Discussion of

Can bonds hedge volatility risk in the U.S. Treasury market? A specification test for affine term structure models

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This paper does two things

1. Derives implications of affine term structure models relating realized variance to variance of state variables in term structure models.
2. Provides tests of these implications and finds rather strong results.

Central question: What is "*volatility*?"

- 3 different answers/potential sources of information on this
  1. It is a state variable in term structure models labeled "volatility"

$$A_1(3):\ dr_t = \kappa_r(\theta_t - r_t) + \sqrt{V_t}\,dW_t$$

$$Piazzesi:\ s_t = r_t - \theta_t$$

$$ds_t = -\kappa_s s_t dt + \sqrt{V_t}\,dW_t$$

  2. Something that comes from a Black-Scholes like formula: implied volatility
     - Short dated: bond futures, Eurodollar futures?; Long dated: swaptions; Complicated mixtures: caps
  3. Statistical evidence: the second moment of yield changes.
     - Realized volatility
     - Parametric time series volatility.
- Crucially depends on what you are using "volatility" for.

Problems with all of them.

1. **State variables** in term structure models
   - Subject to model misspecification and estimation error

2. **Implied volatility**
   - Subject to liquidity issues, data recording issues (what are the underlying forward rates), model misspecification, risk premia, forward-looking, etc…

3. “**Statistical**” or time series volatility
   - Subject to estimation error: volatility is not observed, rather it is estimated.
   - Subject to descriptive time series model assumptions.

Statistical volatility and estimation error

- There are a number of sources of errors:

$$QV_{t,\Delta} = \int_t^{t+\Delta} V_s ds = \int_t^{t+\Delta} (\alpha + \beta X_s) ds$$

$$\sum_n \left(y_{t+\Delta n^{-1}} - y_{t+(\Delta-1)n^{-1}} \right)^2 = a + \sum_j b_j y_{t,\tau_j} + \varepsilon_t$$

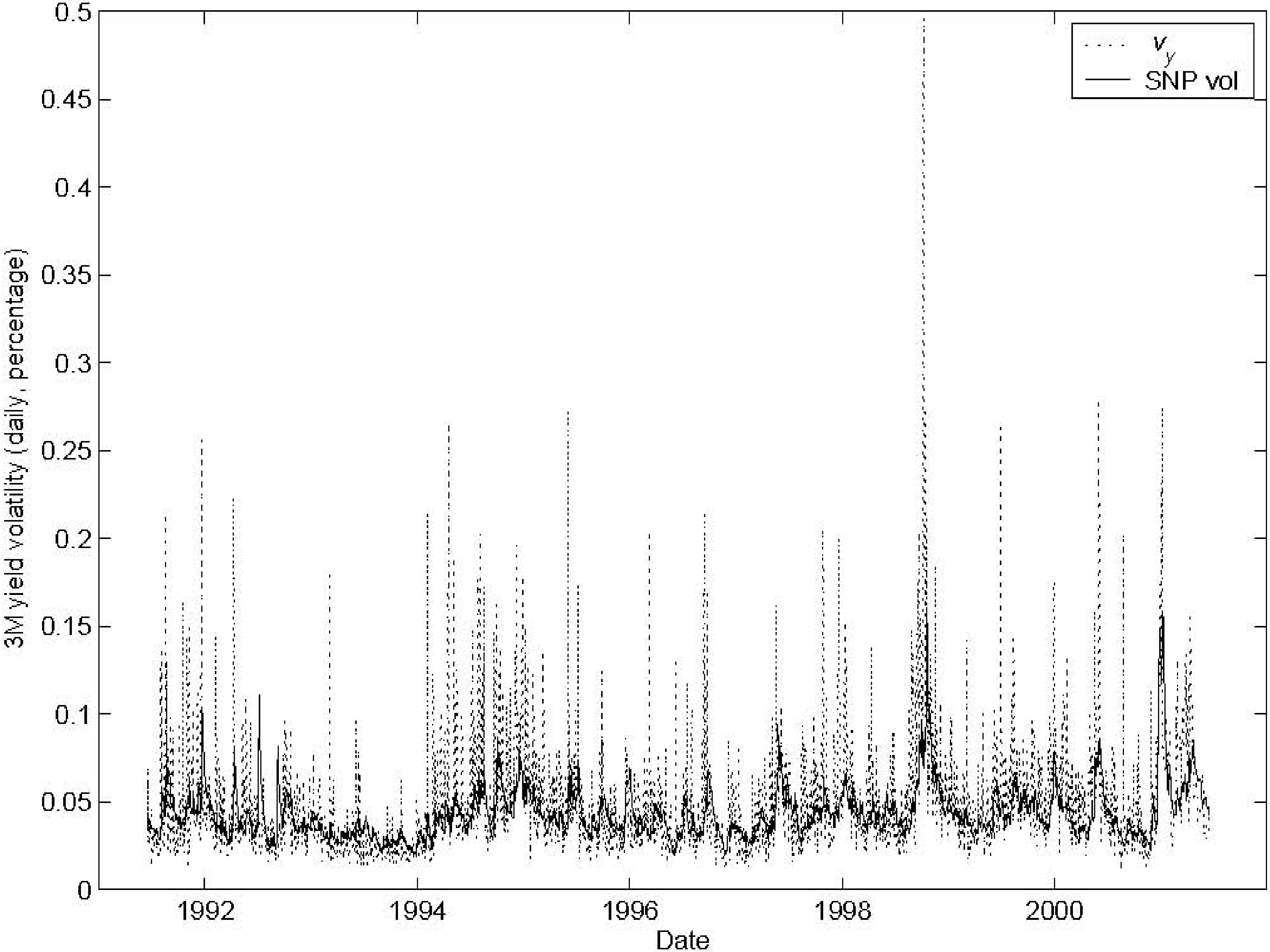
- Errors:
  1. Theory: how closely does RV measure true QV: I did some simulations in a pure SV model and found correlation about 80-90%.
  2. Can you invert? Not if there are pricing errors. Where do they come from? Microstructure movements (bid-ask) or, more likely, idiosyncratic movements of on-the-run Treasury securities.
  3. Yields and average yields are measured with errors. Interpolated yields are noisy (see Dai, Singleton, and Yan (2004)).

