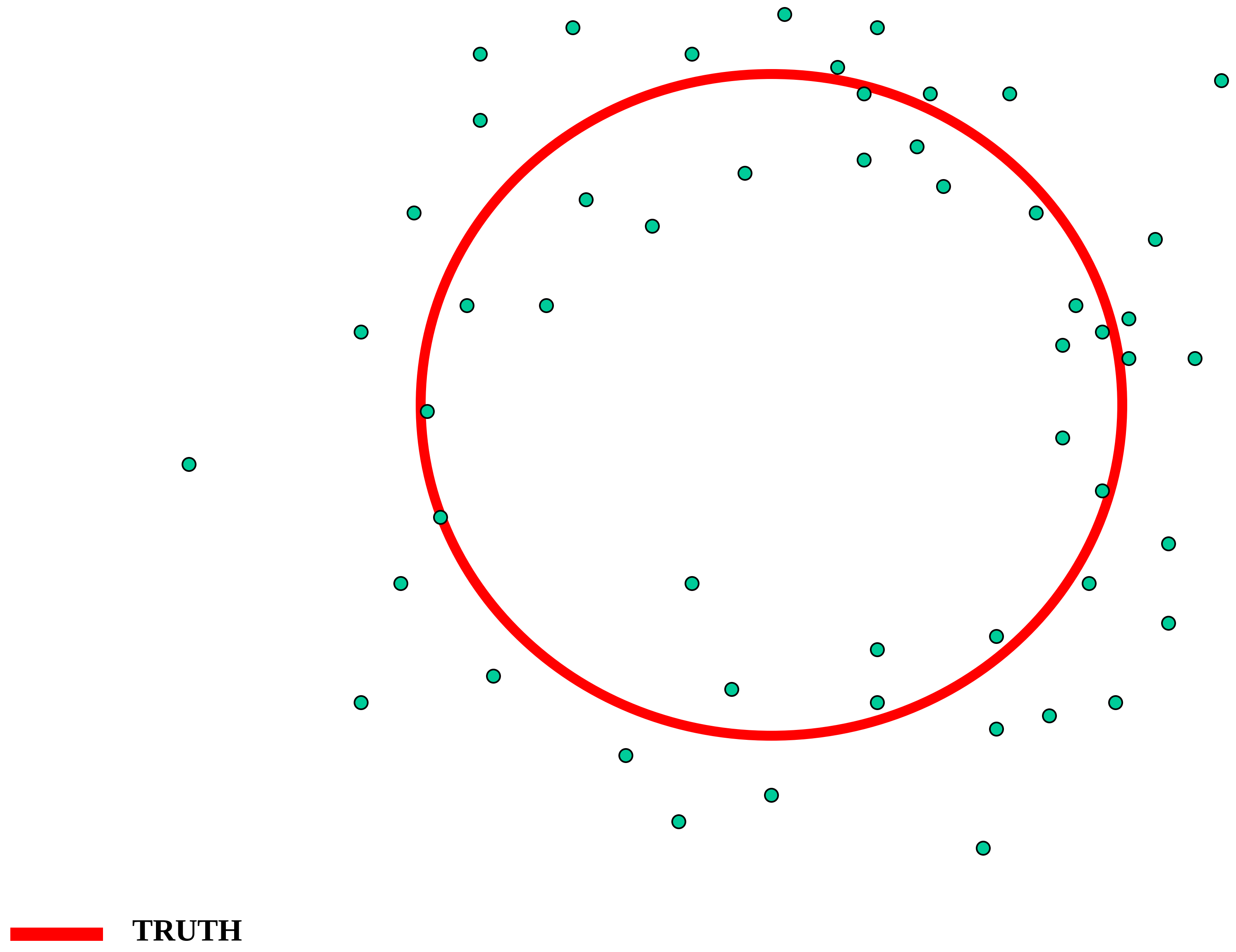
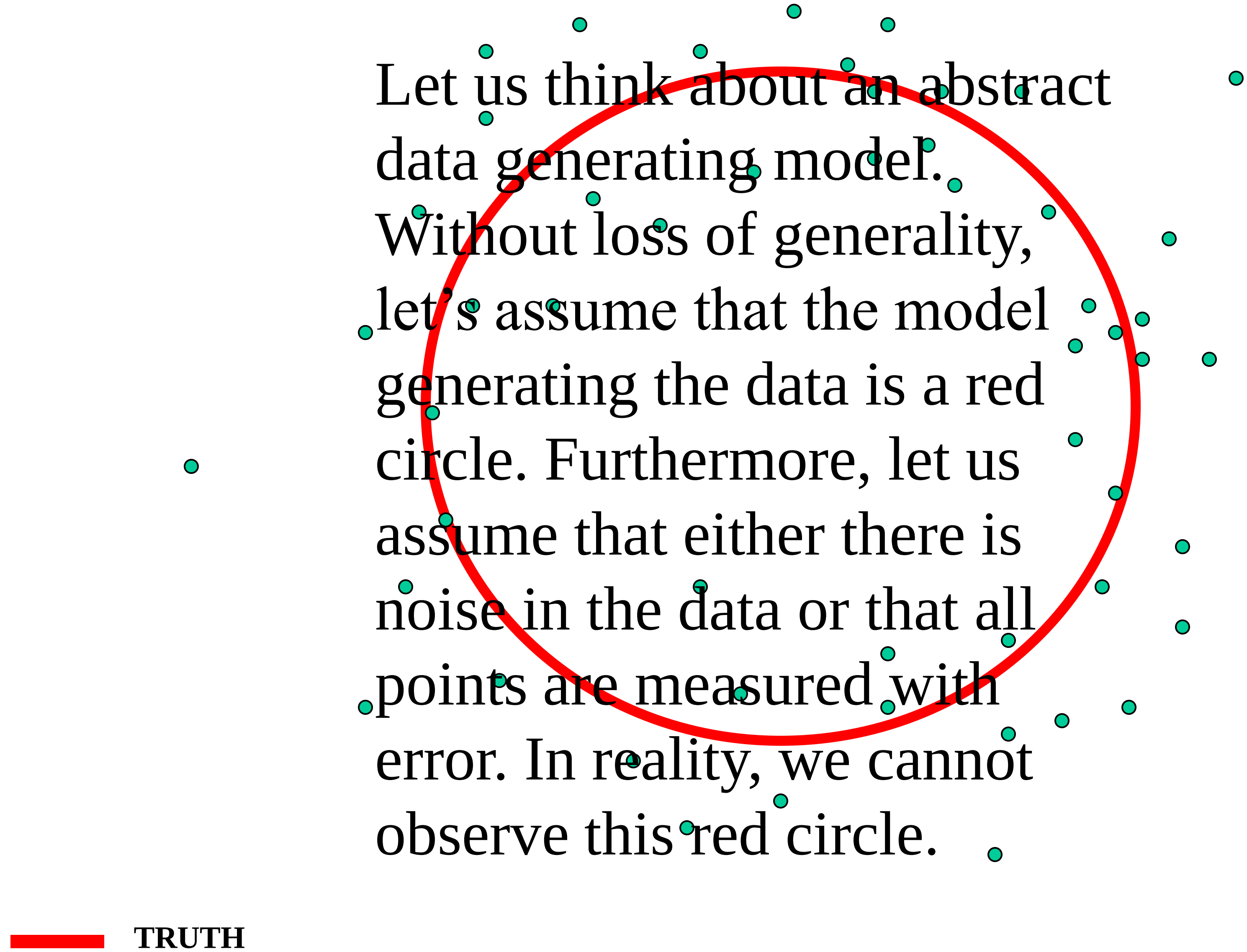
