Offshore Settlement, Collateral, and Interest Rates

Charles M. Kahn
Department of Finance
University of Illinois at Urbana-Champaign

November, 2008

Abstract

Recent developments in private payments arrangements, particularly at the wholesale level, challenge central banks' longstanding monopoly on the provision of the ultimate means of settlement for financial transactions. This paper examines competition between public payments arrangements and private intermediaries, and the effect on central banks' role in monetary policy. Central to the issue is the role of collateral both as a requirement for participation in central bank sponsored payments arrangements and as the backing for private intermediary arrangements. The presence of private systems serves as a check on the ability of a monetary authority to tighten monetary policy.

The previous century may turn out to have been the high-water mark of public payment systems. The developments of central banking in the twentieth century may have made it seem inevitable that government fiat money form the base and centerpiece of a modern economy's methods of payment Actually the underlying institutions arose relatively recently; many innovations over the last three decades have made central bank based payment systems seem an aberration.[1]

The significance of some recent payments innovations could be debated: While the rise of credit cards has been an important change in payment for retail economic activity, it might still be argued that it makes no "fundamental" difference to the system, because, at the end of the month, every transaction translates one-for-one into a payment (or almost every transaction—default does occur), and that payment passes through an account in some commercial bank somewhere. And since commercial banks "ultimately" depend on central bank reserves and "ultimately" settle with one another through central bank payments systems, central banks are still at the core of the arrangement—with only a money multiplier as a minor caveat.

[1]For an introduction, see Kahn and Roberds (2008); for an extensive history of institutions see Manning et al (2008).

Other developments, however, cannot so easily be dismissed. In the UK, the increased concentration of payments into a handful of major settlement banks through "tiering" has meant that an increasing proportion of economic activity is payed through "on-us" transfers within a bank's accounts, never reaching the central system. Intrabank activity is increasingly likely to take place through private settlement arrangements. In the US, CHIPS—a private, cooperatively-owned arrangement among major financial institutions—has activity equal to something like 80% of the value of the activity on Fedwire, the Federal Reserve's system. And although "ultimate" settlement on CHIPS passes through Fedwire at the end of the day, the techniques used by CHIPS to effect netting of payments among its participants means that the amounts appearing on Fedwire are orders of magnitude smaller than the actual activity. The story is the same for CLS, the recently developed private system for settling international, multicurrency transactions. Throughput on this system is now fifteen times world GDP, and one hundred times the ultimate settlement of these transactions on central bank books. Not only is transactions velocity staggering on these advanced netting systems, it varies radically with economic conditions—in recent unsettled periods, daily value of transactions on CLS were double the "normal" high-volume days. In other words, the variations are unaffected by central bank activity.

Perhaps most ominous from the point of view of central bankers is the rise in private "offshore" settlement arrangements. In Hong Kong, in particular, two major banks have set up entirely private arrangements for making payments in both dollars and sterling. These systems have no connection—legal or regulatory—to the U.S. or the U.K. Value and volume on these systems does not depend on the institutions holdings of British or American central bank reserves; instead it is determined by demands for the service in the Far East, and its capacity is constrained only by limits to the reputation of the banks running the arrangements. (Similar arrangements are now becoming available in the rest of China).

For several years theoretical monetary economists have pondered what it would mean for central bank reserves no longer to serve as the "ultimate" means of settlement.[2] But the developments have made clear that central banks reserves could cease to be relevant long before they cease to be ultimate. In other words, the capacity of a private payments system to carry out transactions relies less and less on the degree to which the system has access to government reserves, and more and more on the credibility of the institution as an ongoing entity. It is not reserves which back the system and limit its scale but attachable assets of reliable value—collateral.

Collateral is also central to participation in public payments arrangements. In CHAPS and TARGET—the large value systems of, respectively, the Bank of England and the European Central Bank—collateral must be posted by participants in order to have access to overdrafts on their accounts needed for engaging in transactions during the course of the day. While interest rates on

[2] For opposing views see Goodhart (2000), Woodford (2000).

the collateralized borrowing are extremely low, they still carry an opportunity cost: collateral tied up in central bank systems cannot be used for backing other activity.

Since the amount of money (overdrafts and reserves) in a system during the daylight hours far exceeds the reserves on central bank books overnight, a disconnect has begun to arise between the two types of monetary policy—daylight and overnight, for which government collateral policy and interest rates differ. For major particpants these intraday collateral requirements are likely to be more significant than the reserve requirements established by the central banks on overnight positions. (Fedwire, unusually among central bank systems, allows uncollateralized intraday overdrafts, but in effect makes the same restrictions through more informal means.) Evidence of the significance of the collateral requirements for participants in CHAPS and TARGET comes from the pressure these banks have brought on the institutions to allow greater flexibility in collateral requirements.

And while this is going on within individual national payment systems, increasingly global banking conglomerates are handling payments around the world on a 24 hour basis. The latest twist in this environment is the increasing ease with which collateral can be transferred into and out of national payment system arrangements. Within the trading day, banks now can increase or decrease the collateral in a system, readjusting its use for other activities. Central banks have aided the process by increasingly allowing members to use as collateral securities of foreign governments. And the rise of agreements between central banks for easy shifting of collateral from one national system to another means that the day is not far off when collateral could be shifted around the world following the trading activities of payments systems around the clock.

How can we make sense of these changes? How do financial institutions decide on the use of their collateral and their participation in these systems? What are the consequences for operation of payments systems and for the effectiveness of central bank monetary policy? In this paper we will make a start at answering these questions by developing a model of competition between public and private payments arrrangements. While a monetary authority will have interest rate policies available to it, a central role in the model will be played by collateral, and the real effects of the system will be related to costs of generating collateral.

Sometimes the costs of collateral have been ignored in analyzing payments arrangements or monetary policy—as if there were a sea of the stuff out there, so large that it can be pressed into service as needed for free. In periods of liquidity crisis, as we have seen recently, this is clearly not the case. But even in normal times, when the costs of putting existing collateral into service are small, they are still part of an asset allocation system that a bank's managers must examine—and in particular the choice of investing in assets that are collateralizable should the need arise as opposed to assets that might be more lucrative investments economically but which cannot, for reasons perhaps of opacity, be used as backing, can have significant effects on an economy.

It is probable that over recent decades, the cost of provision of collateral has decreased, and the elasticity of supply of collateral has increased. But what the effects of this on payments activity and monetary policy are is less clear, particularly in the presence of private payments competition. The models of this paper are intended to help answer this question as well.

The model we develop is an extension of Berentsen-Monnet (2007) which in turn is based on Lagos-Wright (2005)'s "day-night" models. As this is a first attempt to address these issues, many simplifications will be included. We will focus on tradeoffs between the costs of collateralization and current consumption; the possibility of additional productive investments will be ignored. We will focus on the situation where central banks use "channel systems" to carry out monetary policy—that is, they establish nominal lending and borrowing rates for central bank funds. However, we will compare the results at several points with the case where at least part of the money supply is outside money. Since we adopt the quasi-linearity of the Lagos-Wright framework, there will be no long-term risk aversion motivating the holding of liquid assets. The role of money is solely a means of payment and the need for a means of payment arises solely from the problem of limited enforcement. Individuals face uncertainty about demand for consumption, which leads to a precautionary motive for money holding. There is no aggregate uncertainty—an extension which will be important for linking the model to more macroeconomic issues Nonetheless, competition between private and public payments arrangements will have important consequences for policy, even in this extremely simple set-up.

