Simple Rules in the M1-VECM*

Denise Côté and Jean-Paul Lam

Department of Monetary and Financial Analysis
Bank of Canada
234 Wellington Street
Ottawa, ON K1A OG9
Canada

dcote@bank-banque-canada.ca
jplam@bank-banque-canada.ca


Abstract

This paper analyses various simple interest rate rules using a vector error correction forecasting model of the Canadian economy that is anchored by long-run equilibrium relationships suggested by economic theory. Dynamic and stochastic simulations are performed using several interest rate rules, including money based rules and their properties are analysed. Among the class of rules we consider in this model, we find that a simple rule with interest rate smoothing minimizes the volatility of output, inflation and interest rate. This rule dominates Taylor-type, Ball and other simple rules.

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1. Introduction

Research on monetary policy rules has exploded in the last few years. Much of this research has focused on finding a simple benchmark rule that the central bank can use in its decision making process. This increased interest in monetary policy rules, particularly Taylor-type rules, among academics, central bankers and other researchers, has produced a voluminous amount of information. In recent years, research on Taylor rules has focused primarily on finding a robust rule that works well across different models. Uncertainty about the structure of the economy has motivated researchers to try to find a rule that is robust to changes in model specifications. These simple rules, though not optimal, seem to perform better across models compared to more complicated rules which are usually model specific. Complex rules can be fine-tuned to account for the specific dynamics of a model but usually perform poorly when tested in other models. A simple rule that works well across models, does not only provide more robustness in the face of uncertainty, but is also easier to understand and communicate and is thus conducive in establishing and maintaining credibility. However, while it is generally true that simple rules are more robust than complicated rules in the face of uncertainty, not all simple rules perform equally well. Hence among the class of simple rules, the authorities face the challenge of identifying the type of rules or class of simple rules which perform well in a wide range of model specifications.

The path taken by researchers on finding a robust Taylor-type or other simple interest rate rule has been remarkably similar. Small estimated or calibrated models with or without rational expectations and large econometric models with rational expectations are usually used to conduct computer simulations. These models vary in level of complexity, details, degree of forward lookingness

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1. Since central banks see themselves as controlling interest rate, this work has attracted a lot of attention. The usefulness of such type of rule to policymakers however has not been well established.
2. For a comprehensive survey on feedback and Taylor rules, see Armour and Côté (1999).
3. The objective of the NBER 1998 conference on Taylor rules was precisely this (Taylor (1999)). Participants were given a set of rules and were asked to perform computer simulations. The Riksbank-IIESA conference in 1998 also looked at interest rate rules in different models. A forthcoming workshop at the Bank of Canada in October 2001 will look at Taylor rules in models of the Canadian economy.
4. By simple rule, we mean a rule which is linear and which is a function of only a small number of state variables.
5. For more details, see Levin, Wieland and Williams (1999). They show that more complex rules are less robust to model uncertainty than simpler rules. More complex rules have often to be recalibrated following changes in a model, whereas it is less the case for simpler rules.
and openness but all are general equilibrium models, have some form of nominal rigidity, either price and/or wage rigidity and are either dynamic or stochastic in nature. Moreover, rules are usually evaluated according to their ability to minimize the variability of key economic variables, in particular the inflation rate, output and interest rate.

Most of the studies looking at Taylor-type rules have been conducted on models of the US economy with few studies done for Canada. Some notable exceptions are Amano (1997), Atta-Mensah (1998), Armour, Fung and Maclean (2000) and Weymark (2000). Amano (1997) estimates a Taylor-type rule for Canada for the period 1974 to 1997 and argues that the rule provides a useful benchmark for comparison with actual movements of the overnight rate but should not be considered as a viable policy alternative. Atta-Mensah (1998) compares the historical behaviour of short-term interest rate with a Taylor rule and argues that such a rule explains recent history fairly well. Armour et al (2000) investigates several Taylor-type rules in the Bank of Canada’s Quarterly Projection Model (QPM) and compare them with QPM’s base case rule which is of an inflation forecast based rule type (IFB). Their results show that Taylor-type rules induce more variability in output, inflation and interest rates as compared to the base case rule and hence do not perform well. However, in the class of Taylor-type rules that minimizes the variability of key economic variables, they recommend a simple Taylor rule with a coefficient of 2 on the contemporaneous inflation gap and 0.5 on the contemporaneous output gap. Weymark (2000) uses a model à la Svensson (1997) and Woodford (1999) and derives efficient classes of rules and the efficient range for the coefficients on the output and inflation gap for various countries including Canada. She argues that for the period 1983-96, the original Taylor rule would not have been an efficient rule for Canada. 8

This paper looks at Taylor, Ball type and other simple rules in a different model of the Canadian economy which is used to forecast inflation at the Bank. It is in the same spirit as the

6. The yield spread and not the overnight rate is used as the instrument.
7. These models basically consist of an expectational IS curve and a New Keynesian Phillips curve. They have reasonable micro foundations and are considered by some as a workhorse model when it comes to monetary policy evaluation. For more details on these models, see Kerr and King (1996), Goodfriend and King (1997), McCallum and Nelson (1999) and Haldane and Batini (1999).
8. For the six countries she looks at (France, Germany, Canada, Italy, UK, US), she concludes that the US was the only country to have implemented an efficient interest-rate policy. However, it seems that her results may seriously depend on the Lucas type Phillips curve she assumes in the paper. This type of Phillips curve may not be appropriate for the countries she looked at and since the original Taylor rule is sensitive to model specification, her results should be interpreted with caution.
paper by Armour et al (2000) and can be regarded as part of a series of papers that will attempt to find a robust Taylor rule in models of the Canadian economy. Various Taylor type and other simple rules are explored in the context of a vector error correction model (VECM)\(^9\) and the properties of each rule is assessed. The rules examined in this paper do not exhaust all possibilities but rather are a useful benchmark with which other studies can be compared to.\(^{10}\) We also experiment with some rules involving a monetary aggregate.\(^{11}\) Money supply rules are particularly interesting in the context of the M1-VECM since the role of money is highlighted in such a model. It will not only provide an alternative view of the formulation of policy but can also be used to compliment interest rate rules. The objective of this paper is simple. We compare the performance of many simple interest rate rules with the existing reaction function in the VECM. Among the class of simple rules, we also identify which one works reasonably well and hence can possibly be used for policy advice.

