

# Endogenous Rule-of-Thumb Price Setters and Monetary Policy\*

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## Abstract

Studies evaluating the efficacy of monetary policy rules and regimes are often based a benchmark new Keynesian model where the parameters are assumed to be policy invariant. It is possible, however, that some key parameters may not be invariant to changes in monetary policy. In this paper, we use a hybrid new Keynesian Phillips curve to examine the inflation versus price-level targeting debate when the proportion of rule-of-thumb price setters is allowed to change endogenously with the monetary regime. Although there are other factors that may also be endogenous, we focus on inflation inertia since it has been identified in the literature as a crucial parameter affecting the performance of monetary policy.

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## 1. Introduction

The new Keynesian Phillips curve with lagged inflation has become a workhorse model for applied monetary policy analysis. Its wide-spread use reflects the ability of the canonical new Keynesian Phillips curve to provide a structural framework for understanding inflation dynamics and its lagged extension to account for the degree of inflation persistence observed in the data. This hybrid inflation model has been used to study, among other issues, the efficacy of different monetary policies.<sup>1</sup> Steinsson (2003), for example, follows Gali and Gertler (1999) by assuming the presence of rule-of-thumb price-setters and uses this type of hybrid new Keynesian Phillips curve to study optimal monetary policy. He finds a purely forward-looking new Keynesian Phillips curve admits price-level stability as optimal monetary policy, whereas optimal policy in the same model augmented with rule-of-thumb behaviour allows for price-level drift that increases with the proportion of rule-of-thumb price-setters. This is an especially interesting result for monetary authorities as it suggests that the case for price-level targeting is weakened as the proportion of rule-of-thumb price-setters increases. Other research comparing the efficacy of inflation versus price-level targeting in a new Keynesian framework extended to include rule-of-thumb price-setters often conclude that price-level targeting dominates inflation targeting. One reason for this result is that the proportion of rule-of-thumb price-setters is typically set at a relatively low value (0.1 to 0.3), and as Steinsson (2003) shows this value has important implications for comparisons between inflation and price-level targeting. While this line of research has improved our understanding of the effectiveness of different monetary policies, it has the potential drawback that it treats the importance of rule-of-thumb behavior as fixed across policy experiments. The loss in profits associated with following a simple rule-of-thumb is likely to be sensitive to the policy regime so policy experiments that hold the proportion of rule-of-thumb price-setters fixed are vulnerable to the Lucas critique.

In an effort to examine the relevance of this concern, we examine the inflation versus price-level targeting question when the proportion of rule-of-thumb price-setters is endogenous. We work in the context of a hybrid new Keynesian Phillips developed by Gali and Gertler (1999) and allow the proportion of rule-of-thumb

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<sup>1</sup>The new Keynesian Phillips curve with rule-of-thumb price setters has been used to study other important monetary issues such as optimal monetary policy rules, zero bound on nominal interest rates, welfare costs of inflation, desirability of inflation targeting, optimal policy under discretion and commitment, etc.

price-setters to be a function of the monetary policy regime. Our focus on rule-of-thumb behaviour and inflation persistence reflects its importance in influencing monetary policy. Rudebusch (2002), for instance, finds the effectiveness of nominal income targeting to be an inverse function of the degree of inflation inertia. Similarly, Walsh (2003) finds the performance of price-level targeting deteriorates significantly as inflation becomes more persistent. Levin and Williams (2003) show that a policy rule that is optimal in a model with a low degree of inflation persistence can lead to disastrous results if the true model is characterized by a high degree of inflation inertia.

To preview our results, we find the relative performance of price-level targeting deteriorates or, even reverses in favour of inflation targeting, when the proportion of rule-of-thumb price-setters is allowed to be endogenous. The intuition for this result is that the effectiveness of price-level targeting in stabilizing inflation causes rule-of-thumb behaviour to become more attractive. This leads to an increase in the proportion of rule-of-thumb price-setters and, in turn, makes inflation more intrinsically persistent and thereby undermines the effectiveness of price-level targeting.

The remainder of the paper is organized as follows. Section 2 outlines the main features of our model and Section 3 describes its calibration. Section 4 presents results comparing inflation versus price-level targeting when the proportion of rule-of-thumb price-setters is endogenously determined. Concluding remarks are provided in Section 5.

## 2. Model

The model consists of a continuum of households with an infinite planning horizon, a collection of monopolistic competitive firms that produce differentiated intermediate goods, sluggish price adjustment, and a monetary authority that sets the short-term interest rate according to a prespecified rule. The derivations in the forthcoming four subsections follow closely those in Steinsson (2003) so we will provide only a high-level sketch of the benchmark rule-of-thumb price setter model. The fifth section provides detail on our extension of the benchmark model to an environment where the proportion of rule-of-thumb price-setters is endogenous.

## 2.1. Households

Households maximize expected utility given by:

$$E_t \sum_{s=t}^{\infty} \beta^s [u(C_s; \xi_s) - v(y_s(z); \xi_s)]$$

where  $\beta \in (0, 1)$  is a discount factor, and  $\xi$  is a vector of shocks to household preferences and production. Each household is assumed to specialize in the production of one differentiated good denoted  $y_t(z)$ , and consume a composite consumption good represented by  $C_t$ . The latter is combined via a Dixit-Stiglitz consumption index given by,

$$C_t \equiv \left( \int_0^1 c_t(z)^{\frac{\theta_t-1}{\theta_t}} dz \right)^{\frac{\theta_t}{\theta_t-1}}$$

and  $z$  indexes differentiated goods and  $\theta_t$  is the elasticity of substitution between differentiated goods in period  $t$ . It is standard practice to assume that  $\theta_t$  is constant but we follow Steinsson and allow the parameter to be stochastic. Note that we have suppressed the household index on consumption since the assumption of complete markets will eliminate consumption heterogeneity in equilibrium.

