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by Oleksiy Kryvtsov and Yang Zhang

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Abstract

ToTEM – the Bank of Canada's principal projection and policy-analysis model for the Canadian economy – is extended to include inventories. In the model, firms accumulate inventories of finished goods for their role in facilitating the demand for goods. The model is successful in matching procyclical and volatile inventory investment behaviour. The authors show that the convex cost of stock adjustment is key to the model's ability to match the inventory data quantitatively.

JEL classification: E31, E32
Bank classification: Economic models; Business fluctuations and cycles

Résumé

Le modèle TOTEM – principal modèle utilisé par la Banque du Canada pour l'analyse de politiques et l'élaboration de projections concernant l'économie canadienne – est élargi de manière à inclure les stocks. Dans le modèle, les entreprises accroissent leurs stocks de produits finis en raison de l'effet favorable qu'exerce l'importance des stocks sur la demande de leurs produits. Le modèle rend bien compte de la procyclicité et de la volatilité du comportement des investissements en stocks. Les auteurs montrent que le coût d'ajustement convexe des stocks est déterminant dans la capacité du modèle de reproduire les données relatives aux stocks sur le plan quantitatif.

Classification JEL : E31, E32
Classification de la Banque : Modèles économiques; Cycles et fluctuations économiques

1 Introduction

Inventory fluctuations are an inherent part of the business cycle. In a companion paper (Kryvtsov and Zhang 2010), we provide main business cycle facts for inventory dynamics in Canada. Although inventory investment is small on average (less than 1 per cent of output), it is highly volatile and co-moves significantly with other macro variables. During an inflation-targeting period, the volatility of inventory investment (expressed as a fraction of output) in manufacturing and trade is about one-third of that for output, and the correlation with output is positive at 0.6. The important role of inventories in the Canadian business cycle can thus be summarized by the fraction of output growth variance explained by inventories, which is around one-quarter for Canada.

Having recognized the importance of modelling inventories, one of the priorities for the Terms-of-Trade Economic Model (ToTEM) II, the next generation of the Bank of Canada's main projection and policy-analysis model for the Canadian economy, is to add inventories. In this paper, we describe the framework in which inventories are used in projection and policy analysis. In the model, firms accumulate inventories of finished goods for their role in facilitating the demand for their output.[1] Firms' stock adjustment decisions are determined by the cost of production, storage cost (i.e., depreciation and discounting), and adjustment cost. When such costs are low, firms will find it optimal to increase their stocks.

We find that the model generates procyclical and volatile fluctuations of inventory investment, which are robust features of inventory fluctuations in Canada. As is typical in the inventory literature, inventory investment is very sensitive to the change in the cost of production. Therefore, in order to match the volatility of inventories observed in the data, large adjustment costs are required. Simply enhancing the role of the adjustment cost is unlikely to be sufficient for the overall success of the model, since it is likely that the inflation dynamics will be significantly affected. We leave work on the quantitative success of ToTEM-II with inventories to future research.

In our companion paper (Kryvtsov and Zhang 2010), we develop a partial-equilibrium model in which firms hold stocks of goods to buffer against stockouts. In booms, firms boost their inventories to avoid stocking out due to the rise in demand. The model combines the real marginal cost estimated by ToTEM with the convex cost of adjusting inventories to match

[1] See Bils and Kahn (2000), Jung and Yun (2005), and Iacoviello, Schiantarelli, and Schuh (2007).

the dynamics of the inventory-sales ratio in the data.[2] The model is able to capture all of the salient business cycle facts for inventory behaviour in Canada during the inflation-targeting period. The main determinant of inventory behaviour in the model is the expected growth of the real marginal cost combined with the large cost of inventory adjustment. The partial-equilibrium model of stockout behaviour in our companion paper complements the work in this paper by extending the analysis to modelling inventories as a buffer against stockouts. It can also be used as a satellite model of inventories in ToTEM backing up projections produced by ToTEM-II.

The only study of inventory behaviour in Canada that we are aware of is by Chacra and Kichian (2004). They estimate an error-correction model of short-run inventory investment behaviour. Our approach differs from theirs mainly in that we provide a structural interpretation of fluctuations in inventories that are based on observed aggregate time series for sales and real interest rates, as well as the estimated current and expected future real marginal costs. It is our view that this model should be used to provide tractable scenarios for the 1- to 2-year-ahead behaviour of inventories given ToTEM's forecasts of other pertinent variables over the same horizon. In this sense, our model is a complement to empirical models, such as that estimated by Chacra and Kichian (2004), which are designed for short-term forecasts.

The rest of this paper is organized as follows. Section 2 provides an overview of ToTEM and extends it to include inventories. Section 3 describes the extended model's parameterization, and generates and discusses impulse responses to the main shocks in the model. Section 4 offers some conclusions.

2 Adding Inventories to ToTEM

ToTEM is the Bank of Canada's principal projection and policy-analysis model for the Canadian economy. It is a medium-scale open-economy dynamic stochastic general-equilibrium (DSGE) model with multiple goods and an endogenous monetary policy rule followed by the central bank. Optimizing behaviour from households, firms, and the central bank yields a set of first-order conditions that dictate how these agents behave. This set of first-order conditions, combined with market-clearing conditions, yields a system of dynamic

[2] Adding a stockout model to the full version of ToTEM is a daunting task, since the cyclical behaviour of stockouts affects inventory and pricing decisions for firms in the model.

non-linear equations that characterize the behaviour of the economy (see Murchison and Rennison 2006). Since ToTEM is used not only for policy analysis but also for projections at the Bank of Canada, it is more elaborate than most standard models. The dynamics of 193 state variables is driven by 29 exogenous shock processes. Section 2.1 provides a brief non-technical summary of ToTEM based on Cayen, Corbett, and Perrier (2006).

