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MONETARY POLICY, UNCERTAINTY AND THE PRESUMPTION OF LINEARITY

by

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CONTENTS

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ABSTRACT

This report shows that extreme conditions and volatility in markets are much more likely to result from systematic policy errors in gauging and responding to inflationary pressures in an economy than from unfortunate random shocks.

We describe a simple model that incorporates the key features of the policy control process. We use two versions of the model to define two hypothetical economies, one where inflation responds linearly to the state of excess demand and one that introduces an asymmetry, with excess demand having faster and stronger effects on inflation than does excess supply. Using stochastic simulations of the two economies, we study the consequences of errors in the model that is used by the monetary authority in formulating policy to keep inflation close to a target level. For each economy, we consider two cases: one where the monetary authority knows the true structure; the other where it mistakenly assumes that the other version of the model describes the economy.

The results indicate that when a monetary authority cannot know the true structure of the economy, it minimizes risks of cumulative errors and volatility in markets by assuming it faces the more difficult task of controlling inflation in a non-linear environment. If this assumption is wrong, there are costs—for example, output is slightly lower than it could have been, on average. However, the costs of incorrectly assuming linearity are much greater, because this error tends to permit outbursts of inflation, which are followed by relatively severe corrections.

RÉSUMÉ

Le présent rapport montre que les conditions extrêmes et la volatilité observées sur les marchés sont beaucoup plus susceptibles de résulter d'erreurs systématiques faites dans l'évaluation des pressions inflationnistes et dans le choix de mesures anti-inflationnistes que de chocs aléatoires défavorables.

Nous présentons un modèle simple qui incorpore les caractéristiques principales du processus de gestion de la politique monétaire. Nous utilisons deux versions du modèle pour définir deux économies hypothétiques. Dans la première, l'inflation réagit de façon linéaire au degré de demande excédentaire; dans la seconde, il y a asymétrie, la demande excédentaire ayant des effets plus rapides et plus puissants sur l'inflation que l'offre excédentaire. À l'aide de simulations stochastiques des deux économies, nous étudions les incidences des erreurs inhérentes au modèle dont se sert l'autorité monétaire pour formuler une politique visant à maintenir l'inflation près d'un objectif cible. Pour chaque économie, nous étudions deux cas : dans un cas, l'autorité monétaire connaît la structure véritable de l'économie; dans l'autre, elle suppose, à tort, que l'autre version du modèle décrit l'économie.

Les résultats montrent que, dans le cas où l'autorité monétaire ne peut connaître la structure véritable de l'économie, elle minimise les risques d'erreurs cumulatives et la volatilité sur les marchés en supposant qu'elle fait face à la tâche plus ardue de contenir l'inflation dans un contexte non linéaire. Si cette hypothèse est erronée, il y a des coûts qui se présentent, par exemple sous la forme d'une production en moyenne légèrement inférieure à ce qu'elle aurait pu être. Toutefois, les coûts que comporte une hypothèse incorrecte de linéarité sont beaucoup plus élevés, parce que cette erreur tend à donner lieu à des poussées d'inflation, qui sont suivies d'ajustements économiques assez difficiles.

1 INTRODUCTION

Monetary policy decisions must be taken in a very uncertain environment. The uncertainty comes in many forms. In this report, we focus on uncertainty about the structure of the economy and, in particular, on uncertainty about the form of the link between excess demand and inflation. We suppress any uncertainty about the way policy affects the economy, assuming a known and stable link between the policy lever and output. The presumed world is stochastic, however, and the outcomes for both output and inflation are influenced by shocks that cannot be known in advance.

We use the approach of stochastic simulation to study the effects of incorrect presumptions about the structure of the economy. Specifically, with the unknown future shocks drawn from hypothetical distributions, we conduct repeated trials to construct the properties of the outcomes of policy decisions made under two different assumed structures for the economy.

Not surprisingly, we find that the performance of the monetary authority is enhanced when its actions are based on a correct perception of the structure, regardless of what that structure is. However, it would be imprudent to assume that such fortunate circumstances do, in fact, prevail. It is interesting, therefore, to explore the consequences of misspecification of the model used for policy decisions and the implications of this aspect of uncertainty for policy modelling.

The difficulties caused by uncertainty have long been recognized and have featured prominently in debates about the nature of optimum policy rules,[1] the use of rules versus discretion[2] and the merits of simple versus more complex rules. For example, it has been argued that if the unknown structure of the economy is incorrectly modelled, any gains in credibility from having established a rule may be outweighed by losses from having specified the wrong rule. A literature has therefore emerged concerning the identification of simple rules that may be applicable to a broad range of

1. See, for example, Tinbergen (1956), Theil (1964), Brainard (1967), Poole (1971) and Holbrook (1973).

2. There are, of course, other arguments for the use of rules having little to do with uncertainty per se. See Englander (1990) for a recent survey of these issues.

economic structures and objectives, as opposed to the identification of the specific control rule that may be optimal, in some sense, based on a particular model of the economy.[3] Our work takes the latter perspective.

A second, complementary line of inquiry would be to consider optimal policy responses to a range of stochastic specifications of the economy. This would be somewhat like extending the Longworth-Poloz (1986) analysis to a world of model uncertainty. In this way one could consider the optimal policy rule for a favoured model specification and the robustness of that and other rules to plausible changes in the model. These issues are both interesting and worthwhile; they are, however, beyond our scope here.

In recent years, it has become common for economists to impose an entirely linear structure on macroeconometric models. This is in part due to the desire to have analytical tractability, but it also reflects a general inability of econometric tests to reject linearity in favour of particular non-linear alternatives. However, major improvements in computer hardware and software over the past decade have reduced the importance of analytical tractability in model building; today, numerical solution procedures are accepted as part of the standard economist's tool bag. The same technological developments have allowed more widespread use of Monte Carlo experiments, which confirm that standard tests have low power to reject linearity.[4]

The low power of tests for non-linearity suggests that there is currently little hope of providing a conclusive empirical rejection of linearity in favour of non-linearity. However, we cannot be too confident that the apparent rejections of non-linearity are correct. This being the case, it is prudent to consider alternative criteria upon which to form our judgments.