Utility maximization subject to a standard budget constraint leads to household demand for individual varieties of:

$$c_t(z) = \left( \frac{p_t(z)}{P_t} \right)^{-\theta_t} C_t \quad \forall z \in [0, 1]$$

where the aggregate price index is given by:

$$P_t = \left( \int_0^1 p_t(z)^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}} \quad (1)$$

In equilibrium, markets must clear so  $c_t(z) = y_t(z)$  and  $C_t = Y_t$  for all  $t$  and  $z$ . Combining these market clearing conditions with the household's first-order conditions for consumption and asset holdings yields the familiar Euler equation:

$$\beta E_t \left\{ \frac{U_C(Y_{t+1}; \xi_{t+1})}{U_C(Y_t; \xi_t)} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{1 + i_t} \quad (2)$$

where  $i_t$  is the risk-free short-term nominal interest rate.

Household pricing decisions are based on Calvo (1983) nominal price contracts. Following Calvo, we assume that in any given period each household has a fixed probability  $1 - \alpha$  that it may adjust its price and, hence, a probability  $\alpha$  that the household must leave its price unchanged. In an effort to better match the persistence found in inflation data, we depart from the standard Calvo structure by having two types of price-setters within the cohort allowed to change their price<sup>2</sup>. One type, which is a fraction  $1 - \omega$  of price changing households, behave like standard Calvo price-setters in the sense they set prices optimally given the constraints of the model and use all available information. We refer to these price-setters as "forward looking." The remaining  $\omega$  households within  $1 - \alpha$ , which we refer to as "rule-of-thumb" price-setters, use a simple rule to adjust their prices.

Forward-looking (FL) price-setters choose prices to solve:

$$\max_{p_t^{f*}} E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left\{ u_C(Y_s; \xi_s) (1 - \tau) \left( \frac{\bar{\pi}^{s-t} p_t^{f*}}{P_s} \right) y_s(z) - v(y_s(z); \xi_s) \right\}$$

subject to,

$$y_s(z) = \left( \frac{\bar{\pi}^{s-t} p_t^{f*}}{P_s} \right)^{-\theta_s} Y_s$$

where  $\tau = -(\bar{\theta} - 1)^{-1}$  is a tax rate and  $p^{f*}$  is the optimal price which satisfies the first-order condition:

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left( \frac{\bar{\pi}^{s-t} p_t^{f*}}{P_s} \right)^{-\theta_s} Y_s (1 - \theta_s) \left\{ \begin{array}{l} u_C(Y_s; \xi_s) (1 - \tau) \left( \frac{\bar{\pi}^{s-t} p_t^{f*}}{P_s} \right) \\ - \left( \frac{\theta_s}{\theta_s - 1} \right) v_y \left( \left( \frac{\bar{\pi}^{s-t} p_t^{f*}}{P_s} \right)^{-\theta_s} Y_s; \xi_s \right) \end{array} \right\} = 0 \quad (3)$$

Note that we suppress the  $z$  index on the reset price since all resetting FL firms choose the same price in a given period. Also note that, in the case of fully flexible prices, (3) collapses to the standard static condition:

$$\left( \frac{p_t^\diamond}{P_t} \right) = \left( \frac{1}{1 - \tau} \right) \left( \frac{\theta_t}{\theta_t - 1} \right) \frac{v_y \left( \left( \frac{p_t^\diamond}{P_t} \right)^{-\theta_t} Y_t; \xi_t \right)}{u_C(Y_t; \xi_t)} \quad (4)$$

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<sup>2</sup>See Steinsson (2003) and Galí and Gertler (1999) for a detailed description of this type of pricing model.

where  $p_t^\diamond$  is the price chosen under flexible prices. So, under flexible prices, the firm will set optimally its relative price as a markup over real marginal cost (adjusted for the tax rate).

We assume that rule-of-thumb (RT) price-setters set their price according to:

$$p_t^{b*} = \mu_t^\zeta p_{t-1}^* \Pi_{t-1}^\gamma \quad (5)$$

where  $\Pi_{t-1} \equiv P_{t-1}/P_{t-2}$  is gross inflation,  $\mu_t \equiv \theta_t/(\theta_t - 1)$  is the gross markup, and  $\gamma$  governs the degree of indexation.<sup>3</sup> An index of prices set at time  $t$  is given by

$$\log p_t^* = (1 - \omega) \log p_t^{f*} + \omega \log p_t^{b*} \quad (6)$$

Given that all FL price-setters will choose the same price and all RT price-setters will choose the same price in any given period, we can write the overall price index (1) as:

$$P_t = \left[ \alpha (\bar{\pi} P_{t-1})^{1-\theta_t} + (1 - \alpha) (1 - \omega) \left( p_t^{f*} \right)^{1-\theta_t} + (1 - \alpha) \omega \left( p_t^{b*} \right)^{1-\theta_t} \right]^{\frac{1}{1-\theta_t}} \quad (7)$$

## 2.2. Log-linearization of the model

The log-linear approximation of (2) yields the IS curve:

$$x_t = E_t x_{t+1} - \sigma \left( \hat{i}_t - E_t \pi_{t+1} - r_t^n \right)$$

where  $x_t = \log(Y_t/Y_t^n)$  and  $Y_t^n$  is the level of output consistent with an economy with fully flexible prices, perfectly competitive markets and no distortionary taxes. Moreover,

$$\begin{aligned} \hat{i}_t &= \log \left[ 1 + \frac{i_t}{1 + \bar{i}} \right] \\ \pi_t &= \log(\Pi_t) \\ \sigma &= -\frac{u_C}{u_{CC} \bar{Y}} > 0 \\ r_t^n &= E_t \left[ \sigma^{-1} \log \left( \frac{Y_{t+1}^n}{Y_t^n} \right) - \frac{u_{C\xi}}{u_C} (\xi_{t+1} - \xi_t) \right] \end{aligned}$$

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<sup>3</sup>Note that this is a generalization of the Galí and Gertler (1999) rule-of-thumb rule: It allows the weight on lagged inflation to be different from unity and the weight on the markup to be different from zero.

