

# Term Premium Dynamics and the Taylor Rule<sup>1</sup>

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## **Abstract**

We explore the bond-pricing implications of an endowment economy where (i) preference shocks result in time-varying term premiums in real yields, and (ii) a monetary policy Taylor rule determines inflation and nominal term premiums. A calibrated version of the model matches the observed term structure of both the mean and volatility of yields; although, the preference parameters required do not admit an external habit-based interpretation. In addition, unlike a comparable model with exogenous inflation, a Taylor rule that matches the properties of observed inflation creates nominal term premiums that remain volatile even at long maturities. Experiments with different parameter values for the Taylor rule demonstrate that the nominal term premiums can be highly sensitive to monetary policy, and that the recent decrease in the level and volatility of the nominal yields could be the result of a more aggressive monetary policy.

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# 1 Introduction

A serious challenge in financial economics is understanding if macroeconomic variables are important factors explaining the term structure of interest rates. Empirical features such as an average upward-sloping term structure, time-varying term premiums, and volatile long-term interest rates are not well captured by standard macroeconomic models. While these findings do not necessarily imply that the link between the macroeconomy and financial markets is weak, they do suggest the value of exploring different specifications of macroeconomic models to better understand the linkage between the term structure of interest rates and macroeconomic variables. We explore the ability of an economic model that incorporates a monetary policy rule to simultaneously capture several macroeconomic features, an upward sloping average yield curve, and the high volatility of long-term rates. In particular, we investigate whether monetary policy can be seen as a significant source of fluctuations in the equilibrium yield curve.

Campbell (1986) makes it clear that the standard constant relative risk aversion specification for preferences implies a downward-sloping real term structure on average, unless consumption growth is counterfactually negatively autocorrelated. The ability of a model with standard preferences to explain a positive upward-sloping nominal term structure then entirely relies on the specification of the inflation process. We explore an endowment economy where the representative agent has a preference shock that leads to a stochastic price of risk, similar to the representative agent in Gallmeyer et al. (2005). Here the representative agent's preference shocks are sensitive to a latent variable and lagged consumption growth. We provide a simple structure for the exogenous state variables in the economy such that the model implies an equilibrium affine term structure. Interest rates are maturity-dependent linear combinations of state variables with a macroeconomic interpretation. We also ask if these calibrated preferences can be interpreted as a form of external habit formation as in Abel (1990), Campbell and Cochrane (1999), and Wachter (2006).

We analyze the ability of the model to capture empirically both an average upward-sloping nominal term structure and the volatility of long-term nominal rates by studying two different characterizations of the inflation process. Specifically, we model inflation as an exogenous process, and also make inflation endogenous by incorporating monetary policy through a Taylor rule into

the analysis.

Our equilibrium affine model with exogenous inflation is unable to simultaneously reproduce an average upward sloping nominal term structure and the volatility of long-term nominal rates as seen in the data. Affine term structure models with stationary factors require highly persistent factors to avoid a rapid decline in the volatility of rates across maturities. This is not achieved by our exogenous inflation model. The factors in the affine model are consumption growth, a latent preference variable, and inflation. These factors are not persistent enough when calibrated to observed macroeconomic dynamics; interest rate volatility dies out too fast with bond maturity. However, the endogenous inflation model can be simultaneously calibrated to observed macroeconomic dynamics, the average nominal term structure and the maturity pattern in nominal interest rate volatilities.

Inflation becomes an endogenous variable by incorporating monetary policy into the analysis. Monetary policy is specified as a Taylor rule — an interest-rate policy rule in which the monetary authority sets the short-term risk-free rate as an affine function of current consumption growth, current inflation, and a policy shock. For simplicity, monetary policy only influences the nominal side of the economy. We solve for the equilibrium inflation process and show that in equilibrium, the inflation process is driven by consumption growth, a latent preference variable, and a policy shock. In contrast to exogenous inflation, long-term interest rates are now also driven by a policy shock. When the policy shock is highly autocorrelated, the model simultaneously captures the volatility of long-term rates and the persistence of the observed inflation process.

The two models for inflation have different implications for asset pricing. In the exogenous inflation model, the price of inflation risk is independent of the other sources of risk in the economy by construction. In the endogenous inflation model, the dependence of inflation on consumption growth induced by the policy rule leads to inflation influencing the nominal price of consumption growth risk. Here, a negative shock to consumption growth increases inflation as is exogenously imposed in Wachter (2006) and Piazzesi and Schneider (2007). As a result, the interest-rate monetary policy rule affects the hedging properties and the riskiness of longer-term nominal bonds.

To capture desired empirical properties of term structures while still retaining a tractable en-

ogenous inflation specification, our calibrated representative agent preference specification is no longer easily interpreted as a form of external habit formation. This is driven by the inability to simultaneously capture a countercyclical price of consumption growth risk and real interest rates that are negatively correlated with consumption growth as in a habit formation model. Additionally, our preference specification includes a latent taste shock that aids in explaining the structure of bond volatilities.

We use the endogenous inflation model to conduct a policy experiment in order to ask whether recent developments in the term structure and inflation are consistent with changes in the monetary policy rule. Specifically, we explore two different policy regimes. The first regime is characterized by an increase in the sensitivity of the short-term interest rate to consumption growth, and the second regime is characterized by an increase in the sensitivity of the short-term interest rate to inflation. The policy rule that is more sensitive to inflation is more consistent with recent data. Increasing the sensitivity to inflation in the policy rule lowers the correlation between inflation and consumption growth as observed in the data. If the policy rule is instead more sensitive to consumption growth, the correlation between inflation and consumption growth increases leading to a counterfactual downward-sloping nominal yield curve.

Our work is related to Gallmeyer et al. (2007). The model in that paper differs from the one presented here in the specification of preferences and by the sources of time-variation in term premiums. Gallmeyer et al. (2007) uses a recursive utility specification and obtains time variation in term premiums through stochastic volatility in the endowment process. Here, time-varying term premiums are the result of a stochastic market price of risk driven by stochastic preference shocks. The model with preference shocks does a better job than the model with stochastic volatility in terms of matching properties of macroeconomic variables and the term structure.

## **2 Affine Term-Structure Models with Stochastic Price of Risk**

To explore the relationship between monetary-policy-induced changes in short-term interest rates and the entire term structure, we study an equilibrium model that implies the Duffie and Kan (1996) affine term-structure model. In the Duffie and Kan (1996) class of arbitrage-free term-

structure models, the prices of multi-period default-free pure discount bonds are affine functions of the model's state variables  $\mathbf{s}_t$ .

The Duffie and Kan (1996) class of models is based on a  $k$ -dimensional vector of state variables  $\mathbf{s}_t$  that follows a first-order vector autoregression<sup>1</sup>

$$\mathbf{s}_{t+1} = (\mathbb{I} - \Phi)\theta + \Phi\mathbf{s}_t + \Sigma(\mathbf{s}_t)^{1/2}\varepsilon_{t+1}, \quad (1)$$

where  $\{\varepsilon_t\} \sim \text{IID}\mathcal{N}(0, \mathbb{I})$ ,  $\Phi$  is a  $k \times k$  matrix of autoregressive parameters assumed to be stable with positive diagonal elements, and  $\theta$  is a  $k \times 1$  vector of drift parameters. The conditional covariance matrix,  $\Sigma(\mathbf{s}_t)$ , can depend on the state vector in specific functional forms considered below.

Prices for real and nominal default-free bonds are given by the fundamental equation of asset pricing

$$b_t^{(n)} = \mathbb{E}_t[M_{t+1}b_{t+1}^{(n-1)}], \quad (2)$$

where  $b_t^{(n)}$  is the price at date  $t$  of a default-free pure-discount bond that pays 1 at date  $t+n$  where  $b_t^{(0)} = 1$ . The positive random variable  $M_{t+1}$  is the asset-pricing kernel or the stochastic discount factor. In our structural model below,  $M_{t+1}$  will be interpreted as the equilibrium marginal utility of the representative household.

By specifying a particular functional form for the pricing kernel and the variance-covariance matrix  $\Sigma(\mathbf{s}_t)$ , bond prices of all maturities are log-linear functions of the state,

$$-\log b_t^{(n)} = \mathcal{A}^{(n)} + \mathcal{B}^{(n)\top} \mathbf{s}_t,$$

where  $\mathcal{A}^{(n)}$  is a scalar, and  $\mathcal{B}^{(n)}$  is a  $k \times 1$  vector. Equivalently, continuously compounded yields,  $i_t^{(n)}$ , defined by  $b_t^{(n)} \equiv \exp(-n i_t^{(n)})$ , are also affine functions of the state variables,

$$i_t^{(n)} = \frac{1}{n} \left[ \mathcal{A}^{(n)} + \mathcal{B}^{(n)\top} \mathbf{s}_t \right].$$

Empirical work by Duffee (2002) and Dai and Singleton (2002, 2003) shows that an affine model

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<sup>1</sup>The discrete version of the Duffie and Kan (1996) model is derived in Backus et al. (1998).

in which the state dependence of the risk premium is driven by the price of risk provides a better empirical model of the term structure than a model in which the risk premium is driven by stochastic volatility. Duffee (2002) and Ang and Piazzesi (2003) show in continuous and discrete-time settings, respectively, that such a model, known as an essentially affine model, is tractable. Here we use the discrete-time version of the essentially affine model and assume that the variance-covariance matrix of the state vector  $\mathbf{s}_t$  is a constant  $\Sigma(\mathbf{s}_t) \equiv \Sigma$ .<sup>2</sup>

In order to capture the state dependence of the market price of risk in an essentially affine framework, the pricing kernel takes the form

$$-\log M_{t+1} = \Gamma_0 + \Gamma_1^\top \mathbf{s}_t + \frac{1}{2} \lambda(\mathbf{s}_t)^\top \Sigma \lambda(\mathbf{s}_t) + \lambda(\mathbf{s}_t)^\top \Sigma^{1/2} \varepsilon_{t+1}. \quad (3)$$

The  $k \times 1$  vector  $\Gamma_1$  represents the “factor loadings” for the pricing kernel and the  $k \times 1$  vector  $\lambda(\mathbf{s}_t)$  is the state-dependent price of risk which is also assumed to be affine in the state vector

$$\lambda(\mathbf{s}_t) = \lambda_0 + \lambda_1 \mathbf{s}_t, \quad (4)$$

where  $\lambda_0$  is a  $k \times 1$  vector of constants and  $\lambda_1$  is a  $k \times k$  matrix of constants. The quadratic term  $\frac{1}{2} \lambda(\mathbf{s}_t)^\top \Sigma \lambda(\mathbf{s}_t)$  in (3) is a correction term that preserves the linearity of interest rates.

