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Parameter Uncertainty and the Central Bank's Objective Function

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Abstract

In this paper, we analyze the problem of parameter uncertainty in micro-founded models in which the central bank's ultimate goal is to maximize household welfare and hence has a loss function with weights that are directly related to the deep structural parameters of the model. In such a setting, the central bank faces uncertainty not only about the dynamic behavior of the economy, but also about how much relative weight to assign to each of the variables that enter its loss function. In effect, the microeconomic foundations of the model imply cross-equation restrictions between the behavioral equations and the central bank's objective function. Using both Bayesian and robust control methods, we show that these cross-equation restrictions can be crucial in determining the policy implications of parameter uncertainty. In the Bayesian context, we find that the presence of uncertainty about the loss function weights can overturn the classic attenuation result of the earlier literature; that is, the optimal policy may exhibit certainty-equivalence or even anti-attenuation. Similarly, the degree of aggressiveness of the robust control policy may be quantitatively or even qualitatively different from that obtained when the loss function is treated as fixed and known.

1. Introduction

Beginning with the seminal work of Brainard (1967), a large body of literature has considered the monetary policy implications of uncertainty regarding the structure of the economy. Many researchers have followed a Bayesian approach (as in Brainard), minimizing the expected value of the central bank's loss function for a given prior distribution of the structural parameters.[1] This approach typically implies that the optimal policy response to shocks exhibits attenuation compared with the case of no parameter uncertainty—a result which has significantly influenced the perspective of some central bankers (cf. Blinder 1998). More recent work on robust control methods has been aimed at protecting against worst-case scenarios by minimizing the maximum loss over a specific set of possible model perturbations, and can imply that optimal policy is more aggressive than in the certainty-equivalent case (an outcome referred to as "anti-attenuation").[2] Nevertheless, a common characteristic of ***all*** of the existing studies has been to assume that the central bank has a fixed loss function with known weights on the variances of a specific set of variables such as the output gap and inflation rate.

In this paper, we analyze the problem of parameter uncertainty in micro-founded models in which the central bank's ultimate goal is to maximize household welfare and hence has a loss function with weights that are directly related to the deep structural parameters of the model. In such a setting, the central bank faces uncertainty not only about the dynamic behavior of the economy, but also about how much weight to assign to each of the variables that enter its loss function. In effect, the microeconomic foundations of the model imply cross-equation restrictions between the behavioral equations and the central bank's objective function.

Using both Bayesian and robust control methods, we show that these cross-equation restrictions can be crucial in determining the policy implications of parameter uncertainty. In the Bayesian context, we find that the presence of uncertainty about the loss function weights can overturn the classic attenuation result of the earlier literature; that is, the optimal policy may exhibit certainty-equivalence or even anti-attenuation. Similarly, the degree of aggressiveness of the robust control policy may be quantitatively or even qualitatively different from that obtained when the loss function is treated as fixed and known.

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[1] Recent examples include Wieland (2000), Sack (2000), Rudebusch (2001, 2002), Söderström (2002), and Aoki and Nikolov (2003).

[2] See Sargent (1999), Tetlow and von zur Muehlen (2000), Giannoni (2002), Onatski and Stock (2002), Hansen and Sargent (2003), and Onatski and Williams (2003).

Our analysis begins by revisiting Brainard's highly-stylized static framework, in which the central bank seeks to minimize the variance of a single target variable and faces uncertainty regarding the sensitivity of the target variable with respect to the policy instrument. Given that the loss function involves the square of the target variable, the classic result of policy attenuation reflects the extent to which the central bank seeks to avoid outcomes in which a high setting of the instrument coincides with strong policy effectiveness. Of course, a fixed leading coefficient can be incorporated into the loss function in order to quantify the loss associated with a unit variance of the target variable, but such a monotonic transformation has no influence on the optimal policy. But now suppose that the magnitude of the leading coefficient is ***inversely*** related to the effectiveness of the policy instrument. In that case, parameter realizations with strong policy effectiveness also imply relatively low costs of target variability, hence dampening or even reversing the optimal degree of attenuation.