Next, consider the supply-side of the economy, where the log-linear approximations of (5)-(7) are:

$$\widehat{p}_t^{b*} = \varsigma \widehat{\mu}_t + \widehat{p}_{t-1}^* - \pi_t + \gamma \pi_{t-1} \quad (8)$$

$$\widehat{p}_t^* = (1 - \omega) \widehat{p}_t^{f*} + \omega \widehat{p}_t^{b*} \quad (9)$$

$$\pi_t = \frac{1 - \alpha}{\alpha} \left( (1 - \omega) \widehat{p}_t^{f*} + \omega \widehat{p}_t^{b*} \right) \quad (10)$$

where (as in Steinsson)  $\widehat{p}_t^{f*}$ ,  $\widehat{p}_t^{b*}$  and  $\widehat{p}_t^*$  denote percent deviations of  $p_t^{f*}/P_t$ ,  $p_t^{b*}/P_t$  and  $p_t^*/P_t$ , respectively, from their steady-state values of unity. Combining the first-order condition (3) with equations (8), (9) and (10) yields a Phillips curve of the form<sup>4</sup>:

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa x_t + \varphi_\theta \widehat{\theta}_t \quad (11)$$

where,

$$\begin{aligned} \chi_f &\equiv \frac{\alpha}{\omega [1 - \alpha + \alpha\beta (\alpha + \gamma (1 - \alpha))] + \alpha} \\ \chi_b &\equiv \frac{\omega (\alpha + \gamma (1 - \alpha))}{\omega [1 - \alpha + \alpha\beta (\alpha + \gamma (1 - \alpha))] + \alpha} \\ \kappa &\equiv \frac{(1 - \omega) (1 - \alpha) (1 - \alpha\beta) (\sigma + \psi)}{\sigma (\omega [1 - \alpha + \alpha\beta (\alpha + \gamma (1 - \alpha))] + \alpha) (\psi + \bar{\theta})} \\ \varphi_\theta &\equiv - \left( \frac{1}{\bar{\theta} - 1} \right) \left( \frac{(1 - \omega) (1 - \alpha)}{\omega [1 - \alpha + \alpha\beta (\alpha + \gamma (1 - \alpha))] + \alpha} \right) \left[ \left( \frac{\varsigma \omega}{1 - \omega} \right) + \frac{(1 - \alpha\beta)}{(1 + \psi^{-1} \bar{\theta})} \right] \end{aligned}$$

The log-linear approximation to (4) is:

$$\widehat{p}_t^\diamond = \zeta_\theta \widehat{\theta}_t + \zeta_x x_t \quad (12)$$

where,

$$\begin{aligned} \zeta_\theta &\equiv - \frac{1}{(1 + \bar{\theta} \psi^{-1}) (\bar{\theta} - 1)} \\ \zeta_x &\equiv \frac{\sigma + \psi}{\sigma (\psi + \bar{\theta})} \end{aligned}$$

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<sup>4</sup>We have used the fact that the markup shock is calibrated to be i.i.d., to eliminate the lead of the shock from the Phillips curve.

### 2.3. Monetary Policy

As a baseline we assume monetary policy is conducted according to a simple rule of the form:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \gamma_\pi \pi_t + \varepsilon_t^{mp} \quad (13)$$

where  $\rho_i$  is a parameter governing the persistence of interest rate movements,  $\gamma_\pi$  reflects the aggressiveness of monetary policy to inflation fluctuations and  $\varepsilon_t^{mp}$  is an i.i.d. shock with zero mean and constant variance.

### 2.4. Welfare

A quadratic approximation to social welfare in this economy is proportional to:

$$\sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O(\|\xi\|^3) \quad (14)$$

where *t.i.p.* represents the so-called "terms independent of policy," and

$$L_t = \pi_t^2 + \lambda_1 x_t^2 + \lambda_2 [\pi_t - (\gamma + \alpha(1 - \gamma)) \pi_{t-1}]^2 + \lambda_3 \widehat{\theta}_t (\pi_t - \pi_{t-1}) + \lambda_4 \widehat{\theta}_t^2 \quad (15)$$

$$\begin{aligned} \Omega &\equiv \frac{\bar{Y} u_C}{2} \left[ \frac{\alpha \bar{\theta} (1 + \bar{\theta} \psi^{-1})}{(1 - \alpha)(1 - \alpha\beta)} \right] \\ \lambda_1 &\equiv \frac{(1 - \alpha)(1 - \alpha\beta)(\sigma + \psi)}{\alpha \sigma \bar{\theta} (\psi + \bar{\theta})} \\ \lambda_2 &\equiv \frac{\omega}{\alpha(1 - \omega)} \\ \lambda_3 &\equiv 2 \left[ \frac{\omega(1 - \alpha)}{\alpha(1 - \omega)} \right] \left( \frac{\varsigma}{\bar{\theta} - 1} \right) \\ \lambda_4 &\equiv (1 - \alpha) \left[ \frac{\omega(1 - \alpha)}{\alpha(1 - \omega)} \right] \left( \frac{\varsigma}{\bar{\theta} - 1} \right)^2 \end{aligned}$$

Note that this social loss function is identical to that in Steinsson (2003) when  $\gamma = 1$  and  $\varsigma = 0$ .



## 2.5. Endogenizing the Proportion of RT Price-Setters

In the preceding section, we followed standard practice and treated the proportion of RT price-setters,  $\omega$ , as a fixed parameter with respect to changes in policy regime. This is potentially problematic because the costs of following a simple rule-of-thumb (or, equivalently, the benefits of being rationally forward-looking) are likely to change with the policy regime. If price-setters re-evaluate their decision procedures in light of changes to the regime, then the proportion of RT price-setters will be regime-dependent. In our analysis, we assume that price-setters incur a fixed cost each time they rationally re-optimize their price. They can avoid paying this fixed cost by instead using the simple rule-of-thumb in (5). We treat the decision to be RT or FL as regime-dependent: Price-setters make a rational decision to be RT or FL so as to maximize the unconditional expectation of profits, given the policy regime. That is, price-setters condition this decision on the policy regime, but not on the state of the economy.