From the assumption on the structure of the pricing kernel, the parameters defining the bond yields,  $\mathcal{A}_n$  and  $\mathcal{B}_n$ , can be found recursively by substituting into the bond pricing equation (2) yielding

$$\begin{aligned} \mathcal{A}_n &= \Gamma_0 + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}^\top [(\mathbb{I} - \Phi)\theta - \Sigma\lambda_0] - \frac{1}{2} \mathcal{B}_{n-1}^\top \Sigma \mathcal{B}_{n-1}, \\ \text{and } \mathcal{B}_n^\top &= \Gamma_1^\top + \mathcal{B}_{n-1}^\top [\Phi - \Sigma\lambda_1]. \end{aligned} \quad (5)$$

Since  $b^{(0)} = 1$ , the initial conditions for the recursions are  $\mathcal{A}_0 = 0$  and  $\mathcal{B}_0 = \mathbf{0}$ .

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<sup>2</sup>Other applications of the essentially affine model in discrete-time include Brandt and Chapman (2003) and Dai and Philippon (2004).

## 2.1 Some Properties of the Affine Term-Structure Model

A successful term-structure model must be able to capture salient properties of interest rates such as time-varying expected returns on bonds, an upward-sloping average yield curve, and volatile long-term interest rates.

The fundamental pricing equation (2) tells us that long-term bonds can be seen as one-period instruments with the uncertain payoff  $b_{t+1}^{(n-1)}$ . It implies that, from a one-period holding period perspective, long-term bonds might involve compensations for risk that must be reflected in expected excess returns over the one-period risk-free rate  $i_t$ . Define the one-period term premium involved in an  $n$ -period bond as

$$\xi_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n} \left[ i_t + (n-1) \mathbb{E}_t i_{t+1}^{(n-1)} \right]. \quad (6)$$

Using the recursive equations (5), the term premium of an  $n$ -period bond can be written in the affine form

$$\xi_t^{(n)} = \frac{1}{n} \left[ \xi_{\mathcal{A},n} + \xi_{\mathcal{B},n}^\top \mathbf{s}_t \right], \quad (7)$$

with coefficients given by

$$\xi_{\mathcal{A},n} = -\mathcal{B}_{n-1}^\top \Sigma \left( \lambda_0 + \frac{1}{2} \mathcal{B}_{n-1} \right)$$

and

$$\xi_{\mathcal{B},n}^\top = -\mathcal{B}_{n-1}^\top \Sigma \lambda_1.$$

From these equations, we can infer that term premiums in the affine framework are time-varying as long as the market price of risk is not constant ( $\lambda_1 \neq \mathbf{0}$ ). This characteristic is essential to capture deviations from the expectations hypothesis. To see this, consider the Campbell and Shiller (1991)



coefficients,  $\beta^{(n)}$ , associated with the regression

$$i_{t+1}^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \frac{\beta^{(n)}}{n-1}(i_t^{(n)} - i_t) + \varepsilon_{CS,t}^{(n)}. \quad (8)$$

Under the expectations hypothesis, the  $\beta^{(n)}$  coefficients are equal to 1. Using equation (6), these coefficients are

$$\beta^{(n)} = 1 - n \frac{\text{COV}(i_t^{(n)} - i_t, \xi_t)}{\text{var}(i_t^{(n)} - i_t)}.$$

Deviations from the expectations hypothesis are explained by time-varying term premiums whose variation is correlated with the variability of interest-rate spreads. Such a pattern is entirely driven by the existence of time variation in the market price of risk.

The term premium  $\xi_t^{(n)}$  multiplied by maturity is equal to the expected one-period holding period return of an  $n$  period bond in excess of the one period rate. To see this, denote by  $xr_{t,t+1}^n$  as the one-period holding period return from time  $t$  to  $t+1$  of an  $n$  period bond in excess of the one period rate. The return is given by

$$xr_{t,t+1}^{(n)} = \log \left( \frac{b_{t+1}^{(n-1)}}{b_t^{(n)}} \right) - i_t = -(n-1)i_{t+1}^{(n-1)} + ni_t^{(n)} - i_t$$

and from equation (6) it follows that  $\mathbb{E}_t [xr_{t,t+1}^{(n)}] = n\xi_t^{(n)}$ .

Historically, long-term nominal bond yields are on average higher than short-term nominal yields. From the affine specification, the average spread between an  $n$ -period bond yield and a one-period interest rate is

$$\mathbb{E}[i_t^{(n)} - i_t] = \frac{n-1}{n}\mathbb{E}[i_t^{(n-1)} - i_t] + \frac{1}{n}\mathbb{E}[xr_{t,t+1}^{(n)}] = \frac{1}{n}\mathbb{E} \left[ \sum_{j=2}^n xr_{t,t+1}^{(j)} \right].$$

The recursive representation for the average spread shows that the unconditional spread associated with a specific maturity can be expressed as the weighted average of the unconditional spread linked to a bond with a shorter maturity and a maturity-specific holding-period expected excess return.

When interest rates are represented by stationary state variables, expected excess returns must be positive enough to obtain an upward-sloping average yield curve. This imposes restrictions on the parameters of the market price of risk.

To obtain the volatility of long-term interest rates, consider the non-recursive solution for the vector of factor sensitivities in equation (5),

$$\mathcal{B}_n = \left[ (\mathbb{I} - \Phi_\lambda)^{-1} (\mathbb{I} - \Phi_\lambda^n) \right]^\top \mathcal{B}_1,$$

where

$$\Phi_\lambda = [\Phi - \Sigma \lambda_1].$$

The matrix  $\Phi_\lambda$  can be interpreted as the autoregressive matrix for the state variables under the risk-neutral measure. This matrix is different from the autocorrelation matrix under the actual measure as long as the market price of risk is time varying. From this representation, the unconditional variance of interest rates is

$$\text{var}(i_t^{(n)}) = \frac{1}{n^2} \mathcal{B}_1^\top (\mathbb{I} - \Phi_\lambda)^{-1} (\mathbb{I} - \Phi_\lambda^n) \text{var}(\mathbf{s}_t) \left[ (\mathbb{I} - \Phi_\lambda)^{-1} (\mathbb{I} - \Phi_\lambda^n) \right]^\top \mathcal{B}_1.$$

For the one state variable case ( $k = 1$ ), the volatility of interest rates simplifies to

$$\sigma(i_t^{(n)}) = \frac{1}{n} \frac{1 - \Phi_\lambda^n}{1 - \Phi_\lambda} \sigma(i_t).$$

Figure 1 presents the volatility of long-term interest rates implied by the formula above for different coefficients  $\Phi_\lambda$  as a proportion of the volatility of the one-period interest rate. From the figure, volatility dies out quickly unless  $\Phi_\lambda$  is very close to one. For models with a constant market price of risk  $\Phi_\lambda = \Phi$ , the volatility of interest rates depends entirely on the autocorrelation of the state variables. Thus, in order to capture a slow declining volatility across maturities, stationary state variables need to be very persistent. This is consistent with the result in Backus and Zin (1994) that the volatility of interest rates converges to zero under stationary state variables. The existence of a state-dependent market price of risk,  $\lambda_1$ , such that  $\Phi_\lambda - \Phi$  is positive definite, potentially overcomes

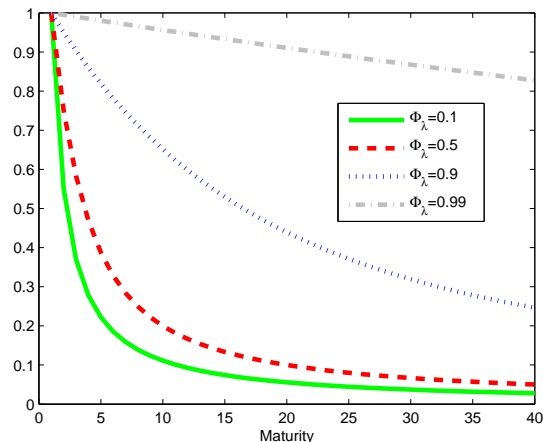


Figure 1: Long-term rate volatility as proportion of short-term rate volatility.

the lack of persistence in the state variables and helps increase the volatility of longer term interest rates.

### 3 An Equilibrium Essentially Affine Economy

To better understand the features of bond prices that can be captured in the context of a pure-exchange economy with a time-varying market price of risk, we derive both real and nominal term structures in a pure exchange economy when the representative household's preferences contains a stochastic preference shock. The stochastic preference shock takes the form of stochastic risk aversion and depending on its calibration can be interpreted as a form of external habits. The model adapted here is a variation of the one considered in Gallmeyer et al. (2005), but without modeling staggered price setting to derive inflation.