A traditional goal of monetary policy has been to avoid major extremes or cumulative excesses of whatever kind. Unsettled or extreme conditions in markets can present policy makers with unpalatable choices in instrument settings. In a stochastic environment, such unfortunate circumstances can

3. See, for example, McCallum (1988, 1990), Judd and Motley (1991), Levine (1991).
4. See, for example, Laxton, Rose and Tetlow (1993a).

arise simply as a consequence of a run of bad luck, even if the policy response was formulated based on the correct model and was ideal *ex ante*. However, as we shall show, such circumstances can also arise when policy decisions are based on incorrect perceptions about the structure of the economy.[5]

We wish to consider the interaction of errors of perception about the structure of the economy and the stochastic nature of the environment. Specifically, allowing that policy makers cannot know with any reasonable certainty whether the Phillips curve is linear or non-linear, we ask which view would be better for them to accept as an a priori position, from the perspective of minimizing policy errors. The choice of the word *position* rather than the word *belief* reflects the fact, explained in detail later in the report, that one may in fact be better off holding a position that one does not believe to be the most likely truth, if the frequency and cost of errors is less than would be the case under the alternative position.

Our question and this paper arose from research we have been doing regarding the appropriate structure to assume for a policy-oriented macroeconomic model. Having found that for simple, expectations-augmented Phillips curves estimated for the G-7 countries, RESET tests could not reject the possibility of non-linearity, we proceeded to consider the issue directly.

In one paper, we reported Monte Carlo experiments that show just how difficult it is to find econometric evidence of non-linearity in a Phillips curve *even when the true specification is non-linear by construction and the precisely correct functional form is estimated*.[6] We found that the econometrician was quite likely to reject the true non-linear specification when potential output was measured using conventional univariate techniques. We therefore concluded that any evidence of rejection of the linear form must be seen as exceptional and that failure to reject linearity should be treated with caution.

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5. Our focus on the cost of errors and the implications of asymmetries is not new. See, for example, Granger (1969) and the extensions in Rose (1976).

6. See Laxton, Rose and Tetlow (1993a).

In another paper, we reported econometric tests for specific non-linear forms of simple, expectations-augmented Phillips curves for Canada.[7] Our tests used output gaps derived from conventional univariate methods as well as gaps derived using a multivariate approach we designed for the purpose,[8] and two different representations of expected inflation, including a proxy measure based on published forecasts. Notwithstanding the low power of the tests, our results showed clear statistical support for a non-linear specification. In particular, we found support for an asymmetric formulation, where excess demand acts more quickly and more strongly to push inflation up than does excess supply to push it down. It is this form of non-linearity that we consider here, and we use the terms asymmetry and non-linearity interchangeably in the text.

This paper looks at the choice of structure for a model from a somewhat different perspective. We show that by maintaining the position that the Phillips curve is asymmetric, in the sense described above, a monetary authority would reduce the risk of major policy errors. We argue that this creates a case for adopting a non-linear specification as a risk-minimizing strategy in the face of uncertainty about the true structure.

In Section 2, we describe a simple structure that we think incorporates the key features of a contemporary model of the macroeconomy and the policy control process. This system provides a concise encapsulation of the properties of QPM (Quarterly Projection Model), the model we are developing for use in projections and policy analysis at the Bank of Canada. We use two versions of this simple model to describe two hypothetical economies, one with a linear Phillips curve and one with an extra term, which adds the asymmetry such that inflationary pressure from excess demand acts more rapidly and more strongly than does disinflationary pressure from excess supply.

We study the properties of these hypothetical economies using stochastic simulations. There are four cases in all. In two of the cases, we assume that the monetary authority's actions are based on a correct view of the inflation

7. See Laxton, Rose and Tetlow (1993b).
8. See Laxton and Tetlow (1992a).

process. These two cases are relatively straightforward stochastic control problems. The monetary authority has a forward-looking rule and "controls" an economy, the structure of which it knows. In one case, the economy is linear; in the other it is non-linear. But it is only the shocks and the dynamics of the economy, and not systematic policy errors, that create cycles and explain deviations from the policy target. In the other two cases, the monetary authority operates under a mistaken impression of the structure of the economy. In one, the monetary authority attempts to control the economy as if it were linear, when in fact it is non-linear. In the other, the monetary authority attempts to control the economy as if it were non-linear, when in fact it is linear. In these cases, the systematic policy errors also contribute to the nature of cycles. These experiments provide us with the information we need to evaluate the impact and importance of uncertainty about model structure in a stochastic environment.

The rest of the report proceeds as follows. Following the description of the hypothetical economies and the policy control process in Section 2, the third section describes our simulation techniques and some of the key differences between a stochastic and a deterministic environment for policy analysis. We have developed some new procedures to implement the desired controlled experiments. For example, in cases where the monetary authority uses the wrong model, the two models are, in effect, simulated in parallel, with the monetary authority operating *as if* the Phillips curve were linear when in fact it is non-linear, and vice-versa. Period by period, outcomes from the true structure are fed into the model used for policy settings, which then influence the next outcome, and so on. Details on how this is done are provided in Section 3. The simulation results are reported and analysed in Section 4. A fifth section sums up and concludes the report.

2 A SIMPLE STOCHASTIC MODEL

We use a simple model of how the economy functions and how monetary policy works at a very high level of abstraction. The model describes four aspects of the macroeconomy: inflation, inflation expectations, output, and the policy control process.

Inflation is driven by a mixture of expectations and forces based on market conditions. Demand conditions play an important role in the determination of inflation, but expectations of inflation are themselves, to a point, an independent determinant of inflation. Hence, there are potentially two channels of effect for monetary policy: through output gaps and through expectations.[9] Of course, generally speaking, these two channels are not mutually exclusive.