Next, we consider these issues in the benchmark New Keynesian model with staggered price contracts, flexible wages, and exogenous cost-push shocks.[3] The log-linearized form of this model consists of two behavioral equations: a forward-looking IS curve, and a forward-looking Phillips curve. The unconditional expectation of household welfare can be expressed (to a second-order approximation) in terms of the variances of the output gap and the inflation rate, with weights involving the same parameters that enter the behavioral equations.[4]
Our analysis focuses on the implications of uncertainty regarding three key structural parameters, namely, the intertemporal substitution elasticity of consumption, the mean duration of price contracts, and the intratemporal elasticity of substitution between differentiated goods.
We obtain analytic results for the case of idiosyncratic shocks and simple Taylor-style rules (involving the contemporaneous output gap and inflation rate), and then use numerical methods to document the extent to which similar results are obtained in more general cases.

For the benchmark New Keynesian model, the presence of structural parameters in the central bank's loss function dramatically influences the characteristics of optimal policy. Just as in our analysis using the stylized Brainard framework, raising the intertemporal elasticity parameter increases the responsiveness of the output gap to the policy instrument (that is, the short-term interest rate) while reducing the welfare cost of output gap variability; hence, this

[3] See Clarida et al. (1999) and Benigno and Woodford (2003).
[4] See Rotemberg and Woodford (1997).

cross-equation restriction implies ***de-attenuation*** relative to the case of a fixed and known loss function. Similarly, raising the contract duration parameter not only reduces the slope of the Phillips curve but also increases the extent to which movements in aggregate inflation generate welfare costs (due to greater dispersion in relative prices across firms), and hence implies that aggregate inflation variability should receive greater weight in the central bank's loss function. In fact, in this case the two effects exactly cancel each other, so that the optimal Bayesian policy exhibits ***certainty-equivalence*** with respect to the contract duration parameter. Finally, uncertainty regarding the intratemporal elasticity parameter can induce ***anti-attenuation*** of the optimal Bayesian policy while inducing somewhat greater attenuation of the robust control rule.

We then go beyond the benchmark New Keynesian model to consider the effects of uncertainty regarding the structure of overlapping price contracts. The benchmark model assumes that all contracts have random duration *á la* Calvo (1983); this specification implies that some contracts last much longer than the mean duration and hence that dispersion in relative prices is highly persistent. As a result, the welfare cost of aggregate inflation variability is very high, with a weight about 100 times larger than that on output gap variability. In contrast, when all price contracts have fixed duration *á la* Taylor (1980), relative price dispersion exhibits much less persistence, and the weight on aggregate inflation variability in the welfare function is markedly smaller, with a magnitude roughly similar to that on output gap variability.[5] Thus, uncertainty about the fraction of price contracts with fixed vs. random duration has crucial implications for the relative weights in the central bank's loss function, and hence for the characteristics of optimal Bayesian and robust policy rules.

Finally, we briefly highlight these issues in models with more sophisticated dynamics. When household preferences incorporate habit persistence in consumption, then output tends to exhibit hump-shaped dynamics, while the welfare function involves the variance of the first-difference of the output gap.[6] When some prices are either indexed to the lagged aggregate inflation rate or determined by a simple rule-of-thumb, the log-linearized "hybrid" Phillips curve involves both lagged and expected inflation and thereby generates intrinsic inflation persistence, while the welfare function places weight on the variance of the first-difference of inflation.[7] Thus, the degree of habit persistence and the degree of intrinsic inflation persistence have

[5] See Erceg and Levin (2002).
[6] See Amato and Laubach (2001).

significant implications for the specification of the loss function and hence for the determination of monetary policy when the central bank faces uncertainty about these characteristics of the economy.

The remainder of this paper is organized as follows. Section 2 introduces the key issues using the highly-stylized framework of Brainard (1967). Section 3 briefly reviews the microeconomic foundations for the behavioral equations and welfare function of the benchmark New Keynesian model. Section 4 analyzes optimal Bayesian policies for this model, while Section 5 obtains corresponding results for robust control rules. Section 6 investigates the implications of uncertainty about the proportion of fixed vs. random duration contracts. Section 7 considers these issues in models with habit formation and intrinsic inflation persistence. Section 8 summarizes our conclusions and highlights areas for further research.