In this discrete-time economy, households derive lifetime consumption from a single good. The supply of this good is exogenously specified. To isolate the asset pricing role of the representative household with a preference shock, real and nominal bonds are in zero net supply. We abstract away from models with price rigidities that would lead to inflation influencing the real side of the economy; instead, nominal prices are constructed in our economy either by exogenously specifying an inflation process or by imposing a monetary policy rule that links the inflation process to the

nominal short-rate in the economy.

### 3.1 Consumption and Households

Consumption  $C_t$  is exogenous in our pure-exchange setting. The process for logarithmic consumption growth,  $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$ , is given by

$$\Delta c_{t+1} = (1 - \phi_c)\theta_c + \phi_c\Delta c_t + \sigma_c\varepsilon_{c,t+1} \quad (9)$$

with  $\{\varepsilon_{c,t+1}\} \sim \text{IID}\mathcal{N}(0, 1)$ .

The representative household is infinitely lived and derives utility from consumption. The representative household solves the intertemporal optimization problem

$$\max_{\{C_t\}_{t=0}^{\infty}} \mathbb{E} \left[ \sum_{t=0}^{\infty} e^{-\delta t} \frac{C_t^{1-\gamma}}{1-\gamma} Q_t \right]$$

subject to the intertemporal budget constraint

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} M_t C_t \right] \leq w_0. \quad (10)$$

Here  $\delta$  denotes the time preference parameter,  $\gamma$  is the local curvature of the utility function,  $Q_t$  is the time  $t$  preference shock, and  $w_0$  is the household's initial wealth.

The preference shock is taken as exogenous by the representative household. The stochastic preference shock, expressed as the change in the logarithm of the shock  $\Delta q_{t+1} \equiv \log Q_{t+1} - \log Q_t$ , is linearly related to shocks in consumption growth  $\Delta c_{t+1}$ , with a coefficient that varies linearly with the current level of consumption growth and an exogenous variable  $\nu_t$  interpreted as a pure taste shock:

$$-\Delta q_{t+1} = \frac{1}{2} (\eta_c \Delta c_t + \eta_\nu \nu_t)^2 \text{var}_t \Delta c_{t+1} + (\eta_c \Delta c_t + \eta_\nu \nu_t) (\Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1}).$$

The preference shock allows for an exogenously varying stochastic risk aversion through an external habit formation and a pure taste shock interpretation. The representative household's overall

sensitivity to consumption growth is  $\gamma + (\eta_c \Delta c_t + \eta_\nu \nu_t)$ , where  $(\eta_c \Delta c_t + \eta_\nu \nu_t)$  can be interpreted as the stochastic part of the representative household's risk aversion. To complete the specification of the preference shock,  $\nu_t$  has autoregressive dynamics given by

$$\nu_{t+1} = \phi_\nu \nu_t + \sigma_\nu \varepsilon_{\nu,t+1} \quad (11)$$

with  $\{\varepsilon_{\nu,t+1}\} \sim \text{IID}\mathcal{N}(0, 1)$ . The shock  $\varepsilon_{\nu,t+1}$  is independent of the consumption growth shock  $\varepsilon_{c,t+1}$ .

The term  $-\frac{1}{2}(\eta_c \Delta c_t + \eta_\nu \nu_t)^2 \text{var}_t \Delta c_{t+1}$  in the stochastic preference shocks implies that the conditional mean of the growth of the preference shock is

$$E_t \left[ \frac{Q_{t+1}}{Q_t} \right] = 1,$$

implying that the process for the preference shock is a martingale. The coefficient  $\eta_c$  measures the sensitivity of the representative household's level of risk-aversion to the current growth rate of aggregate consumption. The coefficient  $\eta_\nu$  measures the sensitivity of the representative household's level of risk aversion to the process  $\nu_t$  which is independent of consumption growth.

From the household's first-order conditions we obtain a real pricing kernel  $M_{t+1}$  given by the intertemporal marginal rate of substitution<sup>3</sup>

$$M_{t+1} = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{Q_{t+1}}{Q_t} \right). \quad (12)$$

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<sup>3</sup>The term  $\frac{Q_{t+1}}{Q_t}$  can be seen as a Radon-Nikodym derivative that represents a change of measure from the pricing kernel of a CRRA economy. This representation for the pricing kernel is isomorphic to the Epstein-Zin pricing kernel presented in Gallmeyer et al. (2007) or the model-uncertainty adjusted pricing kernel in Hansen and Sargent (2007). Although the economic underpinnings in these models are different, they share the purpose of shifting the marginal utility of consumption.

Therefore, the logarithmic real pricing kernel  $m_{t+1} \equiv \log M_{t+1}$  is

$$\begin{aligned}
-m_{t+1} &= \delta + \gamma \Delta c_{t+1} - \Delta q_{t+1} \\
&= \delta + \gamma (1 - \phi_c) \theta_c + \gamma \phi_c \Delta c_t + \frac{1}{2} (\eta_c \Delta c_t + \eta_\nu \nu_t)^2 \sigma_c^2 \\
&\quad + (\gamma + \eta_c \Delta c_t + \eta_\nu \nu_t) \sigma_c \varepsilon_{c,t+1}.
\end{aligned} \tag{13}$$

The real pricing kernel in the habit model can be seen as a 2-factor stochastic price of risk affine model with state variables  $\mathbf{s}_t = (\Delta c_t, \nu_t)^\top$ . Proposition 1 summarizes the link between the equilibrium for this economy and the affine framework presented above.

**Proposition 1.** *The equilibrium characteristics of the economy and its associated real pricing kernel are represented by equations (1), (3), and (4) where*

$$\mathbf{s}_t = (\Delta c_t, \nu_t)^\top$$

and

$$\begin{aligned}
\Phi &= \text{diag}\{\phi_c, \phi_\nu\}, \quad \theta = (\theta_c, 0)^\top, \quad \Sigma^{1/2} = \text{diag}\{\sigma_c, \sigma_\nu\}, \quad \varepsilon = (\varepsilon_c, \varepsilon_\nu)^\top, \\
\Gamma_0 &= \delta + \gamma (1 - \phi_c) \theta_c - \frac{1}{2} \gamma^2 \sigma_c^2, \quad \Gamma_1 = [\gamma (\phi_c - \eta_c \sigma_c^2), \quad -\gamma \eta_\nu \sigma_c^2]^\top, \\
\lambda_0 &= [\gamma, \quad 0]^\top, \quad \text{and} \quad \lambda_1 = \begin{bmatrix} \eta_c & \eta_\nu \\ 0 & 0 \end{bmatrix}.
\end{aligned}$$

*Proof.* It follows from characterizing the state vector process (1) using equations (9) and (11) and expressing (13) in matrix form.  $\square$

This representation allows us to price real discount bonds using equation (5). The equilibrium continuously compounded  $n$ -period real interest rate,  $r_t^{(n)}$ , must satisfy the household's first-order condition for  $n$ -period real bond holdings

$$e^{-nr_t^{(n)}} = \mathbb{E}_t [M_{t+n}] = \mathbb{E}_t \left[ M_{t+1} e^{-(n-1)r_{t+1}^{(n-1)}} \right]. \tag{14}$$

Therefore, real interest rates can be expressed as linear combinations of consumption growth and the exogenous variable  $\nu_t$ , with loadings given by functions of deep economic parameters.

Relative to a general essentially-affine model, the model's structural parameters significantly reduce the dimensionality of the parameter space. From the structure of the price of risk  $\lambda(\mathbf{s}_t)$ , innovations in the pricing kernel are solely driven by shocks to consumption growth  $\varepsilon_{c,t+1}$ . The preference shock  $\nu_t$  does however contribute to time variation in the price of risk as long as  $\eta_\nu \neq 0$ . The preference parameters  $\eta_c$  and  $\eta_\nu$  affect the sensitivity of interest rates to the state variables. In particular, a negative value for  $\eta_c$  increases the response of real interest rates to consumption growth and implies a counter-cyclical price of consumption growth risk. Such a feature can lead to an upward-sloping average yield curve. To see this, consider the spread between a 2-period bond and a one-period bond,  $r_t^{(2)} - r_t$ . Using (14) and Proposition 1, the average spread is

$$\begin{aligned} \mathbb{E}[r_t^{(2)} - r_t] &= -\frac{1}{2}\mathbb{E}[\text{var}_t(r_{t+1})] + \mathbb{E}[\text{cov}_t(m_{t+1}, r_{t+1})] \\ &= -\frac{1}{2}\mathbb{E}[\text{var}_t(r_{t+1})] - \gamma(\gamma + \eta_c\theta_c)(\phi_c - \eta_c\sigma_c^2)\sigma_c^2. \end{aligned}$$

Therefore, the model's potential to generate positive spreads depends entirely on its ability to capture a positive covariance between the real price kernel and interest rates. Under power utility,  $\eta_c = 0$  and the real yield curve is always downward sloping, unless we assume a counterfactual negative autocorrelation of consumption growth. In contrast, since long-term consumption growth,  $\theta_c$ , is positive, a positive spread can be obtained when  $\eta_c < -\frac{\theta_c}{\gamma}$ .

### 3.2 Nominal Bond Pricing

We are interested in pricing nominal bonds that pay in units of money. To take into account the different numeraire, we need to transform the real pricing kernel described by equation (12) into a nominal pricing kernel. Defining  $P_t$  as the nominal price level at time  $t$ , the nominal pricing kernel is

$$M_{t+1}^{\$} = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{Q_{t+1}}{Q_t} \right) \left( \frac{P_{t+1}}{P_t} \right)^{-1}. \quad (15)$$

Denoting by  $i_t^{(n)}$  the continuously compounded  $n$ -period nominal interest rate, the household's first-order condition for  $n$ -period nominal bond holdings is

$$e^{-i_t^{(n)}} = \mathbb{E}_t \left[ M_{t+n}^{\$} \right].$$

The logarithm of the nominal pricing kernel is then  $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$ , where  $\pi_{t+1} \equiv \log P_{t+1} - \log P_t$  is the log of the money price of goods at  $t + 1$  relative to  $t$ , or inflation.