We find that in general, the Taylor rule does not perform as well as the base case reaction function\(^{12}\) since it induces more variability in output, inflation and especially in interest rates. A possible intuition behind this result is as follows: by considering only the output and inflation gap when setting interest rates, the Taylor rule ignores important features of the model, in particular the money gap. Unlike in many other models, the money gap affects the structural equations of the model and since money plays an important role in the transmission mechanism in this model, it is not surprising to see that the Taylor rule does not perform as well as the base case rule. However, in the class of Taylor-type rules, we tend to find that the original Taylor rule i.e., a rule with a coefficient of 0.5 on the contemporaneous output gap and contemporaneous inflation gap, performs reasonably well.\(^{13}\) Our findings also tend to show that the Ball and change rules do

\(^{9}\) For a detailed exposition about the M1-VECM, see Hendry(1995), Adam and Hendry((1998).

\(^{10}\) There are certainly other combinations that will perform better. The objective of this paper is not only to see if a Taylor rule would work well in the M1-VECM but also to find a robust rule that can be used in other models of the Canadian economy.

\(^{11}\) The performance of money supply rules may be better than interest rate rules especially when inflation is high and volatile. In such circumstances, inflationary expectations and the output gap are harder to measure and hence this reduces the usefulness of interest rate rules.

\(^{12}\) We sometimes refer to the base case reaction function as the base case rule. The existing reaction function in the VECM is estimated over history and is not a rule in the strict sense. However, we can regard such reaction function as representing an unconstrained rule.

\(^{13}\) We also find a similar result when we consider rules involving the yield spread, not the overnight rate, as the instrument. In some cases we investigated, rules using the yield spread slightly outperform the original Taylor rule.
not perform as well as simple Taylor rules. This may largely be explained by the uncertainty surrounding the calculation of the equilibrium exchange rate.\textsuperscript{14} Because of the added uncertainty about the equilibrium exchange rate and since it performs poorly, we do not recommend using the Ball rule in this context.

We also find that a rule involving the growth rate of money performs relatively well compared to the Taylor rule but is inferior to a rule with smoothing. Like Clarida, Gali and Gertler (2000) and Levin et al (1999), we find that rules with interest rate smoothing tend to perform relatively well compared to the base case rule and other simple rules. In some cases, rules with smoothing, significantly reduce the variability of interest rates without inducing any increase in the volatility of output and/or inflation. The smoothing parameter was obtained by estimating some reaction functions for Canada using the same method as in Clarida, Gali and Gertler (2000).

The paper is organized as follows. Section 2 offers a very brief overview of the literature on Taylor type rules and motivates our paper. Section 3 gives a short description of the M1-VECM, some of the shocks we consider. Section 4 explains how Taylor type rules are incorporated in the VECM. Section 5 describes how Taylor-type rules behave in the model when similar shocks are considered and presents our results, section 6 presents our results from some stochastic simulations performed using the “best” set of rules and section 7 finally concludes.

2. Literature review

Taylor (1993) has shown that the following rule can predict the behaviour of the federal funds rate from 1987 to 1992 reasonably well.

(1) \[ i_t = 2 + \pi_t + \alpha(\pi_t - \pi^*) + \beta(y_t - y^*) \]

where \( i_t \) is the overnight rate

\( (y_t - y^*) \) is the output gap

\( \pi_t \) is the four quarter inflation rate

\( \pi^* \) is the inflation target

\textsuperscript{14} When using the Ball rule, the equilibrium exchange rate has to be estimated. However, this model does not impose any steady state condition on the level of the exchange rate. To circumvent this problem, we assume a constant value for the equilibrium exchange rate. We also experiment with rules involving the growth rate of the exchange rate.
According to the Taylor rule, the overnight rate responds to the contemporaneous deviation of inflation from its target and to the contemporaneous output gap.\(^\text{15}\) Taylor assumes a long-run equilibrium value of 2 for the real interest rate and a constant weight of 0.5 was imposed on both gaps.\(^\text{16}\) This simple rule which has been modified in a number of ways, continues to attract a lot of attention and has performed reasonably well in a wide variety of models and across countries.

The inflation and output gap term reflects some of the objectives of the central bank which is low and stable inflation and promoting sustainable output growth. The contemporaneous output gap term is also often regarded as bringing a forward-looking dimension to the Taylor rule since it is viewed as capturing future increases in inflation. This view relies heavily of course on the assumption that the measure of the output gap is reliable and has substantial information content on future inflation, an assumption which can be weak.

In recent years, research on Taylor rules have focused more heavily on finding a rule which can be used across a wide range of models and which can be applied to different countries. There is now more evidence that simple rules like Taylor’s perform better in a wide variety of models than more complex rules and are also robust across countries.\(^\text{17}\) For example Levin et al (1999), using four different structural macroeconomic models\(^\text{18}\), find that simple rules are more robust to model changes than more complicated rules. The four models used are different in many respects in terms of level of aggregation, specification of output and price dynamics, degree of openness and forward lookingness and estimation technique and period. They find that a rule which reacts to the current output gap, the one to three year moving average of the inflation rate

\(^\text{15}\) McCallum (1999) has argued that such type of rules may not be operationally feasible as the inflation and output gap are usually not known at the time the authorities have to set interest rates. This type of operationality issue has also been emphasized by Orphanides (1997). Orphanides (1997) argues that the current period values for the output gap are not observed and are usually subject to frequent and substantial revisions after they are initially reported. He argues that these problems are so severe that the Taylor rule would not have prevented the inflation of the 1970s as claimed by Taylor if revised data on the output gap is used. We experimented with a more “operational” Taylor rule by including the lagged values of the output and inflation gap instead of their contemporaneous values but found it made little difference to our results.

\(^\text{16}\) Taylor (1993) deduced his rule from the observed behaviour of the FOMC. However, such rule can also be derived from a simple IS-PC model.

\(^\text{17}\) Although results on the robustness of the Taylor rule come mostly from closed economy models with similar framework.

\(^\text{18}\) The four models used are Fuhrer and Moore model, Taylor’s Multi-Country Model, The FRB Staff model and the MSR model of Orphanides and Wieland.
and which contains a high degree of interest rate smoothing performs very well in all four models while more complicated rules perform much worse when tested in all four models. This result is rather intuitive as more complicated rules are generally fine-tuned to account for the specific dynamics of a model and when tested out, they are likely to perform poorly as the dynamics or specifications of different models are not the same. They also show that a simple rule with a high degree of smoothing outperforms both the original Taylor rule and inflation forecast based rules and is more robust to model changes.