This depiction of the determinants of inflation is represented in our stylized model by equation (1), where $Y - \bar{Y}$ and Y^* are the linear and the non-linear components of the function linking inflation to the output gap and where Π and Π^e are inflation and expected inflation. We define the output variables more precisely below. For the moment, the reader can think of $Y - \bar{Y}$ as the output gap, with $\bar{Y}$ defined as the equilibrium level of output in an economy with the same structure as we are describing, but without shocks—that is, the deterministic mean. We will show that the mean value in the stochastic world will differ from the deterministic mean if the economy is non-linear.

$$\Pi_t = \gamma_1 \cdot \Pi_{t-1} + \gamma_2 \cdot \Pi_t^e + \gamma_3 \cdot (Y_{t-1} - \bar{Y}) + \gamma_4 \cdot Y^*{}_t + \varepsilon_t \qquad (1)$$

Equation (1) has an inertia term included to pick up the effects of intrinsic propagation of inflation from contracts, habits, trading relationships or other factors. Purely expectational dynamics are added to this. We consider only the case where there is no permanent trade-off between inflation and output, that is, the case where $\gamma_1 + \gamma_2 = 1$.

9. The influence of monetary policy through the exchange rate adds another relative price dimension to the general discussion; we do not deal explicitly with open-economy effects in this paper.

Equation (1) represents the most general form of the Phillips curve considered here. The linear model is identical, except for the imposition of the restriction $\gamma_4 = 0$. In particular, both the linear and non-linear formulations of the Phillips curve include the *same* stochastic error term, $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$.

Equation (2) is our representation of expectations formation:

$$\Pi_t^e = \alpha_1 \cdot \Pi_{t-1} + \alpha_2 \cdot \Pi_{t,t+1}^e + (1 - \alpha_1 - \alpha_2) \cdot \Pi^{SS}. \tag{2}$$

We model expectations of future inflation as a linear combination of information from the past, the model's predictions of future inflation, and the steady-state rate of inflation.[10] In this simple representation, a single lag of inflation captures the extent to which past information influences expectations.[11]

The inclusion of the steady-state rate of inflation, Π^{SS}, reflects a world where the public understands and gives some credibility to the monetary authority's commitment to a target level of inflation.[12] Note that we are not considering here a process of *changing* the rate of inflation; rather we are looking at the *maintenance* of a fixed target rate of inflation in a stochastic environment. Technically, putting some weight on Π^{SS} tends to tie down the response to shocks somewhat and prevent major cumulative errors from developing.[13] In any case, the assumed coefficient is very small, 0.05 (calibration is discussed in subsection 3.2).

The forward-looking term, $\Pi_{t,t+1}^e$, the expected value at time *t* of the next period's inflation rate, is provided by solving the model in the stochastic environment. Its appearance here (in addition to the steady-state rate of inflation) can be interpreted as a recognition by economic agents that the

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10. This formulation is taken from our QPM work. It is similar to the forward- and backward-components approach in Buiter and Miller (1985).

11. Note that the effect of the lagged inflation rate in equation (2) is quite apart from the intrinsic inertia from the same variable in equation (1), although the latter can be used to justify the former.

12. In cases where the monetary authority acts based on an incorrect assumption about the economy, it will not generally hit the target for inflation on average. In these cases, we use the actual mean inflation rate and not the target rate in modelling the expectations of private-sector agents.

13. Indeed, when the true model is non-linear and the policy model is linear the results become much less stable without this term. Thus, allowing private expectations to put some weight on the target limits the consequences of the monetary authority's incorrect assumption of linearity.

inertia in the system makes it too costly for a monetary authority to try to maintain $\Pi_t = \Pi^{SS}$ on a continuous basis.

Equation (3) defines the non-linearity we use for this paper:

$$Y_t^* = \begin{pmatrix} (Y_t - \bar{Y}),\ Y_t \geq \bar{Y} \\ 0,\ otherwise \end{pmatrix}. \tag{3}$$

In this formulation, Y^* is zero if the output gap is negative, and it is equal to the output gap when the latter is positive. We therefore sometimes refer to this variable in the text as the "positive output gap." Note that Y^* enters equation (1) contemporaneously, whereas the gap itself enters with a lag. This is consistent with the empirical evidence reported in Laxton, Rose and Tetlow (1993b). It means that excess demand acts both more quickly and more powerfully in creating upward pressure on inflation than does excess supply in putting downward pressure on inflation.[14]

The third aspect of the model concerns the properties of the output cycle and the way that policy affects output. An accepted stylized fact is that shocks to aggregate demand have persistent effects, reasonably described by a low-order autoregressive process. Moreover, the impact of a change in the policy instrument is not felt immediately; we assume that the order of this lag is about the same as the order of the autoregressive process for output. For our purposes here, the policy instrument, R, can be thought of as any control variable. These relationships are shown in a stylized "IS curve," equation (4):

$$Y_t - \tilde{Y} = \beta_1 \cdot (Y_{t-1} - \tilde{Y}) - \beta_2 \cdot R_{t-1} + \eta_t. \tag{4}$$

The control variable, R, is measured relative to an equilibrium level, which is not determined in this simple model. Finally, note that the IS curve includes a stochastic disturbance term, $\eta_t \sim N\left(0, \sigma_\eta^2\right)$. For our experiments, we assume that this shock is independent of the inflation disturbance.

14. Equation (3) defines a particular form of non-linearity in the Phillips curve—asymmetric response to excess demand and excess supply—but the function is locally linear in the two segments. In some of the regressions reported in Laxton, Rose and Tetlow (1993b), there was a marginal improvement in the historical fit when a quadratic curvature was used in the region of excess demand.

