  $\theta^*(\phi)$ , and hence the hedge payments  $(\theta^*(\phi) - \theta^D) F(K)$  are an increasing function of  $\phi$ . In this sense, the firm uses the financial hedge to smooth its profits. To see this, observe that the left hand side of (17) is decreasing in  $\theta^*$  and increasing in  $\phi$  while the right hand side of (17) is increasing in  $\theta^*$  and decreasing in  $\phi$ . Under these assumptions, the firm's optimal capital structure includes a financial hedge against  $\phi$ , the observable signal of the firm's expected profitability.

## 7. The Dynamic Contracting Problem

We now consider the optimal contracting problem in a dynamic extension of our one period model. In this section we show that the optimal dynamic contract is quite similar to the optimal static contract in that it can be broken down into four payments: base pay and performance pay for the manager, debt, and outside equity. We then show that the manager's pay is non-decreasing over time. To begin, we keep our problem simple by assuming that all managers are equally productive in running the firm. In a later section, we then introduce the possibility that the incumbent manager in the firm may, at random, become unproductive at running the firm. With this further extension of the model, we show how a “golden parachute” forms part of the optimal package of executive compensation. Finally, we discuss several issues concerning the interpretation of an optimal contract in this dynamic setting as a theory of the capital structure of the firm.

The extension of our one period model to a dynamic setting is as follows. Each period consists of three sub-periods as described in our one-period model. In the first sub-period, the outside investors hire a manager to run the project and put forward capital  $K$ . In the second sub-period, the current productivity shock  $\theta$ , and hence current output  $y = \theta F(K)$ , is realized, and these values are observed only by the manager. The manager also makes some payment  $v$  to the outside investors in this second sub-period. At the end of the second sub-period, the manager has the option of investing up to fraction  $\tau$  of the remaining output of the firm into perks that he consumes and otherwise he reinvests the remaining output of the firm at gross rate of return one. In the third sub-period, the realized value of the shock  $\theta$  becomes public information, as well as the manager's division of the firm's output between perks and productive reinvestment. The manager is paid  $x$  in this third sub-period.

This production process is then repeated in subsequent periods. We interpret this cycle of information about production as corresponding to an accounting cycle or a capital budgeting cycle within the firm.

We assume that all managers not running a project have an outside opportunity to enjoy consumption  $\bar{c}$  each period. Corresponding to this constant consumption flow is a reservation expected discounted utility level  $U_0$ . Individual rationality requires that new managers can expect utility of at least  $U_0$  under a contract and that incumbent managers can expect a utility of at least  $U_0$  in the continuation of any contract.

We present a recursive characterization of the optimal dynamic contract. While we allow for persistence in the productivity shocks hitting the firm, because there is complete resolution of uncertainty at the end of each period, the persistence of the shocks does not generate any informational incompleteness in the model. Accordingly, we assume that the outside investor's contract with the incumbent manager is indexed by a utility level  $U$  promised him from this period forward and the prior realization of the productivity shock  $\theta_{-1}$ . This utility level is a contractual state variable carried over from the previous period and hence is determined before the realization of the productivity shock  $\theta$ . We let  $V(U, \theta_{-1})$  denote the expected discounted value of payments to outside investors given utility promise of  $U$  to the incumbent manager and the prior shock  $\theta_{-1}$ . We assume that the p.d.f. and c.d.f. for  $\theta$  in are given by  $p(\theta_t; \theta_{t-1})$  and  $P(\theta_t; \theta_{t-1})$  respectively.

Because the distribution of current productivity shocks depends upon the prior period's shocks, the conditional mean of the shocks will not be 1, as in the simple one period model. Just as before, we assume that monitoring costs are proportional to the expected size, which now are given by  $\gamma E(\theta | \theta_{-1}) F(K)$ .

A dynamic contract has the following elements. Given the utility  $U$  promised to the incumbent manager as a state variable, the contract specifies a monitoring region  $M(U, \theta_{-1})$  with indicator function  $m(\hat{\theta}; U, \theta_{-1})$  indicating the monitoring decision as a function of the manager's report, payments from the manager to the outside investors in the second sub-period  $v_0(\hat{\theta}; U, \theta_{-1})$  if there is no monitoring and  $v_1(\hat{\theta}, \theta; U, \theta_{-1})$  if there is monitoring, and payments from the outside investors to the manager in the third subperiod  $x(\hat{\theta}, \theta; U, \theta_{-1})$ . The recursive representation of the contract also specifies continuation utilities  $Z(\hat{\theta}, \theta; U, \theta_{-1})$  for the incumbent manager. In what follows, we suppress reference to  $U$  and  $\theta_{-1}$  where there is no risk of confusion.

These terms of the contract are chosen subject to the limited liability constraints (1). Since the incumbent manager can always quit and take his outside opportunity in the next period, we have an individual rationality constraint

$$Z(\hat{\theta}, \theta) \geq U_0 \text{ for all } \hat{\theta}, \theta \tag{18}$$

We require that the contract deliver the promised utility  $U$  to the incumbent manager

$$\int [u(x(\theta, \theta)) + \beta Z(\theta, \theta)] p(\theta; \theta_{-1}) d\theta = U. \tag{19}$$

The incumbent manager must be induced to truthfully report  $\theta$  in the second sub-period. Hence we have incentive constraints, for all  $\theta$  and  $\hat{\theta} \notin M$  such that  $\hat{\theta}$  is feasible in that

$$\theta \geq v_0(\hat{\theta})/F(K),$$

$$u(x(\theta, \theta)) + \beta Z(\theta, \theta) \geq u(x(\hat{\theta}, \theta)) + \beta Z(\hat{\theta}, \theta). \quad (20)$$

Finally, there is a dynamic analog to the no-perks constraint (2) arising from the assumption that the manager can spend fraction  $\tau$  of whatever resources are left in the project at the end of the second sub-period on perks that he enjoys. This constraint is given by

$$u(x(\hat{\theta}, \theta)) + \beta Z(\hat{\theta}, \theta) \geq u(\tau(\theta F(K) - v_1(\hat{\theta}, \theta))) + \beta U_0 \text{ if } \hat{\theta} \in M \quad (21)$$

$$u(x(\hat{\theta}, \theta)) + \beta Z(\hat{\theta}, \theta) \geq u(\tau(\theta F(K) - v_0(\hat{\theta}))) + \beta U_0 \text{ if } \hat{\theta} \notin M$$

for all  $\theta$  and for all  $\hat{\theta} \notin M$  such that  $v_0(\hat{\theta}) \leq \theta F(K)$ . Here we have used the requirement that the manager's continuation utility  $Z(\hat{\theta}, \theta)$  cannot be driven down below  $U_0$  in the left-hand side of (21) to compute the manager's utility in the event that he invests in perks and then is fired as a consequence.