2.1 Overview of ToTEM

The production side of ToTEM is as follows. There are four types of finished goods produced by domestic firms: consumption, investment, government, and non-commodity export goods. To produce these goods, firms use a constant elasticity of substitution technology that combines capital with labour services, imported intermediate goods, and commodities. There is also a commodity sector. The commodities are produced by domestic firms by combining labour services with capital goods and a fixed factor that we refer to as land. All firms are allowed to vary their utilization rate, but at a cost in terms of foregone output. The firms also face adjustment costs on the level of employment and on the change in investment, and in terms of foregone output. It is assumed that finished-good producers are monopolistically competitive, which allows them to fix prices for more than one period, as in Calvo (1983). The Calvo pricing framework is also used to model wage rigidities and import price rigidities, as in Smets and Wouters (2002).

The demand side of ToTEM can be summarized as follows. Domestic households buy the finished consumption goods as well as bonds from the (domestic) government and foreigners. They earn (after-tax) labour income from the labour services that they provide to the domestic firms, and income from their holding of domestic and foreign bonds in the form of interest payments. They also receive transfers from the government. The government buys the finished government goods from the domestic firms with tax revenues and distributes transfers to the domestic households. These expenditures are financed with the tax revenues from labour income and indirect taxes. The model assumes that the government targets a desired level for the debt-to-GDP ratio, with some smoothing, and uses the tax rate on labour income as the policy instrument. Foreigners buy the commodities exports as well as the final non-commodity export goods. They also sell intermediate imported goods to the domestic importers, and they buy and sell bonds.

Foreign variables in ToTEM are currently generated with a semi-structural model. This model is exogenous with respect to the core of ToTEM in that there is no feedback from domestic variables to the foreign variables. This is consistent with the assumption that Canada is a small open economy. The foreign variables that enter in ToTEM are output and the output gap, the inflation rate, interest rates (real and nominal), and real commodity prices.

Monetary policy in ToTEM is determined by the generalized Taylor rule, which prescribes a path for the nominal interest rate (set by the monetary authority) as a function of the lagged rate, output gap, and expected rate of inflation. It is assumed that the monetary authority in ToTEM can fully commit to its future policy actions. This implies that, for any future history of shock realizations, the path of the nominal interest rate will be consistent with the policy rule. This is one of the key assumptions that we relax. In this paper, we deviate from the full-commitment assumption in that the monetary authority is choosing its policy on a period-by-period basis, optimizing its current-period objectives and considering the private expectations of the future variables to be beyond its control. The remainder of this section describes the overall set-up of ToTEM, which is then employed to solve the monetary policy problem under discretion.

2.2 Adding inventories

We assume that three sectors that produce finished goods in ToTEM – the consumption, investment, and manufactured export sectors – now have technology that allows them to carry inventories. Since the government and imports sectors have only small amounts of inventories, we do not model inventories in those sectors. Let them be indexed by $j = \{C, I, X\}$. Each sector consists of a continuum of monopolistically competitive firms, indexed by i. Demand for inventories is introduced by modifying the demand function for the goods sold by firm i in sector j:

$$S_{is}^{j} = \left(\frac{A_{is}^{j}}{A_{s}^{j}}\right)^{\theta} \left(\frac{P_{it}^{j}}{P_{t}^{j}}\right)^{-\varepsilon} S_{s}^{j}, \tag{1}$$

where S_{is}^{j}, A_{is}^{j}, P_{it}^{j} are the sales, pre-sale stocks, and prices for firm i, and S_{s}^{j}, A_{s}^{j}, and P_{t}^{j} are the respective aggregates for sector j. This demand specification differs from the one used in the original ToTEM by a multiplier $\left(\frac{A_{is}^{j}}{A_{s}^{j}}\right)^{\theta}$, which implies that a 1 per cent increase in the firm's stock (relative to the average stock in the sector) generates an extra θ per cent of sales. Such an effect of stock on sales can be interpreted as stemming from households' preferences for buying goods from firms who have relatively larger stocks of finished goods, as in Bils and Kahn (2000) and Jung and Yun (2005), or from firms' desire to hold buffer stocks of goods to avoid stockout, as in Wen (2008) and Kryvtsov and Midrigan (2009, 2010).

Let MC_{it}^{j} denote firms' nominal marginal costs in sector j in period t. In ToTEM, firms buy labour, capital, and other material inputs in the competitive markets and use them to produce finished goods according to a constant-returns-to-scale production technology. In this case, firm i's total variable costs are equal to $MC_{it}^{j}Y_{it}^{j}$, where Y_{t}^{j} is firm i's output level in period t. Denote Ξ_t as the vector of marginal utilities of consumption across states in period t and P_t as the price of the consumption good in period t. The firm's optimization problem consists of choosing sequences of sale prices $\left\{P_{it}^{j}\right\}$ and pre-sale stocks $\left\{A_{it}^{j}\right\}$ to maximize the present discounted sum of period profits:

$$E_t \sum_{s=0}^{\infty} \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[P_{it+s}^{j} S_{it+s}^{j} - MC_{it+s}^{j} Y_{it+s}^{j}\right],$$

subject to (1), sticky-price constraints on $P_{t+s}^{j}(i)$, and the law of motion for the pre-sale level of stock:

$$\begin{aligned} A_{it}^{j} &= (1-\delta)\left(A_{it-1}^{j}(i) - S_{it-1}^{j}\right) + Y_{it}^{j} \\ &\quad -\frac{\eta}{2}\left(Y_{it}^{j} - Y_{it-1}^{j}\right)^2 \left(Y_{it}^{j}\right)^{-1}, \end{aligned}$$

where δ is the stock depreciation rate in sector j, which reflects the storage costs, and the last term represents the quadratic costs of stock adjustment that punish deviations of its output growth from zero. This law of motion says that firm i's stock available for sale in period t is the sum of undepreciated stock carried over from the previous period and the current output less the costs of adjustment.

