

Estimating the Term Structure with Macro Dynamics in a Small Open Economy*

Fousseni Chabi-Yo
Bank of Canada

Jun Yang
Bank of Canada

April 18, 2006

Preliminary work. Please do not quote without permission.

*The paper represents the views of the authors and should not be interpreted as reflecting the views of the Bank of Canada.

Abstract

We study the joint dynamics of bond yields and macroeconomic variables in a New-Keynesian small open economy model complemented with a no-arbitrage term structure model. With Canadian data, we are able to study the impact of domestic and foreign (US) shocks on the yield curve. The unconditional variance decomposition of the yield level show that the movement of expected short rates is mainly driven by US macroeconomic shocks. The majority of the variation of the yield risk premium are also driven by US macroeconomic shocks. However, the Canadian monetary policy shocks can explain a small proportion of the variation of the short to medium yield risk premium. In addition, the Canadian monetary policy shocks and US aggregate demand shocks explain a majority of the variation of the expected excess holding period returns of short to medium bonds. The expected excess holding period returns of long term bonds are mainly driven by US aggregate supply shocks.

1. Introduction

This paper investigates the economic determinants of the movement of the term structure of the interest rates in a small open economy (SOE). We estimate a no-arbitrage term structure model with the dynamics of macroeconomic variables in a new-Keynesian small open economy framework. The macro-finance modeling strategy developed by Ang and Piazzesi (2003) is implemented with both Canadian (proxy for a SOE) and US (proxy for the rest-of-world) data. We find that the US macroeconomic shocks contribute to a larger proportion of the variation of the yield curve and the yield premium than the Canadian macroeconomic shocks. In addition, the Canadian monetary policy shocks and US aggregate demand shocks explain a majority of the variation of the expected excess holding period returns of short to medium bonds. The expected excess holding period returns of long term bonds are mainly driven by US aggregate supply shocks.

Many finance models have used latent variables to explain term structure fluctuations. For example, Litterman and Scheinkman (1991) find that three principle factors can explain most of the variation in bond returns, and they label these factors "level", "steepness", and "curvature". Chen and Scott (1993) estimate one-, two-, and three-factor CIR models, and find that only the three-factor model can capture the changes in the level, slope and curvature of the yield curves. Pearson and Sun (1994) estimate an extended two-factor CIR model by utilizing the conditional distribution of the state variables. They label the state variables "short rate" and "inflation" even though they do not use inflation data to estimate these factors. Recent stochastic volatility models, such as Balduzzi, Das, Foresi, and Sundaram (1996), Anderson and Lund (1998), and Dai and Singleton (2000), introduce one or two state variables to capture the conditional volatility of the short-term interest rate. Consequently, they call these state variables "volatility factors". All of the models described above are developed under the assumption of no-arbitrage, and they can capture some important features of the short-term interest rate by using the latent factors. However, they fail to explain what macroeconomic variables directly affect these latent variables, and hence determine the movement of the term structure of interest rates.

In a different approach, many empirical studies use Vector Autoregressive (VAR) models to

explain the joint behavior of the term structure of interest rates and macroeconomic variables. For example, Campbell and Ammer (1993) use a VAR model to study the excess stock and bond returns, and their results show that stock and bond returns in US are driven largely by news about future excess stock returns and inflation. Evans and Marshall (2001) also use a VAR model to investigate the impacts of monetary and real shocks on various interest rates. They find that the shocks to monetary policy have a pronounced but transitory impact on short-term interest rates, with almost no effect on long-term interest rates. In contrast, the shocks to employment have a long-lived impact on interest rates across the maturity spectrum. The VAR model enables them to examine the impacts of macroeconomic variables on various interest rates through impulse response functions. However, there are several disadvantages to using the VAR models to study the term structure of interest rates. First, one can only study the effects of macroeconomics variables on those yields of maturities that are included in the model. The VAR models do not describe how yields of maturities not included will respond to changes in the macroeconomic variables. Second, the predicted movements of the yields with different maturities in the VAR models may not rule out arbitrage, since the unrestricted VAR models do not require that the movement of various interest rates provide no-arbitrage opportunities.

An “arbitrage-based” term structure model provides a complete description of how the yields of all maturities respond to the shocks to the underlying state factors, although it cannot identify the sources of those shocks. In contrast, the empirical VAR models can identify the economic sources of the shocks to the selected yields, but they cannot tell how the entire yield curve will respond to those shocks. Recently, some authors have tried to combine the strength of both the “arbitrage-based” term structure models and the VAR models to describe the movement of the yield curve. Ang and Piazzesi (2003) incorporate both macroeconomic variables and latent variables into a Gaussian diffusion model of the term structure of interest rates. They find that macro variables explain a significant amount of the variation in bond yields, and that incorporating macro variables into the model with latent variables improves the out-of-sample forecast. Other papers include Dewachter and Lyrio (2004), Rudebush and Wu (2004), Ang, Piazzesi, and Wei

(2004), Ang, Dong, and Piazzesi (2005), Hördahl, Tristani, and Vestin (2003), Dai and Philippon (2004), and Bakaert, Cho, and Moreno (2003). All these papers study the joint dynamics of bond yields and macroeconomic variables in a closed economy framework.

In this paper, we investigate the joint dynamics of bond yields and macroeconomic variables in a small open economy framework. In an open economy, the real exchange rate movements play an important role in the transmission process that links foreign disturbances to domestic output and inflation movements. The real exchange rate movements induce substitution effects between domestic and foreign goods, thereby influencing aggregate demand and supply. In addition, the monetary authorities may systematically adjust short-term interest rate according to the real exchange rate movements (Ball(1999), Clarida et al. (2001), and Svensson(2001)). To understand the effects of foreign shocks on the domestic economy, one needs to investigate the interaction between the real exchange rate and domestic output, inflation and interest rate. We construct a small scale linear macro model to study the dynamics between domestic and foreign macroeconomic variables. The domestic yield curve is modeled in the affine term structural framework with essential affine risk premium. The price of risk depends on both domestic and foreign macroeconomic variables. Dong (2005) incorporates macro variables as factors in a two-country term structure model. The movement of the exchange rate is pinned down by no-arbitrage condition in the domestic and foreign bond markets. In his setup, the short-term interest rate does not response to the movement of exchange rate. In addition, he focuses on explaining exchange risk premium instead of identifying economic determinants of the movement of the domestic yield curve.

Our main findings are as follows. The variance decomposition results show that the expected movement of the Canadian short rate is mainly explained by US macroeconomic shocks. In short horizons, it is mainly driven by US aggregate demand and monetary policy shocks. In long horizon, it is mainly driven by US aggregate supply shocks. The Canadian monetary policy shocks explain from 30-60% of the one-quarter ahead variation of risk premium embedded in yields. The explanatory power is reduced to 10-25% range in long horizons. On the contrary, the explanatory power of the US macroeconomic shocks increases with forecasting horizons. They explain up to 75% of

the unconditional variation of risk premium embedded in Canadian yields. The same result holds for expected excess holding period returns of Canadian bonds.

The remainder of the paper is organized as follows. Section 2 and 3 outline the structural macroeconomic model the term structure model respectively. Section 4 describes the data used in the paper. We present our estimation methods and results in section 5. Section 6 concludes.

2. Macroeconomic Model

Our structural model contains seven equations. The first three equations are: The rest-of-world (ROW) aggregate supply equation, the aggregate demand equation and monetary policy rule. The fourth equation characterize the exchange rate dynamic. The last three equations are: The SOE aggregate supply equation, the aggregate demand equation and monetary policy rule. The ROW is considered as the closed economy with an assumption that the SOE shocks do not affect the ROW, whereas the ROW shocks affect the SOE. As shown in Woodford (2003), the ROW set of equations can be formulated with explicit micro-foundations as a general equilibrium model. Indeed, Sevansson (1998) shows that the SOE set of equations can also be obtained with micro-foundations as a general equilibrium model.

2.1 Closed Economy

2.1.1 Aggregate Supply

The aggregate supply equation is the generalization of the supply equation developed by Calvo (1983)¹.

$$\pi_t^* = \alpha_0^* + \alpha_{\pi^*} \pi_{t-1}^* + (1 - \alpha_{\pi^*}) E_t \pi_{t+1}^* + \alpha_{g^*} g_t^* + \varepsilon_t^{\pi^*} \quad (1)$$

where π_t^* is the inflation between $t - 1$ and t , g_t^* is the output gap. $\varepsilon_t^{\pi^*}$ is the aggregate supply structural shock. The aggregate supply dynamics (1) is derived in a pricing framework with monopolistic competition in the intermediate good markets. The AS equation links inflation to future expected inflation and the real marginal cost with an assumption that the output gap is proportional to the marginal cost. The endogenous persistence in the AS equation is obtained with an

assumption that the fraction of price-setters which does not adjust prices optimally indexes their prices to past inflation. The coefficient α_{y^*} is the Phillips curve parameter.

2.1.2 Aggregate Demand

In a closed economy, the aggregate demand is usually derive from the first order conditions for a representative agent in a general equilibrium model such as Lucas (1978). Since standard approaches fail to match the well-known persistence in the output gap. To match the persistence in the output gap and pin down the risk aversion parameter, recent studies, among others, Fuhrer (2000) and Cho and Moreno (2005) derive an alternative IS equation from a utility maximizing framework with external habit formation. We follow Fuhrer (2000) and Cho and Moreno (2005) and specify the aggregate demand dynamics as:

$$g_t^* = \beta_0^* + \beta_g^* g_{t-1}^* + (1 - \beta_g^*) E_t g_{t+1}^* - \beta_r^* (r_t^* - E_t \pi_{t+1}^*) + \varepsilon_t^{g^*} \quad (2)$$

where r_t^* is the short-term interest rate. The residual $\varepsilon_t^{g^*}$ is the IS or aggregate demand shock. In equation (2), the habit formation specification imparts endogenous persistence to the output gap. The forward-looking parameter β_g^* depends on the level of habit persistence and the risk aversion parameter.