The conference on monetary policy rules held at the NBER in 1998 also tested several simple rules in nine different models. The findings from this conference tend to confirm the robustness of simple rules, although there are disagreements about the type and specification of the simple rule. For example, Haldane and Batini (1999) and Rudebusch and Svensson (1999) find that forecast based rules are preferred to the simple Taylor-type rule. Ball (1999) argues that an MCI based rule is preferred to the simple Taylor rule and a rule with some interest rate smoothing. He also shows that rules with interest rate smoothing not only generated a higher variance for output and inflation but also for interest rates. On the other hand, Rotemberg and Woodford (1999) propose a rule with a small weight on the output gap and a very high weight on the lagged interest rate.

When dealing with Taylor rules, there are other type of uncertainties which are often ignored. The Taylor rule is made up of four components: the output gap, the inflation gap, the equilibrium interest rate and the weights assigned to the two gaps. As mentioned previously, there is a lot of uncertainty surrounding the output gap and how it is calculated. Moreover, the rule assumes a constant value for the equilibrium interest rate. The equilibrium interest rate will most likely fluctuate in response to real disturbances and assuming a constant value may be incorrect. There is also some uncertainty about which measure of inflation to use. For example, the results may be sensitive to whether total CPI or any other measure of inflation is used. Kozicki (1999) presents evidence on how these Taylor type rules may not be robust when some assumptions

19. The participants were given 5 different rules to test in their models. For more details, see Taylor (1999).
20. Ball’s results should be interpreted with caution as his model is a backward looking model. In backward looking models, the expectational channel is shut off, hence movements in short-term rates usually lead to substantial changes in the long-term rate and hence real output. As a result, efficient policy is characterized by very little persistence in interest rates.
21. Estimates of the equilibrium interest rate may also not be reliable and share some of the problems associated with estimating the output gap.
about the rule are altered. She argues that policy recommendations can vary depending on how the output gap is measured and which measures of the equilibrium interest rate and inflation are used. We do not address many of these issues in this paper but are aware that this type of uncertainty is important.

3. A Brief Description of the M1-VECM

The model which is used in this paper is very different from the traditional sticky price-’intertemporal IS’ model usually used to analyse interest rate rules and monetary policy questions. Most of the models used to evaluate the robustness of Taylor type rules do not have an explicit role for money. In these models, the money supply is endogenous and money has a passive role (the monetary authority simply has to supply the right amount of money to achieve the desired level of interest rate.) Moreover, since changes in money growth do not enter in any of the structural equations and hence do not feed back into the model, money can often be ignored in these models. In this paper, we investigate the performance of Taylor type rules in a model which has at its heart a long-run money demand function and where the role of money in the transmission mechanism is highlighted. The M1-VECM estimates a long-run relationship between M1, prices, output and short-term interest rate and this vector is regarded as the long-run demand for money. In this model, even with an interest rate rule, money does not play a passive role. As the money supply has to change to achieve the desired level of interest rate, this leads to changes in the money gap, the difference between actual money supply and estimated long-run money demand, and since the money gap feeds into the output and price equation, it has an effect on these variables.

22. Models with an IS curve, a Phillips curve and an interest rate rule often ignore the LM curve. In these models, once the interest rate is determined, the LM curve only tells us the amount of money that must be supplied to achieve the desired level of interest rate and hence is residually determined.
23. For money to have a role in these sticky price IS-LM models with a Taylor type interest rate rule, a monetary aggregate can be included in the IS function for example. McCallum (2001) derives such a function but shows that the inclusion of a monetary aggregate in such a framework is not quantitatively important and can therefore be excluded. The same conclusion is reached by Ireland (2000).
24. The long-run equilibrium relationships in the model are combined with a short-run dynamic structure on which much uncertainty lies. Money, in particular the money gap feeds into the structural equations of the model.
The M1-based vector-error-correction model (M1-VECM), which currently provides the analytical background against which the Monetary and Financial Department’s policy recommendations are designed, is a model which gives a primary role to the money gap, the difference between money demand and supply. In most large scale economic models, inflation is largely driven by the level of excess demand in the goods and labour market and since inflation growth depends on excess demand, usually through a Phillips curve relationship, this excess demand causes inflation. Unlike these models, the M1-VECM model, first developed by Hendry (1995), uses an active-money paradigm in which disequilibrium between actual money supply and estimated long-run money demand, called the money gap, causes inflation.

Using the Johansen-Juselius (1990) methodology, the model estimates a unique and stable long-run cointegrating vector between M1, CPI, real GDP and short-term interest rate. This relationship is interpreted as the long-run money demand function. The long-run money demand function is then used to calculate the money gap. Adams and Hendry (1999) show that the money gap has moved very closely with inflation over the last forty years and hence may help to predict inflation. In this model, the cointegrating vectors determine the steady-state behaviour of the variables while the dynamic response of the main variables of the model is determined by the VAR portion. This model can be regarded as addressing the classic problem of combining long-run equilibrium relationships on which one has a high degree of confidence with a short-run dynamic structure on which much uncertainty lies.

The model has four main forecasting equations and the money gap enters in each of these equation, indicating the role money plays in this framework. The reduced-form system represented below defines the main macroeconomic linkages that determine money growth, inflation, output growth and the change in overnight rate.

\[
\begin{bmatrix}
\Delta M_1 \\
\Delta CPI \\
\Delta Y \\
\Delta R
\end{bmatrix} = \Gamma(L) \begin{bmatrix}
\Delta M_1 \\
\Delta CPI \\
\Delta Y \\
\Delta R
\end{bmatrix} + \begin{bmatrix}
\alpha^{M_1} \\
\alpha^{CPI} \\
\alpha^{Y} \\
\alpha^{R}
\end{bmatrix} MGAP_{t-1} + \Theta_t Z_t
\]
where \( G(L) \) is a matrix of fourth-order lag operator, \( \text{MGAP}_{t-1} \) is the money gap derived using the long-run money demand function and \( Z_t \) is a matrix of predetermined and dummy variables.\(^{25}\) The model is then estimated with some equilibrium conditions imposed on output, inflation, money growth and the overnight rate.