The terms of the dynamic contract are chosen to maximize the expected discounted payments to the outside investors. This problem is to choose  $K$ ,  $m(\hat{\theta})$ ,  $v_0(\hat{\theta})$ ,  $v_1(\hat{\theta}, \theta)$ ,  $x(\hat{\theta}, \theta)$ , and  $Z(\hat{\theta}, \theta)$  to maximize

$$V(U, \theta_{-1}) = \max \int \left\{ (\theta - \gamma m(\theta)) E(\theta | \theta_{-1}) F(K) - x(\theta, \theta) + \frac{1}{R} V(Z(\theta, \theta)) \right\} p(\theta; \theta_{-1}) d\theta - rK \quad (22)$$

subject to the constraints (1), (18), (19), (20), and (21).

In the remainder of this section, we characterize elements of an efficient dynamic contract. In proposition 4, we show that the optimal dynamic contract is similar to the optimal static contract in that the monitoring set is an interval from 0 to  $\theta^*$ , and payments in the second sub-period are given by  $v_1(\hat{\theta}, \theta) = \theta F(K)$  if there is monitoring and  $v_0(\hat{\theta}) = \theta^* F(K)$  where  $\theta^* \equiv \inf \{ \hat{\theta} | \hat{\theta} \notin M \}$  if there is no monitoring. The line of argument here is similar to that in proposition 1.

In proposition 5, we discuss the manager's pay and retention. We show that the manager's pay consists again of a constant base pay level denoted  $\bar{w}$  and then an increasing portion of pay if a sufficiently high realization of  $\theta$  occurs. The line of argument on compensation is similar to that for Proposition 2. We also show that, under the optimal dynamic contract, if  $\beta \geq 1/R$ , then the pay to the incumbent manager is non-decreasing over time.

**PROPOSITION 4.** *There is an efficient contract with the following properties: (i)  $v_1(\hat{\theta}, \theta) = \theta F(K)$  for all  $\hat{\theta} \in M$  and  $v_0(\hat{\theta}) = \theta^* F(K)$ , where  $\theta^* \equiv \inf \{ \hat{\theta} | \hat{\theta} \notin M \}$ , (ii)  $M$  is an interval ranging from 0 to  $\theta^*$ , and (iii)  $x(\hat{\theta}, \theta) = 0$  and  $Z(\hat{\theta}, \theta) = U_0$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} \in M$  and  $x(\hat{\theta}, \theta) = \tau(\theta - \theta^*)F(K)$  and  $Z(\hat{\theta}, \theta) = U_0$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} \notin M$ .*

**Proof:** The proof here is quite similar to the proof of proposition 1. For all  $\hat{\theta} \in M$ , setting  $v_1(\hat{\theta}, \theta) = \theta F(K)$  relaxes the constraint (21) as much as possible and has no effect on the objective (22) nor on any other constraint. Again define  $v_0^* = \inf \{ v_0(\hat{\theta}) | \hat{\theta} \notin M \}$  and  $\theta^* = \inf \{ \theta | \theta \notin M \}$ . Observe that to relax the constraint (20) as much as possible, the manager's utility following a misreporting of  $\hat{\theta} \neq \theta$  should be set as low as possible. Given (1), (18),

and (21), this gives  $x(\hat{\theta}, \theta) = 0$ ,  $Z(\hat{\theta}, \theta) = U_0$  for  $\hat{\theta} \neq \theta$  and  $\hat{\theta} \in M$ , and

$$u(x(\hat{\theta}, \theta)) + \beta Z(\hat{\theta}, \theta) = u(\tau(\theta F(K) - v_0^*)) + \beta U_0 \quad (23)$$

for  $\hat{\theta} \notin M$ , and  $\theta \geq v_0^*/F(K)$ . Again, holding fixed the monitoring set, setting  $v_0^*$  as high as is feasible relaxes this constraint as much as possible. Since feasibility requires that  $\theta^* F(K) \geq v_0^*$ , this gives us that under an optimal contract,  $v_0^* = \theta^* F(K)$ . That  $M$  is an interval follows from the argument that including some  $\theta > \theta^*$  in the monitoring set does nothing to relax (23) and does require resources for monitoring. That  $x(\hat{\theta}, \theta) = \tau(\theta - \theta^*)F(K)$  for  $\hat{\theta} \neq \theta$ ,  $\hat{\theta} \notin M$  follows from the result that  $v_0^* = \theta^* F(K)$ . *Q.E.D.*

With this proposition, we can write our optimal contracting problem more simply as one of choosing capital  $K$ , the upper support of the monitoring set  $\theta^*$ , current managerial pay  $w(\theta) = x(\theta, \theta)$ , and continuation values  $W(\theta) = Z(\theta, \theta)$  to maximize the payoff to the outside investors

$$V(U, \theta_{-1}) = \max \int \left\{ \theta F(K) - w(\theta) + \frac{1}{R} V(W(\theta)) \right\} p(\theta; \theta_{-1}) d\theta \quad (24)$$

$$- \gamma P(\theta^*; \theta_{-1}) E(\theta | \theta_{-1}) F(K) - rK \quad (25)$$

subject to the promise-keeping constraint

$$\int [u(w(\theta)) + \beta W(\theta)] p(\theta; \theta_{-1}) d\theta = U \quad (26)$$

and the dynamic no-perks constraint

$$u(w(\theta)) + \beta W(\theta) \geq u(\tau(\theta - \theta^*)F(K)) + \beta U_0. \quad (27)$$

We then have the following proposition.

**PROPOSITION 5.** *If  $\theta$  is i.i.d. there is an optimal contract under which there is a cutoff  $\bar{\theta}$  together with a level of base pay  $\bar{w}$  for the manager such that  $w(\theta) = \bar{w}$  and the continuation values  $W(\theta)$  are also constant at  $\bar{W}$  for  $\theta \leq \bar{\theta}$ . Both  $w(\theta)$  and  $W(\theta)$  are increasing for  $\theta > \bar{\theta}$ . If  $\beta R \geq 1$ , then managerial pay  $w(\theta)$  is non-decreasing over time.*