To close the nominal side of the model, we need to derive a process for the evolution of inflation in the economy. For simplicity, we consider two approaches to pin down inflation. Our first approach is to directly specify its dynamics. Our second approach is to specify a Taylor rule describing Fed policy which links the nominal short-rate and inflation.

### 3.3 Exogenous Inflation Nominal Pricing Kernel

By expanding the state space to include an exogenous inflation process  $\pi_t$ , the nominal state vector is  $\mathbf{s}_t^{\$} = (\Delta c_t, \nu_t, \pi_t)^\top$ . Further, we assume that the stochastic process for inflation is given by

$$\pi_{t+1} = (1 - \phi_\pi)\theta_\pi + \phi_\pi\pi_t + \sigma_\pi\varepsilon_{\pi,t+1}, \quad (16)$$

where  $\{\varepsilon_{\pi,t+1}\} \sim \text{IIDN}(0, 1)$  and is independent of all other shocks in the model. Given the conditional variance of inflation,  $\text{var}_t(\pi_{t+1}) = \sigma_\pi^2$ , is constant, the nominal state vector still conforms to the essentially affine setting described above.

Based on the equilibrium real and nominal pricing kernels given by equations (13) and (15), the equilibrium nominal term structure from our habit-based pure exchange economy can be expressed as a 3-factor stochastic price of risk affine model characterized in Proposition 2.

**Proposition 2.** *The equilibrium characteristics of the economy under the exogenous inflation process and its associated nominal pricing kernel are represented by equations (1), (3), and (4) where*

$$\mathbf{s}_t^{\$} = (\Delta c_t, \nu_t, \pi_t)^\top$$



and

$$\Phi^{\$} = \text{diag}\{\phi_c, \phi_\nu, \phi_\pi\}, \quad \theta^{\$} = (\theta^\top, \theta_\pi)^\top, \quad \Sigma^{1/2, \$} = \text{diag}\{\sigma_c, \sigma_\nu, \sigma_\pi\}, \quad \varepsilon^{\$} = (\varepsilon^\top, \varepsilon_\pi)^\top,$$

$$\Gamma_0^{\$} = \Gamma_0 + (1 - \phi_\pi)\theta_\pi - \frac{1}{2}\sigma_\pi^2, \quad \Gamma_1^{\$} = [\Gamma_1, \quad \phi_\pi]^\top,$$

$$\lambda_0^{\$} = \left[ \lambda_0^\top, \quad 1 \right]^\top, \quad \text{and} \quad \lambda_1^{\$} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

*Proof.* It follows from characterizing the state vector process (1) using equations (9), (11), and (16), substituting them into the nominal pricing kernel (15) and expressing it in matrix form.  $\square$

From Proposition 2, the market prices of risk related to consumption growth and the exogenous taste shock  $\nu_t$  are the same as in Proposition 1. Equivalently, the compensations for the risks associated with consumption growth and the exogenous preference variable are the same for assets with real and nominal payoffs. The last term of  $\lambda(\mathbf{s}_t)$  contains the price of inflation risk, which in this setting is constant.

### 3.4 A Monetary-Policy Consistent Nominal Pricing Kernel

Alternatively, we can derive the nominal pricing kernel by imposing a monetary policy rule linking inflation to the nominal short rate. Assume that monetary policy follows a nominal interest rate rule of the form

$$i_t = \bar{i} + \iota_c \Delta c_t + \iota_\pi \pi_t + u_t, \tag{17}$$

where  $u_t$  is a policy shock capturing the non-systematic component of monetary policy. This policy shock follows an autoregressive process with dynamics given by

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \tag{18}$$

where  $\{\varepsilon_{u,t+1}\} \sim \text{IID}\mathcal{N}(0, 1)$  and is independent of all other shocks in the model.

The policy rule (17) is similar to the one proposed in Taylor (1993). The evident difference

between the two rules is that, while under the original Taylor (1993) specification the short-term interest rule depends on the output gap level, the rule here reacts to consumption growth. The absence of a production sector with frictions in this endowment economy does not admit an interpretation of an output gap. Therefore, with slight abuse of terminology, we refer to the policy rule as the Taylor rule for the model.

Given that the Taylor rule (17) must be consistent with the nominal pricing kernel (15), we can use the two equations to solve for an internally consistent process for inflation. This process is given by

$$\pi_t = \bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t.$$

The equilibrium constraint imposed by the price of the one-period nominal bond (16) implies loading coefficients for the equilibrium inflation that satisfy

$$\bar{\pi} = \frac{1}{1 - \iota_\pi} \left[ \bar{i} - \delta - (\gamma + \pi_c)(1 - \phi_c)\theta_c + \frac{1}{2}(\gamma + \pi_c)^2 \sigma_c^2 + \frac{1}{2}\pi_\nu^2 \sigma_\nu^2 + \frac{1}{2}\pi_u^2 \sigma_u^2 \right],$$

$$\pi_c = \frac{\gamma(\phi_c - \sigma_c^2 \eta_c) - \iota_c}{\iota_\pi - \phi_c + \sigma_c^2 \eta_c}, \quad \pi_\nu = -\frac{(\gamma + \pi_c)\sigma_c^2 \eta_\nu}{\iota_\pi - \phi_\nu}, \quad \text{and} \quad \pi_u = -\frac{1}{\iota_\pi - \phi_u}.$$

These coefficients show that the sensitivity of inflation to the state variables are determined by the response of the monetary authority to consumption growth and inflation.

Substituting the monetary-policy consistent inflation process into the nominal pricing kernel (15), we obtain a 3-factor essentially affine nominal term structure model. The nominal state vector is given by  $\mathbf{s}_t^\$ = (\Delta c_t, \nu_t, u_t)^\top$ . The dynamics of the nominal state variables and the nominal pricing kernel are characterized in Proposition 3.

**Proposition 3.** *The equilibrium characteristics of the economy under the endogenous inflation process and its associated nominal pricing kernel are represented by equations (1), (3), and (4) where*

$$\mathbf{s}_t^\$ = (\Delta c_t, \nu_t, u_t)^\top$$

and

$$\Phi^{\$} = \text{diag}\{\phi_c, \phi_\nu, \phi_u\}, \quad \theta^{\$} = (\theta^\top, 0)^\top, \quad \Sigma^{1/2,\$} = \text{diag}\{\sigma_c, \sigma_\nu, \sigma_u\}, \quad \varepsilon^{\$} = (\varepsilon^\top, \varepsilon_u)^\top,$$

$$\Gamma_0^{\$} = \delta + \bar{\pi} + (\gamma + \pi_c)(1 - \phi_c)\theta_c - \frac{1}{2}(\gamma + \pi_c)^2\sigma_c^2 - \frac{1}{2}\pi_\nu^2\sigma_\nu^2 - \frac{1}{2}\pi_u^2\sigma_u^2,$$

$$\Gamma_1^{\$} = [(\gamma + \pi_c)(\phi_c - \eta_c\sigma_c^2), \quad \pi_\nu\phi_\nu - (\gamma + \pi_c)\eta_\nu\sigma_c^2, \quad \pi_u\phi_u]^\top,$$

$$\lambda_0^{\$} = [\gamma + \pi_c, \quad \pi_\nu, \quad \pi_u]^\top, \quad \text{and} \quad \lambda_1^{\$} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

*Proof.* It follows from characterizing the state vector process (1) using equations (9), (11), and (18), substituting them into the nominal pricing kernel (15) and expressing it in matrix form.  $\square$

The vector  $\lambda_0^{\$}$  shows that the constant component of the market prices of risk related to consumption growth and  $\nu_t$  are affected by the inflation process. Given in equilibrium, the inflation process is determined by consumption growth and  $\nu_t$ , the nominal compensation for risk depends on the response of inflation to these two processes.

## 4 Analysis

We compare the term structure implications of the exogenous and endogenous inflation economies presented above. This analysis is useful in understanding what we can learn about interest rate dynamics when a monetary policy rule is imposed. In addition, we conduct a policy experiment by changing the monetary policy rule to analyze its implications for the term structure and macroeconomic variables.

### 4.1 Data

To understand the main differences in the term-structure dynamics between the exogenous and endogenous inflation models, we calibrate the two models to selected statistics of the U.S. data. We use quarterly U.S. data from 1971:3 to 2005:4 for interest rates, consumption, and consumer prices. The zero-coupon yields for yearly maturities from 1 to 10 years are obtained using the

Svensson (1994) methodology applied to off-the-run Treasury coupon securities by the Federal Reserve Board.<sup>4</sup> The short-term nominal interest rate is the 3-month T-bill from the Fama-Bliss risk-free rates database. The consumption growth series is constructed using quarterly data on real per capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series is obtained following the methodology in Piazzesi and Schneider (2007). These data capture inflation related only to nondurable consumption and services. Therefore, it is consistent with the consumption data. The construction of the inflation data and a comparison to log-changes in the CPI are presented in the Appendix.

## 4.2 Calibration: Exogenous inflation vs. Endogenous inflation

For comparison purposes, the two models are calibrated such that they share the same real dynamics. That is, the parameters describing the real side of the economy in the two models are the same. The parameters are chosen by calibrating the endogenous inflation model to match the average level and volatility of nominal interest rates as well as the average, volatility, and first order autocorrelation of consumption growth and inflation. The resulting parameter values for the real side of the economy are used in the exogenous inflation model. The parameters for its exogenous inflation process are chosen to match selected moments of the observed inflation process. Analytical representations of macroeconomic and term structure model-implied statistics for the two models are reported in the Appendix.