In this paper, the model and the various rules are analysed under different shocks. We assume that the economy can be hit by a demand, nominal exchange rate, money or US interest rate shock. Except for the US interest rate shock, we assume that the economy is hit by a one percent shock in period 1\(^{26}\) and this shock declines by a factor of 0.25% each quarter and dies out after four quarters. In the case of a US interest rate shock, we assume that interest rates are reduced by 25 basis points for four quarters starting in period 1.

### 4. Simple Rules in the M1-VECM

We investigate several simple interest rate rules in the model. These simple rules are imposed only over the forecast or projection period while the existing base case rule is used over the historical period. The rationale behind this is simple. When some of these simple rules were imposed over history, the model had some problems converging to its steady state and it was decided to use these rules only over the projection period. More importantly, as we are mostly interested in comparing the performance of simple rules with each other and the existing base case rule, it is not unreasonable to impose the new rule only over the forecasting period and determine whether or not it would be optimal to use such type of rules. The properties of each rule are assessed in terms of the resulting variability of output, inflation and interest rates and the speed at which the economy returns to its steady state. The latter will not play a significant role in influencing our results as these simple rules are imposed in such manner to ensure that the model eventually converge to its steady state.

As already mentioned, we look at many different type of rules, e.g open economy rules, generalized Taylor rules or rules with smoothing, rules with the yield spread as the instru-

\(^{25}\)Please note that this is a very raw representation of the M1-VECM. For a more accurate description of the actual model, see Adams and Hendry (1999).

\(^{26}\)Our simulations with the shocks start in the last quarter of 2000.
ment, rules with more than one lag of the output and inflation gap and rules containing the money growth rate. Some of these rules have performed well in one or many models. Since we want to compare our results with other studies, the coefficients selected for the different types of rules were of similar magnitude. In addition, we also investigate many other different combinations and those which were inferior to the benchmark rule were rejected. For ease of comparison and exposition, the “best” rule in each category is selected and these rules are then compared to each other and to the base case reaction function.

Open economy rules are motivated by the findings of Ball (1999) who argues that the simple Taylor rule should be modified in open economy models to account for exchange rate effects. According to Ball (1999) adding the exchange rate to the original Taylor rule improves macroeconomic performance in small open economy models. In our case, we look at several types of open economy rules. In the first case, we look at the Ball rule where the exchange rate gap, i.e. the difference between the actual exchange rate and the equilibrium rate, is added to the benchmark rule. The Ball rule is given by

\[
i_t = \hat{i} + \alpha(\pi_t - \bar{\pi}) + \beta\tilde{y}_t + \gamma(e_t - \bar{e})
\]

where \(\pi_t\) is core inflation  
\(e_t\) is the nominal exchange rate  
\(\bar{e}\) is the equilibrium exchange rate

In addition to the uncertainty surrounding the output gap, the Ball rule introduces other uncertainties. When using the Ball rule, the equilibrium exchange rate has to be calculated. As the M1-VECM model does not have a steady state or equilibrium level for the nominal exchange rate, we have to make some assumptions about the equilibrium value of the exchange rate. As in Armour et al (2000), we assume a constant value for the steady state value of the exchange rate throughout our exercise. Moreover, since we do not know the degree of exchange rate pass-through, we investigate with two measures of inflation when using open economy rules, core inflation and core-core inflation, i.e., direct exchange rate pass-through effects are removed.

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27. Ball (1999) argues that the inclusion of the exchange rate as a policy variable in his model, reduces output variability by around 17% for the same amount of inflation variability. Since most models are closed economy models, the robustness of such results needs further checking.
from core inflation. To construct our core-core inflation series, we follow Armour et al (2000)\textsuperscript{29} and subtract a weighted average of ten lags of the exchange rate from the level of core prices. Because of the above uncertainties, the Ball rule is often avoided.\textsuperscript{30}

We also look at another open economy rule which is similar to the Ball rule, the change rule. When the change rule is used, the exchange rate gap is replaced by the change in the exchange rate and the equilibrium exchange rate need not be estimated. The change rule is given by

\begin{equation}
\hat{i}_t = \hat{i} + \alpha (\pi_t - \bar{\pi}) + \beta \hat{y}_t + \gamma (e_t - e_{t-1})
\end{equation}

To ensure that the model converges, we assume that any gap in the exchange rate disappears after fifty years. While the change rule avoids some of the uncertainties surrounding the Ball rule, it has the drawback of not being formally derived from a model. We also conducted some experiments with rules containing the change in the rate of growth of the exchange but found that these rules perform poorly.

We also investigate rules with interest rate smoothing as in Clarida et al (2000) and Levin et al (1999). According to them, it is common practice among central banks to adjust interest rates only gradually and the desired level is usually achieved after several small and smooth changes. Clarida et al (2000) estimate reaction functions for different countries and show that the coefficient on the lagged interest rate is relatively high, indicating a very high degree of smoothing. When rules with interest rate smoothing are introduced in the four models Levin et al (1999) consider in their study, they show that the volatility of output, inflation and interest rate was significantly reduced. They argue that from the numerous permutations of simple rules they investigate, rules with a high degree of smoothing provide the most significant gains.

Interest rate smoothing rules are given by

\begin{equation}
\text{28. A constant value for the equilibrium exchange rate is a gross oversimplification of the real world and in the case of Canada, this is very unrealistic. We are fully aware of this limitation and this is why the results for the Ball rule should be treated with caution. We experiment with several different values for the equilibrium exchange rate. In almost all cases, the Ball rule performs very poorly and should not be used in this model.}
\text{29. For more details, see page10 in Armour et al (2000).}
\text{30. Even when these uncertainties are partially accounted for, many studies do not find any gain from using the Ball rule over the Taylor rule even in an open economy model.}
\end{equation}
Combining (4a) and (4b), we get equation 4

\[(4) \quad i_t = \rho i_{t-1} + (1 - \rho)i_t^{**} \]

\[(4b) \quad i_t^{**} = \alpha(\pi_t - \pi^*) + \beta(y_t - y^*) + i^* \]

There are several reasons to explain why interest rate smoothing may be common practice among central bankers.\(^{31}\) With smoothing, large volatility in interest rates can often be avoided, therefore minimizing disruptions in the financial markets. It may also reflect an element of prudence on the part of central bankers. Moreover, frequent reversals in interest rates usually undermine the credibility of a central bank and are usually avoided. More importantly, a policy rule which contains a lagged interest rate, implicitly responds not only to the contemporaneous output and inflation gap but also to lagged output and the inflation gap. In other words, a rule with interest rate smoothing provides some measure of the historical behaviour of the economy and thus incorporates more information about the economy. Hence, instead of reflecting partial adjustment, the lagged interest rate term may be evidence of persistent special factors or events not properly accounted for in the rule. This is interesting in the context of this model. Lagged interest rates may not only contain information on lagged output and inflation but also on the money gap. Thus a rule with inertia or smoothing implicitly takes into account the information content of the money gap. This is a possible reason why this type of rule works well in this model.