2.1.3 Monetary Policy Rule

The monetary authority set short-term interest rate according to a simple Taylor rule (Taylor(1999)):

$$r_t^* = \gamma_{MP}^* + (1 - \rho^*)(\gamma_{\pi^*} \pi_t^* + \gamma_{y^*} g_t^*) + \rho^* r_{t-1}^* + \varepsilon_t^{r^*}. \quad (3)$$

where γ_{MP} is a constant, ρ^* is the smoothing parameter. $\varepsilon_t^{r^*}$ is the monetary policy shock. The Central Bank reacts to high inflation and to deviations of output from its trend. The parameter γ_{π^*} measures the response of the Central Bank to inflation, while γ_{y^*} describes its reaction to output gap fluctuations.

2.2 Small Open Economy

2.2.1 Aggregate Supply

The aggregate supply equation (Phillips curve) describes the short run inflation dynamics. The aggregate supply equation is of the Phillips curve type estimated by Svensson (1998):

$$\pi_t = \alpha_0 + \alpha_\pi \pi_{t-1} + (1 - \alpha_\pi) E_t \pi_{t+1} + \alpha_g g_t + \alpha_q (q_t - q_{t-1}) + \varepsilon_t^\pi \quad (4)$$

where π_t is inflation between $t - 1$ and t , q_t is the real exchange rate and g_t is the output gap. A higher q_t denotes a depreciation of the SOE currency. This supply type equation is derived in Svensson (1998), from the first order condition of an optimization problem and hence, with some microfoundations. Inflation depends on lagged inflation, expected future inflation, the current output gap and the change in the real exchange rate. It is similar to Fuhrer and Moore (1995) type Phillips curve in that inflation depends on both lagged inflation and expected future inflation. The timing on exchange rate changes reflects an assumption of instant pass-through. The rest-of-the world (hereafter ROW) shocks are transmitted to the SOE inflation mainly through the exchange rate. A zero value of α_q can be interpreted as perfect pricing to market.

2.2.2 Aggregate Demand

The aggregate demand equation is an aggregate demand type of equation developed by Svensson (2000):

$$g_t = \beta_0 + \beta_g g_{t-1} + (1 - \beta_g) E_t g_{t+1} - \beta_r (r_t - E_t \pi_{t+1}) + \beta_q (q_t - q_{t-1}) + \beta_g^* y_t^* + \varepsilon_t^g \quad (5)$$

where g_t is the output gap, r_t is the short-term interest rate. ε_t^g is the aggregate demand shock. The aggregate demand equation is derived, from a first order condition consistent with optimization and hence with some microfoundations, and discussed in further detail in Svensson (1989). The output gap equation provides a description of the dynamics of aggregate demand, which is assumed to be affected by movements in the short term real interest rate, the real exchange rate and the foreign output gap. The forward looking term captures the inter-temporal smoothing motives characterizing consumption. A similar specification was recently used by Giordani (2004) to

evaluate New-Keynesian models of a SOE. The rest-of-the world shocks are transmitted to the SOE aggregate demand through the exchange rate and the ROW macro variables..

2.2.3 Monetary Policy Rule

We assume that the monetary authority specifies the short-term interest rate according to the following reaction function

$$r_t = (1 - \rho)(\gamma_\pi \pi_t + \gamma_g g_t + \gamma_q (q_t - q_{t-1}) + \gamma_{r^*} r_t^*) + \rho r_{t-1} + \varepsilon_t^r \quad (6)$$

The lagged interest rate captures the well known tendency of the monetary authority towards smoothing interest rate. This formulation assumes that the ROW shocks are transmitted to the SOE interest rate through the ROW monetary policy.

2.2.4 Real Exchange Rate

Uncovered Interest Rate Parity (UIRP) predicts that high yield currencies should be expected to depreciate. It also predict that, *ceterius paribus*, a real interest rate increase should appreciate the currency. Nevertheless, there appears to be overwhelming empirical evidence against UIRP (see Hodrick (1987) and Engel (1996)). Furthermore, in New Open Economy Macroeconomic Models, domestic and foreign macro-variables enter the exchange rate equation in differences: Engel and West (2004) assume that exchange rate is a function domestic and foreign macro variables. Given the empirical evidence against UIRP and that SOE and the ROW macro-variables enter the real exchange rate equation, we consider the following exchange rate dynamic:

$$q_t = q_{t-1} + \delta_r [(r_t - E_{t-1}\pi_t) - (r_t^* - E_{t-1}\pi_t^*)] + \varepsilon_t^q \quad (7)$$

where ε_t^q captures the real exchange rate shock. Both domestic and foreign structural shocks are assumed to be independent and identically distributed with homoscedistic variances.

Bringing together all the macroeconomic equations: (5), (7), (4), (6), (2), (1), and (3), we

obtain a seven variables system:

$$\begin{aligned}
\pi_t^* &= \alpha_0^* + \alpha_{\pi^*} \pi_{t-1}^* + (1 - \alpha_{\pi^*}) E_t \pi_{t+1}^* + \alpha_{g^*} g_t^* + \varepsilon_t^{\pi^*} \\
g_t^* &= \beta_0^* + \beta_g^* g_{t-1}^* + (1 - \beta_g^*) E_t g_{t+1}^* - \beta_r^* (r_t^* - E_t \pi_{t+1}^*) + \varepsilon_t^{g^*} \\
r_t^* &= \gamma_{MP}^* + (1 - \rho^*) (\gamma_{\pi^*} \pi_t^* + \gamma_{y^*} g_t^*) + \rho^* r_{t-1}^* + \varepsilon_t^{r^*} \\
q_t &= q_{t-1} + \delta_r [(r_t - E_{t-1} \pi_t) - (r_t^* - E_{t-1} \pi_t^*)] + \varepsilon_t^q \\
\pi_t &= \alpha_0 + \alpha_{\pi} \pi_{t-1} + (1 - \alpha_{\pi}) E_t \pi_{t+1} + \alpha_g g_t + \alpha_q (q_t - q_{t-1}) + \varepsilon_t^{\pi} \\
g_t &= \beta_0 + \beta_g g_{t-1} + (1 - \beta_g) E_t g_{t+1} - \beta_r (r_t - E_t \pi_{t+1}) + \beta_q (q_t - q_{t-1}) + \beta_{g^*} y_t^* + \varepsilon_t^g \\
r_t &= \gamma_{MP} + (1 - \rho) (\gamma_{\pi} \pi_t + \gamma_g g_t + \gamma_q (q_t - q_{t-1}) + \gamma_{r^*} r_t^*) + \rho r_{t-1} + \varepsilon_t^r
\end{aligned}$$

We summarize our macroeconomic model in matrix form:

$$A_{11} X_t = \alpha + B_{11} E_t X_{t+1} + B_{12} X_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \rightsquigarrow N(0, \Sigma \Sigma^\top) \quad (8)$$

where $X_t = (\pi_t, g_t, r_t, q_t, \pi_t^*, g_t^*, r_t^*)'$ and $\varepsilon_t = (\varepsilon_t^{\pi}, \varepsilon_t^g, \varepsilon_t^r, \varepsilon_t^q, \varepsilon_t^{\pi^*}, \varepsilon_t^{g^*}, \varepsilon_t^{r^*})'$. The coefficients of matrix A_{11} , B_{11} and B_{12} are defined by the structural equations of the domestic and foreign country macro-economic variables. These coefficients are given by $\Sigma \Sigma^\top$ is diagonal matrix with constant variances. As can be seen in this model, the ROW shocks are transmitted to the SOE mainly through the real exchange rate and the ROW macro variables.

Under regularity conditions, the solution of (8) is based on the Schur decomposition can be obtained numerically following the methodology described in McCallum (1998). Following an Undetermined Coefficient (UC) approach (see McCallum 1998), the solution of (8) is:

$$X_t = c + \Omega X_{t-1} + \Gamma \varepsilon_t \quad (9)$$

The reduced form (9) implied by the structural model (8) is a VAR of order 1 with nonlinear parameters. We now add the term structure to the model described by equation (9).

3. Adding the Term Structure Information to the Macro Model

We follow the standard dynamic arbitrage-free term structure literature and define the SOE nominal pricing kernel as

$$m_{t+1} = \exp(-r_t) \frac{\xi_{t+1}}{\xi_t} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)$$

where ξ_{t+1} is assumed to follow the log-normal process with:

$$\xi_{t+1} = \xi_t \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)$$

where λ_t are the time-varying market prices of risk associated with the source of uncertainty ε_{t+1} in the economy. The market price of risk parameter is commonly assumed to be constant in Gaussian models or proportional to the factor volatilities (e.g. Dai and Singleton, 2000). However, recent research has highlighted the benefits in allowing for a more flexible specification of the market price of risk. We therefore decide to parameterize λ_t as a linear function of X_t

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{10}$$

where X_t is defined by (9). (10) relates the shocks in the underlying macro to ξ_{t+1} . The last equation shows that the source of uncertainty in the SOE pricing kernel is driven by the shocks in macro variables and short-term interest rate. X_t is a vector containing seven variables. Note that in a micro-founded framework (Bekaert, Cho and Moreno (2005)), the pricing kernel would be linked to consumer preferences rather than being postulated exogenously as in (10). We prefer (10) because the affine prices of risk specification in (10) has been used by, among others, Ang and Piazzesi (2003), Dai and Singleton (2002). The later authors demonstrate that the flexible affine price of risk specification is able to capture patterns of expected holding period returns on bonds that we observe in data.