It is important to note that equation (4) is written in terms of Y relative to a constant $\tilde{Y}$. The fact that $\tilde{Y}$ is time invariant is merely a notational simplification. We could easily add a growth element to the model; indeed, we prefer to think of the model as describing fluctuations around a potential growth path. However, the analysis in this paper concerns only cycle effects and it simplifies the exposition to suppress the growth element. The fact that we use $\tilde{Y}$ and not the $\bar{Y}$ from equations (1) and (3) is important. The stochastic process for output is written in terms of a deviation from a mean value—$\tilde{Y}$ is always the mean value of Y. We shall show in the next section that the mean value of Y in a stochastic, non-linear economy must lie below $\bar{Y}$, the point that defines the kink of the non-linear Phillips curve.[15]

In the linear world, the introduction of uncertainty into a model that describes the theoretical foundations for our macroeconomic model might well change the equilibrium for the level of the real interest rate and the level of potential output. However, our simple model cannot address the impact of uncertainty in this sense. In effect, we normalize the level of output at $\bar{Y}$ for the linear economy, and $\tilde{Y}$ is the same as $\bar{Y}$ in this case. We then consider the impact of adding non-linearity to this world (as defined by the rest of the model structure and the properties of the shocks). One result is a different mean value for Y. We will generally reserve the symbol $\tilde{Y}$ for this non-linear case, but it is correct to think of $\tilde{Y}$ as the stochastic mean, regardless of the model.

The simple model sketched above is best thought of as an annual model; a quarterly representation would require more lags and more complicated representations of how dynamic processes interact. Of course, the real-world monetary authority often gets considerable information about shocks in the year they occur. This is offset, however, by the fact that the more complete model would have a somewhat longer lag for full influence

15. This point is not peculiar to our particular choice of a functional form. Given any function with this sort of asymmetric response to excess demand, the stochastic mean must lie below the mean for a deterministic economy with the same structure.

of the control variable on output.[16] What is important for our work is that both the monetary authority and private agents consider the future in making current choices, and that policy and demand shocks affect inflation with a lag. For our purposes, there is little to be gained from the use of a more complex quarterly model and a good deal to be lost in terms of the computational costs of the exercise.

The delayed effect of the policy instrument on output is one reason why it is not appropriate for a monetary authority to aim to keep inflation precisely on target in a stochastic environment. If the monetary authority cannot know the future shocks it cannot, except by chance, hit the inflation target precisely. Moreover, the lags in the effect of policy and the other sources of inertia in the system limit the speed at which it is reasonable to try to bring inflation back towards the target following a shock. For these experiments, the model is closed with the following policy rule:

$$R_t = \delta \cdot \left[\sum_{i=1}^{3} \left(\lambda_i \cdot \Pi GAP^{e}_{t,t+i} \right) + \lambda_4 \cdot (Y_t - \tilde{Y}) \right] + (1 - \delta) \cdot R_{t-1} . \qquad (5)$$

Our choice of a policy rule requires some motivation. It is, of course, an ad hoc rule, in the sense that it was not derived as the solution to an optimal control problem. In this regard, it is like the current Canadian targets for reducing inflation and virtually all other popular candidates for monetary targets and operating rules.[17] In general, "optimal" rules are only optimal in the context of a particular model, and for a particular specification of the objective or loss function. The rule we use for this work is similar to those we have used in developing the QPM model. It works fairly well as a control rule over a wide variety of typical simulation experiments, providing the necessary nominal anchor to a system experiencing shocks and returning the solution to control within a reasonable time in the absence of further shocks.[18]

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16. Some readers of an earlier draft questioned the shortness of the policy lag we have assumed. In making this lag shorter than it might in fact be, we are biasing the results against cumulative errors from an incorrect presumption about the economy, because the effects of mistakes can be offset more quickly in this model than may in fact be feasible.

17. See, for example, McCallum (1988, 1990) and Judd and Motley (1991).

18. This rule ensures that, on average in the linear world, inflation returns to control within three years.

The policy rule has two parts. The first term in the square brackets is the policy-targeting part. It triggers response to *expected* future inflation gaps, ΠGAP^{e}_{t+i}, while recognizing that closing these gaps quickly would be costly. The use of *future* inflation gaps reflects the fact that policy can affect output only with a lag. That these are *expected* gaps reflects our simulation methodology, wherein future shocks are not known when the policy instruments are set. Note that the monetary authority's perception of the structure plays a role here in the policy response to a shock. From given initial conditions, a shock that creates excess demand will elicit stronger response if the authority views the economy as having non-linear structure, because of the forward-looking expectations of the impact of the shock on inflation. The inclusion of $Y_t - \tilde{Y}$ reflects the output gap's usefulness as an additional indicator of latent inflationary pressure.[19]

The specific measure of the inflation gaps used in the control rule must differ in the two hypothetical economies we consider. In the linear case, we can define $\Pi GAP^{e}_{t+i} = \Pi^{e}_{t,t+i} - \Pi^{ss}$, that is, the monetary authority simply compares its projected inflation values with the target, steady-state rate. In the non-linear world this would be an inconsistent rule. To see why, consider again the form of the Phillips curve. In a notional stochastic equilibrium, with respect to output, the linear gap term, $Y_t - \bar{Y}$, will be negative (output below the deterministic mean, the kink point). This implies that, as a matter of course, the monetary authority must act as though it expects inflationary pressures to be operating in the immediate future, since it knows that there will eventually be inflationary shocks with asymmetric effects. To anchor the system in a long-term average sense to the target rate of inflation, the short-term operating target must be slightly lower than the long-term target. Given the non-linear world described above and this reaction function, it turns out that the monetary authority must aim at a short-term target rate of inflation about 0.8 percentage points below the desired long-term average rate. We return to this issue later in the report.

19. The monetary authority is presumed to use a measure of the gap consistent with its view of the world. Thus, when attempting to control an economy it believes to be non-linear, the monetary authority takes into account the fact that the sustainable level of output is lower than in an equivalent deterministic world.

The second part of equation (5) is the R_{t-1} term, which represents a desire, on the part of the monetary authority, to avoid excessive volatility in the instrument settings. This desire may reflect uncertainty regarding the controllability of instruments, or concern that measured point elasticities of a model may not be valid over larger variations in instrument settings. Alternatively, it may reflect other implicit objectives or constraints on the monetary authority's behaviour.

3 METHODOLOGY

3.1 Simulation technique

Our goal is to illustrate some of the implications of uncertainty for policy. We focus on two aspects of the uncertainty faced by a monetary authority—the world is stochastic (and the shocks cannot be known in advance), and the structure of the economy cannot be known with certainty. The stochastic nature of the problem and the lags in the system make perfect control impossible; however, in repeated trials (or over very long time periods) the monetary authority can aim to control the economy well enough to achieve its goal *on average*.