This problem yields the following optimality condition for optimal stock holdings:

$$\frac{A_{it}^j}{S_t^j} = \theta \cdot \frac{P_{it}^j - (1-\delta)E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} \Lambda_{it+1}^j}{\Lambda_{it}^j - (1-\delta)E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} \Lambda_{is+1}^j}, \tag{2}$$

where $\Lambda_{it}^j = MC_{it}^j / \left[1 - \eta \left(1 - \frac{Y_{it-1}^j}{Y_{it}^j}\right) + \frac{\eta}{2}\left(1 - \frac{Y_{it-1}^j}{Y_{it}^j}\right)^2\right]$ is the marginal cost of producing and adjusting inventories. The left-hand side is the firm's pre-sale stock-to-sales ratio. Equation (2) shows that the marginal cost of producing and delivering an additional unit of inventories, $\Lambda_{it}^j - (1-\delta)E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} \Lambda_{is+1}^j$, must equal the marginal revenue from extra sales, $\theta \frac{S_t^j}{A_{it}^j} \left[P_{it}^j - (1-\delta)E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} \Lambda_{it+1}^j\right]$.

We next turn to analyzing the effect of inventories on the firm's pricing decision. Without inventories, the optimal price for the flexible-price firm is a markup over the current marginal production cost, MC_{it}^j. When the firm holds inventories, the first-order condition with respect to price is (under the assumption of flexible prices):

$$P_{it}^j = \frac{\varepsilon}{\varepsilon - 1} \left[(1-\delta)E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} \Lambda_{t+1}^j\right]. \tag{3}$$

Hence, in the model with inventories, the optimal price is a markup over the cost of replacing inventories; i.e., the next period's marginal cost of producing and adjusting inventories, discounted by the depreciation rate and stochastic discount factor.

The remaining step of adding inventories to ToTEM is to linearize (2) and a sticky-price version of (3) and aggregate them over firms in each sector.[3] ToTEM with inventories, then, contains one new equation for optimal pre-sale inventories for each sector and one Phillips curve equation modified by replacing $\log\left(\frac{MC_t^j}{P_t^j}\right)$ with $E_t \log\left(\frac{\Xi_{t+1} P_t}{\Xi_t P_{t+1}} \frac{P_{t+1}^j}{P_t^j} \frac{\Lambda_{t+1}^j}{P_{t+1}^j}\right) = E_t \left[\log\left(\frac{\Xi_{t+1}}{\Xi_t}\right) + \log\left(\frac{P_{t+1}^j / P_{t+1}}{P_t^j / P_t}\right) + \log\left(\frac{MC_{t+1}^j}{P_{t+1}^j}\right) + \eta \left(\log Y_{it+1}^j - \log Y_{it}^j\right)\right]$. The first term is the negative of the real interest rate, the second term is the growth rate of the sectoral price level relative to the consumption sector's price level, the third term is the next period's real marginal cost, and the last term is the log adjustment cost that is proportional to the expected growth rate of the sector's output.

[3] Aggregation is relatively straightforward under linear approximation; see Jung and Yun (2005).

3 Parameterization and Impulse Responses

3.1 Parameterization

There are three new parameters in the model: θ, δ, and η. We use Equation (2) to pin down the ratio of the end-of-period stock to sales in steady state, $\frac{A^j - S^j}{S^j}$, to 0.8 at a quarterly frequency, which is a standard value in the data (Khan and Thomas 2007). The corresponding value of θ is 0.467. The depreciation rate δ is 1 per cent per quarter, which is in the middle of the plausible range: between the relatively small values to match the ratio of the mean change in the end-of-period inventories to output (Kryvtsov and Midrigan 2009, 2010) and the relatively large values reported in the inventory literature (Richardson 1995). Finally, the adjustment cost parameter η is set to 0.5 so that a 1 per cent change in the firm's output increases the cost of adjusting one extra unit of stock by 0.5 per cent. Eventually, this parameter can be calibrated to match the standard deviation of the aggregate inventory-to-sales ratio in the data.

3.2 Impulse responses

Figures 1 through 3 provide impulse responses in ToTEM with inventories to three major shocks: a negative consumption shock, a positive nominal interest rate shock, and a positive productivity shock. We focus on the per cent responses to six variables in the core consumption sector: the annualized nominal interest rate (named R1N_ANN in the ToTEM code), output and sales in the core consumption sector (LCX8 and SALES), annualized core inflation (INF), the shadow cost of inventories (SHCINVT) defined as $\log\left(\Xi_t \Lambda_t^C\right)$, and the ratio of the end-of-period inventory stock to sales in the core consumption sector (R_LSINVT). To isolate the effect of adding inventories, impulse responses are compared to those in the original ToTEM.

Figure 1 shows responses to a negative real consumption shock. The shock increases the current marginal utility of consumption, so that households prefer to decrease their current consumption. The resulting decrease in output and the price level is accompanied by a high real interest rate and hence a high cost of carrying inventories. This results in firms cutting down their inventory stocks, thus augmenting the decrease of total output after the shock.