Before explaining how we define the equilibrium proportion of RT price-setters, it will be helpful to note some preliminary relationships. As in Romer (1990), we can show that a quadratic approximation to the loss in profits from having a suboptimal price is given by:

$$\widehat{q}(\widehat{p}_t(z), Y_t, \theta_t, \xi_t) \approx -\frac{|\widehat{q}_{11}|}{2} (\widehat{p}_t(z) - \widehat{p}_t^\diamond)^2 + \text{terms independent of } \widehat{p}_t(z)$$

where,

$$\widehat{q}_{11} = -(\bar{\theta} - 1) [\psi^{-1}\bar{\theta} + 1] \bar{Y}$$

Let  $\widehat{p}_{t-1}^f(z)$  denote the lagged relative price of a FL price-setter and  $\widehat{p}_{t-1}^b(z)$  denote the lagged relative price of a RT price-setter. Then we can write the value functions for FL price-setters as:

$$\begin{aligned} V_t^{nr}(z) &= \widehat{q}(\widehat{p}_{t-1}^f(z) - \pi_t, Y_t, \theta_t, \xi_t) + \beta\alpha E_t \{V_{t+1}^{nr}(z)\} + \beta(1-\alpha) E_t \{V_{t+1}^r(z)\} \\ V_t^r(z) &= \widehat{q}(\widehat{p}_t^{f*}, Y_t, \theta_t, \xi_t) - c + \beta\alpha E_t \{V_{t+1}^{nr}(z)\} + \beta(1-\alpha) E_t \{V_{t+1}^r(z)\} \end{aligned}$$

where  $c$  is the fixed of rational re-optimization. Define  $W$  as the unconditional expected value of a FL price setter:

$$W \equiv \alpha E[V_t^{nr}(z)] + (1-\alpha) E[V_t^r(z)]$$

and define the FL price gap as:

$$\widehat{p}_t^{f,gap}(z) = \begin{cases} \widehat{p}_{t-1}^f(z) - \pi_t - \widehat{p}_t^\diamond & \text{in the "nr" state} \\ \widehat{p}_t^{f*} - \widehat{p}_t^\diamond & \text{in the "r" state} \end{cases}$$

Then we can write,

$$W = -\frac{|\hat{q}_{11}|}{2} \left( \frac{1}{1-\beta} \right) \text{var} \left( \hat{p}_t^{f,gap}(z) \right) - \left( \frac{1-\alpha}{1-\beta} \right) c \quad (16)$$

The value functions for RT price-setters takes the form:

$$\begin{aligned} \tilde{V}_t^{nr}(z) &= \hat{q}(\hat{p}_{t-1}^b(z) - \pi_t, Y_t, \theta_t, \xi_t) + \beta\alpha E_t \left\{ \tilde{V}_{t+1}^{nr}(z) \right\} + \beta(1-\alpha) E_t \left\{ \tilde{V}_{t+1}^r(z) \right\} \\ \tilde{V}_t^r(z) &= \hat{q}(\hat{p}_t^{b*}, Y_t, \theta_t, \xi_t) + \beta\alpha E_t \left\{ \tilde{V}_{t+1}^{nr}(z) \right\} + \beta(1-\alpha) E_t \left\{ \tilde{V}_{t+1}^r(z) \right\} \end{aligned}$$

Define  $\tilde{W}$  as the unconditional value of a RT price-setter:

$$\tilde{W} \equiv \alpha E \left[ \tilde{V}_t^{nr}(z) \right] + (1-\alpha) E \left[ \tilde{V}_t^r(z) \right]$$

and the RT price gap as:

$$\hat{p}_t^{b,gap}(z) = \begin{cases} \hat{p}_{t-1}^b(z) - \pi_t - \hat{p}_t^\diamond & \text{in the "nr" state} \\ \hat{p}_t^{b*} - \hat{p}_t^\diamond & \text{in the "r" state} \end{cases}$$

Then,

$$\tilde{W} = -\frac{|\hat{q}_{11}|}{2} \left( \frac{1}{1-\beta} \right) \text{var} \left( \hat{p}_t^{b,gap}(z) \right) \quad (17)$$

We can now turn to the definition of the equilibrium value of  $\omega$ , which we will denote by  $\omega^*$ . Note that in our model, and with the calibrated values we use below, the condition  $W'(\omega) - \tilde{W}'(\omega) > 0$  holds for all  $\omega \in [0, 1]$ . That is, the marginal benefit of being an FL price-setter is increasing in  $\omega$ . This ensures that we always have a unique equilibrium value of  $\omega$ . However, we have no reason to believe that this condition will hold in other models or for other calibrations of our model. Thus, in other environments it may be possible to find situations with multiple equilibria. Given that  $W'(\omega) - \tilde{W}'(\omega) > 0$ , an equilibrium will satisfy one of the following conditions:

- (i) If  $W(\omega^*) = \tilde{W}(\omega^*)$  for some  $\omega^* \in [0, 1]$  then  $\omega^*$  is an equilibrium.
- (ii) If  $W(\omega) > \tilde{W}(\omega) \forall \omega \in [0, 1]$  then  $\omega^* = 0$  is an equilibrium.
- (iii) If  $W(\omega) < \tilde{W}(\omega) \forall \omega \in [0, 1]$  then  $\omega^* = 1$  is an equilibrium.

Type (i) equilibria require that the values of being FL and RT are equated at  $\omega^*$ . Figure 1 illustrates this type of equilibrium. Type (ii) equilibria arise when firms are better off being FL for all permissible values of  $\omega$ . This is illustrated in Figure 2. Finally, type (iii) equilibria occur when firms are better off being RT for all values of  $\omega$ . Figure 3 provides examples of type (iii) equilibria.

### 3. Calibration

In order to generate results, we need to assign numerical values to the parameters under consideration. For the benchmark model, we follow Steinsson and set  $\beta = 0.99$ ,  $\sigma = 5$ ,  $\alpha = 0.7$ ,  $\psi = 2$ , and  $\bar{\theta} = 5$ . These values are consistent with those reported in Rotemberg and Woodford (1997) and Gali and Gertler (1999). On the monetary policy side, we set  $\rho_i = 0.8$  and  $\gamma_\pi = 1.5$  which is broadly in line with the Taylor rule estimation results reported in Orphanides (2003). Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001) provide a range of empirical estimates for the proportion of RT price-setters but the average full-sample value appeared centered on 0.3; thus we set  $\omega = 0.3$  as a baseline. We set the weight on lagged inflation in the rule-of-thumb,  $\gamma$  equal to unity. In addition, we will set  $\zeta = 1 / (1 + \bar{\theta}\psi^{-1})$ , so that the RT price-setters react to markup shocks as if they were solving a static problem. The shock process are calibrated using the post-1982 results reported in Rabanal and Rubio-Ramirez (2005), *viz.*,  $\rho_\theta = 0$ ,  $\rho_{r,n} = 0.7$ ,  $\sigma_\theta = 0.44$ ,  $\sigma_{r,n} = 0.0244$  and  $\sigma_{mp} = 0.002$ . We view the current calibration as capturing the midpoint of the ranges for the parameters under consideration.