1 The model

All agents are risk neutral and have a common discount factor β per day. Each day is divided into two periods; in the first ("morning") all individuals trade in a Walrasian market. The second ("afternoon") has only anonymous trading; thus agents will need a means of payment to make purchases in this period. For convenience there is no discounting between periods within a day.

There are two goods; one can be produced and consumed in the mornings and the other can be produced and consumed in the afternoons. All agents can produce morning goods at a cost of 1 per unit. If consumed immediately, the morning good gives a utility of 1 per unit. If a unit is produced one morning and placed in a storage technology, it provides R units of utility the following morning. We will assume that $R\beta < 1$, so that agents will not desire, in the absence of other considerations, to produce for storage.

Each period an individual faces uncertainty about preferences and productivity with respect to afternoon good. In each period a fraction n of the agents can produce but not consume afternoon good. For such agents the cost of production is 1 per unit. A fraction $1 - n$ can consume and not produce. For such agents utility is $u(q)$, a function satisfying the normal convexity and Inada conditions. Individuals learn which group they belong to in any period at the beginning of the afternoon; these draws are serially uncorrelated. The afternoon good is not storable.

We denote periods by $t = \{0, 1, 2, ...\}$, even numbers denoting morning periods. For ease of presentation, we will focus on three periods, a morning (period 0) an afternoon (period 1) and the following morning (period 2). For t even, let h_t be an agent's net production of newly produced morning good at time t (production less consumption), y_{t+1} his production of afternoon good if the agent is an afternoon producer and q_{t+1} consumption of afternoon good if the agent is an afternoon consumer at time $t+1$, and x_t^{t+2} consumption in period $t+2$ of morning good stored from the previous day. Then an agent's expected utility over the three periods is

$$-h_0 - ny_1 + (1-n)u(q_1) - \beta h_2 + \beta R x_0^{0+2}$$

The quantities h_t can be positive or negative; y_t, q_t, and x_t^{t+2} must be non negative. Consumption in period 2 can depend on the period 1 realizations; we will let subscripts b and s denote period 2 choices conditional on the agent turning out to be a buyer or seller respectively in period 1. The so-called "quasi-linearity" of the utility function (Lagos-Wright, 2005) allows us to isolate the problem to these three periods when convenient.

2 Non-Monetary Equilibria

We begin by considering equilibria in this economy in the absence of monetary instruments. We will consider two possibilities: one in which agents are "trustworthy," so that afternoon trades can be handled by uncollateralized credit, and another in which the storable commodity is exchanged to resolve the problem of anonymity in afternoon markets. We will describe all prices relative to the price of the most recently produced morning good: for t even, let f_{t+1} represent the period $t+1$ price of afternoon good relative to the price of morning good, and let p_t^{t+2} represent the period $t+2$ price of morning good produced in period t relative to newly produced morning good.

2.1 Trustworthy agents

Proposition 1 *If agents are trustworthy, then in any equilibrium, afternoon consumers consume q^* units of afternoon good, where*

$$u'(q^*) = 1. \tag{1}$$

No storage occurs in equilbrium, and for t even,

$$\begin{aligned} p_t^{t+2} &\leq R \\ f_{t+1} &= \beta p_t^{t+2}. \end{aligned}$$

We will call q^* the efficient or "full-trust" level of output. An equilibrium with trustworthy agents is equivalent to a Walrasian equilibrium. In this equilbrium, prices *deflate* period by period at the rate β, meaning that individuals are indifferent between choices of working one period or the next, or of consuming newly produced morning good one period or another. Because of the linearity of costs and of preferences for morning goods, individual consumptions and productions of morning good are indeterminate. However, when comparing this economy with the rest of our examples in which agents are not trustworthy, it is natural to focus on the allocation in which all "debts" are paid the next period. That is, afternoon consumers provide output the next morning equal in value to their previous afternoon consumption, and vice versa for producers. This means that ex-consumers provide

$$\frac{q^*}{\beta}$$

units per person of morning good, and ex-producers receive, on average,

$$\frac{1-n}{n}\frac{q^*}{\beta}.$$

2.2 Commodity payment

Given that agents are not trustworthy, it will not be possible to borrow for consumption in the afternoon market. An afternoon consumer could, nonetheless, pay by trading with stored good. We can think of each agent as producing the amount h_t for storage; if he is an afternoon consumer, he will pay for consumption with stored good; if he is an afternoon producer he will hold his stored good plus any afternoon receipts for consumption the next morning. In this case, the resultant equilibria contain the analogue of a cash-in-advance constraint: each agent maximizes utility subject to period-by-period budget constraints, including the requirement that

$$f_{t+1}h_t \geq q_{t+1}$$

for t even, where h_t is the amount of period t morning good stored. Note that h_t must be measurable with respect to the information the agent receives—that is, it cannot depend on whether the agent turns out to be a consumer or producer in period $t+1$.

Determination of equlibrium in this case is aided by the following considerations: As of the period in question afternoon producers value the stored good at $R\beta$ per unit. Given constant marginal costs, sellers make zero profits in the afternoon. Since agents do not know whether they will be sellers or buyers, they choose a storage level h in the morning to solve the following problem:

$$\max_h -h + (1-n)u(R\beta h) + nR\beta h.$$

In other words, if buyers, they sell their storage for afternoon good; if sellers, they hold their own storage until the next period. Since $q = R\beta h$, we have the first order condition:

$$-1 + (1-n)R\beta u'(q) + nR\beta = 0.$$

Armed with this information one can quickly verify

Proposition 2 *If agents can only pay for afternoon consumption with stored morning goods then in the competitive equilibrium afternoon consumers consume* $\widetilde{q}$ *where*

$$\frac{(R\beta)^{-1} - n}{1 - n} = u'(\widetilde{q}). \tag{2}$$

In equilibrium, for t *even*

$$p_t^{t+2} = R$$

$$f_{t+1} = \frac{1 - n}{1 - R\beta n}$$

We will call $\widetilde{q}$, the level of consumption under commodity payment ("the barter level"). Since there is no intertemporal market on which afternoon good can be sold we only have spot rates of exchange between the two goods available for trade. Note that the left side of the equation defining $\widetilde{q}$ is greater than 1 so that

$$\widetilde{q} < q^*.$$

and afternoon good becomes expensive relative to stored morning good. Note that agents anticipate a capital loss on the stored good. They are willing to store the good despite the fact that in present value terms each unit will be only worth at maturity the fraction $R\beta$ of its initial cost. The difference is the liquidity premium on the morning good.

2.3 Collateralized Borrowing

This equilibrium can be given a second interpretation: suppose rather than using the stored good as an outright payment, the agents treat it as collateral; the good is held by the seller until period 2 when it is returned to the buyer in return for new morning good of equal value. Clearly this interpretation makes no substantive change in the account. But it does allow us to extend the analysis to the case where the collateral value is greater or less than the value of the goods purchased with it. It also allows us explicitly to consider interest rates for borrowing or lending between periods 1 and 2. We will include that possibility in considering the individual maximization problem, with two different rates. Of course, in the competitive equilibrium, borrowing and lending rates will be the same, but by treating them separately we will be able to use the analysis for more general situations later.