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[7] See Steinsson (2000) and Benigno and Lopez-Salido (2002), respectively.

2. Revisiting the Brainard Framework

2.1 The Classic Attenuation Result

- Static framework with linear relationship between target variable $\boldsymbol{Y}$ and policy instrument $\boldsymbol{X}$

$$\boldsymbol{Y} = \boldsymbol{b}\,\boldsymbol{X} \tag{1}$$

- Policymaker faces uncertainty regarding the sensitivity parameter $\boldsymbol{b}$, and has a Gaussian prior with mean $\bar{\boldsymbol{b}}$ and variance $\boldsymbol{\sigma_b^2}$.

- Loss function involves squared deviations of the target variable:

$$\mathcal{L} = \lambda\,(\boldsymbol{Y} - \boldsymbol{a})^2 \tag{2}$$

- In Brainard (1967), the objective function weight is known and fixed (at $\lambda = 1$), and hence has no effect on the determination of optimal policy.

- Minimizing expected loss yields the classic attenuation result:

$$\mathbf{E}(\mathcal{L}) = \boldsymbol{a}^2 - 2\boldsymbol{a}\bar{\boldsymbol{b}}\,\boldsymbol{X} + \left(\bar{\boldsymbol{b}}^2 + \boldsymbol{\sigma_b^2}\right)\boldsymbol{X}^2 \tag{3}$$

$$\boldsymbol{X}^* = \frac{\boldsymbol{a}\bar{\boldsymbol{b}}}{\bar{\boldsymbol{b}}^2 + \boldsymbol{\sigma_b^2}} \tag{4}$$

2.2 Uncertainty Regarding the Objective Function Weight

- Now suppose that the cost of target variability is related to the sensitivity parameter:

$$\lambda = \bar{\lambda} + \lambda_b\left(\boldsymbol{b} - \bar{\boldsymbol{b}}\right) \tag{5}$$

- Thus, the central bank faces uncertainty about the objective function weight λ.

- The expected loss can be expressed as follows:

$$\mathbf{E}(\mathcal{L}) = \bar{\lambda}\,\boldsymbol{a}^2 - 2\boldsymbol{a}\left(\bar{\lambda}\,\bar{\boldsymbol{b}} + \lambda_b\boldsymbol{\sigma_b^2}\right)\boldsymbol{X}$$

$$+ \left(\bar{\lambda}\,\bar{\boldsymbol{b}}^2 + \bar{\lambda}\boldsymbol{\sigma_b^2} + 2\lambda_b\bar{\boldsymbol{b}}\,\boldsymbol{\sigma_b^2}\right)\boldsymbol{X}^2 \tag{6}$$

- Optimal policy reflects uncertainty regarding the objective function weight:

$$X_t^* = \frac{a\bar{b}}{\bar{b}^2 + \left[1 + \frac{\lambda_b}{\lambda}\left(a + 2\bar{b}\right)\right]\sigma_b^2} \tag{7}$$

$\lambda_b = 0$: Classic Brainard Attenuation

$\lambda_b > 0$: Stronger Attenuation

$\lambda_b < 0$: De-attenuation or Anti-Attenuation

3. The Benchmark New Keynesian Model

3.1 Microeconomic Foundations

"New Keynesian" dynamic general equilibrium model; cf. Kerr and King (1996), Bernanke and Woodford (1997), Clarida, Gali, and Gertler (1999), Woodford (1999).

- Monopolistically-competitive firms produce differentiated goods using identical production technology with capital share α.

$$Y_t(f) = A_t K_t(f)^{\alpha} L_t(f)^{1-\alpha} \tag{8}$$

- Each firm faces downward-sloping demand curve with elasticity θ

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t \tag{9}$$

- Competitive labor and capital rental markets. Physical capital is fixed at the aggregate level, and mobile across firms, so that all firms face identical marginal cost $MC_t = W_t \,/\, MPL_t$.

