Term Premium Dynamics and the Taylor Rule

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Motivation

- Empirical success of NA affine term-structure models.
  - Essentially vs. completely affine: Essentially more flexible.
  - Limited economic interpretations of these models.
- Ideal to link back to the macroeconomy.
  - Identify the latent state variables.
    - Macro aggregates.
    - Monetary policy state variables.
  - Determine the pricing kernel through g.e. restrictions.
  - Model monetary authority setting a short-term nominal rate,

\[ i_t^{(1)} = f(\text{macro variables}), \]

...imposes additional restrictions.
Questions

- Can we provide an economic interpretation in conjunction with an interest rate policy rule to an essentially affine model?

- What can we learn about term premiums when inflation is determined by an interest rate policy rule?

- Is monetary policy an important source of long-term interest rate variability?

- Can we learn about policy regimes from long-term rates?
Approach and Findings

Endowment economy with:
- preference shocks,
- an interest rate policy rule to pin down inflation,

Leads to an essentially affine equilibrium model for yields.

- The interest rate rule helps capture an upward-sloping yield curve, volatile long-term yields, & macroeconomic dynamics.

- Recent features of interest rates are consistent with a more aggressive response to inflation in monetary policy.
Related literature


Nominal Yields Across Maturity

Mean Nominal Yields

Std. Dev. Nominal Yields

Term Premium Dynamics and the Taylor Rule
Completely vs. Essentially Affine Models

- Completely affine pricing kernel:
  \[- \log M_{t+1} = \Gamma_0 + \Gamma_1^\top s_t + \lambda \Sigma(s_t)^{1/2} \varepsilon_{t+1}.\]

- Essentially affine pricing kernel:
  \[- \log M_{t+1} = \Gamma_0 + \Gamma_1^\top s_t + \frac{1}{2} \lambda(s_t)^\top \Sigma \lambda(s_t) + \lambda(s_t)^\top \Sigma^{1/2} \varepsilon_{t+1}\]
  with \(\lambda(s_t) = \lambda_0 + \lambda_1 s_t.\)

- Interest rates:
  \[e^{-n_i^{(n)} t} = \mathbb{E}_t [M_{t+n}] \implies i_t^{(n)} = \frac{1}{n} \left[ A_n + B_n^\top s_t \right].\]
Long Rate Volatility in Essentially Affine Models

\[
\frac{\sigma(i_t^{(n)})}{\sigma(i_t^{(1)})} = \frac{1 - \Phi^n}{n(1 - \Phi)}, \quad \Phi_\lambda = [\Phi - \Sigma \lambda_1].
\]

Φ: Autocorrelation of state variables.

λ₁: Price-of-risk sensitivity to state variables.
Essentially Affine Economic Model - Real Part

- **Utility:** \( \mathbb{E} \left[ \sum_{t=0}^{\infty} e^{-\delta t} \frac{c_{t}^{1-\gamma}}{1-\gamma} Q_t \right] \).

- **Consumption Growth** \((c \equiv \log C)\):
  \[
  \Delta c_{t+1} = (1 - \phi_c) \theta_c + \phi_c \Delta c_t + \sigma_c \varepsilon_{c,t+1}.
  \]

- **Preference Shock** \((q \equiv \log Q)\):
  \[
  -\Delta q_{t+1} = \frac{1}{2} \left( \eta_c \Delta c_t + \eta \nu_t \right)^2 \sigma_c^2 + (\eta_c \Delta c_t + \eta \nu_t) \sigma_c \varepsilon_{c,t+1}.
  \]

- **Essentially Affine Pricing Kernel:**
  \[
  -\log M_{t+1} = \delta + \gamma \Delta c_{t+1} - \Delta q_{t+1}.
  \]
Essentially Affine Economic Model - Nominal

Nominal Pricing Kernel:

\[ \log(M^S_{t+1}) = \log(M_{t+1}) - \pi_{t+1} \]

