The Production Function Approach to Calculating Potential Growth and Output Gaps
Estimates for EU Member States and the US

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Abstract

This paper presents the current methodology used by the EU Commission to calculate potential output and output gaps. The first section of the paper provides an overview of the methodology, with special emphasis on how trends for the input components of the production function are calculated. It also discusses the medium term projections which currently extend to the year 2010. In the following the paper discusses the NAIRU and TFP trends in the euro area which underlie the potential growth estimates and draws some conclusions concerning the medium term outlook for the euro area.
Introduction

Despite the relatively encouraging performances of a number of countries, including the EU’s new Member States, Europe’s overall growth performance since the mid-1990’s has been the subject of increasing scrutiny, with this specific time period having witnessed some important changes in international growth patterns. In particular, while the EU15 and here especially the euro area is continuing on a downward sloping labour productivity path, the US has managed to reverse its productivity fortunes.

These growing divergences in growth patterns are provoking an active ongoing debate regarding the implications of recent trends for future prospects. The “pessimistic view”, largely supported by the Sapi report, suggests that a large number of Member States have as yet failed to recognise the extent of the reforms which need to be introduced given the challenges posed by an acceleration in the pace of technological progress, by globalisation and by ageing. This view suggests that the EU might be unable to achieve a shift in its resources to sectors with high productivity growth prospects and will continue with production in areas where it has traditionally held a global advantage, namely medium-technology manufacturing industries. This overall strategy appears increasingly threatened with the emergence of a number of strong competitors around the world in these more traditional manufacturing industries, most notably China and India. In addition, future productivity growth rates will potentially be dampened by the likely continuation of the ongoing trend towards the relatively less productive service sectors, with the latter possibly accelerated and reinforced by the emerging demographic patterns.

The “less pessimistic view”, as enunciated by Blanchard amongst others, is that part of the explanation for Europe’s poor performance could be measurement problems / adjustment lags, with perhaps the basis for future growth already firmly established due to the labour, capital and product market reforms which have already been introduced. Under this view the EU may now simply be in a transition phase whereby some of the negative effects of those reforms (e.g. a temporary decrease in productivity due to labour market changes) are visible, whilst the gains to be reaped in the future are not. This view is corroborated by the experience of a large number of the smaller EU Member States which have successfully achieved the essential sectoral restructuring needed to move from an imitation to an innovation-based economy, with more emphasis on knowledge based, high human capital intensive, production. It is also supported by Europe’s ability in the 1960’s and 1970’s to achieve a relatively strong process of technological convergence towards the US. Repeating the successes of the past however might be more difficult in an increasingly globalised environment and in the specific case of more knowledge-based industries.

This paper looks at recent growth and productivity trend in the EU and the US by using the production function approach for calculating potential growth in the EU. This has been used by the EU commission since July 2002, it constitutes the reference method for the calculation of output gaps and trend growth when assessing the stability and convergence programmes for EU Member States. This method has replaced a simple HP filter approach for calculating trend output. Based on this methodology DG ECFIN monitors potential output in 25 EU member states and produces medium term projections which extent three years beyond the short term forecast. This paper presents the methodology underlying these calculations and analysis current economic trends based on this method.
1. The Production Function Methodology

Instead of making statistical assumptions on the time series properties of trends and their correlation with the cycle, the production function approach makes assumptions based more strongly on economic theory. This latter approach focuses on the supply potential of an economy and has the advantage of giving a more direct link to economic theory but the disadvantage, is that it requires assumptions on the functional form of the production technology, returns to scale, trend technical progress (TFP) and the representative utilisation of production factors. As shown in the diagram below, with a production function, potential GDP can be represented by a combination of factor inputs, multiplied with the technological level or total factor productivity (TFP). The parameters of the production function essentially determine the output elasticities of the individual inputs. With the Cobb-Douglas specification, it is necessary to estimate the trend components of the individual production factors, except capital. Since the capital stock is not detrended, estimating potential output amounts therefore to removing the cyclical component from both labour and TFP.
**Cobb-Douglas Production Function**: In more formal terms, with a production function, GDP (Y) is represented by a combination of factor inputs - labour (L) and the capital stock (K), corrected for the degree of excess capacity (U_L, U_K) and adjusted for the level of efficiency (E_L, E_K). In many empirical applications, including the Quest II model, a Cobb-Douglas specification is chosen for the functional form. This greatly simplifies estimation and exposition. Thus potential GDP is given by:

\[ Y = (U_L L E_L)^\alpha (U_K K E_K)^{1-\alpha} = L^\alpha K^{1-\alpha} * TFP \]

where total factor productivity (TFP), as conventionally defined, is set equal to:

\[ TFP = (E_L^\alpha E_K^{1-\alpha} (U_L^\alpha U_K^{1-\alpha}) \]

which summarises both the degree of utilisation of factor inputs as well as their technological level. Factor inputs are measured in physical units. An ideal physical measure for labour is hours worked which we use as our labour input. For capital we use a comprehensive measure which includes spending on structures and equipment by both the private and government sectors.

Various assumptions enter this specification of the production function, the most important ones are the assumption of constant returns to scale and a factor price elasticity which is equal to one. The main advantage of these assumptions is simplicity. However these assumptions seem broadly consistent with empirical evidence at the macro level. The unit elasticity assumption is consistent with the relative constancy of nominal factor shares. Also, there is little empirical evidence of substantial increasing/decreasing returns to scale (see, e.g. Burnside et al. for econometric evidence).