The constant risk premium parameter λ_0 is a 7×1 vector column while the time varying risk premium parameter λ_1 is a 7×7 matrix. We assume that time varying risk premium parameter λ_1 is a diagonal matrix. This reduces the number of parameters to be estimated.

If $p_t^{(n+1)}$ represents the price at t of an $n + 1$ -period zero coupon bond in the SOE, then the bond price in the SOE can be computed recursively using the relationship:

$$p_t^{(n+1)} = E_t \left[m_{t+1} p_{t+1}^{(n)} \right] \quad (11)$$

with

$$p_t^{(n)} = \exp \left(\bar{A}_n + \bar{B}'_n X_t \right) \quad (12)$$

where $\bar{A}_1 = 0$ and $\bar{B}'_1 = [0_{1 \times 2}, 1, 0_{1 \times 4}]$ and:

$$\begin{aligned} \bar{A}_{n+1} &= \bar{A}_1 + \bar{A}_n + \bar{B}'_n (c - \Gamma \lambda_0) + \frac{1}{2} \bar{B}'_n \Gamma \Gamma^\top (\bar{B}'_n)^\top \\ \bar{B}'_{n+1} &= \bar{B}'_n (\Omega - \Gamma \lambda_1) + \bar{B}'_1 \end{aligned} \quad (13)$$

Therefore, the bond yields are an affine function of the state variables:

$$y_t^n = -\frac{\log p_t^{(n)}}{n} = A_n + B'_n X_t \quad (14)$$

where

$$A_n = -\frac{\bar{A}_n}{n} \text{ and } B'_n = -\frac{\bar{B}'_n}{n}$$

Let Y_t represents the vector containing the SOE bond yields. Then,

$$Y_t = A_y + B_y X_t \quad (15)$$

Consequently the model that need to be estimated is the following:

$$X_t = c + \Omega X_{t-1} + \Gamma \varepsilon_t \quad (16)$$

$$Y_t = A_y + B_y X_t \quad (17)$$

We define the one-period excess holding period return as

$$rx_{t+1}^{(n)} = \log \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} - r_t = n y_t^n - (n-1) y_{t+1}^{n-1} - r_t \quad (18)$$

and compute the conditional expected excess holding period returns as

$$E_t \left(rx_{t+1}^{(n)} \right) = A_n^x + B_n^x X_t$$

with:

$$A_n^x = -\frac{1}{2}\overline{B}_{n-1}'\Gamma\Gamma^\top\left(\overline{B}_{n-1}'\right)^\top + \left(\overline{B}_{n-1}'\right)^\top\Gamma\lambda_0 \quad \text{and} \quad B_n^x = \Gamma^\top\overline{B}_{n-1}'\lambda_1$$

The excess expected return has two components: The first A_n^x component is not time varying while the second component $B_n^x X_t$ is time varying. The unconditional excess expected holding period return can be computed as $E(A_n^x + B_n^x X_t)$.

In the empirical illustrations, we assume that Canada is the SOE whereas US is the closed economy.

4. Data

We estimate the model with quarterly Canadian yields and Canadian and US macroeconomic data. The macroeconomic data are from 1978:Q4 to 2005:Q3. The macroeconomic variables include inflation, output gap, interest rate and the real exchange rate. The real exchange rate is constructed from the nominal exchange rate and CPI indexes of both countries. The core CPI index data is also used to compute the inflation. The inflation rate is computed as the log difference of the core CPI index between the end and the beginning of each quarter. We measure output gap as the difference between the real GDP and quadratically detrended real GDP. The 3-month T-bill rates are used as the monetary policy instruments in both countries. The yield data are from 1978:Q4 to 2005:Q3, and include zero coupon bond yields of maturities 2, 4, 8, 12, 20, 28, 40 and 60 quarters. A description of the methodology used to derive the yield curves can be found in Bolder, Johnson, and Metzler (2004). Figure 1 plots the macroeconomic variables and Table 1 presents some sample statistics of macroeconomic variables and bond yields. The table shows that the average yield curve is slightly upward sloping during the sample. The standard deviations of yields generally decrease with maturity, and yields are highly persistence. Table 1 also shows that persistence exists in the macroeconomic variables.

5. Estimation and Results

5.1 Estimation Methodology

We implement maximum likelihood estimation technique to estimate macro structural parameters and the time-varying risk premium parameters. Because of the estimation difficulty involved with high dimension maximizing problem, we use two-step estimation technique. In the first step, we estimate macro structural parameters with both US and Canadian data. In the second step, we fix these parameters, and estimate the risk premium parameters with Canadian yield data. The estimation result are presented in Table 2. All the reported standard errors are based on a 3-lag Newey and West (1987) consistent covariance estimator.

5.2 Macro Results

5.2.1 Parameter Estimates

Table 2 presents the parameter estimates of the model and their standard errors. Our estimation yielded a stationary unique solution. Panel A shows the parameter estimates for Canadian macro-variable dynamics.

The first row of Panel A shows the parameter estimates of the Canadian Phillips curve. The Phillips curve parameter estimate has the expected sign, but not statistically significant from zero. The real exchange rate parameter estimate has the wrong sign and not statistically significant from zero. Using US data, previous studies, except Gali and Gertler (1999) and Bekaert, Cho and Moreno (2005), fail to obtain reasonable estimate of the Phillips curve parameter α_g . The forward-looking parameter in the AS equation is estimated close to 0.55 which is consistent to previous finding in the literature. The second row of Panel A shows that the parameter estimates for the Canadian aggregate demand equation. The real interest rate parameter estimates has the wrong sign, and the real exchange rate parameter estimate has the expected sign. They are not statistically significant. The US output gap parameter estimate is positive and statically significant. The US output has a direct positive effect on Canadian out put. The parameter β_g is almost indistinguishable from 0.5 implying that agents put similar weights on expected and past output gap.

The third row shows the Canadian short rate equation parameters. Canadian short rate loads positively on the Canadian inflation, output gap, real exchange rate change, and the US short rate with coefficients of 0.042, 0.042, 0.024, and 1.145 respectively. They are statistically significant. This suggests that the Canadian monetary authority responds strongly to US short rate movement. A 1% contemporaneous inflation increase leads to only 4 bp increase in the Canadian short rate. On the contrary, a 1% US short rate increase leads to 1.17% increase in the Canadian short rate.

Panel B in table 2 shows parameter estimates in the exchange rate equation. The real interest rate differential parameter estimate is negative and statistically significant. It is consistent with many empirical findings that the uncovered interest rate parity does not hold.

Panel C shows the parameter estimates for the US aggregate demand, aggregate supply and short rate equation. All the parameter estimates have the expected sign. The first row shows that the Phillips curve parameter, 0.001, is small and not significant. Fuhrer and Moore (1995), Ireland (2001) and Cho and Moreno (2005) obtained estimates of similar magnitudes. This reflects the weak link between detrended output and inflation in the data. This finding is consistent with the previous literature.

The second row shows the parameter estimates for the US output gap equation. The output gap can be forecasted by the lagged US output gap which is consistency with previous studies. The US short rate loads negatively on the US output gap but the coefficient of the US output gap in the short rate equation is not statistically significant. In the US monetary policy equation, the smoothing parameter ρ^* is around 0.74, reflecting the persistence in the short-term interest rate. The coefficient of inflation is around 1.08, suggesting strong response of the FED to inflation.

Figure 2 presents the recovered structural shocks for Canadian and US. It shows there is no major Canadian (US) AS shocks during the sample period. The Canadian (US) IS shocks exhibits some persistence. The US monetary policy shocks were of small magnitude after 1983. this results are consistent with Taylor (1999) and Leeper and Zha (2000).

5.2.2 Impulse Response of Macro Variables

To gauge the effect of the various shocks on Canadian macro variables, we compute impulse response functions. Figure 3 shows the impulse response functions of Canadian macroeconomic variables to the structural shocks.

The first row of graphs in Figure 3 shows the responses of Canadian macro variables to one standard deviation Canadian AS shock. The inflation shock is a negative technology or supply shock which decreases the productivity of firms. As expected the Canadian aggregate supply shock pushes Canadian inflation almost 30 bp above its steady state, but it soon returns to its original level, given the forward-looking nature of the aggregate supply equation (the coefficient of Canadian inflation in the Canadian Phillips curve equation is 0.55). The monetary authority increases the interest rate by 1 bp following the supply shock. The output exhibits a hump-shaped decline for few quarters. The real exchange rate depreciates after the AS shock.

The second row of graphs in Figure 3 shows the responses of Canadian macro variables to one standard deviation Canadian IS shock. The IS shock is a demand shock which can also be interpreted as a preference shock (see Woodford (2003)). The IS shock initially increases output, inflation and interest rate. Canadian output gap initially increases about 50 bp, but it soon returns to its steady state. The IS shock has no initial impact on the real exchange rate.