Note that the inventory stock decreases faster than sales; i.e., the stock-to-sales ratio goes down. In the data, the ratio is counter-cyclical. Hence, despite the stock adjustment cost, inventories still seem to be more flexible than in the data. This excessive sensitivity of inventories is due to the high elasticity of the timing of inventory investment. The cost of delaying or accelerating investment in inventories by one period is determined by the depreciation rate and the real interest rate. Since both are quite small, even little changes in the cost of producing and adjusting inventories between the current and future periods will lead firms to shift the timing of most of their investment in inventories.[4]

Figure 2 shows responses to a positive nominal demand shock–expansionary monetary policy shock. All the main effects of the inventories on impulse responses after a real demand shock remain in this case. The initial expansion is augmented by the procyclical response in inventories. Inventories increase because their shadow costs are low after the shock. The increase is quite large, as the inventory-sales ratio also increases.

Responses to a positive productivity shock are shown in Figure 3. This is a time of high demand for output and low production costs, and so inventories rise. The need to adjust the stock increases the adjustment cost, which offsets the effect of low marginal cost on inflation. Note that this implies an interesting feature of inventory adjustment costs: they amplify inflation responses to demand shocks and stabilize them after supply shocks.

Section 3.3 investigates in more detail the determinants of inventory dynamics by conducting a sensitivity analysis with respect to parameter values chosen in the benchmark model.

3.3 Sensitivity analysis

Figures 4, 5, and 7 show the effect of changing the values of our inventory parameters δ, θ, and η on impulse responses to the productivity shock. From the linearized version of (2) it can be shown that inventories are less volatile if the depreciation rate is higher, elasticity θ is higher, or adjustment cost η is higher.

First, we show the effect of having a depreciation rate that is twice as high: 0.02. The higher depreciation rate increases the cost of shifting the timing of the inventory investment and hence makes it less volatile (Figure 4). The new impulse responses are compared to those in the benchmark ToTEM with inventories. Quantitatively, the effect of doubling the rate of depreciation is small. Jung and Yun (2005) show that a depreciation rate consistent with the

[4] House (2008) explains the extra sensitivity of investment in physical capital.

observed behaviour of inventories has to be implausibly large, around 0.6.

Figure 5 demonstrates responses when θ corresponds to a lower average stock-to-sales ratio of 0.5. According to demand equation (1), lower elasticity θ means that inventories are less efficient in generating sales at given prices of goods. Hence, in order to sell extra output produced during a time of low marginal cost, a firm that cannot lower its prices by very much, due to price-stickiness, has to move its inventories more. As a result, inventories are even more flexible, with the stock-to-sales ratio and output rising faster than in the benchmark model. A comparison of Figure 5 with Figure 7 reveals that eliminating the inventory adjustment cost ($\eta = 0$) produces effects that are quantitatively similar to decreasing θ above.

Finally, we isolate the effect of inventories on inflation by substituting the Phillips curve equation in the benchmark model with the Phillips curve equation from the original ToTEM. The result is shown in Figure 6. Since the current price does not reflect the cost of adjusting inventories, at the time of the shock, inflation falls sharply, due to the fall in the real marginal cost. Hence, the effect on inflation from adding inventories is noticeable if the cost of stock adjustment is large.

4 Conclusions

In this paper, we describe how to add inventories to ToTEM. In this regard, we employ a standard model in which inventories facilitate sales, as in Bils and Kahn (2000) and Jung and Yun (2005). The model generates procyclical and volatile fluctuations of inventory investment, which are robust features of inventory behaviour in Canada. As is typical in the inventory literature, inventory investment is very sensitive to a change in the cost of production. Therefore, in order to match the volatility of inventories observed in the data, large adjustment costs are required.

Future work on ToTEM with inventories will focus on matching the volatility of inventory investment. We have demonstrated that increasing the adjustment cost can slow down the response of inventories to shocks. But since prices reflect the adjustment cost, this means that predicted inflation dynamics is also significantly affected.[5] There are two alternative ways to reduce the sensitivity of inventories without increasing the volatility of

[5] Kryvtsov and Midrigan (2009, 2010) argue that this is a fundamental problem for sticky-price models with a large degree of real rigidities.

inflation. First, one can relax the assumption that a representative firm holds stock, and consider a model in which a subsector of firms does not carry inventories, as in Jung and Yun (2005). Second, alternative specifications of the adjustment cost can be more useful. For example, Jung and Yun (2005) propose a cost that punishes deviations of the inventory-sales ratio from the target level.

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Figure 1. Benchmark vs. ToTEM

