The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

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Outline

1. Motivation and Background
2. The Bond Premium in the Standard New Keynesian Model
3. Epstein-Zin Preferences
4. Long-Run Risks
5. Conclusions
The bond premium puzzle: excess returns on long-term bonds are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Backus, Gregory, and Zin, 1989).
The **equity premium puzzle**: excess returns on stocks are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Mehra and Prescott, 1985).

The **bond premium puzzle**: excess returns on long-term bonds are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Backus, Gregory, and Zin, 1989).

Note: Since Backus, Gregory, and Zin (1989), DSGE models with nominal rigidities have advanced considerably.
The bond premium puzzle: excess returns on long-term bonds are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Backus, Gregory, and Zin, 1989).

Note:
- Since Backus, Gregory, and Zin (1989), DSGE models with nominal rigidities have advanced considerably.
Fig. 1 10-year Treasury bond yield and inflation expectations

Data are quarterly. The 10-year zero-coupon Treasury bond yield is the end-of-quarter yield from Gurkaynak, Sack, and Wright (2007). 10-year inflation expectations are from the Federal Reserve Board, which is from three sources: from 1991 onward, the data are inflation expectations from 5 to 10 years ahead from the Survey of Professional Forecasters; from 1981 to 1991, the data are inflation expectations from 5 to 10 years ahead from the Blue Chip Survey of forecasters; prior to 1981, this series was extended backward by Federal Reserve Board staff using multiple data sources and the FRB/US model.

Fig. 2 Affine, no-arbitrage model decomposition of 10-year bond yield

Data are quarterly, sampled at the end of each quarter. Source: Kim and Wright (2005).
Why Study the Term Premium?

The term premium is important:

- DSGE models increasingly used for policy analysis; total failure to explain term premium may signal flaws in the model.
- Many empirical questions about term premium require a structural DSGE model to provide reliable answers.
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The equity premium has received more attention in the literature, but the term premium:

- provides an additional perspective on the model
- tests nominal rigidities in the model
- only requires modeling short-term interest rate process, not dividends
- applies to a larger volume of U.S. securities
Some Recent Studies of the Bond Premium Puzzle

- Wachter (2005)
  - can resolve bond premium puzzle using Campbell-Cochrane preferences in endowment economy
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  - the term premium is far too small in a standard New Keynesian model, even with Campbell-Cochrane habits
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We examine to what extent the Piazzesi-Schneider results generalize to the DSGE case
Related Strands of the Literature

The Bond Premium in a DSGE Model:

Epstein-Zin Preferences and the Bond Premium in an Endowment Economy:

Epstein-Zin Preferences in a DSGE Model:

Epstein-Zin Preferences and the Bond Premium in a DSGE Model:
2 The Bond Premium in the Standard New Keynesian Model
- Define Standard New Keynesian DSGE Model
- Review Asset Pricing
- Solve the Model
- Results with the Standard Model
New Keynesian Model (Very Standard)

Representative household with preferences:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - h_t)^{1-\gamma}}{1 - \gamma} - \chi_0 \frac{l_t^{1+\chi}}{1 + \chi} \right)$$
New Keynesian Model (Very Standard)

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standard model: $h_t \equiv bC_{t-1}$

Stochastic discount factor:

$$m_{t+1} = \frac{\beta(C_{t+1} - bC_t)^{-\gamma}}{(C_t - bC_{t-1})^{-\gamma}} \frac{P_t}{P_{t+1}}$$
New Keynesian Model (Very Standard)

Representative household with preferences:

\[
\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - h_t)^{1-\gamma}}{1 - \gamma} - \chi \frac{l_t^{1+\chi}}{1 + \chi} \right)
\]

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m_{t+1} = \frac{\beta (C_{t+1} - bC_t)^{-\gamma}}{(C_t - bC_{t-1})^{-\gamma}} \frac{P_t}{P_{t+1}}
\]

Parameters: \( \beta = .99, b = .66, \gamma = 2, \chi = 1.5 \)
New Keynesian Model (Very Standard)

Continuum of differentiated firms:
- face Dixit-Stiglitz demand with elasticity $\frac{1+\theta}{\theta}$, markup $\theta$
- set prices in Calvo contracts with avg. duration 4 quarters
- identical production functions $y_t = A_t\bar{k}^{1-\alpha}l_t^\alpha$
- have firm-specific capital stocks
- face aggregate technology $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