The third row of graphs in Figure 3 shows the responses of Canadian macro variables to one standard deviation Canadian monetary policy shock. The monetary policy shock reflects shifts to the interest rate unexplained by the state of the economy. Given our monetary transmission mechanism, the interest rate increases by 18 bp following the monetary policy shock, but then decreases to its steady state level. The impacts on domestic inflation and output gap are weak.

The fourth row of graphs in Figure 3 shows the responses of Canadian macro variables to one standard deviation US AS shock. Canadian inflation responds weakly to the US AS shock. The Canadian output gap responds negatively to the US AS shock, and stay in the negative region for a long time. The Canadian interest rate responds strongly to US AS shock.

The fifth row of graphs in Figure 3 shows the responses of Canadian macro variables to one

standard deviation US IS shock. The US IS shock has no effect on Canadian inflation. It has strong effects on Canadian inflation and short-term interest rate.

The sixth row of graphs in Figure 3 shows the responses of Canadian macro variables to one standard deviation US monetary policy shock. Canadian inflation and output gap response weakly to the US monetary policy shock. The Canadian interest rate responses strongly by increasing 20 bp following the US monetary policy shock.

The last row of graphs in Figure 3 shows the responses of Canadian macro variables to one standard deviation real exchange rate shock. There is not much response of Canadian inflation, output gap and interest rate to the real exchange rate shock.

5.3 Yield Results

5.3.1 Parameter Estimates

Panel D of Table 3 reports the estimates of the market prices of risk with the restriction that the matrix parameter λ_1 is diagonal. The risk premia in λ_1 indicate that expected excess returns vary significantly over time. All the diagonal elements of λ_1 are all statistically significant except for the time varying component due to the real exchange rate.

5.3.2 Impulse Response of Yields

Our structural model allows us to compute impulse response functions of Canadian bond yields to the 7 structural shocks. Figure 4 shows the impulse responses of the 1-Year, 5-Year, and 15-Year yields to the structural shocks. Canadian aggregate supply shock initially raises the level of all yields. The initial response is highest for the long yield (15-Year yield), at 7 bp1, while the initial response of the short yield, 1-year yield is small. The US aggregate supply shock initially raises all yields. The initial responses to US aggregate supply shock is about the same for all yields. They peak after four quarters, then decreases to their steady state.

Canadian aggregate demand shock has no impact on all yields, whereas the US aggregate demand shock initially decreases the 15-Year yield by 10 bp. The impact of US aggregate demand shock on Canadian short and medium yield is small.

Canadian monetary policy shock initially raises all the yields but the initial response is highest for the short yield at 15 bp, while the initial response of the medium and long yields is small. However, the US monetary policy shock has an immediate high positive impact on all yields, almost 25 bp, then the responses decline during the first 5 quarters and reach the steady state.

The real exchange rate shock has no impact on all yields.

5.3.3 Variance Decomposition

Yield Levels In our model, equation (15) states that the variables in X_t explain all yield dynamics. To complement impulse response functions we present the analyze based on unconditional variance decomposition of yields from equation (15) and the data at different horizons. These decompositions are based on Cholesky decompositions of the innovation variance in the order: $X_t = (\pi_t, g_t, r_t, q_t, \pi_t^*, g_t^*, r_t^*)'$. We ignore observation error in the yields when computing variance decompositions. The results are reported in Table 3 for horizon $h = 1, 4, 100$ quarters. In the column under the heading “EH ” (Expectation Hypothesis), we compute the proportion of the forecast variance attributable to Expectation Hypothesis. In the column under the heading “UPRP” (Unconditional Pure Risk Premia), we compute the proportion of the forecast variance attributable to time-varying risk premia.

To compute the proportion of forecast variance attributable to time-varying risk premia, we follow Ang, Dong and Piazzesi (2005) and partition the bond coefficient B'_n on X_t in equation (17) into an Expectation Hypothesis component and into a risk premia component:

$$B'_n = B_n'^{EH} + B_n'^{RP}$$

where the $B_n'^{EH}$ bond pricing coefficient is computed by setting $\lambda_1 = 0$. Since the yield dynamics are given by $y_t^n = A_n + B_n' X_t$, we have

$$y_{t+h}^n = A_n + B_n'^{EH} X_{t+h} + B_n'^{RP} X_{t+h}$$

Let $\Omega^{F,h}$ represent the forecast variance of the factors X_t , at horizon h . The forecast variance of

the n -quarter yield at horizon h is given by

$$Var(y_{t+h}^n) = B_n' \Omega^{F,h} B_n = \underbrace{B_n'^{EH} \Omega^{F,h} B_n^{EH}}_{(1)} + \underbrace{2B_n'^{EH} \Omega^{F,h} B_n^{RP}}_{(2)} + \underbrace{B_n'^{RP} \Omega^{F,h} B_n^{RP}}_{(3)}. \quad (19)$$

In (19), we ignore the component (2) which is the covariance of the risk premia with the state variables and then compute the proportion of the variance of yields attributable to time-varying risk premia as follows:

$$\text{Risk Premia Proportion} = \frac{B_n'^{RP} \Omega^{F,h} B_n^{RP}}{B_n' \Omega^{F,h} B_n}.$$

Note that the model implied unconditional pure risk premia proportion is actually a ratio (can be higher than 100% if the risk premia and state variables are negatively correlated). We also compute the proportion of forecast variance attributable to the expectation hypothesis as follows:

$$\text{Expectation Hypothesis Proportion} = \frac{B_n'^{EH} \Omega^{F,h} B_n^{EH}}{B_n' \Omega^{F,h} B_n}.$$

Panel A of Table 3 shows the variance decomposition for yield levels for one-quarter ahead horizon. It shows that risk premia play important role in explaining the level of yields. The expectation hypothesis proportion of the 1-Year yield is 104% while the risk premia proportion of the 1-Year yield is 39%. As the yield maturity increases, the expectation hypothesis proportion decreases (96% for 2-Year yield and 9% for 15-Year yield) while the risk premia proportion increases (73% for 2-Year yield and 109% for 15-Year yield). Under the lines “Expectation Hypothesis” and “Risk Premia”, Panel A shows the variance decompositions for the variance of the expectation hypothesis component, $B_n'^{EH} \Omega^{F,h} B_n^{EH}$ and the risk premia variance $B_n'^{RP} \Omega^{F,h} B_n^{RP}$ respectively. For one quarter ahead horizon, Canadian inflation and output gap cannot explain the forecast variance of all yields. In the risk premia components, the Canadian monetary policy shock explains a smaller proportion of the forecast variance of short and long yields than medium yields. The range is from 30% for the 1-year and 15-year yields to 56% for the 5-year yields. The rest are explained by US macroeconomic shocks.

Panel B and C reports the variance decomposition for yield levels for four-quarter and 100-quarter ahead horizon respectively. The results are similar to those obtained in Panel A. The

unconditional variance decomposition results show that the expectation component of all yields are totally explained by US macroeconomic shocks. The explanatory power of Canadian monetary policy shock on the risk premium components declines to a range of 10-14%.

Yield Spreads We repeat the same analysis for yield spreads of maturity n quarters in excess of the one-quarter yield, $y_t^n - y_t^1$. Table 4 shows that risk premia matter even more for yield spreads.

Panel A, B, and C reports the variance decompositions of the expectation hypothesis component, $B_n^{EH} \Omega^{F,h} B_n^{EH}$ and the risk premia variance $B_n^{RP} \Omega^{F,h} B_n^{RP}$ at 1-quarter, 4-quarter, and 100-quarter ahead respectively. In the expectation hypothesis term, the proportion of forecast variance attributable to the Canadian monetary policy shock is about 56-73% for all yield spreads while the US macroeconomic shocks explain the rest. The Canadian monetary policy shock explains a smaller proportion of the forecast variance of risk premia components in longer horizons. The US macroeconomic shocks explain about 77-95% of unconditional risk premia variance.

Expected Excess Holding Period Returns Table 5 reports the variance decomposition of expected excess holding period returns. The expected excess holding period is the risk premia. Thus, time varying risk premia is equivalent to time varying expected excess holding period returns.

Panel A, B, and C reports the variance decomposition of expected excess holding period returns at 1-quarter, 4-quarter, and 100-quarter ahead respectively. It reveals that US macroeconomic shocks explain a majority of the expected excess holding period return variance. The Canadian monetary policy shock and the US aggregate supply shock explain a majority of the variation of the expected excess holding period returns of short-term yields. But the US aggregate demand shocks explains up to 95% of the variation of the expected excess holding period returns of long yields.

5.3.4 Characterizing Excess Return

In Panel D, we report the means and standard deviations of the approximate excess returns computed from yield data and implied by our model. This panel shows that the standard deviation of

excess returns computed from the model are nearly identical to their approximate counterparts for 4, 8 and 20-quarter yields. The model overestimate the standard deviation of excess returns for 40 and 60-quarter yields and underestimate the mean of excess returns for all yields.

6. Conclusion

We estimate the joint dynamics of macroeconomic variables and bond yields in a small open economy framework complemented with an affine term structure model. With Canadian and U.S. data, we are able to study the impact of domestic and foreign (US) shocks on the yield curve. The unconditional variance decomposition of the yield level show that the movement of expected short rates is mainly driven by US macroeconomic shocks. The majority of the variation of the yield risk premium are also driven by US macroeconomic shocks. However, the Canadian monetary policy shocks can explain a small proportion of the variation of the short to medium yield risk premium. In addition, the Canadian monetary policy shocks and US aggregate demand shocks explain a majority of the variation of the expected excess holding period returns of short to medium bonds. The expected excess holding period returns of long term bonds are mainly driven by US aggregate supply shocks.

References

- [1] Ang, Andrew, Dong, S., and Piazzesi, M., 2005, No-arbitrage Taylor rules, Working paper.
- [2] Ang, Andrew, and Piazzesi, M., 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics*, 50, 745-787.
- [3] Ang, Andrew, Piazzesi, M., and Wei, M., 2004, What does the yield curve tell us about GDP growth?, Working paper.
- [4] Bekaert, Geert, Cho, Seonghoon, and Moreno, Antonio, 2005, New-Keynesian macroeconomics and the term structure, Working paper.

- [5] Ball Laurence, 1998, Policy rules for open economies. NBER working paper 6760.
- [6] Clarida, Richard, Galí, J., and Gertler, M., 1999, The science of monetary policy: a new Keynesian perspective, *Journal of Economic Literature*, 37, 1661-1707.
- [7] Dewachter, Hans, and Lyrio, Macro, 2004, Macro factors and the term structure of interest rates, Working paper.
- [8] Diebold, Francis X., Rudebusch, Gleen D., and Aruoba, S. Boragan, 2004, The macroeconomy and the yield curve: a dynamic latent factor approach, NBER working paper 10616.
- [9] Evans, Charles L., and Marshall, David, 2001, Economic determinants of the nominal treasury yield curve, Working paper.
- [10] Fuhrer, Jeffrey C., 2000. Habit formation in consumption and its implications for monetary-policy models. *American Economic Review*, 90, 367-89.
- [11] Galí, Jordi, and Gertler, M., 1999, Inflation dynamics: a structural econometric analysis, *Journal of Monetary Economics*, 44, 195-222.
- [12] Giordani, Paul, 2004. Evaluating New-Keynesian models of a small open economy. *Oxford Bulletin of economics and Statistics*, 66, 713-733.
- [13] Hördahl, Peter, Tristani, O., and Vestin, D., 2004, A joint econometric model of macroeconomic and term structure dynamics, Working paper.
- [14] Lane, P. R. and Ganelli, G. 2003, Dynamic general equilibrium analysis: the open economy dimension. in Altug S., Chaddha J. and Nolan C. (eds), *Dynamic Macroeconomic Analysis*, Cambridge University Press, cambridge, pp, 308-334.
- [15] Leeper, Eric M., and Zha. T., 2000, Assessing simple policy rules: a view from a complete macro model, Federal Reserve Bank of Atlanta Working Paper 19.
- [16] McCallum, Bennett T., 1998, Solutions to linear rational expectations models: a compact exposition. *Economics letters*, 6, 143-147.

- [17] Rudebusch, Glenn D., and Wu, T., 2003, A macro-finance model of the term structure, monetary policy, and the economy, Working paper
- [18] Svensson, Lars E. O., 2000. Open Economy Inflation Targeting. *Journal of International Economics*, 50, 155-183.
- [19] Taylor, John B., 1999, A historical analysis of monetary policy rules in John B. Taylor, Ed. *Monetary Policy Rules*, Chicago University of Chicago Press, 319-341.

Table 1: Descriptive Statistics

| Description of macroeconomic variables | | | | | | | | |
|--|----------|----------|----------|----------|-----------|---------|----------|----------|
| | Mean | Variance | Skewness | Kurtosis | | | | |
| Canadian Inflation | 0.00911 | 0.0005 | 1.14690 | 3.48750 | | | | |
| Canadian Output Gap | -0.00171 | 0.00128 | -0.37393 | 1.71803 | | | | |
| Canadian Short term interest rate | 0.01904 | 0.00010 | 0.53734 | 2.54576 | | | | |
| Exchange Rate | 0.67470 | 0.01505 | 0.38957 | 2.04160 | | | | |
| US inflation | 0.01023 | 0.00005 | 2.01569 | 7.16917 | | | | |
| US Output Gap | -0.00418 | 0.00054 | -0.26397 | 3.10233 | | | | |
| US Short term interest rate | 0.01520 | 0.0007 | 0.77437 | 3.5357 | | | | |
| Aurocorrelations | | | | | | | | |
| | π_t | g_t | r_t | q_t | π_t^* | g_t^* | r_t^* | |
| Lag1 | 0.82815 | 0.96464 | 0.94740 | 0.97225 | 0.72887 | 0.91932 | 0.91051 | |
| Lag2 | 0.79869 | 0.90557 | 0.88962 | 0.94045 | 0.66738 | 0.81307 | 0.85566 | |
| Description of Yields | | | | | | | | |
| | Mean | Variance | Skewness | Kurtosis | | | | |
| 6 month | 0.07565 | 0.00147 | 0.51341 | 2.60954 | | | | |
| 1 Year | 0.07584 | 0.00132 | 0.51074 | 2.76772 | | | | |
| 2 Years | 0.07766 | 0.00115 | 0.52162 | 2.88436 | | | | |
| 3 Years | 0.07920 | 0.00104 | 0.49077 | 2.82475 | | | | |
| 5 Years | 0.08180 | 0.00093 | 0.44498 | 2.66646 | | | | |
| 7 Years | 0.08416 | 0.00090 | 0.46756 | 2.63516 | | | | |
| 10 Years | 0.08579 | 0.00086 | 0.47495 | 2.73532 | | | | |
| 15 Years | 0.08895 | 0.00088 | 0.47941 | 2.65510 | | | | |
| Autocorrelations | | | | | | | | |
| | 6 Month | 1 Years | 2 Years | 3 Years | 5 Years | 7 Years | 10 Years | 15 Years |
| Lag 1 | 0.95225 | 0.94935 | 0.94608 | 0.94777 | 0.95259 | 0.95871 | 0.96139 | 0.96374 |
| Lag 2 | 0.89573 | 0.89204 | 0.88925 | 0.89309 | 0.90260 | 0.91414 | 0.91566 | 0.92314 |

Note: This table shows the summary statistics for macro variables and yields. The sample period is from 1978:Q4 to 2005: Q3

Table 2: Domestic country macro dynamics

Panel A: SOE Macro Dynamics

| π_t | α_π | α_g | α_q | | | $\sigma_{\varepsilon_t^\pi} \times 10^2$ | | |
|---------|--------------|------------|------------|----------------|---------|--|--|--|
| | 0.446 | 0.000 | -0.003 | | | 0.268 | | |
| | (0.032) | (0.041) | (0.016) | | | | | |
| g_t | β_g | β_r | β_q | β_{g^*} | | | $\sigma_{\varepsilon_t^{\pi^*}} \times 10^2$ | |
| | 0.468 | -0.005 | 0.0002 | 0.010 | | | 0.343 | |
| | (0.027) | (0.032) | (0.057) | (0.003) | | | | |
| r_t | γ_π | γ_g | γ_q | γ_{r^*} | ρ | | | $\sigma_{\varepsilon_t^r} \times 10^2$ |
| | 0.042 | 0.042 | 0.024 | 1.145 | 0.616 | | | 0.214 |
| | (0.013) | (0.012) | (0.002) | (0.044) | (0.019) | | | |

Panel B: Exchange rate Equation Coefficients

| q_t | δ_r | $\sigma_{\varepsilon_t^r} \times 10^2$ |
|-------|------------|--|
| | -0.525 | 2.312 |
| | (0.063) | |

Panel C: ROW Macro dynamics

| π_t^* | α_{π^*} | α_{g^*} | | | $\sigma_{\varepsilon_t^{\pi^*}} \times 10^2$ | |
|-----------|------------------|----------------|----------|--|--|--|
| | 0.418 | 0.001 | | | 0.278 | |
| | (0.025) | (0.012) | | | | |
| g_t^* | β_g^* | β_r^* | | | $\sigma_{\varepsilon_t^{g^*}} \times 10^2$ | |
| | 0.483 | 0.005 | | | 0.357 | |
| | (0.019) | (0.009) | | | | |
| r_t^* | γ_{π^*} | γ_{g^*} | ρ^* | | | $\sigma_{\varepsilon_t^r} \times 10^2$ |
| | 1.086 | 0.029 | 0.735 | | | 0.299 |
| | (0.072) | (0.002) | (0.012) | | | |

Panel D: Market Price of Risk

| | π_t | g_t | r_t | q_t | π_t^* | g_t^* | r_t^* |
|-------------|---------|----------|----------|-------|-----------|----------|----------|
| λ_0 | -0.116 | 0 | -0.065 | 0 | 0 | 0 | 0 |
| | (2.475) | | (0.521) | | | | |
| λ_1 | -50.445 | 300.535 | -166.300 | 0 | 381.760 | 289.434 | 269.220 |
| | (3.625) | (10.915) | (31.437) | | (17.612) | (24.843) | (38.111) |

Observation Error Standard Deviation

| $\sigma^{(n)}$ | $n = 2$ | $n = 4$ | $n = 8$ | $n = 12$ | $n = 20$ | $n = 28$ | $n = 40$ | $n = 60$ |
|----------------|---------|---------|---------|----------|----------|----------|----------|----------|
| | 0.009 | 0.021 | 0.026 | 0.028 | 0.030 | 0.033 | 0.031 | 0.0306 |

Note: This table list parameter estimates of the model. Panel A reports parameter values for the domestic country as in equations (4), (5) and (6). Standard errors are in parenthese. Panel B reports the parameter value for the exchange rate dynamic as in equation (7). Panel C reports parametervalue for the domestic country as in equations (1), (2) and (3). Panel D lists market prices of risk estimates for the model as in equation (10).

Table 3: Variance Decompositions:: Yield levels

| Q | Expectation Hypothesis | | | | | | | | | Risk Premia | | | | | | |
|---------------------|------------------------|-------|-----|------|-----|---------|-------|-------|------|-------------|-----|------|-----|---------|-------|-------|
| | EHP | π | g | r | q | π^* | g^* | r^* | UPRP | π | g | r | q | π^* | g^* | r^* |
| Panel A: $h = 1Q$ | | | | | | | | | | | | | | | | |
| 4 | 104 | 0.1 | 0.4 | 23.6 | 0.3 | 29.8 | 1.2 | 44.6 | 39 | 0.2 | 0.5 | 32.8 | 0.3 | 54.5 | 3.3 | 8.3 |
| 8 | 96 | 0.1 | 0.5 | 9.3 | 0.1 | 49.6 | 5.2 | 35.2 | 73 | 0.9 | 0.3 | 41.8 | 0.4 | 48.6 | 6.9 | 1.0 |
| 20 | 46 | 0.1 | 0.6 | 4.3 | 0.6 | 48.1 | 26.7 | 20.2 | 85 | 7.0 | 0.1 | 55.5 | 0.5 | 17.1 | 16.2 | 3.5 |
| 40 | 20 | 0.1 | 0.6 | 3.4 | 0.0 | 35.4 | 45.9 | 14.7 | 94 | 20.2 | 0.0 | 48.7 | 0.3 | 3.5 | 19.6 | 7.6 |
| 60 | 9 | 0.1 | 0.5 | 3.2 | 0.0 | 32.7 | 49.7 | 13.7 | 109 | 21.9 | 0.0 | 28.6 | 0.1 | 0.1 | 43.6 | 5.7 |
| Panel B: $h = 4Q$ | | | | | | | | | | | | | | | | |
| 4 | 109 | 0.1 | 0.5 | 9.2 | 0.1 | 48.3 | 3.9 | 37.9 | 18 | 0.5 | 0.5 | 29.0 | 0.2 | 52.9 | 8.7 | 8.1 |
| 8 | 91 | 0.1 | 0.5 | 4.0 | 0.0 | 57.7 | 9.5 | 28.0 | 36 | 1.6 | 0.2 | 35.1 | 0.3 | 37.2 | 13.7 | 11.8 |
| 20 | 43 | 0.1 | 0.6 | 1.9 | 0.0 | 47.8 | 33.9 | 14.7 | 58 | 7.4 | 0.1 | 34.4 | 0.3 | 11.9 | 18.7 | 27.3 |
| 40 | 20 | 0.1 | 0.5 | 1.5 | 0.0 | 34.4 | 52.4 | 11.1 | 78 | 17.4 | 0.1 | 26.3 | 0.1 | 10.2 | 18.1 | 27.8 |
| 60 | 10 | 0.1 | 0.5 | 1.4 | 0.0 | 31.7 | 55.9 | 10.3 | 101 | 17.7 | 0.0 | 14.8 | 0.1 | 8.4 | 41.3 | 17.7 |
| Panel C: $h = 100Q$ | | | | | | | | | | | | | | | | |
| 4 | 99 | 0.1 | 0.5 | 4.4 | 0.0 | 46.6 | 25.9 | 22.5 | 10 | 0.6 | 0.4 | 23.7 | 0.2 | 48.8 | 13.7 | 12.6 |
| 8 | 80 | 0.1 | 0.5 | 2.0 | 0.0 | 46.5 | 34.4 | 16.5 | 22 | 1.5 | 0.2 | 25.1 | 0.2 | 40.9 | 14.2 | 17.8 |
| 20 | 42 | 0.1 | 0.4 | 0.9 | 0.0 | 30.6 | 60.0 | 8.1 | 44 | 5.3 | 0.1 | 19.8 | 0.1 | 34.3 | 13.7 | 26.6 |
| 40 | 23 | 0.0 | 0.4 | 0.6 | 0.0 | 20.3 | 73.4 | 5.3 | 65 | 12.2 | 0.2 | 15.2 | 0.1 | 33.8 | 13.2 | 25.3 |
| 60 | 14 | 0.0 | 0.4 | 0.6 | 0.0 | 18.5 | 75.6 | 4.9 | 96 | 13.2 | 0.1 | 9.1 | 0.0 | 26.3 | 33.8 | 17.3 |

Note: The table reports unconditional variance decompositions of forecast variance for yield levels y_t^n . In each panel, the numbers under the line "Expectation Hypothesis" report the variance decompositions for the component of the variance yields that is due to Expectation Hypothesis, $B_n^{EH} \Omega^{F,h} B_n^{EH}$. The numbers under the line "Risk Premia" report the variance decompositions for the pure risk premia variance, $B_n^{RP} \Omega^{F,h} B_n^{RP}$. The number under the line "EHP" reports the proportion of forecast variance attributable to Expectation Hypothesis. The number under the line "UPRP" reports the proportion of forecast variance attributable to time-varying risk premia. We ignore observation error for computing variance decompositions for yield levels and yield spreads.

Table 4: Variance Decompositions:Yield spread levels

| Q | Expectation Hypothesis | | | | | | | | | Risk Premia | | | | | | |
|---------------------|------------------------|-------|-----|------|-----|---------|-------|-------|------|-------------|-----|------|-----|---------|-------|-------|
| | EHP | π | g | r | q | π^* | g^* | r^* | UPRP | π | g | r | q | π^* | g^* | r^* |
| Panel A: $h = 1Q$ | | | | | | | | | | | | | | | | |
| 4 | 115 | 0.0 | 0.2 | 56.4 | 0.6 | 28.6 | 5.5 | 0.0 | 74 | 0.2 | 0.5 | 32.9 | 0.3 | 54.5 | 3.3 | 8.3 |
| 8 | 103 | 0.0 | 0.1 | 65.2 | 0.7 | 28.6 | 5.5 | 0.0 | 77 | 0.9 | 0.3 | 41.8 | 0.4 | 48.6 | 6.9 | 1.0 |
| 20 | 93 | 0.0 | 0.0 | 77.0 | 0.8 | 2.5 | 8.2 | 11.4 | 69 | 7.0 | 0.1 | 55.5 | 0.5 | 17.1 | 16.2 | 3.5 |
| 40 | 91 | 0.0 | 0.0 | 75.4 | 0.8 | 0.4 | 3.5 | 19.9 | 74 | 20.2 | 0.0 | 48.7 | 0.3 | 3.5 | 19.6 | 7.6 |
| 60 | 78 | 0.0 | 0.1 | 73.9 | 0.8 | 1.5 | 1.4 | 22.3 | 86 | 21.8 | 0.0 | 28.5 | 0.1 | 0.1 | 43.6 | 5.7 |
| Panel B: $h = 4Q$ | | | | | | | | | | | | | | | | |
| 4 | 140 | 0.0 | 0.1 | 51.2 | 0.6 | 29.6 | 5.1 | 13.4 | 89 | 0.5 | 0.5 | 29.0 | 0.3 | 52.9 | 8.7 | 8.1 |
| 8 | 148 | 0.0 | 0.1 | 49.8 | 0.5 | 18.8 | 8.2 | 22.6 | 98 | 1.6 | 0.2 | 35.1 | 0.3 | 37.2 | 13.7 | 11.8 |
| 20 | 187 | 0.1 | 0.1 | 45.8 | 0.4 | 15.5 | 6.1 | 39.2 | 100 | 7.4 | 0.1 | 34.4 | 0.3 | 11.9 | 18.7 | 27.3 |
| 40 | 199 | 0.1 | 0.3 | 32.6 | 0.3 | 23.5 | 1.6 | 41.6 | 110 | 17.4 | 0.0 | 26.3 | 0.1 | 10.2 | 18.1 | 27.7 |
| 60 | 164 | 0.1 | 0.3 | 30.8 | 0.3 | 26.2 | 0.6 | 41.7 | 128 | 17.7 | 0.0 | 14.8 | 0.0 | 8.4 | 41.3 | 17.7 |
| Panel C: $h = 100Q$ | | | | | | | | | | | | | | | | |
| 4 | 155 | 0.1 | 0.2 | 36.4 | 0.4 | 37.2 | 5.8 | 20.0 | 97 | 0.6 | 0.4 | 23.7 | 0.2 | 48.8 | 13.7 | 12.6 |
| 8 | 184 | 0.1 | 0.2 | 29.4 | 0.3 | 36.8 | 7.8 | 25.5 | 100 | 1.5 | 0.2 | 25.1 | 0.2 | 40.9 | 14.2 | 17.8 |
| 20 | 218 | 0.1 | 0.3 | 19.1 | 0.2 | 41.2 | 8.4 | 30.3 | 86 | 5.3 | 0.1 | 19.8 | 0.1 | 34.3 | 13.7 | 26.6 |
| 40 | 221 | 0.1 | 0.4 | 15.3 | 0.2 | 43.8 | 11.5 | 28.8 | 86 | 12.2 | 0.2 | 15.2 | 0.1 | 33.8 | 13.2 | 25.3 |
| 60 | 166 | 0.1 | 0.4 | 14.1 | 0.2 | 43.5 | 14.0 | 27.7 | 98 | 13.2 | 0.1 | 9.1 | 0.0 | 26.3 | 33.8 | 17.3 |

Note: The table reports unconditional variance decompositions of forecast variance for yield spread levels $y_t^n - y_t^1$. In each panel, the numbers under the line "Expectation Hypothesis" report the variance decompositions for the component of the variance yields that is due to Expectation Hypothesis, $B_n^{'EH} \Omega^{F,h} B_n^{EH}$. The numbers under the line "Risk Premia" report the variance decompositions for the pure risk premia variance, $B_n^{'RP} \Omega^{F,h} B_n^{RP}$. The number under the line "EHP" reports the proportion of forecast variance attributable to Expectation Hypothesis. The number under the line "UPRP" reports the proportion of forecast variance attributable to time-varying risk premia. We ignore observation error for computing variance decompositions for yield levels and yield spreads.