Consumption shock, LC_SHK

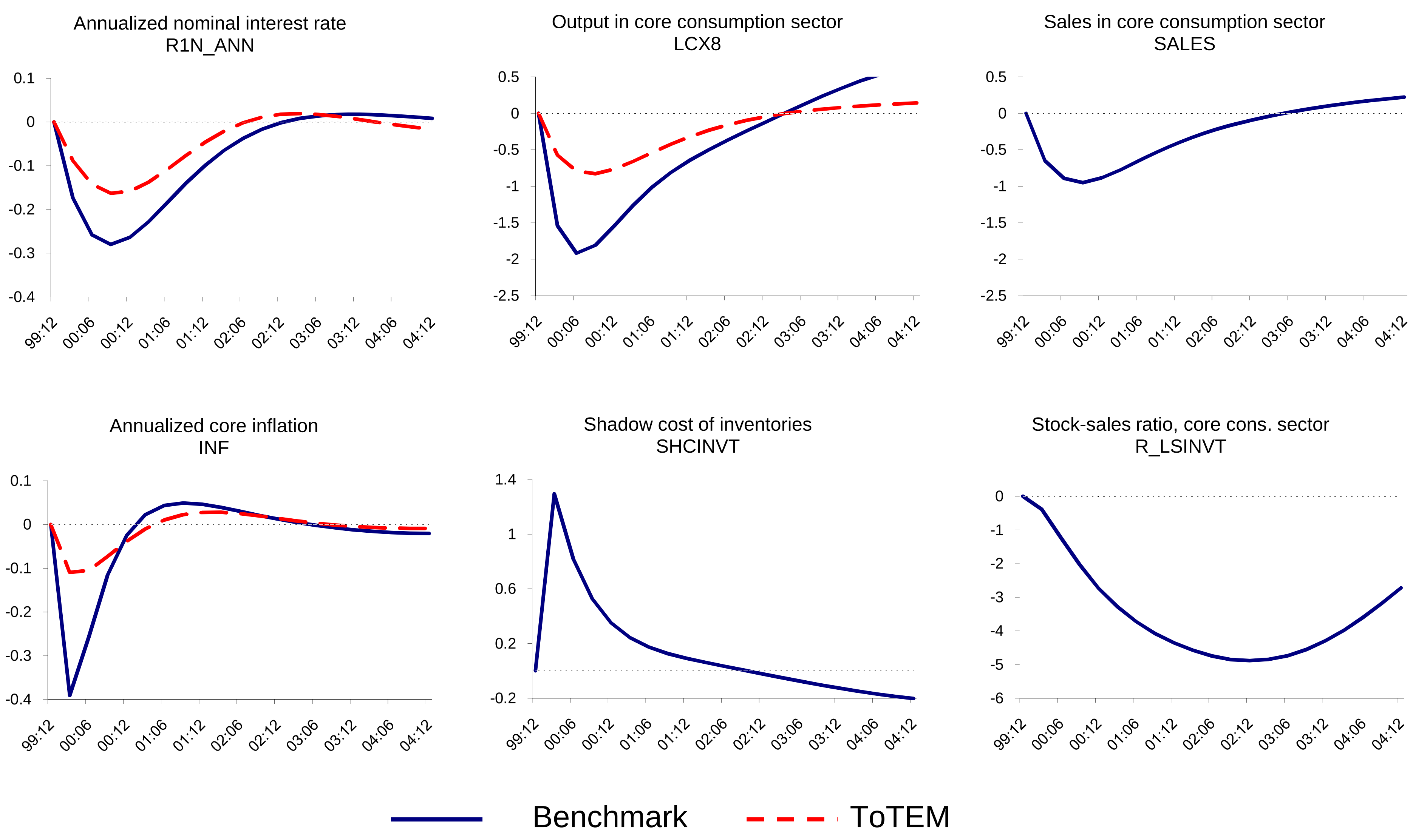

Figure 2. Benchmark vs. ToTEM

Policy shock, R1N_TRAN_SHK

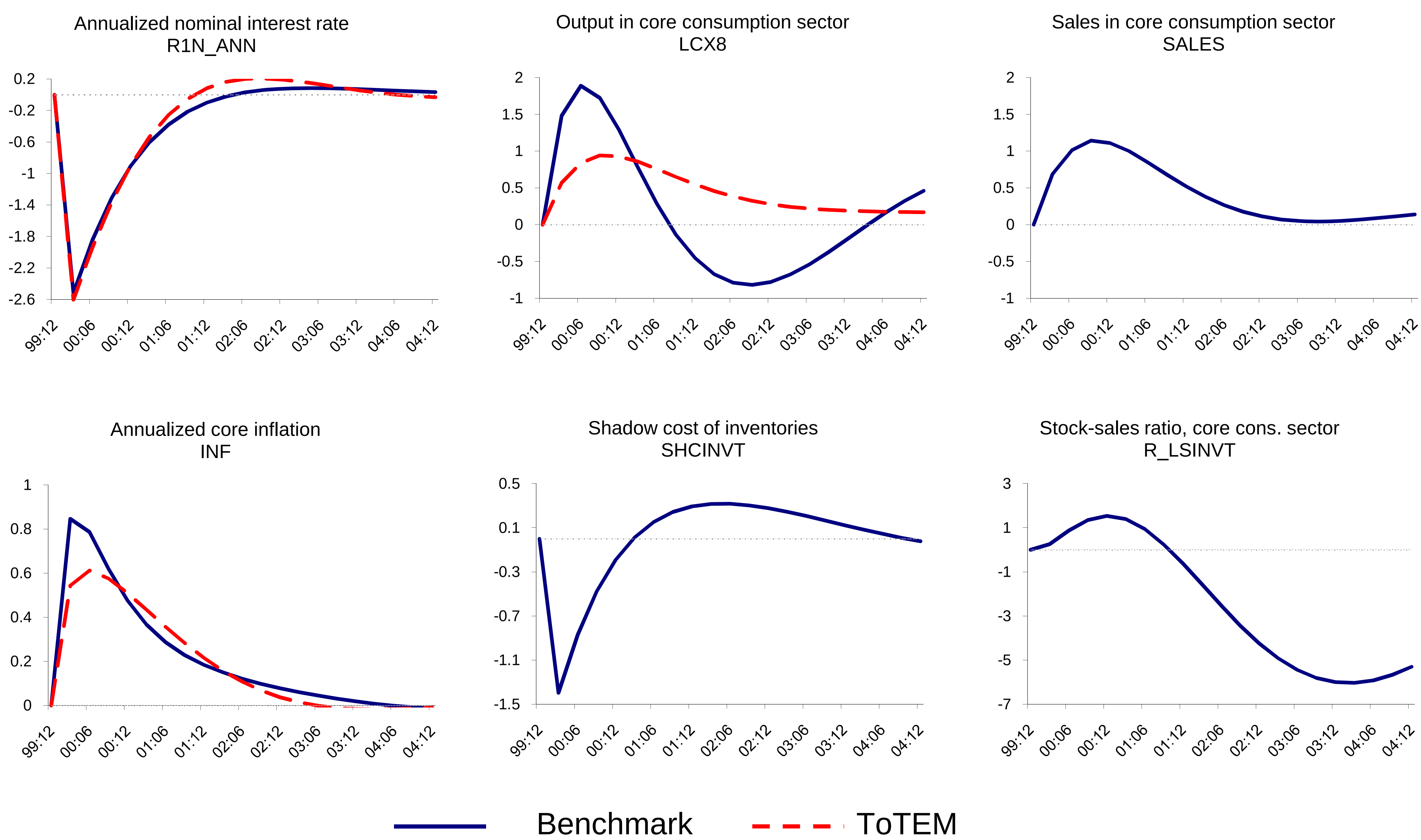