In the simulation experiments, the main objective of the monetary authority is to influence the economy with a view to maintaining the underlying target rate of inflation in the face of shocks to aggregate demand (the η_t in the output equation, equation 4) and to inflation directly (the ε_t in the Phillips curve, equation 1). It is important to understand that, while the model is forward-looking, the monetary authority is *not* given knowledge of the future shocks. We do *not* simulate the model using perfect-foresight methods. At each point in time, the monetary authority knows only what has happened up to that point.

In the two cases where the authority is assumed to know the true structure, the economy is clearly controllable in the above sense and the monetary authority can achieve its goal on average. The main interest in the results is in higher moments—for example, the variability and skewness of the distributions of inflation and output.

The other two cases are more complex. For these cases, we use two models operating in parallel. The two models are identical with the exception that in one there is a non-linearity in the Phillips curve. The monetary authority chooses the current instrument setting and a notional path for future values of the instrument, and also computes expected outcomes for inflation and output from one model, but the instrument settings influence the actual results through the other model.

Private-sector agents do not misinterpret the economy. Moreover, they place some weight on the true steady-state rate of inflation in forming their expectations, so the solution has a nominal anchor. Note that the actual mean inflation rate in these cases will not be the target rate; the monetary authority does not achieve its goal on average. But there is a stable mean inflation rate in each case, and we assume that private agents know and use that mean value in forming their expectations.[20] We use these simplifying assumptions deliberately to keep the nature of the uncertainty in the problem clear and limited.

Note, furthermore, that we do not allow active learning by the monetary authority in these experiments. We assume that the monetary authority cannot *know* the true structure of the economy. While it would make little sense to ignore learning if the monetary authority could get all the information in the repeated draws of a Monte Carlo experiment on the economy, in reality there would be only one sequence of draws, and our results suggest that the systematic errors that arise could easily be attributed to chance, even after 50 years.[21]

In the linear economy, with additive, zero-mean, symmetrically distributed shocks, the mean of the distribution of outcomes in a stochastic environment is the same as the deterministic solution to the same model. In other words, the model as written has the property of certainty equivalence—everything looks the same in the stochastic environment because the random variation is mean-preserving and behaviour is assumed to be such that other aspects of the uncertainty do not change the behavioural equations.[22]

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20. The private agents do not formally solve a problem that includes the monetary authority's expected future behaviour in response to the predictions of the true model. In this sense, private agents do not know that the monetary authority is using an erroneous model. Private agents act as if they receive a forecast for interest rates from the monetary authority, and they give some weight to the stochastic steady-state inflation rate from their own model (which happens to be the truth in this case) as if it were the monetary authority's target inflation rate.

21. A more ambitious alternative would be to allow the monetary authority to establish prior beliefs as to the likelihood of the alternative specifications and to update these weights in Bayesian fashion as new information arrives. While undoubtedly interesting, this would be challenging to implement with available algorithms.

22. Recall, however, that this is a normalization assumption. We do not wish to assert that certainty equivalence truly holds in the linear world. Our model is not designed to deal with that question.

In the non-linear stochastic world, however, things are more complicated. In the Phillips curve, the expected value of the positive gap term is strictly positive. Thus, if inflation is to be stable, the mean of the distribution of output in the stochastic world must be lower than the deterministic solution of the same model.[23] This is not a feature of our particular choice of functional form for the non-linearity. Any economically similar form of asymmetry in the Phillips curve would result in this property. In effect, the "natural" level of output must lie below the deterministic steady-state level of output by just enough to offset the inflationary bias induced by the non-linearity. Technically, there must be an average negative effect from the symmetric term to offset the average positive effect of the other term or inflation will not be stable. In our model, the mean value for output in a stochastic world, $\tilde{Y}$, must lie below the kink in the non-linear Phillips curve at $\bar{Y}$. In the stochastic simulations of non-linear economies, we must take this into account. As explained in the description of the model, we do so in the output equation by interpreting it as generating the dynamics of output, *relative to the correct stochastic mean*.[24]

We assume that the monetary authority makes decisions that are fully consistent with its assumption about the structure of the economy. This implies that when the monetary authority uses an incorrect model, there are two sources of error in the policy settings—the inflation predictions come from the wrong structural model, and the output gap is mismeasured. The structural error will be compounded in the reaction function by the use of an inappropriate short-term target rate of inflation in determining the instrument settings.

In summary, we have four cases: (1) the monetary authority assumes the economy is linear and it is, in fact, linear—we designate this as $E(L|L)$, where the expected model is before the bar and the truth is indicated after

23. These points are developed more fully in the Appendix.

24. We cannot determine this mean analytically. We use an iterative numerical procedure. Beginning with a guessed starting value for the mean, we solve the non-linear control problem with a fixed set of antithetically paired random shocks. This gives us an outcome for the mean. We then repeat the experiment and continue this process until we have a stable estimate of the mean of output, consistent with the non-linearity in the Phillips curve. The use of antithetical shocks allows us to reduce substantially the computing required for this step. The main experiment is then performed using standard random shocks.

the bar; (2) the authority assumes the economy is linear when it is, in fact, non-linear—$E(L|N)$; (3) the expected economy is non-linear and this is correct—$E(N|N)$; and (4) the monetary authority assumes the economy is non-linear when it is, in fact, linear—$E(N|L)$.

We get the shocks, ε_t and η_t, using the quasi-random number generator routines in TROLL to draw values from their presumed independent normal distributions. In each trial, we simulate the model, calibrated as described in the next subsection, over a sequence of 50 "years." In each of the four cases, there are 60 trials or replications of the experiment. The same shocks are used for each case, so that variation across the cases comes solely from the structure and policy control process.

In each period, the simulator solves the control problem for the "expected" model to determine the instrument setting. This solution takes as known the current-period shocks, ε_t and η_t, and the predetermined state variables, Y_{t-1}, Π_{t-1} and R_{t-1}, determining R_t based on equation (5) and the presumed laws of motion of the economy. This means that the monetary authority forms estimates of Π_{t+i} without knowledge of the values of future shocks and, in some cases, with incorrect assumptions about the structure of the economy.