Table 5: Variance Decompositions: Conditional expected excess holding period returns

| Q | UPRP | Risk Premia | | | | | | |
|--|------|-------------|-------|---------------|--------|---------|-------|-------|
| | | π | g | r | q | π^* | g^* | r^* |
| Panel A: $h = 1Q$ | | | | | | | | |
| 4 | 100 | 0.1 | 0.6 | 32.9 | 0.3 | 47.3 | 2.0 | 16.8 |
| 8 | 100 | 0.3 | 0.4 | 37.2 | 0.4 | 41.4 | 2.1 | 18.2 |
| 20 | 100 | 1.2 | 0.4 | 37.8 | 0.3 | 37.6 | 4.7 | 17.9 |
| 40 | 100 | 2.7 | 0.4 | 27.0 | 0.2 | 24.3 | 33.4 | 11.9 |
| 60 | 100 | 0.8 | 0.2 | 4.9 | 0.0 | 0.8 | 91.6 | 1.7 |
| Panel B: $h = 4Q$ | | | | | | | | |
| 4 | 100 | 0.3 | 0.5 | 29.4 | 0.3 | 51.7 | 6.3 | 11.5 |
| 8 | 100 | 0.7 | 0.4 | 33.8 | 0.3 | 45.8 | 6.2 | 12.8 |
| 20 | 100 | 2.1 | 0.3 | 33.4 | 0.3 | 40.1 | 11.4 | 12.4 |
| 40 | 100 | 3.1 | 0.3 | 18.5 | 0.1 | 19.4 | 51.7 | 6.8 |
| 60 | 100 | 0.7 | 0.1 | 2.4 | 0.0 | 0.4 | 95.4 | 0.9 |
| Panel C: $h = 100Q$ | | | | | | | | |
| 4 | 100 | 0.4 | 0.5 | 25.1 | 0.2 | 47.8 | 11.5 | 14.4 |
| 8 | 100 | 0.8 | 0.3 | 28.8 | 0.3 | 43.7 | 9.8 | 16.2 |
| 20 | 100 | 2.2 | 0.3 | 27.3 | 0.2 | 37.8 | 16.7 | 15.4 |
| 40 | 100 | 2.5 | 0.2 | 11.4 | 0.1 | 15.7 | 62.9 | 7.2 |
| 60 | 100 | 0.4 | 0.1 | 1.2 | 0.0 | 2.0 | 95.1 | 1.3 |
| Panel D: Characterizing excess returns | | | | | | | | |
| | | Data | | Model Implied | | | | |
| | | mean | std | mean | std | | | |
| 4 | | 0.056 | 0.845 | 0.042 | 0.925 | | | |
| 8 | | 0.219 | 1.885 | 0.096 | 1.978 | | | |
| 20 | | 0.552 | 4.087 | 0.245 | 4.638 | | | |
| 40 | | 0.780 | 6.861 | 0.449 | 9.391 | | | |
| 60 | | 1.475 | 9.938 | 0.494 | 27.747 | | | |

Note: Panel A and B of this table reports unconditional variance decompositions of the conditional expected excess holding period returns $E_t \left(rx_{t+1}^{(n)} \right)$. The numbers under the line "Risk Premia" report the variance decompositions for the pure risk premia variance, $B_n'^{RP} \Omega^{F,h} B_n^{RP}$. The number under the line "UPRP" reports the proportion of forecast variance attributable to time-varying risk premia. The maturities are in quarters.

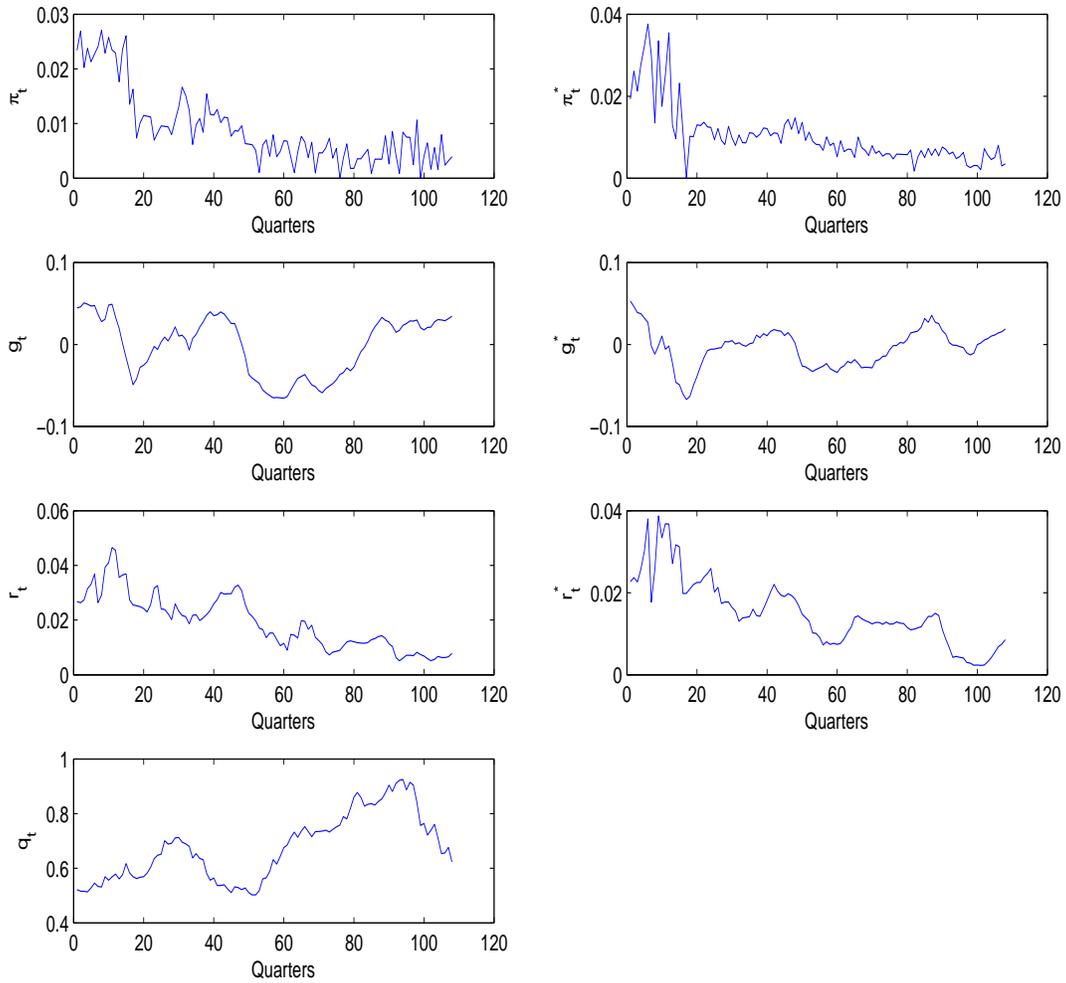


Figure 1: This Figure shows the values of the time series of the macro-variables for both foreign and domestic country. The sample period is from 1978: Q4 to 2005: Q3.