In order to calibrate the cost of rational forward-looking reoptimization,  $c$ , we ask what value of  $c$  would be consistent with the benchmark calibration described earlier (that is, what value of  $c$  would rationalize  $\omega = 0.3$  given the calibration for the other parameters). This procedure yields  $c = 0.0035$ .

### 4. IT versus PLT with an Endogenous Proportion of RT Price-Setters

A general result of models with lagged inflation is that some degree of price-level drift is optimal, even if the monetary authority can commit to its future policies. Steinsson (2003) demonstrates that the optimal degree of price-level drift is related positively to the proportion of RT price-setters. Thus, when studies use values of  $\omega$  that are relatively low (0.1 to 0.3), it is not surprising that they often conclude that price-level targeting dominates inflation targeting. The low value of  $\omega$ , however, has been based on samples where central banks have explicitly or implicitly targeted inflation so it is possible that the proportion of RT price-setters may change with a shift to an explicit price-level targeting regime. We explore this possibility and its implications in this section.

We consider two simple monetary policy rules of the form:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) (r_t^n + \gamma_\pi \pi_t)$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) (r_t^n + \gamma_p p_t)$$

where the first rule is referred to as an inflation targeting (IT) rule and the second as a price-level targeting (PLT) rule. We choose the parameters of these rules to minimize the unconditional expectation of the welfare loss (14).

Table 1 reports results for the case in which  $\omega$  is held constant at its benchmark value while the policy rule is optimized over  $\rho_i$  and  $\gamma_\pi$  (or  $\gamma_p$ ). The parameter  $\gamma_\pi$  changes only little relative to its benchmark calibration. For both IT and PLT, the policy inertia parameter is zero, reflecting the relative importance of shocks to the natural rate of interest and the absence of an interest rate term in the loss function. As the benchmark calibration of  $\omega$  is relatively low, PLT dominates IT as expected (the welfare loss associated with PLT relative to IT is 0.81). The table also reports several other statistics that help us better understand the subsequent results. In particular the value of the loss function weight on quasi-differenced inflation,  $\lambda_2$ , is reported as it is a function of  $\omega$ . In Table 1, therefore, the value of  $\lambda_2$  remains constant across the two policy experiments as the proportion of RT price setters is fixed. Nevertheless, changes in  $\lambda_2$  will play an important role in the subsequent results. In addition, we report the standard deviations of inflation and the output gap. In Table 1, these statistics are reported relative to the benchmark policy rule calibration, so we see reductions in both inflation and output gap variability from the "optimized" policy rule parameters. These unconditional moments will play an important role in households' decisions between pricing in a FL or RT manner.

To gain some intuition into the role played by these unconditional moments, it is useful to think about the determinants of the variance of  $\widehat{p}_t^{b,gap}(z)$ . Recall that if  $\widehat{p}_t^{b,gap}(z)$  is more volatile, then the losses associated with being a RT price-setter are greater, *ceteris paribus*. The definition of  $\widehat{p}_t^{b,gap}(z)$  implies:

$$var\left(\widehat{p}_t^{b,gap}(z)\right) = \alpha var\left(\widehat{p}_{t-1}^b(z) - \pi_t - \widehat{p}_t^\diamond\right) + (1 - \alpha) var\left(\widehat{p}_t^{b*} - \widehat{p}_t^\diamond\right)$$

We can gain further insights by focusing on the second term on the right-hand side. Using the expressions for  $\widehat{p}_t^{b*}$  and  $\widehat{p}_t^\diamond$  we can write:

$$var\left(\widehat{p}_t^{b*} - \widehat{p}_t^\diamond\right) = var\left(-\left(\frac{\varsigma}{\theta - 1}\right)\widehat{\theta}_t + \widehat{p}_{t-1}^* - \pi_t + \pi_{t-1} - \zeta_\theta \widehat{\theta}_t + \zeta_x x_t\right)$$

Using the fact that  $\zeta_\theta = -\varsigma/(\bar{\theta} - 1)$  and that  $\widehat{p}_{t-1}^*$  is proportional to  $\pi_{t-1}$  we can rewrite this as:

$$\text{var}(\widehat{p}_t^{b*} - \widehat{p}_t^\diamond) = \text{var}\left(\left(\frac{1}{1-\alpha}\right)\pi_{t-1} - \pi_t + \zeta_x x_t\right)$$

or,

$$\begin{aligned} \text{var}(\widehat{p}_t^{b*} - \widehat{p}_t^\diamond) &= \left[ \left(\frac{1}{1-\alpha}\right)^2 + 1 \right] \text{var}(\pi_t) + \zeta_x^2 \text{var}(x_t) \\ &\quad - \left(\frac{2}{1-\alpha}\right) \text{cov}(\pi_t, \pi_{t-1}) + \left(\frac{2\zeta_x}{1-\alpha}\right) \text{cov}(x_t, \pi_{t-1}) - 2\zeta_x \text{cov}(\pi_t, x_t) \end{aligned} \quad (18)$$

Thus,  $\text{var}(\widehat{p}_t^{b*} - \widehat{p}_t^\diamond)$  depends on a number of variances and covariances. The two unconditional moments identified in the tables are those that we found to be most important for the results. In particular, it is possible to show that  $\text{var}(\widehat{p}_t^{b, \text{gap}}(z))$  is increasing in  $\text{var}(\pi_t)$  and  $\text{var}(x_t)$ . That is, as inflation and the output gap become more volatile, it becomes more costly to be an RT price-setter, *ceteris paribus*. As will become apparent from the tables, output gap volatility is the single most important factor in determining the equilibrium proportion of RT price-setters. This is because the output gap term in (18) is generally an order of magnitude greater than the inflation term, so it tends to be the primary determinant of  $\text{var}(\widehat{p}_t^{b*} - \widehat{p}_t^\diamond)$ .

Turning again to Table 1, note that both reported standard deviations decline relative to the benchmark calibration. This implies that the performance of the rule-of-thumb has improved. On this basis, one might conjecture that, were we to hold the policy parameters constant at the values reported in Table 1 while endogenizing  $\omega$ , the equilibrium proportion of RT price-setters would exceed 0.3. This exercise is reported in Table 2.