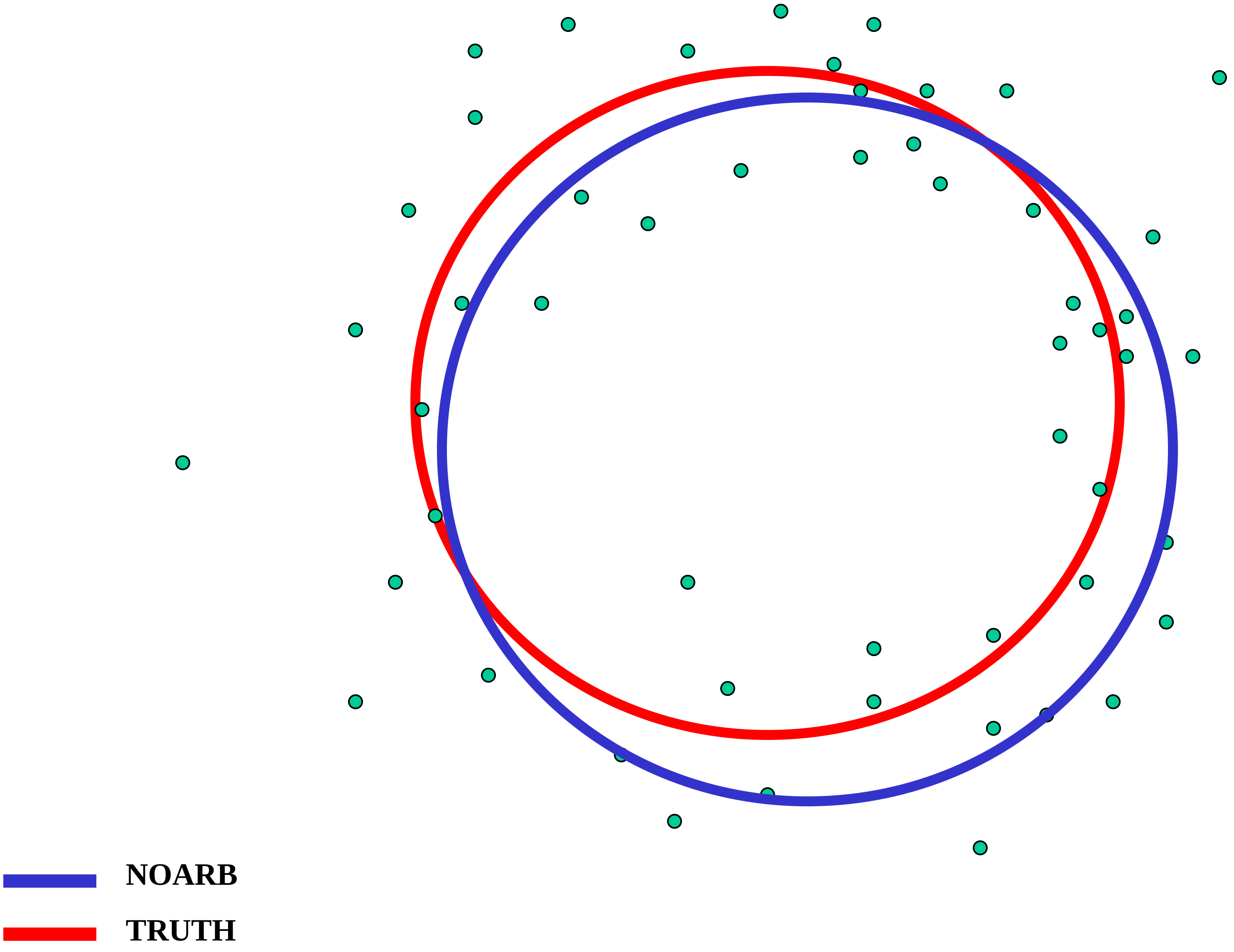
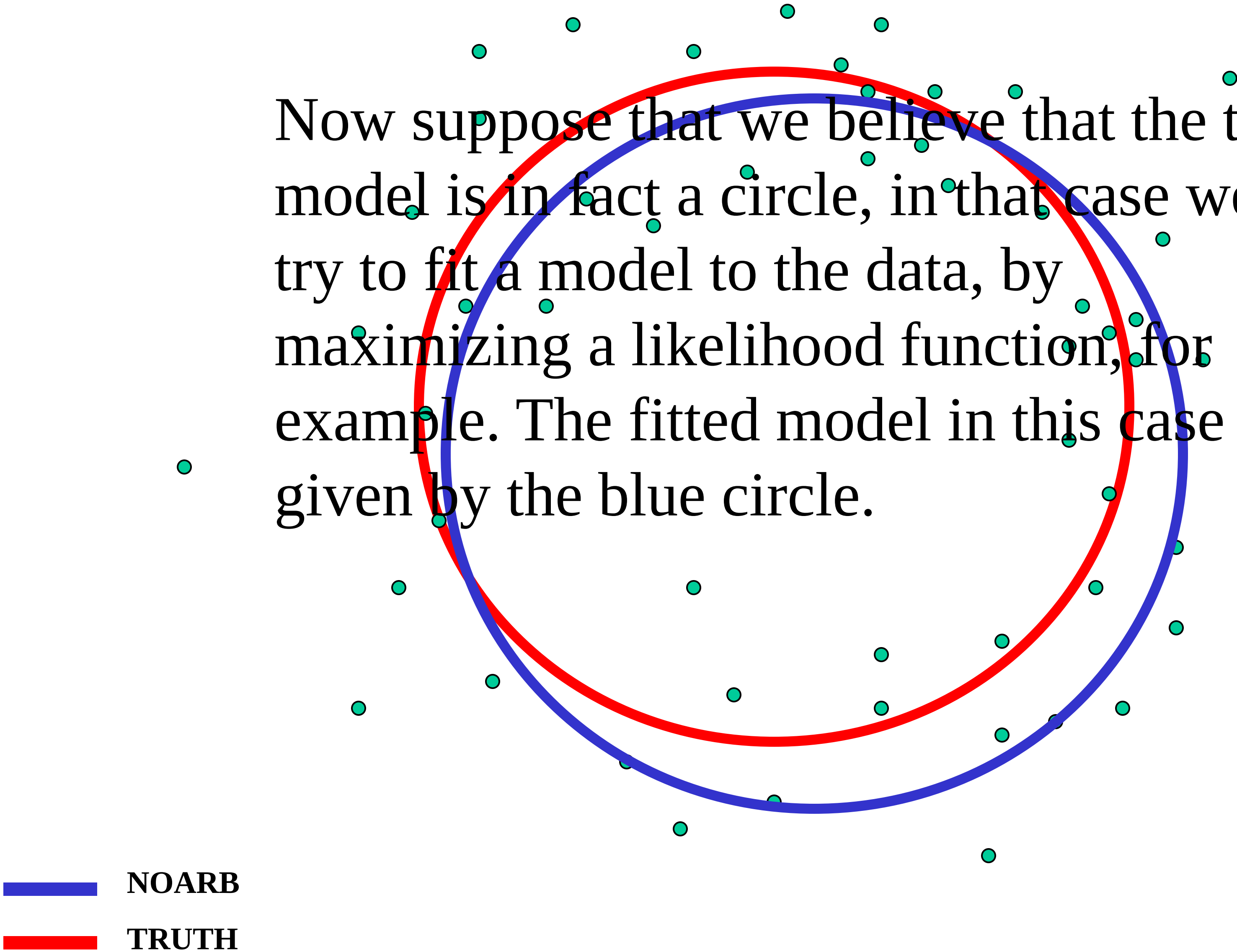