The output elasticities of labour and capital are represented by \( \alpha \) and \( (1-\alpha) \) respectively. Under the assumption of constant returns to scale and perfect competition, these elasticities can be estimated from the wage share. The same Cobb-Douglas specification is assumed for all countries, with the mean wage share for the EU15 over the period 1960-2003 being used as the estimate for the output elasticity of labour, which gives a value of .63 for \( \alpha \) for all Member States and, by definition, .37 for the output elasticity of capital. While the output elasticity for labour may deviate somewhat from the imposed mean coefficient in the case of

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1. **Choice of Production Technology – Why use Cobb-Douglas?** One of the big advantages of using Cobb-Douglas is undoubtedly its simplicity, in that it is easy to make sense out of the coefficients imposed. The Cobb-Douglas assumption greatly simplifies estimation of output elasticities, conditional on an assumption on returns to scale. With a high average degree of competition in the goods market, the output elasticities can be equated to their respective factor shares. Thus, there is only one parameter to estimate. While a large variety of views on alternative specifications to the Cobb-Douglas approach of constant factor shares are available, one needs to be aware of the implications associated with these alternatives. For example, if one chooses to adopt an elasticity of less than 1, one is left with the problem of explaining why wage shares have fallen recently. If one goes for the alternative assumption of using an elasticity of greater than 1, then the lack of econometric evidence to support such a function needs to be taken into account. Consequently, given the difficulties associated with the alternatives, the Cobb-Douglas assumption of unity appears to be a reasonable compromise. In addition, of course, if one were to use a CES function with an elasticity of 0.8 or 1.2 the results would not differ very strongly from Cobb-Douglas. Finally, the aggregation problem associated with having a mixture of low and high skilled workers in the workforce would also appear to lend support to the Cobb-Douglas view. In this regard, if you aggregate over both sets of workers one would come close to Cobb-Douglas, with low skilled workers having a high elasticity of substitution (EoS) with capital (EoS > 1) balancing out the low EoS associated with high skilled workers (EoS < 1). High skilled workers have generally a low EoS since such workers are regarded as being more complementary to K. This view regarding the distinction between low and high skilled workers is supported by a paper by Krussell et al. published in *Econometrica* in September 2000.
individual Member States, such differences should not seriously bias the potential output results.

**CAPITAL**: With respect to capital this task of defining potential factor use is straightforward since the maximum potential output contribution of capital is given by the full utilisation of the existing capital stock in an economy. Since the capital stock is an indicator of overall capacity there is no justification to smooth this series in the production function approach. In addition, the unsmoothed series is relatively stable for the EU and the US since although investment is very volatile the contribution of capital to growth is quite constant since net investment in any given year is only a tiny fraction of the capital stock figures. In terms of the measurement of the capital stock, the perpetual inventory method is used which makes an initial assumption regarding the size of the capital / output ratio.

\[ I = i_s \ast YPO T \]

\[ K = I + (1 - dep)K(-1)^2 \]

**LABOUR**: The definition of the maximum potential output contribution of labour input is more involved since it is more difficult to assess the "normal" degree of utilisation of this factor of production. Labour input is defined in terms of hours. Determining the trend of labour input involves several steps. In defining the trend input we start from a maximum possible level, namely the population of working age \( POPW \). We obtain the trend labour force by mechanically detrending (using an HP filter) the participation rate \( parts \). In a next step we calculate trend un/employment to be consistent with stable, non accelerating, (wage) inflation (NAIRU). The NAIRU is identified via a Phillips curve relationship

\[ \Delta^2 w_t = \phi^w \Delta^2 (pr_t) + \phi^w \Delta^2 ws_t + \phi^w \Delta^2 tot, - \beta (u_t - nairu_t) + v_w \]

where \( w \) is the log of nominal wages, \( pr \) is the log of labour productivity, \( ws \) is the log of the wage share, \( tot \) is the log of the terms of trade, and \( u \) is the unemployment rate.

The Phillips curve shows the short run response of nominal wages to labour productivity, labour demand shocks and the unemployment gap. The response to the unemployment gap is intuitively plausible. Whenever unemployment is above the NAIRU, nominal wage growth will decelerate and vice versa. However, this link is not perfect but is disturbed by observed and unobserved shocks to the wage rule and the labour demand equation. How nominal wage growth responds to productivity and labour demand shocks depends on a variety of factors, this is discussed in the appendix. This Phillips curve specification is used to extract an estimate for the NAIRU by using a Kalman filter approach.

Though there is considerable diversity of the estimated coefficients of the Phillips curve over EU MS it is interesting to point out that the average EU Phillips curve elasticities do not differ substantially from those in the US. The elasticity of wage inflation to the unemployment gap is identical in both zones. The response of wage inflation to productivity shocks or changes in

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2 The depreciation rate is assumed to remain constant over the projection period.
3 Since Eurostat and the OECD have agreed that the national accounts (as opposed to the labour force survey) is the preferred source for labour input data, the production function approach now uses the national accounts for the labour input variables i.e. for hours worked and employment.
4 See the appendix for a more detailed discussion of the labour market specification.
the wage share are similar. Only the response to changes in the terms of trade is markedly different (higher) in the US compared to the euro area.