Table 1 contains the common parameter values across the two models. The parameters  $\theta_c$ ,  $\phi_c$ , and  $\sigma_c$  were chosen to match the mean, standard deviation, and first-order autocorrelation of consumption growth. The habit parameters  $\eta_c$  and  $\eta_\nu$  were calibrated in the endogenous inflation model. They were chosen to match the shape of the average nominal yield curve and its volatility. The negative sensitivity of the habit to consumption growth,  $\eta_c < 0$ , generates an upward sloping yield curve. It can be interpreted as counter-cyclical risk aversion shifting marginal utility to obtain positive average risk premiums for long-term bonds.

The autoregressive parameter  $\phi_\nu$  is set to capture the volatility of interest rates for intermediate

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<sup>4</sup>The data series are available at <http://www.federalreserve.gov/pubs/feds/2006/200628/feds200628.xls>.

Table 1: Common parameter values in the two models.

Parameter	Description	Value
$\delta$	Subjective discount factor	$1.7766 \times 10^{-4}$
$\gamma$	Curvature parameter	0.65
$\theta_c$	Average consumption growth	$4.938 \times 10^{-3}$
$\phi_c$	Autocorrelation of consumption growth	0.4146
$\sigma_c$	Conditional volatility of consumption growth	$3.962 \times 10^{-3}$
$\eta_c$	Habit sensitivity to consumption growth	-28805
$\phi_\nu$	Autocorrelation of latent variable	0.10
$\sigma_\nu$	Conditional volatility of latent variable	0.055
$\eta_\nu$	Habit sensitivity to consumption growth	-12500

maturities with  $\sigma_\nu$  fixed at 0.055. When  $\phi_\nu = 0$ , we can capture the volatility of short-term and long-term rates, but it implies a quick decline in volatilities for intermediate maturities that is not observed in the data. Allowing for a positive autocorrelation of the latent taste shock variable helps to overcome this limitation. The magnitude of  $\sigma_\nu$  is such that  $\eta_\nu$  has the same order of magnitude as  $\eta_c$ . The sensitivity of the habit to the latent taste shock variable,  $\eta_\nu$ , allows us to capture the volatility of the short-term rate. When  $\eta_\nu = 0$  and the model matches the shape of the yield curve, the endogenous inflation model implies a lower volatility for the short-term rate than observed in the data.

Given both the exogenous and endogenous inflation models are calibrated such that they share the same real asset pricing dynamics, Figure 2 presents the common properties of the the real yield curve — its average shape, volatility, and term premium structure. In addition to requiring a latent taste shock variable  $\nu$  that helps to fit the structure of bond volatilities, our calibrated preference structure is not easily interpreted as habit formation preferences. This is driven by our desire to capture an upward-sloping average real yield curve and still generate an affine-based nominal term structure with endogenous inflation.

Habit-based models such as Campbell and Cochrane (1999)<sup>5</sup> and Wachter (2006) generate a countercyclical price of consumption growth risk and real interest rates that are negatively correlated with consumption growth. Our preference specification does not allow us to capture these two

<sup>5</sup>The published version of Campbell and Cochrane (1999) assumed that real interest rates are constant, but the working paper version also studied the case of time-varying interest rates.

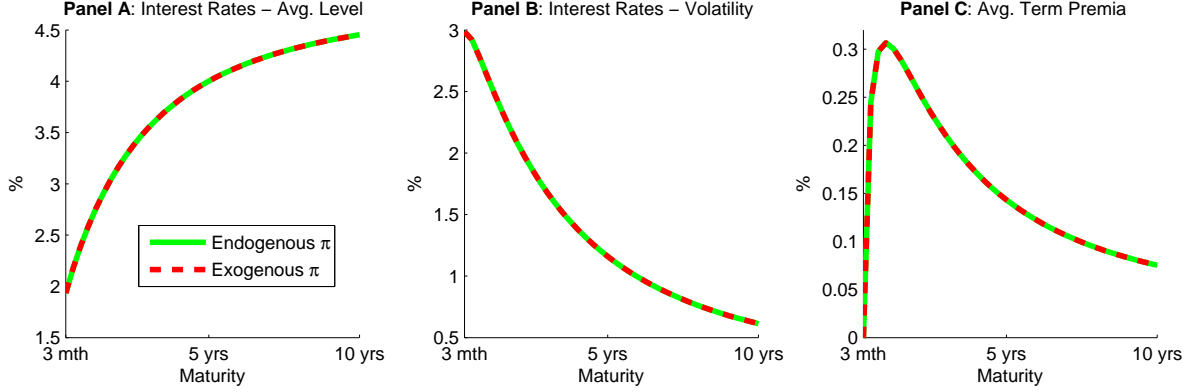


Figure 2: Real Interest Rate Properties - Exogenous and Endogenous Inflation

properties simultaneously and so cannot be interpreted as a habit-based preference specification. To see this, consider the average spread between a 2-period bond and a one-period bond previously examined in section 3.

$$\begin{aligned}
 \mathbb{E}[r_t^{(2)} - r_t] &= -\frac{1}{2}\mathbb{E}[\text{var}_t(r_{t+1})] + \mathbb{E}[\text{cov}_t(m_{t+1}, r_{t+1})] \\
 &= -\frac{1}{2}\mathbb{E}[\text{var}_t(r_{t+1})] - \gamma(\gamma + \eta_c\theta_c)(\phi_c - \eta_c\sigma_c^2)\sigma_c^2.
 \end{aligned}$$

Ignoring the term for the conditional variance of the short-term rate, this specification allows for an upward sloping average curve if either  $\phi_c - \eta_c\sigma_c^2 < 0$  or  $\gamma + \eta_c\theta_c < 0$ .

For  $\phi_c - \eta_c\sigma_c^2 < 0$  to hold,  $\eta_c$  has to be positive enough ( $\eta_c > \frac{\phi_c}{\sigma_c^2}$ ), since the autocorrelation of consumption growth  $\phi_c$  in the data is positive. Therefore  $\gamma + \eta_c\Delta c_t$  is high for high consumption growth and low for low consumption growth, which implies a cyclical price of consumption growth risk rather than a countercyclical one as in a habit formation specification. In addition,  $\phi_c - \eta_c\sigma_c^2 < 0$  implies that the autocorrelation of consumption growth under the risk neutral measure is negative. This leads to instability in the consumption growth process and generates a choppy yield curve where real rates are negatively correlated with consumption growth.

Alternatively, our calibration leads to a positive sloping real yield curve through  $\gamma + \eta_c\theta_c < 0$ . For this to hold,  $\eta_c < 0$  which captures the idea of countercyclical risk aversion. However, in a habit-based interpretation, it also implies a negative average risk aversion precluding interpreting

our preference specification as a form of habit formation. Additionally, a negative  $\eta_c$  delivers a positive correlation between real rates and consumption growth, in contrast to what is implied by the usual habit models.

Table 2 contains the model-specific parameters. For the endogenous inflation model, the policy rule parameters imply positive responses of the monetary authority to consumption growth and the level of inflation. To capture the volatility of long term rates, the autoregressive coefficient of the policy shock,  $\phi_u$ , is set close to one. The inflation process in the endogenous inflation model is given by

$$\pi_t = 0.012 - 0.28\Delta c_t + 0.047\nu_t - 1.48u_t,$$

where the negative loading on consumption growth induces the negative correlation between consumption growth and inflation that is observed in the data. For the exogenous inflation model, the parameters  $\theta_\pi$ ,  $\phi_\pi$ , and  $\sigma_\pi$  are chosen to match the mean, standard deviation, and first order autocorrelation of inflation.

Table 2: Model-specific parameter values.

Endogenous- $\pi$		Exogenous- $\pi$	
$\bar{i}$	-0.007	$\theta_\pi$	$1.115 \times 10^{-2}$
$\nu_c$	0.79	$\phi_\pi$	0.84
$\nu_\pi$	1.68	$\sigma_\pi$	$3.593 \times 10^{-3}$
$\phi_u$	0.9982		
$\sigma_u$	$2.5 \times 10^{-4}$		

Table 3 reports some model-implied statistics for both models. Panel A of the table shows that both models are able to capture important properties of the dynamics of consumption growth and inflation. As mentioned above, the endogenous inflation model has the advantage of capturing the negative correlation between consumption growth and inflation. This correlation is zero by construction under the exogenous inflation model.

Panel B of Table 3 and Figure 3 present selected properties of nominal interest rates. The average level of the yield curve implied by the endogenous inflation model match its empirical counterpart. The average nominal short-term rate and the slope of the curve for the calibrated

exogenous inflation model are higher than in the data. Panel C in Figure 3 shows that the higher spreads in the exogenous inflation model are explained by differences in risk premiums. The risk premiums in the endogenous inflation model imply expected excess returns that increase monotonically with maturity and vary from 0.22% for the 6-month rate to 2.10% for the 10-year bond yield. In the exogenous inflation model, the implied expected excess returns are 0.48% for the 6-month rate, and reach 2.90% for the 10-year bond yield.

Panel B in Figure 3 demonstrates that the volatilities of the short-term rate and the 10-year rate in the endogenous inflation model match the data. The volatility of the nominal short-term rate in the exogenous inflation model is higher than in the data and the volatility of the 10-year interest rate is significantly lower. While the ratio of the volatility of the 10-year rate to the short-term rate is 78% in the data as well as the endogenous inflation model, it is only 19% in the exogenous inflation model. This failure of the exogenous inflation model is driven by the lack of persistence in the consumption growth and inflation processes. The time-varying prices of risk ( $\lambda_1 \neq \mathbf{0}$ ) given by the habit parameters  $\eta_c$  and  $\eta_\nu$  is not strong enough to increase the volatility of long rates when inflation is an exogenous process.