Parameters $\theta = .2$, $\rho_A = .9$, $\sigma_A^2 = .01^2$

Perfectly competitive goods aggregation sector
Government:
- imposes lump-sum taxes $G_t$ on households
- destroys the resources it collects

$$\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \epsilon_t^G$$

Parameters $\bar{G} = .17 \bar{Y}$, $\rho_G = .9$, $\sigma^2_G = .004^2$
New Keynesian Model (Very Standard)

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- imposes lump-sum taxes $G_t$ on households
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$$\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon_t^G$$

Parameters $\bar{G} = .17 \bar{Y}, \ \rho_G = .9, \ \sigma^2_G = .004^2$

Monetary Authority:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ \frac{1}{\beta} + \pi_t + g_Y (y_t - \bar{y}) + g_\pi (\pi_t - \pi^*) \right] + \varepsilon_i^i$$

Parameters $\rho_i = .73, g_Y = .53, g_\pi = .93, \pi^* = 0, \sigma^2_i = .004^2$
Asset Pricing

Asset pricing:

\[ p_t = d_t + E_t[m_{t+1}p_{t+1}] \]
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Zero-coupon bond pricing:

\[ p_t^{(n)} = E_t[m_{t+1}p_{t+1}^{(n-1)}] \]

\[ i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)} \]

Notation: let \( i_t \equiv i_t^{(1)} \)
The Term Premium in the Standard NK Model

In DSGE framework, convenient to work with a default-free consol, a perpetuity that pays $1, $1 + $1 + $1 + ... (nominal)

Price of the consol:

$\tilde{p}(n) = 1 + \delta c_e^n t + 1 \tilde{p}(n) t + 1$

Risk-neutral consol price:

$\hat{p}(n) = 1 + \delta c e^{-it} E \hat{p}(n) t + 1$

Term premium:

$\psi(n) t \equiv \log(\delta c \tilde{p}(n) t / \tilde{p}(n) t - 1) - \log(\delta c \hat{p}(n) t / \hat{p}(n) t - 1)$
The Term Premium in the Standard NK Model

In DSGE framework, convenient to work with a default-free *consol*,

\[ \text{Price of the consol:} \quad \tilde{p}(n) = 1 + \delta c E_t m_t + 1 \tilde{p}(n) + 1 \]

\[ \text{Risk-neutral consol price:} \quad \hat{p}(n) = 1 + \delta c e^{-it} E_t \hat{p}(n) + 1 \]

\[ \text{Term premium:} \quad \psi(n) t \equiv \log(\delta c \tilde{p}(n) - 1) - \log(\delta c \hat{p}(n) - 1) \]
In DSGE framework, convenient to work with a default-free *consol*, a perpetuity that pays $1, \( \delta_c \), \( \delta_c^2 \), \( \delta_c^3 \), ... (nominal)
The Term Premium in the Standard NK Model

In DSGE framework, convenient to work with a default-free *consol*, a perpetuity that pays $1, \( \delta_c, \delta_c^2, \delta_c^3, \ldots \) (nominal)

Price of the consol:

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\tilde{p}_t^{(n)} = 1 + \delta_c E_t m_{t+1} \tilde{p}_{t+1}^{(n)}
\]
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\psi_t^{(n)} \equiv \log \left( \frac{\delta_c \tilde{p}_t^{(n)}}{\tilde{p}_t^{(n)} - 1} \right) - \log \left( \frac{\delta_c \hat{p}_t^{(n)}}{\hat{p}_t^{(n)} - 1} \right)
\]
Solving the Model

The standard NK model above has a relatively large number of state variables: $C_{t-1}, A_{t-1}, G_{t-1}, i_{t-1}, \Delta_{t-1}, \bar{\pi}_{t-1}, \varepsilon^A_t, \varepsilon^G_t, \varepsilon^i_t$.
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We solve the model by approximation around the nonstochastic steady state (perturbation methods)
Solving the Model

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We solve the model by approximation around the nonstochastic steady state (perturbation methods)

- In a first-order approximation, term premium is zero
- In a second-order approximation, term premium is a constant (sum of variances)
- So we compute a *third*-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes $n$th order approximations
Results