Specifically, assume traders in the afternoon engage in a "repo" transaction: buyers borrow by making a *loan* of morning good which will then be returned the following morning when the borrowing is repaid. Now buyers rather than sellers consume the old morning good, and instead buyers produce new morning good to make their payments. With linear technologies this exchange is a wash. Now we can consider "haircuts"—transactions in which the value of the collateral exceeds the value the goods received—and "loans on margin"—in which the collateral only represents a fraction of the loan value. To the extent that there are non-pecuniary costs to default, it is not necessary to require full collateral to ensure repayment. To the extent that there may be adverse selection in the collateral posted, collateral value on average will have to exceed the value of the loan.

We will let α denote the fraction of the loan value which must be collateralized; thus $\alpha < 1$ represents an incompletely collateralized loan, and $\alpha > 1$ represents a haircut. Thus $\alpha = 0$ is the equivalent of trustworthy agents; $\alpha = \infty$ is an economy where commodities cannot be used to make purchases (in other words, autarky, in the absence of government-provided money).

At a cost of 1 an individual manufactures a collateral good in the morning. He can use it to guarantee payment for purchase in the afternoon and will, in any case consume the collateral good the next day, at a present value of βR per unit. Thus the net cost of collateral provision is $(1 - \beta R)$. In the afternoon suppliers produce and demanders purchase afternoon good. The collateral good gives an inferior amount of consumption in period 2, but relaxes the constraint on afternoon consumption. The agent's problem becomes

$$\max_{h,q,y \geq 0} -h + (1-n)u(q) - ny + \beta R h - \beta(1-n)(1+r_b)q + \beta n(1+r_s)y$$

subject to

$$\alpha^{-1} R h \geq (1 + r_b) q \tag{3}$$

Here $1 + r_b$ and $1 + r_s$ are the number of units of period 2 morning good that must be given in exchange for an afternoon loan to buy 1 unit of afternoon good. In other words r_b and r_s are the "real" interest rates—or more precisely, interest rates adjusted for the relative price of the two goods.

First order conditions for this problem are as follows, using λ as the Lagrange multiplier for the constraint (3):

$$\begin{aligned} 1 - \beta R &= \lambda \alpha^{-1} R \\ (1-n)(u'(q) - \beta(1+r_b)) &= \lambda(1+r_b) \\ \beta(1+r_s) &= 1 \end{aligned}$$

The third condition means that, given the constant returns to scale for production of afternoon good, in equilibrium the relative price of afternoon good and good the subsequent morning must be equal to the marginal rate of substitution. Eliminating λ, the remaining conditions say

$$u'(q) = \left(\beta + \alpha \frac{1 - \beta R}{R(1-n)}\right)(1 + r_b)$$

If $r_b = r_s$, as will occur if lending is competitive, and if $\alpha = 1$, this condition reduces to the condition (2) determining the level of output under commodity payment. As α approaches 0, the condition approaches (1) and consumption approaches the trustworthy agents case. In general consumption decreases with increasing costs of using the system (increases in r_b or α).

3 Government Monopoly on Money

Next we consider a government which has a monopoly on the provision of means of payment in the economy. Let P_t, (t even) denote the nominal period t price of newly produced morning good, and F_t (t odd) denote the period t price of afternoon good.

3.1 Inside Money

Suppose the government is able to issue as much nominal money as the public wants. This money will be "inside money" —issued in the afternoon to be paid back the next day.

The level of prices is indeterminate. In other words, for arbitrary positive P_2, the government can make an announcement of a willingness to buy or sell P_2 units of money in return for one unit of morning goods in period 2. While government supply of money is then completely elastic at this price, private agents' aggregate supply of and demand for money in period 2 are completely inelastic and equal. Thus money trades at the government's specified price. However the real money supply is independent of the stated price: The price of afternoon goods in period 1 is $F_1 = P_2/\beta$, and each buyer will borrow enough to purchase q^* units. No storage of morning goods takes place, and the real per capita money supply in the economy overnight is $(1-n)q^*/\beta$ valued at period 1 prices, or $(1-n)q^*$ valued at period 2 prices. The marginal rate of substitution between morning and afternoon goods is 1, so that $P_0 = P_2/\beta$, that is, prices deflate in line with the discount rate. If the process is repeated then in each period a smaller nominal amount of inside money is borrowed.

In this economy the government can conduct a monetary policy by establishing a (nominal) interest rate for money it lends to the public. Let the interest rate be denoted i_ℓ. For completeness and clarity we can also consider that anyone holding money at the end of the afternoon can deposit it with the government overnight, and receive a deposit rate, i_d also determined by the government. Clearly, all money supplied by the government will end up in overnight deposits. Equally clearly, the government is restricted to combinations of (i_ℓ, i_d) such that

$$i_\ell \geq i_d;$$

otherwise there will be arbitrage opportunities.

If the government sets the two rates to be equal (call it i), then there is no real effect. Again, the government can announce an arbitrary value for money on the following morning; given this value, the price of afternoon goods in period 1 is $F_1 = (1+i)^{-1}P_2/\beta$, and again each individual borrows enough to purchase

q^*. Valued at period 2 prices and including interest, the real value of the money supply (call it the "overnight money supply") is unchanged. Valued at period 1 prices, it is smaller by the anticipated interest payments. The interest payments are also built into the inflation rate:

$$\frac{P_2}{P_0} = \beta(1+i)$$

and if $1+i = \beta^{-1}$ prices remain constant, period to period.

On the other hand, a spread between the interest rates does have real effects. First note that with a spread in interest rates, the public must in aggregate pay back more money on any day than is available to it. The difference is assumed to be distributed lump-sum by the government to the population as a whole; thus each pair of interest rates entails an associated (negative) tax policy.[3]

As the interest rate spread increases, the use of money decreases. In this case as we will see in detail in the following sections

$$u'(q) = \frac{1+i_\ell}{1+i_d}$$

and

$$\frac{P_2}{P_0} = \beta(n(1+i_d) + (1-n)(1+i_\ell)).$$

In other words, inflation is determined by the average of interest rates faced by buyers and sellers, and economic activity is reduced by the spread in rates.

In the United States until very recently, that the monetary authorty did not pay interest on monetary assets, so that $i_d = 0$. Then increases in the borrowing rate on monetary assets decreases economic activity and reduces prices today relative to future prices.

3.1.1 Extensions

As long as the interest rate spread remains low, no agent would actually find it useful to attempt to use commodity money. However, as the interest spread increases beyond a critical level

$$\frac{(R\beta)^{-1} - n}{1-n}$$

an incentive arises to develop private alternatives to government money.

Of course a monetary authority could also require that participants provide collateral in return for borrowing. An individual who borrows one dollar from the government must repay $1+i_\ell$ the next morning. He must post γ dollars worth of collateral value per dollar owed. He will pay F_1 per unit of good bought. Thus he must post $\gamma F_1(1+i_\ell)/(RP_2)$. As a result, the level of consumption of afternoon good falls further.

We will consider both of these extensions in greater detail in subsequent sections.

[3] By considering the possibility of rolling over borrowings of money, we could extend the government's space of policies, but without signifciant effect on our analysis.

3.2 Outside Money

Suppose that the monetary authority provides "outside money." Our strategy to analyze this situation is to *assume* money has a particular value $(1/P_2)$ as of period 2, and to work backward to determine its value in periods 1 and 0.

For example, in the case of a constant nominal stock of money, the analysis of this section will show that as long as the price level in period 2 is low enough such that

$$M_0 \geq \frac{P_2}{\beta} q^*$$

the amount of money in the system is sufficient to achieve efficiency: agents use money to purchase the full trust level of output.

We now present the analysis in detail; the inside money results described in the previous section will be shown to be a special case of this general presentation.