In contrast, the endogenous inflation model is able to capture short-term rate and long-term rate volatility simultaneously since the policy rule allows us to describe inflation, and thus, interest rates, in terms of a very persistent process, the policy shock. That is, the volatility of interest rates does not die out quickly with bond maturity because the non-systematic component of the Taylor rule exhibits significant persistence.

One way to increase the volatility of long-term rates relative to the short-term rate in the exogenous inflation model is to increase the autoregressive parameter for the latent preference variable  $\phi_\nu$ . However, increasing this parameter leads to counterfactual implications. When the 10-year rate volatility is matched, a hump-shaped pattern for volatility across maturities is obtained: the volatility of interest rates for some intermediate maturities is significantly higher than the volatility of short and long term rates. Therefore, the exogenous inflation model is unable to jointly capture macroeconomic behavior and the average level and volatility of nominal interest rates.



Table 3: Data and model-implied descriptive statistics.

	Data	Endogenous- $\pi$	Exogenous- $\pi$
<b>Panel A</b>			
$\mathbb{E}[\Delta c_t] \times 4$ (%)	1.98	1.98	1.98
$\mathbb{E}[\pi_t] \times 4$ (%)	4.46	4.42	4.46
$\sigma(\Delta c_t) \times 4$ (%)	1.74	1.74	1.74
$\sigma(\pi_t) \times 4$ (%)	2.66	2.69	2.67
$\text{corr}(\Delta c_t, \Delta c_{t-1})$	0.41	0.41	0.41
$\text{corr}(\pi_t, \pi_{t-1})$	0.84	0.85	0.84
$\text{corr}(\Delta c_t, \pi_t)$	-0.33	-0.18	0
<b>Panel B</b>			
$\mathbb{E}[i_t] \times 4$ (%)	6.11	6.11	6.39
$\mathbb{E}[i_t^{(20)}] \times 4$ (%)	7.31	7.36	8.40
$\mathbb{E}[i_t^{(40)}] \times 4$ (%)	7.68	7.65	8.83
$\sigma(i_t) \times 4$ (%)	3.04	3.04	3.73
$\sigma(i_t^{(20)}) \times 4$ (%)	2.61	2.48	1.35
$\sigma(i_t^{(40)}) \times 4$ (%)	2.38	2.37	0.71
$\text{corr}(i_t, i_{t-1})$	0.92	0.69	0.39
$\text{corr}(i_t, i_t^{(20)})$	0.86	0.93	0.99
$\text{corr}(i_t, i_t^{(40)})$	0.82	0.88	0.99
<b>Panel C</b>			
$\text{corr}(i_t, \Delta c_t)$	-0.10	0.19	0.26
$\text{corr}(i_t, \pi_t)$	0.60	0.91	0.59

Table 3 also shows other properties of the endogenous inflation model implied by the calibration. The first-order autocorrelation of the short-term interest rate is higher in the endogenous inflation model, but it is still too low relative to the autocorrelation implied by the data. The correlation between short-term and long-term rates is also too high relative to the data. The endogenous inflation model is also unable to fully capture the correlation structure between the short rate, consumption growth, and inflation. The correlation between the short rate and consumption growth is positive in the endogenous inflation model, while in the data it is negative. The correlation between the short rate and inflation is also higher in the endogenous inflation model than in the data.

To understand the differences across the two models, we can compare the dynamics of real and

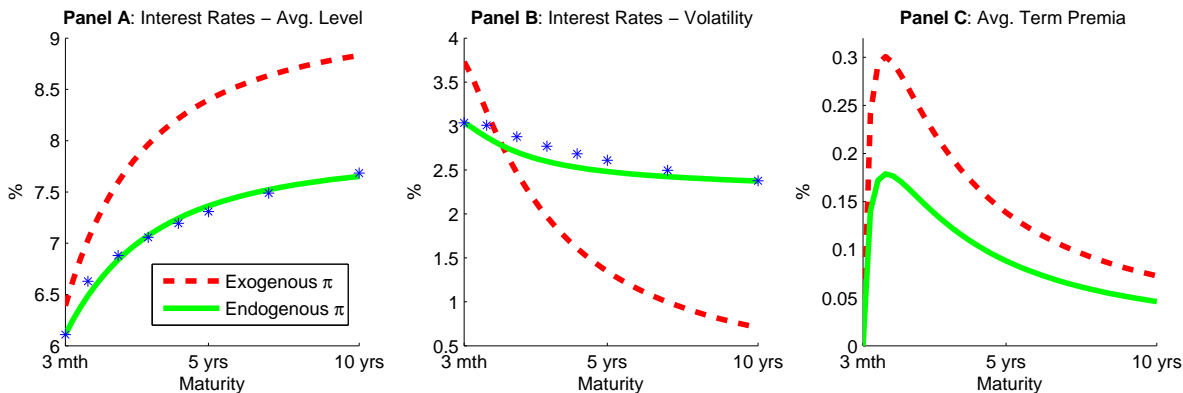


Figure 3: Nominal Interest Rate Properties - Exogenous and Endogenous Inflation. The (\*) denotes data.

nominal interest rates. Since the two models share the same parameters for the real side of the economy, the dynamics for real interest rates in the two models are the same. By comparing the real yield curve given in Figure 2 with the nominal yield curve in the exogenous inflation model given in Figure 3, note that the shape of the two average curves, their volatilities, and their risk premiums are very similar. This is not the case if we compare the real yield curve and the nominal yield curve in the endogenous inflation model.

These differences can be understood by comparing the prices of risk in Propositions 1 through 3. The prices of risk and the loading coefficients associated to consumption growth and the exogenous preference variable for assets with real payoffs are the same as those for nominal payoffs in the exogenous inflation model. Here, inflation is modeled as a process that is uncorrelated with these two factors so that the prices of risk in the nominal term structure are the same as in the real term structure. This is no longer true in the endogenous inflation model. Here, inflation depends on consumption growth and the exogenous preference variable. Since  $\pi_c < 0$ , it implies that the price of consumption growth risk for real payoffs is higher than the price for nominal payoffs. It translates into lower nominal risk premiums than real risk premiums in the calibration. A monetary policy rule generates a negative correlation between consumption growth and inflation that reduces the price of consumption risk in the nominal pricing kernel. This reduction provides hedging properties for nominal bonds and investors require lower expected excess returns to hold them.

The effects of the policy rule are also reflected in differences in the volatilities of nominal and

real rates. While the volatility of the short-term nominal rate is similar to that of the short-term real rate, the volatilities of long-term real rates are significantly lower than the volatilities of long-term nominal rates. This difference is explained by the persistence of the policy shock that do not influence real rates and drives the higher volatility of nominal rates.

We can also compare the sensitivity of interest rates and term premiums to the two common state factors in the models: consumption growth and the exogenous preference variable  $\nu_t$ . Figures 5 and 6 in the Appendix show that nominal loadings in the endogenous inflation model for interest rates and risk premiums are significantly lower than in the exogenous inflation model.

## 5 A Policy Experiment

The endogenous inflation model is useful to analyze the effects of monetary-policy changes on the dynamics of interest rates. Such policy experiments can be captured by changes in the functional form of the policy rule or changes in the reaction coefficients of the policy rule presented in Section 3.4. Here we follow the latter. We analyze the effects on the dynamics of interest rates to changes in the reaction coefficients on inflation and consumption growth in the policy rule. The motivation for this exercise is provided by empirical evidence presented in Clarida et al. (2000). They estimate reaction functions for monetary policy in the U.S. for different periods and find that the policy rule that describes the most recent period in the U.S. economy has a higher reaction coefficient to the level of inflation than in previous periods. Our objective is to analyze the implications of changes in the reaction to macroeconomic variables on the dynamics of interest rates and try to determine whether these changes are consistent with the evolution of interest rates in recent years.

Table 4 presents regression results for the policy rule (17) for the periods analyzed in Clarida et al. (2000). The table shows that the coefficient of inflation in the rule is significantly higher during the Volcker-Greenspan period (1979-2005) than in the pre-Volcker era (1960-1979).

We use the calibration for the endogenous inflation model in Section 4.2 as the baseline calibration. We conduct two policy experiments. In each experiment we modify one reaction coefficient,  $\iota_\pi$  or  $\iota_c$ , to match the average level of the short-term rate for the Greenspan (1987-2005) period, keeping all the other parameters as in the baseline calibration. We refer to these two experiments,

Table 4: Regressions  $i_t = \bar{i} + \iota_c \Delta c_t + \iota_\pi \pi_t + u_t$  for different samples.

Sample	$\bar{i}$	$\iota_c$	$\iota_\pi$	$R^2$	$\text{corr}(u_t, u_{t-1})$
1960 : 1 – 2005 : 4	0.01 (0.00)	0.07 (0.10)	0.74 (0.07)	0.42	0.80
1960 : 1 – 1979 : 3	0.00 (0.00)	0.13 (0.06)	0.60 (0.04)	0.73	0.58
1979 : 4 – 2005 : 4	0.00 (0.00)	0.21 (0.16)	1.12 (0.12)	0.49	0.67

Standard errors are reported in parenthesis.

as  $\Delta \iota_\pi$  and  $\Delta \iota_c$ , respectively. These experiments allow us to see the term-structure implications of changes in the reaction coefficient to consumption growth and inflation.

Table 5 shows some descriptive statistics associated with the two experiments. Experiment  $\Delta \iota_\pi$  requires an increase in  $i_\pi$  to 2.14 from 1.67 to match the average short term interest rate for the Greenspan era. Experiment  $\Delta \iota_c$  requires an increase in  $i_c$  to 1.07 from 0.79 to do the same. However, the implications for the dynamics of interest rates are significantly different.