TRUTH

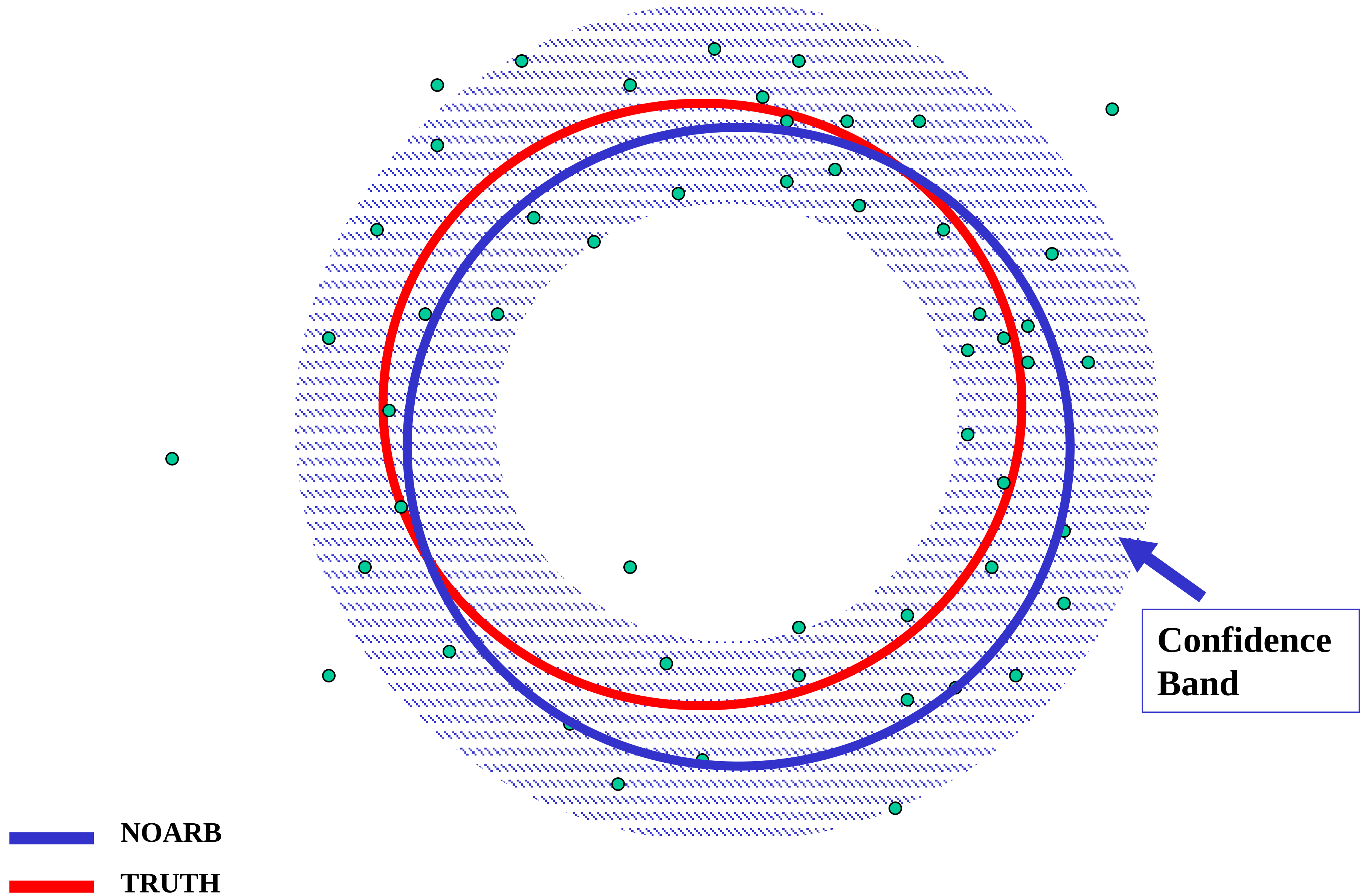
Let us think about an abstract data generating model. Without loss of generality, let's assume that the model generating the data is a red circle. Furthermore, let us assume that either there is noise in the data or that all points are measured with error. In reality, we cannot observe this red circle.



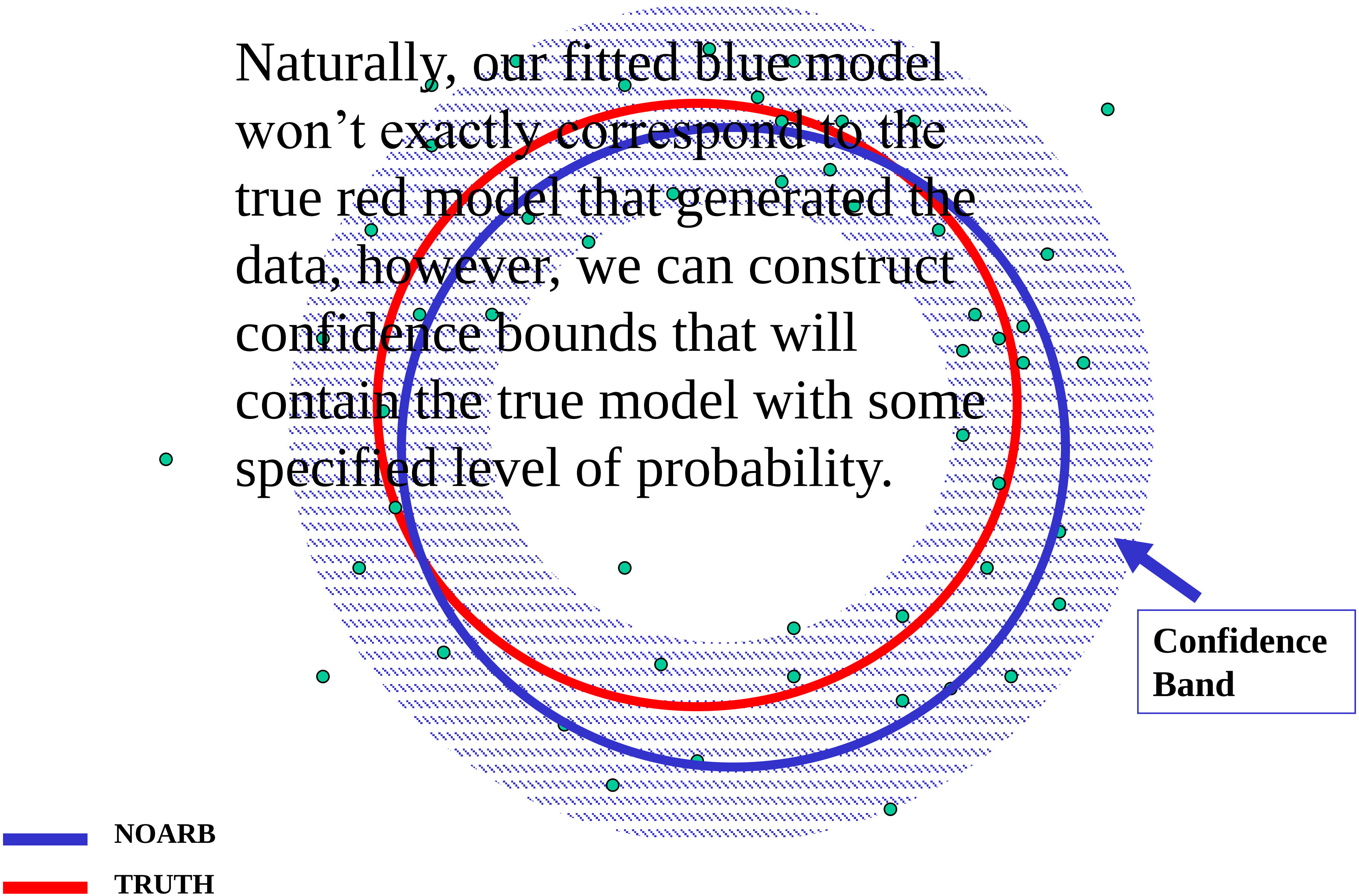

NOARB
TRUTH

Now suppose that we believe that the true model is in fact a circle, in that case we can try to fit a model to the data, by maximizing a likelihood function, for example. The fitted model in this case is given by the blue circle.

