A Model of Tiered Settlement Networks*

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Abstract

This paper develops a model of settlement system to study the endogenous structure of settlement networks, and the welfare consequences of clearing agent failure. The equilibrium degree of tiering is endogenously determined by the cost structure and the information structure. The degree of tiering is decreasing in the fixed cost of operating the second-tier network and the availability of public credit history. Furthermore, the welfare effects of clearing agent failure can be decomposed into operational inefficiency and the loss of private information.

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1 Introduction

Payment and settlement networks typically involve various tiers of intermediation. Some banks participate and clear directly in a "first-tier" network. A subset of these direct clearers (DC) then act as clearing agents (CA) by operating internal "second-tier" networks and provide settlement accounts to downstream indirect clearers (IC) who belong to these second-tier networks. Tiered structures are observed in most large-value payments systems and retail payment systems of industrialized countries. In Canada, for example, both large-value payments systems, LVTS and ACSS, exhibit a high degree of tiering.[1] The goal of this paper is to build an analytically tractable model to study the endogenous degree of tiering and the welfare effects of the failure of a clearing agent. In particular, this paper focuses on the following questions: (i) Why do settlement networks generally exhibit a tiered structure? (ii) How is the equilibrium degree of tiering determined? (iii) What are the welfare consequences of the failure of a clearing agent?

Standard Walrasian models abstract from the mechanism through which payments and settlement take place and are thus not suitable tools for answering questions about settlement. While there has been a large amount of recent work in the modelling of payments and settlements the majority of the work dealing with participation in a settlement system is practitioner oriented dealing with very specific policy issues.[2] In particular, there is still little theoretical work studying the tiered structure in settlement systems.[3] One exception is Kahn and Roberds (2002); who model tiering as an efficient outcome in a static model with limited enforcement. Our paper builds on the same idea to develop a dynamic equilibrium model for studying the degree of tiering and welfare effects of clearing agent failure. Other related papers include Lai, Chande, and O'Conner (2006) and Chapman and Martin (2007).

[1]For example in 2006 their were only fourteen private participants in LVTS compared to the universe of 85 financial institutions (Committee on Payment and Settlement Systems 2008, page. 27)

[2]Much of the practitioner-oriented literature on payment system design is based on payment system simulators such as the one developed by the Bank of Finland (BoF-PSS2). Because they do not model system participants' behavior, these tools are not appropriate for studying the endogenous formation of tiered networks and how behaviour changes given changes in tiering.

[3]For an overview of recent work on the economics of payment and settlement the interested reader is referred to the surveys Kahn and Roberds (2008) and Lai and Chiu (2007)

In a partial equilibrium model Lai, Chande, and O'Conner (2006) analyzes the contractual arrangements between clearing agents and indirect clearers. In a general equilibrium with moral hazard, Chapman and Martin (2007) studies efficient arrangements for liquidity provision by central banks. They show that the optimal arrangement may exhibit a tiered structure.

We construct a model of a settlement system that is analytically tractable in which the settlement structure is determined endogenously. Our model builds on two basic components: *private information* regarding participants' credit-worthiness, and *economies of scale* in the participation of the settlement system.[4] The model economy consists of a trading sector and a settlement sector. In the trading sector, agents meet bilaterally to trade consumption goods financed by private liabilities. In the settlement sector, agents interact to clear and settle these payment instruments. Due to consumption and production shocks, underlying transactions in the trading sector generate the bilateral random payments flows in a settlement network. In this environment, the mode of settlement (i.e. real-time vs. deferred) and the structure of settlement networks (i.e. direct or indirect participation) are endogenously determined by underlying agents, subject to the fundamental cost structure and information structure.

Let us explain the basic idea of the model. The choice of settlement mode between real-time and deferred settlement involves the fundamental trade-off between liquidity costs and default risk. On the one hand, since real-time settlement imposes a higher liquidity cost, payment senders (debtors) prefer to choose deferred settlement. On the other hand, owing to the settlement risk involved, payment recipients (creditors) are willing to accept deferred settlement only from creditworthy payment senders. So the choice of settlement mode depends critically on whether creditors possess reliable information about debtors'

[4]To motivate the relevance of private information, note that in Canada while all clearing agents and direct clearers have high credit ratings, many indirect clearers have relatively low ratings or do not have ratings. Also, many participants in the ACSS indicate that "the familiarity that one direct participant has with its counterparts creates the 'mutual trust' that is critical to a system that operates on implicit interbank credit" (Tripartite Study Group 2006). To see the relevance of economies of scale, note that direct clearers indicated that "they need sufficient volume to achieve the operating economies necessary to lower their internal per payment costs" (Tripartite Study Group 2006).

credit history. This informational constraint is particularly binding for trades involving "small" agents who lack publicly observable credit history, and who have only infrequent trading among each other. As a result, some of the small, but safe, debtors without public history will be forced to use early settlement. This is inefficient due to the unnecessary liquidity costs lead to suboptimal allocation of resources.

This inefficiency can be resolved by having exogenous "large" institutions who can choose to provide as clearing agents for other small agents. By having a portfolio of diversified projects large agents' own creditworthiness are public information. By having frequent dealings with debtors these large institutions can monitor their clients' credit history and optimally choose the settlement mode for each of them. This is the information role of clearing agents.[5] In addition to this information role, when there are fixed costs associated with settlement system participation, clearing agents also enjoy economies of scale and play a cost-saving role in a settlement network. We should note that while large equals safe in the model (except for section 4) this is to keep the model tractable and not the point we are trying to make. We instead want to make the point that the large agents can accumulate information about their clients through long-run trading relationships with them. Since this information is the private information large agent, a failure of one a clearing agent can lead to a large and persistent welfare loss to the economy to this lost information.

The main findings of our paper can be summarized as follows. First, we show that, in the presence of a non-convex cost structure and information imperfection, the tiered structure can improve efficiency by supporting cost saving and inter-bank monitoring. Second, the degree of tiering is decreasing in the fixed cost of operating the second-tier network and the availability of public credit history. Third, clearing agent failure leads to welfare costs, which can be decomposed into an information loss and an operational inefficiency. In particular, the failure will lead to a persistent loss of valuable private information regarding indirect clearers' creditworthiness. The welfare and trades will recover gradually as indirect clearers rebuild their reputation gradually over time.

[5] This is related to the adverse selection models of reputations such as Diamond (1989) and Mailath and Samuelson (2001). The interested reader is referred to Mailath and Samuelson (2006).

Here is the plan of the paper. Section (2) establishes the inefficiency of direct settlement. Section (3) derives the equilibrium degree of tiering and shows that indirect settlement can improve welfare. Section (4) derives the welfare effects of clearing agent failure and decomposes the welfare cost of the failure and section (5) concludes the paper.

2 Direct Settlement

In this section, we will first consider a benchmark economy in which agents settle their debt directly (i.e., all agents are DCs). In this economy, all agents are "small" and are subject to idiosyncratic shocks. In the next section, we will introduce large agents into the economy to work as clearing agents and allow indirect settlement.

2.1 The Economic Environment

Time is discrete and denoted $t = 0, 1, 2, ...$ Each period is further divided into three sub-periods, denoted by $s = 1, 2, 3$. We use time subscript "ts" to denote the s^{th} subperiod of time period t. There is a measure N continuum of agents, indexed by $i \in [0, N]$. The economy consists of a home island, a trading island and a settlement island. There are two types of goods, x and y. In sub-period $s = 1$, agents trade good x in the trading island. In sub-period $s = 2, 3$, agents settle their debt by using good y in the settlement island. We will use $x_{ts}(i)$ and $y_{ts}(i)$ to denote the production of good x and y respectively by agent i in sub-period ts. Similarly, we will use $x^c_{ts}(i)$ and $y^c_{ts}(i)$ to denote the consumption of good x and y respectively by agent i in sub-period ts.

Home Island

At the beginning of a period, agents start on the home island. Each agent i faces an i.i.d. random shock, $\alpha_t(i)$, regarding his consumption and production pattern in the current period. With a probability $\sigma = \frac{1}{2}$, an agent is able to produce good x and willing to consume good y. We label this agent as a *creditor*. With a probability $1-\sigma = \frac{1}{2}$, an agent is willing to consume good x and is endowed with a production project that produces good y (discussed below). We label this agent as a *debtor*. We use $\alpha_t(i) = 1$ and $\alpha_t(i) = 0$ to denote respectively the trading status of being a debtor and a creditor.

Trading Island

In sub-period 1, after the shock $\alpha_t(i)$ realized, agents leave the home island for the trading island. Agents are subject to pairwise random matching in this island: each creditor is randomly matched with a debtor. We denote the match partner of agent i by $\iota_t(i)$. When debtor i is paired with creditor $j = \iota_t(i)$, debtor i may issue a personal IOU which promises to repay $d_{ts}(i)$ units of good y to the holder in sub-period $s = 2$ or $s = 3$. After trade, agents go back to the home island.

Settlement Island

In both sub-period 2 and 3, agents start on the home island where debtors' production projects yield good y. The settlement island then opens. In the settlement island, debtors settle their IOUs using good y. In particular, debtor i pays $y_{t2} = d_{t2}(i)$ to the holder of his debt in sub-period 2, and pays $y_{t3} = d_{t3}(i)$ to the holder of his debt in sub-period 3. There is an enforcement technology to ensure debtors will redeem their IOUs issued in the current period subject to their resource constraint.[6] Attending this settlement island is costly. There is a per period fixed utility cost, C, for an agent to attend the settlement island. The decision to participate in the settlement island is made at the beginning of each period.

Debtors

A debtor wants to consume good x and has a project that produces good y. Denote the quantity of good x consumption in sub-period $t1$ by $x^c_{t1}(i)$. Debtor i's period t payoff is given by:

$$x^c_{t1}(i) - \nu_t(i)C.$$

Here, the utility function for good x is linear. Also, $\nu_t(i) = 1$ and $\nu_t(i) = 0$ denote

[6] In this model, agents are able to commit to settle their IOUs issued in the current period. Strategic default is not a feature that this paper aims to study. The issue of enforcement is left for future research.

respectively whether or not agent i attends the settlement island.