The  $\Delta \iota_\pi$  experiment is successful in reducing the level of inflation, its volatility, and autocorrelation, as well as the less negative correlation between inflation and consumption growth. The  $\Delta \iota_c$  experiment does not capture these features of the inflation process seen in the data.

With respect to the term structure properties, despite the fact that  $\iota_\pi$  is the only parameter that is changed in the  $\Delta \iota_\pi$  experiment, Figure 4 shows that the implied average yield curve resembles the one observed in the Greenspan era. In particular, an increase in the reaction coefficient to inflation increases the slope of the curve.

The  $\Delta \iota_c$  experiment delivers a flat yield curve. The difference between the two experiments can be explained by observing Panel C of Figure 4. The term premiums associated with the  $\Delta \iota_\pi$  experiment are positive and those of the  $\Delta \iota_c$  experiment are close to zero. While an increase in the  $\iota_\pi$  coefficient decreases the negative sensitivity of inflation to consumption growth to -0.25 from -0.43, the increase in the  $\iota_c$  coefficient increases the negative correlation to -0.81. A stronger reaction of short-term interest rates in monetary policy to inflation therefore increases the riskiness of longer bonds. In contrast, a stronger reaction of short-term interest rates to consumption growth

Table 5: Data and model-implied descriptive statistics for the policy experiments.

	Data		Policy Experiment		
	(1971-2005)	(1987-2005)	Baseline	$\Delta\iota_\pi$	$\Delta\iota_c$
<b>Panel A</b>					
$\mathbb{E}[\Delta c_t] \times 4$ (%)	1.98	1.83	1.98	1.98	1.98
$\mathbb{E}[\pi_t] \times 4$ (%)	4.46	2.95	4.42	2.71	3.11
$\sigma(\Delta c_t) \times 4$ (%)	1.74	1.35	1.74	1.74	1.74
$\sigma(\pi_t) \times 4$ (%)	2.66	1.26	2.69	1.80	2.67
$\text{corr}(\Delta c_t, \Delta c_{t-1})$	0.41	0.28	0.41	0.41	0.41
$\text{corr}(\pi_t, \pi_{t-1})$	0.84	0.54	0.85	0.70	0.90
$\text{corr}(\Delta c_t, \pi_t)$	-0.33	-0.17	-0.28	-0.23	-0.41
<b>Panel B</b>					
$\mathbb{E}[i_t] \times 4$ (%)	6.11	4.49	6.11	4.49	4.49
$\mathbb{E}[i_t^{(20)}] \times 4$ (%)	7.31	5.83	7.36	6.04	4.56
$\mathbb{E}[i_t^{(40)}] \times 4$ (%)	7.68	6.40	7.65	6.39	4.56
$\sigma(i_t) \times 4$ (%)	3.04	2.05	3.04	2.70	2.43
$\sigma(i_t^{(20)}) \times 4$ (%)	2.61	1.73	2.65	1.66	2.39
$\sigma(i_t^{(40)}) \times 4$ (%)	2.38	1.50	2.39	1.47	2.35
$\text{corr}(i_t, i_{t-1})$	0.92	0.97	0.69	0.38	0.99
$\text{corr}(i_t, i_t^{(20)})$	0.86	0.89	0.93	0.89	0.99
$\text{corr}(i_t, i_t^{(40)})$	0.82	0.79	0.88	0.77	0.99
<b>Panel C</b>					
$\text{corr}(i_t, \Delta c_t)$	-0.10	0.08	0.19	0.26	0.01
$\text{corr}(i_t, \pi_t)$	0.60	0.44	0.91	0.84	0.91

increases the hedging benefits of longer bonds.

Panel B of Figure 4 shows the implications of the experiments on the volatility of interest rates. The  $\Delta\iota_\pi$  experiment implies a higher volatility for short-term rates than implied in the Greenspan period and a quick decline in volatility with maturity. The ratio of 10-year rate volatility to short-term rate volatility decreases to 55% from 78%. This ratio is low in comparison to the 73% ratio observed on average during the Greenspan era. Therefore, policy shocks lose some of their ability to generate long-term rate volatility. The reason is a reduced response in inflation to policy shocks that is also reflected in the reduced persistence in inflation observed during the period. The  $\Delta\iota_c$  experiment reduces the volatility of short-term rates, but long-term rate volatility is unaffected.

Other implications of the  $\Delta\iota_\pi$  that are consistent with interest rate developments during the

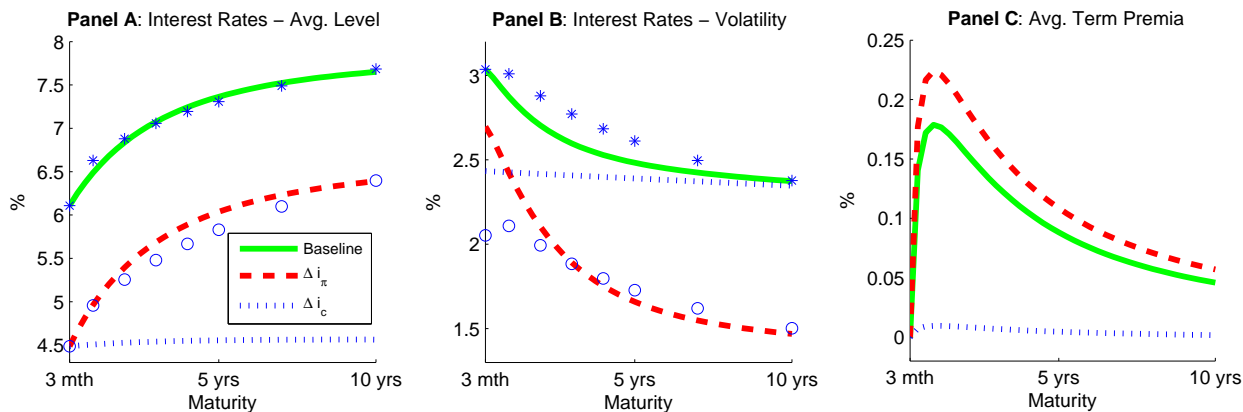


Figure 4: Nominal Interest Rate Properties - Policy Experiment. The (\*) denotes 1971-2005 data and the (o) denotes 1987-2005 data.

Greenspan era are the increase in the correlation between consumption growth and the short-term interest rate, and a decrease in the correlation between inflation and the interest rate. The autocorrelation of the short-term rate decreases in the policy experiment while it increased during the Greenspan era.

## 6 Conclusion

We show that a consumption-based affine term-structure model is able to capture an important property of long-term interest rates — that they are almost as volatile as short-term rates. We do this by incorporating preferences that lead to a stochastic price of risk and an interest-rate rule for monetary policy. Affine term structure models require, in general, highly persistent state factors to avoid a quick decline in volatility across maturities. This requirement apparently disqualifies macroeconomic variables such as consumption growth or inflation as explanatory variables in these models. However, when a monetary policy rule endogenously makes inflation correlated to real economic activity and a highly autocorrelated monetary policy shock, it is possible to simultaneously obtain a high volatility of long-term rates and reproduce the observed persistence in inflation dynamics.

Our model allows for the analysis of bond-pricing implications of policy changes. This feature provides term structure restrictions that could be potentially used to identify changes in policy

regimes. We show that a policy rule with a higher reaction to inflation better captures recent macroeconomic and term structure developments than a policy rule with a stronger reaction to consumption growth.

Two features of the model suggest further study. First, given our preference specification used cannot be directly interpreted as a form of external habit formation, it instead might be more fruitful to directly understand how our endogenous inflation specification influences term premium dynamics in a no-arbitrage term structure model that is more general than an essentially affine model. Our preliminary analysis shows that a monetary policy rule can be tractably incorporated into the exact discrete-time affine term structure models studied by Dai et al. (2006) under reasonable parameter restrictions. Such an analysis would help better understand the linkage between real and nominal term structures induced by a monetary policy rule. Specifically, one could ask what class of term structure model jointly best captures empirical features of real and nominal term structures in the presence of a monetary policy rule.

Second, the model requires a latent variable, the policy shock, with no evident economic interpretation to reproduce the observed level of volatility of the short-term rate. This is an undesirable feature if our purpose is to obtain a full general equilibrium model of the term structure. A natural question to ask is what are the economic underpinnings behind the policy shock?

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# Appendix

## A Macroeconomic Data

We present a comparison of statistical properties of two different data sets for aggregate consumption and inflation. We use quarterly U.S. data from 1971:3 to 2005:4. In the first set, inflation is constructed using quarterly data on the consumer price index from the Center for Research in Security Prices (CRSP) and the consumption growth series was constructed using quarterly data on real per capita consumption of nondurables and services from the Bureau of Economic Analysis. This data set considers inflation related to aggregate output, and therefore includes durable goods. In the second set, inflation is obtained following the methodology in Piazzesi and Schneider (2007). This data set captures inflation related only to non-durables and services consumption. Therefore, it represents the adequate measure of inflation for the representative agent economy considered here. Inflation is computed as the log-difference in the price index,  $PI$ ,

$$PI_t = PI_{t-1} \sqrt{\frac{P_t Q_{t-1}}{P_{t-1} Q_{t-1}} \frac{P_t Q_t}{P_{t-1} Q_t}}.$$

The details of the construction of  $P$  and  $Q$  can be found in <http://faculty.chicagogsb.edu/monika.piazzesi/research/macroannual/>.

The second series for consumption growth was constructed following the Piazzesi and Schneider methodology, but adjusting it to extract the effect of population growth. Consumption growth is the log-difference in the quantity index,  $QI$ , given by

$$QI_t = QI_{t-1} \frac{N_{t-1}}{N_t} \sqrt{\frac{P_{t-1} Q_t}{P_{t-1} Q_{t-1}} \frac{P_t Q_t}{P_t Q_{t-1}}}.$$

where  $N$  denotes population. The population series is obtained from the Bureau of Economic Analysis.