Figure 3. Benchmark vs. ToTEM

Productivity shock, LA_SHK

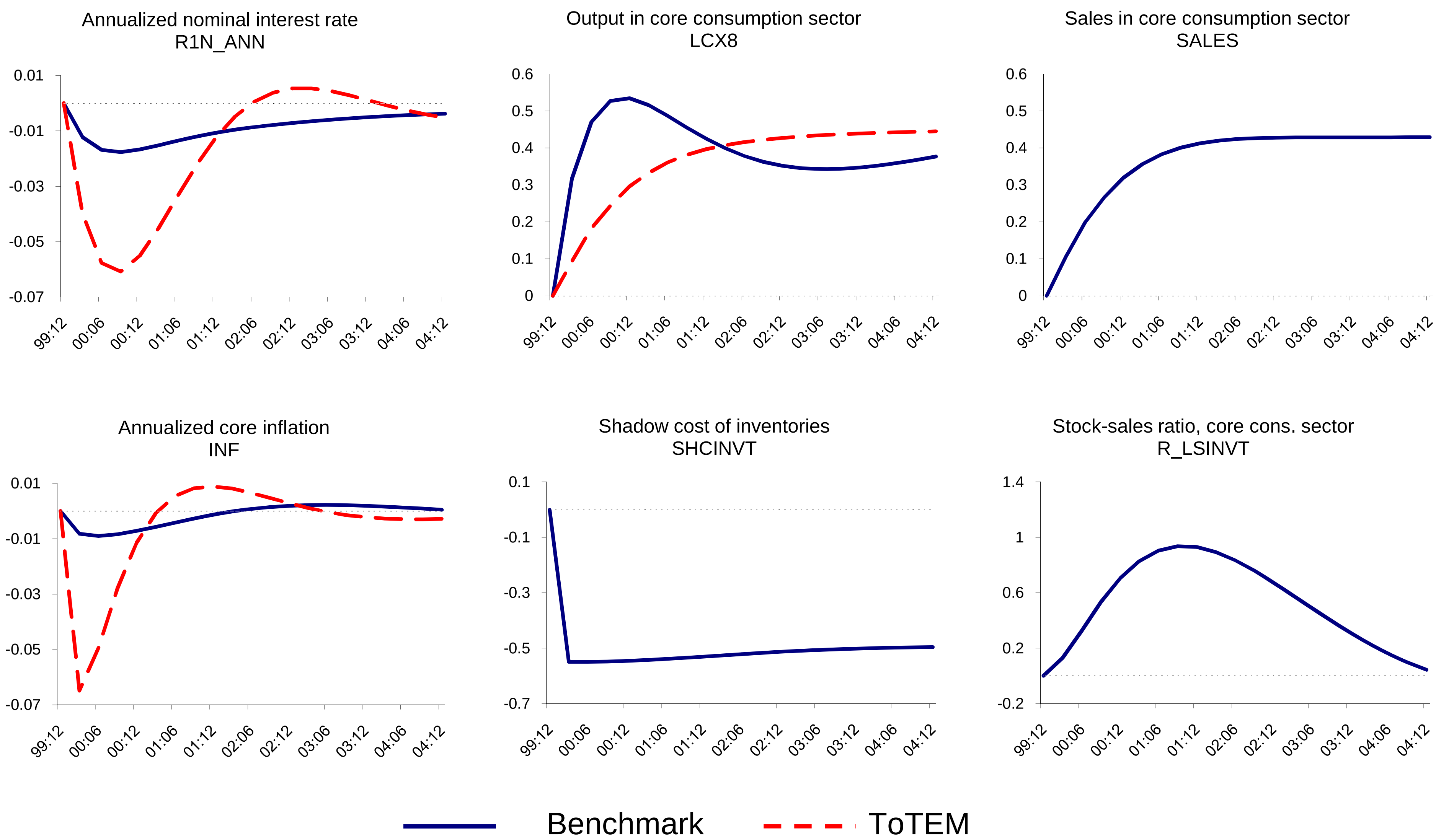

Figure 4. Higher depreciation rate

Productivity shock, LA_SHK