- Exogenous inflation - a benchmark:

\[ \pi_{t+1} = (1 - \phi_{\pi})\theta_{\pi} + \phi_{\pi}\pi_t + \sigma_{\pi}\varepsilon_{\pi,t+1}, \quad \varepsilon_{\pi,t+1} \perp \text{other shocks.} \]

\[ \Rightarrow i^{(n)}_t = A^S_n + B^S_{n,c} \Delta c_t + B^S_{n,\nu} \nu_t + B^S_{n,\pi} \pi_t. \]

- Endogenous inflation via a “Taylor Rule.”
Economic Model - Endogenous Inflation via “Taylor Rule”

Monetary policy sets the 1-period nominal yield:

\[ i_t = \bar{i} + \kappa_c \Delta c_t + \kappa_\pi \pi_t + u_t \]

with the “monetary policy shock” given by

\[ u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t}. \]

\( \pi_t \) must simultaneously satisfy:

1. the “Taylor Rule,”
2. the NA bond pricing equation.
Equilibrium Inflation Process: “Guess and Verify”

\[
\bar{r} + \nu_c \Delta c_t + \nu_\pi \left( \bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t \right) + u_t
\]

guess for \( \pi_t \)

\[
\log M_{t+1}^s = -\log E_t \left[ \exp \left\{ \log M_{t+1}^s - \left( \bar{\pi} + \pi_c \Delta c_{t+1} + \pi_\nu \nu_{t+1} + \pi_u u_{t+1} \right) \right\} \right]
\]

guess for \( \pi_{t+1} \)

\[
\pi_c = \frac{\gamma (\phi_c - \sigma_c^2 \eta_c) - \nu_c}{\nu_\pi - (\phi_c - \sigma_c^2 \eta_c)}, \quad \pi_\nu = -\frac{(\gamma + \pi_c) \sigma_c^2 \eta_\nu}{\nu_\pi - \phi_\nu}, \quad \pi_u = -\frac{1}{\nu_\pi - \phi_u}.
\]

\[
\Rightarrow i_t^{(n)} = A_n^s + B_{n,c}^s \Delta c_t + B_{n,\nu}^s \nu_t + B_{n,u}^s u_t.
\]
Prices of Risk

Shocks: $\varepsilon = (\varepsilon_c, \varepsilon_\nu, \varepsilon_u \text{ or } \varepsilon_\pi)$.

- Real

$$\lambda(s_t) = (\gamma + \eta_c \Delta c_t + \eta_\nu \nu_t, 0, 0)^\top.$$ 

- Nominal - exogenous $\pi$

$$\lambda^\$ (s_t) = \lambda(s_t) + (0, 0, 1)^\top.$$ 

- Nominal - endogenous $\pi_t = \bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t$

$$\lambda^\$ (s_t) = \lambda(s_t) + (\pi_c, \pi_\nu, \pi_u)^\top.$$
Inflation & Term Premiums Driven by Monetary Policy

\[ \mathbb{E}[i_t - r_t] = \ldots + \mathbb{E}[\text{cov}_t(\log M_{t+1}, \pi_{t+1})], \text{ where} \]
\[ \mathbb{E}[\text{cov}_t(\log M_{t+1}, \pi_{t+1})] = -\pi_c(\gamma + \eta_c \theta_c)\sigma_c^2 \]

\[ \mathbb{E}[i_t^{(2)} - i_t] = \ldots + \frac{1}{2} \mathbb{E}[\text{cov}_t(\log M_{t,t+1}^$, t+1], \text{ where} \]
\[ \mathbb{E}[\text{cov}_t(\log M_{t,t+1}^$, t+1)] = -(\gamma + \pi_c)(\gamma + \pi_c + \eta_c \theta_c)(\phi_c - \eta_c \sigma_c^2)\sigma_c^2 + (-) \text{ Term.} \]

- \[ \pi_c = \frac{\gamma(\phi_c - \sigma_c^2 \eta_c - \nu_c)}{\nu_c - (\phi_c - \sigma_c^2 \eta_c)} < 0 \text{ if} \]
  - A weak response to inflation or
  - A strong response to consumption growth.