Moreover, Goodfriend (1991) has argued that smooth changes in short-term interest rates provide greater control over long-term interest rates. This is important especially if long-term rates play an important role in economic decisions. By controlling the long-term rate, a central bank can thus have a better control on output and inflation. If such is the case, then central banks engage in smoothing not because they care about short-term interest rates but rather about having greater control on output and inflation.

Recently Woodford (1999) has put forward another reason why rules with interest rate smoothing perform well in forward looking models.\(^{32}\) According to Woodford (1999), inter-

\[^{31}\] See Sack and Wieland (1999) for example.
est rate smoothing rules can help a central bank improve its performance when it is operating under discretion. Woodford (1999) shows that when a central bank is acting under commitment, the optimal outcome displays a large degree of policy inertia. Since policy under commitment is not realistically feasible\(^{33}\), a rule that displays a large degree of inertia under discretion can help the central bank move closer to the commitment outcome. This argument can be better appreciated with the following example. In an economy with forward looking agents, current inflation depends also on expected inflation. Suppose the economy is hit by an inflationary shock that requires a contractionary monetary policy. If the contraction is expected to persist, expected inflation will fall and this will in turn reduce current inflation, improving the trade-off between output and inflation. Hence a given reduction in inflation can be achieved at a smaller but more persistent reduction in the output gap.

When rules with smoothing are used, Taylor’s original coefficients are kept but they are adjusted accordingly to reflect the degree of interest rate smoothing. We follow Clarida et al (2000) and assume that the coefficients on the output and inflation gap are adjusted by a factor of \((1-\rho)\) where \(\rho\) is the degree of smoothing. The smoothing parameter was obtained by performing some GMM estimation for the period 1980 to 1999 using the same method advocated by Clarida et al (2000).

The literature on simple rules has paid very little attention to money based rules. In fact, rules which give a prominent role to money have become a rarity in economics and advocates of a monetary based rule are rare. Some notable exceptions are McCallum (2000) and Meltzer (1999). The disappearance of money as an instrument or intermediate target is mostly due to the perceived instability of money demand and also to new views on the transmission mechanism. Moreover, as compared to a money based rule, the Taylor rule is much more attractive since it is expressed in terms of the instrument actually used by central banks. Recent empirical evidence on the performance of money based rules is also rare. McCallum (2000) compares the historical performance of a monetary based rule with the Taylor rule for Japan, the UK and the US. He shows that in a number of cases, especially for

\(^{32}\) Although the M1-VECM is not a forward looking model, the money gap can be regarded as providing information about future inflation.

\(^{33}\) Policy under commitment is not time consistent. A central bank always has the incentive to renege once the effects of a shock have dissipated.
Japan, a money based rule would have given better recommendations to policymakers than the Taylor rule. Despite this claim, McCallum’s rule has still not attracted a lot of attention.\footnote{McCallum (1999) argues that policymakers “view discussions of the monetary base with about the same enthusiasm as somebody would have for the prospect of being locked in a telephone booth with someone who had a bad cold or some other infectious disease.”} Since money has an important role in this model, we look at rules involving the change in the growth rate of adjusted M1. We experiment with such rules by allowing $\beta$ to take on different values, $\lambda$ to be 0.5 and $\alpha$ to be 1.5 in equation (6). This rule with money growth targeting can be regarded as a hybrid rule of Taylor’s (1993) and McCallum’s (2000).

(6) \[ i_t = i^* + \alpha(\pi_t - \pi^*) + \lambda(y_t - y^*) + \beta(\Delta M_t - \Delta M^*) \]

where $\Delta M_t$ is the growth rate of adjusted M1 and $\Delta M^*$ is the target for the growth rate of adjusted M1.

We also tested rules involving different weights for $\lambda$, $\beta$ and $\alpha$. For example, we experimented with a rule involving only the money growth term, hence setting $\alpha$ and $\lambda$ to be zero in equation (6). We also tested a rule with $\lambda$ set to zero and $\alpha$ and $\beta$ taking on different values. Note that for stability reasons, the weight on $\beta$ was restricted to be less than 0.75. We only report the results for some of the combination of rules we tried in the model. We also tested a rule involving the money gap. However, the model did not converge when such a rule was used. Moreover, we do not experiment with a purely money based rule, i.e., a rule with the monetary base as the instrument and not the interest rates, as the monetary base is not the actual instrument used by central banks and is thus not realistic.

Finally, we also experimented with other types of rules. These include nominal income targeting rules, rules containing more than one lag of either the output or the inflation gap and rules with the yield spread as the instrument. These rules were restricted to be closed economy rules. In general, these rules did not perform as well as the original Taylor rule and this is why we do not report any of these results.
5. Our Results.

To compare the different rules, we look at the variability of all key macroeconomic variables, especially at output, inflation and interest rates. We first compare the original Taylor rule with other Taylor type rules with different coefficients on the output and inflation gap. We find that the original Taylor rule, with a coefficient of 0.5 on both gaps performs reasonably well compared to other Taylor-type rules. As seen in Figure 1, when hit with a demand shock the original Taylor rule induces a slightly lower volatility of output and inflation and much lower interest rate volatility compared to the other Taylor-type rules. This result is also robust for the various shocks we consider. Moreover, rules with a coefficient greater than 2 on either the output or inflation gap tend to perform very poorly and are even unstable in certain circumstances.