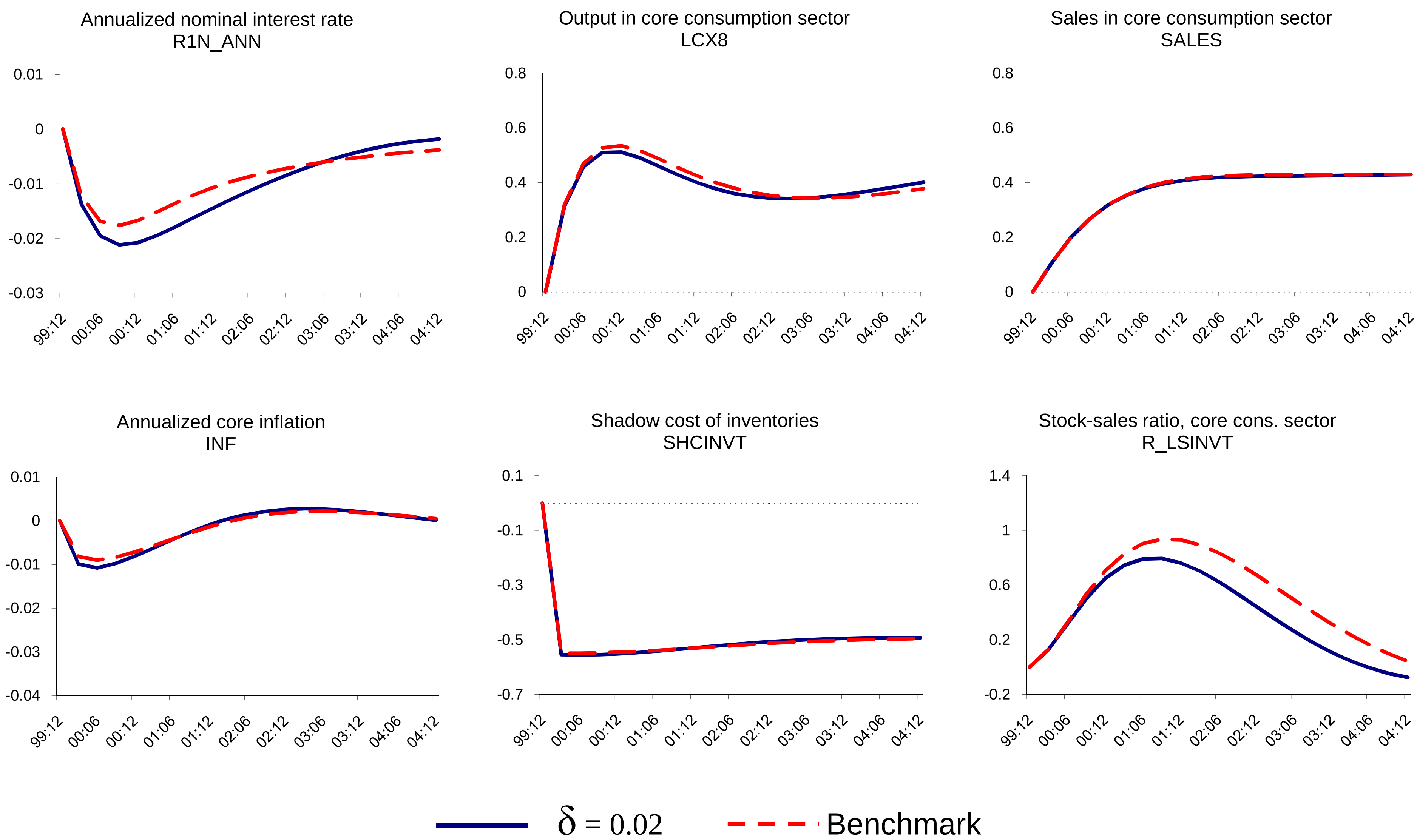

Figure 5. Smaller average stock-to-sales ratio

Productivity shock, LA_SHK

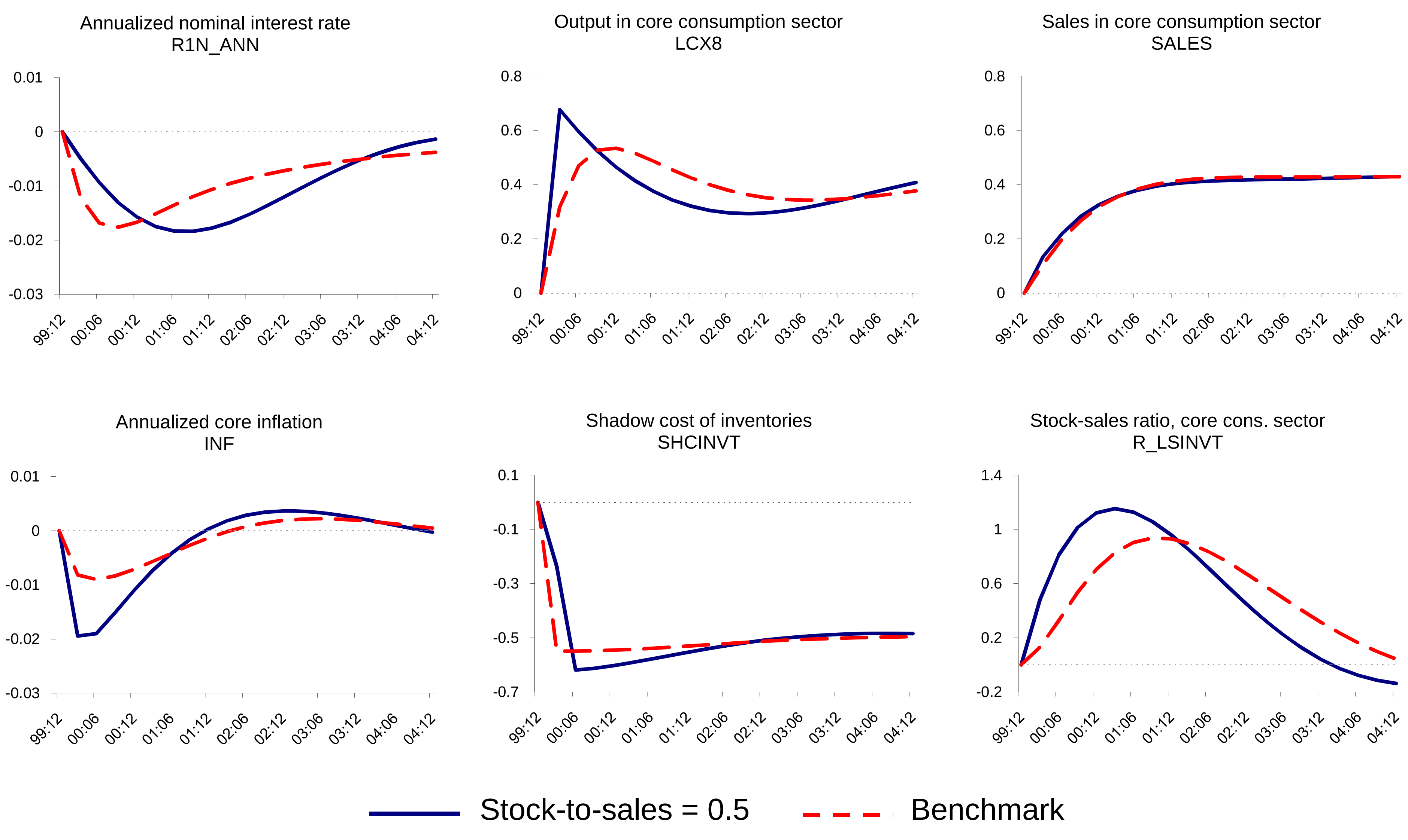