With interest paid in money, there is no longer a guarantee that the nominal supply of outside money is the same from period to period. For completeness therefore we need to consider the possibility that the government distributes or collects money balances from the population. Assume that at the beginning of the morning, the government collects (distributes, if negative) in a lump sum fashion T nominal units of money per person. Given the quasilinearity of agent preferences, this has no effect on decisions other than the rebalancing of money holdings that occurs each morning.

Following Berentsen and Monnet, it is convenient to use the proportional spread between interest rates

$$\Delta = \frac{1 + i_\ell}{1 + i_d}$$

as one indicator of the government's policy. To avoid arbitrage opportunities, $\Delta \geq 1$.[4]

In this environment, an agent's maximization problem is as follows:

$$\max_{h,q,y,M_3,x,c} -h + (1-n)u(q) - ny + \beta Rc + (1-n)\beta(x_b + \frac{M_{3b}}{P_2}) + n\beta(x_s + \frac{M_{3s}}{P_2})$$

subject to

$$(M_0 + P_0(h-c) + F_1 y)(1+i_d) - T \geq P_2 x_s + M_{3s} \quad (4)$$

$$(M_0 + P_0(h-c) - F_1 q)(1+i_\ell) - T \geq P_2 x_b + M_{3b} \quad (5)$$

$$(M_0 + P_0(h-c) - F_1 q) + (1+i_\ell)^{-1}\gamma^{-1} c P_2 R \geq 0 \quad (6)$$

[4] Strictly speaking, this is only necessary if $\gamma = 0$. Otherwise, arbitrage may not be possible even if the deposit rate exceeds the lending rate. In general the condition is

$$\frac{1 + i_\ell}{1 + i_d} \geq \frac{1}{1 + \gamma(R^{-1} - \beta)}$$

Here h is the net production of morning good in period 0; $c > 0$ is the amount stored for use as collateral against borrowed money; x is the net consumption of good produced in period 2; and M_{3s} and M_{3b} are final holdings of money at the end of period 2 by sellers and buyers respectively. The fact that M_3 is in the objective function is in this case innocuous: if we were to work through a recursive dynamic programming problem, we would discover that $1/P_2$ is simply the shadow value of money in next period's value function. P_2 is exogenous to the problem; consider it to be set by having the monetary authority manage expectations of future prices. Note therefore, that the policy of the monetary authority establishes a real supply of money as of period 2. For each possible value of P_2 the equilibrium will establish values for period 0 and period 1 prices. The problem then can be repeated for periods 2 and 3 given a real money supply expected for period 4 and so forth. The dynamics of the money supply are therefore

$$M_3 = qF_1(1 + i_d - i_\ell) - T$$

Constraints (4-5) are wealth constraints on sellers and buyers respectively. Constraint (6) is the collateral constraint: a buyer's purchases of afternoon good are limited to any initial holdings of money plus any profits made from morning sales, plus the value that can be purchased given the agent's collateral. For each dollar to be repaid (including interest), an agent must post collateral worth γ dollars in period 2. A unit of collateral is worth P_2R dollars in period 2.

Let $(\lambda_1, \lambda_2, \lambda_3)$ be the Lagrange multipliers for the constraints (4-6) respectively. The first two clearly bind. Given the Inada conditions on u, the first order conditions are as follows:

$$\begin{gathered}
-1 + P_0(\lambda_1(1 + i_d) + \lambda_2(1 + i_\ell) + \lambda_3) = 0 \\
(1 - n)u'(q_b) - (\lambda_2(1 + i_\ell) + \lambda_3)F_1 = 0 \\
-n + \lambda_1 F_1(1 + i_d) = 0 \\
n\beta = \lambda_1 P_2 \\
(1 - n)\beta = \lambda_2 P_2 \\
\frac{n\beta}{P_2} \leq \lambda_1;\ M_{3s} \geq 0 \\
\frac{(1 - n)\beta}{P_2} \leq \lambda_2;\ M_{3b} \geq 0 \\
\beta R - P_0(\lambda_1(1 + i_d) + \lambda_2(1 + i_\ell) + \lambda_3) + \lambda_3(1 + i_\ell)^{-1}\gamma^{-1}P_2R \leq 0;\ c \geq 0
\end{gathered} \tag{7}$$

(where paired inequalities hold with complementary slackness). These conditions simplify to

$$\begin{aligned}
P_0(n\beta(1+i_d)+(1-n)\beta(1+i_\ell)+\lambda_3 P_2) &= P_2 && (8)\\
(1-n)(u'(q_b)-\frac{1+i_\ell}{1+i_d}) &= \frac{\lambda_3 P_2}{\beta(1+i_d)} && (9)\\
F_1 &= \frac{P_2}{\beta(1+i_d)} && (10)\\
\lambda_1 &= \frac{n\beta}{P_2} && (11)\\
\lambda_2 &= \frac{(1-n)\beta}{P_2} && (12)\\
M_{3s} &\geq 0 && (13)\\
M_{3b} &\geq 0 && (14)\\
(1+i_\ell)\gamma(R^{-1}-\beta) &\geq \lambda_3 P_2;\ c\geq 0. && (15)
\end{aligned}$$

In order to analyze these conditions, first suppose $\lambda_3=0$. Then $c=0$ and

$$\begin{aligned}
\frac{P_2}{P_0} &= \beta n(1+i_d)+\beta(1-n)(1+i_\ell) && (16)\\
u'(q) &= \Delta && (17)
\end{aligned}$$

If $\lambda_3=0$, then there is no liquidity premium for morning goods. As Berentsen and Monnet note, the spread between lending and borrowing rates has a real effect: wider spreads reduce economic activity. The levels of interest rates, however, simply affect inflation. Since the money supply must be sufficient to purchase the afternoon goods, we have

$$M_0 > qF_1 = q\frac{P_2}{\beta(1+i_d)}$$

—that is, this case is the equilibrium solution whenever period 2 prices are anticipated to be sufficiently low, or, in terms of the monetary dynamics, when

$$\frac{M_3+T}{P_2} \geq q\frac{(1+i_d-i_\ell)}{\beta(1+i_d)}$$

(keeping in mind that q in this condition is a function of i_d and i_ℓ according to 17). Again, we can think of the left side of this inequality as the "overnight" real money supply, with the taxes collected the next morning. If $i_d=i_\ell$, then $q=q^*$, the full-trust level of production, and prices fall according to the Friedman rule.

Next consider the case where $c>0$—that is, collateral is used. Then the left inequality in (15) binds and

$$\begin{aligned}
\frac{P_2}{P_0} &= n\beta(1+i_d)+(1-n)\beta(1+i_\ell)+\beta\gamma((\beta R)^{-1}-1)(1+i_\ell) && (18)\\
u'(q) &= \Delta(1+\gamma\frac{(\beta R)^{-1}-1}{1-n}) && (19)
\end{aligned}$$

and by (6),

$$\frac{M_0\beta(1+i_d)}{P_2} < q$$

If collateral is used, it must be that the initial real money supply is below a critical level q. When $i_d = i_\ell = 0$ and $\gamma = 1$, this level is the barter level $\tilde{q}$, although the availability of outside money will reduce the amount of collateral needed to reach this production, relative to the case of commodity payment. The critical level decreases as the spread between rates grows; it also decreases as collateral requirements increase. If the money supply is below this critical level, no matter how far, the level of afternoon production q remains fixed at the particular lower level defined by (19). The overnight money supply cannot sink below the critical level; instead collateralized borrowing makes up the difference. In this region, even if $i_d = i_\ell = 0$, inflation is greater than the Friedman rule.

Finally, if the money supply starts at an intermediate level, so that

$$\Delta(1+\gamma\frac{(\beta R)^{-1}-1}{1-n}) > u'\left(\frac{M_0\beta(1+i_d)}{P_2}\right) > \Delta$$

it must be the case that $c = 0$ (no collateralized borrowing takes place) and $\lambda_3 > 0$ (the liquidity constraint is binding). In this case

$$q = \frac{M_0\beta(1+i_d)}{P_2} \tag{20}$$

$$\frac{P_2}{P_0} = ((1-n)u'(q)+n)\beta(1+i_d) \tag{21}$$

Thus the rate of inflation and the level of production both depend on the interest rate paid on deposits (but not on the interest rate paid for collateralized borrowing). Higher deposit rates increase the value of money today (holding its value tomorrow constant), and thus increase economic activity. The effect on inflation is ambiguous; the interest rate on deposits contributes directly to its increase, but by reducing the collateral constraint, it incresases prices today relative to tomorrow.

Inside money is simply the special case of $M_0 = 0$ and $T = qF_1(1+i_d-i_\ell)$. In this case, as noted in the previous section, the expected price level in period 2 has no real effect on the economy. When there is outside money in the system, the level of P_2 has real effects, since the real supply of money reduces the need for collateral.

3.3 Steady state money growth

Berentsen and Monnet focus on the case where the real money supply remains constant. Buyers enter the afternoon with $(1-n)M_0$ and sellers with nM_0. In principle agents could increase or decrease their money balances by making trades of morning good in period 0, but in the aggregate these cancel. In the afternoon buyers use the money they have plus any they borrow to make purchases. They borrow $(1-n)\max\{(F_1q - M_0), 0\}$. Thus their total holdings, are $(1-n)\max\{F_1q, M_0\}$. If there is an excess, the buyers deposit it. The sellers deposit any they receive plus their initial holdings. Thus the total money funds deposited overnight are $\max\{(1-n)F_1q + nM_0, M_0\}$. The next day, interest is paid and received in money, so the total nominal holdings are $M_0(1+i_d) - T$ if $F_1q \leq M_0$, and $(1+i_d)((1-n)F_1q + nM_0) - (1-n)(1+i_\ell)(F_1q - M_0) - T$ otherwise.

Define $\tau = T/M_0$. There can be a steady state with no use of collateral. If such a steady state exists, then (by equation (21)):

$$P_2/P_0 = ((1-n)u'(q) + n)\beta(1+i_d) = M_2/M_0 = (1+i_d) - \tau$$

or simplifying

$$((1-n)u'(q) + n)\beta = 1 - \tau(1+i_d)^{-1}$$

Consider the case where transfers are zero. In this case, only one level of economic activity is consistent with a steady state path when collateral is not used. Call it $\hat{q}$, implicitly defined by

$$u'(\hat{q}) = \frac{1-\beta n}{\beta - \beta n}.$$

Thus for this to be a steady state, a necessary and sufficient condition is that

$$\Delta(1 + \gamma\frac{(\beta R)^{-1} - 1}{1-n}) \geq u'(\hat{q}) \geq \Delta$$

or, equivalently

$$\frac{\beta^{-1} - n}{1-n} \geq \Delta \geq \frac{\beta^{-1} - n}{1 - n + \gamma((\beta R)^{-1} - 1)}$$

Since the per capita money stock in this middle case is spent on afternoon consumption $\hat{q}$ then determines the real supply of money:

$$\frac{M_0}{P_2} = \frac{\hat{q}}{\beta(1+i_d)}$$

In this case i_ℓ is irrelevant (no one borrows). The interest rate paid on deposits directly affects the level of inflation, but does not affect the level of economic activity.

Returning to the general case for τ, if

$$\Delta > \frac{\beta^{-1}(1 - \tau(1+i_d)^{-1}) - n}{1-n}$$

then the only candidate for a steady state equilibrium is one with no liquidity premium for morning goods. But then we conclude that

$$\begin{aligned}\frac{M_2}{M_0} - \frac{P_2}{P_0} &= (1+i_d) - \tau - (\beta n(1+i_d) + \beta(1-n)(1+i_\ell)) \\ &= (1+i_d)(1 - \beta n - \beta(1-n)\Delta) - \tau \\ &< 0.\end{aligned}$$

In other words in this region, in the absence of government transfers, the real money supply must always grow; it cannot be constant.

On the other hand, if

$$\Delta < \frac{\beta^{-1}(1 - \tau(1+i_d)^{-1}) - n}{1 - n + \gamma((\beta R)^{-1} - 1)}$$

Then the steady state will involve the use of collateral. The condition for a steady state is now

$$((1+i_d) - (1+i_\ell))(1-n)\frac{F_1 q}{M_0} + n(1+i_d) + (1-n)(1+i_\ell) - \tau = \frac{P_2}{P_0}$$

where P_2/P_0 and q satisfy (18-19). Simplifying, this condition becomes

$$\frac{(i_\ell - i_d)(1-n)}{-\gamma(R^{-1} - \beta)(1+i_\ell) + (n(1+i_d) + (1-n)(1+i_\ell))(1-\beta) + \tau} \qquad (22)$$

$$= \frac{M_0 \beta(1+i_d)}{P_2 q} < 1 \qquad (23)$$

In other words, the condition picks a particular real money supply and inflation rate for each i_d, i_ℓ, τ, and γ.

Finally, we can also consider the limiting, but natural, case of no outside money ($M_0 = 0$). As M_0 shrinks, the steady state can only occur if

$$T = -(i_\ell - i_d)F_1 q_b$$

that is, the government must provide money each period equal to the excess interest paid by borrowers over that received by lenders. Provided this is the case, then the level of interest rates determine the rate of inflation via (19) and the spread and haircut determine the real activity of the economy via (18).

4 Competition between private and public systems

Now we consider competition between publicly provided money and private collateralized loans. As a warm-up, we will examine the situation for a fixed nominal money supply. Then we will consider the case of collateralized interest-bearing inside money.

4.1 Fixed Nominal Money Supply

The economy has a fixed money supply M_0. An individual can either purchase afternoon goods with money or purchase on credit by posting collateral equal to α times the value to be repaid. The problem for the individual is therefore

$$\max_{h,q,y,M_3,x,b} -h + (1-n)u(q) - ny + \beta Rb + (1-n)\beta(x_b + \frac{M_{3b}}{P_2}) + n\beta(x_s + \frac{M_{3s}}{P_2})$$

subject to

$$\begin{aligned} M_0 + P_0(h-b) + F_1 y &\geq P_2 x_s + M_{3s} && (24) \\ M_0 + P_0(h-b) - F_1 q &\geq P_2 x_b + M_{3b} && (25) \\ M_0 + P_0(h-b) + \alpha^{-1} F_1 \beta Rb &\geq F_1 q && (26) \end{aligned}$$

Here, variables are as before; recall that b is the amount stored by an agent for use as collateral against private loans. Condition (26) is the collateral constraint; to verify it consider the shortfall in units of afternoon good purchased relative to the money available to purchase them:

$$q - \frac{1}{F_0}(P_0(h-b) + M_0).$$

For each such unit of shortfall, the seller will require that the buyer post collateral equal in value to α times the cost of produciton. The cost of production is 1 per unit. In terms of period 2 morning good, this is worth β^{-1}. A unit of collateral is equivalent to R units of period 2 morning good. Thus for each unit of afternoon good received against a loan, the seller will require $\alpha(R\beta)^{-1}$ units of collateral.

Let $(\lambda_1, \lambda_2, \lambda_3)$ be the Lagrange multipliers for constraints (24-26).

First order conditions are analogous to the set (7) from before, and now simplify to

$$\begin{aligned} P_0(1 + F_1\lambda_3) &= F_1 && (27) \\ (1-n)u'(q) - 1 + n - F_1\lambda_3 &= 0 && (28) \\ F_1\lambda_1 &= n && (29) \\ F_1\lambda_2 &= 1-n && (30) \\ F_1\beta &= P_2 && (31) \\ F_1\lambda_3 &\leq \alpha(\beta R)^{-1} - 1; b \geq 0. && (32) \end{aligned}$$

If $\lambda_3 = 0$, the conditions are as before; in the special case of no interest charge and no collateral requirement for money they reduce to

$$\begin{aligned} P_0 &= F_1 = \frac{P_2}{\beta} \\ u'(q) &= 1 \\ b &= 0 \end{aligned}$$

In other words, q^* the full-trust level of afternoon good is produced, and from (26) we conclude that

$$M_0 \geq \frac{P_2}{\beta} q^*.$$

That is, the per capita money supply is sufficient to purchase the efficient level of output.

On the other hand, if $b > 0$, so that the left inequality in (32) binds, the conditions simplify to

$$u'(q) = \frac{\alpha(\beta R)^{-1} - n}{1 - n} \tag{33}$$

$$F_1 = P_2/\beta \tag{34}$$

$$P_0 = P_2 R/\alpha \tag{35}$$

Let condition (33) implicitly define the decreasing function $\tilde{q}(\alpha)$, so that $\tilde{q}(1) = \tilde{q}$ and $\tilde{q}(\beta R) = q^*$. Then as long as the money supply is adequate to purchase $\tilde{q}(\alpha)$, agents will not produce collateral. Note that when measured in prices tomorrow, this critical money supply varies monotonically with α:

$$M_0 = \frac{\tilde{q}(\alpha) P_2}{\beta}$$

as collateral becomes more efficient (that is, as α decreases), it gets used in more circumstances—that is, for larger and larger levels of period 2 money balances (lower and lower period 2 prices). However, when measured in today's morning prices

$$M_0 = \frac{\tilde{q}(\alpha)\alpha P_0}{\beta R}$$

the relationship is ambiguous: The more purchases that collateral can back, the more valuable morning good becomes.

Of course an increase in α beyond 1 (that is, as haircuts become more stringent) has the opposite effect. Collateral is used more rarely, at more extreme shortages of money. The money price of production today falls, as it becomes less valuable as collateral; and when collateral is used inflation is more severe.

4.2 Collateralized inside money competing with private collateral

We now consider competition between collateralized inside money and private collateralized loans. As before, we assume that the money is issued one afternoon and must be repaid the next day. Initially, we ignore interest rates. Now the agent's problem is

$$\max_{h,q,y,M_3,x,b,c} -h + (1-n)u(q) - ny + E\beta(x + Rb + Rc)$$

subject to

$$\begin{aligned} P_0(h-b-c)+F_1 y &\geq P_2 x_s \\ P_0(h-b-c)-F_1 q &\geq P_2 x_b \\ P_0(h-b-c)+\alpha^{-1} b F_1 \beta R+\gamma^{-1} c P_2 R &\geq F_1 q \end{aligned}$$

Recall that c is the amount of collateral placed in the public facility and b is the amount of collateral used in private loans. The collateral constraint says that any shortfall in payment for afternoon good that is not met by private collateralized loans must be met by borrowed money.

The first order conditions are

$$\begin{aligned} P_0(1+F_1\lambda_3) &= F_1 \\ (1-n)u'(q)-1+n-F_1\lambda_3 &= 0 \\ F_1\lambda_1 &= n \\ F_1\lambda_2 &= 1-n \\ F_1\beta &= P_2 \\ \gamma^{-1}F_1\lambda_3 &\leq (\beta R)^{-1}-1; c\geq 0 \\ \alpha^{-1}F_1\lambda_3 &\leq (\beta R)^{-1}-1; b\geq 0 \end{aligned}$$

Now the choice of use of private or public payment simply boils down to the question of which requires the more expensive haircut. Holding second period prices fixed, an increase in the haircut on borrowing money lowers the demand for money and reduces afternoon consumption. The reduction in the afternoon consumption reduces demand for collateral and thus morning prices of goods. However, once the haircut exceeds that required for private borrowing, demand for money falls to zero, and further increases in haircuts have no effect on the economy. This does not affect the money price of goods in period 2; the government still continues to be willing to redeem money from any holder at a price P_2. Thus the amount of consumption of afternoon good in the economy is

$$\tilde{q}(\min\{\alpha,\gamma\})$$

4.2.1 Adding initial money balances

If in addition, agents start with a stock of money balances M_0 per capita and these are believed to have a shadow value of P_2 in period 2, the equilibrium will be altered very little: If the money balances exceed the amounts needed to pay for $\tilde{q}(\min\{\alpha,\gamma\})$, then no collateral is used. Otherwise, collateral is used to make up the difference between the money stock and the amounts needed to make this purchase.

4.3 Interest Bearing, Collateralized Inside Money

Now consider the case where agents can either use interest bearing inside money, or make private arrangements on their own. In either case they must post collateral in advance. Money interest rates are a policy variable of the government; terms for private arrangements are set competitively. Let C be the money price in period 2 that a private borrower agrees to pay for a unit of afternoon good purchased in period 1. The equivalent value in collateral in period 2 is $C/(RP_2)$. A private borrower must post collateral $b = \alpha C/(RP_2)$ per unit of afternoon good purchased in a private loan. An individual who borrows one dollar from the government must repay $1 + i_\ell$ the next morning. He must post γ dollars of collateral value per dollar owed. He will pay F_1 per unit of good bought. Thus he must post $c = \gamma F_1(1 + i_\ell)/(RP_2)$.

A seller who receives a dollar in period 1 will deposit it overnight and have $(1 + i_d)$ dollars in period 2. Thus a seller who sells a unit for money will have $F_1(1 + i_d)$ dollars in period 2. A seller who receives a promise to pay for a unit will have C dollars in period 2. Thus for a seller to be indifferent between methods

$$C = F_1(1 + i_d) \tag{36}$$

For the two methods to co-exist it must be that

$$\alpha(1 + i_d) = \gamma(1 + i_\ell)$$

otherwise put, if

$$\Delta > \alpha/\gamma$$

only private payment arrangements are used; if the inequality is reversed only public payment arrangements are used. We focus on the case of no initial outside money. (We will be casual about the medium of exchange used in period 0, because in equilibrium no exchange will actually take place.)

If only public payment arrangements are used, then the equilibrium is as in section 3. If only private arrangements are used, then the equilibrium is an in section 2. To get a feel for the effects, return to the steady state case. As the government increases the spread Δ, activity in the economy falls, until the spread reaches the level α/γ. From then on, further spreads have no effect, since the economy substitutes private payments arrangements for public ones. Similarly, increasing interest rate levels affects inflation. However, once the critical level is exceeded, then this has no significance: since public money is not actually used, the private loans could be denominated in any real good, and inflation would be irrelevant.

4.4 Variable Collateral Requirements for Private Loans

So far we have not addressed the issue of the source of the collateral requirements. While public requirements are largely a policy variable, private requirements depend on reliability, information, incentives and enforcement considerations. In practice, payments arrangements have collateral requirements which vary with the identity of the participants and the amount of their participation.

Private systems place a variety of restrictions on membership and collateral requirements for participants, including differentiation between various classes of participants. As a result, only the larger and (presumably) better collateralized institutions participate in the private systems directly.

The important consequence is that changes in the collateral requirements of the public system yield continuous responses in the use of the private system. For example, increased collateral requirements in the public system induce a move to the private system by some institutions who would formerly have found the private requirements too stringent.

Suppose that in order to borrow at date 1 an amount equivalent to bRP_2 in nominal value at date 2, it is necessary to post an amount of collateral equal to $\kappa\alpha(b)$, where α is an increasing convex function of b, satisfying the Inada conditions. Over time we expect κ to fall. The problem becomes

$$\max_{h,q,y,,M_3,x,c} -h + (1-n)u(q) - ny + E\beta(x + Rc + Rb + \frac{M_3}{P_2})$$

subject to

$$(P_0(h-c-b) + F_1 y)(1+i_d) - T \geq P_2 x_s + M_{3s} \quad (37)$$

$$(P_0(h-c-b) - F_1 q)(1+i_\ell) - T \geq P_2 x_b + M_{3b} \quad (38)$$

$$(P_0(h-c-b) - F_1 q) + ((1+i_\ell)^{-1}\gamma^{-1}c + (1+i_d)^{-1}\kappa^{-1}\alpha^{-1}(b))P_2 R \geq 0 \quad (39)$$

where we have used the condition (36) for the seller to be indifferent between the two methods.

The first order conditions

$$-1 + P_0(\lambda_1(1+i_d) + \lambda_2(1+i_\ell) + \lambda_3) = 0$$
$$(1-n)u'(q) - (\lambda_2(1+i_\ell) + \lambda_3)F_1 = 0$$
$$-n + \lambda_1 F_1(1+i_d) = 0$$
$$n\beta = \lambda_1 P_2$$
$$(1-n)\beta = \lambda_2 P_2$$
$$\frac{n\beta}{P_2} \leq \lambda_1;\ M_{3s} \geq 0$$
$$\frac{(1-n)\beta}{P_2} \leq \lambda_2;\ M_{3b} \geq 0$$
$$\beta R - P_0(\lambda_1(1+i_d) + \lambda_2(1+i_\ell) + \lambda_3) + \lambda_3(1+i_\ell)^{-1}\gamma^{-1}P_2 R \leq 0;\ c \geq 0$$
$$\beta R - P_0(\lambda_1(1+i_d) + \lambda_2(1+i_\ell) + \lambda_3) + \lambda_3(1+i_d)^{-1}\kappa^{-1}[(\alpha^{-1})'(b)]P_2 R = 0$$

simplify to

$$\begin{aligned}
P_0(n\beta(1+i_d)+(1-n)\beta(1+i_\ell)+\lambda_3 P_2) &= P_2 \\
(1-n)(u'(q)-\Delta) &= \frac{\lambda_3 P_2}{\beta(1+i_d)} \\
F_1 &= \frac{P_2}{\beta(1+i_d)} \\
(1+i_\ell)(R^{-1}-\beta) &\geq \gamma^{-1}\lambda_3 P_2;\ c \geq 0 \\
(1+i_d)(R^{-1}-\beta) &= \kappa^{-1}[(\alpha^{-1})'(b)]\lambda_3 P_2 \\
\frac{\Delta q}{R\beta} &= \gamma^{-1}c+\Delta\kappa^{-1}\alpha^{-1}(b).
\end{aligned}$$

There are two cases to consider: $c > 0$:

$$\begin{aligned}
P_2/P_0 &= n\beta(1+i_d)+(1-n)\beta(1+i_\ell)+\gamma(1+i_\ell)(R^{-1}-\beta) \\
u'(q) &= \Delta(1+\frac{\gamma(R^{-1}-\beta)}{\beta(1-n)}) \\
F_1 &= \frac{P_2}{\beta(1+i_d)} \\
\kappa^{-1}[(\alpha^{-1})'(b)]\gamma\Delta &= 1 \\
c &= \frac{\alpha^{-1}(b)}{[(\alpha^{-1})'(b)]}-\frac{\gamma\Delta q}{R\beta}
\end{aligned}$$

and $c = 0$:

$$\begin{aligned}
P_0(n\beta(1+i_d)+(1-n)\beta(1+i_\ell)+\lambda_3 P_2) &= P_2 \\
(1-n)(u'(q)-\Delta) &= \frac{\lambda_3 P_2}{\beta(1+i_d)} \\
F_1 &= \frac{P_2}{\beta(1+i_d)} \\
(1+i_\ell)(R^{-1}-\beta) &\geq \gamma^{-1}\lambda_3 P_2;\ c \geq 0 \\
(1+i_d)(R^{-1}-\beta) &= \kappa^{-1}[(\alpha^{-1})'(b)]\lambda_3 P_2 \\
\frac{\Delta q}{R\beta} &= \gamma^{-1}c+\Delta\kappa^{-1}\alpha^{-1}(b)
\end{aligned}$$

In other words, if γ exceeds a critical level, public means of payment are not used. As γ falls below that level, use of private means of payment shrinks and use of public means increases.

5 Monetary Policy when Private Payment Competes

The above analysis makes clear that, at least to the extent that monetary policy is intended to affect the real economy, it can do so by changing the margin along which agents divide between the use of private and public payment systems. In particular, in this model when payment requires the use of costly collateral, agents will increase their production of such collateral, distorting the trade-off between collateral dependent and collateral independent purcahses. In a more general model this would translate to a "liquidity preference" decision—a choice of dividing investments between collateralizable and (presumably higher return) uncollateralizable assets.

However, in the model, this power to affect the real economy is self-limiting. As the monetary authority increases the expense of using the public system, by increasing collateral requirements or by increasing the interest rate on borrowed money, agents abandon the public system. In the extreme, all activity takes place through private payments and the monetary authority is unable to effect further tightening on the economy. Monetary policy retains the ability to affect the price level—that is, the posted money price for goods. But it is a Pyrrhic victory: the activity of the economy can actually be carried out in real terms, with private lending based on real interest rates and repayments specified in real terms. The "official" nominal prices are only of academic interest.

Several important limitations remain on the analysis as presented in this paper. Probably the most important one is the lack of aggregate shocks, given that monetary policy is a tool for stabilization; introduction of such shocks into the model is a priority. The model also ignores the central feature of neo-Keynesian analysis: price stickiness. To the extent that prices are posted and sticky nominally, the authority retains some power. But as the private payments arrangements take over, we could easily imagine that the pricing of the private system is indexed to something other than the official currency, and that agents in the economy find it more convenient to price in those terms. Of course this is the situation observed in chronic inflations, where prices become pegged to a stable foreign currency. Governments will take comfort in the fact that this movement to indexation of everyday prices does not appear to occur for moderate levels of inflation.

6 Literature

The Berentsen Monnet model can be regarded as a formalization of the ideas of Woodford (2000 et seq.) about conducting monetary policy in a world with no outside money. The macroeconomic role of money as a medium of exchange has also been explored in numerous cash-in-advance models; in most of them, however, there is no flexibility in the use of publicly provided cash in payment for so-called cash goods. A recent partial exception is Sauer (2008), which examines the trade in which investors can sell illiquid shares or liquidate assets in order to trade by making payments on a goods market. In his model the central bank can prevent this liquidation by entering a repo market.

The issue of private competition with public payments arrangements is, of course, not new. In an important early paper Wallace (1983) argues, in the context of retail payment systems, that the only reasons that U.S. government issued interest bearing securities do not replace non-interest bearing Federal Reserve Notes as a transaction medium are their non-negotiability (in the case of savings bonds) and limitation to large denominations (in the case of treasury bills). But private intermediaries could solve the latter problem in particular, and make a profit, by establishing narrow banks which hold large value treasuries and issue smal denomination, riskless private notes suitable for payment. The lack of such notes in the U.S. is clearly due to legal restriction (notably, in Scotland, such legal restrictions are still not in place, and commercial banks do issue their own circulating notes). In an intriguing footnote (p.4), Wallace asks if checking accounts might in effect play the same role. He then states that "interest ceilings, reserve requirements, zero marginal-cost check clearing by the Federal Reserve and the failure to tax income in the form of transaction services ... explain the way checking account services have been priced." In the context of retail banking in the U.S. nowadays,it is hard to argue that any of these considerations make a significant difference. Thus the following sentences of the footnote become the relevant ones: "In the absence of these forms of government interference, most observers predict that checking accounts would pay interest at the market rate with charges levied on a per transaction basis"—a prediction that seems largely to have come true.[5]

But then, in Wallace's view, provided the public and private arrangements have the same ability to effect payments, an open market operation which reduces the available reserves of treasury bills to commercial banks and substitutes central bank money simply shifts payments services from private to public arrangements, without affecting interest rates, prices or economic activity. This is equivalent to our arrangement in which $\alpha = \gamma$ and interest rates are nil. Wallace assumes, unlike us, that the government has the possibility of restricting the payments in the system through legal requirements. On the other hand he assumes that the government system is constrained not to incur losses. Under these circumstances, there is an upper bound on the interest rate on default free securities when they co exist with non interest bearing government currency.

Sargent and Wallace (1982) use the overlapping generations framework of Samuelson (1958) to examine the "real-bills doctrine." In their framework, a fiat currency can compete with private credit instruments. Differences in endowments in alternating generations lead to a natural variation in relative prices of consumption good in adjacent periods. If fiat money and private lending coexist, then the return on the two must be the same, that is, the nominal interest rate on lending must be zero. When a monetary equilibrium exists there are a continuum of equilibria in general, each consistent with a different initial value of a unit of fiat money. Monetary equilibria exist as long as the population is not "too impatient." In all of these monetary equilibria but one, the value of money goes asymptotically to zero. In the remaining equilibrium, the value of money remains stationary, fluctuating with the periodicity of endowments; goods prices and money stock are positively correlated. (In addition there

[5] More questionable, however, is Wallace's view, that this effectively puts checking accounts on the "non-cash" side in inventory models of money demand.

is always a nonmonetary equilibrium, in which private borrowing and lending occurs, but money does not effect intergenerational changes.) Sargent and Wallace then consider a restriction so that some households cannot engage in private lending (because of a minimum restriction on the size of privately issued securities), forcing them to hold government issued securities. If these securities have lower return than private securities in equilbrium, rich savers hold the private securities, and the difference in returns implies suboptimal equilibria, despite the fact that by constraining the poor lenders from the market, price fluctuation can be eliminated. Sargent and Wallace argue that use of government borrowing at low levels will undo the restriction on small bills.

Goodhart (2000) considers the role of central bank in a world where electronic payments have become dominant. He has two arguments in favor of the continuing importance of the central bank: the first is that currency and electronic moneys are imperfect substitutes, particularly with regard to privacy. The second, which he contrasts to "free banking" approaches of the papers described above, is that a central bank, as a bank for a government, is able to run losses financed by the govenment's tax levying powers. Using the government's deep pockets, the central bank can always wrest control of the money supply from the private provides by standing ready to engage in loss-making open market operations. The public's knowledge of the bank's power to do so, means that in fact these activities do not need to be carried out much of the time; instead the bank can engage in "open-mouth" operations. Goodhart has in mind the exchange of central bank notes for government debt, or possibly the purchase of private bank debt. However, as we have seen, in a world where provision of private bank debt is only constrained by the availability of collateralizable assets the crucial determinant of the power of a central bank to restrict the money supply is the elasticity of the supply of collateralizable assets.

The issue of the role of cross-border collateral has been examined in several papers by central bankers. Manning and Willison examine cross-country provision of collateral,when collateral is expensive, banks engage in activity in multiple countries, and delay in payment is costly. They show that in many circumstances permitting cross-border collateral induces banks to increase the pool of collateral available for backing payments. This becomes important in the case where there is uncertainty in the overall demand for payment.

7 Conclusions

This paper has developed a model in which technologies for effecting payment provide real benefits to an economy. Ultimately through its powers of taxation, a government may have a natural comparative advantage in generating assets that can be used for payment. To the extent that it has a monopoly power over these assets, its choices for pricing them—in effect, its monetary policy—will have real effects on an economy. This power becomes the leverage with which a monetary authority can encourage or discourage economic activity in order to achieve policy goals.

However, when there exist alternative, non-public means of effecting payments, the central authority's power to affect economic activity becomes limited. When, in an attempt to reduce activity, a central bank makes payments assets more expensive, either by increasing the spread between borrowing and lending rates on the asset, or increasing the haircut required in terms of collateral, agents readjust by substituting away from public payment systems into private ones. As private systems become more effective at handling payment services, the leeway available to central banks in maintaining restrictive monetary policies is reduced.

In the framework as analyzed thus far, we have assumed that a single public entity competes with private payment providers. In the world today, in fact, the situation is more complicated: rather than a single public entity, we in fact have multiple public entities providing payment services, in effect, sequentially, through the course of the day. Extensions to this model will examine the implications for cross-border use of collateral in a world of round-the-clock payments activity.

References

[1] Berentsen, Aleksander, and Cyril Monnet (2007), "Monetary Policy in a Channel System" CESIFO working paper 1929.

[2] Goodhart, Charles (2000) "Can Central Banking Survive the IT Revolution?" *International Finance*, July,2000, Vol. 3 Issue 2, 189.

[3] Kahn, Charles M. and William Roberds (2008), "Why Pay? An Introduction to Payments Economics," forthcoming, *Journal of Financial Intermediation.*

[4] Lagos, R. and R. Wright (2005), "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463-484

[5] Manning, Mark, Erlend Nier and Jochen Schanz, eds. (2008) *A Central Bank's Perspective on Large-Value Payments and Settlement: Economic Theory and Policy Issues*, forthcoming.

[6] Samueslon, Paul A.(1958),"An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy 66, no. 6. Dec. 1958, pp 467-82.

[7] Sargent, Thomas J. and Neil Wallace (1982) "The Real-Bills Doctrine versus the Quantity Theory: A Reconsideration," Journal of Political Economy, Vol 90, No. 6, Dec 1982 pp 1212-1236.

[8] Wallace, Neil (1983), "A Legal Restrictions Theory of the Demand for 'Money' and the Role of Monetary Policy," Federal Reserve Bank of Minneapolis Quaterly Review, Winter 1983, Vol 7, no.1 pp. 1-7.

[9] Woodford, Michael "Monetary Policy in a World Without Money" *International Finance*, July 2000, Vol. 3 Issue 2, 229.