The solution is built up sequentially. For the first period, given the starting conditions[25] and the shock drawings, an instrument setting is determined according to the control rule. Owing to the forward nature of the control problem, however, this necessitates a complete solution for the projected path of the return of the economy to the steady state, in the absence of further shocks. When the monetary authority is using an incorrect model, there is an additional step. The control variable settings from the false model are fed into the true model to generate the actual outcome for the first period. Given the first-period solution, the process moves on and the shocks for the second period are incorporated. This moves the economy off the previously expected solution, even in those cases where the instrument

25. The starting conditions vary only to the extent that we start the process off from a notional steady state two years back in order that all lagged variables have non-steady-state values in the first year of the simulation.

was set based on a correct assumption about the structure. The policy process is repeated, providing another projected path for the policy instrument.

This process is continued for 50 years.[26] The whole sequence is replicated 60 times for each case. Each replication can be thought of as a drawing of what postwar data might have looked like, given a particular set of shocks. The replications allow us to measure how experience might have varied over such a period, given the stochastic nature of the world and the presumed policy response rule.

These simulations require a lot of computing, particularly in the cases where the authority is mistaken in its view of the economy. The 60 trials with 50 periods for each of the four cases amount to 12,000 time points; with two cases involving false models, the equivalent of 18,000 full simulations is required.[27] This could not be reduced very much. It is important to have roughly normal frequency distributions for the shocks themselves within trials, and enough replications that we can have confidence that the resulting sample statistics reflect reasonably well the properties of the stochastic world with the presumed policy behaviour. In all cases, the first three moments of the distributions of inflation and output settled down on roughly constant values by the 60th trial. It is an important lesson, however, that particular sequences of "random" shocks to a model with considerable inertia and an imperfect controller can produce quite different outcomes. There is much of interest in the distributions of the results, both across and within trials.

3.2 Calibration of the model

The parameters chosen for this study are shown in Table 1. The model is calibrated to reflect the Canadian data, based on a variety of evidence. Where information is available to guide our judgment, we use estimates

26. Each simulation is actually allowed to proceed for some time after the 50th year, during which time no further shocks are drawn, in order to ensure that the policy rule is, in fact, controlling the economy. With the model and policy rule used here, loss of control was not a problem.

27. Many additional simulations are required to identify the mean shift in the non-linear economy and the corresponding shift in the short-run target rate of inflation in the reaction function. Also, in the two cases with false models, still more simulations are needed to get consistency with respect to the steady-state rate of inflation used by private-sector agents in forming their expectations.

from other work. In particular, the estimation results of Laxton, Rose and Tetlow (1993b) are used to calibrate the Phillips curve, equation (1). In calibrating the equation for expectations, we assume that agents place a relatively large weight (70 per cent) on recent history, some weight on model-consistent forecasts (25 per cent) and a small weight on the steady-state inflation rate (5 per cent). The coefficients for the output equation, equation (4), come from an estimation using annual data for Canada from 1956 to 1990. The standard deviations assumed for the shocks are based on the properties of the errors from the respective estimated models.

Table 1
Calibration information

Equation number	**Dependent variable**	**Parameters**	
		Symbol	**Value**
1	Inflation	γ_1	0.32
	π	γ_2	0.68
		γ_3	0.315
		γ_4	0.81
	Shock	σ_ε	1.06
2	Expected inflation	α_1	0.70
	π^e	α_2	0.25
		$1 - \alpha_1 - \alpha_2$	0.05
3	Output gap	β_1	0.61
	$Y - \tilde{Y}$	β_2	0.98
	Shock	σ_η	0.89
5	Control variable	δ	0.50
	R	λ_1	2.00
		λ_2	1.00
		λ_3	0.50
		λ_4	1.00

Note: The standard deviations are measured in percentage points.

The parameters chosen for the reaction function, equation (5), are based on pragmatic considerations. The coefficient on R_{t-1} (that is, $1 - \delta$) measures the monetary authority's tolerance for wide movements in instrument settings. In the spirit of the exercise, we hypothesize that an authority that is unsure as to the true structure of the economy would not feel confident enough to allow extreme swings in instrument settings; hence, we incorporate a significant amount of smoothing by choosing $\delta = 0.5$. Laxton and Tetlow (1992a) argue that a forward-looking policy rule with discounting and uncertainty should include weights on future values of the target variables that decline roughly geometrically with lead length. The parameters λ_1, λ_2 and λ_3 have been chosen to approximate this pattern. Finally, λ_4 was fixed at unity. This value seems to work well in that control is never lost and inflation has a tendency to return to the target level over a reasonable horizon. For the linear world, inflation returns to the target level within three years, on average, with this rule.

As mentioned previously, our policy rule is formally ad hoc, albeit designed to keep inflation reasonably close to the target. An alternative approach would be to construct the optimal control rule for both the linear and the non-linear economies, under some specification of the objectives of the monetary authority, and apply that rule to the expected model. However, since we are interested in the issue of model uncertainty, it is not obvious what optimality should mean. Most economists and policy makers do not think of their models as literally true representations of the economy. In recognition of this fact, policy economists are often interested in formulating rules that are robust to changes in the model rather than rules that are technically optimal with respect to a particular specification.[28] While we find the prospect of considering the nature of optimal policy rules in an uncertain stochastic environment intriguing, we do not think that this approach is necessary for this work.

28. See, for example, McCallum (1988, 1990), Judd and Motley (1991).

4 SIMULATION RESULTS

The purpose of a Monte Carlo exercise is to derive, through repeated experiments, the properties of a hypothetical environment, when those properties cannot be derived analytically. It is the properties of the distributions of the outcomes that are of particular interest. We begin by summarizing our results with respect to the first two moments of the distributions of inflation, output and the control variable. We add later some information on the skewness of the distributions.