<table>
<thead>
<tr>
<th>Phillips curve coefficients estimates</th>
<th>Unemployment gap</th>
<th>Terms of trade</th>
<th>Productivity</th>
<th>Wage share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coincident</td>
<td>lagged</td>
<td>coincident</td>
<td>lagged</td>
</tr>
<tr>
<td>Euro area</td>
<td>-0.6%</td>
<td>0.3</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>(Student)</td>
<td>-4.0</td>
<td>1.9</td>
<td>5.2</td>
<td>0.6</td>
</tr>
<tr>
<td>US</td>
<td>-0.5%</td>
<td>0.3</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>(Student)</td>
<td>-4.1</td>
<td>1.7</td>
<td>5.5</td>
<td>9.0</td>
</tr>
</tbody>
</table>

This suggests a similar cyclical response of wages in the EU and the US. Of course it is well known that the EU exhibits a much higher volatility of the structural unemployment rate. As explained in the appendix, a stronger response of wages to the wage share is an indication that wages respond more to the demand wage for labour in the US and more to average productivity in the EU. This difference could be one reason why the US labour market responds differently to structural disturbances.

Finally we obtain trend hours worked or trend labour input \( (LP) \) by multiplying trend employment with the trend of average hours worked \( (HOURST) \) as follows

\[
(6) \quad LP = (POPW * parts*(1−NAIRU)) * HOURST .
\]

**Trend Efficiency**: Within the production function framework, potential output refers to the level of output which can be produced with a "normal" level of efficiency of factor inputs, with this trend efficiency level being measured as the HP filtered Solow Residual. TFP is regarded a exogenous in our framework and is assumed to follow a stochastic trend. The growth rate of TFP is specified as an AR process

\[
(7) \quad \Delta tfp_t = g + \alpha(L)\Delta tfp_{t-1} + \epsilon_t^{fp} .
\]

Based on projections of the population of working age, the participation rate, average hours worked and TFP the above set of equations is used to calculate medium term projections of GDP in EU member states.

### 2. Recent Growth Trends and Outlook

Table 1 shows period averages of potential growth rates for various EU regions and the US. Compared to the mid 80s, potential growth is on a downward trend in the EU. For the euro area in particular potential growth has declined from 2.5% in the mid 80s to 1.9% over the period 2001-2005. This compares with a fairly stable potential growth trend for the US of about 3.0% over the same time periods. The situation is substantially better for the new Member States (NMS) where potential growth rates well in excess of those of the US have been achieved since the late 1990’s. This higher overall EU10 performance relative to the EU15 and the US reflects of course the generally low initial starting positions for the majority
of these countries and consequently what we are witnessing is the signs of a normal catching-up process, with this convergence process expected to stretch over a period of decades.

**Table 1: Potential Growth Rates** (period averages)

<table>
<thead>
<tr>
<th>Year</th>
<th>EA</th>
<th>EU15</th>
<th>NMS</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981-1985</td>
<td>2.2</td>
<td>2.1</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>1986-1990</td>
<td>2.5</td>
<td>2.5</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>1991-1995</td>
<td>2.3</td>
<td>2.2</td>
<td></td>
<td>2.9</td>
</tr>
<tr>
<td>1996-2000</td>
<td>2.2</td>
<td>2.3</td>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td>2001-2005</td>
<td>1.9</td>
<td>2.0</td>
<td>3.5</td>
<td>3.0</td>
</tr>
<tr>
<td>2006-2010</td>
<td>1.9</td>
<td>2.0</td>
<td>4.1</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Source DG ECFIN

In terms of a decomposition of the various components of growth, graph 1 gives the cumulative contributions to potential growth over the period 2001-2005 from labour (hours), capital accumulation and total factor productivity (TFP). Regarding the EU15 countries, the graph shows the degree of heterogeneity amongst the Member States, with respect to the individual inputs. In terms of labour, the contributions range from a positive growth impulse of 9 ½ % in Spain to a negative contribution of 1% in the case of Germany. A similar picture emerges with regard to capital accumulation, with Germany again joint lowest (with Finland) with a contribution of 1.8% points compared with gains of 8-9% points for Spain, Ireland and Luxembourg. Finally, for TFP, Spain is lowest with only a ½% point contribution, with Greece, Ireland, Finland and Sweden at the other end of the spectrum with gains ranging from 9 to 14% points.

Regarding the new member states, what is particularly striking is the extremely weak labour market performance, with all countries registering poor outturns. However, overall potential growth rates have held up well in the EU10 grouping due to strong contributions from both capital accumulation and TFP. Regarding capital, all of the EU10 countries produced good performances over the period. In the case of TFP, with the exception of Malta, all countries made strong efficiency gains, with particularly impressive results for the Baltic countries, Poland and Slovakia.
The following figure gives a more complete picture of potential growth in the euro area and the US, by graphically summarising the different elements that enter our potential growth calculations.
The above graph allows the following observations. First, since the beginning of the 90s the employment contribution to growth has become positive, due to rising participation rates and a fall in structural unemployment since the mid 90s. These two labour market trends dominate the negative contributions arising from a continuous decline in average hours worked and of
the growth of population of working age. Second, the contribution of capital to growth has been fairly stable and third, TFP growth shows a slight tendency to decline over time.