In the standard NK model:
- mean term premium: 1.4 bp
- unconditional standard deviation of term premium: 0.1 bp
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- mean term premium: \(1.4 \text{ bp}\)
- unconditional standard deviation of term premium: \(0.1 \text{ bp}\)

Intuition:
- shocks in macro models have standard deviations \(\approx .01\)
- 2nd-order terms in macro models \(\sim (.01)^2\)
- 3rd-order terms \(\sim (.01)^3\)
Results

In the standard NK model:

- mean term premium: 1.4 bp
- unconditional standard deviation of term premium: 0.1 bp

Intuition:

- shocks in macro models have standard deviations ≈ .01
- 2nd-order terms in macro models ∼ (.01)^2
- 3rd-order terms ∼ (.01)^3

To make these higher-order terms important,

- need “high curvature” modifications from finance literature
- or shocks with standard deviations ≫ .01
Additional Robustness Checks

This basic finding is extremely robust:

- Campbell-Cochrane habits: $\bar{\psi}^{(10)} = 2.4 \text{ bp}, \text{sd}(\psi^{(10)}) = 0.1 \text{ bp}$
- “best fit” parameters: $\bar{\psi}^{(10)} = 10.6 \text{ bp}, \text{sd}(\psi^{(10)}) = 1.3 \text{ bp}$
- larger models (CEE): $\bar{\psi}^{(10)} = 1.0 \text{ bp}, \text{sd}(\psi^{(10)}) = 0.1 \text{ bp}$
- models with investment
- internal habits
- markup shocks
- nominal wage rigidities
- real wage rigidities
- time-varying $\pi_t^*$ (long-run risk)
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- real wage rigidities
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Basic problem: even if agents in these habit-based models are very risk averse, in a DSGE setting they are able to offset the risk that they hate (high-frequency variation in $C$)
Epstein-Zin Preferences

Modify the standard NK model to incorporate Epstein-Zin preferences.

The model then has three key ingredients:

1. Intrinsic nominal rigidities
   - makes bond pricing interesting

2. Epstein-Zin preferences
   - makes households risk averse

3. Long-run risk (productivity or inflation)
   - introduces a risk households cannot offset
   - makes bonds risky
Epstein-Zin Preferences

Standard preferences:

\[ V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1} \]
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Note:

- need to impose \( u \geq 0 \)
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Note:
- need to impose \( u \geq 0 \)
- or \( u \leq 0 \) and \( V_t \equiv u(c_t, l_t) - \beta (E_t (-V_{t+1})^{1-\alpha})^{1/(1-\alpha)} \)
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- or \( u \leq 0 \) and \( V_t \equiv u(c_t, l_t) - \beta (E_t (-V_{t+1})^{1-\alpha})^{1/(1-\alpha)} \)

We’ll use standard NK utility kernel:

\[ u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1 + \chi}, \]
Epstein-Zin Preferences

Household optimality conditions with EZ preferences:

\[ \mu_t u_1 \big|_{(c_t, l_t)} = P_t \lambda_t \]
\[ -\mu_t u_2 \big|_{(c_t, l_t)} = w_t \lambda_t \]
\[ \lambda_t = \beta E_t \lambda_{t+1} (1 + r_{t+1}) \]
\[ \mu_t = \mu_{t-1} \left( E_{t-1} V_t^{1-\alpha} \right)^{\alpha/(1-\alpha)} V_t^{-\alpha}, \quad \mu_0 = 1 \]
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\[ \mu_t = \mu_{t-1} \left( E_{t-1} V_t^{1-\alpha} \right)^{\alpha/(1-\alpha)} V_t^{-\alpha}, \quad \mu_0 = 1 \]

Stochastic discount factor:

\[ m_{t,t+1} \equiv \frac{\beta u_1 \big|_{(c_{t+1}, l_{t+1})}}{u_1 \big|_{(c_t, l_t)}} \left( \frac{V_{t+1}}{(E_t V_t^{1-\alpha})^{1/(1-\alpha)}} \right)^\alpha \frac{P_t}{P_{t+1}} \]
### Table 2: Empirical and Model-Based Unconditional Moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data</th>
<th>EU Preferences</th>
<th>“best fit” EZ Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd[C]</td>
<td>1.19</td>
<td>1.42</td>
<td>2.53</td>
</tr>
<tr>
<td>sd[L]</td>
<td>1.71</td>
<td>2.56</td>
<td>2.21</td>
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<tr>
<td>sd[w']</td>
<td>0.82</td>
<td>2.08</td>
<td>1.52</td>
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<tr>
<td>sd[π]</td>
<td>2.52</td>
<td>2.25</td>
<td>2.71</td>
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<tr>
<td>sd[i]</td>
<td>2.71</td>
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</tr>
<tr>
<td>sd[i(10)]</td>
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<tr>
<td>mean[ψ(10)]</td>
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<td>0.010</td>
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<tr>
<td>sd[ψ(10)]</td>
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<td>0.000</td>
<td>0.184</td>
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<tr>
<td>mean[i(10) − i]</td>
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<td>−0.047</td>
<td>0.99</td>
</tr>
<tr>
<td>sd[i(10) − i]</td>
<td>1.33</td>
<td>1.43</td>
<td>1.33</td>
</tr>
<tr>
<td>mean[x(10)]</td>
<td>1.76</td>
<td>0.015</td>
<td>1.04</td>
</tr>
<tr>
<td>sd[x(10)]</td>
<td>23.43</td>
<td>6.56</td>
<td>9.02</td>
</tr>
</tbody>
</table>

memo: quasi-CRRA


Long-Run Risks

- Long-Run Inflation Risk
- Long-Run Real Risk
Introduce long-run inflation risk to make long-term bonds more risky:

- same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk
- long-term inflation expectations more observable than long-term consumption growth
- other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary
Motivation
Bond Premium in a DSGE Model
EZ Preferences
Long-Run Risks
Conclusions

Long-Run Inflation Risk

Fig. 1 10-year Treasury bond yield and inflation expectations

10-year zero-coupon yield
Survey-based 10-year inflation expectations

Data are quarterly. The 10-year zero-coupon Treasury bond yield is the end-of-quarter yield from Gurkaynak, Sack, and Wright (2007). 10-year inflation expectations are from the Federal Reserve Board, which is from three sources: from 1991 onward, the data are inflation expectations from 5 to 10 years ahead from the Survey of Professional Forecasters; from 1981 to 1991, the data are inflation expectations from 5 to 10 years ahead from the Blue Chip Survey of forecasters; prior to 1981, this series was extended backward by Federal Reserve Board staff using multiple data sources and the FRB/US model.

Fig. 2 Affine, no-arbitrage model decomposition of 10-year bond yield

Data are quarterly, sampled at the end of each quarter. Source: Kim and Wright (2005).
Suppose:

\[ \pi_t^* = \rho_{\pi} \pi_{t-1}^* + \varepsilon_t^* \]
Long-Run Inflation Risk

Suppose:

\[ \pi_t^* = \rho_{\pi} \pi_{t-1}^* + \varepsilon_t^* \]

Then:

- inflation is volatile, but not risky
- in fact, long-term bonds act like insurance:
  - when \( \pi^* \uparrow \), then \( C \uparrow \) and \( p^{(10)} \downarrow \)
- result: term premium is negative
Consider instead:

\[ \pi_t^* = \rho_{\pi}^* \pi_{t-1}^* + (1 - \rho_{\pi}^*) \theta^* (\pi_t^* - \pi_t^*) + \varepsilon_t^* \]
Consider instead:

\[ \pi_t^* = \rho_{\pi}^* \pi_{t-1}^* + (1 - \rho_{\pi}^*) \theta_{\pi}^* (\bar{\pi}_t - \pi_t^*) + \varepsilon_{\pi_t}^* \]