- Calvo-style staggered price contracts with average duration $1/(1 - \xi)$. Maximization of expected present discounted profits yields the following first-order condition:

$$\underset{t}{E} \sum_{j=0}^{\infty} \xi^j \delta_{t,t+j} \left((1+\tau_t) P_t(f) - \left(\frac{\theta}{\theta-1}\right) MC_{t+j} \right) Y_{t+j}(f) = 0 \tag{10}$$

- Time-varying production subsidy τ_t induces transitory fluctuations in desired markups.

• Households maximize expected utility over consumption and leisure, yielding standard FOCs in terms of the intertemporal substitution elasticities of consumption *(η)* and leisure *(χ)*:

$$C_t^{-1/\eta} = \underset{t}{E}\, \beta R_t C_{t+1}^{-1/\eta} \tag{11}$$

$$W_t / P_t = \left(1 - L_t\right)^{-\chi} C_t^{1/\eta} \tag{12}$$

3.2 Log-Linearized Behavioral Equations

• We assume that the production subsidy offsets the steady-state monopolistic distortion, and that the central bank targets zero steady-state inflation. Thus, the steady state is Pareto-optimal.

• Log-linearizing the behavioral equations around the steady state yields the following expressions involving the inflation rate π_t , the output gap y_t , the nominal interest rate i_t , the equilibrium real interest rate r_t^* and the exogenous aggregate supply disturbance ε_t :

$$y_t = \underset{t}{E}\, y_{t+1} - \eta\left(i_t - \underset{t}{E}\, \pi_{t+1} - r_t^* \right) \tag{13}$$

$$\pi_t = \beta \underset{t}{E}\, \pi_{t+1} + \kappa\left(v + \eta^{-1}\right) y_t + \varepsilon_t \tag{14}$$

where

$$\kappa = \frac{\left(1-\xi\right)\left(1-\beta\xi\right)}{\xi} \tag{15}$$

$$v = \frac{\alpha + \chi^{-1} s_L}{1-\alpha} \tag{16}$$

3.3 The Social Welfare Function

• We assume that the monetary policymaker's objective is to maximize the unconditional expectation of household welfare.

• The welfare function is obtained by taking a second-order approximation of household welfare in the actual economy and in the Pareto-optimal economy, and measuring the difference in terms of certainty-equivalent consumption.

- Thus, the policymaker's loss function can be expressed in terms of the unconditional variances of inflation and the output gap, with weights that depend on the structural parameters:

$$\mathcal{L} = \frac{1}{2}\left[\frac{\theta}{\kappa}Var(\pi) + \left(v + \eta^{-1}\right)Var(y)\right] \tag{17}$$

- For fixed values of the structural parameters, the loss function can be equivalently expressed

$$\mathcal{L} = \Omega\,[\,\mathrm{Var}(\pi) + \lambda\,\mathrm{Var(y)}\,] \tag{18}$$

where $\Omega = \theta/\kappa$ and $\lambda = (v + \eta^{-1})\,\kappa/\theta$.

3.4 Monetary Policy Rules

- In the following analysis, we will generally focus on simple Taylor-style rules:

$$i_t = c_\pi \pi_t + c_y\, y_t \tag{19}$$

- We impose the constraint that $c_\pi > 1$, which is sufficient to ensure a unique stationary rational expectations equilibrium.

- For ease of exposition, we assume that the aggregate supply disturbance ε_t is i.i.d. (0, σ_ε^2).

- Given the absence of any intrinsic inertia in the behavioral equations, the reduced-form solution of the rational expectations equilibrium will only involve the current disturbance ε_t , with $\underset{t}{E}\,\pi_{t+1} = \underset{t}{E}\,y_{t+1} = 0$.

- Following the approach of Woodford (1999) and Giannoni (2002), we first determine the optimal coefficient ϕ in the reduced-form equation $i_t = \phi\,\varepsilon_t$ and then determine the corresponding c_π and c_y .