Figure 6. Effect of inventories on inflation

Productivity shock, LA_SHK

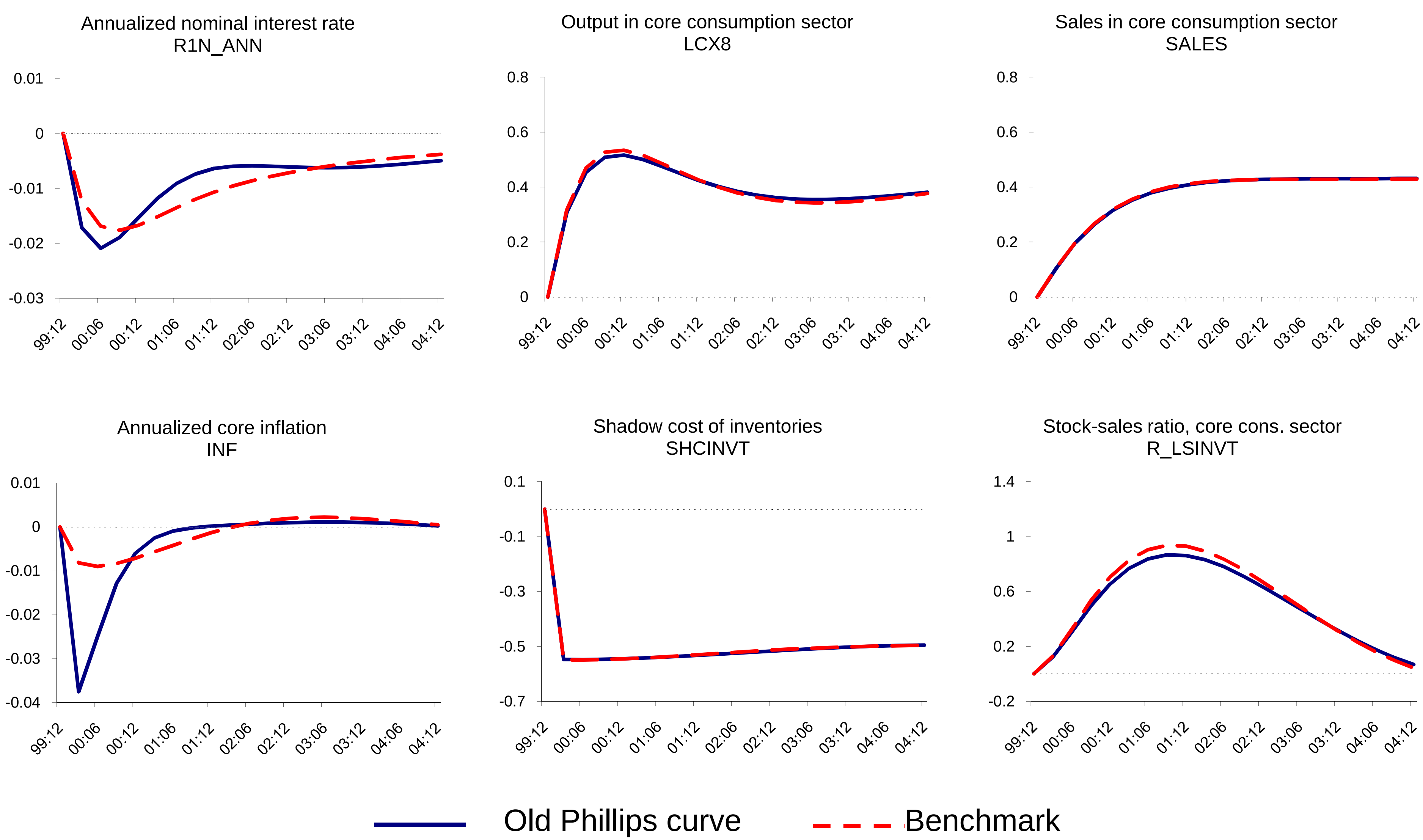

Figure 7. Effect of adjustment cost

Productivity shock, LA_SHK