The results in Table 2 can be interpreted as an exercise where the central bank chooses its policy parameters assuming that  $\omega$  will remain at 0.3 regardless of what it does to the policy rule, but, in practice,  $\omega$  reacts endogenously. As expected the proportion of rule-of-thumb price-setters rises substantially relative to Table 1. Note that the relative standard deviations reported in Table 2 do not provide information on the sources of the increase in  $\omega$ . These standard deviations are outcomes of an equilibrium in which  $\omega$  is endogenous, so they are consistent with price-setters being indifferent between the RT and FL options.

Rather, the increase in  $\omega$  can be traced to the decline in the standard deviations reported in Table 1 (which make RT behaviour relatively more attractive).

The increase in  $\omega$  causes the performance of both the PLT and IT rules to deteriorate. However, the PLT rule deteriorates more severely, reversing its dominance over IT. The primary source of this relative deterioration is the direct effect of the change in  $\omega$  on the weight ( $\lambda_2$ ) on the squared quasi-difference of inflation in the loss function. Although the values of  $\omega$  reported in Table 2 may not appear to be very different across IT and PLT, the small difference translates into a large differences in  $\lambda_2$  (because  $\lambda_2 = \omega / (\alpha(1 - \omega))$ ). In particular,  $\lambda_2$  increases from 0.61 in Table 1 to 7.5 under IT and to 12.86 under PLT in Table 2. Thus, failing to account for the impact of policy on the proportion of RT price-setters can lead to outcomes that are significantly worse than anticipated.

The fact that IT outperforms PLT when the proportion of RT price-setters is high, as in Table 2, is not surprising. As noted earlier, Steinsson has shown that the degree of price-level drift that is optimal is increasing in  $\omega$ . In light of this, one may be tempted to conclude that our assumption of full indexation ( $\gamma = 1$ ) in the rule-of-thumb is driving our results, as indexation affects the weight on lagged inflation in the new Keynesian Phillips curve. That is, one might presume that RT price-setters introduce optimal price-level drift primarily through the lagged inflation term in their price-setting rule. Amano, Mendes and Murchison (2009) find, however, that the degree of indexation does not play an important role in determining optimal monetary policy. Even for extreme indexation values of  $\gamma = 0$  and  $\gamma = 1$ , the changing weight on lagged inflation in the new Keynesian Phillips curve plays only a small part in determining optimal price-level drift and inflation undershooting. In other words, inflation persistence via lagged inflation plays only a small role in determining the efficacy of monetary policy.

Rather, Amano, Mendes and Murchison show that the new Keynesian Phillips curve with RT price-setters implies an overdiscounting effect: Future output gaps are discounted at a rate greater than the discounting of future losses in the social welfare function. To see this, note that the forward solution of the new Keynesian Phillips curve in (11) is:

$$\pi_t = \lambda\pi_{t-1} + \frac{\kappa}{1 - \chi_f\beta\lambda} \sum_{i=0}^{\infty} \left( \frac{\chi_f\beta}{1 - \chi_f\beta\lambda} \right)^i E_t x_{t+i} + \frac{\varphi_\theta}{1 - \chi_f\beta\lambda} \hat{\theta}_t$$

where,

$$\lambda = -\frac{1}{2\chi_f\beta} \left( -1 + \sqrt{1 - 4\chi_b\chi_f\beta} \right)$$

If  $\omega = 0$ , then  $\chi_f = 1$ ,  $\chi_b = 0$  and therefore  $\lambda = 0$ . In this case,  $\chi_f\beta / (1 - \chi_f\beta\lambda) = \beta$ . That is, the discount factor in the forward solution is equal to the discount factor in the welfare function. But, if  $\omega > 0$ , then  $\chi_f\beta / (1 - \chi_f\beta\lambda) < \beta$ . This overdiscounting reduces the impact of the expectations channel and, therefore, leads to greater optimal drift. In numerical experiments, Amano, Mendes and Murchison find the overdiscounting effect to be more important in determining optimal monetary policy than the degree of indexation in the RT.

Finally, we consider the case where the central bank adjusts the parameters of its policy rule to minimize the social loss function while taking into account the reaction of  $\omega$ . These results are reported in Table 3. The values of  $\rho_i$  increase substantially to 0.9 and 0.8 under IT and PLT, respectively. However, the aggressiveness parameters do not behave similarly under IT and PLT. The value of  $\gamma_\pi$  remains largely unchanged, but the value of  $\gamma_p$  increases from 0.21 to 1.72. The three parameters that change (both  $\rho_i$ 's and  $\gamma_p$ ) induce greater undershooting of inflation as they increase (this is not true of the one parameter that remains unchanged,  $\gamma_\pi$ ). When  $\omega$  is endogenous, several factors influence the central bank's incentive to induce undershooting. First, as  $\omega$  approaches unity the coefficient on the markup shock in the Phillips curve increases by a factor of  $1/(1 - \alpha\beta)$ . This occurs because RT price-setters react to markup shocks in a statically optimal manner – they ignore the fact that they might not have the opportunity to change their price for several periods. Second, as we describe in more detail below, the fact that  $\lambda_2$  has increased relative to the benchmark makes a greater degree of inflation undershooting desirable. Third, increases in undershooting tend to increase output gap volatility which reduces the incentive to be RT. This leads to a welfare-improving reduction in the proportion of RT price-setters, *ceteris paribus*.

Note that the performance of PLT relative to IT does not improve. However, the performance of both the PLT and IT rules improve relative to the case in which the central bank erroneously treats  $\omega$  as fixed (under both IT and PLT the losses are smaller by a factor of roughly 0.55). Most of these welfare gains come from the reductions to  $\lambda_2$  and inflation volatility.