The outlook for the next five years until 2010 is characterised by some mean reverting properties of the model. First, concerning the NAIRU it is mechanically assumed that structural unemployment will stabilise over a period of five years. The stable investment rate is projected forward. Concerning TFP, a recovery to historic trends is implied by the time series specification of the TFP series.

3. Discussion of recent Trends

The discussion so far shows that the recent trend of potential output is characterised by two divergent trends, namely a rising contribution of labour and a declining growth of TFP. This section discusses in more detail how these trends have been determined. Concerning the employment trend we will focus on the structural unemployment rate.

NAIRU:
According to our NAIRU estimates the rising trend was stopped and reversed after the mid 90s. Unfortunately the concept used only provides indirect evidence for the level of structural unemployment because two crucial determinants of the NAIRU, namely the reservation wage and the wage mark up. These two variables are not observable, partly because they relate to non-market activities involving prices or shadow prices that are not reported to statistical offices, and partly because they reflect the labour market structure. For instance, the reservation wage involves the value of leisure, the value of home production and the wage that can be earned in the shadow economy, all of them not observable to the econometrician. The mark up of wages over the reservation wage involves, among other factors, on the bargaining strength of trade unions, on the structure of the bargaining process between workers and firms, on labour market regulations such as hiring and firing rules, on labour taxes and, under certain conditions, on technical progress. These measurement problems and the fact that explicit proxies such as the benefit replacement rate only partially account for the reservation wage have led to the use of unobserved components models for estimating the NAIRU; see for instance Apel and Jansson (1999a, 1999b), Fabiani and Mestre (2001), Llaudes (2005), and the time-varying specification in Gordon (1997) and in Staiger et al. (1997). The non-observable character of the structural determinants of unemployment is thereby explicitly acknowledged. Within this approach inferences about the structural unemployment are essentially made by identifying the cyclical component. Therefore one can ask whether the unemployment gap (as identified by the NAIRU) is consistent with fluctuations in wage inflation when one additionally controls for observed cyclical variations in productivity, the terms of trade and the wage share.

The following graph shows the NAIRU estimate and the corresponding unemployment gap. Since the early 80s, there are three episodes with unemployment above the NAIRU. The first period extends practically over the 80s, the second period lasts from 93 to 99 and the third period is from 2003. Both the second and third figure indicate that our Phillips curve estimate is not inconsistent with such a dating of negative and positive unemployment gaps.
TFP:

Our methodology regards TFP as an exogenous variable and our TFP trend estimates are therefore based on a simple time series model. Time series analysis generally favour I(1) specification for TFP, i.e. the growth rate of TFP is stationary. This specification implies that over the medium term, TFP projections show some mean reversion. Most likely this is too optimistic in the case of the euro area, where we have been observing steadily declining trend growth over the last two decades.

Table 2: Trend TFP Growth (period averages)

<table>
<thead>
<tr>
<th>Year</th>
<th>EA</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-90</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>1991-95</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>1996-00</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>2001-05</td>
<td>0.8</td>
<td>1.4</td>
</tr>
<tr>
<td>2006-10</td>
<td>0.9</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Source DG ECFIN

According to these trend estimates trend TFP growth has declined between .2 and .3% points over periods of 5 years. This suggests that it is unlikely that there will be a recovery as predicted by the time series model. Notice also, that in the case of the US we observe exactly the opposite trend.

When making predictions one should not only rely on time series evidence but one should ask what are the underlying forces which drive these trends in TFP. There has been some debate on the possible reasons of this declining trend. Especially the following explanations have been put forward:

- Labour market reforms in many Euro area countries may be temporarily depressing TFP because of an increase in low productivity sectors;
- The EU’s existing production structure, which is excessively focussed on low and medium technology industries, may be driving the trends;
- Divergences between EA and US may be due to the superior performance of the US in terms of the production and use of ICT.

All three explanations have a certain validity. Labour market reforms in a number of Euro area Member States, aimed at boosting overall employment rates, may have had a temporary negative effect on productivity. This view is supported by estimates quoted in the 2004 EU Review which on the basis of different methods (a VAR analysis, Quest II simulations and growth regressions) suggested that the increase in employment could only explain 10 to 35% of the productivity slowdown in the EU. This suggests that labour market reforms are only part of the explanation.

Sectoral analysis provides some evidence which favours the second and third hypothesis. As shown in Annex 2. US productivity acceleration has been largely due to extraordinary advances in the production and use if ICT, while productivity growth in other sectors has been similar to the EU. Annex 2 shows that the EU has been doing reasonably well compared with the US in a wide range of manufacturing and service industries over the period 1996-2003. However, the problem is that most of these industries are not making big contributions to overall productivity growth, with the graph indicating a contribution of much less than 0.1%
for most of the industries concerned. For example, while Annex 2 shows that the EU’s chemical industry contributed substantially more to the EU’s overall productivity growth rate as did the equivalent US industry, it nevertheless still contributed only 0.11% points to the EU’s overall total. This is only 1/4 of the contribution of the semiconductor industry to overall US productivity growth. This latter industry in fact contributed nearly 20% of all US productivity growth over the period 1996-2003. This basic story is replicated right across the 56 industries. In the 30, mainly traditional and medium tech, industries where the EU has equalled or outperformed the US since the second half of the 1990’s, apart from communications, all of the remainder are either low productivity growth industries or do not have a large enough share of EU output to alter the EU’s overall productivity performance.