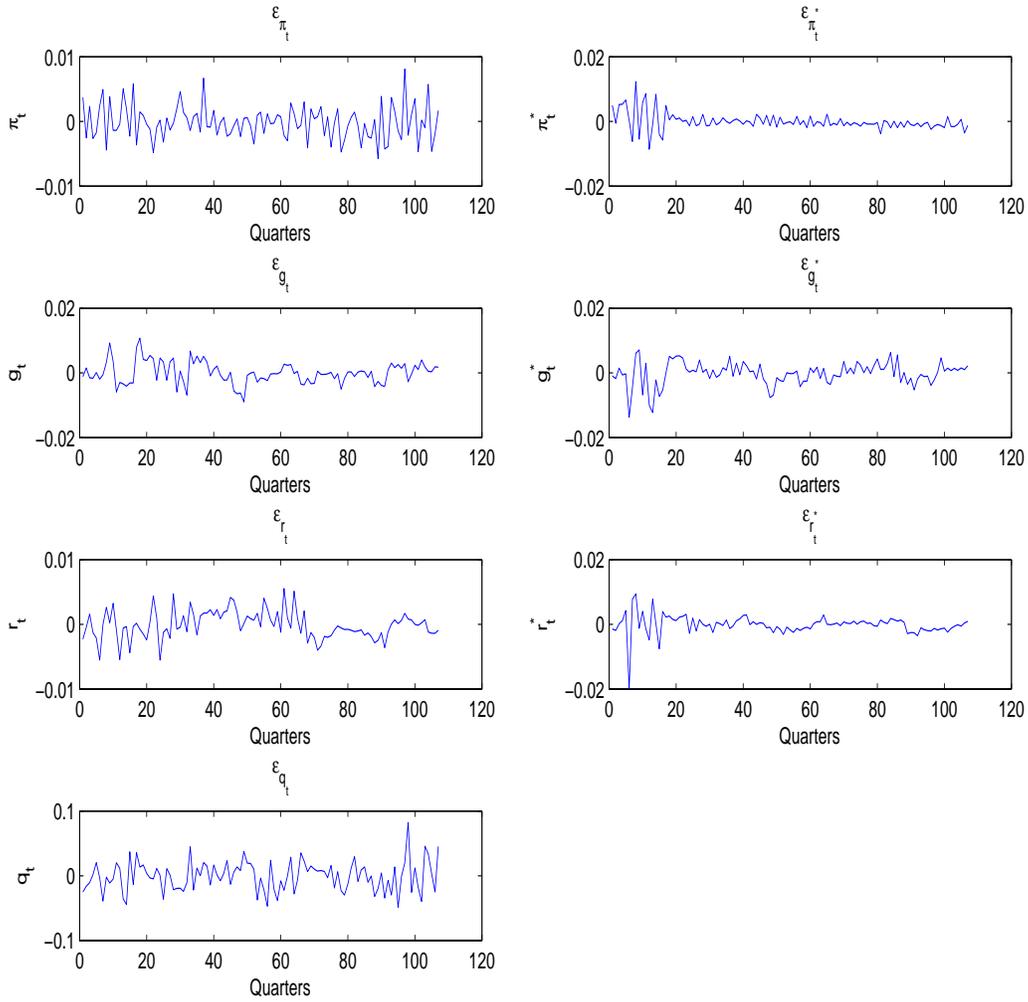


Figure 2: This Figure shows the values of the time series of the macro-variable errors for both foreign and domestic country. The sample period is from 1978: Q4 to 2005: Q3.

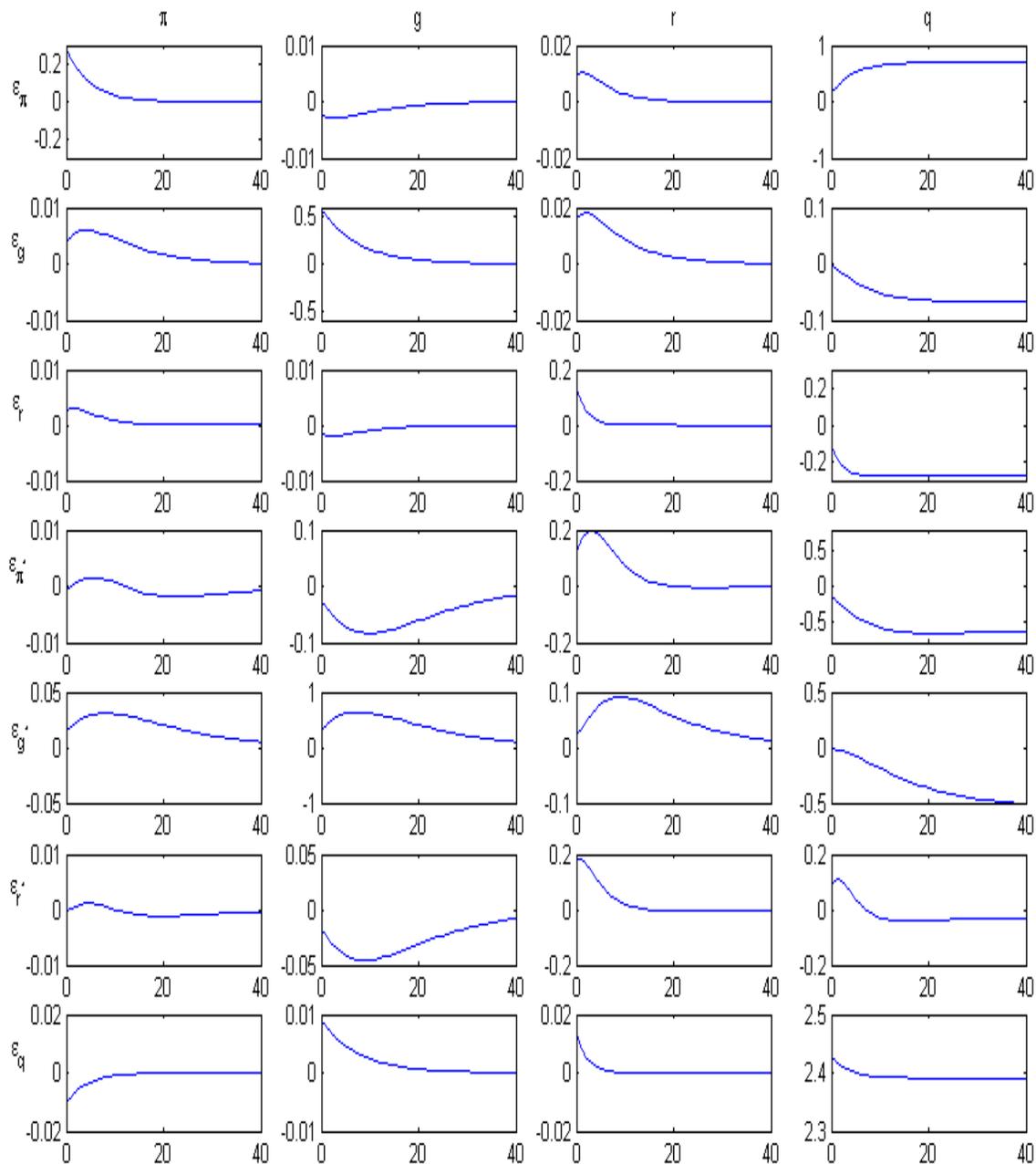


Figure 3: The panels show response of the one-, four- and fourty-quarter domestic country macro-variables to one standard deviation shock to macro variables.

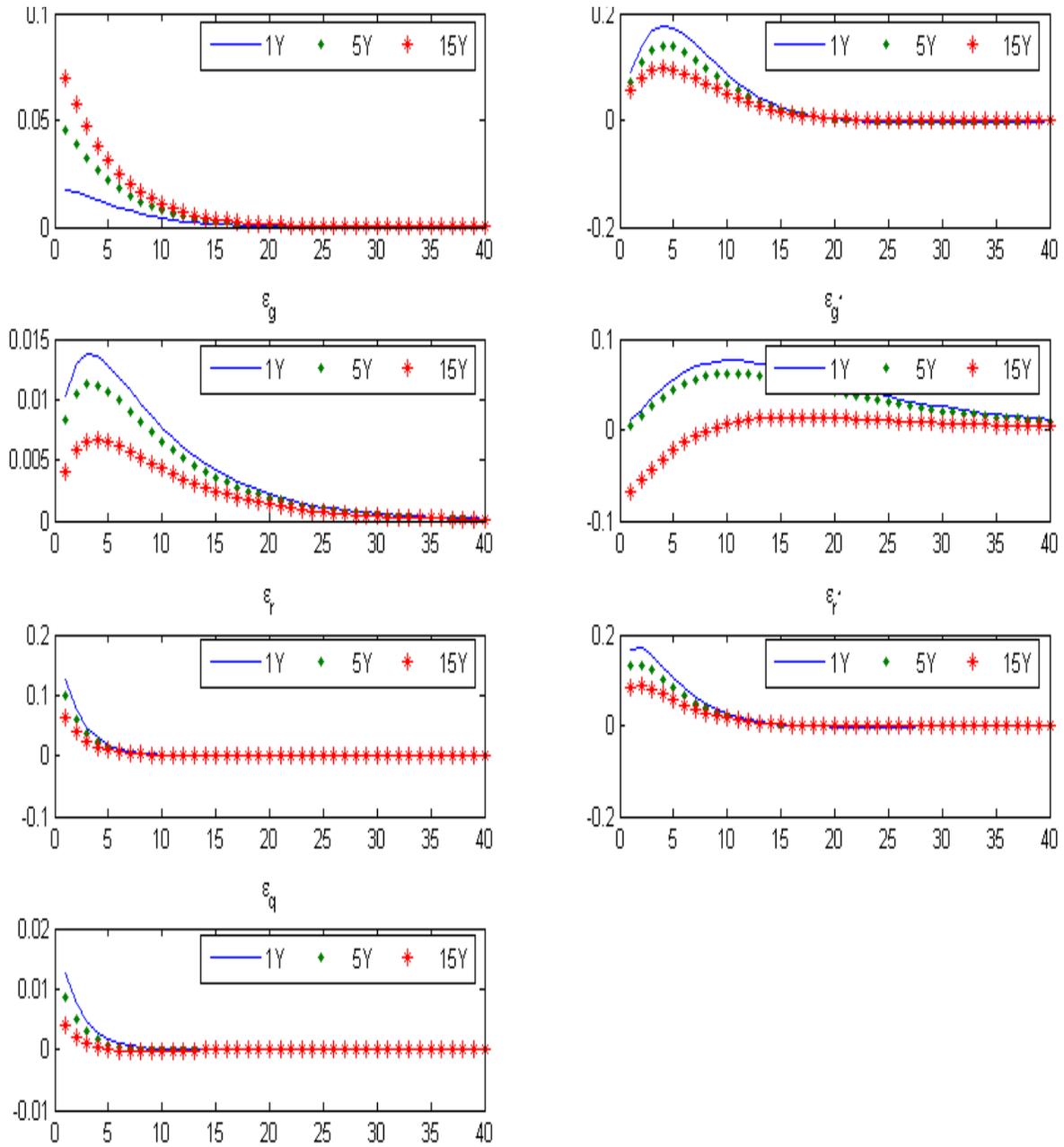


Figure 4: The panels show response of the 1 Year, 5-Year and 15-Year yield level to one standard deviation shock to macro variables.