**Proof:** The proof is quite similar to that of proposition 2 and follows from the first order conditions of the optimal contracting problem. Let  $\lambda$  be the Lagrangian multiplier on the constraint (26) and  $\delta(\theta)p(\theta)$  the Lagrangian multipliers on the constraints (27). The first order conditions of this problem with respect to  $w(\theta)$  and  $W(\theta)$  are

$$1 = (\lambda + \delta(\theta)) u'(w(\theta)),$$

and

$$-\frac{1}{R} V_1(W(\theta), \theta) = \beta(\lambda + \delta(\theta)),$$

except that in this case,  $V_1$  is independent of  $\theta$ . These first order condition imply that  $w(\theta)$  and  $W(\theta)$  are constant unless the constraint (27) binds. Note that since  $u(\tau(\theta - \theta^*)F(K)) + \beta U_0$  is increasing in  $\theta$ , there is a cutoff  $\bar{\theta}$  such that this constraint does not bind for  $\theta \leq \bar{\theta}$  and

binds for  $\theta > \bar{\theta}$ . When the constraint (27) binds, the optimal choices of  $w(\theta)$  and  $W(\theta)$  satisfy

$$1 = -\frac{1}{\beta R} V_1(W(\theta), \theta) u'(w(\theta)) \quad (28)$$

and (27) as an equality. Hence,  $w(\theta)$  and  $W(\theta)$  are both increasing in  $\theta$  when this constraint binds. To prove that if  $\beta R \geq 1$ , then  $w(\theta)$  is non-decreasing over time, we use the envelope condition  $-V'(U) = \lambda$ . Plugging this into the first order conditions for  $w(\theta)$  and  $W(\theta)$  and using  $\beta R \geq 1$  gives

$$u'(w(\theta))/u'(\bar{w}') = \beta R \geq 1 \quad (29)$$

where  $\bar{w}'$  and  $F(K')$  are next period's values of these variables. Hence  $\bar{w}' \geq w(\theta)$ , which gives our result. *Q.E.D.*

In our dynamic model, high realizations of  $\theta > \bar{\theta}$  in the current period lead to the no-perks constraint binding, and raise both current compensation and the future promised utility for next period. In this case  $w(\theta)$  and  $W(\theta)$  are chosen to satisfy the current period no-perks constraint (27) and (28). However, this optimal spreading of performance bonuses forward in time leads to some significant differences between optimal compensation in the dynamic model as opposed to the one-period model. Unlike the one-period model where  $w(\theta)$  is linearly increasing in  $(\theta - \theta^*)$  for all  $\theta > \bar{\theta}$ , here,  $w(\theta)$  increases less than linearly since  $u'(\tau(\theta - \theta^*)F(K)) < u'(w(\theta))$  for all  $\theta > \bar{\theta}$ . Moreover,  $w(\theta)$  is likely to become less and less sensitive to  $\theta$  as  $\theta$  gets large since  $u'(\tau(\theta - \theta^*)F(K))$  will be shrinking relative to  $u'(w(\theta))$ .

The manager's compensation is constant at  $w(\theta) = \bar{w}$  and  $W(\theta) = \bar{W}$  for low realiza-

tions of the productivity shock  $\theta \leq \bar{\theta}$  for which the no-perks constraint doesn't bind. The pair  $(\bar{w}, \bar{W})$  are chosen to satisfy (28) and the promise-keeping constraint (26). With respect to the promise-keeping constraint, note that whenever  $\theta < \bar{\theta}$ , the ex post payoff of the agent is given by  $u(\bar{w}) + \beta\bar{W}$ , and whenever  $\theta > \bar{\theta}$  it is given by  $u(\tau(\theta - \theta^*)F(K)) + \beta U_0$ , while  $\bar{\theta}$  is such that these two schedules coincide. Hence  $(\bar{w}, \bar{W})$  determines a floor on the manager's ex post payoff, and this floor is chosen so as to satisfy his promise-keeping constraint.

If  $\beta R = 1$ , and since  $V'(U) = 1/u'(\bar{w})$ , then tomorrow's base wage  $\bar{w}'$  is equal to today's wage  $w(\theta)$  from (29), and  $U = \bar{W}$ . Hence, if  $\theta < \bar{\theta}$ , the only change in the capital structure of the firm will be coming through the persistence of  $\theta$ .

Our dynamic model the first order conditions governing the choice of the capital stock and optimal monitoring are essentially unchanged from the one-period version of the model (see the appendix).

The one technical requirement needed to extend the results in proposition 6 is that  $d^2V(U, \theta_{-1})/dUd\theta_{-1} \leq 0$ . This ensures that if

$$-\frac{1}{R}V'(\bar{W}(\bar{\theta}), \bar{\theta}) = \beta\lambda, \tag{30}$$

then  $\bar{W}(\theta)$  is weakly decreasing in  $\theta$ , and hence, if the no-perks constraint (27) just binds at  $\bar{\theta}$ , it binds at all  $\theta > \bar{\theta}$ , and doesn't bind for all  $\theta < \bar{\theta}$ . Constructing conditions on primitives to ensure this condition is satisfied is much easier in a two period model, which we develop below in the comparative statics section, where we explore the implications of our model further in the next section.

## 8. Comparative Statics

Our dynamic model delivers a dynamic theory of the capital structure of the firm. In particular, the capital structure of the firm is predicted to change in response to the evolution of executive compensation and the investment opportunities of the firm. We explore these implications of this theory in greater detail here. We show that a two-period version of our model has two dynamic implications that are consistent with Myers' (1984) "pecking-order" theory of the dynamics of capital structure. First, holding fixed the firm's investment opportunities, the more profitable is the firm in the first period, the lower is the debt in the second period. Second, holding fixed the firm's profitability in the first period, the greater the firm's investment opportunity in the second period, the higher is its debt in that period.

To begin, we consider several special cases of our one period model and show how the terms of the optimal contract for financing the firm depend upon the parameters governing the severity of the agency problem,  $\tau$ , and the costs of monitoring the output of the firm,  $\gamma$ , the compensation due the manager,  $U$ , as well as the size of the project as measured by  $F(K)$ . We do these comparative statics under the assumptions that the manager's utility is CRRA and that the distribution of shocks  $\theta$  is uniform. In much of what we do, we assume that the optimal choice of capital  $K$  is fixed, however, we will derive some results on  $K$  for the risk neutral case.

We then apply these comparative static results developed in the one-period version of the model to study the dynamics of capital structure in a two period version of our model. We show how the debt of the firm in the second period responds to shocks to firm profitability in the first period and to shocks to the firm's investment opportunities in the second period.

## A. Static Case 1: Capital Fixed

With  $K$  fixed, the contract terms that vary are base pay  $\bar{w}$ , the set of states  $\theta < \theta^*$  for which monitoring occurs, and the set of states  $\theta > \bar{\theta}$  for which the manager receives a performance bonus. These contract terms maximize

$$\max_{\bar{w}, \bar{\theta}, \theta^*} F(K) \int_0^{\infty} \theta p(\theta) d\theta - \bar{w} P(\bar{\theta}) - \tau F(K) \int_{\bar{\theta}}^{\infty} (\theta - \theta^*) p(\theta) d\theta - \gamma P(\theta^*) F(K) - rK \quad (31)$$

subject to the constraints that

$$u(\bar{w}) P(\bar{\theta}) + \int_{\bar{\theta}}^{\infty} u(\tau(\theta - \theta^*) F(K)) p(\theta) d\theta = U_0 \quad (32)$$

and

$$\bar{w} = \tau(\bar{\theta} - \theta^*) F(K) \quad (33)$$

PROPOSITION 6. *Assume that  $u(c) = (c^{1-\phi} - 1)/(1 - \phi)$ ,  $\theta$  is uniformly distributed, and  $F(K) = \min\{K, 1\}$ . If  $\bar{w}$  and  $\theta^*$  are an interior solution of the problem (31) and the optimal choice of  $K$  is fixed at 1, then*

1. *monitoring  $\theta^*$  and the bonus cutoff  $\bar{\theta}$  are increasing in the perks parameter  $\tau$ . The effect on base pay  $\bar{w}$  is ambiguous.*
2. *base pay  $\bar{w}$ , monitoring  $\theta^*$  and the bonus cutoff  $\bar{\theta}$  are decreasing in the monitoring cost  $\gamma$ ,*
3. *base pay  $\bar{w}$  is increasing and monitoring  $\theta^*$  and the bonus cutoff  $\bar{\theta}$  are decreasing in the*

reservation utility  $U_0$ .

**Proof:** We proof this proposition by differentiating the first order conditions determining the optimal choice of contract terms. Details are given in the appendix.

Since the value of the debt net of monitoring costs is given by

$$D = \left[ \int_0^{\theta^*} \theta p(\theta) d\theta + (1 - P(\theta^*))\theta^* \right] F(K),$$

it is increasing in  $\theta^*$ . This implies that under the optimal contract, the expected value of the debt payment (net of monitoring costs) is increasing in the perks parameter  $\tau$ , decreasing in the monitoring cost parameter  $\gamma$ , and decreasing in the reservation utility  $U_0$ .

## B. Static Case 2: Risk Neutrality

In the risk neutral case where  $\phi = 0$ , the optimal choice of  $\bar{w} = 0$  if the no-perks constraint ever binds. The analytics of the risk neutral case are complicated by the fact that it is possible for the no-perks condition not to bind for high enough levels of the utility promise to the manager even though  $\theta$  has unbounded upward support. With risk aversion the no-perks case always binds with unbounded support for  $\theta$ , in particular, for any  $\theta$  such that  $p(\theta) > 0$  and  $\tau\theta F(K) > \bar{c}$ . We characterize the nonbinding and binding cases respectively in the following two propositions for the risk-neutral case.

PROPOSITION 7. *If  $u(c) = c$  and if with  $K = F'^{-1}(r)$ ,*

$$U_0 \geq \tau F(K), \tag{34}$$

capital is first-best,  $\theta^* = 0$ , and  $\bar{w}$  is set to satisfy the utility condition.

**Proof:** See the Appendix.

PROPOSITION 8. *If (34) doesn't hold, then  $\bar{w} = 0$ , and*

1. *increases in  $\tau$  will lower  $K$  and raise  $\theta^*$ ,*
2. *increases in  $\gamma$  will lower  $K$  and  $\theta^*$ ,*
3. *reductions in  $\bar{c}$  will lower  $K$  and raise  $\theta^*$ .*

**Proof:** See the Appendix.

Note here that when  $\bar{w} = 0$ , then all compensation of the manager comes in the form of a performance based payment. Increases in either  $\theta^*$  or  $K$  raise the value of the expected debt payment of the firm, and increases in  $\theta^*$  raise the debt-to-value ratio of the firm.

### C. Dynamic Case 1: Shocks to profitability

Next we consider a two period version of our model. As before, the manager's utility is CRRA. We also assume that the shocks are i.i.d. over time. This assumption implies that the only source of dynamics in the model is the response of the second period outcomes to the first period shock via the response in promised utility for the second period. We use a subscript 1 to denote the first period and 2 to denote the second period.

The structure of the analysis is very simple in light of our earlier results. The optimal contract has a recursive structure in promised utility. Hence, we solve the second period problem for the efficient combinations of  $[\bar{w}_2(U), \theta_2^*(U), \bar{\theta}_2(U)]$  just as in the static one period model. We have shown that  $\bar{w}_2$  is increasing in  $U$ , while  $\theta_2^*$  and  $\bar{\theta}_2$  are decreasing. Second

period  $U_2$  is the first period continuation utility  $W_1(\theta_1)$ . And, we have already shown that

$$u'(w_1(\theta_1))u'(\bar{w}_2(W(\theta_1))) = \beta R,$$

and that  $W(\theta)$  is weakly increasing in  $\theta$ , strictly increasing for all  $\theta > \bar{\theta}$ , and constant for all  $\theta \leq \bar{\theta}$ . Hence, we can conclude that

PROPOSITION 9. *In a two period version of our static case I, where the distribution of shocks  $\theta$  is uniform, and the optimal choice of capital  $K$  is fixed. with i.i.d. shocks,*

1. *The terms of the second period contract do not vary across first period shocks  $\theta_1 \leq \bar{\theta}_1$ .*
2. *For all  $\theta_1 > \bar{\theta}_1$ , the second period base pay  $\bar{w}_2$  is increasing in  $\theta_1$ . The second period monitoring probability  $\theta_2^*$  is decreasing in  $\theta_1$ , and hence the so is the expected value of the second period debt payment net of monitoring costs. The second period bound at which the performance bonus kicks in,  $\bar{\theta}_2$ , is also decreasing in  $\theta_1$ .*

PROPOSITION 10. *In a two period version of our model with a risk-neutral manager (in which  $\phi = 0$ ), if (34) does not hold, then  $\bar{w}_2 = 0$ , and increases in  $\theta_1 > \bar{\theta}_1$  will raise  $K$  and lower  $\theta^*$ .*

We interpret these findings as implying that temporary productivity shocks raise profits today, and have persistent effects on capital structure through their effect on the agency frictions within the firm. If the productivity shock is large enough to induce a performance bonus for the manager, then it leads to an increase in his future compensation which relaxes the agency frictions within the firm, and induces a reduction in both the overall debt load and the riskiness of that debt.

## D. Dynamic Case 2: Shocks to profitability and future investment opportunities

Finally, we consider a two period version of our first static case under the assumption that the productivity shocks are persistent and hence alter the firm's investment opportunity in the second period. We simplify the form in which first period shocks effect second period output by assuming that output in the second period is given by

$$A(\theta_1)\theta_2 F(K_2),$$

where  $A(\theta_1)$  is an increasing function meant to capture the effects of a persistent effect of a first period productivity shock, and, as in the static model, we assume that  $E\{\theta_2\} = 1$ . Let  $\tilde{w}$  be the fraction of expected output going to the manger in terms of base pay

$$\tilde{w}A(\theta_1)F(K_2) = \bar{w}.$$

Then, because of our assumption of CRRA preferences, the second period promise-keeping, and the optimal monitoring first order conditions take the following simple forms.

$$u(\tilde{w})P(\bar{\theta}) + \int_{\bar{\theta}}^{\infty} u(\tau(\theta - \theta^*))p(\theta)d\theta = \frac{U}{[A(\theta_1)F(K_2)]^\phi},$$

and

$$\tau \int_{\bar{\theta}}^{\infty} \left(1 - \frac{u'[\tau(\theta - \theta^*)]}{u'(\tilde{w})}\right) p(\theta)d\theta = \gamma p(\theta^*).$$

Since this is exactly the same mathematical structure, modulo the slight change in the denominator of the r.h.s. of the promise-keeping constraint, that we had before, we can conclude that the impact of a first period shock  $\theta_1$  depends upon the ratio  $W_1(\theta_1)/A(\theta_1)^\phi$ . If this ratio falls as  $\theta_1$  rises, this is equivalent to a reduction in promised utility relative to the project size, which leads to a fall in the fraction of expected output  $\tilde{w}$  being paid to the manager in the form of base pay. It also leads to a rise in the second period monitoring threshold  $\theta_2^*$  and hence a rise in the expected value of the second period debt payment net of monitoring costs. Lastly, it leads to a rise in the second period bound at which the performance bonus kicks in  $\bar{\theta}_2$ . Note that these results are reinforced if we assume that  $K_2$  also increases with  $\theta_1$ .

We interpret these findings as implying that a large positive and persistent productivity shock today generates positive profits today, and if it leads to a sufficiently large increase in the size of the firm tomorrow, it also leads to an increase in the debt load of the firm and an increase in the riskiness of that debt. With respect to compensation, the threshold for the performance bonus and base pay both rise.

## 9. Retention and Golden Parachutes

Thus far, our dynamic model has had the feature that in equilibrium the manager is never fired. We now extend our dynamic model to include a decision about whether to retain the manager. This decision is contingent on the realization of an observable shock that affects the productivity of the incumbent manager in running the firm. Outside investors have an incentive to retain incumbent managers with high productivity and replace those with low productivity. We show that a golden parachute type payment to an incumbent manager who becomes unproductive and is fired is part of the optimal compensation scheme. We also show

that the optimal policy with respect to retention is to employ a cut-off rule in which the manager is retained if his productivity is above the cut-off and he is fired if it is below the cut-off. This cut-off is decreasing in the manager's promised utility.

We consider a two period version of our model and for simplicity, we assume that the  $\eta$  shock only occurs during the second period. In particular, we assume that for an incumbent manager  $\eta$  is drawn according to a p.d.f.  $h(\eta)$  and a c.d.f  $H(\eta)$ . We also assume that a new manager has  $\eta_0 = 1$  for sure.

The payoff in the second period from having a manager who was promised ex ante utility  $U$ ,  $V^2(U)$ , is given by the solution to the maximization problem with respect to the choice of retention in the second period. This payoff can be expressed as

$$V^2(U) = \max_{\delta(\eta) \in [0,1], U(\eta)} \int_{\eta} \{ \delta(\eta) V^R(U(\eta), \eta) + [1 - \delta(\eta)] [V^R(U_0, 1) - w^F(\eta)] \} h(\eta) d\eta \quad (35)$$

subject to

$$\int_{\eta} \{ \delta(\eta) U(\eta) + [1 - \delta(\eta)] u(w^F(\eta)) \} h(\eta) d\eta = U,$$

where  $V^R(U(\eta), \eta)$  is the payoff function to the investors from having a manager of ex ante productivity  $\eta$  and promised utility  $U(\eta)$ . The payoff from hiring a new manager is given  $V^R(U_0, 1)$  since the outside reservation value of a new manager is taken to be  $U_0$  and his productivity is 1 for sure by assumption. The variable  $\delta(\eta)$  denotes the probability that a manager is retained given that his ex ante productivity is  $\eta$ . The function  $V^R(U(\eta), \eta)$  is

given by the solution to

$$V^R(U, \eta) = \max_{\bar{w}, \bar{\theta}, \theta^*} F(K) \int_0^\infty \theta \eta p(\theta) d\theta - \bar{w} P(\bar{\theta}) - \tau F(K) \int_{\bar{\theta}}^\infty (\theta - \theta^*) p(\theta) d\theta - \gamma P(\theta^*) \eta F(K) - rK \quad (36)$$

subject to the constraints that

$$u(\bar{w}) P(\bar{\theta}) + \int_{\bar{\theta}}^\infty u(\tau(\theta - \theta^*) \eta F(K)) p(\theta) d\theta = U,$$

and

$$\bar{w} = \tau(\bar{\theta} - \theta^*) F(K).$$

Note here that it is optimal to condition the choices of  $\bar{w}$ ,  $\theta^*$ ,  $\bar{\theta}$  and  $K$  on  $\eta$ . We can now state our first result on managerial retention.

**PROPOSITION 11.** *It is optimal to set  $\bar{w}(\eta) = \bar{w}$  and to set  $w_F = \bar{w}$ . The optimal choice of  $\delta(\eta)$  is a simple cutoff rule where  $\delta(\eta) = 1$  if  $\eta \geq \bar{\eta}$  and equal to 0 otherwise. If we assume that  $K$  is fixed, then optimal cut-off level is decreasing in initial promised utility  $U$ .*

**Proof:** See the appendix.

The result that the golden parachute payment is the same as the base wage is somewhat special to this formulation of the model. More generally, the parachute payment will smooth individual's marginal utility over the retention vs. non-retention outcomes.

The result that the cut-off is declining in  $U$  implies that the model exhibits a form

of managerial entrenchment. Manager's who have had better performance in the past will have higher continuation utility promises, and these higher continuation utilities will lead to the manager being more likely to be retained in the future. Since managers who have been on the job longer will have a better chance of having had a high productivity shock  $\theta > \bar{\theta}$  leading to a bonus, older managers will on average been less likely to be replaced than newer manager.

The key aspect of the model that delivers our retention result is the fact that higher utility promises reduce the extent of agency frictions within the firm, and lead to it being cheaper to provide the manager with utility on the job than off it, given that a new manager will have to be hired to replace him. As a result, a manger with a high utility promised will be retained at an productivity  $\eta$  at which a manager with a low utility promise would be fired.

## 10. Capital structure in the dynamic model

In this section, we discuss two issues that arise in interpreting an efficient contract in our model as a theory of the capital structure of the firm and the compensation of the firm's manager. The first of these issues concerns our model's implications for the optimal debt-equity ratio of the firm. We show here that while our model does have implications for the payments to debt and equity holders, it does not pin down the debt-equity ratio of the firm. The second of these issues concerns the interpretation of the monitoring of the firm by outside investors as bankruptcy.

Our dynamic model delivers a theory of the division of the gross payments out of an ongoing firm between holders of the firm's debt, outside equity, and the firm's manager.

This division of gross payments is not sufficient, however, to pin down the relative value of the firm's debt and equity. This is because our model does not pin down whether it is the debt holders or the outside equity holders who pay for the investments  $K$  in future periods. This issue is not new to our model and can arise in any financial contract in which there are multiple flows out of the firm.

We illustrate this problem with the following simple example. Imagine that an entrepreneur has created a project that can be operated for 2 periods in which an investment of 1 at the beginning of each period produces an output of 2 at the end of each period. Assume that this entrepreneur sells this project to outside investors after having made the initial investment at the beginning of period 1 and that there is no discounting, so that the total value of this project is equal to 3 units of output (2 units of output in period 1 plus two more units in period 2 less 1 unit of investment in period 2). Hence, it is clear in a competitive capital market, the outside investors must pay the entrepreneur 3 units of output at the beginning of period 1 to purchase this project. What is to be determined is the division of this value between outside investors who hold debt and outside investors who hold equity. Imagine further that a theory such as ours has yielded the implication that, each period, the gross output of 2 is divided equally between debt and equity holders, so 1 unit is paid to the debt holders and 1 unit is paid to the equity holders. As the following examples make clear, the division of the value of this firm between debt and equity is not pinned down under these assumptions, despite the fact that the division of the gross payments to the outside investors is pinned down.

To see this, assume first that the firm is financed with a combination of short-term debt and equity. In particular, assume that short-term debt holders lend one unit at the

beginning of each period and are repaid that one unit at the end of each period. In period 1, the equity holders pay 2 units to the entrepreneur to purchase the project and the remainder of the purchase is financed with the first issuance of 1 unit of short-term debt. The equity holders in this case receive a dividend of one unit each period in exchange for their investment while the investment of 1 unit required at the beginning of the second period is financed by a second issuance of short-term debt at the beginning of period 2. Under these assumptions, in the first period, after the initial investment of 1 unit has been made, the value of the debt is 1 unit and the value of the equity is 2 units.

Next assume that the investment of one unit in the firm at the beginning of the second period is financed by the outside equity holders (through retained earnings) while the debt is long-term debt. In this case, to purchase this project, one group of outside investors puts forward 2 units of output in exchange for a long-term debt claim that pays 1 unit at the end of each of the two periods while another group of outside investors puts forward 1 unit of output in exchange for an equity stake that pays no dividend in the first period and a dividend of 1 unit at the end of the second period. The three units of output raised in this way are used to purchase the project from the entrepreneur. Under this financing scheme, the firm's debt is worth 2 units and the firm's equity is worth 1 unit.

As this simple example makes clear, under different assumptions about the division of responsibility for ongoing investments in the firm, one obtains different implications for the debt-equity ratio of the firm. We conjecture that this issue will arise in any well-specified "trade-off" theory of optimal capital structure.

In interpreting our efficient contract as a theory of capital structure, we associate monitoring with bankruptcy. Monitoring in our model occurs whenever the current gross

output of the firm fall below a threshold  $\theta^*F(K)$  determined by the optimal contract. In our one-period version of the model, in the event that  $\theta \leq \theta^*$ , monitoring occurred, all of the remaining value of the firm was paid to debt-holders and the outside equity holders received nothing. In this sense, in the one-period model, monitoring corresponds to a stylized notion of bankruptcy. In a multi-period version of our model, the division of the value of the firm between debt and equity holders in the event of monitoring is not so stark. In the event that  $\theta \leq \theta^*$ , monitoring occurs, but the firm still has a value to the outside investors as an ongoing concern (denoted by the continuation value  $V(W(\theta))$ ). In the event that this continuation value exceeds the face value of the debt, then the equity holders emerge from this episode of bankruptcy with shares that still have positive value. In this sense, monitoring in the dynamic model does not necessarily correspond to the liquidation of the firm. Of course, the same is true of bankruptcy in the data.

## 11. Concluding Comments

This paper has present a model of capital structure and compensation based upon two internal frictions to the firm: an information friction and an agency friction. These friction operation with an accounting cycle in which the realization of the proceeds of productivity activity is manifest. This simple structure is able to generate a surprisingly complete theory of the financial operation of the firm. They motivate the division of firm's payout into debt and equity payments, and the division of compensation into base pay and a performance bonus coming from internal firm considerations. The trade-off between the monitoring and agency costs associated with these frictions, along with the impact of shocks to productivity, lead to a theory of the dynamics of capital structure and compensation.

This theory links the wedge between the internal return to capital and the external rental rate of capital to the division of executive compensation between base and performance pay. It can explain why firms might find it optimal to use financial hedges to fine tune this trade-off between these two frictions. It can explain nonproductive managers are paid a severance package out of a simple insurance motive. However, it turns out that managers who have a larger claim on the firm are less costly to retain, and hence the model exhibits a form of managerial entrenchment because a manager's claim on his firm accumulates over time in response to positive productivity shocks. The theory has implications for capital structure and compensation across countries due to differences in monitoring costs or agency frictions.

Several surprising findings emerge. Bankruptcy emerges as means of achieving optimal monitoring, not because of solvency. Hedging emerges as optimal to achieve efficient trade-off between bankruptcy risk and agency risk, not to simply hedge bankruptcy risk. Finally, despite pinning down debt-equity share of payments, the frictions in the model do not pin down the actual debt-equity structure. Hence, the theory challenges the conventional wisdom by indicating that an additional friction on the provision of investment funds may be necessary to account for this aspect.

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## 12. Appendix

### A. Additional Optimality Conditions

The first order conditions governing the choice of the capital stock and optimal monitoring in the dynamic model are given by

$$\left\{ \begin{array}{l} E(\theta|\theta_{-1}) [1 - \gamma P(\theta^*|\theta_{-1})] \\ - \int_{\bar{\theta}}^{\infty} \left[ 1 - \frac{u'[\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \right] \tau(\theta - \theta^*) p(\theta; \theta_{-1}) d\theta \end{array} \right\} F'(K) = r,$$

and

$$\tau \int_{\bar{\theta}}^{\infty} \left( 1 - \frac{u'[\tau(\theta - \theta^*)F(K)]}{u'(\bar{w})} \right) p(\theta; \theta_{-1}) d\theta = \gamma E(\theta|\theta_{-1}) p(\theta^*; \theta_{-1}).$$

The only changes in these conditions are to take account of the impact of persistence on the probability distribution over  $\theta$ , and that  $E(\theta|\theta_{-1})$  may no longer equal 1.

## B. Risk Neutral Comparative Statics

The problem can be stated as Choose  $\bar{x}, \theta^*$ , and  $K$  to maximize

$$\left\{ 1 - \tau \int_{\bar{\theta}}^{\infty} (\theta - \theta^*) p(\theta) d\theta + \gamma P(\theta^*) \right\} F(K) - \bar{x}P(\bar{\theta}) - rK \quad (37)$$

subject to the promise-keeping constraint,

$$\bar{x}P(\bar{\theta}) + \tau F(K) \int_{\bar{\theta}}^{\infty} (\theta - \theta^*) p(\theta) d\theta = U_0 \quad (38)$$

the constraint that

$$\frac{\bar{x}}{\tau F(K)} + \theta^* - \bar{\theta} = 0$$

$$\bar{x} \geq 0$$

**Proof of Proposition 7.** This result follows directly from the facts that the promise-keeping constraint requires the outside investors to pay the manager more than he is able to consume in perks even if there is never any monitoring and that the manager is risk neutral. given (34), the outside investors can satisfy the promise-keeping constraint with a payment schedule  $x(\theta, \theta)$  such that  $x(\theta, \theta) \geq \tau \theta F(K)$  for all  $\theta$ . For any such payment schedule, the constraint that the manager not want to take perks is slack for all  $\theta$ , even if there is no monitoring. Since the manager is risk neutral, he is indifferent between any such payment schedule, including the one proposed in the statement of the proposition. With no monitoring and slack constraints that the manager not want to take perks, the optimal capital choice is the one that satisfies

$F'(K) = r$ . *Q.E.D.*

**Proof of Proposition 8.** When condition (34) is not satisfied then, with  $\theta^* = 0$  and capital chosen according to  $K = F'^{-1}(r)$  the promise-keeping condition is slack, and the first-best cannot be supported. We deal with this case in the following proposition. The analysis is complicated by the fact that the promise keeping condition is an equality rather than an inequality, and hence admits the possibility of inefficiently low levels of utility for the manager.

Again, let  $\lambda$  be the multiplier on the first constraint,  $\eta$  the multiplier on the second, and  $\delta$  the multiplier on the third. The first order conditions are as follows. With respect to  $\bar{\theta}$

$$(1 - \lambda) [-\bar{x}p(\bar{\theta}) + \tau(\bar{\theta} - \theta^*)F(K)p(\bar{\theta})] = \eta$$

Since  $\tau(\bar{\theta} - \theta^*)F(K) = \bar{x}$ , this first order condition implies that  $\eta = 0$ . we use this result in the first order conditions that remain. With respect to  $\bar{x}$

$$(\lambda - 1)P(\bar{\theta}) + \delta = 0.$$

Note that there are two possibilities here: one is that  $\lambda < 1$  and  $\delta > 0$ , which implies that  $\bar{x} = 0$ . The other possibility is that  $\delta = 0$ , which would mean that  $\bar{x} \geq 0$  and  $\lambda = 1$ . With respect to  $\theta^*$

$$(1 - \lambda)\tau(1 - P(\bar{\theta}))F(K) - \gamma p(\theta^*)F(K) = 0$$

This first order condition gives us that

$$(1 - \lambda) = \frac{\gamma}{\tau} \frac{p(\theta^*)}{1 - P(\bar{\theta})}.$$

Note that since the hazard is monotonically increasing, if

$$\frac{\tau}{\gamma} > \lim_{\theta^* \downarrow 0} \frac{p(\theta^*)}{1 - P(\theta^*)}, \quad (39)$$

doesn't hold, then monitoring is never efficient even when the value placed on resources to the manager is zero. This implies that monitoring will only be undertaken when the utility level for the manager is inefficiently low and the nonnegativity constraint on  $\theta^*$  can bind.

The first order condition with respect to capital is

$$\left\{ 1 - \gamma P(\theta^*) + (\lambda - 1) \tau \int_{\bar{\theta}}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right\} F'(K) = r$$

which can be written

$$\left\{ 1 - \gamma P(\theta^*) - \frac{\gamma}{\tau} \frac{p(\theta^*)}{1 - P(\bar{\theta})} \tau \int_{\bar{\theta}}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right\} F'(K) = r. \quad (40)$$

Finally, we have the constraints.

To summarize,  $u(c) = c$  and (34) doesn't hold, then  $\bar{x} = 0$  and there are two cases depending upon whether is satisfied (39). If (39) holds the optimal choices of  $\theta^*$  and  $K$  are given by (40) and the promise keeping constraint (38). If (39) doesn't hold, then the levels

of  $\lambda$ ,  $\omega$ ,  $\theta^*$  and  $K$  are given by (38), (40),

$$\tau(1 - \lambda)(1 - P(\theta^*)) - \gamma p(\theta^*) + \omega = 0,$$

and

$$\omega\theta^* = 0,$$

where  $\omega$  is the multiplier on the nonnegativity constraint on  $\theta^*$ .

We now derive our comparative statics results for the case in which (34) doesn't hold.

In this case  $\delta > 0$  and  $\lambda < 1$ , and we can rewrite our first-order conditions for  $\theta^*$  and  $K$  as

$$\left\{ 1 - \gamma P(\theta^*) - \gamma h(\theta^*) \left[ \int_{\theta^*}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right] \right\} F'(K) = r$$

$$\left[ \tau \int_{\theta^*}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right] F(K) = U_0.$$

Taking derivatives of these two equations with respect to the endogenous variables  $\theta^*$  and  $K$ ,

and the parameters  $\tau$ ,  $r$ , and  $U_0$  gives

$$\begin{aligned} dr = & \left\{ 1 - \gamma P(\theta^*) - \gamma h(\theta^*) \left[ \int_{\theta^*}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right] \right\} F''(K) dK - \gamma \{ p(\theta^*) + h'(\theta^*) \} F'(K) d\theta^* \\ & - (P(\theta^*) + \gamma h(\theta^*)) \left[ \int_{\theta^*}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right] F'(K) d\gamma \end{aligned}$$

$$dU_0 = \left[ \tau \int_{\theta^*}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right] F'(K) dK - \tau(1 - P(\theta^*)) F(K) d\theta^* \\ + \left[ \int_{\theta^*}^{\infty} (\theta - \theta^*) p(\theta) d\theta \right] F(K) d\tau$$

In the first equation, the first-order condition for capital implies that the term multiplying  $F'' dK$  is positive, and hence the overall term multiplying  $dK$  is negative. The term in this expression multiplying  $d\theta^*$  is negative given our monotonically increasing hazard assumption. The term multiplying  $d\gamma$  is negative given our increasing hazard assumption. Hence, this equation implies a  $K^A(\theta^*, r, \gamma)$  which depends negatively on  $\theta^*$ ,  $r$  and  $\gamma$ . In the second equation, the promise-keeping condition, the term multiplying  $dK$  is positive, the term multiplying  $d\theta^*$  is negative, and the term multiplying  $d\tau$  is positive. Hence, this expression implies  $K^B(\theta^*, \tau, U_0)$  schedule that depends positively on  $\theta^*$ , negatively on  $\tau$ , and positively on  $U_0$ . The fact that these two schedules have the opposite slope with respect to  $\theta^*$  implies that the solution to

$$\theta^*(r, \tau, U_0) \text{ s.t. } K^A(\theta^*, r, \gamma) = K^B(\theta^*, \tau, U_0),$$

is unique, and that this level of  $\theta^*(r, \tau, U_0, \gamma)$  depends negatively on  $r$ , positively on  $\tau$ , negatively on  $U_0$ , and negatively on  $\gamma$ . Substituting back into our expression for  $K$  and doing a bit more algebra yields our results. *Q.E.D.*

### C. Golden Parachute Proof

**Proof of Proposition 11.** The proof works by noting constructing the first-order condition for the firing threshold, and noting that only  $\eta$  and the cut-off have a first-order

effect on this threshold, and concluding that the threshold must fall due to the signs of these first-order effects.

We start with second period problem of deciding whether or not to retain the manager given by (35). The first-order conditions to the problem of retention and ex post (w.r.t.  $\eta$ ) utility allocation given by (35) include

$$D_1 V^R(U(\eta), \eta) = -\Lambda,$$

and

$$1 = \Lambda u'(w^R),$$

where  $D_i$  denotes the derivative of the function with respect to its  $i^{\text{th}}$  argument, and  $\Lambda$  is the Lagrangian on the promise-keeping constraint. This gives us a simple efficiency condition on how utility will be allocated across the two situations of retaining the manager or firing him

$$D_1 V^R(U(\eta), \eta) = \frac{-1}{u'(w^R)}.$$

To see what determines the marginal cost of utility if the manager is retained on the lhs of the above expression, we utilize the problem of determining the efficient contract conditional on retention given by (36). From the envelope condition, which is given by differentiating the problem in (36) with respect to  $U$ , we that

$$D_1 V^R(U, \eta) = -\lambda,$$

where  $\lambda$  is the promise-keeping constraint in this problem. Since, just as the dynamic contracting section, optimality implies that  $\lambda = 1/u'(\bar{w})$ , we get our first result.

Next note that the derivative w.r.t.  $\delta$  of the Lagrangian associated with retention problem (35) is given by

$$\frac{dL^2}{d\delta} = [V^R(U(\eta), \eta) + \Lambda U(\eta)] - [V^R(U_0, 1) - \bar{w} + \Lambda u(\bar{w})]. \quad (41)$$

This derivative is increasing in  $\eta$ . To see this, assume otherwise: that there exists an  $\eta_1 > \eta_2$  where  $h(\eta_1) = h(\eta_2)$  and  $\delta(\eta_1) = 0$  while  $\delta(\eta_2) = 1$ . Now, consider retaining the manager at  $\eta_1$  and firing him at  $\eta_2$  and paying him  $w_F$ . When retaining him at  $\eta_1$ , simply set  $\theta^*(\eta_1) = \theta^*(\eta_2)$  and  $\bar{w}(\eta_1) = \bar{w}(\eta_2)$  and set  $K(\eta_1)$  to be such that

$$\eta_1 F(K(\eta_1)) = \eta_2 F(K(\eta_2)).$$

Trivially, this satisfies all of our constraint, but leaves the outside investors better off since  $K(\eta_1) < K(\eta_2)$  and hence they are saving on the rental costs of capital. If  $h(\eta_1) < h(\eta_2)$  then we could simply retain the manager in case of  $\eta = \eta_1$  and fire him with probability  $h(\eta_1)/h(\eta_2)$  in the case of  $\eta = \eta_2$ , or vice versa. This contradicts our original assumption. Note also that we could have simply set  $K(\eta_1) = K(\eta_2)$  in this argument. In which case, it follows that both the outside investors and the manager are better off. Then we could have simply noted that since the promise-keeping constraint always binds from below, it would be possible for the outside investors to improve even further on this alternative contract. Hence  $d^2 L^2 / d\eta d\delta > 0$ .

The fact that  $d^2L^2/d\eta d\delta > 0$  implies that there is a unique  $\bar{\eta}$  such that

$$dL/d\delta|_{\bar{\eta}} = 0.$$

The optimal  $\delta(\eta)$  is

$$\delta(\eta) = \begin{cases} 1 & \text{if } \eta > \bar{\eta} \\ [0, 1] & \text{if } \eta = \bar{\eta} \\ 0 & \text{if } \eta < \bar{\eta} \end{cases} .$$

Finally, to we want to show that  $\bar{\eta}$  is declining in the original ex ante promised utility  $U$ .  $V^R(U, \eta)$  is declining in  $U$  since

$$-\Lambda = D_1V^R(U(\eta), \eta) = -1/u'(\bar{w}),$$

and we have previously established in the comparative statics of the one period model in proposition **X** that  $\bar{w}$  is increasing in the utility promise  $U(\eta)$ . It therefore follows that  $D_2V^R(U, \eta) < 0$ . Hence, an increase in  $\Lambda$  will lead to a solution to the optimality conditions for the Lagrangian that raises  $U(\eta)$  and  $w^R$  and leads to higher ex ante utility. Thus higher levels of ex ante utility lead to higher levels of the multiplier  $\Lambda$ . If we differentiate our condition for determining  $\bar{\eta}$ ,

$$[V^R(U, \bar{\eta}) + \Lambda U] - [V^R(U_0, 1) - \bar{w} + \Lambda u(\bar{w})],$$

with respect to  $\Lambda$ , the derivative is positive since  $U > u(\bar{w})$  (since  $w(\theta) \geq \bar{w}$  in the subsequent sub-problem of contracting being  $U$  and  $\eta$ ). Then, note that since we have already established that  $V^R$  is increasing in  $\eta$ , it follows that  $\bar{\eta}$  must fall to response to the rise in  $\Lambda$  induced by the rise in ex ante expected utility. *Q.E.D.*