A similar picture can be obtained when we look across EU countries, especially the relationship between ICT production and aggregate productivity growth is striking.  


Source : GGDC, Ameco

To summarise, both explanations for a declining TFP trend have some validity. The first explanation would be less pessimistic in the longer run, however, given the current pace in which structural unemployment declines in Europe, it would take more than a decade before unemployment rates in the order of magnitude of 5% could be reached, which would imply that TFP growth will continue to decline in the medium term as well. If the second hypothesis is true, then we have to live with the fact that traditional manufacturing industries are unlikely to generate the same technological advances as in the past. Acceleration of aggregate TFP growth would instead require substantial sectoral reallocations. This is unlikely to happen in the medium term.

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5 There is big debate on the relative contribution of ICT production and ICT use for aggregate productivity growth. While the table in annex 2 suggests a strong contribution from ICT use as well, cross country and cross regional analysis favours the ICT production link. For a discussion of these issues see for example EU Economy Review 2004.
Conclusion

This paper has presented the current methodology used by the EU Commission to calculate potential output and output gaps. The paper identifies declining TFP growth trends in the majority of euro area member states which are not fully compensated by an increasing employment content of growth as the major reason for the recent slowdown in growth. It also discusses the medium term projections which currently extend to the year 2010. Given past trends and the history of downward potential growth revisions, the projections are on the optimistic side.

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This annex discusses various issues related to the NAIRU. First of all, it provides a description of the theoretical framework underlying the NAIRU estimates. It starts from a standard model of the labour market with explicitly formulated wage and labour demand equations. In particular it is shown how the Phillips curve, which links the change of wage inflation to the unemployment gap, is shifted by observed and unobserved shocks to the wage rule and the labour demand equation. Within this context the concept of structural unemployment or NAIRU can be discussed more clearly. This derivation also allows one to provide an economic interpretation for differences between the Euro Area and US labour markets.

1. The Labour Market Model

Following standard textbooks, there are broadly four different hypotheses trying to describe the labour market: the neoclassical view, the efficiency wage approach, the wage bargaining theory and the search model. A generic wage rule covering all four hypothesis can be formulated as follows.

\[ w_t - p_t^e = a_0 + (1 - \mu)b_t^e + \mu p_t^e - \beta u_t + a^w_t \]  

Workers/trade unions negotiate a nominal wage \( w_t \) at time t conditional on the price expectation \( p_t^e \), on the expected level of the reservation wage \( b_t \), on expected productivity \( p_t = y_t - l_t \)

\(^6\) and on the unemployment rate \( u_t \). The term \( a^w_t \) is a shock to the wage-setting rule that can be autocorrelated. As shown by Pissarides (1999), the four macroeconomic theories imply certain restrictions on the parameter values of equation (1): both the neoclassical and the efficiency wage models imply \( \mu = 0 \), i.e. wages are not directly linked to productivity. The wage bargaining and the search model allow instead for productivity to play a role. Within this latter class of models, the magnitude of productivity indexation depends crucially on the bargaining strength of workers. In an atomistic labour market without any market power for workers such as in the neoclassical model, wages would be equal to the reservation wage. By contrast, in a highly unionised labour market, \( \mu \) would approach unity.

Theories also differ in the specification of the reservation wage. In the neoclassical model the reservation wage would be the value of leisure, a concept derived from a utility function for workers which is defined in terms of consumption and leisure. Consequently, in the neoclassical model, consumption and leisure time would be the arguments of \( b_t \). While the value of leisure could also play a role under the other hypotheses, these generally stress a non-market wage as an alternative. The non-market wage could be for instance unemployment benefits, the value of home production or the income earned in the shadow economy.

\(^6\) The notion of productivity entering the wage equation will be discussed in more detail later.
Another important element is the concept of productivity entering the wage equation, namely either average labour productivity or “marginal productivity”. Under the neoclassical model, the search and efficiency wage hypothesis, the relevant concept seems to be “marginal productivity” while in bargaining models an average productivity concept applies. As will be shown below in situations where average and marginal productivity diverge, the two productivity concepts have implications for the structural unemployment rate and also for the short run adjustment of wages. This wage rule as expressed in eq (1) is very similar to the rule formulated by Blanchard and Katz (1999). Here two generalisations are introduced, first it is assumed that expectations not only have to be formulated about prices but also about the reservation wage and productivity and we allow for slightly more general expectation formation schemes. The second generalisation concerns the concept of productivity which enters the wage rule. We will explicitly distinguish between the average and marginal product of labour.

In order to close the model, labour demand must be specified. It is assumed that firms set labour demand at its profit maximising level by equating the marginal revenue product of labour to the real wage. The resulting first order condition of the optimisation problem is given by equation (2).

\[ w_t - p_t = (y_t - l_t) + x_t \]  

(2)

It can be interpreted in two directions. Starting from the right hand side, eq. (2) determines the “demand wage for labour”, which is the wage the firm is willing to pay for a given level of marginal productivity. Alternatively, for given real wages it determines the marginal product of labour the firm is aiming for. Notice, marginal and average productivity are not always proportional. The term \( x \) can drive a wedge between marginal and average productivity. One can think of the variable \( x \) as a shock to a (long run) labour demand equation (as implied by the underlying Cobb Douglas PF) by simply rewriting (2) as

\[ l_t = y_t - (w_t - p_t) + x_t \]  

(2')

The variable \( x \) can itself be a function of various factors and it is useful to distinguish between a structural \( (x^*) \) and a cyclical/transitory component \( (\rho) \)

\[ x_t = x^*_t + \rho_t \]  

(3)

After having determined the demand wage of firms one can ask the question what is the productivity concept used by workers in their wage schedule. In particular, do they take into account shocks to labour demand, when setting wages? We are not imposing an a priori restriction about the concept of productivity used by workers in setting wages and define the concept of productivity entering the wage rule as

\[ pr_t = (y_t - l_t) + \psi x_t, \quad 0 \leq \psi \leq 1. \]  

(4)

Marginal productivity and the demand wage for labour are used interchangeably. The term marginal productivity is not entirely correct. Marginal productivity corrected for the mark-up of prices over marginal cost would be the correct expression.
We also express the reservation wage as a fraction of a combination of productivity and \( x \),

\[
b_t = b_t^0 + (y_t - l_t) + \psi x_t
\]

(5)

where \( b_t^0 \) is the logarithm of the replacement rate. Notice that as \( b_t^0 \) is allowed to vary over time, equation (5) is not restricting the dynamics of the reservation wage.

**Adjustment of wages to inflation and productivity**: Adjustment of wages to economic conditions can be delayed because of limited information in the formation of expectations or because of institutional rigidities (e.g. a fixed contract length). With the annual data used here, we try to capture two extremes. Either instantaneous adjustment of wages to both inflation and productivity, i.e. adjustment within the same period (one year) or completely backward looking behaviour where wages only respond with a lag of one year. Such an extreme case could occur for example if wage contracts were negotiated at the beginning of each year with a duration of one year and where workers/trade unions would simply extrapolate inflation or productivity trends from the previous year. Any parameter setting between these two extremes is of course possible and is determined by the coefficients \( a \) and \( c \) in the following expectation formulas

\[
\pi_t^e = a \pi_{t-1} + (1 - a) \pi_{t-1}^e
\]

(6a)

\[
\Delta pr_t^e = c \Delta pr_{t-1} + (1 - c) \Delta pr_{t-1}^e.
\]

(7a)

The degree of nominal rigidity is proportional to \((1 - a)\) while the degree of real rigidity is proportional to \((1 - c)\). However, for some countries the unemployment gap appears in the Phillips curve as a quasi difference. This cannot be generated with the moving average scheme, therefore we also allow for adaptive expectations schemes of the following form

\[
\pi_t^e = a \pi_{t-1} + (1 - a) \pi_{t-1}^e
\]

(6b)

\[
\Delta pr_t^e = c \Delta pr_{t-1} + (1 - c) \Delta pr_{t-1}^e.
\]

(7b)

We also allow for combinations between (6,7a) and (6,7b) in the regressions. Equations (1) to (7) determine the structural unemployment rate which is defined as the level of unemployment when there are no expectation errors, i.e. \( \pi_t^e = \pi_t \), \( pr_t^e = pr_t \) and where the wage share is equal to its long run level, i.e. \( x_t = x_t^* \). Under these conditions, the equilibrium unemployment rate is given by

\[
u_t^* = \left[ a_0 + (1 - \mu) b_t^0 + (\psi - 1) x_t^* \right] / \beta
\]

(8)

Equation (8) shows that the equilibrium level of unemployment depends positively on the reservation wage (which itself is a function of labour taxation, unemployment replacement rate etc.), and negatively on the trend value of the labour demand shock, if workers do not completely take into account \( x_t^* \). This sounds intuitively plausible. Imagine, for example, an increase in the average training costs for workers. This obviously is a cost component for firms related to individual workers. In determining labour input, the firm must take these costs
into account. If wages do not respond to an increase in training costs then effective labour costs increase and firms respond with a decline in labour demand. If the increase in training costs is permanent then the equilibrium level of unemployment will be higher. If training costs are borne by workers in the form of lower wages ($\psi = 1$), equilibrium unemployment will not be affected.

A relationship between the change in nominal wage inflation and the unemployment gap can be derived, with shocks to labour productivity, labour demand and the terms of trade as additional explanatory variables.

$$
\Delta^2 w_t = \phi^{pr} \Delta^2 (y_t - l_t) + \phi^{ws} \Delta^2 w^*_t + \phi^{tot} \Delta^2 \text{tot}_t - \beta (u_t - \text{nairu}_t) + \nu^w_t
$$

and the reduced form coefficients of the Phillips curve linked to the structural coefficients as follows

$$
\phi^{pr} = \frac{(c - a)}{(1 - a)}
$$

$$
\phi^{ws} = \frac{(c - a) + (\psi - 1)(c - 1)}{(1 - a)}
$$

$$
\phi^{tot} \geq 0.
$$

The error term $\nu^w_t$ is a function of $a^w_t$, $\rho_{-1}$ and $\Delta x^*_t$.