- An upward sloping nominal curve is driven by \( \pi_c \).
Calibration

- Calibrate the exogenous & endogenous inflation models to quarterly U.S. data (1971:3 to 2005:4).
  - Zero coupon yields (3 months - 10 years).
  - Per capita consumption of nondurables & services.
- Both models calibrated to share the same real dynamics.
Calibration - Fitted Policy Rule Parameters

- Policy rule responds positively to consumption and inflation.
- Endogenous $\text{corr} (\Delta c_t, \pi_t) < 0$.
- Highly persistent policy shock captures long bond volatility.
Calibration - Fitted Preference Parameters

- Habit $\eta_c < 0$:
  - Upward-sloping yield curve,
  - Countercyclical price of risk.

- Taste shock $\nu_t$:
  - Short rate volatility through $\eta_\nu$,
  - Intermediate maturity volatilities through $\phi_\nu$.

- No external habit model interpretation though:
  - Affine-class restriction invokes tensions on parameters to achieve upward sloping yield curves.
  - Model does not deliver countercyclical real yields.
  - Model requires a taste shock to fit volatilities.
Highly autocorrelated policy shocks explain long rate volatility.

**Panel A**: Interest Rates – Avg. Level

**Panel B**: Interest Rates – Volatility

*: 1971-2005
Two Policy Experiments

Increase the reaction coefficients to (1) inflation & (2) consumption growth to match the average short-term rate (1987-2005).

Baseline:  
\[ i_t = -0.007 + 0.79 \Delta c_t + 1.68 \pi_t + u_t. \]

\[ \Delta \pi: \quad i_t = -0.007 + 0.79 \Delta c_t + 2.14 \pi_t + u_t. \]

\[ \Delta c: \quad i_t = -0.007 + 1.07 \Delta c_t + 1.68 \pi_t + u_t. \]
Two Policy Experiments

Increase the reaction coefficients to (1) inflation & (2) consumption growth to match the average short-term rate (1987-2005).

Baseline: \[ i_t = -0.007 + 0.79\Delta c_t + 1.68\pi_t + u_t. \]

\( \Delta i_{\pi} \): \[ i_t = -0.007 + 0.79\Delta c_t + 2.14\pi_t + u_t. \]

\( \Delta i_{c} \): \[ i_t = -0.007 + 1.07\Delta c_t + 1.68\pi_t + u_t. \]
Changes in the dynamics of inflation are consistent with a more aggressive reaction to inflation.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Policy Experiment</th>
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<tbody>
<tr>
<td></td>
<td>(1971-2005)</td>
<td>(1987-2005)</td>
</tr>
<tr>
<td>$\mathbb{E} [\Delta c_t] \times 4 , (%)$</td>
<td>1.98</td>
<td>1.83</td>
</tr>
<tr>
<td>$\mathbb{E} [\pi_t] \times 4 , (%)$</td>
<td>4.46</td>
<td>2.95</td>
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<tr>
<td>$\sigma (\Delta c_t) \times 4 , (%)$</td>
<td>1.74</td>
<td>1.35</td>
</tr>
<tr>
<td>$\sigma (\pi_t) \times 4 , (%)$</td>
<td>2.66</td>
<td>1.26</td>
</tr>
<tr>
<td>corr ($\Delta c_t, \Delta c_{t-1}$)</td>
<td>0.41</td>
<td>0.28</td>
</tr>
<tr>
<td>corr ($\pi_t, \pi_{t-1}$)</td>
<td>0.84</td>
<td>0.54</td>
</tr>
<tr>
<td>corr ($\Delta c_t, \pi_t$)</td>
<td>-0.33</td>
<td>-0.17</td>
</tr>
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Conclusions

- A policy rule aids a consumption-based bond pricing model.
  - Highly autocorrelated policy shocks needed.
  - Negative correlation between inflation & real activity.
  - Term structure information can help identify the policy regime.

Future Work:

- Role of endogenous inflation a general N.A. affine model.
  - Jointly capture real & nominal term structures.

- Source of the policy shock?
- Inflation & the real side of the economy?