When the original Taylor rule is compared to the base case rule, we can see that in general it does not perform as well. For example, in the case of demand shocks, it is seen that output deviates more from its target under the original Taylor Rule compared to the base case rule and interest rates are much more volatile under the original Taylor rule. We obtain similar results for the other shocks. However, under the Taylor rule, inflation returns to its target at a faster rate compared to the base case. This can be explained to some extent by the fact that interest rates move more aggressively under a Taylor rule, as seen by its higher volatility, and force inflation to move back to its target at a faster rate. Moreover, since the base case rule is estimated from history, it is not surprising that it has a slower path to equilibrium than the original Taylor rule.

When the weight on the output gap is increased, it is seen that it does not reduce the deviation of output from its target nor does it reduce the volatility of output. This is a surprising result to some extent, as one would expect output to deviate less from its target when the weight on the output gap is increased. This counter intuitive result is also obtained in some cases when other rules are used and when stochastic simulations are performed on the model.

Our findings are different from Armour et al (2000). They recommend a simple rule with a coefficient of 2 on the inflation gap and 0.5 on the output gap and argue that within the

35. We do not report results of rules which use the lagged output and inflation gap and not the contemporaneous gap. The results were not very different from the original Taylor rule or were marginally worse in some cases.
36. The base case reaction function also contains many more lags of the output gap and interest rate than the original Taylor rule and hence displays more inertia.
context of QPM, the original Taylor rule does not work well as it does not bring inflation back to its target quickly enough. When their recommended Taylor rule is introduced in this model, it is seen in Figure 2 that it induces more variability in output and especially on interest rates without any noticeable reduction in the variability of inflation. The QPM-Taylor rule performs very poorly in this model and hence offers further evidence on how sensitive simple rules can be to model specifications.

We find similar results when rules with the yield spread (thereafter YS rule) instead of the overnight rate as instrument are investigated. We find that a YS rule which uses the same coefficients as the original Taylor rule performs well and in many cases, gives similar results as the original Taylor rule. This rule even slightly outperforms the original Taylor rule in some cases since output and especially interest rates are less volatile. This small gain is not enough however to outweigh some of the other benefits a simple Taylor rule can bring, e.g being easy to communicate and understand. Moreover, since we want to find a rule which is robust and comparable across models37, we do not opt for a YS rule.38 As it is the case for the original Taylor rule, we find that in the class of YS rule, a coefficient of 0.5 on both gaps is warranted. Higher coefficients on both gaps usually result in higher variability in interest rates, output and inflation and to outright instability in some cases.

To a large extent, our results are similar to Armour et al (2000) on the Ball rule. The Ball rule does not perform well in this model. We experiment with the Ball rule in two ways: first we keep the coefficient on the exchange rate gap fixed at 0.25 and alter the coefficients on both the output and inflation gap and second, the coefficient on the exchange rate gap is altered while the coefficients of the original Taylor rule are used for the output and inflation gap and are kept fixed. When the coefficient on the exchange rate gap is allowed to change while those of the output gap and inflation gap are kept fixed, we find that the Ball rule performs very poorly since it induces excessive volatility in output, inflation and interest rates. As shown in Figure 3, the Ball rule induces secondary cycling in output and wide fluctuations in interest rates. The results were not substantially different when we experimented with different values for the equilibrium

37. To our knowledge, other studies have not considered rules containing the yield spread as the overnight rate is often seen as the instrument a central bank can directly control.
38. There are of course, other practical difficulties associated with a rule involving the yield spread such as being operationally feasible. There is also greater uncertainty surrounding the calculation of the equilibrium yield spread.
exchange rate. The verdict on the Ball rule is not very different when the coefficient on the exchange rate gap is fixed while the other coefficients are altered. As shown in Figure 4, this rule induces more output volatility, although in this case, there is no secondary cycling but significantly more volatility in interest rates. We also obtain poor results when different values for the equilibrium exchange rate were used.

Since the Ball rule explicitly takes into considerations the exchange rate gap, we should expect it to perform well compared to the original Taylor rule when the economy is subjected to exchange rate shocks. However, the results indicate that in this case also, the Ball rule is still dominated by the Taylor rule and is thus not recommended. We have already discussed some of the practical issues which can arise when using such type of rules. The poor performance of Ball type rules can be attributed to some extent to these factors. If such is the case and to reduce this type of uncertainty, it is recommended to put a zero weight on the exchange rate gap. Since it performs poorly and because of the uncertainty surrounding the calculation of the equilibrium exchange rate, we do not recommend using the Ball rule in this context.

The change rule does not perform well in this model also. In many cases, the results from this type of rules are either marginally or noticeably worse as compared to the Ball rule. Rules involving the change in the rate of growth of the exchange rate do not perform well also. In general, they induce more output and interest rate volatility (in some cases excessive secondary cycling) without any reduction in the variability of inflation. Hence, our findings tend to show that open economy rules do not work well in this model. Some of the reasons for this poor performance have already been provided.

Like Levin et al (1999), we find that rules with smoothing perform well compared to the original Taylor rule and other simple rules. As previously mentioned, we use a relatively high value for the smoothing parameter. Figure 5 shows how different rules with various degree of smoothing perform when the economy is hit with a demand shock. As is clearly shown, the volatility of output and interest rates is reduced as the weight increases on the lagged interest rate. It is interesting to note that while the variability of output and inflation is reduced when the degree of smoothing is increased, the volatility of inflation is virtually unchanged. Furthermore, it is seen that a rule with a high degree of inertia performs relatively well compared to the base case. Under this type of rule and with a demand shock, it is seen that output volatility is slightly higher while
interest rates volatility is lower and inflation returns to target at a faster rate. We obtain similar results for other shocks, indicating that this rule is also robust across shocks.

When this rule is compared to the original Taylor rule and the Ball rule, it is seen in Figure 6 that following a demand shock, the volatility of output is lower under such a rule compared to the original Taylor and the Ball rules, while the volatility of interest rates is significantly reduced. In the case of an exchange rate, money and foreign rate shocks, a rule with smoothing also outperforms the other simple rules. In short, rules with smoothing work remarkably well in this model and outperform any other simple rule we investigated.

It is interesting to note that as in Levin et al (1999) and Clarida et al (2000), we also find that such rule can lead to gains in terms of output, inflation and interest rate volatility, indicating that such rule are not only robust across models and shocks but also across countries. If such rules are effectively a good characterization of monetary policy in many countries as documented by many, it is not surprising that it performs well in this model. Hence, it would be interesting to see if such a rule would also perform well in other models of the Canadian economy. If such is the case, it would provide further evidence about the robustness of simple rules with smoothing. We have already given some reasons why such types of rules may work well. As argued before, lagged interest rate may serve as a proxy for lagged output, inflation and other variables in the economy. As a result, this rule incorporates more information about the economy in a sub optimal way and thereby offers policymakers greater control on output and inflation.