The Phillips curve shows the short run response of nominal wages to labour productivity, labour demand shocks and the unemployment gap. The response to the unemployment gap is intuitively plausible. Whenever unemployment is above the NAIRU, nominal wage growth will decelerate and vice versa. However, this link is not perfect but is disturbed by observed and unobserved shocks to the wage rule and the labour demand equation. How nominal wage growth responds to productivity and labour demand shocks (here approximated by changes in the growth rate of the wage share) depends on a variety of factors.

**Short run nominal wage response to productivity shocks**: Nominal wages respond to a shock in productivity via two channels, a productivity channel and an inflation channel. The strength of the response depends on how strongly inflation and productivity expectations respond to the productivity shock within the first year. Everything else equal, wages respond positively to productivity (with an elasticity of $c$) but they respond negatively to the extent that productivity affects inflation (with an elasticity equal to $-a$). Whether the response is positive or negative actually depends on the relative magnitude of $c$ and $a$.

**Short run response of nominal wages to changes in labour demand shocks**: A similar consideration applies to labour demand shocks. The wage rule implies that wages respond positively to labour demand shocks (to the extent they are taken into account by workers (namely by the size of $\psi$)). The difference compared to the productivity response comes from the parameter $\psi$. If wages only respond to productivity but not to the demand wage for
labour \((\psi = 0)\) then there will be no positive transmission of a labour demand shock. Turning to the other extreme case, with wages responding fully to the labour demand shock, then the elasticity of wages w. r. t. labour demand shocks will be equal to the productivity response of wages. However, in general, the magnitude of the positive response of wages to labour demand shocks will be smaller compared to labour productivity shocks. Also notice from the inflation rule that labour demand shocks have a negative effect on inflation. Therefore to the extent wages respond to current inflation (negatively) they also respond negatively to labour demand shocks via the inflation channel.

**Short run response of nominal wages to terms of trade (TOT) shocks**: The theoretical derivation of the wage equation was done in a closed economy context. Obviously open economy aspects are likely to play a role in wage setting, especially if there is a divergence between domestic and import prices and if wages are linked to the consumer price deflator. To the extent that these conditions are fulfilled one would also expect that nominal wages respond positively to the wedge between consumer price and GDP inflation. In order to capture this open economy aspect, a TOT variable is added to the Phillips curve.

2. Comparing the Euro Area and US Phillips Curves

The Phillips curve estimates (Table 1) appear fairly similar for the Euro Area and the US, both in terms of the absolute size of the parameter estimates as well as the statistical fit. The response of nominal wages to the unemployment gap is practically identical. Also labour productivity has a quantitatively similar effect on wages. This result suggests that there is a positive short run response of nominal wages to labour productivity growth. The elasticity within a year is about 0.8 in both regions. This suggests that the productivity response by far exceeds the inflation response. This seems plausible. First of all, central banks tend to accommodate productivity shocks and keep inflation stable. Therefore productivity shocks can lead to higher real wage growth via nominal wage growth. Notice also, we are looking at wages per employee and changes in hours are consequently translated directly into changes in wages. Also to the extent that workers receive piece rate wages, the translation of productivity to changes in wages is direct. Finally, to the extent that wages are negotiated at the sectoral or even the firm level, information about local productivity might be more easily available than information about aggregate inflation.

Interestingly there is a difference in the response of wages to the wage share. US wages respond more to the demand wage for labour, while European wages tend to adjust more strongly to average productivity growth. This could reflect differences in labour market institutions and points to a difference in the response of wages to cyclical versus structural shocks. The US aggregate wage response seems typical of a decentralised labour market, while the European wage response seems more typical of a centralised labour market. Suppose shocks to labour demand result from sectoral shifts, with an increase in production in a low productivity sector (e.g. services). At the aggregate level this can be represented as a decline in the demand wage for labour. However, employment can be sustained if real wages in the low productivity sector declines. If the labour market would ignore the negative demand (productivity shock) then the low productivity sector would not expand. Thus the Phillips curve estimates suggest that the EU and US labour markets respond fairly similarly to typical business cycle movements but they tend to respond differently to structural shocks. There is other micro and macro evidence from the empirical literature which suggests that the
response of wages and prices to the business cycle is similar. In particular some recent micro
studies suggest that the degree of nominal rigidity (e.g. measured by the duration of price
changes by firms as well as the duration of wage contracts) is similar across the two regions.

In terms of statistical criteria, the unemployment gap together with the observed shock
variables, as suggested by the theoretical model, explain about 50% of the observed
fluctuations in the first difference of wage inflation in the case of the EU and about 70% in
the case of the US. A more intuitive way to look at the empirical fit is to compare fluctuations
in the (inverse of the) unemployment gap with fluctuations in wage indicators. The Phillips
curve predicts that episodes in which the NAIRU is above the actual unemployment rate
should be associated with an acceleration in wage inflation (see graph 1). The following table
provides correlations between the unemployment gap as identified by our methodology and
three alternative wage concepts, firstly nominal wages, secondly unit labour costs and thirdly
unit labour costs adjusted for changes in the wage share. All three concepts of wage inflation
show correlations with the (inverse) of the unemployment gap of a similar magnitude for the
two regions.