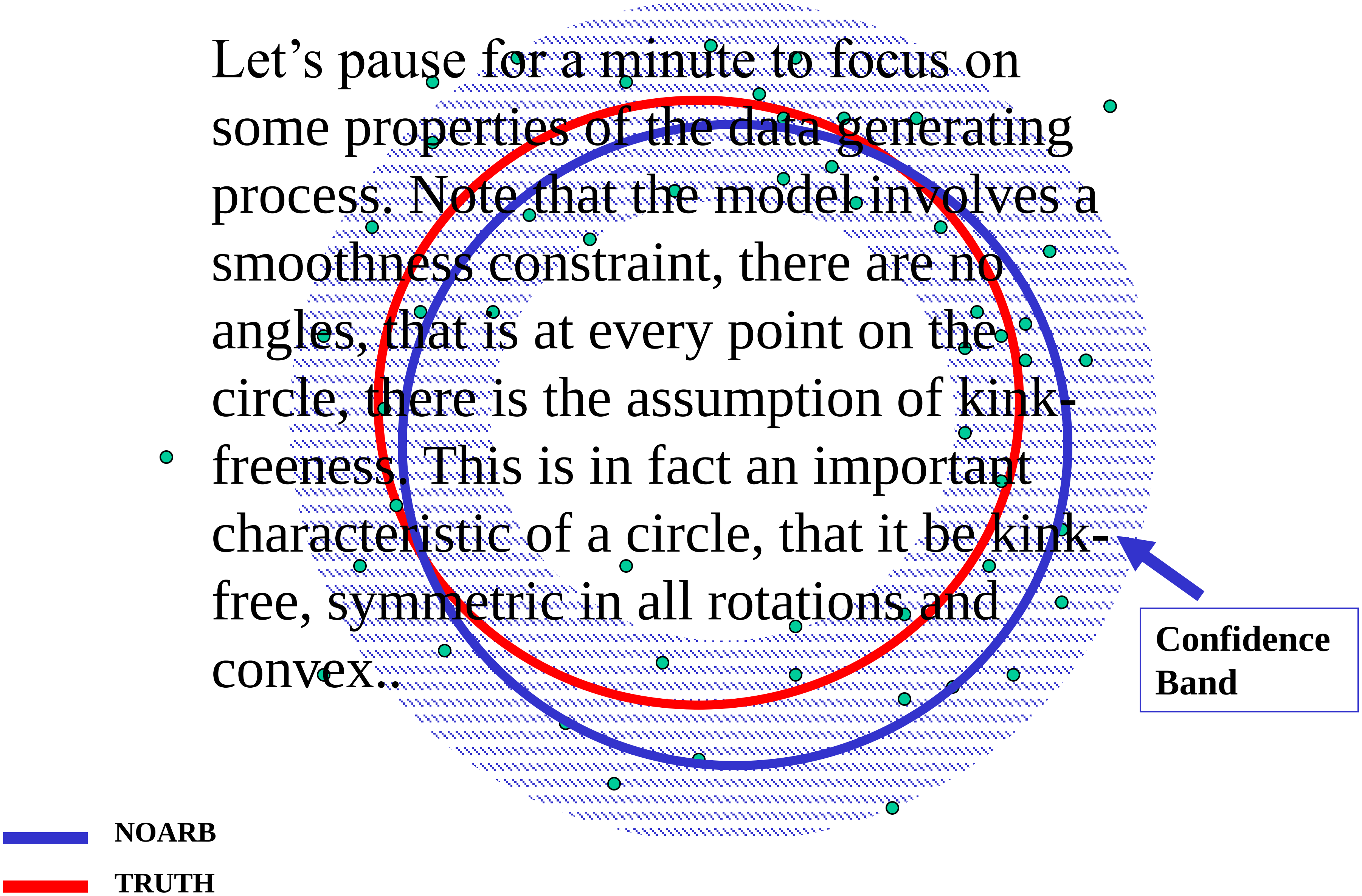


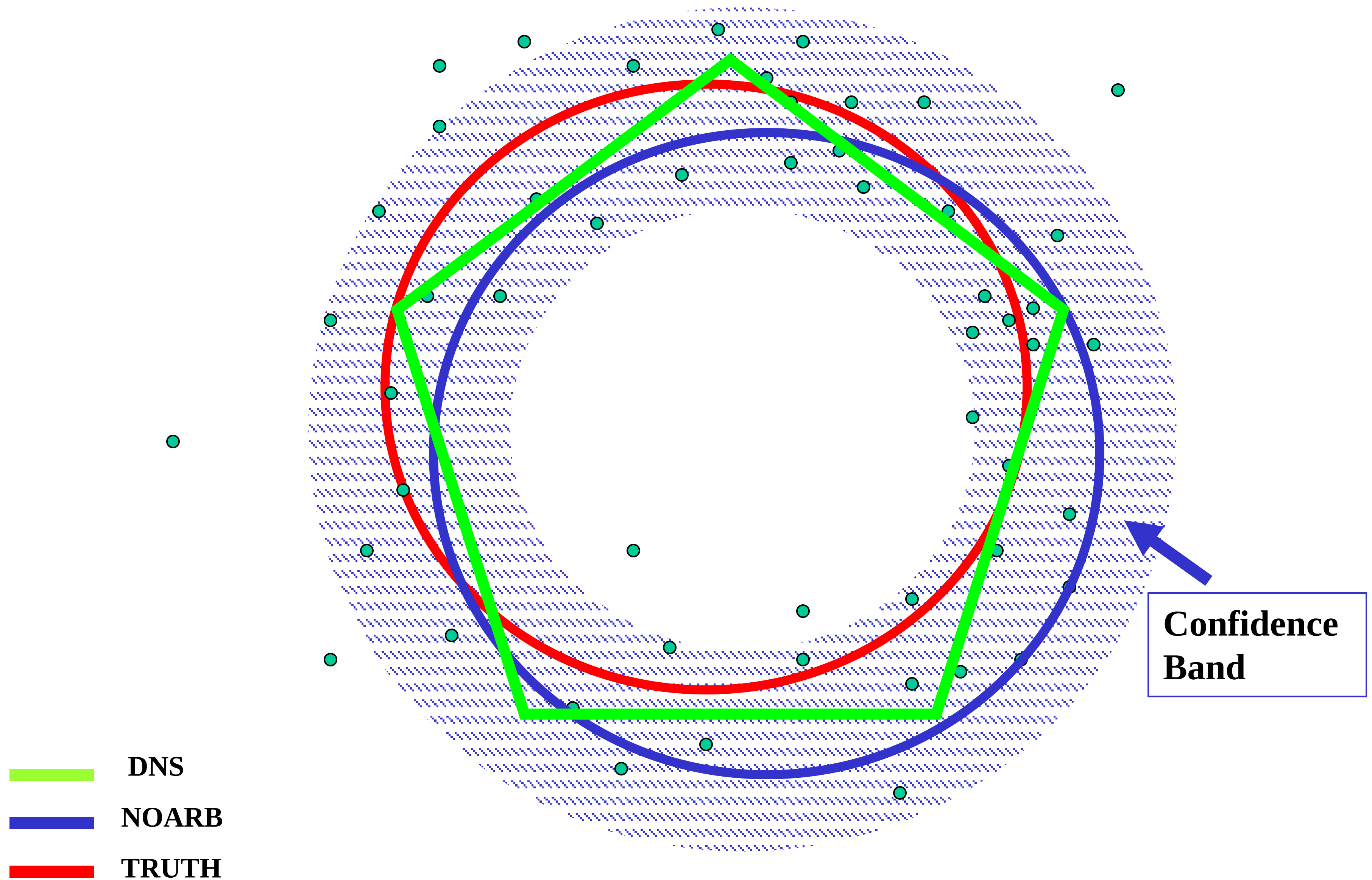

Confidence Band
NOARB
TRUTH

Naturally, our fitted blue model won’t exactly correspond to the true red model that generated the data, however, we can construct confidence bounds that will contain the true model with some specified level of probability.

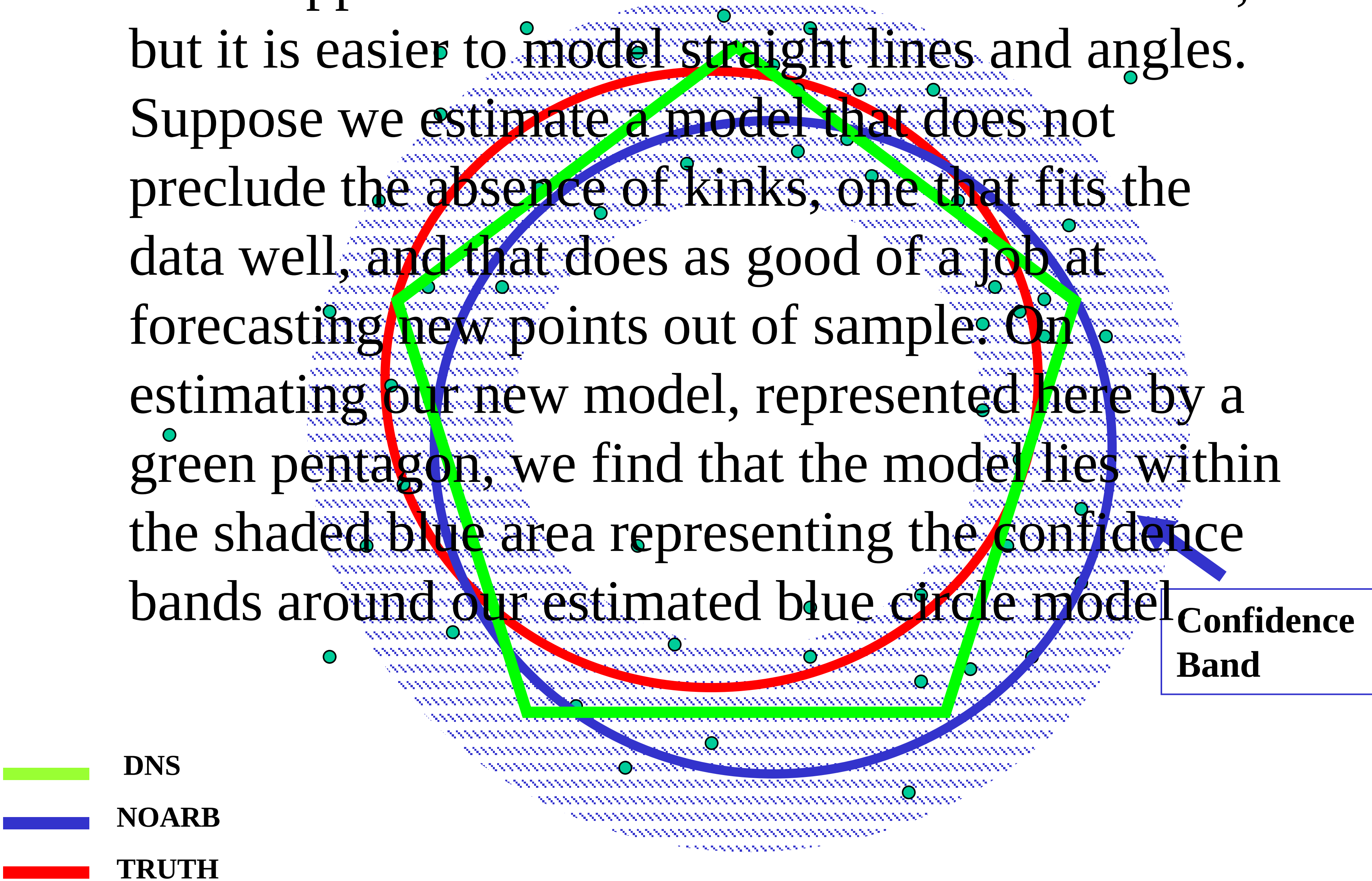


Let’s pause for a minute to focus on some properties of the data generating process. Note that the model involves a smoothness constraint, there are no angles, that is at every point on the circle, there is the assumption of kink-freeness. This is in fact an important characteristic of a circle, that it be kink-free, symmetric in all rotations and convex..

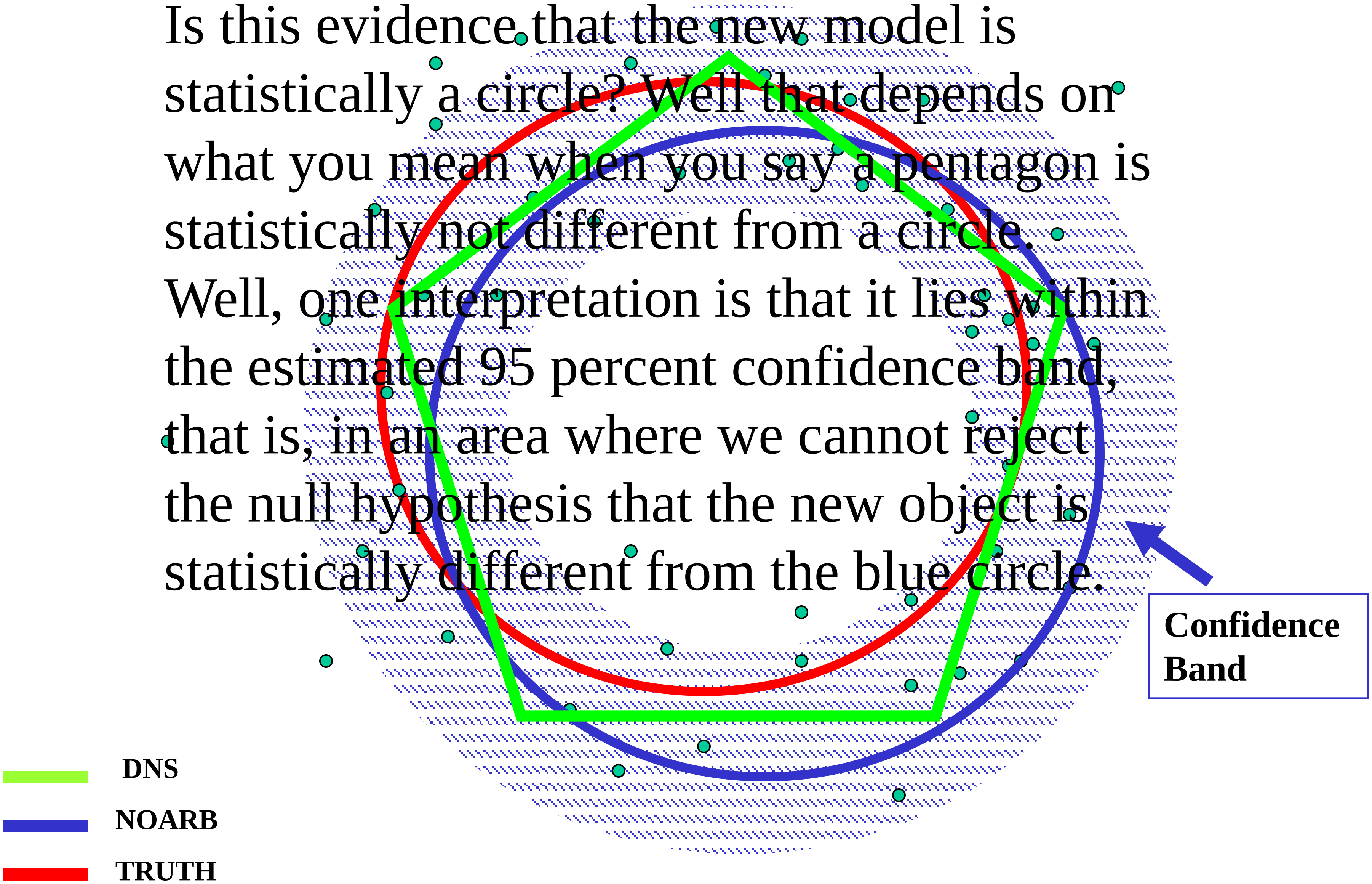



Confidence Band
DNS
NOARB
TRUTH

Now suppose that it is difficult to model a circle, but it is easier to model straight lines and angles. Suppose we estimate a model that does not preclude the absence of kinks, one that fits the data well, and that does as good of a job at forecasting new points out of sample. On estimating our new model, represented here by a green pentagon, we find that the model lies within the shaded blue area representing the confidence bands around our estimated blue circle model.



Is this evidence that the new model is statistically a circle? Well that depends on what you mean when you say a pentagon is statistically not different from a circle. Well, one interpretation is that it lies within the estimated 95 percent confidence band, that is, in an area where we cannot reject the null hypothesis that the new object is statistically different from the blue circle.



Albert Lee CHUN

Discussion of:

How Arbitrage-Free is the Nelson-Siegel Model?

Coroneo, Nyholm and Vidova-Koleva

HEC MONTRĒAL

Department of Finance

In a Nutshell, the Authors...

- Extract the 3 dynamic Nelson-Siegel factors, that correspond to the level, slope and curvature, and then employ these 3 observable factors in a no-arbitrage model.
- Test if the dynamic Nelson-Siegel model is "compatible with arbitrage-freeness" in a statistical sense. Conclusion: Yes.
- Find that both the AFNS and DNS models performs equally well at forecasting out of sample.

Dynamic Nelson-Siegel (DNS) Model

- The Dyamic Nelson-Siegel (DNS) model was developed in Diebold and Li (2005) .

Advantages:

- Arbitrage-free models are quite hard to estimate. DNS models, on the other hand, are easy to estimate.
- DNS models show evidence of having good out of sample forecasting properties.

No Arbitrage and Forecasting

Let's start with Duffee(2002).

Parametric/Hybrid Models: Diebold and Li (2006), Almeida and Vicente (2007), Christensen, Diebold and Rudebusch (2008).

Econometric Tests: Duffee (2008), Giacomini and Carriero (2008)

Not Arbitrage Free

- We know that the parametric model of Nelson Siegel is generally not arbitrage free: Bjork and Christensen (1999), Filipovic (1999).
- A version of DNS can be made arbitrage free by adding an additional factor: Christensen, Diebold and Rudebusch (2007).
- Yet we know in theory, the DNS model will admit arbitrage opportunities. It doesn't impose restrictions on the cross-sectional and time-series properties of yields.

Let's Do an Experiment

- Are the authors really saying something about the arbitrage free-nature of DNS model, or is this simply something that is true by construction, something will be true of any model that fits yields well.
- Would the same be true if we took the principal components, and use these as factors in an AF model?
- I took the DNS factors and estimated linear models using OLS. Are the resulting coefficients compatible with arbitrage-freeness?

Level: Is OLS Arbitrage-Free?

Maturity	2.5	97.5	OLS	OLS+c	DNS
1	0.89	1.14	0.9755	0.9821	1
3	0.89	1.07	1.0062	1.0123	1
6	0.88	1.05	1.0145	1.0111	1
9	0.89	1.05	1.013	1.0095	1
12	0.9	1.06	1.0061	0.9902	1
15	0.91	1.05	1.0007	0.9915	1
18	0.91	1.06	1.0004	1.0014	1
21	0.94	1.06	1.0016	1.0115	1
24	0.95	1.08	0.9966	1.0006	1
30	0.95	1.1	0.9909	0.9879	1
36	0.95	1.11	0.9922	0.9959	1
48	0.94	1.12	0.9955	1.0007	1
60	0.93	1.11	0.9947	1.0006	1
72	0.94	1.08	1.0012	1.0068	1
84	0.94	1.06	1.0012	0.9959	1
96	0.9	1.04	1.0038	1.003	1
108	0.86	1.05	1.0043	1.0067	1
120	0.81	1.07	1.0015	0.9923	1

The estimates all lie within the 95% confidence bounds.

Slope: Is OLS Arbitrage-Free?

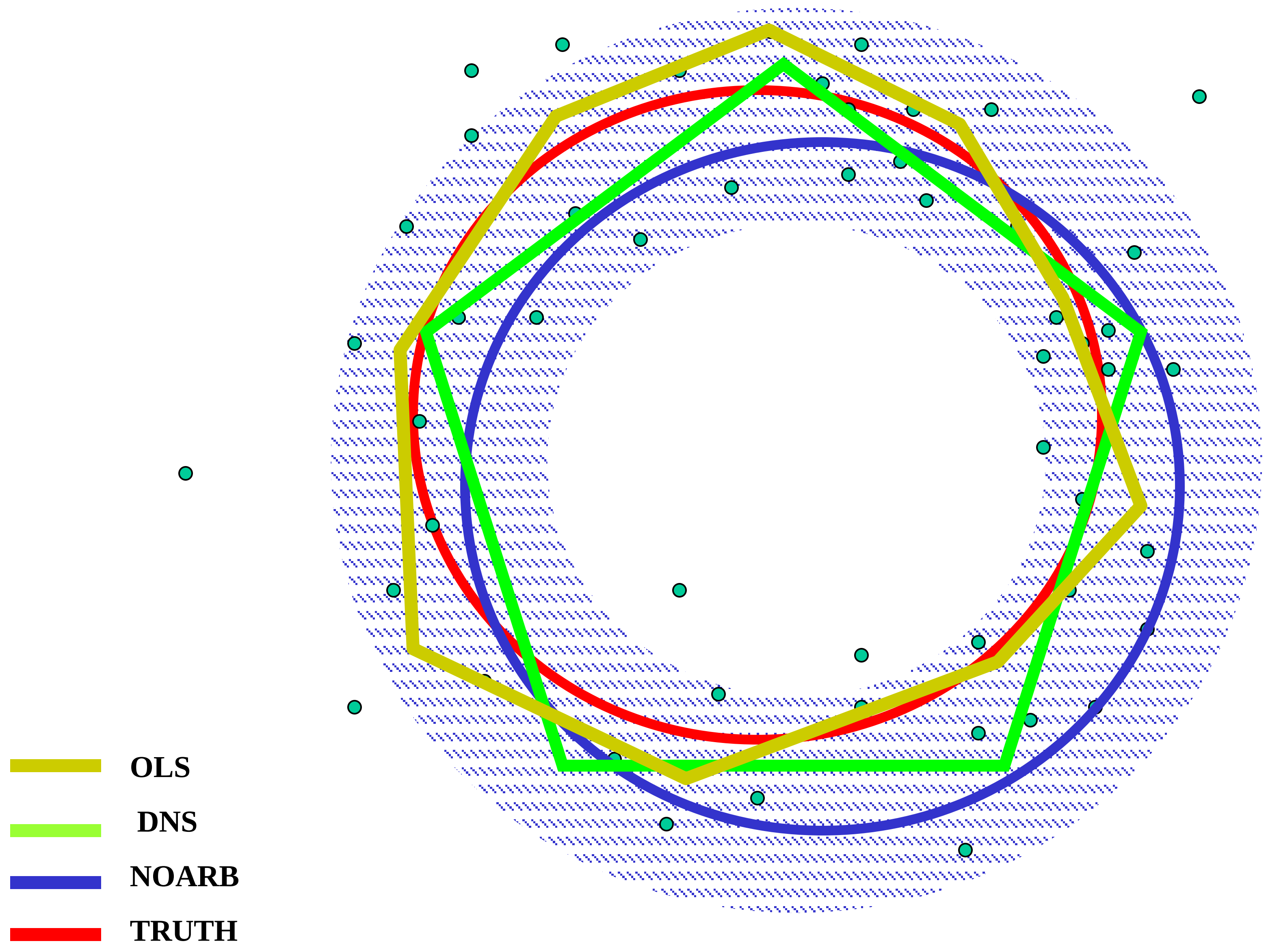
Maturity	2.5	97.5	OLS	OLS+c	DNS
1	0.83	1.07	0.9326	0.9317	0.97
3	0.82	0.99	0.921	0.9202	0.91
6	0.74	0.93	0.8616	0.862	0.84
9	0.67	0.86	0.8	0.8004	0.77
12	0.61	0.78	0.725	0.7271	0.71
15	0.56	0.71	0.6423	0.6436	0.66
18	0.51	0.66	0.5972	0.5971	0.61
21	0.47	0.62	0.5623	0.561	0.56
24	0.44	0.58	0.5277	0.5271	0.53
30	0.37	0.53	0.4454	0.4458	0.46
36	0.32	0.48	0.3984	0.3979	0.41
48	0.25	0.41	0.3204	0.3197	0.32
60	0.2	0.35	0.2674	0.2666	0.27
72	0.17	0.32	0.22	0.2193	0.23
84	0.14	0.28	0.1987	0.1994	0.19
96	0.11	0.26	0.167	0.1671	0.17
108	0.08	0.25	0.1506	0.1503	0.15
120	0.05	0.24	0.1499	0.1511	0.14

The estimates all lie within the 95% confidence bounds.

Curvature: Is OLS Arbitrage-Free?

Maturity	2.5	97.5	OLS	OLS+c	DNS
1	-0.09	0.06	0.0017	0.001	0.03
3	0.03	0.16	0.0661	0.0654	0.08
6	0.1	0.25	0.1599	0.1602	0.14
9	0.13	0.28	0.2127	0.213	0.19
12	0.15	0.29	0.2501	0.2519	0.23
15	0.17	0.3	0.2764	0.2775	0.25
18	0.18	0.3	0.2857	0.2856	0.27
21	0.19	0.31	0.2856	0.2845	0.29
24	0.19	0.31	0.2829	0.2824	0.29
30	0.2	0.31	0.2839	0.2842	0.3
36	0.2	0.3	0.2747	0.2743	0.29
48	0.19	0.29	0.2464	0.2458	0.27
60	0.17	0.27	0.2323	0.2316	0.24
72	0.17	0.25	0.2049	0.2043	0.21
84	0.15	0.24	0.1887	0.1893	0.19
96	0.14	0.23	0.1674	0.1674	0.17
108	0.12	0.23	0.1719	0.1717	0.15
120	0.09	0.22	0.1477	0.1487	0.14

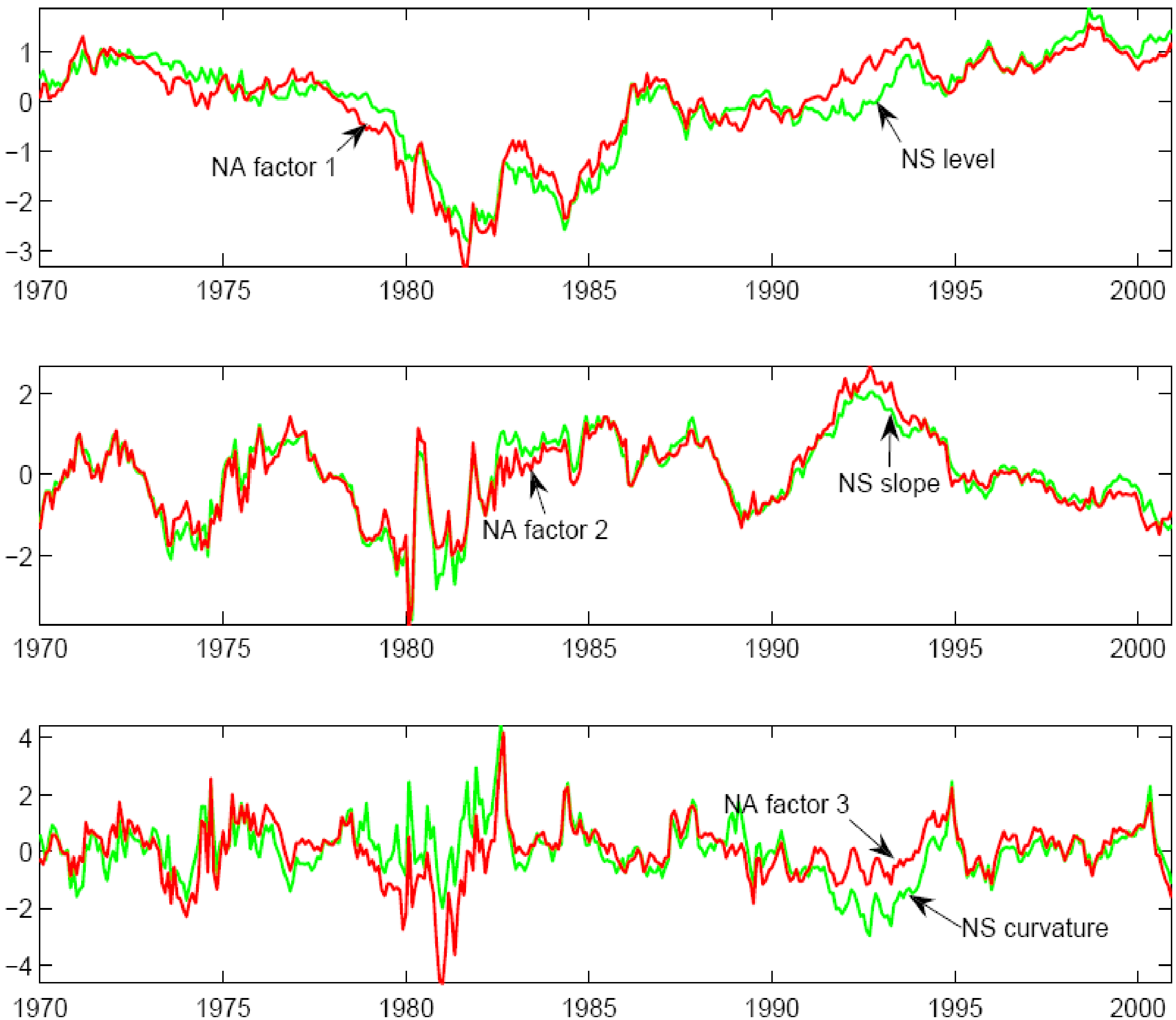
The estimates all lie within the 95% confidence bounds.


OLS
DNS
NOARB
TRUTH

Is OLS Arbitrage-free?

- OLS regressions would tend to over-fit the data.
- Yet, the coefficients all lie within the 95% confidence bounds.
- Is an OLS model "compatible" with AF?
- In fact, by construction, any model that fits yield data well (which is assumed to be arbitrage free), would likely be 'compatible' with an AF version of the model using the same factors.