However, putting all of these things together,
I'm not sure you are going to get R^2's of 5%.

So what do we learn?

- We still don't know exactly what "volatility" is:
  - It doesn't look like it is a state variable labeled V_t in a term structure model.

- Then, what is it. What does *realized volatility* tell us about various models? We know what it isn't:
  - RV isn't GARCH either. ρ=0.44 (R^2=0.19), close to their other regressions!
  - Not very persistent: daily autocorrelations are on the order of 40%
- Picture



0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1992
1994
1996
1998
2000
Date
3M yield volatility (daily, percentage)
v_y
SNP vol

Everything points to … non-Gaussian components

- Jumps
  - Why? RV not highly autocorrelated, high kurtosis of data, spikes in RV estimates, macroeconomic announcements, …
- The authors argue that their tests are robust to "jumps." Is this true? Sort of…
  - QV contains squared jumps.
  - If jumps have a stochastic intensity that depends on state variables, *and* one can take their expectation, then E(QV) = a+bX. Relies on being able to accurately compute E(QV). Not sure their procedure does it.
  - Interestingly, their tests do better when jumps are included.
  - **Recommendation:** with the forecasting model (separate diffusive and jump components), run spanning tests separately with jump and diffusive components.
- Tests are not robust, if jumps are generated by predictable announcements.

$$y_{t,\tau} \neq \alpha(\tau) + \beta(\tau)' X_t$$

  - Coefficients must be solved for recursively and depend on announcement volatility.
  - Time heterogeneous models.

Jumps and hedging “volatility”?

- If “volatility” is diffusive plus jumps, then we cannot hedge. Why?

- Suppose there is an employment report tomorrow at 8:30 a.m. What portfolio of bonds would be immunized to the report? Depends on the announcement…
  - Easy to immunize exact parallel shifts (two bonds to do that)
  - What if the curve steepens or flattens?
  - We don’t know ex-ante which one it is going to be, so we cannot hedge the “volatility” because it arrives as a jump.

- I think this could resolve a number of outstanding issues.

Conclusions

- I think the authors provide some persuasive evidence that we have a poor understanding of volatility, and especially how it should enter into our models.

- Volatility contains many components, and these components may have different implications for "volatility," risk premia, and asset pricing.

- Lots of work to do in figuring out the issues.