Table 2 reports the means and standard deviations of the results for the three variables of interest. These numbers are based on variation across the trials. For each sample of 50 years we compute the moments of the results. Individual data points for these calculations are results relative to some reference level. Inflation is measured relative to the target rate, while output and the control variable are measured relative to their deterministic steady-state values, that is, the steady-state values generated by the same model without shocks. The 60 replications then give us a distribution for each statistic. We report the average values of these outcomes. For example, the value reported for a standard deviation is the average of the standard deviations across the trials.[29]

Case 1, where the monetary authority is correct in its conjecture that the economy is linear—that is, the $E(L|L)$ case—provides a useful benchmark. It shows that, on average, the monetary authority is able to hit its target with the control rule that we have specified. The means of inflation, output and the instrument are all very near their target or steady-state values. The standard deviations of the series are, as is to be expected, larger than the standard deviations of the original shocks, reflecting the propagating mechanisms in the model and policy rule. Nevertheless, they are not huge. For example, if the target inflation rate is zero, then on average in this world, we should expect the inflation to reach or exceed 3.4 per cent, or to reach or fall below -3.4 per cent, in only 2 or 3 years out of 50. An output

29. In each case, the average of the trial means is virtually identical to the mean in the pooled sample of all 3,000 observations. The average standard deviation reported in Table 2 is always slightly smaller than the standard deviation in the pooled sample. Some further information on the pooled sample is provided below.

gap of plus or minus 4.2 per cent or more should be expected with about the same frequency. However, if we look at the issue with respect to bands of, say, plus and minus 1 per cent for inflation, then there would be a substantially higher proportion of years when the outcome drifted outside those bands, about 56 per cent according to these results.

It would be an interesting extension of this work to ask what the requirements would be for a control rule that would keep the outcome for inflation within an arbitrary band with specified probability—and what the consequences would be for the properties of output. However, this type of question is beyond our scope here.

Table 2
Means and standard deviations based on 60 trials
(units are percentage points)

Case	Variable	Mean	Standard deviation
1	Π	-0.02	1.71
$E(L\|L)$	Y	0.03	2.09
	R	0.01	1.82
2	Π	2.24	2.09
$E(L\|N)$	Y	-1.27	2.81
	R	0.56	2.63
3	Π	0.03	1.62
$E(N\|N)$	Y	-0.78	1.77
	R	0.02	1.62
4	Π	-1.04	1.82
$E(N\|L)$	Y	-0.01	1.60
	R	0.02	1.49

Note: Individual trial statistics are based on outcomes measured relative to target or deterministic steady state.

In Case 2, the monetary authority thinks it is controlling a linear economy, but in fact the economy is non-linear—$E(L|N)$. The tendency in this case is for the authority to provide insufficient resistance to inflationary shocks. As a result, inflation is above its target level by well over 2 percentage points, on average. There is, in turn, a monetary reaction to this inflation bias that generates *lower* output, on average, and positive average instrument settings, relative to control.

Positive inflation, relative to target, persists in spite of the deflationary output gaps and instrument settings because the monetary authority's control is imprecise and inappropriately balanced, owing to the modelling error. This imprecision is reflected in the substantially higher standard deviation for each series, as compared with those of Case 1.

It is important to note that higher inflation is *not* associated with higher output in this case. To keep inflation from accelerating away from the target, the monetary authority must run a more restrictive policy than would be necessary if it realized that the economy was non-linear. This is shown in the results for Case 3.

In Case 3, the monetary authority correctly perceives the economy to be non-linear—$E(N|N)$. As in Case 1, the monetary authority is successful in hitting its long-term inflation target, on average.[30] The result for output is important, but requires careful interpretation. It reflects calculations relative to the *deterministic* steady state. It is *not* valid to conclude from Table 2 that there is a permanent output gap of -0.78 per cent in the non-linear world. Rather, the result reflects the fact that the deterministic steady state is *not a feasible outcome* in a non-linear, stochastic economy. The reported statistic measures the extent of the reduction in the effective "potential" output resulting from the interaction of the non-linearity and the uncertainty in the stochastic environment. It is our estimate of $\tilde{Y} - \bar{Y}$ in the notation of Section 2, and shows that average output in our stochastic,

30. Recall that we have adjusted the policy rule such that the monetary authority realizes that the operational short-term target for inflation must lie below the long-term target. We cannot derive the size of the necessary correction analytically, although it is directly related to the extent of the non-linearity, as measured by the size of $\tilde{Y} - \bar{Y}$. We solved the problem numerically, adjusting the short-run target until the inflation bias was removed. The reported estimates are based on a notional short-term target 0.8 percentage points below the long-term target.

non-linear economy is about 0.78 per cent less than it would be in the same economy without continual shocks.

Case 4, where the authority thinks the economy is non-linear when it is actually linear—$E(N|L)$—completes our matrix of possibilities. Analytically, this is the obverse of Case 2. Whereas in Case 2 the monetary authority fails to appreciate fully the inflationary consequences of excess demand, in Case 4 the monetary authority overestimates the potential for inflation and reacts unnecessarily strongly in such circumstances. This results in a *negative* inflation bias and a negative output gap on average.

Note, however, that since the truth is linear in this case, the modelling error does not result in policy mistakes with a tendency to compound, as happens in Case 2. The error is therefore relatively costless. The output loss is substantially smaller in Case 4 than in Case 2. Moreover, whereas mistaken presumption of linearity leads to a relatively wide dispersion of outcomes, the same is not true about mistaken presumption of non-linearity. The average standard deviations are *all* lower in Case 4 than in Case 2. Indeed, the dispersion measure for output and the instrument setting are actually smaller in Case 4 than in Case 3, where the non-linearity is really there.

By taking as benchmarks the *true-model cases*, $E(L|L)$ and $E(N|N)$, and comparing *misinformed cases* against these benchmarks, we can get a measure of the benefits of the monetary authority's having knowledge about the structure of the economy. As shown in Table 3, the consequences of not knowing the true structure of the economy are important. Yet, it is clear that these costs are much larger if a non-linear economy is controlled as if it were linear than if the opposite error is made. Examining the differences in the means, we see that the absolute differences are substantially smaller when the monetary authority incorrectly assumes that the economy is non-linear.