3.5 Rational Expectations Equilibrium

- Thus, the reduced form solution of the rational expectations equilibrium can be expressed as:

$$i_t = \phi\, \varepsilon_t \tag{20}$$

$$y_t = -\eta\phi\varepsilon_t \tag{21}$$

$$\pi_t = [1 - (1 + \nu\eta)\kappa\phi]\varepsilon_t \tag{22}$$

- The unconditional variances of the output gap and inflation rate can then be expressed as:

$$Var(\, y_t\,) = \eta^2\phi^2\sigma_\varepsilon^2 \tag{23}$$

$$Var(\pi_t) = [1 - (1 + \nu\eta)\kappa\phi]^2\,\sigma_\varepsilon^2 \tag{24}$$

3.6 The Reduced-Form Loss Function

- Therefore, the policymaker's loss function can be expressed in terms of the structural parameters $\{\eta, \kappa, \theta, \nu\}$, the reduced-form policy coefficient ϕ, and the innovation variance σ_ε^2 :

$$\mathfrak{L} \;=\; \frac{\sigma_\varepsilon^2}{2}\Big[\, \frac{\theta}{\kappa} - 2(1 + \nu\eta)\theta\,\phi$$

$$+ \Big(\kappa\theta(1 + \nu\eta)^2 + \eta(1 + \nu\eta)\Big)\,\phi^2\,\Big] \tag{25}$$

- For fixed weights Ω and λ, the loss function can be expressed as:

$$\mathfrak{L}_F \;=\; \frac{\sigma_\varepsilon^2}{2}\,\Omega\Big[1 - 2(1 + \nu\eta)\kappa\phi + \Big(\kappa^2(1 + \nu\eta)^2 + \lambda\eta^2\Big)\,\phi^2\Big] \tag{26}$$

4. Optimal Bayesian Policy in the Benchmark Model

- We now proceed to determine the optimal policy coefficient ϕ^* when the policymaker faces uncertainty about η, κ, and/or θ and has ***well-defined prior beliefs*** about the distribution of these parameter(s).

- We first consider the case in which the loss function weights Ω and λ are treated as known, and fixed at values based on the prior mean of each parameter.

- We then determine the optimal policy that accounts for the link between the structural parameters and the objective function weights.

4.1 Uncertainty about η

Prior distribution for η: mean $\tilde{\eta}$, variance σ_η^2 All other parameters treated as known and fixed.

4.1.1 Fixed Loss Function Weights

$$\mathrm{E}_\eta \mathcal{L}_F = \frac{\sigma_\varepsilon^2}{2}\Omega\Big[1 - 2(1+v\tilde{\eta})\kappa\phi + \Big(\kappa^2(1+2v\tilde{\eta}) + (\kappa^2 v^2 + \lambda)(\tilde{\eta}^2 + \sigma_\eta^2)\Big)\phi^2\Big] \tag{20}$$

Minimizing this function with respect to ϕ yields:

$$\phi_F^* = \frac{(1+v\tilde{\eta})\kappa}{\kappa^2(1+2v\tilde{\eta}) + (\kappa^2 v^2 + \lambda)(\tilde{\eta}^2 + \sigma_\eta^2)} \tag{21}$$

Using the relation $\lambda = (v + \tilde{\eta}^{-1})\kappa / \theta$, we obtain:

$$\phi_F^* = \frac{(1+v\tilde{\eta})\theta}{\kappa\theta(1+v\tilde{\eta})^2 + \tilde{\eta} + v\tilde{\eta}^2 + (\kappa\theta v^2 + v + \tilde{\eta}^{-1})\sigma_\eta^2} \tag{22}$$

4.1.2 Structural Loss Function Weights

$$\mathrm{E}_\eta \mathcal{L} = \frac{\sigma_\varepsilon^2}{2}\Big[\frac{\theta}{\kappa} - 2(1+v\tilde{\eta})\theta\phi \tag{23}$$

$$+ \Big(\kappa\theta(1+2v\tilde{\eta}) + \tilde{\eta} + (\kappa\theta v^2 + v)(\tilde{\eta}^2 + \sigma_\eta^2)\Big)\phi^2\Big]$$

Minimizing this function with respect to ϕ yields:

$$\phi^* = \frac{(1+v\tilde{\eta})\theta}{\kappa\theta(1+v\tilde{\eta})^2 + \tilde{\eta} + v\tilde{\eta}^2 + (\kappa\theta v^2 + v)\sigma_\eta^2} \tag{24}$$

4.2 Uncertainty about κ

Prior distribution for κ: mean $\tilde{\kappa}$, variance σ_κ^2 All other parameters treated as known and fixed.

4.2.1 Fixed Loss Function Weights

$$\mathrm{E}_\kappa \mathcal{L}_\mathbf{F} = \frac{\sigma_\varepsilon^2}{2}\Omega\Big[1 - 2(1+v\eta)\tilde{\kappa}\phi + \Big((\tilde{\kappa}^2 + \sigma_\kappa^2)(1+v\eta)^2 + \lambda\eta^2\Big)\phi^2\Big] \tag{25}$$

Minimizing this function with respect to ϕ yields:

$$\phi_F^* = \frac{(1+v\eta)\tilde{\kappa}}{\lambda\eta^2 + (1+v\eta)^2(\tilde{\kappa}^2 + \sigma_\kappa^2)} \tag{26}$$

Using the relation $\lambda = (v+\eta^{-1})\tilde{\kappa}/\theta$, we obtain:

$$\phi_F^* = \frac{\theta}{\eta + \theta(1+v\eta)\tilde{\kappa} + \theta(1+v\eta)\tilde{\kappa}^{-1}\sigma_\kappa^2} \tag{27}$$

4.2.2 Structural Loss Function Weights

$$\mathrm{E}_\kappa \mathcal{L} = \frac{\sigma_\varepsilon^2}{2}\Big[\theta E_\kappa(\kappa^{-1}) - 2(1+v\eta)\theta\phi + \Big(\tilde{\kappa}\theta(1+v\eta)^2 + \eta(1+v\eta)\Big)\phi^2\Big] \tag{28}$$

Minimizing this function with respect to ϕ yields:

$$\phi^* = \frac{\theta}{\eta + \theta(1+v\eta)\tilde{\kappa}} \tag{29}$$

4.3 Uncertainty about θ

Prior distribution for θ: mean $\widetilde{\theta}$, variance σ_{θ}^{2} All other parameters treated as known and fixed.

4.3.1 Fixed Loss Function Weights

$$\mathcal{L}_{\mathrm{F}} = \frac{\sigma_{\varepsilon}^{2}}{2}\Omega\left[1 - 2(1+v\eta)\kappa\phi + \left(\kappa^{2}(1+v\eta)^{2}+\lambda\eta^{2}\right)\phi^{2}\right] \tag{30}$$

Minimizing this function with respect to ϕ yields:

$$\phi_{F}^{*} = \frac{(1+v\eta)\kappa}{\lambda\eta^{2}+(1+v\eta)^{2}\kappa^{2}} \tag{31}$$

Using the relation $\lambda = (v+\eta^{-1})\kappa/\widetilde{\theta}$, we obtain:

$$\phi_{F}^{*} = \frac{\theta}{\eta+\theta(1+v\eta)\kappa} \tag{32}$$

4.3.2 Structural Loss Function Weights

$$\mathbf{E}_{\theta}\mathcal{L} = \frac{\sigma_{\varepsilon}^{2}}{2}\left[\frac{\widetilde{\theta}}{\kappa} - 2\widetilde{\theta}(1+v\eta)\phi + \left(\widetilde{\theta}\kappa(1+v\eta)^{2}+\eta(1+v\eta)\right)\phi^{2}\right] \tag{33}$$

Minimizing this function with respect to ϕ yields:

$$\phi^{*} = \frac{\widetilde{\theta}}{\eta+\widetilde{\theta}\kappa(1+v\eta)} \tag{34}$$

4.4 Uncertainty about κ and θ

Priors: κ and θ are jointly distributed with means $\tilde{\kappa}$ and $\tilde{\theta}$ and variances σ_κ^2 and σ_θ^2, respectively, and with covariance $\sigma_{\kappa,\theta}$. All other parameters are treated as known and fixed.