The comparison of statistics for the two sets of data is presented in Table 6. While the properties of consumption growth are very similar across the two sets, the properties of inflation are

significantly different. The series that captures inflation related only to non-durables and services is much less volatile and much more persistent than the series for changes in the consumer price index.

Table 6: Consumption Growth and Inflation Statistics. 1971 - 2005

	Set I	P & S (adj.)
$\mathbb{E} [\Delta c_t] \times 4$	2.03%	1.98%
$\mathbb{E} [\pi_t] \times 4$	4.58%	4.46%
$\sigma (\Delta c_t) \times 4$	1.70%	1.74%
$\sigma (\pi_t) \times 4$	3.66%	2.66%
$\text{corr} (\Delta c_t, \Delta c_{t-1})$	0.41	0.41
$\text{corr} (\pi_t, \pi_{t-1})$	0.53	0.84
$\text{corr} (\Delta c_t, \pi_t)$	-0.30	-0.34

## B Moment Conditions

Inflation independent processes:

$$\mathbb{E} [\Delta c_t] = \theta_c, \quad \sigma^2 (\Delta c_t) = \frac{\sigma_c^2}{1 - \phi_c^2}, \quad \text{corr} (\Delta c_{t+1}, c_t) = \phi_c.$$

$$\sigma^2 (\nu_t) = \frac{\sigma_\nu^2}{1 - \phi_\nu^2}, \quad \text{corr} (\nu_{t+1}, \nu_t) = \phi_\nu.$$

Exogenous inflation:

$$\mathbb{E} [\pi_t] = \theta_\pi, \quad \sigma^2 (\pi_t) = \frac{\sigma_\pi^2}{1 - \phi_\pi^2}, \quad \text{corr} (\pi_{t+1}, \pi_t) = \phi_\pi$$

$$\text{corr} (\Delta c_t, \pi_t) = 0.$$

$$\mathbb{E} [i_t] = \delta + \gamma \theta_c (1 - \eta_c \sigma_c^2) + \theta_\pi - \frac{1}{2} \gamma^2 \sigma_c^2 - \frac{1}{2} \sigma_\pi^2.$$

Endogenous inflation:

$$\mathbb{E} [\pi_t] = \bar{\pi} + \pi_c \theta_c, \quad \sigma (\Delta \pi_t) = (\pi_c^2 \sigma^2 (\Delta c_t) + \pi_\nu^2 \sigma^2 (\nu_t) + \pi_u^2 \sigma^2 (u_t))^{1/2},$$

$$\text{corr}(\pi_{t+1}, \pi_t) = 1 - (1 - \phi_c)\pi_c^2 \frac{\sigma^2(\Delta c_t)}{\sigma^2(\pi_t)} - (1 - \phi_\nu)\pi_\nu^2 \frac{\sigma^2(\nu_t)}{\sigma^2(\pi_t)} - (1 - \phi_u)\pi_u^2 \frac{\sigma^2(u_t)}{\sigma^2(\pi_t)},$$

$$\text{corr}(\Delta c_t, \pi_t) = \pi_c \frac{\sigma(\Delta c_t)}{\sigma(\pi_t)},$$

$$\sigma^2(u_t) = \frac{\sigma_u^2}{1 - \phi_u^2}.$$

$$\mathbb{E}[i_t] = \delta + \bar{\pi} + (\gamma + \pi_c)\theta_c(1 - \eta_c\sigma_c^2) - \frac{1}{2}(\gamma + \pi_c)^2\sigma_c^2 - \frac{1}{2}\pi_\nu^2\sigma_\nu^2 - \frac{1}{2}\pi_u^2\sigma_u^2.$$

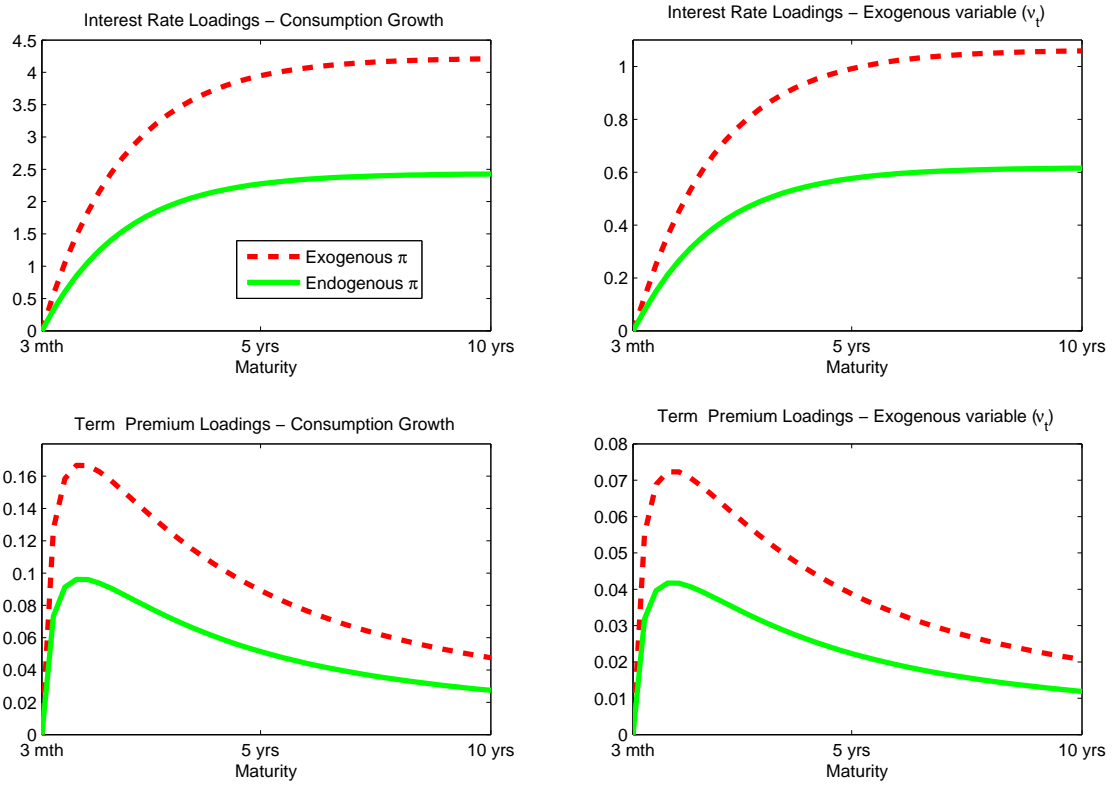


Figure 5: Nominal Interest Rates and Term premiums Loadings

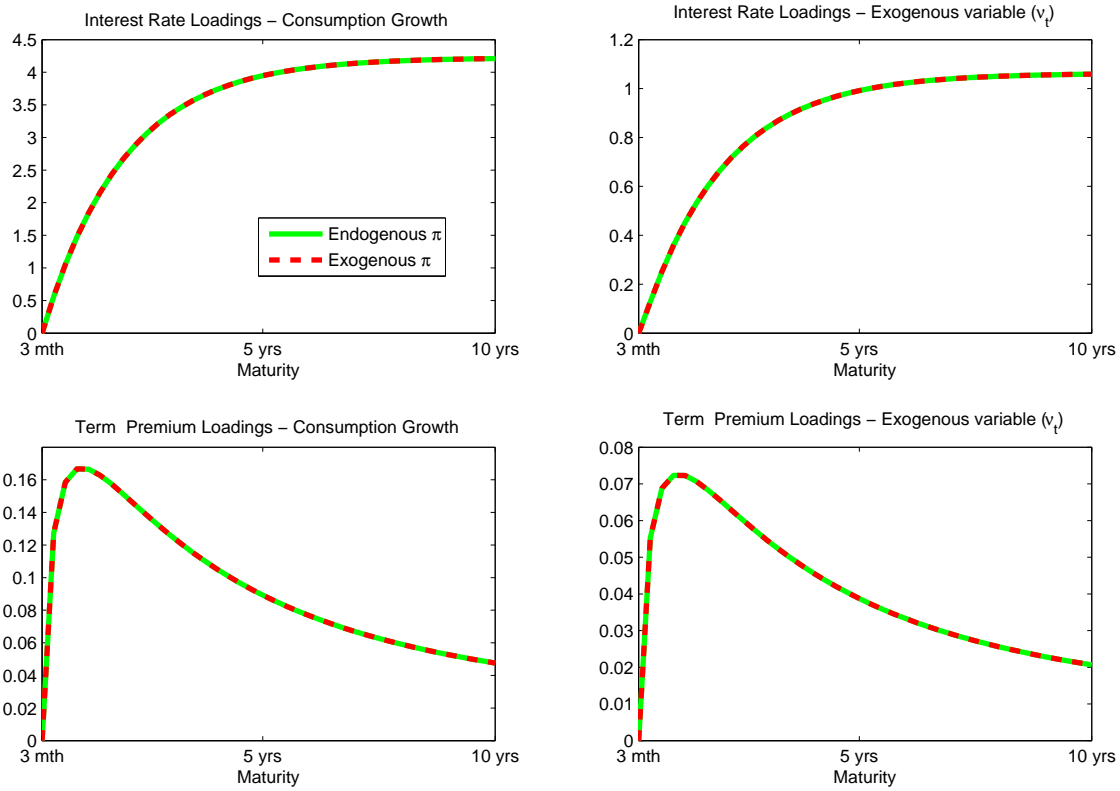


Figure 6: Real Interest Rates and Term premiums Loadings