Creditors

A creditor is able to produce good x and wants to consume good y. Denote creditor j's production of good x in sub-period $t1$ as $x_{t1}(j)$ and the consumption of good y in sub-period $t2$ and $t3$ as $y^c_{t2}(j)$ and $y^c_{t3}(j)$ respectively. Creditor j's one period payoff is given by:

$$y^c_{t2}(j) + y^c_{t3}(j) - x_{t1}(j) - \nu_t(j)C.$$

Here, the cost of producing good x and the utility from consuming good y are both linear.

Projects

Each debtor i is endowed with a project which yields 1 unit of good y in sub-period 3 with a probability $p_t(i)$ (discussed below), and yields 0 unit of good y with the complementary probability. The debtor can also choose to terminate and liquidate the project earlier and yield $\pi < 1$ units of good y in sub-period 2 for sure. The debtor's discrete liquidation choice is captured by $e_t(i) \in \{1, 0\}$. With early liquidation denoted by $e_t(i) = 1$, and no early liquidation denoted by $e_t(i) = 0$, the good y output by debtor i is given by

$$(y_{t2}(i), y_{t3}(i)) = \begin{cases} (0,1) \text{ with probability } p_t(i), & \text{if } e_t(i) = 0 \\ (0,0) \text{ with probability } 1 - p_t(i), & \text{if } e_t(i) = 0 \\ (\pi, 0), & \text{if } e_t(i) = 1 \end{cases}$$

2.2 The Sequence of Events

The following table and Figure (1) summaries the time-line of the sequence of events.

Events	Island	Debtor	Creditor
$s = 1$	home	1. Agent i pays cost C or not	1. Agent j pays cost C or not
	home	2. Agent i receives preference shock	2. Agent j receives preference shock
	trading	3. Buys x^c_{t1} with d_{t2} or d_{t3}	3. Sells x_{t1} for d_{t2} or d_{t3}
$s = 2$	home	4. Terminates project and yields π	4. No Action
	settlement	5. Settles $y_{t2} = d_{t2}$ early	5. Receives $y^c_{t2} = d_{t2}$ early
	home	6. Debtor i dies or not	6. Debtor $\iota_t(j)$ dies or not
$s = 3$	home	7. Project matures and yields 1	7. No Action
	settlement	8. Settles $y_{t3} = d_{t3}$ late	8. Receives $y^c_{t3} = d_{t3}$ late
	home	9. Exogenous exit or not	9. Exogenous exit or not

1. Participation Decision in $s = 1$

In the home island, each agent decides whether to pay C to participate in the settlement island.

2. Preference shock in $s = 1$

In the home island, each agent receives a preference shock that determining whether he is a creditor or a debtor.

3. Debtors making offers in $s = 1$

Agents are subject to pairwise random matching in the trading island. Each debtor is randomly matched with a creditor. The terms of trade are determined by take-it-or-leave-it offers by the debtors which consist of $(x_{t1}, d_{t1}, d_{t2}) \in \Re^3_+$. Here, x_{t1} denotes the quantities of good x the debtor asks the creditor to produce. Also, a debtor offers personal IOUs which promise certain quantities of good y, d_{ts}, to the creditor. Creditor j then chooses whether to accept or reject the offer to maximize her expected payoff. If creditor j accepts the offer, her current period payoff is

$$\nu_t(j)\{d_{t2}(j) + p^j_t(i)d_{t3}(j)\} - x_{t1}(j) - \nu_t(j)C,$$

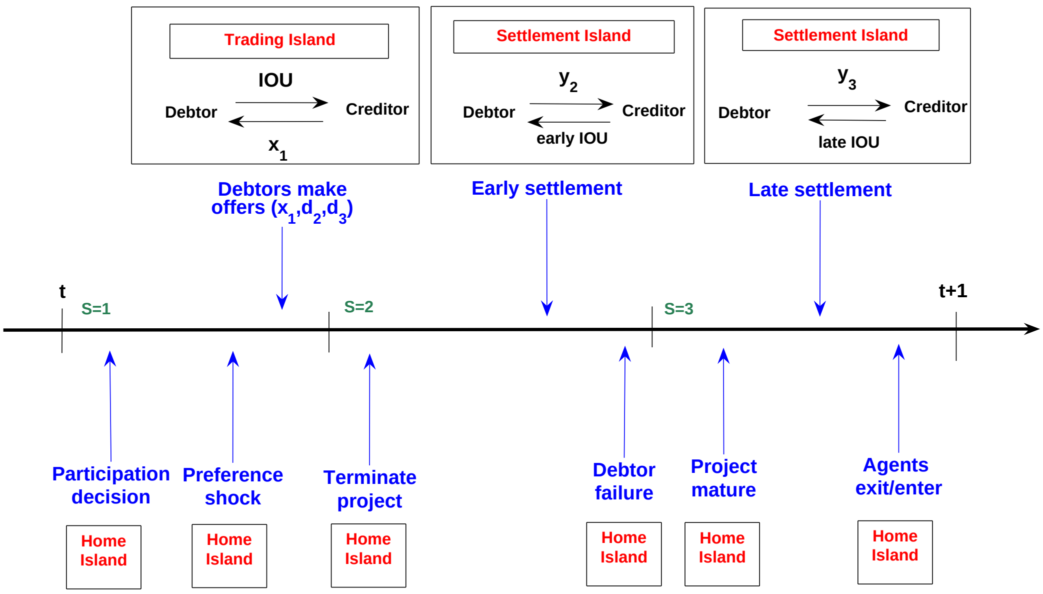

Figure 1: Time-line

where $p_t^j(i)$ denotes creditor j's subjective evaluation of the probability that debtor i will repay in sub-period $t3$.[7] Notice that a creditor can receive the repayment of good y, $y_{ts}^c = d_{ts}$, only if the creditor pays the fixed participation cost C. Therefore, a creditor j will accept an offer if only if $\nu(j) = 1$ was chosen at the beginning of the period, and

$$d_{t2}(j) + p_t^j(i) d_{t3}(j) \geq x_{t1}(j).$$

4. Termination of projects in $s = 2$

At the beginning of sub-period 2, debtors can choose to terminate their projects and get liquidation value π in the home island.

5. Early settlement in $s = 2$

If $d_{t2}(i) > 0$ (i.e., early IOUs are issued), debtor i terminates his project and attends the settlement sector in $s = 2$ to redeem his IOUs from the the holders by paying $y_{t2} = d_{t2}(i)$. A creditor holding early IOUs attends the settlement island to get repayment. Agents return to the home island after settlement.

6. Debtor failure in $s = 2$

At the end of sub-period 2, a *debtor* may die exogenously, and any unliquidated project will fail. This will induce settlement failure if $d_{t3}(i) > 0$ (i.e., deferred settlement is used).[8] The probability of receiving this settlement shock depends on an agent's intrinsic type. There are two types of agents (exogenously determined when they enter the economy). Fraction γ of them will never die at the end of sub-period $s = 2$. We label them as "good" type agents. The remaining fraction $1 - \gamma$ will die at the end of sub-period $s = 2$ with probability 1 when they are debtors.[9] We label them as "bad" type agents. When an agent enters the economy, the true type of this agent is unknown for everyone.

[7] Given the assumption of perfect enforcement, debtors cannot choose to default strategically. Here, debtors fail to repay in sub-period $t3$ only because of the exogenous shock discussed below.

[8] This risk is independent of whether the debtor trades or not.

[9] We make this strong assumption for tractability reasons. We expect that relaxing this assumption will not affect the qualitative result.

Note that the past trading history of an agent can help to predict his probability of being a bad type (i.e., having a settlement problem). In particular, for an agent who is a debtor for the first time, the probability that this agent is of bad type and dies at the end of sub-period 2 is $1-\gamma < 1$. We call it a *risky* agent. For an agent who was a debtor before, the probability that he is of good type is 1. We call this agent a *safe* agent. Therefore, the sequence of past trading statuses of an agent can reveal whether he is safe or risky. In particular, agent i is safe in period t if $\alpha_\tau(i) = 1$ for some $\tau < t$. Otherwise, agent i is risky.

7. Projects mature in $s = 3$

At the beginning of sub-period 3, if a debtor with an unliquidated project survives in sub-period 3, he will be endowed with one unit of good y in the home island.

8. Late settlement in $s = 3$

If $d_{t3}(i) > 0$ (i.e., late IOUs are issued), debtor i will attend the settlement island to redeem his IOUs from the the holders by paying $y_{t3} = d_{t3}(i)$. A creditor holding late IOUs attends the settlement island to get repayment. Agents return to the home island after settlement.

9. Exogenous death in $s = 3$

In order to maintain a stationary distribution, we assume that, at the end of each period, a random fraction δ of agents exits the economy and is replaced by a set of new agents. Also, those debtors who die and default are replaced by new agents.[10] Denote the fraction of safe households by Λ. As illustrated in Figure 2[11], the stationarity of distribution implies that

$$\begin{aligned}\Lambda &= \Lambda(1-\delta) + (1-\Lambda)\sigma\gamma(1-\delta) \\ &= \frac{\sigma\gamma(1-\delta)}{1-(1-\delta)(1-\sigma\gamma)}.\end{aligned}$$

[10]New agents are drawn from the unconditional distribution as follows: (i) a fraction γh are public agents of good type, (ii) a fraction $\gamma(1-h)$ are anonymous agents of good type, (iii) a fraction $(1-\gamma)h$ are public agents of bad type, and (iv) a fraction $(1-\gamma)(1-h)$ are anonymous agents of bad type.

[11]The debtor-creditor nodes under the "safe" branch are not shown in the figure, since it will not affect the flows.