Figures 7, 8 and 8a, show the response of output, interest rates and inflation when a money based rule is used. The three different graphs refer to three different assumptions we make regarding \( \lambda, \beta \) and \( \alpha \) in equation (6). Figure 7 refers to the case where a money growth term is added to the original Taylor rule and where only the weight on the money growth term is allowed to change. In Figure 8, the weight on the output gap is set to zero while that on the money growth term is allowed to change. Figure 8a refers to the case where the weights on both the output and inflation gap are set to zero while that on the money growth term is allowed to vary. In all cases, the weight on the money growth term does not exceed 0.75 for stability reasons. When the money growth term is added to the original Taylor rule, as shown by Figure 7, it is seen that a rule with a weight of 0.25 on the money growth term dominates the other two money rules. Moreover, as compared to the basecase reaction function, output deviates more from its steady state and interest rates are much more volatile. As compared to the original Taylor rule, the addition of the money
growth term does not lead to any improvements. On the contrary, it seems to induce more volatility in output and in interest rates.

Figure 8 shows a similar rule but where the weight on the output gap is set to zero. When this type of rule is compared to the other money based rule (Figure 7), it is seen that it slightly outperforms the latter. The volatility of output, interest rates and inflation are all reduced. There is especially a marked difference in the volatility of interest rates. This result is interesting since it shows that in such a model, a rule which contains the money growth rate, the deviation of inflation from its target and the commonly used output gap would be outperformed by a similar rule which excludes the output gap. If the rule contains only the money growth term, it is seen that it performs slightly better than the two other money based rules. Output deviations from steady state under such a rule are less as compared to the rules in Figures 7 and 8 and are comparable in magnitude to the rule with a high degree of smoothing (Figure 5). Since interest rates are only reacting to the deviations of the growth rate of money from its target, the interest rate profile under such a rule is very different from the other rules we tested in this model.

Figure 9 compares the “best” money growth rule with the rule with smoothing, the Taylor rule and the basecase reaction function. As mentioned previously, the money growth rule performs relatively well when it comes to output stabilization, slightly outperforming both the Taylor and the rule with smoothing. On the other hand, inflation under such a rule deviates more from its target and for a longer period of time as compared to the Taylor rule and the rule with smoothing. Moreover, the money growth rule induces more volatility in interest rates as compared to the rule with smoothing but substantially less volatility than the Taylor rule. Overall, the rule with smoothing seems to outperform both the Taylor rule and the money growth rule but the difference between all three rules are not large


Once we have identified which types of rule work reasonably well in this model, some stochastic simulations were performed using the Taylor-type rule, the money growth based rule and the rule with smoothing. For the latter case, the smoothing parameter was set at 0.8 and 0.9. The parameter on the output gap was set successively at 0, 0.25, 0.5, 1 and 1.5 for the Taylor rule and the rule with smoothing. For the money based rule, the weight on the output gap was set
to zero while that on the money growth term was set successively at 0, 0.25, 0.75 and 1. In all cases, the parameter on the inflation gap was allowed to vary between 0 and 3 during the simulation. The shocks used for the simulation were obtained from the historically estimated errors of the model. The experiment was designed as follows.

Step 1: A set of parameters for the Taylor-type rule is first randomly drawn (for example when the coefficient on the output gap is fixed at 0.5, a parameter is randomly drawn for the inflation gap from a uniform distribution). After the parameters for the rule are drawn, a vector of shocks is then randomly drawn.

Step 2: The model is solved using the randomly drawn shocks and parameters for the simple rule. Once the model is solved, out of sample forecasts were used to compute the standard deviation for output, inflation and interest rates.

Step 3: Another vector of shocks is then drawn but the same coefficients for the simple rule from the first draw are kept. The model is solved using this new vector of drawn shocks and using the same parameters for the rule from draw 1 and once again the standard deviation for output, inflation and interest rate is calculated. This is repeated n times and the standard deviation for output, inflation and the interest rate for this rule is obtained by taking the average of the standard deviations over n draws.

Step 4: Once this is completed, a second set of parameters for the rule is drawn and the same exercise is repeated above using the same set of drawn shocks (we are effectively comparing a set of rules using the same set of shocks). This exercise is repeated m times. In effect, we are actually doing a stochastic simulation within a stochastic simulation. Values for n and m were set at 10 and 1000 respectively.

The results from our stochastic exercise are shown in Figure 10 to 16 whereas Tables 1 and 2 present the loss function calculated from the variance of output, inflation and interest rates for some selected rules. What is surprising about our results is how the stochastic frontiers behave. Figure 10 shows some simulations which were performed using the Taylor rule. It is clearly seen in Figure 10 that when the weight on the output gap is increased, the standard deviation of output increases and does not decrease as one would expect. Overall, we obtain similar results when the rule with smoothing is used. This result is surprising and deserves some explanation. This “perverse” result may come from two sources in the model. It is possible that one of the equations in the model is estimated with the wrong sign or has the wrong sign on some of its
parameters. It may be the case also that the structural errors in the model are not properly identified and this may give rise to this counter intuitive result. At present, fixing this problem is beyond the scope of this paper and we do not attempt to do so here.

Figure 10 shows the stochastic frontiers for the Taylor rule. As seen in Figure 10, the trade-off between inflation and output increases rapidly when the weight on the output gap exceeds 2. It is seen that the original Taylor rule does reasonably well compared to the other set of values. Figures 11 and 12 show the variance frontiers when a rule with smoothing is used. The smoothing parameter was respectively set at 0.8 and 0.9. In this case also, it is seen that for high values on the inflation gap, the trade-off between output variance and inflation variance increases.

When the original Taylor rule is compared to the rule with smoothing (ρ =0.8), it is seen in Figure 13, that the rule with smoothing dominates the Taylor rule. The stochastic frontier of the rule with smoothing for a given value of ygap is always to the left of that of the Taylor rule. Note that as the weight on the output gap is increased, the performance of the Taylor rule deteriorates very quickly vis à vis the rule with smoothing. When the two rules with smoothing are compared, it is seen in Figure 14 that the rule with a smoothing parameter of 0.9 dominates the other smoothing rule. Figures 15 and 16 shows the frontiers for the rule involving only the money growth term. Figure 16 compares the Taylor-type rule with the money growth rule. It is seen that there is not much difference between these two types of rules. It is obvious from Figure 16 and 13 that both the Taylor and money growth rule would be dominated by the rule with smoothing.