4.4.1 Fixed Loss Function Weights

Same as previous case for uncertain κ. Using the relation $\lambda = (\nu + \eta^{-1})\tilde{\kappa} / \tilde{\theta}$, we obtain:

$$\phi_F^* = \frac{\tilde{\theta}}{\eta + (1+\nu\eta)\tilde{\kappa}\tilde{\theta} + (1+\nu\eta)\tilde{\kappa}^{-1}\tilde{\theta}\sigma_\kappa^2} \tag{35}$$

4.4.2 Structural Loss Function Weights

$$\begin{aligned} \mathrm{E}_{\kappa,\theta}\mathcal{L} \;=\; \frac{\sigma_\varepsilon^2}{2}\Bigg[& E_{\kappa,\theta}\left(\frac{\theta}{\kappa}\right) \;-\; 2\left(1+\nu\eta\right)\tilde{\theta}\,\phi \\ & + \left((\tilde{\kappa}\,\tilde{\theta} + \sigma_{\kappa,\theta})(1+\nu\eta)^2 + \eta(1+\nu\eta)\right)\phi^2\Bigg] \end{aligned} \tag{36}$$

$$\phi^* = \frac{\tilde{\theta}}{\eta + (1+\nu\eta)\tilde{\kappa}\tilde{\theta} + (1+\nu\eta)\sigma_{\kappa,\theta}} \tag{37}$$

5. Robust Minimax Policy in the Benchmark Model

- We now proceed to determine the optimal policy coefficient ϕ^* when the policymaker faces Knightian uncertainty about η, κ, and/or θ; that is, the policymaker does ***not*** have well-defined prior beliefs about these parameter(s).

- As above, we first consider the case in which the loss function weights Ω and λ are treated as known and fixed.

- We then determine the optimal policy that accounts for the link between the structural parameters and the objective function weights.

- In each case, we follow the methodology of Giannoni (2002) to determine ϕ^*.

5.1 Knightian Uncertainty about η

5.1.1 Fixed Loss Function Weights

$$\left.\frac{\partial L_F}{\partial \eta}\right|_{\phi^*} = \frac{\Omega \sigma_\varepsilon^2 \phi^* \kappa v}{\kappa v \theta + \kappa v^2 \theta \eta + v \eta} > 0 \tag{38}$$

so the worst-case occurs under maximum η.

5.1.2 Structural Loss Function Weights

$$\left.\frac{\partial L}{\partial \eta}\right|_{\phi^*} = \frac{\sigma_\varepsilon^2 \theta \phi^*}{\kappa \theta (1 + v\eta)} > 0 \tag{39}$$

so the worst-case also occurs under maximum η.

5.2 Knightian Uncertainty about κ

5.2.1 Fixed Loss Function Weights

$$\left.\frac{\partial L_F}{\partial \kappa}\right|_{\phi^*} = \frac{-\Omega\sigma_\varepsilon^2\phi^*\eta(1+\nu\eta)}{\kappa\theta(1+\nu\eta)+\eta} < 0 \tag{40}$$

so the worst-case occurs under minimum κ.

5.2.2 Structural Loss Function Weights

$$\left.\frac{\partial L}{\partial \eta}\right|_{\phi^*} = \frac{-\sigma_\varepsilon^2\theta[\eta^2+2\kappa\theta\eta(1+\nu\eta)]}{2\kappa^2[\kappa\theta(1+\nu\eta)+\eta]^2} < 0 \tag{41}$$

so the worst-case occurs under minimum κ.

5.3 Knightian Uncertainty about θ

5.3.1 Fixed Loss Function Weights

The robust policy is given by:

$$\phi_{FR}^* = \frac{\underline{\kappa}}{\underline{\kappa}^2 + \lambda\overline{\eta}^2} \tag{42}$$

But the relative weight λ is taken as fixed, so this policy does ***not*** protect the economy against the worst-case scenario.

5.3.2 Structural Loss Function Weights

$$\left.\frac{\partial L}{\partial \eta}\right|_{\phi^*} = \frac{\sigma_\varepsilon^2}{2\kappa}(1-\kappa\phi^*)^2 > 0 \tag{43}$$

so the worst-case occurs under maximum θ. Thus, the robust policy is given by:

$$\phi_R^* = \frac{\overline{\theta}}{\overline{\theta}\underline{\kappa} + \overline{\eta} + \nu\overline{\eta}^2} \tag{44}$$

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