Comparison with a latent factor AF Model?



There are slight differences between the latent factors and the DNS factors.

The authors could explore how these differences affect forecasting performance.

They have already estimated the model, so why not report the results.

Out of Sample Forecasting Results

	1-m ahead		6-m ahead		12-m ahead	
τ	NS	NA	NS	NA	NS	NA
1	0.82	**0.67**	0.68	**0.56**	0.67	**0.59**
3	0.91	**0.89**	0.72	**0.70**	0.64	**0.62**
6	1.08	**1.07**	**0.80**	0.82	**0.65**	0.66
9	**1.06***	1.26	**0.79**	0.82	**0.63**	0.66
12	**1.01**	1.02	**0.79**	0.81	**0.63**	0.65
15	1.06	**1.00**	**0.78**	0.79	**0.63**	0.64
18	1.04	1.04	**0.79**	0.80	**0.64**	0.65
21	**1.06**	1.09	0.79	0.79	0.65	0.65
24	**1.09**	1.12	0.79	0.79	0.66	0.66
30	**1.04**	1.05	0.79	**0.77**	0.68	**0.67**
36	0.99	0.99	0.79	**0.77**	0.70	**0.68**
48	**0.98**	0.99	0.83	**0.80**	0.75	**0.73**
60	1.10	**1.04**	0.88	**0.85**	0.81	**0.78**
72	1.02	**1.01**	0.89	**0.88**	0.85	**0.83**
84	1.08	**1.07**	0.91	**0.90**	0.87	**0.86**
96	1.03	1.03	**0.92***	0.94	**0.91***	0.92
108	**1.04**	1.09	**0.94**	0.98	**0.93**	0.96
120	**1.08***	1.35	**1.01**	1.09	**1.00**	1.06

The AFNS is never significantly better than the DNS model.

The results are not conclusive.

Arbitrage restrictions do not generally improve forecasting performance.

Do We Really Want to do Forecasting?

- In my view, it really doesn’t have to do with Arb or No Arb, as this is only a second order effect.
- Most prior claims to benefits from Arbitrage seem to arise from a reduction in the dimensionality of the model.
- To really improve forecasting performance, you need to do a better job of forecasting the underlying factors!
- There is evidence that employing AR(1) or VAR(1) dynamics does not lead to the best forecasts in DNS models.

Do We Really Want to do Forecasting?

Chun (2008): "Forecasting Interest Rates and Inflation: Blue Chip Clairvoyant or Econometrics?"

- I find that survey forecasters from Blue Chip Financial Forecasts do the best at forecasting short-medium maturity yields and for inflation.
- DNS with VAR(3) dynamics + Qrinkage* perform best at long horizon forecast of short maturity yields.
- And for forecasting long yields, the best model is a simple AR(2) model + Qrinkage.

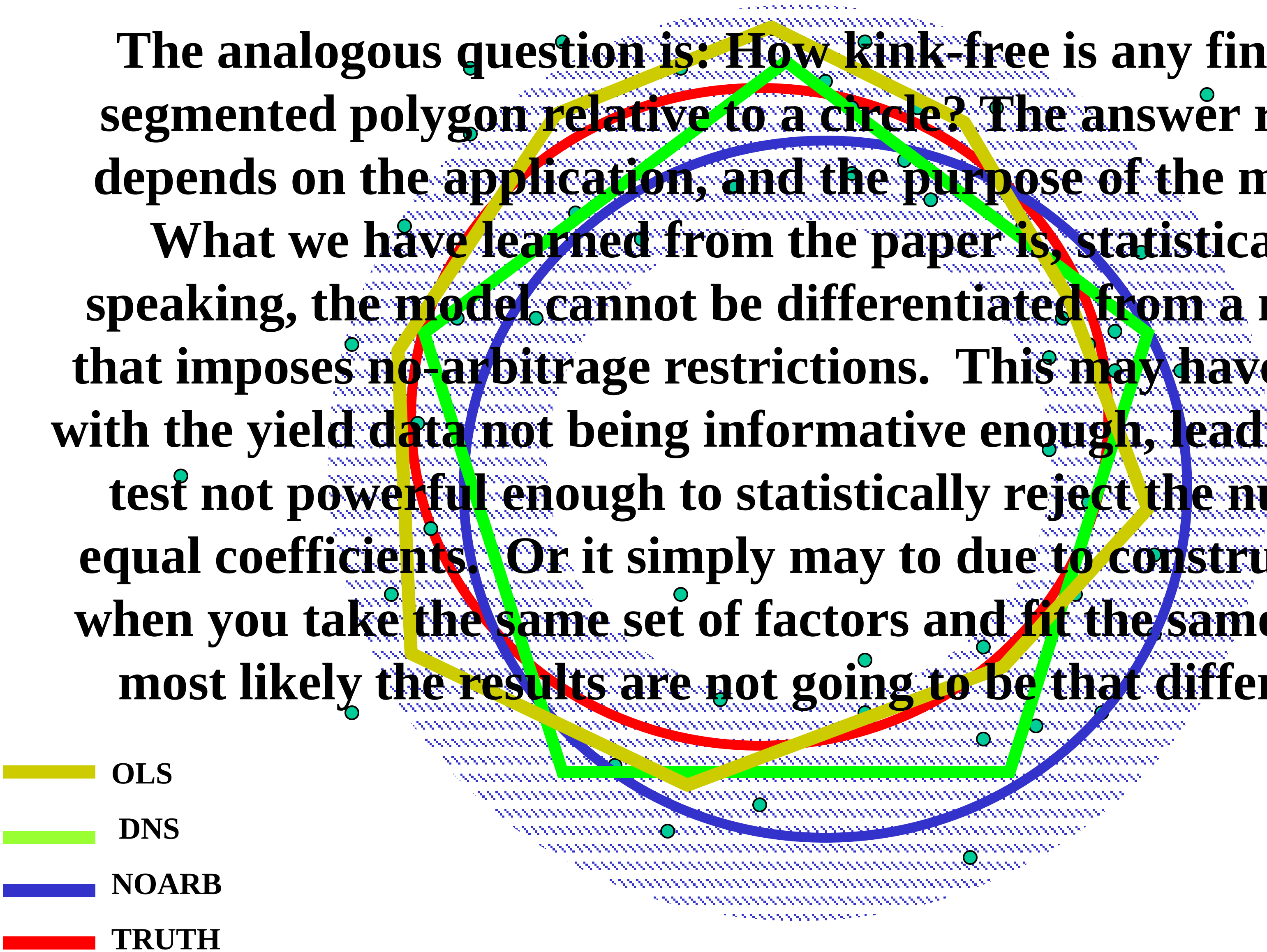
*Qrinkage is a shrinkage method developed by Peter Reinhard Hansen.

Implications

- We can be confident that the DNS model, or any model for that matter that fits yields well, approximates the fit from an AF model.
- This is OK, if we are NOT interested in properties of
  - Risk-premia.
  - Links with the macro-economy and monetary policy, including market expectations of these variables.

How Arbitrage-Free is the Nelson-Siegel Model?

The analogous question is: How kink-free is any finitely segmented polygon relative to a circle? The answer really depends on the application, and the purpose of the model. What we have learned from the paper is, statistically speaking, the model cannot be differentiated from a model that imposes no-arbitrage restrictions. This may have to do with the yield data not being informative enough, leading to a test not powerful enough to statistically reject the null of equal coefficients. Or it simply may to due to construction, when you take the same set of factors and fit the same data, most likely the results are not going to be that different.



How Arbitrage-Free is the Nelson-Siegel Model?