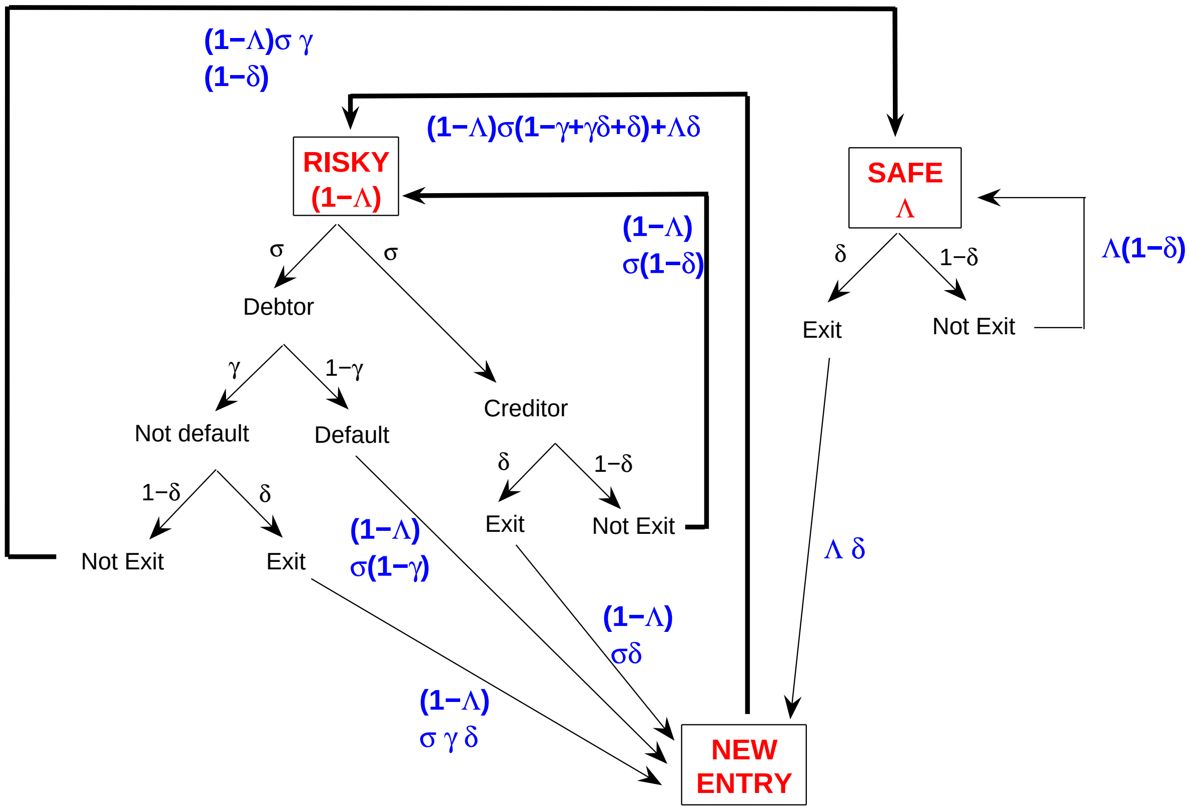

Figure 2: Stationary Distribution

2.3 Equilibrium with Direct Settlement

Public and Anonymous Agents

Here, we assume that agents are different in terms of the observability of whether they are safe or risky. In particular, the population is partitioned into two subsets. For a fraction h of the population, whether they are safe or risky are publicly observable. We label them as *public* agents. This subset of agents is denoted by H.

The remaining fraction $(1-h)$ are *anonymous*. This subset of agents is the complement of H and is denoted by $\tilde{\mathsf{H}}$. To ensure that anonymous agents do not have public trading histories, we make the following technical assumption: at the beginning of each period, each anonymous agent is randomly assigned an identity i from the set $\tilde{\mathsf{H}}$, with this identity i being private information. Since the identity of an anonymous agent changes randomly over time, the past history of his trading statuses is also private information.

Consider a meeting in the trading island in which a debtor offers $d_{t3} > 0$ to a creditor. Denote f as the unconditional probability of this debtor being a good type (i.e., who will not default on this IOU which will be settled in sub-period 3). This will happen if the debtor is either safe (with a probability Λ) or is risky but good (with a probability $(1-\Lambda)\gamma$).

From now on, we will impose the restriction on parameter values such that the liquidation value of a project is higher than the unconditional expected value of a matured project:

Assumption: $\pi > f = \Lambda + (1-\Lambda)\gamma$

Under this assumption, an anonymous debtor is able to promise more good y in expected terms by liquidating the project and settling early at $s = 2$ than by having the project matured and settling late at $s = 3$.

Given the structure of our model, no long term relationship is possible between any two agents. Apparently, the relevant individual state variable for each agent is whether he is risky (denoted by '0') or safe (denoted by '1'). Depending on whether the agent is anonymous or not, this may or may not be public information. For a public debtor, all agents can tell whether he is risky or safe. For an anonymous debtor, his trading partner cannot tell whether he is risky or safe. We will now consider the decision by the public agents and then by the anonymous agents.

Public agents

Denote the expected discounted value of a *safe* public agent by V_1. We have

$$
\begin{aligned}
V_1 = \max_{x_1^c, d_2, d_3, e, \nu} & \left[\sigma x_1^c - \nu C + (1-\delta) V_1\right], \\
\text{s.t.} \qquad & d_2 + d_3 \geq x_1^c \\
& \nu e \pi \geq d_2 \\
& \nu(1-e) \geq d_3
\end{aligned}
$$

Since an agent has bargaining power and gets trade surplus only when he is a debtor, we only need to consider the payoff when he is a debtor. A public agent needs to choose whether to participate in the settlement island (ν), what to offer when he is a debtor (x_1^c, d_2, d_3), and whether to liquidate the project (e) to maximize his current payoff $\sigma x_1^c - \nu C$, plus the continuation value $(1-\delta)V_1$. This public agent will survive to the next period with a probability $1-\delta$. The first constraint is the participation constraint of the trading partner who is a creditor. The second and third constraint are the feasibility constraints for settlement. Since this agent is safe and has public history, he will settle a late IOU for sure when he is a debtor, and his creditor knows that. The optimal decision is given by

$$
\begin{cases}
x_1^c = 1, d_2 = 0, d_3 = 1, e = 0, \nu = 1 & \text{, if } \quad \sigma \geq C \\
x_1^c = 0, d_2 = 0, d_3 = 0, e \in \{0, 1\}, \nu = 0 & \text{, if } \quad \sigma < C
\end{cases}
$$

Denote the expected discounted value of a *risky* public agent by V_0. We have

$$V_0 = \max_{x_1^c, d_2, d_3, e, \nu} \left[\sigma x_1^c - \nu C + \sigma\gamma(1-\delta)V_1 + (1-\sigma)(1-\delta)V_0 \right],$$
$$\text{s.t.} \qquad d_2 + \gamma d_3 \geq x_1^c$$
$$\nu e \pi \geq d_2$$
$$\nu(1-e) \geq d_3$$

Here, this agent is risky, implying that he will settle a late IOU with a probability γ when he is debtor, and his creditor knows that. With a probability $\sigma\gamma(1-\delta)$, the risky agent is a debtor and survives to the following period as a safe agent. With a probability $(1-\sigma)(1-\delta)$, the risky agent is a creditor and survives to the following period as a risky agent. Under the above assumption, we have $\pi > f > \gamma$. Therefore, early settlement yields more consumption than late settlement. As a result,

$$\begin{cases} x_1^c = \pi, d_2 = \pi, d_3 = 0, e = 1, \nu = 1 & \text{, if} \quad \sigma\pi \geq C \\ x_1^c = 0, d_2 = 0, d_3 = 0, e \in \{0,1\}, \nu = 0 & \text{, if} \quad \sigma\pi < C \end{cases}$$

Anonymous agents

Since the trading history of an anonymous debtor is not observable by other agents, the creditor cannot tell whether the debtor is risky or safe. The creditor has to use the unconditional probability, f, to evaluate the probability of receiving a late settlement.

Denote the expected discounted value of a *safe* anonymous agent by $\tilde{V}_1$. We have

$$\tilde{V}_1 = \max_{x_1^c, d_2, d_3, e, \nu} \left[\sigma x_1^c - \nu C + (1-\delta)\tilde{V}_1 \right],$$
$$\text{s.t.} \qquad d_2 + f d_3 \geq x_1^c$$
$$\nu e \pi \geq d_2$$
$$\nu(1-e) \geq d_3$$

The solution is given by

$$\begin{cases} x_1^c = \pi, d_2 = \pi, d_3 = 0, e = 1, \nu = 1 & \text{, if } \quad \sigma\pi \geq C \\ x_1^c = 0, d_2 = 0, d_3 = 0, e \in \{0,1\}, \nu = 0 & \text{, if } \quad \sigma\pi < C \end{cases}$$

Denote the expected discounted value of a *risky* anonymous agent by $\tilde{V}_0$. We have

$$\begin{aligned} \tilde{V}_0 = \max_{x_1^c, d_2, d_3, e, \nu} & \left[\sigma x_1^c - \nu C + \sigma\gamma(1-\delta)\tilde{V}_1 + \sigma(1-\delta)\tilde{V}_0\right], \\ \text{s.t.} \qquad & d_2 + f d_3 \geq x_1^c \\ & \nu e \pi \geq d_2 \\ & \nu(1-e) \geq d_3 \end{aligned}$$

The solution is the same as that of a safe anonymous agent:

$$\begin{cases} x_1^c = \pi, d_2 = \pi, d_3 = 0, e = 1, \nu = 1 & \text{, if } \quad \sigma\pi \geq C \\ x_1^c = 0, d_2 = 0, d_3 = 0, e \in \{0,1\}, \nu = 0 & \text{, if } \quad \sigma\pi < C \end{cases}$$

Here, we focus on *symmetric stationary equilibria (SSE) with direct settlement* in which agents trade, and agents of the same types make the same participation and trading decision.

Therefore,we have the following proposition.

Proposition 1. *There exists a unique SSE with direct settlement when $\sigma\pi \geq C$.*

The good 1 consumption of the three types of debtors are

Types	Measure	x^c
Anonymous debtors	$N(1-h)$	π
Risky public debtors	$Nh(1-\Lambda)$	π
Safe public debtors	$Nh\Lambda$	1

Since the creditors have zero trade surplus, the welfare measured by average current period payoff is given by $W = \sigma(h[\Lambda + (1 - \Lambda)\pi] + (1 - h)\pi) - C$.

Information imperfection and infrequent trading between agents imply that safe anonymous debtors are unable to induce creditors to use deferred settlement. This will imply inefficient use of early settlement with the welfare loss equal to $\sigma(1-h)\Lambda(1-\pi)$. Therefore, whenever past trading history is imperfect ($h < 1$), direct settlement is inefficient because of the pre-matured liquidation.

3 Indirect Settlement

Owing to the lack of public history, allocation in an equilibrium with direct settlement is not efficient because the anonymous safe households are forced to liquidate their projects even though there is no risk of default. This allocation can be improved by introducing clearing agents to keep track of agents' past histories.

3.1 Clearing Agents

We now introduce some large households into the economy to study the role of clearing agents and the tiered settlement structure. Suppose, among the measure N of agents, there are $M < N$ households. Each household consists of a coalition of measure one of agents from a representative sample of the population. Therefore, a household can survive forever, even though at the end of each period a subset of its members (of measure $(1 - \Lambda)\sigma(1 - \gamma + \delta\gamma) + \Lambda\delta < 1$) dies and is replaced by a set of new members.