Since the quasi-difference of inflation term in the loss function plays an important role in the results, it is worth discussing it in more detail. The presence of  $\lambda_2 \neq 0$ , in isolation, will reduce the optimal level of price drift since stabilizing inflation also helps to stabilize changes in inflation. In the limit as  $\lambda_2$  goes to infinity, the central bank will fully stabilize inflation and a mark-up shock will only be reflected in the output gap, just as it would if the level of inflation were the only

argument in the loss function. The blue (dark) line relative to the green (light) line in Figure 4 shows the marginal impact under the optimal state-contingent policy from using the correct loss function, with  $\lambda_2 = \frac{\omega}{\alpha(1-\omega)}$ , rather than zero. Note that having a non-zero value for  $\lambda_2$  causes there to be less price-level drift. This can be explained by expanding the third argument in the loss function:

$$\lambda_2 (\pi_t - \pi_{t-1})^2 = \lambda_2 (\pi_t^2 + \pi_{t-1}^2 - 2\pi_t\pi_{t-1})$$

and in unconditional expectation:

$$\lambda_2 E [(\pi_t - \pi_{t-1})^2] = 2\lambda_2 [var(\pi_t) - cov(\pi_t, \pi_{t-1})]$$

The introduction of the  $-2\lambda_2 cov(\pi_t, \pi_{t-1})$  term implies that the central bank will have an incentive to induce some positive autocorrelation in inflation. This explains the somewhat more drawn out undershooting of inflation relative to the case with  $\lambda_2 = 0$ .

When  $\omega$  is very high, as it is in Table 2, the central bank has a very strong incentive to increase the extent of inflation undershooting. With an IT rule, this is best achieved by increasing the value of the smoothing parameter,  $\rho_i$ . This is in fact what happens in Table 3. This change also causes the standard deviation of inflation to fall and the standard deviation of the output gap to rise. As discussed above, output gap volatility plays a critical role in the decision to be RT or FL. In this case, the increase in output gap volatility is a main driver of the reduction in  $\omega$ .

## 5. Concluding Remarks

In this paper we examined the inflation versus price-level targeting debate when the proportion of rule-of-thumb price setters is allowed to vary with the monetary policy regime. Two tentative results emerged from this research. First, we found that if a monetary authority fails to account for the impact of policy changes on the proportion of rule-of-thumb price-setters, outcomes that are significantly worse than anticipated can occur. Second, in contrast to earlier studies in this literature, we find that allowing the fraction of rule-of-thumb price setters to respond endogenously to the monetary policy framework can lead us to conclude that inflation targeting dominates price-level targeting in minimizing welfare losses. We would like to close by emphasizing that our results are quite preliminary and that a full round of sensitivity analysis is required before firmer conclusions can be drawn.



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**Table 1: Fixed  $\omega = 0.3$** 

Case	$\rho_i$	$\gamma_\pi$	$\omega$	Relative Loss (PLT/IT)	$\lambda_2$	Relative to Benchmark Case	
						$std(\pi_t)$	$std(x_t)$
IT	0	1.51	0.3	–	0.61	0.77	0.85
PLT	0	0.21	0.3	0.81	0.61	0.66	0.91

**Table 2: Fixed Policy Parameters, Endogenous  $\omega$** 

Case	$\rho_i$	$\gamma_\pi$	$\omega$	Relative Loss (PLT/IT)	$\lambda_2$	Relative to Table 1 Cases	
						$std(\pi_t)$	$std(x_t)$
IT	0	1.51	0.84	–	7.50	1.36	0.76
PLT	0	0.21	0.90	1.40	12.86	1.37	0.93

**Table 3: Optimized Policy Parameters with Endogenous  $\omega$** 

Case	$\rho_i$	$\gamma_\pi$	$\omega$	Relative Loss (PLT/IT)	$\lambda_2$	Relative to Table 2	
						$std(\pi_t)$	$std(x_t)$
IT	0.9	1.41	0.64	–	2.54	0.81	1.43
PLT	0.8	1.72	0.75	1.41	4.28	0.79	1.26