<table>
<thead>
<tr>
<th>Table 1: Correlation between unemployment gap and wage indicators</th>
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<tbody>
<tr>
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<td></td>
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<tr>
<td>Δ²(Wage)</td>
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<tr>
<td>Δ²(ULC)</td>
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<tr>
<td>Δ²(ULC-Wage Share)</td>
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</tbody>
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The following graph provides information about the evolution of the NAIRU in the euro area
and the US as well as the fit of the Phillips curve. The two graphs on the top show the NAIRU
for the Euro Area and the US as estimated with our model. Euro Area structural
unemployment has peaked in the mid 1990s in the EU and is slightly declining since. In
contrast, the US NAIRU has been on a steady decline since the early 1980s and is stabilising
at a level of about 5 ½% since 2000. The two graphs in the middle show how the (inverse) of
the unemployment gap is correlated with the change in wage inflation. While there is a
relationship, the Phillips curve specification suggests that there are other factors influencing
the change in wage inflation. This is shown by the two graphs at the bottom which indicate
the fit of the Phillips curve.
Graph 1: Euro Area (EA12) and US NAWRU estimations

Graph showing the comparison of Euro Area (EA12) and US NAWRU estimations over a period from 1980 to 2006.
ANNEX IB: DESCRIPTION OF THE NAIRU ESTIMATION METHOD USED FOR THE NEW MEMBER STATES

For calculating the NAIRU for the new Member States, a methodology proposed by the OECD is used (i.e. the “Elmeskov” method\textsuperscript{8}). A simple model of the labour market predicts that expected real wages will rise whenever the unemployment rate is below the NAIRU and vice versa.

\[ \Delta w_t - \pi_t^e = -\beta (u_t - nairu) + u_t^w \]  

(1a)

Assuming static inflation expectations

\[ \pi_t^e = \Delta w_{t-1} \]  

(2)

gives the following Phillips curve relationship

\[ \Delta^2 w_t = -\beta (u_t - nairu) + u_t^w \]  

(1b)

This is the most simple formulation of the Phillips curve which ignores all other possible influences on wage setting such as productivity, for example. Allowing for productivity shocks may be important for at least two reasons. Firstly, the new Member States show relatively high growth rates of productivity, thus productivity growth may be an important factor for wage growth. Secondly, for the EU15 member states we control for productivity shocks as well. The following paragraphs therefore present a simple extension of the framework presented above.

The wage rule (following the specification that we use for the EU15 member states) is given by

\[ w_t - p_t^e = (y_t - l_t) - \beta (u_t - nairu) + u_t^w \]  

(3a)

This can be rewritten as

\[ \Delta w_t - \pi_t^e = (y_t - l_t) - (w_{t-1} - p_{t-1}) - \beta (u_t - nairu) + u_t^w \]  

(3b)

Labour demand can be formulated as follows

\[ w_t - p_t = (y_t - l_t) + u_t^l \]  

(4)

With static inflation expectations we obtain

\[ \pi_t^e = \Delta w_{t-1} - (\Delta y_{t-1} - \Delta l_{t-1}) \].

Using this expectation rule together with the labour demand schedule one can reformulate the wage equation as follows

\textsuperscript{8} J. Elmeskov (1993)
\[ \Delta^2 w_i = (\Delta^2 y_i - \Delta^2 l_i) - \beta (u_i - \text{NAIRU}) + u_{w}^i - u_{l - 1}^i \]  
\text{(3c)}

or

\[ \Delta^2 ulc = \Delta^2 w_i - (\Delta^2 y_i - \Delta^2 l_i) = -\beta (u_i - \text{NAIRU}) + u_{w}^i - u_{l - 1}^i \]  
\text{(3d)}

This formulation indicates that unemployment is below the NAIRU whenever the growth rate of unit labour costs increases.

Given these alternative expressions (1b) and (3d), the calculation of the NAIRU proceeds in the following steps.

**STEP 1**: Assuming the NAIRU to be constant and further assuming that the shocks have a mean of zero one can calculate \( \beta \) by taking a first derivative (in discrete time) of eq. (1b) and (3d) with respect to the unemployment rate

\[ \beta_{ulc} = \frac{\Delta^2 ulc}{\Delta lur} \quad \text{and} \quad \beta_w = \frac{\Delta^2 w_i}{\Delta lur} \]  
\text{(4)}

**STEP 2**: With these estimates we can solve eq. (1b) and (3d) for the NAIRU (plus shock)

\[ \text{NAIRU} - u_i = lur_i + \frac{\Delta^2 w_i}{\beta_w} \quad \text{or} \quad \text{NAIRU} - u_i = lur_i + \frac{\Delta^2 ulc_i}{\beta_{ulc}} \]  
\text{(5)}

**STEP 3**: Now one can use the right hand sides of eq. (5) to calculate the NAIRU by noticing that the right hand side is equal to the NAIRU and the shocks to the wage setting and labour demand equations. These shocks are eliminated by applying a filter to the right hand side. Notice, this procedure becomes arbitrarily close to applying a filter to the unemployment rate directly for large enough \( \beta \).
Annex 2: Industry Level Analysis


-0.1%
0.0%
0.1%
0.2%
0.3%
0.4%
0.5%

USA
EU15

ICT Producing
ICT Using
Intensive ICT Producing
Intensive ICT Using
Less Intensive ICT Producing
Less Intensive ICT Using

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ICT Using
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