These large households can choose to become clearing agents who offer settlement services to other small agents in the home island. The settlement technology allows each household to interact with a positive mass, n, of small agents in the home island at a utility cost $D(n)$ *per member*, with $\eta(0) = \eta_0 > 0$, $\eta' \geq 0$, $\eta'' \geq 0$. Each client of a clearing agent is given an account, which only the account owner can have access to operate. By having repeated dealings with the account owner and by keeping track of each account's trading history, a clearing agent is able to identify whether the account owner is safe or risky.

A clearing agent provides two basic functions: (1) issues IOUs to clients who are debtors and promise to redeem these IOUs in the settlement island, and (2) settle IOUs issued by other clearing agents for the clients who are creditors. Both these functions are needed to provide the proper incentives for the clearing agents. If the clearing agents were instead credit agencies and only provided some type of identifiable IOU but did not promise to settle these IOUs then the clearing agent would have no incentive to truthfully identify safe creditors.

3.2 The Sequence of Events

The following table and Figure 3 summaries the time-line of the sequence of events when clearing agents are used. Here, we use i and j to denote a typical debtor and creditor respectively, and CA I and CA J to denote their clearing agents respectively.

Time	**Island**	**Events**
$s = 1$	home	1. Households choose to become clearing agents or not
	home	Agents choose direct or indirect settlement
	home	2. Agents receive preference shock
	home	Debtor i trade $d_{ts}(i)$ for $D_{ts}(I)$ with clearing agent I
	trading	3. Debtor i buys $x^c_{t1}(i)$ with $D_{t2}(I)$ or $D_{t3}(I)$ from creditor j
$s = 2$	home	4. Debtor i terminates projects, settles $d_{t2}(i)$ with clearing agents I
	home	Creditor j trades $D_{t2}(I)$ for $D_{t2}(J)$ with clearing agent J
	settlement	5. Clearing agent I settles $D_{t2}(I)$ with clearing agent J
	home	6. Debtor i dies or not
	home	Clearing agent J settles $D_{t2}(J)$ with creditor j
$s = 3$	home	7. Project matures. Debtor i settles $d_{t3}(i)$ with clearing agents I
	home	Creditor j trades $D_{t3}(I)$ for $D_{t3}(J)$ with clearing agent J
	settlement	8. Clearing agent I settles $D_{t3}(I)$ with clearing agent J
	home	Creditor j trades $D_{t2}(I)$ for $D_{t2}(J)$ with clearing agent J
	home	9. Clearing agent J settles $D_{t2}(J)$ with creditor j
	home	Exogenous exit or not

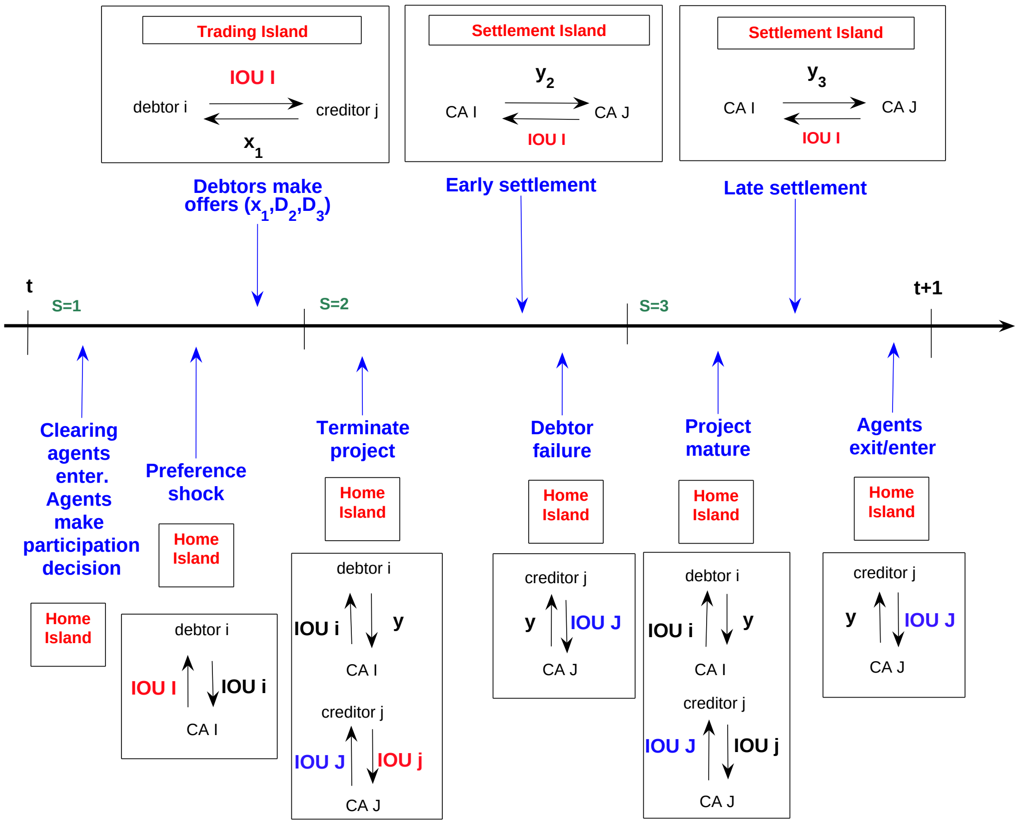

Figure 3: Time-line

1. Participation Decision in $s = 1$

In the home island, each small agent chooses between being a

(i) Small Direct Clearer (SDC): to use direct settlement by paying a fixed cost C,

(ii) Indirect Clearer (IC): to use indirect settlement by attaching to a clearing agent,

(iii) Non-Trader (NT): not to trade.

Each household chooses between being a

(i) Direct Clearer (LDC): to use direct settlement by paying a fixed cost C,

(ii) Clearing Agent (CA): to serve n indirect clearers by paying $\eta(n)$ and the fixed cost C.

(iii) Non-Trader (NT): not to trade.

An agent who chooses to become a small indirect clearer will choose a clearing agent and make a take-it-or-leave-it offer ϕ to that clearing agent (in terms of good x to be paid in sub-period 2). Here, ϕ captures the intermediation fee promised by the indirect clearer to the clearing agent.

2. Preference shock in $s = 1$

In the home island, each agent receives a preference shock that determines whether he is a creditor or a debtor. Debtor i using indirect settlement makes a take-it-or-leave-it offer $(d_{ts}(i), D_{ts}(I))$ to clearing agent I, with $s = 2$ or 3. Here, debtor i asks clearing agent I for $D_{ts}(I)$ units of I' IOUs which promises to pay the holder good y in sub-period s. Similarly, $d_{ts}(i)$ is the quantity of debtor i's IOUs given to clearing agent I, which promises to pay back good y at the beginning of sub-period s. Clearing agent I decides to accept or not.

3. Debtors making offers in $s = 1$

Agents are subject to pairwise random matching in the trading island. Debtors using indirect settlement will trade by offering clearing agents' IOUs. Debtors using direct settlement will trade by issuing their own IOUs. Creditors decide to accept or not.

4. Termination of projects in $s = 2$

At the beginning of sub-period 2, debtors can choose to terminate their projects and get a liquidation value π in the home island. Creditor j using indirect settlement will make a take-it-or-leave-it offer $(D_{t2}(I), D_{t2}(J))$ to clearing agent J. Here, creditor j offers $D_{t2}(I)$ units of clearing agents I's IOUs to clearing agent J. In return, creditor j asks clearing agent J for $D_{t2}(J)$ units of J' own IOUs which promises to pay back good y at the end of sub-period 2. Clearing agent J decides to accept or not.

5. Early settlement in $s = 2$

Agents using direct settlement and clearing agents attend the settlement island to settle early IOUs held by other agents and clearing agents if $d_{t2}(i) > 0$ or $D_{t2}(I) > 0$.

6. Debtor failure in $s = 2$

At the end of sub-period 2, a *debtor* may die exogenously, and any unliquidated project will fail. At the same time, creditor j holding $D_{t2}(J)$ will settle with his clearing agent J.

7. Projects mature in $s = 3$

At the beginning of sub-period 3, If a debtor with an unliquidated project survives in sub-period 3, he will be endowed with one unit of good y in the home island. Creditor j using indirect settlement will make a take-it-or-leave-it offer $(D_{t3}(I), D_{t3}(J))$ to clearing agent J. Clearing agent J decides to accept or not.

8. Late settlement in $s = 3$

Agents using direct settlement and clearing agents attend the settlement island to settle late IOUs held by other agents and clearing agents if $d_{t3}(i) > 0$ or $D_{t3}(I) > 0$.

9. Exogenous death in $s = 3$

At the end of sub-period 3, creditor j holding $D_{t3}(J)$ will settle with his clearing agent J. Also, a random fraction δ of agents exits the economy and is replaced by a set of new agents. Also, those debtors who die and default are replaced by new agents.

<u>Decision Nodes</u>

The trading and settlement decision of direct clearers (SDC) are the same as those in the last section, so the following discussion will only focus on the decision of indirect clearers

(IC). Under the assumption of perfect enforcement, creditors, debtors and clearing agents will always settle their IOUs at point (4), (6), (7), (8), (9) in the above sequence of events. Basically, there are only five decision nodes we need to examine:

(i) **Home island,** $s = 1$ [**(1) in sequence of events**]: Households choose between LDC, CA, and NT. Small agents choose between SDC, IC, and NT.

(ii) **Home island,** $s = 1$ [**(2) in sequence of events**]: Debtor i makes a take-it-or-leave-it offer $(d_{ts}(i), D_{ts}(I))$, to clearing agent I. Given $(d_{ts}(i), D_{ts}(I))$, clearing agent I chooses whether to accept or not agent i's offer.

(iii) **Trading island,** $s = 1$ [**(3) in sequence of events**]: Holding $(D_{t2}(I), D_{t3}(I))$, debtor i makes a take-it-or-leave-it offer $(\tilde{x}_{t1}(j), \tilde{D}_{t2}(I), \tilde{D}_{t3}(I))$ to creditor j, subject to $\tilde{D}_{ts}(I) \leq D_{ts}(I)$. Creditor j chooses whether to accept an offer or not.