Figure 1: Excess Value of Forward-Looking Behaviour

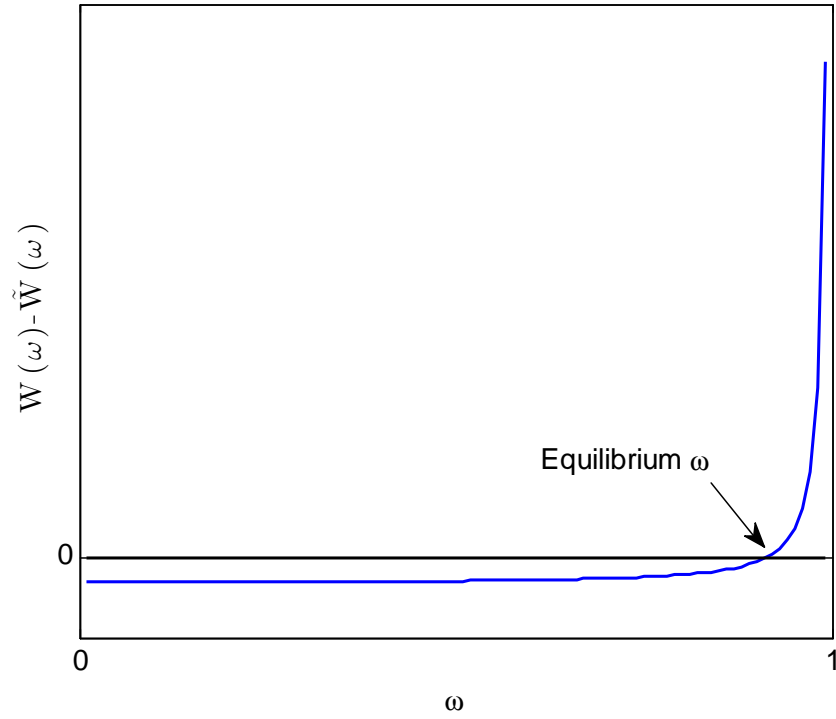


Figure 2: Excess Value of Forward-Looking Behaviour

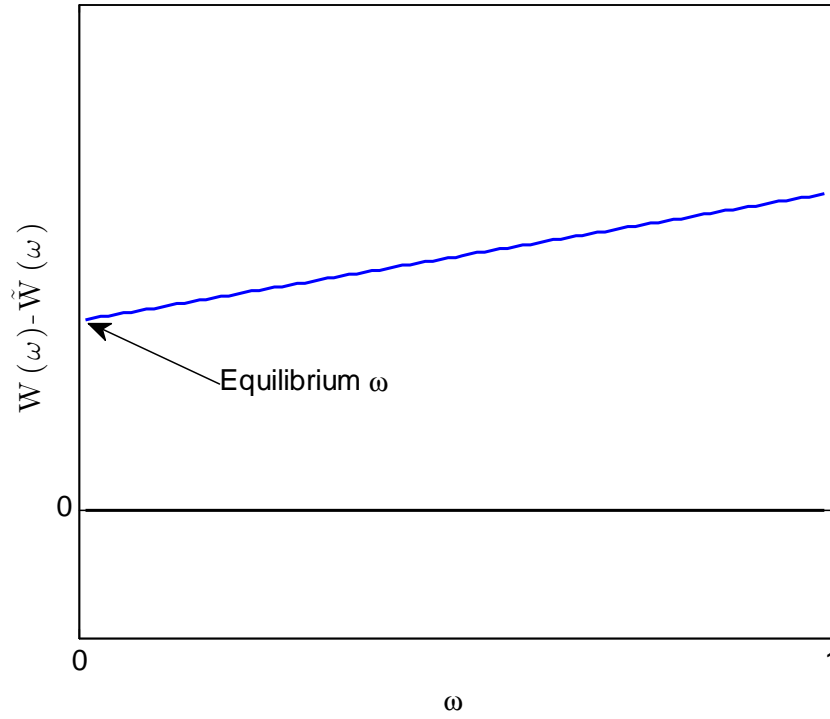
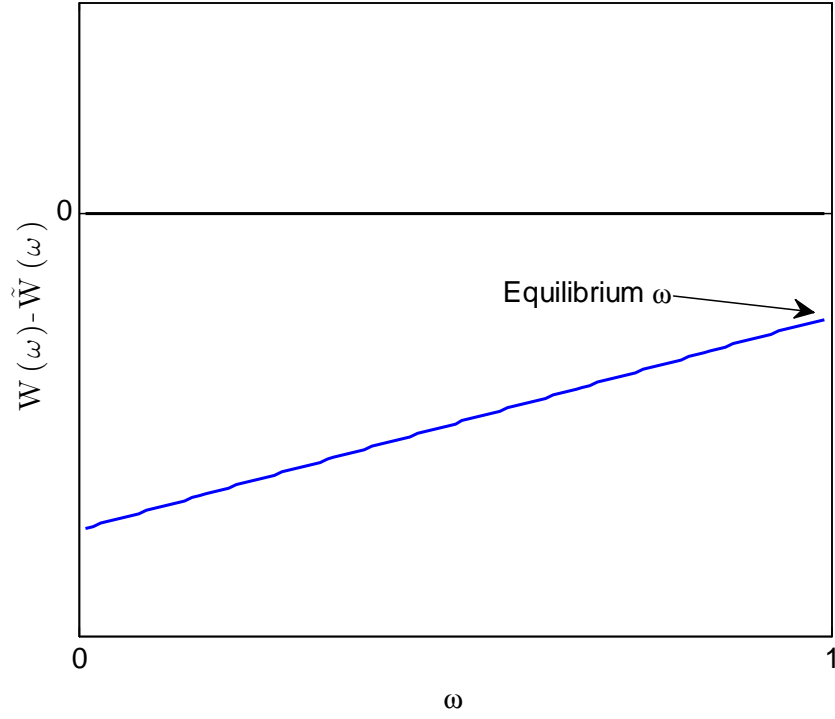


Figure 3: Excess Value of Forward-Looking Behaviour



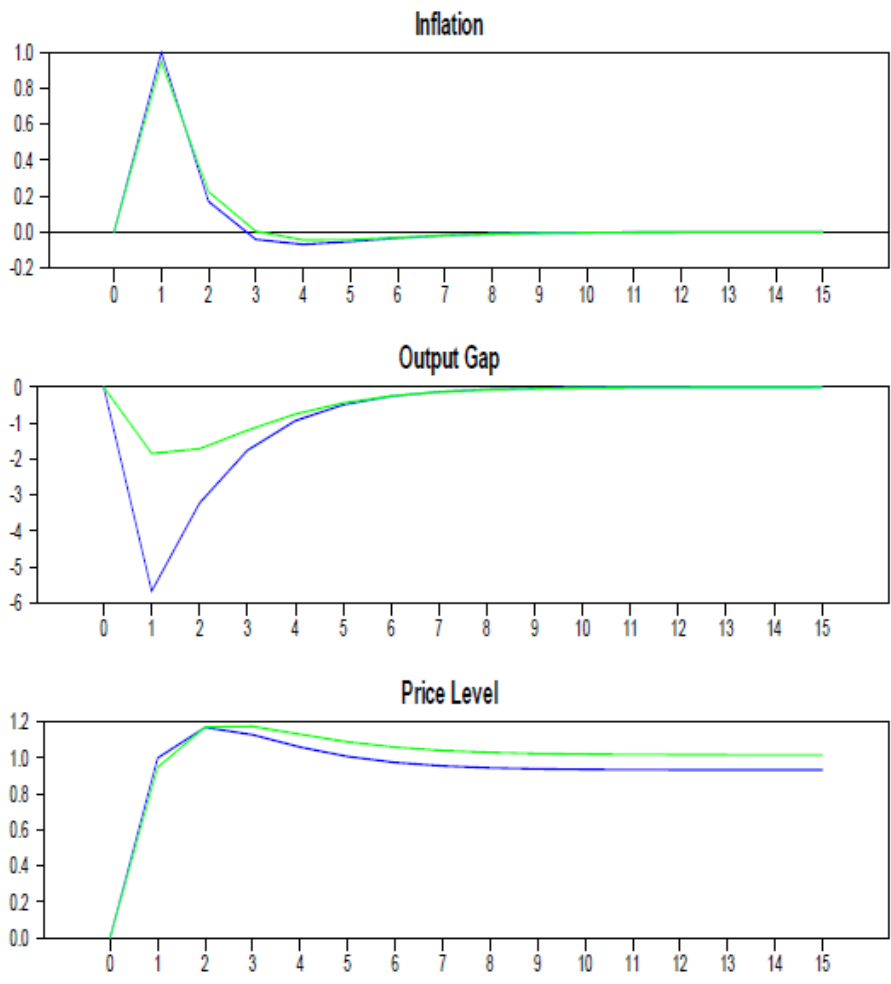


Figure 4: Loss Function Effect