Table 1 and 2 present the loss function under the Taylor rule, the rule with smoothing and the money growth rule. The loss function we used is very typical and is given by

\[
(7) \quad \text{Loss} = Var(\pi) + \gamma Var(\bar{y}) + \mu Var(i)
\]

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Taylor Rule</th>
<th>Money Growth</th>
<th>Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.33</td>
<td>1.44</td>
<td>1.37</td>
</tr>
<tr>
<td>0.25</td>
<td>1.98</td>
<td>2.04</td>
<td>1.98</td>
</tr>
<tr>
<td>0.5</td>
<td>2.63</td>
<td>2.64</td>
<td>2.59</td>
</tr>
<tr>
<td>0.75</td>
<td>3.29</td>
<td>3.24</td>
<td>3.19</td>
</tr>
<tr>
<td>1</td>
<td>3.95</td>
<td>3.85</td>
<td>3.80</td>
</tr>
</tbody>
</table>
Tables 1 and 2 clearly show that a rule with smoothing dominates the other two rules. In all cases, except in Table 1 when $\gamma = 0$, the loss function under such a rule is lower as compared to the other two rules. It is interesting to note that as $\gamma$ grows towards one, the money growth rule does noticeably better than the Taylor rule but is still inferior to the rule with smoothing.

Overall, the results from these simple stochastic simulations confirm our results from the deterministic simulations. A rule with a high degree of smoothing dominates the other simple rules and is robust across shocks.

### 7. Conclusions

Our results are easy to summarize. Out of the class of simple rules we examined in the paper, no rule clearly outperforms the existing base case reaction function of the model. Among the class of simple rules we consider, we find that a rule with a high degree of smoothing performs relatively well. This rule outperforms all the other simple rules we consider in the model, including the original Taylor rule and yields results which are close to the base case rule in many cases. Moreover, when this rule is used, we experience a substantial reduction in the volatility of interest rates without any increase in the volatility in output or inflation. We also tend to find that the original Taylor rule performs relatively well compared to other Taylor type rules and Ball type rules. Furthermore, we do not find that Ball type rules improve macroeconomic performance as claimed by Ball (1999). In this model, it is seen that the Ball rule and the change rule are inferior compared to the Taylor rule and a rule with smoothing. A money growth rule works as well as a simple Taylor rule but is inferior to a rule with smoothing and is therefore not recommended.
In many models, it is seen that a simple rule can be a useful guide for the formulation of policy. Based on our results, it seems that a rule with smoothing may be used in the VECM for indicative purposes and may provide some starting point for discussions of issues relevant to policy makers. However it should not be followed mechanically and its robustness should certainly be investigated in other models of the Canadian economy. It is interesting to note that our results tend to support the findings of Clarida et al (2000) and Levin et al (1999) and to some extent those of Armour et al (2000). Our results, therefore tend to show that rules with smoothing may not only be robust across models but also across countries. However, more work on simple interest rate rules in Canadian models is warranted before any strong conclusion can emerge.
References


Taylor Rule in levels - Demand Shock

Figure 1

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line (basecase), solid line (Original TR), dashed line (ygap=1, pigap=1.5), dotted line (ygap=1.5, pigap=1.5)
Taylor Rule in levels - Demand Shock

Figure 2

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line (basecase), solid line (Original TR), dashed line (QPM rule)
Ball rule - Demand Shock

Figure 3

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line (basecase), solid line (egap=0.25, ygap=0.75), dashed line (egap=0.25, ygap=0.5), dotted line (egap=0.25, ygap=0.25)
Figure 4

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line(basecase), solid line(egap=0.25,pgap=1.5), dashed line(egap=0.5), dotted line(egap=0.75)
Rule with smoothing - Demand Shock

Figure 5

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line(basecase), solid line(rho=0.5), dashed line(rho=0.7), dotted line(rho=0.9)
Comparison of rules- Demand Shock

Figure 6

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line(basecase), solid line(original TR), dashed line(Ball), dotted line(smoothing)
Money-based rule - Demand Shock

Figure 7

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line (basecase), solid line (pigap=1.5, ygap=0.5, m=0.25), dashed line (m=0.5), dotted line (m=0.75)
Money-based rule - Demand Shock

Figure 8

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line(basecase), solid line(pgap=1.5, ygap=0, m=0.25), dashed line(m=0.5), dotted line(m=0.75)
Money-based rule - Demand Shock

Figure 8a

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line(basecase), solid line line(pigap=0,ygap=0,m=0.25), dashed line(m=0.5), dotted line(m=0.75)
Comparison of rules- Demand Shock

Figure 9

Output - deviation from SS

Inflation - deviation from target

% change in interest rate

interest rate

Dark line(basecase), solid line(original TR), dashed line(money), dotted line(smoothing)
Figure 10, Variance Frontier, Taylor rule

\[ \pi = 3 \rightarrow \pi = 2 \rightarrow \pi = 1 \rightarrow \pi = 0.5 \rightarrow \pi = 0 \rightarrow y = 0 \rightarrow y = 0.25 \rightarrow y = 0.5 \rightarrow y = 1 \rightarrow y = 1.5 \]

Figure 11, Variance Frontier, \( \rho = 0.8 \)

\[ \pi = 3 \rightarrow \pi = 2 \rightarrow \pi = 1 \rightarrow \pi = 0.5 \rightarrow \pi = 0 \rightarrow y = 0 \rightarrow y = 0.25 \rightarrow y = 0.5 \rightarrow y = 1 \rightarrow y = 1.5 \]
Figure 12, Variance Frontier, $\rho = 0.9$

Figure 13, Variance Frontier, TR in red and $\rho = 0.8$ in blue
Figure 14, Variance Frontier, \( \rho = 0.9 \) in red and \( \rho = 0.8 \) in blue

Figure 15, Variance Frontier, rule with money

\[ y^{\rho=0.9} = 1.5 \]

\[ y^{\rho=0.8} = 1.5 \]
Figure 16, Variance Frontier, rule with money in red, TR in blue