(iv) **Home island,** $s = 2$ [**(4) in sequence of events**]: Holding $D_{t2}(I)$, creditor j makes a take-it-or-leave-it offer $(\tilde{D}_{t2}(I), \tilde{D}_{t2}(J))$ to clearing agent J, subject to $\tilde{D}_{t2}(I) \leq D_{t2}(I)$. Clearing agent J chooses whether to accept an offer or not.

(v) **Home island,** $s = 3$ [**(7) in sequence of events**]: Holding $D_{t3}(I)$, creditor j makes a take-it-or-leave-it offer $(\tilde{D}_{t3}(I), \tilde{D}_{t3}(J))$ to clearing agent J, subject to $\tilde{D}_{t3}(I) \leq D_{t3}(I)$. Clearing agent J chooses whether to accept an offer or not.

Recall that an agent is *risky* if it has not been a debtor before. An agent is *safe* if it has been a debtor before and did not fail. Besides, we will denote an anonymous agent as having a a *credit history* if there exists a clearing agent who knows that this agent is safe (i.e. the agent was served by the clearing agent in the past when it was a debtor).

The Appendix shows the following results:

- At decision node (ii), the offer from debtor i to clearing agent I is ($d_2(i)$, $d_3(i)$, $D_2(I)$, $D_3(I)) = (\pi, 0, \pi, 0)$, if i is a public risky debtor or if i is an anonymous debtor without credit history. And $(d_2(i), d_3(i), D_2(I), D_3(I))=$ $(0, 1, 0, 1)$ if i is a safe debtor, or if i

is an anonymous debtor with credit history. All these offers are accepted by clearing agents.

- At decision node (iii), the offer from debtor i to creditor j is $(x_1(j), D_2(I), D_3(I)) = (\pi, \pi, 0)$, if i is a public risky debtor or if i is an anonymous debtor without credit history. And $(x_1(j), D_2(I), D_3(I)) = (1, 0, 1)$, if i is a safe debtor, or if i is an anonymous debtor with credit history. All these offers are accepted by creditors.

- At decision node (iv) and (v), the offer from creditor j to clearing agent J is $D_s(I) = D_s(J)$. All these offers are accepted by clearing agents.

Now, we discuss the decisions at decision node (i). Denote the equilibrium number of clearing agents by $A \leq M$. There are two equilibrium outcomes, depending on whether the public agents choose to use clearing agents or not. If all the public agents choose to become indirect clearers by attaching to clearing agents, then the number of clients per clearing agents is given by

$$n = \frac{N - M}{A}. \tag{1}$$

Since small agents make take-it-or-leave-it offers to clearing agents, the equilibrium intermediation fee, ϕ, should be equal to the marginal cost of serving the indirect clearers:

$$\phi = \eta'(\frac{N - M}{A}). \tag{2}$$

The free entry condition of clearing agents is such that clearing agents earns non-negative profit and any one additional clearing agent earns non-positive profit[12]:

$$\frac{N - M}{A}\eta'(\frac{N - M}{A}) - \eta(\frac{N - M}{A}) \geq 0 \geq \frac{N - M}{A+1}\eta'(\frac{N - M}{A+1}) - \eta(\frac{N - M}{A+1}). \tag{3}$$

[12]This is true only when M is sufficiently large such that $A \leq M$ is not binding. From now on, we assume that this is true.

If all the public agents choose to become direct clearers and not to use clearing agents, then the number of clients per clearing agents is given by

$$n = \frac{(N-M)(1-h)}{A}. \tag{4}$$

Also, the equilibrium fee is given by

$$\phi = \eta'(\frac{(N-M)(1-h)}{A}), \tag{5}$$

and the free entry condition of clearing agents implies

$$\begin{aligned} &\frac{(N-M)(1-h)}{A}\eta'(\frac{(N-M)(1-h)}{A}) - \eta(\frac{(N-M)(1-h)}{A}) \geq 0 \\ \geq\ &\frac{(N-M)(1-h)}{A+1}\eta'(\frac{(N-M)(1-h)}{A+1}) - \eta(\frac{(N-M)(1-h)}{A+1}). \end{aligned} \tag{6}$$

Now, we consider the decision between direct settlement (SDC), indirect settlement (IC) and no trade (NT) of small agents at node (i). We first consider the decision of public agents and then that of anonymous agents.[13]

Small public agents

Since public agents possess public history of trading statuses, they will get the same terms of trade whether they use clearing agents or direct settlement. So their only concern is whether using clearing agents can reduce the settlement cost. So they will prefer IC rather than SDC if $C \geq \phi$. Also, a public safe agent will choose not to trade when the gain from trade is lower than the cost of using direct or indirect settlement (i.e., $\sigma \leq \min\{\phi, C\}$). Similarly, a public risky agent will choose not to trade when $\sigma\pi \leq \min\{\phi, C\}$). Therefore, a public safe agent's optimal choice at node (i) is

[13]Note that large households do not have incentives to use clearing agents because the total cost of direct settlement is C and thus the *average* cost of direct settlement per member is zero. Therefore, large households will always prefer being a LDC to NT.

$$\begin{cases} \text{IC} & \text{, if } \min\{C, \sigma\} \geq \phi, \\ \text{SDC} & \text{, if } \min\{\phi, \sigma\} \geq C, \\ \text{NT} & \text{, if } \min\{\phi, C\} \geq \sigma. \end{cases} \tag{7}$$

A public risky agent's optimal choice at node (i) is

$$\begin{cases} \text{IC} & \text{, if } \min\{C, \sigma\pi\} \geq \phi, \\ \text{SDC} & \text{, if } \min\{\phi, \sigma\pi\} \geq C, \\ \text{NT} & \text{, if } \min\{\phi, C\} \geq \sigma\pi. \end{cases} \tag{8}$$

So, all public agents trade if

$$\min\{\phi, C\} \leq \sigma\pi. \tag{9}$$

Small anonymous agents

An anonymous agent's decision depends on whether they are safe or risky, and whether he has a credit history (i.e., some clearing agents know that the agent is safe). As a result, at the beginning of a period, there are three types of anonymous agents:
(i) risky agents (denoted by subscript "00"): who has not been a debtor before;
(ii) safe agents without credit history (denoted by subscript "10"): who are safe but no clearing agents know
(iii) safe agents with credit history (denoted by subscript "11"): who are safe and some clearing agents know that.

We denote the beginning-of-the-period expected payoff of these three types by $\tilde{V}_{00}, \tilde{V}_{10}, \tilde{V}_{11}$ respectively:

$$
\begin{aligned}
\tilde{V}_{00} &= \max\{(1-\sigma)(1-\delta)\tilde{V}_{00} + \sigma[\pi + (1-\delta)\gamma\tilde{V}_{11}] - \phi, \\
&\quad (1-\sigma)(1-\delta)\tilde{V}_{00} + \sigma[\pi + (1-\delta)\gamma\tilde{V}_{10}] - C, \\
&\quad (1-\delta)(1-\sigma(1-\gamma))\tilde{V}_{00}\} \\
\tilde{V}_{10} &= \max\{(1-\sigma)(1-\delta)\tilde{V}_{10} + \sigma[\pi + (1-\delta)\tilde{V}_{11}] - \phi, \\
&\quad (1-\sigma)(1-\delta)\tilde{V}_{10} + \sigma[\pi + (1-\delta)\tilde{V}_{10}] - C, \\
&\quad (1-\delta)\tilde{V}_{10}\} \qquad (10) \\
\tilde{V}_{11} &= \max\{(1-\sigma)(1-\delta)\tilde{V}_{11} + \sigma[1 + (1-\delta)\tilde{V}_{11}] - \phi, \\
&\quad (1-\sigma)(1-\delta)\tilde{V}_{11} + \sigma[\pi + (1-\delta)\tilde{V}_{11}] - C, \\
&\quad (1-\delta)\tilde{V}_{11}\}
\end{aligned}
$$

Here, each agent needs to decide whether to pay ϕ to become a IC, to pay C to become a SDC, or choose not to trade (NT). For example, the first equation represents the choice of a risky debtor between direct settlement, indirect settlement, and no trade. The tradeoff is that a direct clearer pays a cost C while an indirect clearer pays a cost ϕ and may get a higher continuation value $\tilde{V}_{11}$ when he is a creditor and survives. Note that, implicitly, we assume that an agent is always served by those clearing agents possessing his credit history.

In the Appendix, it is shown that small anonymous agents always choose indirect settlement if

$$\Delta - \phi \geq \max\{C, -\sigma\pi\}, \qquad (11)$$

where $\Delta = \sigma(1-\delta)\gamma\frac{\sigma(1-\pi)}{1-\sigma(1-\delta)} \geq 0$. In particular, small anonymous agents prefer being a IC to a SDC if

$$\Delta + (C - \phi) \geq 0.$$

Here, the term in the inequality, Δ, captures the gain from building up the credit history through the use of clearing agents. The second term, $C - \phi$, captures the potential cost-saving by using clearing agents. The informational role and cost-saving role are the two basic functions provided by clearing agents.

3.3 Equilibrium

We will focus on an equilibria in which (i) all agents trade, (ii) all small anonymous agents choose to use indirect settlement, (iii) there are $A \in \{1, 2, ..., M\}$ clearing agents.

The previous derivation suggests that an equilibrium with indirect settlement can be defined as follows:

1. A symmetric stationary equilibrium with indirect settlement (without small direct clearers) is given by (A^*, n^*, ϕ^*) satisfying , (1), (2), (3), (9), (11), $C \geq \phi$, and $1 \leq A^* \leq M$.

2. A symmetric stationary equilibrium with indirect settlement (with small direct clearers) is given by (A^*, n^*, ϕ^*) satisfying , (4), (5), (6), (9), (11), $\phi \geq C$, and $1 \leq A^* \leq M$.

The first definition refers to case in which all small agents choose indirect settlement (IC). Conditions (1), (2), (3) define (A^*, n^*, ϕ^*). Conditions (9), (11), $C \geq \phi$ imply that anonymous agents choose IC. The second definition has the reversed condition $C \leq \phi$ to ensure that public agents prefer SDC to IC. (see Figure (4))

Under these two equilibria, the payoffs of different agents are given by

Type	Measure	Payoff
Clearing agents (CA)	A	$\sigma[1-(1-\Lambda)(1-\pi)]+n\phi-\eta(n)$
Large direct clearers (LDC)	M-A	$\sigma[1-(1-\Lambda)(1-\pi)]$
Public creditors	$(1-\sigma)(N-M)h$	$-\min\{\phi, C\}$
Anonymous creditors	$(1-\sigma)(N-M)(1-h)$	$-\phi$
Public risky debtors	$\sigma(N-M)(1-h)(1-\Lambda)$	$\pi-\min\{\phi, C\}$
Public safe debtors	$\sigma(N-M)(1-h)\Lambda$	$1-\min\{\phi, C\}$
Anonymous risky debtors	$\sigma(N-M)h(1-\Lambda)$	$\pi-\phi$
Anonymous safe debtors	$\sigma(N-M)h\Lambda$	$1-\phi$

Apparently, the use of indirect settlement makes anonymous agents better off. Public agents will be better off whenever $C > \phi$. Again, there are two potential sources of welfare gain: informational gain (Δ) and cost-saving ($C-\phi$). The average payoff is given by:

$$\bar{W} = \begin{cases} \sigma[1-(1-\Lambda)(1-\pi)] - \frac{A\eta(n)}{N} & \text{, if } C \geq \phi \\ \sigma[1-(1-\Lambda)(1-\pi)] - \frac{A\eta(n)+(N-M)hC}{N} & \text{, if } C < \phi \end{cases} \tag{12}$$

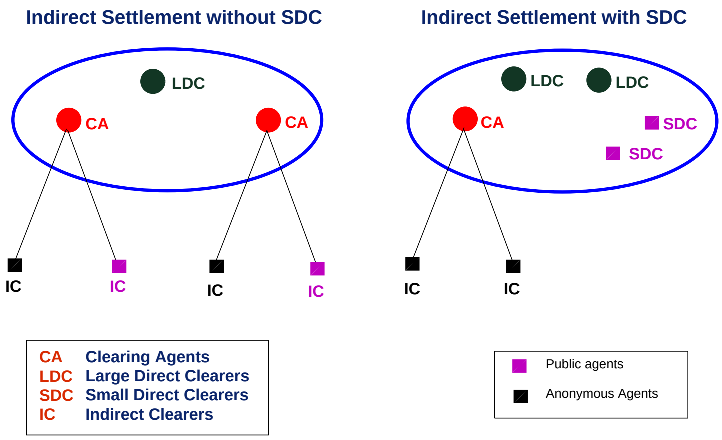

Figure 4: Two Equilibrium Outcomes

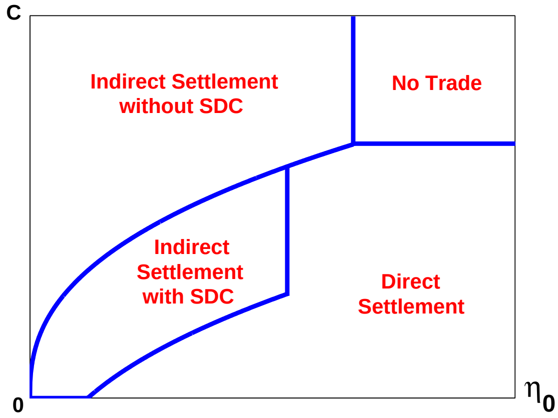

Figure 5: Distribution of SSE

Defining the fixed cost of providing clearing agent services by $\eta_0 = \eta(0)$, one can derive the distribution of equilibria over the parameter space (η_0, C). As shown in Figure (5), direct settlement is used when C is relatively low. As C increase, anonymous agents choose to use indirect settlement. When C is high, both anonymous and public agents use indirect settlement.[14]

3.4 Degree of Tiering

In this subsection, we derive the model implications regarding some empirical measures of the degree of tiering. We may measure the degree of tiering by the share of settlement going through clearing agents.[15]

The number of clearing agents is given by

$$A = \begin{cases} \frac{N-M}{n} & \text{, if } C > \phi \\ \frac{(N-M)(1-h)}{n} & \text{, if } C < \phi \end{cases}$$

The total volume of settlement is

[14]See the Appendix for the details of the distribution of equilibria.

[15]For simplicity, we will treat the number of clearing agents, A, as a real number, instead of an integer.

$$\text{TV} = \begin{cases} = N - 1 - (N-M)\frac{n+1}{N} & \text{, if } C > \phi \\ = N - 1 - (N-M)\frac{n+1}{N} + h(N-M)\frac{2+n}{N} & \text{, if } C < \phi \end{cases}$$

Clearing agents' volume of settlement is given by

$$\text{CAV} = \begin{cases} (1 - \frac{1+n}{N})(A + N - M) & \text{, if } C > \phi \\ (1 - \frac{1+n}{N})(A + (1-h)(N-M)) & \text{, if } C < \phi \end{cases}$$

Clearing agents' share of settlement is given by

$$\text{Share} = \text{CAV}/\text{TV} = \begin{cases} 1 - (M-A)\frac{N-1}{N}/\text{TV} & \text{, if } C > \phi \\ 1 - [h(N-M) + (M-A)\frac{N-1}{N}]/\text{TV} & \text{, if } C < \phi \end{cases}$$

The effects of the fixed cost associated with indirect settlement (η_0) and the availability of public information (h) on the equilibrium degree of tiering are summarized in the following table.

	Parameter	**n**	**A**	**CAV**	**TV**	**Share**
$C > \phi$	$\eta_0 \uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$C > \phi$	$h \uparrow$	0	0	0	0	0
$C < \phi$	$\eta_0 \uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$C < \phi$	$h \uparrow$	0	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$

Note that the degree of tiering captured is decreasing in the fixed cost of operating a settlement network. A rise in fixed cost ($\eta_0 \uparrow$) will require each clearing agent to operate in a larger scale ($n \uparrow$), reducing the equilibrium number of clearing agents ($A \downarrow$). The total volume of settlement goes up because less settlement is internalized by the clearing agents (TV$\downarrow$), but the total volume settled by clearing agents drops even more (CAV$\downarrow$). As a result, the clearing agents' share of settlement goes down (Share$\downarrow$).

Also, when $C < \phi$, the degree of tiering is decreasing in the fraction of small households with public history. A rise in the availability of public credit history ($h \uparrow$) does not affect the size of each clearing agent, but reduces the number of clearing agents ($A \downarrow$). Since less settlement flows are internalized by the clearing agents, total volume goes up (TV↑) and clearing agents' settlement volume goes down (CAV↓). As a result, the clearing agents' share of settlement goes down (Share↓). When $C > \phi$, all small households use indirect settlement, so changes in h do not affect the two measures.

To consider a simple numerical example, assume $\eta(n) = \eta_0 + n^\rho$, with $\rho > 1$, we have $n = (\frac{\eta_0}{\rho-1})^{\frac{1}{\rho}}$ and $\phi = \rho(\frac{\eta_0}{\rho-1})^{\frac{\rho-1}{\rho}}$. Figure 6 plots the two measures in the case when $C < \phi$.

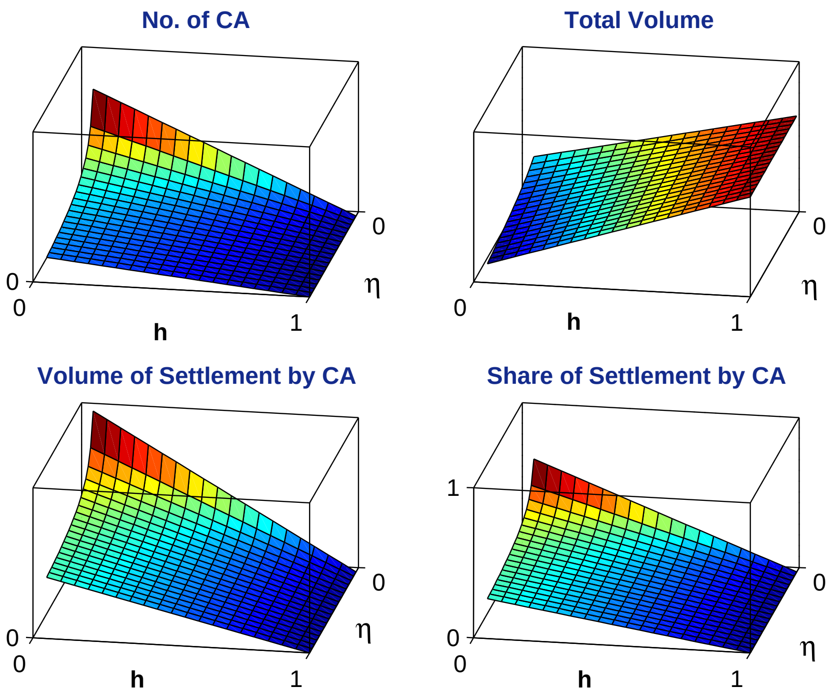

Figure 6: Degree of Tiering ($C > \phi$)

4 Failure of Clearing Agents

In this section, we study the consequence of the failure of clearing agents.[16] The economy is initially in a SSE with $A \geq 1$ clearing agents. Suppose at the beginning of each period, a clearing agent may fail exogenously with a probability θ. Suppose clearing agent failure is a random i.i.d. event and after a failure, clearing agents shut down and exit the economy. Since clearing agent failures are low probability events, we assume that θ is close to zero. Also, we assume that a failed clearing agent is replaced by a new household. The memory regarding its clients' credit histories associated with the clearing agent will vanish.[17] Apparently, in this case agents do not have incentive to use more than one clearing agent, and the equilibrium outcome is the same as in the previous section.[18]

For small public agents, clearing agent failure will not affect the distribution of the safe and risky types.

Distributional change for anonymous agents

Now, we look at the distributional change among the small anonymous agents after a *one-time* failure at the beginning of period 1. We use $\{\Lambda_{00}(t), \Lambda_{11}(t), \Lambda_{10}(t)\}_{t=0}^{\infty}$ to denote respectively the sequences of fractions of risky agents, safe agents with credit history, and safe agents without credit history. Before the period 1 failure, the distribution for $t < 1$ is given by

$$
\begin{aligned}
\Lambda_{00}(t) &= 1 - \Lambda, \\
\Lambda_{11}(t) &= \Lambda, \\
\Lambda_{10}(t) &= 0.
\end{aligned}
$$

[16]For simplicity, we focus on the case with C sufficiently large so that all small agents always use clearing agents.

[17]This is without loss of generality. While some information may be salvageable from a failed credit agent not all of it will be either verifiable or tradable; either because it is relationship specific or not allowed to be traded due to privacy laws.

[18]In particular, if agents can only use one clearing agent in each period, then using a second clearing agent costs an extra liquidation cost $1 - \pi$ without yielding any diversification gains when $\theta \approx 0$.

Denote the fraction of households affected by the failure as $\epsilon = \tilde{A}/A$. In period 1, as illustrated in (7), the distribution is given by

$$\begin{aligned}\Lambda_{00}(1) &= 1-\Lambda,\\ \Lambda_{11}(1) &= (1-\Lambda)(1-\epsilon)\sigma\gamma(1-\delta)+\Lambda(1-\epsilon)(1-\delta),\\ \Lambda_{10}(1) &= (1-\Lambda)\epsilon\sigma\gamma(1-\delta)+\Lambda\epsilon(1-\delta).\end{aligned}$$

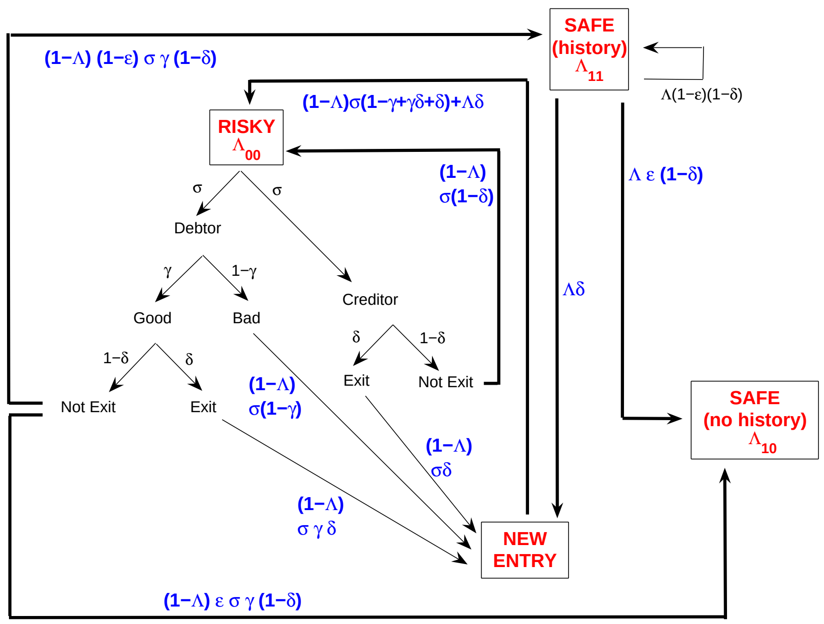

Figure 7: Distributional change in period $t = 1$

As illustrated in (8), the distribution in period $t > 1$ is given by

$$\begin{aligned}\Lambda_{00}(t) &= 1-\Lambda,\\ \Lambda_{11}(t) &= (1-\Lambda)\sigma\gamma(1-\delta)+\Lambda_{11}(t-1)(1-\delta)+\Lambda_{10}(t-1)(1-\delta)\sigma,\\ \Lambda_{10}(t) &= \Lambda_{10}(t-1)(1-\delta)\sigma.\end{aligned}$$

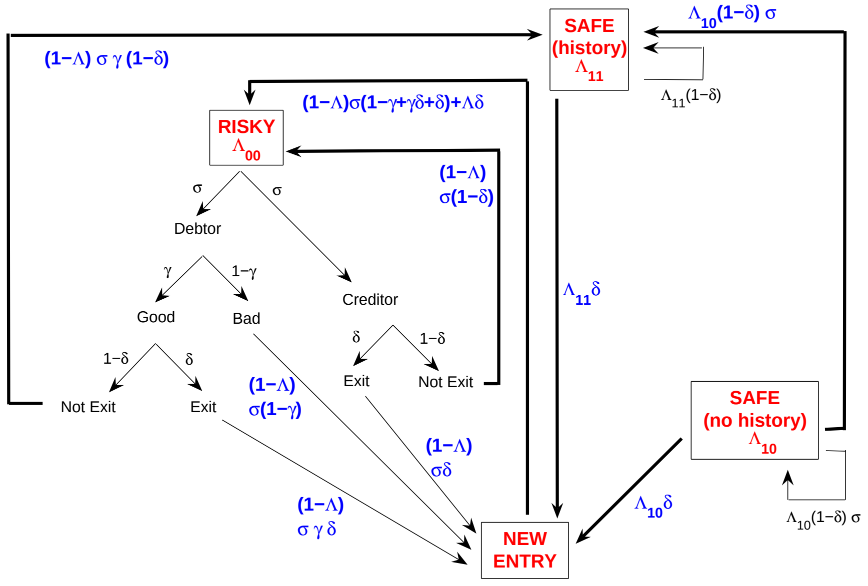

Figure 8: Distributional change in period $t > 1$

4.1 Welfare consequence of failure

The average payoff of all agents in period $t \geq 0$ is given by

$$\bar{W}(0) = \sigma(1-(1-\Lambda)(1-\pi)) - \frac{A}{N}\eta(\frac{N-M}{A}))$$

$$\bar{W}(t) = \sigma(1-(1-\Lambda)(1-\pi)) - \underbrace{\frac{\tilde{A}}{N}\Lambda_{10}(t)(1-h)}_{\text{loss of private info.}} - \underbrace{\frac{A-\tilde{A}+\Delta A(t)}{N}\eta(\frac{N-M}{A-\tilde{A}+\Delta A(t)})}_{\text{operational inefficiency}}, \text{ for } t \geq 1$$

where $\Delta A(t)$ is the total number of large households turning into new clearing agents (exogenously) from period 1 to period t. As illustrated in this equation, there are two potential sources of welfare loss: loss of information concerning anonymous indirect clearers' credit history, and operational inefficiency. Figures (9), (10) and (11) illustrate the effects of clearing agent failure on the number of clearing agents, agents' credit history, volume of trade, as well as the average payoffs of direct clearers, indirect clearers, and clearing agents.

Case (1): No Informational Role ($h = 1$) and Immediate Entry of Clearing Agents

Suppose after the failure of clearing agents (marked by the cross in the figure), they are replaced by the new clearing agents immediately. Since clearing agents have no informational roles when $h = 1$, there is no loss of credit history in the economy. Also, the cost-saving role of clearing agents is not affected when they are replaced immediately. Therefore, a failure of clearing agents has no short-term or long-term impact.

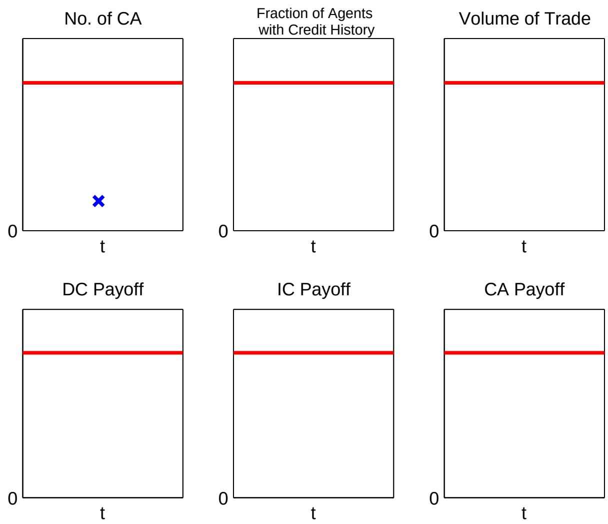

Figure 9: $h = 1$ and Immediate Entry of New Clearing Agents

Case (2): Informational Role ($h < 1$) and Immediate Entry of Clearing Agents

The failure of clearing agents implies loss of credit history of anonymous agents when $h < 1$, lowering the average trades and the average welfare of indirect clearers. After the failure in period 0, the anonymous indirect clearers whose clearing agents failed need to switch to some other clearing agents. Since the new clearing agents do not know their credit history, they need to rebuild their reputation again. Because households may not be debtors every period, it takes time for all of them to rebuild their credit history.

Case (3): Informational Role ($h < 1$) and Gradual Entry of Clearing Agents

The failure of clearing agents implies loss of credit history, lowering the average trades and the average welfare of indirect clearers. The gradual entry of new clearing agents implies

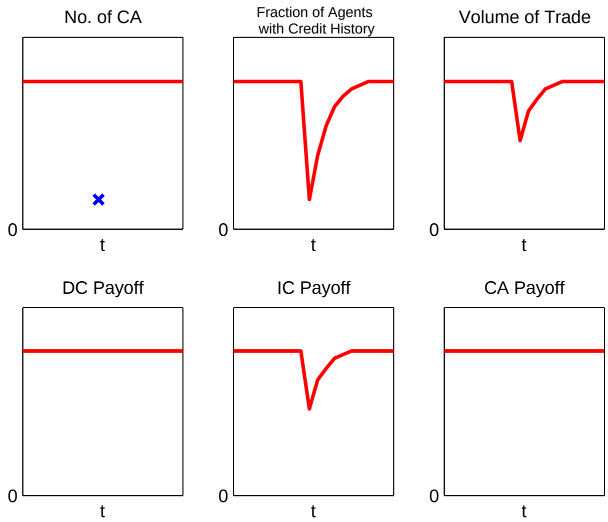

Figure 10: $h < 1$ and Immediate Entry of New Clearing Agents

inefficient operation of existing clearing services. Given the properties of $\eta(n)$, this will lower the payoffs of all indirect clearers (due to higher fee) and raise the (monopoly) profit of the clearing agents.

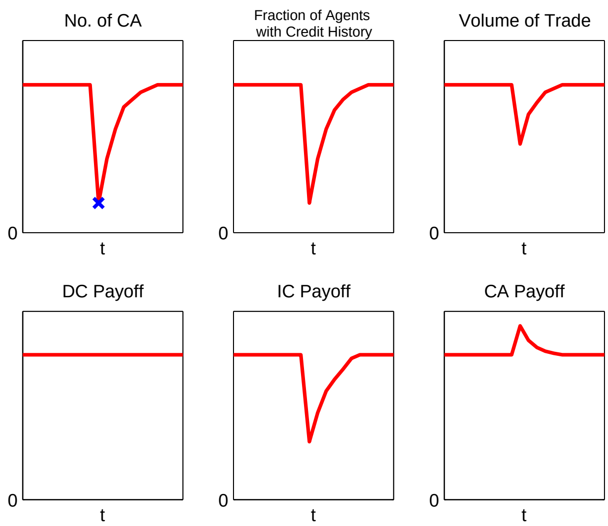

Figure 11: $h < 1$ and Gradual Entry of New Clearing Agents

Note that the special informational and cost-saving roles of clearing agents imply that the effects of a failure of clearing agents is different from that of a large direct clearer. In particular, a failure of clearing agents leads to persistent loss of credit history of indirect clearers and operational inefficiency which will not happen in the case of large direct clearer failure.

5 Conclusion

Our paper shows that, in the presence of information imperfection and non-convex cost structure, a tiered structure can improve efficiency by supporting inter-bank monitoring and cost-saving. A policy implication is that, restricting the degree of tiering in settlement systems (such as LVTS and ACSS in Canada) may distort the efficient monitoring structure in the payment system.[19] However, we also show that settlement failure may generate negative spill over on other participants. Therefore, the equilibrium concentration and degree of tiering may not optimally diversify the risk of clearing agent failure. The optimal payment system policy of the central bank is a left for future research.

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Appendices

(1) **To solve for optimal decision in SSE**

Given the number of clients served by each clearing agent is n, we can solve for the optimal decision backward:

- Decision node (v)

  Clearing agent J: Clearing agent J accepts an offer, $(D_3(I), D_3(J))$, from creditor j iff $D_3(I) \geq D_3(J)$

  Creditor j: Denote the fee charged by clearing agents as ϕ. Given $(D_3(I), x_1(j))$, creditor j chooses $\tilde{D}_3(J)$ to maximize $D_3(J) - x_1(j) - \phi$ subject to $D_3(I) \geq D_3(J)$. So the optimal offer is $D_3(J) = D_3(I)$.

- Decision node (iv)

  Clearing agent J: Clearing agent J accepts an offer, $(D_2(I), D_2(J))$, from creditor j iff $D_2(I) \geq D_2(J)$

  Creditor j: Given $(D_2(I), x_1(j))$, creditor j chooses $D_2(J)$ to maximize $D_2(J) - x_1(j) - \phi$ subject to $D_2(I) \geq D_2(J)$. So the optimal offer is $D_2(J) = D_2(I)$.

- Decision node (iii)

  Creditor j accepts an offer $(D_s(I), x_1(j))$ from debtor i iff $D_s(J) = D_s(I) \geq x_1(j)$.

Debtor i: Given $D_s(I)$ in hand, debtor i chooses $(\tilde{D}_s(I), x_1^c(i))$ to maximize $x_1^c(i)$ subject to $D_s(I) \geq \tilde{D}_s(I)$ and $\tilde{D}_s(I) \geq x_1^c(i)$. So the optimal offer is $x_1^c(i) = D_s(I)$ and $\tilde{D}_2(I) = D_2(I)$.

- Decision node (ii)

  Clearing agent I: Clearing agent I will accept an offer, $(d_2(i), D_2(I))$ from debtor i iff $d_2(i) \geq D_2(I)$. He will accept an offer, $(d_3(i), D_3(I))$ from debtor i iff $p^I(i)d_3(i) \geq D_3(I)$, where $p^I(i)$ is equal to 1 or γ depending on whether i is a safe or a risky debtor (for an anonymous agent i, depending on whether he has been a debtor with I before).

  Debtor i: Debtor i chooses ($d_2(i)$, $d_3(i)$, $D_2(I)$, $D_3(I)$, $e(i)$) to

$$\max x_1^c(i) - \phi = D_2(I) + D_3(I) - \phi$$

$$\begin{aligned} \text{subject to} \\ d_2(i) \geq D_2(I) \\ p^I(i)d_3(i) \geq D_3(I) \\ e(i)\pi \geq d_2(i) \\ (1 - e(i)) \geq d_3(i) \end{aligned}$$

The optimal offer for debtor i is

$$\begin{cases} d_2(i) = \pi, d_3(i) = 0, D_2(i) = \pi, D_3(i) = 0, e(i) = 1 & \text{, for risky debtors} \\ d_2(i) = 0, d_3(i) = 1, D_2(i) = 0, D_3(i) = 1, e(i) = 0 & \text{, for safe debtors} \end{cases}$$

(2) To derive the conditions for SSE with indirect settlement

Risky debtors choose IC rather than SDC iff $\sigma(1-\delta)\gamma(\tilde{V}_{11} - \tilde{V}_{10}) \geq \phi - C$. Safe debtors without credit history choose IC rather than SDC iff $\sigma(1-\delta)(\tilde{V}_{11} - \tilde{V}_{10}) \geq \phi - C$. Safe debtors with credit history choose IC rather than SDC iff $\sigma(1 - \pi) \geq \phi - C$. Also, in a SSE with indirect settlement,

$$\begin{aligned} \tilde{V}_{11} - \tilde{V}_{10} &= (1-\sigma)(1-\delta)\tilde{V}_{11} + \sigma[1 + (1-\delta)\tilde{V}_{11}] - (1-\sigma)(1-\delta)\tilde{V}_{10} - \sigma[\pi + (1-\delta)\tilde{V}_{11}] \\ &= (1-\sigma)(1-\delta)[\tilde{V}_{11} - \tilde{V}_{10}] + \sigma(1-\pi) \\ &= \frac{\sigma(1-\pi)}{1-(1-\sigma)(1-\delta)} < 1 - \pi. \end{aligned}$$

Note that the incentive to use indirect settlement is the lowest for the risky debtors. Therefore, to support an equilibrium with indirect settlement, the parameters need to satisfy:

$$\sigma(1-\delta)\gamma\frac{\sigma(1-\pi)}{1-\sigma(1-\delta)}+(C-\phi)=\Delta+(C-\phi) \quad > \quad 0.$$

Similarly, agents of all types choose IC rather than NT iff

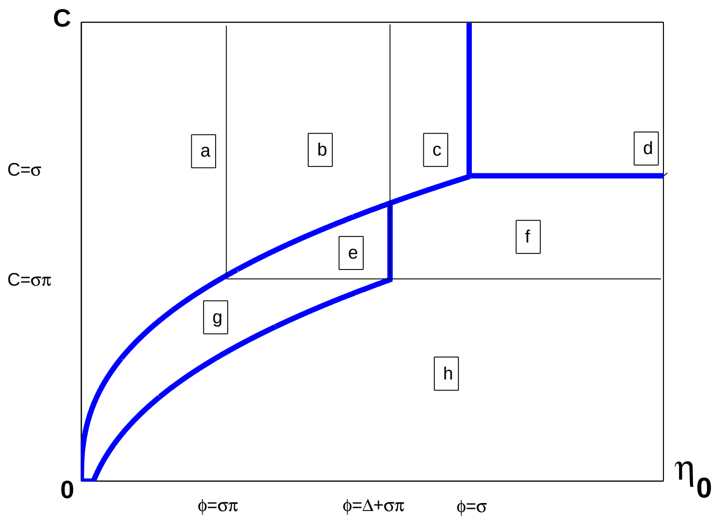
$$\sigma\pi+\Delta-\phi \quad > \quad 0.$$

And agents of all types choose SDC rather than NT iff

$$\sigma\pi-C \quad > \quad 0.$$

(3) To derive the distribution of equilibria in the parameter space

Anonymous agents choose IC if $C \geq \phi-\Delta$, $\phi \leq \Delta+\sigma\pi$. Choose SDC if $C \leq \phi-\Delta$, $C \leq \sigma\pi$. Choose NT if $\phi \geq \Delta+\sigma\pi$,$C \geq \sigma\pi$. Public risky agents choose IC if $\phi \leq \sigma\pi$, $C \geq \phi$. Choose SDC if $C \leq \sigma\pi$, $\phi \geq C$. Choose NT if $\phi \geq \sigma\pi$, $C \geq \sigma\pi$. Public safe agents choose IC if $\phi \leq \sigma$, $C \geq \phi$. Choose SDC if $C \leq \sigma$, $\phi \geq C$. Choose NT if $\phi \geq \sigma$, $C \geq \sigma$. Using these conditions, we can plot the following graph:



	Anonymous	Public risky	Public safe
a	IC	IC	IC
b	IC	NT	IC
c	NT	NT	IC
d	NT	NT	NT
e	IC	NT	DC
f	NT	NT	DC
g	IC	DC	DC
h	DC	DC	DC

(4) To derive the effects of η and h on the degree of tiering

1. The number of clearing agents:

   For $C > \phi$: $A = \frac{N-M}{n}$ which is decreasing in η.

   For $C < \phi$: $A = \frac{(N-M)(1-h)}{n}$ which is decreasing in η and h.

2. The total volume of settlement:

   For $C > \phi$: TV$= nA(\frac{N-1-n}{N}) + A(\frac{N-1-n}{N}) + (M-A)(\frac{N-1}{N}) = N - 1 - \frac{(N-M)(n+1)}{N}$ which is decreasing in η.

   For $C < \phi$: TV$= h(N-M) + nA(\frac{N-1-n}{N}) + A(\frac{N-1-n}{N}) + (M-A)(\frac{N-1}{N}) = N - 1 - \frac{(N-M)(n+1)}{N} + h(N-M)\frac{2+n}{N}$

   $= N - 1 - \frac{(N-M)(n+1)}{N}(1-h) + \frac{h(N-M)}{N}$ which is decreasing in η and increasing in h.

3. Clearing agents' volume of settlement:

   For $C > \phi$: CAV$= nA(\frac{N-1-n}{N}) + A(\frac{N-1-n}{N}) = A(1+n)(\frac{N-1-n}{N}) = (A+N-M)(1-\frac{1+n}{N})$ which is decreasing in η.

   For $C < \phi$: CAV$= nA(\frac{N-1-n}{N}) + A(\frac{N-1-n}{N}) = A(1+n)(\frac{N-1-n}{N}) = (A+(1-h)(N-M))(1-\frac{1+n}{N})$ which is decreasing in η and h.

4. Clearing agents' share of settlement:

   For $C > \phi$: Share$= 1 - [(M-A)(\frac{N-1}{N})]/[N - 1 - \frac{(N-M)(n+1)}{N}]$ which is decreasing in η.

   For $C > \phi$: Share$= 1 - [h(N-M) + (M-A)\frac{N-1}{N}]/[N - 1 - \frac{(N-M)(n+1)}{N}(1-h) + \frac{h(N-M)}{N}]$ which is decreasing in η and h.