- \( \theta_{\pi}^* \) describes pass-through from current \( \pi \) to long-term \( \pi^* \)
- Gürkaynak, Sack, and Swanson (2005) found evidence for \( \theta_{\pi}^* > 0 \) in U.S. bond response to macro data releases
- makes long-term bonds act less like insurance: when technology/supply shock, then \( \pi \uparrow \), \( C \downarrow \), and \( p^{(10)} \downarrow \)
- supply shocks become very costly
- The term premium is positive, closely associated with \( \theta_{\pi}^* \)
### Table 4: Model-Based Moments with Long-Run Inflation Risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data</th>
<th>EU Preferences &amp; LR Risk</th>
<th>EZ Prefs &amp; LR Risk</th>
</tr>
</thead>
<tbody>
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<td>sd[C]</td>
<td>1.19</td>
<td>1.92</td>
<td>1.86</td>
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<td>sd[L]</td>
<td>1.71</td>
<td>3.33</td>
<td>1.73</td>
</tr>
<tr>
<td>sd[wfr]</td>
<td>0.82</td>
<td>2.55</td>
<td>1.45</td>
</tr>
<tr>
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<td>2.52</td>
<td>5.00</td>
<td>3.22</td>
</tr>
<tr>
<td>sd[i]</td>
<td>2.71</td>
<td>4.74</td>
<td>2.99</td>
</tr>
<tr>
<td>sd[i^{10}]</td>
<td>2.41</td>
<td>3.32</td>
<td>1.94</td>
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<tr>
<td>mean[ψ^{10}]</td>
<td>1.06</td>
<td>.002</td>
<td>.748</td>
</tr>
<tr>
<td>sd[ψ^{10}]</td>
<td>0.54</td>
<td>.001</td>
<td>.431</td>
</tr>
<tr>
<td>mean[i^{10} - i]</td>
<td>1.43</td>
<td>-.062</td>
<td>.668</td>
</tr>
<tr>
<td>sd[i^{10} - i]</td>
<td>1.33</td>
<td>1.60</td>
<td>1.11</td>
</tr>
<tr>
<td>mean[x^{10}]</td>
<td>1.76</td>
<td>.003</td>
<td>.737</td>
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<td>23.43</td>
<td>16.96</td>
<td>11.83</td>
</tr>
<tr>
<td>memo: quasi-CRRA</td>
<td>2</td>
<td>65</td>
<td></td>
</tr>
</tbody>
</table>
Following Bansal and Yaron (2004), introduce long-run real risk to make the economy more risky:

Assume productivity follows:

\[
\log A_t^* = \rho_{A^*} \log A_{t-1}^* + \varepsilon_t^{A^*}
\]

\[
\log A_t = \log A_t^* + \varepsilon_t^A
\]

where \( \rho_{A^*} = .98 \), \( \sigma_{A^*} = .002 \), and \( \sigma_A = .005 \).

- makes the economy much riskier to agents
- increases volatility of stochastic discount factor
## Results

### Table 3: Model-Based Moments with Long-Run Productivity Risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data</th>
<th>EU Preferences &amp; LR Risk</th>
<th>EZPrefs &amp; LR Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd[C]</td>
<td>1.19</td>
<td>0.92</td>
<td>2.95</td>
</tr>
<tr>
<td>sd[L]</td>
<td>1.71</td>
<td>1.03</td>
<td>1.32</td>
</tr>
<tr>
<td>sd[w']</td>
<td>0.82</td>
<td>1.43</td>
<td>1.90</td>
</tr>
<tr>
<td>sd[π]</td>
<td>2.52</td>
<td>1.12</td>
<td>3.14</td>
</tr>
<tr>
<td>sd[i]</td>
<td>2.71</td>
<td>1.17</td>
<td>2.88</td>
</tr>
<tr>
<td>sd[i(10)]</td>
<td>2.41</td>
<td>0.65</td>
<td>1.84</td>
</tr>
<tr>
<td>mean[ψ(10)]</td>
<td>1.06</td>
<td>.005</td>
<td>.872</td>
</tr>
<tr>
<td>sd[ψ(10)]</td>
<td>0.54</td>
<td>.000</td>
<td>.183</td>
</tr>
<tr>
<td>mean[i(10) − i]</td>
<td>1.43</td>
<td>−.018</td>
<td>.758</td>
</tr>
<tr>
<td>sd[i(10) − i]</td>
<td>1.33</td>
<td>0.64</td>
<td>1.15</td>
</tr>
<tr>
<td>mean[x(10)]</td>
<td>1.76</td>
<td>.005</td>
<td>.859</td>
</tr>
<tr>
<td>sd[x(10)]</td>
<td>23.43</td>
<td>4.39</td>
<td>11.59</td>
</tr>
</tbody>
</table>

**memo:** quasi-CRRA 2 35
Conclusions

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Conclusions

1. The term premium in standard NK DSGE models is very small, even more stable.

2. Habit-based preferences can solve bond premium puzzle in endowment economy, but fail in NK DSGE framework: although agents are risk-averse, they can offset that risk.

3. Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework: agents are risk-averse and cannot offset long-run real or nominal risks.

4. Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments.