

# Credit Exposure and Valuation of Revolving Credit Lines

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19 July 2009

**Preliminary**

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The revolving credit line—an arrangement under which customers may borrow and repay at will subject to a maximum outstanding—is the dominant form of bank commercial lending. The interest rate paid on borrowings, or the *spread* over some reference rate in the case of floating rate loans, is typically fixed for the term of the contract. This grants a valuable option to the borrower: As credit quality, and hence rate that would be paid on alternative borrowings, subsequently fluctuates, he can raise or lower borrowings on the line. The loan spread, likelihood of default, and line utilization from the perspective of *initial* credit quality may thus present a misleading picture of both loan profitability and life-of-contract credit exposure.

This paper examines credit lines using the tools of contingent claims analysis. It provides a broadly applicable, computationally tractable, arbitrage-free framework for defaultable securities when creditworthiness evolves independently of the security being valued. I.e., the security in question does not itself influence solvency of the issuer.

In the terminology of Duffie and Singleton (1997), we use a reduced-form rather than structural model. Credit quality is assumed to be a one-dimensional latent factor following a mixed jump-diffusion process. Parameters of the credit quality process can be chosen to represent a wide range of default models, including ratings transition, pure diffusion to barrier and Poisson jump to default. Model parameters are estimated by fitting to a limited set of US bond prices. These estimates form the basis for a range of numerical experiments on hypothetical credit lines. Readily computable are forward looking properties of the line of concern to lenders such as current fair market value (profitability), probability of default, expected default losses and sensitivity of market value to fluctuations in credit quality. Further, the role of contract features such as facility and origination fees, loan term and linking of loan spread to credit quality (borrower CDS rates) are examined.

Borrower behaviour is characterized as *quasi-rational*. It can range from sharp-pencilled, fully rational option exercise, for borrowers with liquid alternative capital market funding sources, to those with non-existent alternatives, or loan utilization dictated by business considerations outside the model.

We introduce a novel measure of credit exposure to render results meaningful to credit managers and regulators: Exposure under a revolving credit line can be expressed in terms of its *bond equivalent*. This is the principal amount of a constant balance floating rate loan, of same term as the credit line, which would have the same upfront cost of protection via credit default swap (CDS) as would a custom CDS compensating the bank for default losses under the line. Essentially, this constant balance loan and

the revolving credit line have the same expected present value of default losses under the risk-neutral valuation measure. For example, a 100mm 3 year credit facility, of which 20mm is initially drawn and with expected utilization rate of 40% over its life, might be characterized as having credit exposure equivalent to a 75mm fixed term loan. Such metrics are clearly relevant to both bank management and to regulators incorporating loan commitments into capital adequacy requirements.

Several features of credit lines become apparent. First, credit line value depends critically on borrower drawdown behaviour. Higher loan spreads are required for a lender to break even with borrowers who adjust drawdown and repayment according to their cost of borrowing elsewhere. Second, for borrowers with line utilization highly sensitive to alternative borrowing costs, there may not exist any finite spread sufficient for the lender to break even. Hence other fees such as originatin fees, facility fees, standby fees, or a combination of them are necessary. Third, value to the lender does not necessarily increase monotonically with loan spread. Higher spreads may decrease utilization enough to reduce loan value. Last, we find that the presence or absence of the jump component in credit quality dramatically influences bond equivalent credit exposure. Ignoring potential for jumps can lead to overvaluation of credit lines and understatement of risk.

Valuation and measurement of risk exposure of credit lines is relevant to both bank management and regulators incorporating loan commitments into capital adequacy requirements. The findings of this paper should prove useful to banks with a large proportion of loans in this format. The model provides a tool for banks to assess pricing policies and to evaluate credit exposure of RCLs for purpose of designing effective risk management strategies. Regulators might benefit from the framework to gain better understanding of the risk profile of RCLs and appropriate capital ratios. Current Bank for International Settlements (BIS) rules measures loan exposure for a commercial credit line as the currently drawn amount plus 50 percent of the undrawn amount (0 if original maturity 1 year or less). Such uniform treatment for RCLs does not accurately reflect their risk. Our model provides a more rigorous framework for its computation.<sup>1</sup>

The paper is organized as follows: Section 1 presents the model of credit quality evolution and default. As is, it is applicable not only revolving to revolvers but also to bonds, credit default swaps, CDS swaptions and other credit derivatives that might be written on a firm or used to hedge credit exposure to it. Section 2 describes the credit line contract, assumptions regarding borrower behaviour, and numerical solution for its value. Section 3 presents the various numerical experiments. Section 4 discusses background and related literature. Section 5 concludes.

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<sup>1</sup>Basel II regulations being phased in are somewhat more complex.

# I. The default process and security valuation

## 1. Credit quality evolution

Any operational framework for analyzing the full range of default and credit contingent claims should recognize certain empirical realities: (i) credit spreads on firm debt<sup>2</sup> are positive even at short maturities, (ii) credit spreads vary with term, (iii) credit spreads fluctuate over time, (iv) default can occur either without warning (jump) or be seen to be coming, (v) credit spreads can jump without default occurring.

With this in mind, let us treat credit quality as a scalar  $s(t)$  that follows a calendar time independent jump-diffusion process. As a Markov process, this provides computational tractability, facilitates treatment of securities with American option features, and allows room for a second factor that may influence contractual payouts or values (e.g., riskfree interest rates, variable entering swap or option contract, credit quality of counterparty, etc.). Default is associated with the credit state crossing an exogenously specified barrier, here  $s = 0$ . Thus

$$ds = \alpha dt + \sigma dz + \mu d\pi \tag{1}$$

in which  $z(t)$  is a standard Brownian motion,  $\pi(t)$  counts the jumps in a Poisson process with instantaneous intensity  $\lambda$ , independent of  $z$ ,  $\alpha$  is the diffusion drift,  $\sigma$  the diffusion volatility, and  $\mu$  the (random) jump size conditional upon a jump occurring.  $\alpha$ ,  $\sigma$ ,  $\lambda$  are permitted to be ‘nice’ functions of  $s$ . Jump sizes are specified by a conditional distribution function

$$F(x, x') \equiv \mathbf{prob}\{s(t^+) \leq x \mid s(t^-) = x', d\pi(t) = 1\} \tag{2}$$

with  $\mu$  otherwise independent of  $z$  and  $\pi$ .<sup>3</sup> That is,  $F$  is the cumulative probability distribution function for credit quality  $s$  immediately after a jump. The resulting default time—first  $t$  such that  $s(t) \leq 0$ —will be denoted by  $\tau$ . Default states are treated as absorbing: i.e.,  $ds(t) = 0$  for  $t > \tau$ .

## 2. Theoretical security values

Assume there is a continuously functioning, frictionless market in default-free bonds. Let  $r(t)$  denote the interest rate on instantaneously maturing bonds at time  $t$ , with the process for  $r$  being independent of  $s$ .

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<sup>2</sup>Difference between par coupon rate and default-free rates of same maturity

<sup>3</sup>Appropriate choice of the functions  $\alpha$ ,  $\sigma$ ,  $\lambda$ ,  $F$  allows representation of a wide range of reduced-form default models, such as ratings-based specifications ( $\alpha, \sigma = 0$ ,  $F$  a step function), pure diffusion to barrier ( $\lambda = 0$ ), jumps are always to default ( $F(x, x')$  steps up from 0 to 1 at  $x = 0 \ \forall x'$ ), fixed distribution of jump sizes ( $F(x + y, x' + y) = F(x, x') \ \forall x, x', y$ ), and so on.

Consider financial securities—or contracts between two parties—of the following form: There is a finite set  $\{t_i\}_{i=0,1,\dots}$  of contractual payment dates. Contract maturity  $T$  is the last payment date; contract origination  $t_0 = 0$  is the first. One party is designated the purchaser or lender, and treated as default free; the other, the seller or borrower, has credit quality evolve as in the previous section. Security valuation is from the perspective of the lender.

The contract specifies net payments to the lender on payment dates of  $q(s, r, t_i)$  if no prior default, where  $s, r$  are values at  $t_i$ . At each  $t$  there is also a contractual *recovery balance*  $B(s, r, t)$ . If default occurs at time  $\tau$ , the lender receives  $\rho B(s, r, \tau)$  at that time.  $\rho$  is a *recovery rate* assumed constant in the current application.<sup>4</sup> Initial loan advance, origination fees or purchase price are subsumed in the (possibly negative) origination date payment  $q(s, r, 0)$ .<sup>5</sup>

Consistent with equilibrium in the absence of arbitrage opportunities, we assume existence of a (risk-neutral) probability measure  $Q$  over realizations such that the time  $t$  fair market value of the contract, conditional on no prior default, is the expected discounted cash flows remaining:

$$V(s', r', t) = \mathbf{E}^Q \left[ \sum_{t \leq t_i < \tau} R(t_i) q(s, r, t_i) + i_{\tau < T} R(\tau) \rho B(s, r, \tau) \mid s(t) = s', r(t) = r', \tau > t \right] \quad (3)$$

where  $i_{\tau < T}$  is an indicator taking value 1 if  $\tau < T$ , otherwise 0, and

$$R(t_i) \equiv e^{-\int_t^{t_i} r(x) dx} \quad (4)$$

is the riskless rate discount factor along the realized path.

We treat  $r$  as deterministic for the experiments of the current paper. Discount factors  $R(t_i)$  can thus be calculated directly and consideration of the  $r$  state will be suppressed. Numerical solution for  $V(s, r, t)$  is obtained by recursively working back from contract maturity  $T$  in a discretized state space. Details of the procedure are provided in Appendix A.

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<sup>4</sup>Though  $B$  will be interpreted in what follows as the contractual loan balance, it could readily be the netted fair market value of a portfolio of derivatives in which the ‘borrower’ is the counterparty, payment received under a credit default swap based on the borrower, etc. Depending on the application, the  $s$  referred to might be  $s(\tau^+)$ , embodying say a dependence of recoveries on how far  $s$  jumped into a default region, or it might be  $s(\tau^-)$ , embodying the influence of prior credit state on borrower actions such as drawdowns on a credit line.

<sup>5</sup>Contractual payments  $q$  and recovery balance  $B$  will generally depend on *actions* taken by borrower or lender that are permissible under the contract—e.g., drawdown of a credit line or exercise of an option. These plausibly depend on the entire time path of  $r, s$  up to the date of such action(s). The current specification assumes that actions influencing contract payments and recovery balance at time  $t$  are functions of the then current  $s, r$  only, that the contingent actions of both parties are known to the lender, and that all this is already incorporated in functions  $q, B$ . That said, some dependence of contractual payments on prior dates’ states will be accommodated, reflecting for example the payment of interest in arrears on loans or swaps.

### 3. Process specification

After some experimentation with fit to bond price data (see below), and desiring to keep the number of parameters small, the following specification was adopted for the processes involved:

$$\begin{aligned}\alpha(s) &= \kappa(\bar{s} - s) \\ \sigma(s) &= \sigma \\ \lambda(s) &= \lambda_0 \max\left\{0, \frac{e^{(10-s)\delta} - 1}{e^{10\delta} - 1}\right\} \\ F(s, s') &= \begin{cases} 0 & s < a \\ (s - a)/(b - a) & s \in [a, b] \\ 1 & s > b \end{cases}\end{aligned}$$

That is, the diffusion volatility of credit quality  $s$  is constant; the diffusion drift is mean-reverting; the instantaneous jump intensity is an exponential function with maximum value  $\lambda_0$  at the default boundary, declining to 0 for  $s$  at 10 or above, with curvature parameter  $\delta$ ; and  $s$ -levels after a jump are uniformly distributed over a fixed interval  $[a, b]$ .<sup>6</sup>

For this specification there are thus eight time-invariant model parameters, including the recovery rate in default:  $\theta \equiv \{\kappa, \bar{s}, \sigma, \lambda_0, \delta, a, b, \rho\}$ .

### 4. Parameter estimation

We obtain estimates of the model parameters from US corporate bond prices. The data set available to us consisted of month-end bond price quotes, obtained from Interactive Data Corporation, for 737 US firms with publicly traded debt. Prices were available for the period May 1993 to December 1997. Even within this period, observations were missing much of the time for a majority of firms. Prices for two months, May 1996 and January 1997, were missing for all bonds. Altogether there were 114614 price observations.

From this, a subset of 105 firms was selected which (i) had price observations every month (except for May 96 and Jan 97 as noted above), and (ii) had multiple bond issues outstanding during the period.<sup>7</sup>

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<sup>6</sup>The choice of 10 as the instantaneously default-free credit quality can be viewed as just a scaling parameter given the rest of the specification. Regarding  $F$ , experiments were conducted with uniform and normally distributed relative and absolute jumps.  $\lambda(s)$  approaches linear as  $\delta \rightarrow 0$ .

<sup>7</sup>Since the default model gives an exact fit to any single market price quote on a given day by construction, firms with single debt issues were excluded as not legitimately testing the model.

Only non-callable, fixed rate bonds were considered. This left 40,955 price observations. The included bonds had agency credit ratings ranging from B to AAA, and maturities ranging from a few months to over 25 years. Average number of bonds outstanding per firm at month end was 8.

Although this subset has many thousands of price observations, certain of its aspects qualify any inference drawn. First, 1993-97 was a period of continuous economic growth. It misses the recessions of 1991, 2001 and 2008 and consequent evidence on the behavior of corporate bond prices in such circumstances. Closely related is the fact that no firms in the sample actually defaulted during this time period. Second, the data set ends before the dramatic mid-1998 rise in credit spreads, which has been characterized by some observers as more a drying-up of liquidity than a widespread deterioration in credit quality. Such phenomena are of obvious importance for both portfolio and single-issuer analysis, and inclusion of such periods could quantitatively affect conclusions.

Estimation was exclusively of the *risk-neutral process* parameters. I.e., those which, when used to generate a probability measure  $Q$  over realizations, make equation (3) a predictor of observed market prices. These are the parameters investors behave as if they believed in, were they actually risk-neutral; they typically differ from those describing the *objective process* of  $s$  over time, since investors are risk-averse and not all risks are diversifiable. Additionally, our abstract credit quality is not explicitly observable and must itself be inferred from observed bond prices.

The estimation procedure and resulting parameter estimates are described in Appendix B. These parameters give quite a good fit to the observed bond prices. Standard deviation of the *price* residuals is .70 (on par value of 100). Expressed as *yield to maturity* residuals, 53% of observations are below 5 basis points, 77% are below 10 bp, 95% are below 25 bp, 99% are below 50 bp.

## II. Revolving credit line

### 1. The loan contract

The loan contract grants the borrower for a fixed *term* the right—if no default to date—to borrow and repay at will up to fixed *amount*. We further specialize this as follows: There is a fixed *payment interval* associated with the loan (e.g., monthly) during which interest accrues. Drawdowns and repayments can only occur on these ‘payment dates’. Default, though it may be reached between such dates, will only be recognized as of these dates.

Interest accrues between payment dates at a fixed *spread* above the level of a default-free reference rate  $r_{\text{ref}}$  as of the beginning of the interval.<sup>8</sup> The reference rate is the then prevailing market rate (simple interest) on a default-free zero-coupon instrument of term *reference rate maturity* (e.g., 90 day LIBOR rate). Additionally, the balance due may accrue a *facility fee* payable on the full credit line amount, and/or a *standby fee* payable on the amount undrawn during the interval. The latter are expressed as (simple) interest rates per year. Finally, an up-front *origination fee* expressed as percent of maximum loan amount, may be charged.<sup>9</sup>

Contract parameters are thus: the fixed term  $T$ , maximum loan amount  $A$ , set of repayment/drawdown dates  $\{t_i\}$ , reference rate maturity  $T_{\text{ref}}$ , loan spread  $c$ , facility  $c_f$ , standby  $c_s$  and origination  $c_o$  fees. These parameters, together with drawdowns and the reference rate at the beginning of a payment interval, determine a contractual *balance owed* at the end of an interval. This is the basis for recoveries if default occurs within the interval; this is the amount assumed paid in full if default does not occur, to be then followed by a new drawdown.<sup>10</sup>

A relatively recent development in credit line contracts is *grid-pricing*, the linking of loan spread to either the agency credit rating or the credit default swap spread of the borrower. We incorporate one variant of the latter. The contractual rate at which loan interest accrues is assumed to take the form

$$r_{\text{con}} = r_{\text{ref}} + c + \beta \max\{0, r_{\text{opp}} - r_{\text{ref}} - c\} \quad (5)$$

I.e., if the borrower's current CDS spread  $r_{\text{opp}} - r_{\text{ref}}$  exceeds the base loan spread, then a proportion  $\beta$  of that excess is added to the current period loan rate. Note that although  $\beta$  would typically be between 0 and 1, setting it to infinity could be used to effectively suspend the credit line whenever CDS spread exceeds  $c$ . The role of such provision is to reduce the moral hazard associated with the borrower's drawdown option.

## 2. Borrower drawdown behaviour

There is little empirical evidence regarding drawdown behaviour on credit lines. Kaplan and Zingales (1997) find that the undrawn portion of credit lines decreases when firms are more liquidity constrained.

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<sup>8</sup>Fixed rate rather than fixed spread credit lines, typical for most credit cards, can be easily accommodated but are not done so here.

<sup>9</sup>Since any given standby fee can be re-expressed as an equivalent change in the facility fee and spread, it will be suppressed in the remaining sections of the paper.

<sup>10</sup>Realistically, of course, these two transactions occur simultaneously and show up as just a net change in drawdowns.



Gatev and Strahan (2006) report that drawdowns increase when the commercial paper–Tbill rate spread rises. Both studies suggest interest incentives at work. This motivates the following.

Let  $r_{\text{opp}}$  denote the fair rate (i.e., opportunity cost) at which the borrower could currently issue commercial paper for term of one payment interval, given his then-current credit quality. We assume that the proportion of the credit line drawn is an increasing function  $f(g)$  of the difference,  $g \equiv r_{\text{opp}} - r_{\text{con}}$ , between this alternative opportunity rate and the marginal cost of line borrowings. This means that when the contractual interest rate is not fully adjusted to reflect the deterioration of a borrower’s credit quality, i.e.  $\beta < 1$ , the closer is the borrower to default, the higher his utilization of the line. Moral selection—in the sense that line utilization is higher when credit quality is lower—is present.

Were borrower behaviour completely rational, he should use nothing from the credit line when  $g$  is negative, and draw the maximum  $g$  is positive. But in practice, firms seldom use credit lines this way. We wish to account for the fact that borrower behavior may be rational to varying degrees—or influenced by factors other than just immediate borrowing costs—by specifying a flexible functional form that includes full ‘rationality’ as a special case. Hence, we assume the drawdowns take the following form:

$$f(g) = d_{\min} + (d_{\max} - d_{\min}) N(d_{\text{sen}} g) \tag{6}$$

Here  $N(\cdot)$  is the standard normal distribution function, which varies between 0 and 1. Behavioural parameters  $d_{\min}$ ,  $d_{\max}$  and  $d_{\text{sen}}$  represent respectively minimum and maximum drawdowns and sensitivity to  $g$ . The amount borrowed for interval  $[t_i, t_{i+1}]$  is thus  $Af(g_{t_i})$ . Fully rational behavior in this model is characterized by  $d_{\text{sen}} = \infty$ . Constant borrowing is characterized by  $d_{\text{sen}} = 0$ . Although this is a deterministic relationship, note that for valuation purposes we may also interpret  $f(g)$  as *expected* drawdown, with the expectation taken over operational and cash-flow factors excluded in model as long as such factors are statistically independent of credit quality.

Because of the complications of credit quality jump and fluctuating line utilization, numerical methods must be used to determine loan value. This proceeds as described in Appendix A but with modification to accommodate the fact that drawdowns are determined by credit state at the start of each payment interval but re-payment is determined by credit state at the end. Appendix C details how this is done.

### III. Numerical experiments

We now explore credit line values and risk exposures. Credit exposure is quantified as the cost of a CDS that compensates the lender for default losses, and a bond-equivalent measure by converting that to a size of standard term loan with identical CDS cost. We also solve for the fair loan spread that makes initial loan value 0 to the lender. Of primary interest is how credit quality, loan spread, credit quality jumps and optionality of drawdowns affect these measures.

Our base case is a 3 year credit line with contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{\text{opp}} = T_{\text{ref}} = .0833$  yr); contractual spread over reference rate  $c = 2\%/yr$ ; and origination, facility and standby fees of 0. Borrower credit quality ranges between  $s_{\text{max}} = 15$  and  $s_{\text{min}} = 0$ . Drawdown behavior parameters  $d_{\text{min}} = 0, d_{\text{max}} = 1$  in all cases. Recovery rate in default is  $\rho = .5$ . The default-free term structure is assumed flat at 5%/yr continuously compounded.

Poisson jump intensity at the default boundary is set at  $\lambda_0 = 0.48/yr$ , as estimated from the bond data. However for some experiments we eliminate jumps by setting  $\lambda_0 = 0$ . This implies a pure diffusion-to-barrier model of default. This of course alters default probabilities for any given maturity. However, since it alters CDS values for both the credit line and the standard loan in similar fashion, the change in bond equivalent credit exposure isolates the significance of jump effects. We proceed through a variety of cases.

#### 1. Zero drawdown sensitivity

Consider first the case of  $d_{\text{sen}} = 0$ : Drawdown becomes fixed at 50% regardless of credit state or contractual spread. The borrower ignores the value of the credit line as an option against credit quality deterioration. The first two rows of Table 1 give, for initial credit state of  $s(0) = 6$  and various maturities, the probability of default (under the risk-neutral pricing measure) within the contract term and the fair contractual spread. Also given is the fair contractual spread for other drawdown sensitivities.

It is not surprising to see the fair loan spread and probability of default both positively related to loan term. The longer the loan term, the more likely that a default event will occur. Moreover, when starting credit quality is high, this probability rises more than in proportion to the term. Hence, higher loan spread is required to break even. The fair contractual spread also increases with drawdown sensitivity. Higher

drawdown sensitivity implies that the borrower’s utilization of the line varies more strongly with credit quality. The borrower is taking advantage of the optionality of credit lines—to the lender’s detriment. A higher contractual spread is needed to compensate.

Panel A of Table 2 reports loan values for different loan spread and initial credit quality combinations. For example, when loan spread is 4% and borrower has initial credit quality of 6, the loan is worth \$3.35 to the lender. Loan values are positively related to the credit spread and the credit quality. The intuition is straight-forward. When the loan pays a higher rate spread or is issued to higher quality borrowers, the loan—funded at the riskfree rate—offers higher expected net present value to the bank.

Panel B of Table 2 reports loan values assuming no credit quality jump. The loan values here are noticeably higher than those in Panel A. For example, suppressing sudden credit quality changes, a loan with contractual spread of 4% to a borrower with initial credit state of 6 is worth \$5.54, which is 65.4% higher than the value in Panel A. This difference illustrates that ignoring the possibility of sudden credit quality change can result in overvaluation of credit lines.

Table 3 reports two credit exposure measurements: bond-equivalent and CDS values. The bond-equivalent converts the credit line to a constant balance floating rate loan. Under the assumption of zero drawdown sensitivity, drawdown is constant at \$50. It is unsurprising that the bond-equivalent is also \$50. Panel B shows that CDS value decreases with an increase in borrower’s credit quality. Borrowers with higher credit quality have lower default probability, therefore it costs less to compensate the lender for default risk. Here the value of CDS does not change with the credit spread because of the assumption made that the CDS only compensates the borrower for the risk-free interest rate, not the full contractual interest rate. In unreported numerical experiments, we also find that CDS value lower under assumption of no jumps and higher for longer term loans.

## 2. High drawdown sensitivity

We next examine loan values and drawdown rates with extremely high drawdown sensitivity. Under this assumption, when the contract spread is below the opportunity loan spread, the borrower draws down 100% of the line. Otherwise, the borrower draws down nothing. In the numerical experiment, we set  $d_{sen} = 1000$  to approximate infinite drawdown sensitivity.

Panel A of Table 4 displays net value of the credit line for various initial credit states and contractual

spreads. One striking feature found is that the loan value is negative even with high loan spreads. Numerous other calculations, not reported due to the limitation of space, show that the line value stays negative even under spreads up to 100%. This is because highly rational drawdown behaviour creates a no-win situation for the lender: Borrowing only occurs—and the loan spread is only collected—in states where it is insufficient to compensate for the risk of default. In this case, loan origination and/or loan standby fees *must* be used for the loan to be a profitable investment for the bank. This explains the common practice of multiple-fee structures for credit lines reported by Shockley and Thakor (1997). Loan value is less negative the higher is  $c$  since that shrinks the set of credit states for which any borrowing occurs. Loan value is also less negative the higher is the borrower’s initial credit quality. Note that , loan values are lower than with zero drawdown sensitivity as reported in Table 2. For example, the loan value with a contractual credit spread of 4% to a borrower of credit state of 6 is worth -\$2.76, while the same loan under zero sensitivity is worth \$3.35. With high drawdown sensitivity, the borrowers are exercising the embedded credit quality option to the maximum extent—at the cost of the lenders—while with zero drawdown sensitivity the embedded option is not used at all.

Panel B of Table 4 present drawdown proportions as function of credit quality and loan spread. We see drawdown (almost) jumping between 0 and 100% as credit spread  $c$  crosses the fair credit spread level. Panel C of the Table reports line values with credit quality jumps suppressed. As in the last case, ignoring the possibility of jumps leads to overvaluation of loans. For example, the loan with contractual credit spread of 4% to a borrower of credit state of 6 is worth -\$0.03 instead of -\$2.76.

### 3. Intermediate drawdown sensitivity

Neither zero nor extreme high drawdown sensitivity is consistent with empirical evidence on loan drawdowns. The evidence (Saidenberg and Strahan (1999); Gatev and Strahan (2006)) suggests that credit line drawdowns are related both to interest rate spread and to other unrelated factors. This section considers the more realistic case of noisy, or quasi-rational, drawdown behaviour. We set  $d_{sen} = 5$  for this experiment. Panel A of Table 5 displays the net value of the credit line for various initial states  $s$  and spreads  $c$ .

Figure 2 graphs line values. Unlike the extremes considered so far, line value is no longer monotonic in contractual spread, as seen from the curved line value curve along the loan spread dimension. For borrowers with credit quality of 6, line value reaches a maximum value of \$0.86 at  $c$  of approximately 6%.

For borrowers with credit quality of 8, line value reaches a maximum value of \$2.76 at  $c$  of approximately 6%. Above these levels, the reduction in balances on which the spread is received more than outweighs the increase in spread and reduction in default losses. The reverse happens as spreads are reduced. Similar to the previous cases, line value is positively related to the borrower’s initial credit quality. The loan value in this case is lower than in the zero drawdown scenario, but higher than in the high drawdown case. Under an intermediate drawdown rate, the borrower only exercises part of the embedded option. Hence the loan presents a higher cost to the lender than the no optionality case (no sensitivity) and lower cost than the total optionality case (high sensitivity).

Panel B of Table 5 reports loan values suppressing jumps in credit quality. Similar to the last two cases, ignoring jumps leads to overvaluation.

Table 6 gives credit exposure measures and initial drawdown ratios of loans. Figure 3 gives some sense of how much drawdowns vary with loan spread and credit quality.

Figure 4 depicts the bond-equivalent exposure measure for different loan spread and borrower credit qualities. Noteworthy here is that the bond-equivalent measure is typically far larger than the initial drawdown. E.g., for a borrower with credit quality of 6 and a contractual spread of 4%, the bond-equivalent risk exposure is \$80.34, while the initial drawdown is only \$32.4. The CDS values in Panel B report that it would cost \$3.22 to compensate the lender for default risk for this specific borrower. An important lesson learned is that the credit exposure over the life of credit lines is far greater than the initial drawdown would suggest. This suggests that it would be inaccurate to use either the current drawdown or the full line limit as measure of the line’s credit exposure.

The drawdowns in Panel C confirm smaller line utilization the higher is either borrower credit quality or loan spread.

#### 4. Break-even loan spreads

This experiment investigates how break-even loan spreads relate to drawdown sensitivity. We get a different perspective from looking at the lowest contractual spread for which line value is 0. Panel A of Table 7 displays this spread for a range of  $d_{sen}$  values and initial credit qualities. For each credit quality, however, there is a size of  $d_{sen}$  beyond which there is no spread that can make the line break even for the lender. In other words, without origination, facility, standby fees, or a shortening of maturity, there

are no terms under which the lender can rationally grant the borrower a revolving credit line. This again explains why a multiple fee structure is typically used in practice.

Panel B reports the corresponding fair spreads when an origination fee  $c_u = \$2$  is added. Comparison of Panel A and B shows that the addition of origination fees permits a definitely negative value loan to become zero value. I.e., the non-existence of the fair credit spread problem is overcome by addition of the origination fee. For instance, when  $d_{sen} = 3$  and the borrower has credit quality of 4, without origination fee no fair credit spread exists. This indicates that the loan poses negative value to the lender no matter how high a spread is charged. However, with the addition of the origination fee, the fair credit spread becomes 6.21%. Confirmed by many unreported numerical experiments, this shows the complementarity of fees and also justifies the necessity of multiple fees.

Panel C of Table 7 reports credit line values for different drawdown sensitivities and credit quality combinations assuming a loan spread of 2%. Two interesting features are confirmed. First, loan value is negatively related to drawdown sensitivity. E.g., for a loan made to borrower with a credit quality of 6, loan value is \$-0.74 when  $d_{sen} = 1$  and \$-1.45 when  $d_{sen} = 11$ . Second, the loan values are negative when the credit spread is lower than the fair credit spread, and positive otherwise. For cases in which the fair credit spread is less than 2%, the lender is over-pricing the loan, hence the loan delivers positive value. For cases in which fair credit spread is more than 2%, the lender is under-pricing the loan, and the loan delivers negative value. For example, the fair credit spreads with a drawdown sensitivity of 3 (Panel A) are 2.88% for credit quality of 6 and 0.91% for credit quality of 8. In Panel B for the same drawdown sensitivity of 3, the credit line made to borrowers with credit quality of 6 is under-priced and has negative loan values of \$-0.91. The loan made to borrowers with credit quality of 8 are over-priced and has a positive loan value of \$1.27.

#### IV. Conclusion

Revolving lines of credit are the most important channel for banks to provide business loans. We propose a broadly applicable, computationally tractable, credit line valuation framework, that considers both gradual and sudden change in credit quality. Through numerical examples, we illustrate important characteristics of credit line pricing. First, credit line value depends critically on drawdown sensitivity. The higher the sensitivity, the more costly the loan. Second, the break-even spread increases with drawdown sensitivity. For each credit quality, however, there is a level of drawdown sensitivity beyond

which there is no spread that can make the line break even for the lender. Hence multiple fees may be required for a mutually beneficial contract to be reached. Third, loan value does not necessarily monotonically increase with loan spread. A higher credit spread can reduce credit line value due to reduction in line usage. Lastly, it can be costly to the lender to ignore the possibility of a sudden credit quality change.

The setup of the model is sufficiently flexible to allow it to reflect different fee structures and loan characteristics. Parameters of the model can be calibrated using empirical bond pricing data.

Our model also produces two important measures for a revolving credit line's exposure to default risk. The first measure is the line's bond-equivalent, which expresses the credit exposure as the principal amount of constant balance floating rate loan of the same term with identical credit default swap cost. This measure is lower than the loan limit for credit lines, but typically much higher than current draw-downs. This measure more accurately reflects the risk exposure of credit lines over the life of the loan. It generally lay between 70 and 100% of the loan limit. Second, we calculate credit exposure using the up-front cost of an associated CDS. Though very small for borrowers of high quality, it rapidly escalates with declines in credit quality. This provides an exposure measure from a different perspective as well as providing useful information for financial professionals to price such derivative products.

## Appendix A: Numerical solution for security values

We obtain an approximate numerical solution for  $V(s, r, t)$  by recursively working back from contract maturity  $T$  in a discretized state space. We will treat  $r$  as deterministic in the current paper. Discount factors  $R(t_i)$  can thus be calculated directly and consideration of the  $r$  state is suppressed. We solve for  $V$  on a uniformly spaced rectangular grid on  $[0, T] \times [s_{\min}, s_{\max}]$  with mesh size  $h$  in the  $s$  direction and  $k$  in the  $t$  direction, chosen so that payment dates lie at gridpoints. Since we also take recoveries to be independent of how negative is  $s$ , we let  $s_{\min} = 0$  correspond to the default barrier.  $s_{\max}$  is chosen to be a large enough that exceeding it from initial  $s(0)$  of interest within time  $T$  is sufficiently unlikely.<sup>11</sup> Let  $V_{ij}$ ,  $i = 0 \dots I$ ,  $j = 0 \dots J$  be the solution at credit quality step  $i$  up from the default barrier and time step  $j$  away from origination. Vector  $V_j$  denotes the solution at time step  $j$ . Its length is the number of states considered in the  $s$ -direction.

The maturity value  $V(s, r, T)$ , conditional no prior default, is assumed known from the contract terms and used to fill in values in the terminal vector  $V_J$ . We work backward one timestep  $k$  by treating the jump and diffusion changes in  $s$  as occurring in sequence: I.e., within each interval all changes in  $s$  from diffusion over time  $k$  occur first, with no jumps; then all jumps in  $s$  over time  $k$  occur, with no diffusion.

The diffusion part is handled by applying the Crank-Nicholson finite difference method to a the vector of  $s$ -contingent values, just as if the jump process were not present, with boundary condition at  $s_{\min}$  imposed by recovery value  $\rho B$ . The partial differential equation being solved is what one gets from the Feynman-Kac formula for conditional expected value,

$$\frac{1}{2}\sigma^2 V_{ss} + \alpha V_s + V_t = 0 \tag{7}$$

At the upper boundary we use the (technically incorrect) expedient of assuming  $V$  to be quadratic in  $s$ . I.e., the value of  $V_j$  at  $s_{\max}$  is the quadratic extrapolation of its values at the three neighbouring interior gridpoints. In certain cases, if the value of the security were it default-free is known, it works better to suppose that default is effectively inaccessible from  $s_{\max}$  and impose that as a known-value condition at that edge. The result of this step is to multiply vector  $V_j$  by some matrix  $C$ .

The jump part is handled by constructing a transition matrix  $M$  over  $s$ -states approximating the changes from this source over interval  $k$ . This matrix is pre-calculated as follows. Choose a modest sized integer  $n$ , and approximate the Poisson process over a subinterval of length  $k/2^n$  by supposing at most

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<sup>11</sup>This obviously will depend on the particulars of the processes involved and the relative magnitude of cash flows in different regions of the state space.



one jump occurs in that subinterval. From a particular level  $s_i$  on the grid, the probability of one jump is  $\lambda(s_i)k/2^n$ . If that occurs, the probability of moving to level  $s_l$  is taken to be  $F(s_l + \frac{1}{2}h, s_i) - F(s_l - \frac{1}{2}h, s_i)$ . I.e., it is assumed that jumps are exclusively to gridpoints. The subinterval jump transition probabilities are thus<sup>12</sup>

$$\mathbf{prob}\{s_l|s_i\} = \begin{cases} 1 - (1 - F(s_i + \frac{1}{2}h, s_i) + F(s_i - \frac{1}{2}h, s_i))\lambda(s_i)k/2^n & \text{for } l = i \\ (F(s_l + \frac{1}{2}h, s_i) - F(s_l - \frac{1}{2}h, s_i))\lambda(s_i)k/2^n & \text{for } l \neq i \end{cases} \quad (8)$$

The resulting matrix is then squared  $n$  times to get the transition matrix  $M$  (which allows for up to  $n$  jumps) over interval  $k$ .<sup>13</sup>

Discounting by the default-free rate over time interval  $[(j-1)k, jk]$  is accomplished by multiplying the result (for the non-default  $s$  levels) by the  $s$ -independent quantity

$$d_j \equiv e^{-\int_{(j-1)k}^{jk} r(x) dx}$$

We thus get

$$V_{j-1} = d_j C M V_j \quad (9)$$

In practice, computation costs are reduced by treating as 0 elements of  $M$  below some threshold (e.g.,  $10^{-10}$ ), storing the rescaled result as a sparse matrix, and using a sparse matrix multiplication routine. The ‘multiplication by  $C$ ’ is done by solving the tridiagonal equation system that comes out of the Crank-Nicolson algorithm. Computation costs per time step are thus typically much less than a single matrix multiplication of  $V_j$ .<sup>14</sup>

One proceeds thus back from time  $T$ , pausing at each payment date  $t_i$  to add to each element of  $V_j$  the  $s$ -contingent contractual payment received by the lender in non-default states. Security values for  $s$  not at gridpoints is found by (cubic) interpolation on  $V_0$ .

## Appendix B: Parameter estimation

A fixed rate non-callable bond is a simple contract. In valuation equation (3),  $\{t_i\}$  are the coupon payment dates remaining,  $q(s, r, t_i)$  equal the fixed coupon payments (plus maturity value at  $T$ ), and  $B$

<sup>12</sup>Jumps to below  $s_{\min}$  are treated as being to  $s_{\min}$  and to above  $s_{\max}$  as being to  $s_{\max}$ .

<sup>13</sup>The alternative of allowing for arbitrarily large number of jumps in interval  $k$  would require calculating the eigenvalues and eigenvectors of the  $I \times I$  matrix of infinitesimal transition probabilities, which would be computationally more costly.

<sup>14</sup>We are losing something by ignoring the interaction of jump and diffusion within the interval when the components of each depends on  $s$ . A more ‘centered’ approximation over interval  $k$  can be achieved by doing  $M^{1/2}CM^{1/2}$  rather than  $CM$ .

is the par value plus (linearly) accrued interest at default time  $\tau$ . For given model parameters  $\theta$  and credit state for firm  $i$  at month  $j$  of  $s_{ij}$ , let  $V_{ijk}$  denote the (numerically computed) theoretical value of bond  $k$  of that firm.<sup>15</sup> Let  $P_{ijk}$  denote the corresponding actual market price quote (plus accrued interest to transaction settlement date). Let  $d_{ijk}$  denote the Hicks/Macauly duration of bond  $ijk$  (sensitivity of value to its own annualized yield to maturity). The parameter estimation criterion was to find  $\theta, \{s_{ij}\}$  minimizing the weighted sum-squared-residuals

$$S \equiv \sum_{i,j,k} \left( \frac{P_{ijk} - V_{ijk}}{d_{ijk}} \right)^2 \quad (10)$$

subject to the constraints

$$\sum_k \frac{P_{ijk} - V_{ijk}}{d_{ijk}} = 0 \quad \forall i, j \quad (11)$$

The deflating of price residuals by duration expresses the residuals (approximately) in terms of difference between quoted yield to maturity and the model’s forecast; if not done, the short end of the maturity spectrum would have little influence. Constraints (11) assert that market and model yields, averaged over all bonds of a given firm outstanding on a given day, agree exactly. This identifies the credit state, and renders  $s_{ij}$ , conditional on  $\theta$ , observable without error—one way of handling the latent variable problem were we to bring time series properties of  $s$  into the estimation.<sup>16</sup> Minimization of  $S$  over  $\theta$  was done using Marquardt’s algorithm, extended to incorporate bounds on parameter values to make the search better behaved.

Notice that we make no use of actual time series properties of bond prices here. Data dates could be shuffled with no effect. The basic question is whether a small set of fixed parameters—common to all firms and all points in time—combined with a *single* firm-and-date-specific variable  $s$  can reasonably capture all ‘term structures of credit spreads’ observed. Equivalently, since it is only the risk-neutral density of time to default that matters for the bond values, can the *one-dimensional* family of density functions implied by the specification adequately explain observed prices?

For the data set available, within the constraints of the specification chosen, not all parameters were well identified. To cut the story short (and the computational intensity), we imposed the following: recovery rate  $\rho = .5$ , roughly consistent with Altman and Kishore’s (1996) reported average for US

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<sup>15</sup>Also needed for this calculation is that date’s default-free yield curve, taken from the constant maturity US Treasury yield curve as mentioned earlier.

<sup>16</sup>See Honore (1998) and Jones and Wang (1996) for previous use of this method.

corporate bonds;<sup>17</sup>  $\kappa = 0$ , so no mean reversion;  $\sigma = 1.0$  (per year), viewed as an almost free scaling parameter. The four remaining parameters were estimated with the following results:

$$\begin{array}{rcl}
 \lambda_0 & .48 & \kappa & 0.0 \\
 \delta & .38 & \bar{s} & \text{—} \\
 \bar{s}_a & 1.10 & \sigma & 1.0 \\
 \sigma_a & .80 & \rho & 0.5
 \end{array} \tag{12}$$

Here  $\bar{s}_a$  and  $\sigma_a$  are the mean and standard deviation of the uniform distribution on absolute jump destination. I.e.,  $F$  is uniform on the interval  $[\bar{s}_a - 3^{1/2}\sigma_a, \bar{s}_a + 3^{1/2}\sigma_a] = [-.286, 2.49]$ .

These parameters give quite a good fit to the observed bond prices. Standard deviation of the *price* residuals is .70 (on par value of 100). Expressed as *yield* to maturity residuals, 53% of observations are below 5 basis points, 77% are below 10 bp, 95% are below 25 bp, 99% are below 50 bp.

The credit state  $s(t)$  can be given more tangible interpretation in terms of the fair credit spread on instantaneously maturing loans. Very short term loans can only default through jump. Expected default losses over an interval of length  $dt$  are thus  $(1 - \rho)\lambda(s)F(0, s) dt$ , the product of the loss rate in default, probability of a jump occurring, and probability that the jump would land in the default region.

Fair credit spreads for longer maturities must allow for diffusion of  $s$  both to default directly and to levels with different jump intensities. They are typically much larger than the instantaneous spread above. For currently high quality borrowers, credit spread increases with maturity; for low quality borrowers, after an initial hump occurring somewhere before two years, credit spread declines with maturity.

Time series analysis of the estimated  $s_{ij}$  showed them to have small drift (.072/yr), slight negative first order serial correlation (-.067 for monthly observations), and diffusion volatility of  $\sigma$  of 0.57. The latter was noticeably below the value of 1.0 assumed, so  $\sigma$  was reset to 0.75 and the parameters re-estimated, with better consistency between the assumed and realized  $\sigma$ .

## Appendix C: Numerical solution for line value

Valuation of the credit line proceeds as in section IV. with some modification. We must accomodate the fact that the size of drawdown is determined by  $s$  at the beginning of a payment interval, but whether one gets paid is determined by  $s$  at the end. Moreover the interest rate charged over the interval is

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<sup>17</sup>They report mean recovery rate for senior secured notes of 57.94%, with standard deviation of 23.12%; and mean recovery rate for senior unsecured notes of 47.70%, with standard deviation of 26.60%.

based on  $r_{\text{ref}}$  prevailing at the start. Thus cash flows at  $t_i$  are determined by the  $r, s$  state at both  $t_i$  and  $t_{i-1}$  jointly.<sup>18</sup> This is handled as follows. Between payment dates, the vector  $V_j$  recursively solved for is the value of the credit line's cash flows from the *next* payment date onwards, as a function of current  $s$ , conditional on no default prior to that time. If  $s = 0$  currently, then all future cash flows will be zero, captured by imposing boundary condition  $V(0, r, t) = 0$  for  $t$  not in  $\{t_i\}$ . Let  $\Delta \equiv t_{i+1} - t_i$  be the payment interval. If  $t$  is a payment date  $t_i$ , then for each  $s$  in our discretized state space we calculate  $r_{\text{ref}}, r_{\text{opp}}, \text{gap } g$ , and the fair value  $p$  of a unit discount bond of maturity  $\Delta$  issued by the borrower with the same recovery rate as the credit line. We add to  $V_j(s)$  the amount

$$-Af(g) + pA(f(g)(1 + (r_{\text{ref}} + c - c_s)\Delta) + (c_s + c_f)\Delta) \quad (13)$$

First term is the cash paid out as current drawdown; second term is expected<sup>Q</sup> discounted value of the contractual payments now due at  $t_{i+1}$ . At  $s = 0$  this treats the borrower as drawing down, defaulting right away, and providing immediate recoveries a proportion of the balance due at the *next* payment date. For equal length payment intervals the vector of risky bond values  $p(s)$ , up to a riskless rate discount factor independent of  $s$ , need only be computed once.

Solution for the breakeven loan spread is done by iterative search (using secant method) over values of  $c$  until spread giving 0 loan value is found.

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<sup>18</sup>The payment of interest in arrears is not a problem in the current deterministic interest rate setting, but would become so if the default-free rate were treated as stochastic.

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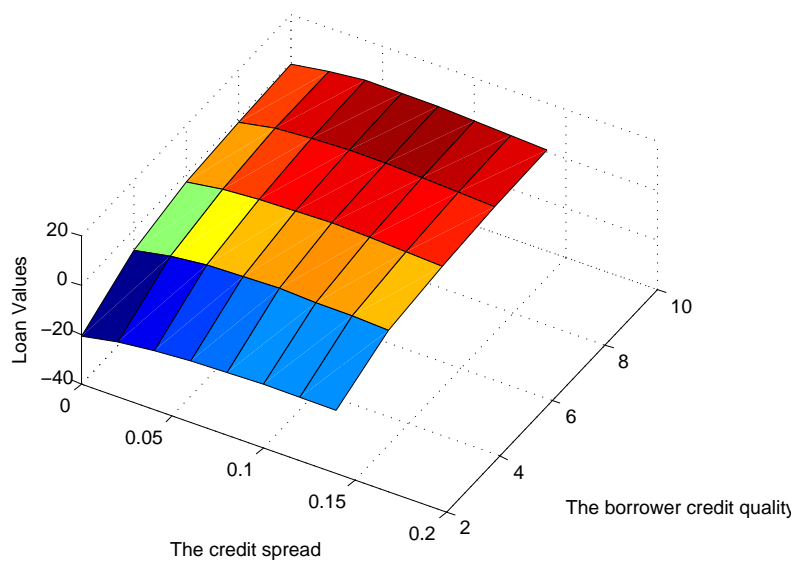


Figure 1: This figure depicts the credit line values assuming an intermediate drawdown rate. The credit line value first increases with the rising of the credit spread, and then decreases after the value reaches a maximum.

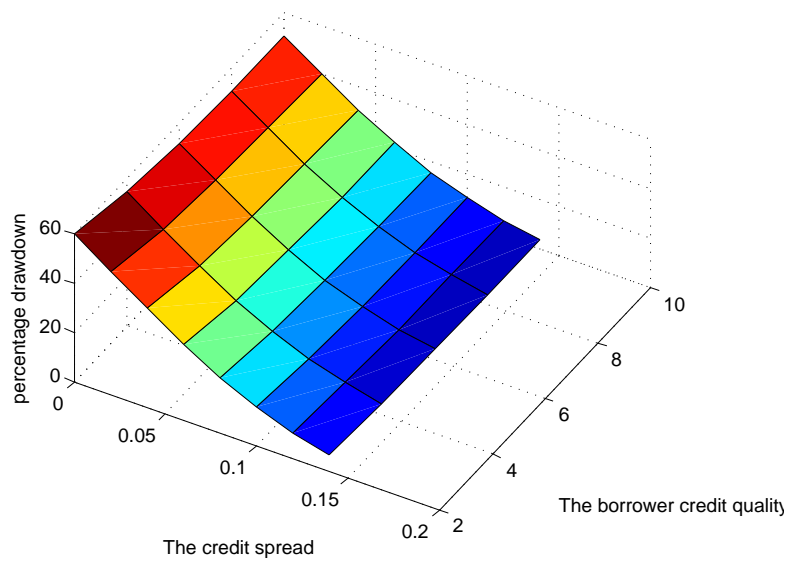


Figure 2: This figure depicts the credit line drawdown proportion assuming an intermediate drawdown rate. The drawdown proportion is negatively related to the credit spread of the credit lines as well as the borrower's initial credit quality.



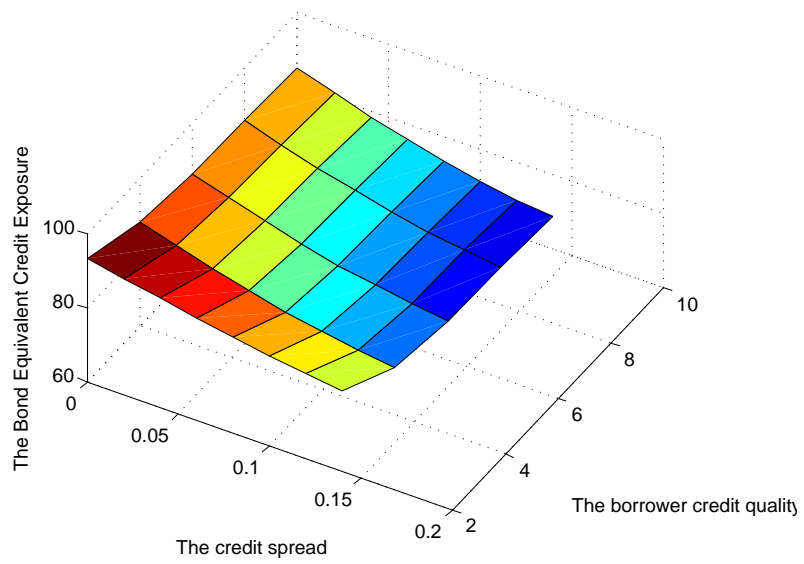


Figure 3: This figure depicts the credit line bond-equivalent risk exposure assuming an intermediate drawdown rate. The bond-equivalent, similar to the initial drawdown, is negatively related to the credit spread of the credit lines as well as t

Table 1: Term structure of credit spreads

	1 mo.	6 mo.	1 yr.	2 yr.	3 yr.	5 yr.	10 yr.	20 yr.
prob. default (%)	0.06	0.51	1.26	3.24	5.7	11.66	27.81	50.32
$d_{sen}=0$	0.36	0.86	1.06	1.31	1.50	1.77	2.08	2.12
$d_{sen}=1$	0.36	1.36	1.75	2.24	2.59	3.14	3.78	3.87
$d_{sen}=2$	0.36	1.38	1.80	2.33	2.73	3.35	4.12	4.23
$d_{sen}=3$	0.36	1.41	1.85	2.44	2.88	3.61	4.59	4.75
$d_{sen}=4$	0.36	1.44	1.91	2.56	3.08	4.01	5.52	5.88

Note: The table reports the fair contractual spread and default probabilities for credit lines of various maturities and drawdown sensitivities. The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded. The initial credit state of the borrower  $S(0) = 6$ .

Table 2: Line Value with Zero DrawDown Sensitivity

$c$	2%	4%	6%	8%	10%	12%	14%
$s$	A: Line Values With Credit Quality Jump (\$)						
2	-8.98	-6.86	-4.73	-2.61	-0.49	1.64	3.76
4	-2.12	0.42	2.96	5.5	8.04	10.57	13.11
6	0.67	3.35	6.03	8.71	11.39	14.06	16.74
8	2.04	4.78	7.53	10.27	13.02	15.76	18.72
10	2.66	5.43	8.21	10.98	13.76	16.53	19.31
	B: Line Values Without Credit Quality Jump (\$)						
2	-4.46	-2.04	0.38	2.8	5.22	7.64	10.06
4	2.15	4.92	7.68	10.45	13.21	15.97	18.74
6	2.76	5.54	8.32	11.1	13.88	16.66	19.44
8	2.78	5.56	8.34	11.12	13.9	16.68	19.46
10	2.78	5.56	8.34	11.12	13.9	16.68	19.46

Note: The table reports credit line values calculated numerically under different credit spreads  $c$  and the borrower's initial credit quality  $s$ , assuming zero drawdown sensitivity. The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded.

Table 3: Credit exposures: Bond-equivalent and CDS value ( $d_{sen} = 0$ )

$c$	2%	4%	6%	8%	10%	12%	14%
$s$	A: Bond Equivalent (\$)						
2	50	50	50	50	50	50	50
4	50	50	50	50	50	50	50
6	50	50	50	50	50	50	50
8	50	50	50	50	50	50	50
10	50	50	50	50	50	50	50
	B: Value of credit default swap (\$)						
2	11.1	11.1	11.1	11.1	11.1	11.1	11.1
4	4.66	4.66	4.66	4.66	4.66	4.66	4.66
6	2	2	2	2	2	2	2
8	0.71	0.71	0.71	0.71	0.71	0.71	0.71
10	0.12	0.12	0.12	0.12	0.12	0.12	0.12

Note: The table reports the bond-equivalent and the cost of credit swap for credit lines with different credit spreads  $c$  and the borrower's initial credit quality  $s$ , assuming zero drawdown sensitivity. The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded.

Table 4: Line Value with High DrawDown Sensitivity

$c$	2%	4%	6%	8%	10%	12%	14%
$s$	A: Line Values With Credit Quality Jump (\$)						
2	-18.58	-17.86	-17.51	-17.24	-16.96	-16.69	-16.42
4	-6.82	-6.64	-6.55	-6.47	-6.39	-6.31	-6.23
6	-2.82	-2.76	-2.72	-2.7	-2.67	-2.64	-2.61
8	-0.99	-0.97	-0.96	-0.95	-0.94	-0.93	-0.92
10	-0.16	-0.16	-0.16	-0.15	-0.15	-0.15	-0.15
	B: DrawDown Proportion of Loan (%)						
2	100	100	100	100	66	0	0
4	100	0.	0	0	0	0	0
6	100	0	0	0	0	0	0
8	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0
	C: Line Values Without Credit Quality Jump (\$)						
2	-13.23	-12.96	-12.7	-12.43	-12.16	-11.89	-11.77
4	-1.17	-1.14	-1.12	-1.09	-1.07	-1.04	-1.03
6	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
8	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0

Note: The table reports credit line values and the loan drawdown percentages calculated numerically under different credit spreads  $c$  and the borrower's initial credit quality  $s$ , assuming extremely high drawdown sensitivity. The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded.

Table 5: Line Value with Intermediate DrawDown Sensitivity

$c$	2%	4%	6%	8%	10%	12%	14%
$s$	A: Line Values With Credit Quality Jump (\$)						
2	-18.03	-16.16	-15.02	-14.49	-14.39	-14.55	-14.82
4	-5.63	-4.01	-3.24	-3.14	-3.46	-3.98	-4.54
6	-1.05	0.34	0.86	0.76	0.28	-0.36	-0.99
8	1.08	2.35	2.76	2.55	2.00	1.31	0.66
10	2.04	3.25	3.61	3.36	2.77	2.07	1.40
	B: Line Values Without Credit Quality Jump (\$)						
2	-11.42	-10.1	-9.51	-9.43	-9.67	-10.04	-10.42
4	1.04	2.25	2.62	2.40	1.83	1.15	0.51
6	2.20	3.39	3.74	3.48	2.89	2.18	1.51
8	2.23	3.43	3.77	3.51	2.92	2.21	1.54
10	2.23	3.43	3.77	3.51	2.92	2.21	1.54

Note: The table reports credit values calculated numerically under different credit spread  $c$  and the borrower's initial credit quality  $s$ , assuming intermediate drawdown sensitivity. The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded.

Table 6: Bond-equivalent, CDS value and initial drawdown( $d_{sen} = 5$ )

$c$	2%	4%	6%	8%	10%	12%	14%
$s$	A: Bond Equivalent (\$)						
2	91.29	89.41	87.56	85.81	84.22	82.83	81.65
4	85.38	82.63	80.05	77.75	75.8	74.21	72.97
6	83.27	80.34	77.64	75.29	73.34	71.79	70.60
8	82.61	79.67	76.99	74.67	72.76	71.25	70.11
10	81.57	78.52	75.75	73.37	71.43	69.91	68.77
	B: Value of credit default swap (\$)						
2	20.27	19.85	19.44	19.05	18.7	18.39	18.13
4	7.96	7.7	7.46	7.25	7.07	6.92	6.80
6	3.34	3.22	3.11	3.02	2.94	2.88	2.83
8	1.17	1.13	1.09	1.06	1.03	1.01	0.99
10	0.19	0.18	0.18	0.17	0.17	0.16	0.16
	C: Initial drawdown (\$)						
2	49.9	40.0	30.7	22.5	15.7	10.4	6.6
4	44.4	34.8	26.0	18.6	12.6	8.2	5.0
6	41.8	32.4	24.0	16.9	11.3	7.2	4.4
8	40.7	31.3	23.0	16.1	10.8	6.8	4.1
10	40.1	30.8	22.6	15.8	10.5	6.6	4.0

Note: The table reports the bond-equivalent, the cost of credit swap, and the initial drawdown for credit lines with different credit spreads  $c$  and the borrower's initial credit quality  $s$ , assuming drawdown sensitivity  $d_{sen} = 5$ . The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded.

Table 7: Fair Credit Spread and Line Values

$d_{sen}$	1	3	5	7	9	11
s	A: Fair Credit Spread (%)					
2	x	x	x	x	x	x
4	6.82	x	x	x	x	x
6	2.59	2.88	3.37	x	x	x
8	0.87	0.91	0.94	0.97	1.02	1.07
10	0.14	0.14	0.15	0.15	0.15	0.15
s	B: Fair Credit Spread Under Positive Origination Fee (\$)					
2	25.81	x	x	x	x	x
4	4.90	6.21	x	x	x	x
6	1.02	1.07	1.10	1.14	1.17	1.21
8	-0.58	-0.55	-0.53	-0.51	-0.50	-0.48
10	-1.27	-1.21	-1.16	-1.12	-1.08	-1.05
s	C: Credit Line Values (\$)					
2	-17.54	-17.9	-18.03	-18.09	-18.13	-18.16
4	-5.38	-5.53	-5.63	-5.70	-5.77	-5.83
6	-0.74	-0.91	-1.05	-1.19	-1.32	-1.45
8	1.47	1.27	1.08	0.90	0.72	0.55
10	2.47	2.26	2.04	1.84	1.63	1.44

Note: The table reports fair credit spreads without and with \$2 origination fee, and credit values calculated numerically under different drawdown sensitivities  $d_{sen}$  and the borrower's initial credit quality  $s$ . A fair credit spread of x indicates that no fair credit spread can be found. The base case assumes a 3 year credit line with the contractual maximum balance  $A = 100$ ; payment interval, reference and opportunity cost rate maturities of one month ( $T_{opp} = T_{ref} = .0833$  yr); origination, facility and standby fees are 0. Recovery rate in default is  $\rho = 0.5$ . The default-free term structure is assumed flat at 5%/yr, continuously compounded.