Moving to the standard deviations, we again find that incorrectly assuming linearity is more costly than incorrectly assuming non-linearity. In fact, incorrectly assuming non-linearity causes the standard deviation of output to *fall* relative to the no-error case, albeit at the cost of a slightly lower average level of output. The reason for this is that because the deflationary bias in instrument settings leaves the economy operating on average with a slightly lower level of output and inflation below the target, the sequences of shocks that would otherwise have resulted in an inflationary boom and subsequent monetary contraction do not trigger such phenomena as frequently or strongly.

Table 3
Comparison of the consequences of misperceiving the structure of the economy
(differences of percentage points)

Comparison	Variable	Mean	Standard deviation
(4) - (1)	Π	-1.02	0.11
$E(N\|L) - E(L\|L)$	Y	-0.04	-0.49
	R	0.01	-0.33
(2) - (3)	Π	2.21	0.47
$E(L\|N) - E(N\|N)$	Y	-0.49	1.04
	R	0.54	1.01

The policy instrument is also less variable in Case 4 than in Case 1, but there is a small increase in the standard deviation of inflation. To the extent that there are costs associated with greater variability in inflation, these might outweigh the benefits from greater predictability of output—we have no standard within this model to assess the relative importance of these effects.

When the model is truly non-linear, assuming linearity results in a deterioration of economic performance as measured by both moments of all three state variables. The differences are large enough that one would have to be very certain that the world is truly linear before assuming such a position in a policy model of this type.

Our conclusion is, therefore, that if a central bank cannot be sure of whether the economy is non-linear or linear, it is better off maintaining the a priori position that the economy is non-linear. Operating in this way, it would incur less risk that errors in combatting inflationary pressures would compound, resulting in more deeply ingrained inflationary expectations and consequently higher output costs of getting inflation back under control. Moreover, opting for a non-linear position reduces the variability of output and the policy instrument, even when this position is wrong.

Let us now consider the incidence of skewness in inflation from these experiments. It is interesting to examine skewness for what it tells us about the episodic nature of bouts of inflation. Figures 1 and 2 show the distributions of the outcomes from the pooled samples (that is, all 3,000 outcomes for each case) for the two misperception experiments. There is clear evidence of skewness in Case 2. What this means is that in repeated trials the inflation bias would not be especially severe in most instances. Periodically, however, there would be episodes when the confluence of the modelling error and sequences of inflationary shocks would trigger a burst of relatively high inflation, despite the endogenous policy response. With the mistake in Case 4, there is much less tendency to induce asymmetry in the distribution of the inflation. There is some skewness towards outcomes with lower inflation, but it is much less pronounced than the opposite result in Case 2.[31]

Table 4 provides the standardized third-moment statistic of skewness[32] and incidences of rejection of symmetry for the 60 trials in each of the four cases.[33] We use a 5 per cent significance level for rejection in these tests. The most interesting question is whether, in Case 2, the $E(L|N)$ case, the monetary authority would get clear evidence that its model was wrong. We see that statistical rejections of symmetry would emerge about one-third of the time. In an extreme, real-world experience of the sort described in Case 2, one would expect the monetary authority to abandon either the rule or the linear model or both, but this would happen only after the

31. Symmetry is nevertheless rejected statistically in the pooled sample for Case 4 at the 5 per cent significance level (but not at the 1 per cent significance level).

32. The statistic is the standard measure of skewness, based on the discussion in Kendall and Stuart (1958). It is zero for a symmetric distribution. A positive figure indicates skewness “to the right,” or higher values. Our computations were done using the implementation in the RATS program.

33. These are based on the standard test, which assumes the absence of autocorrelation. This biases the results towards excessive rejection of symmetry.

Figure 1
Frequency distribution for inflation in 3,000 draws
Case 2: Monetary authority uses linear model; the economy is non-linear.
(Inflation is measured relative to target.)

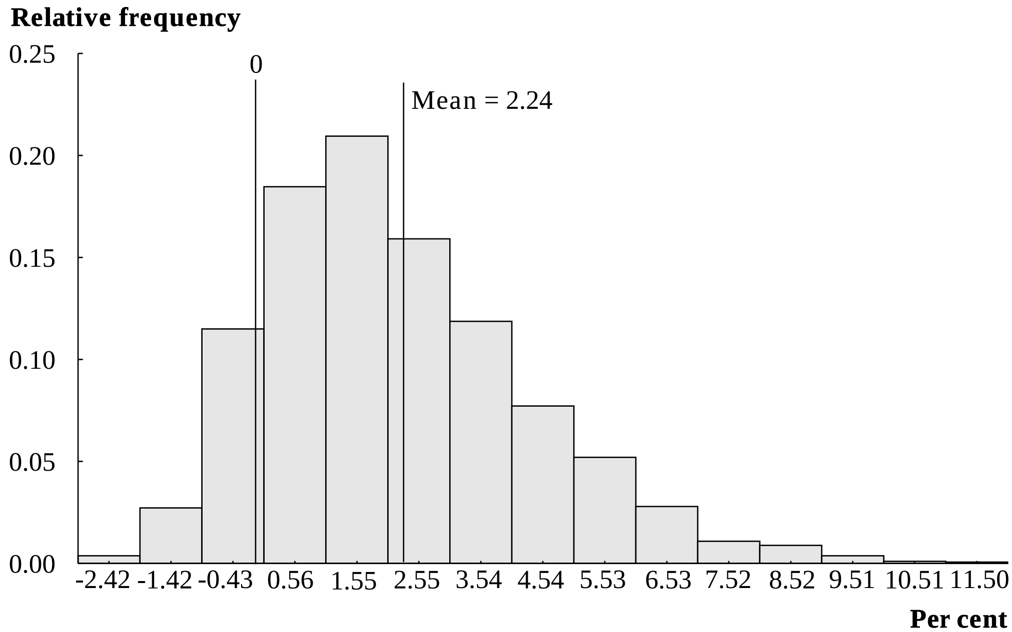
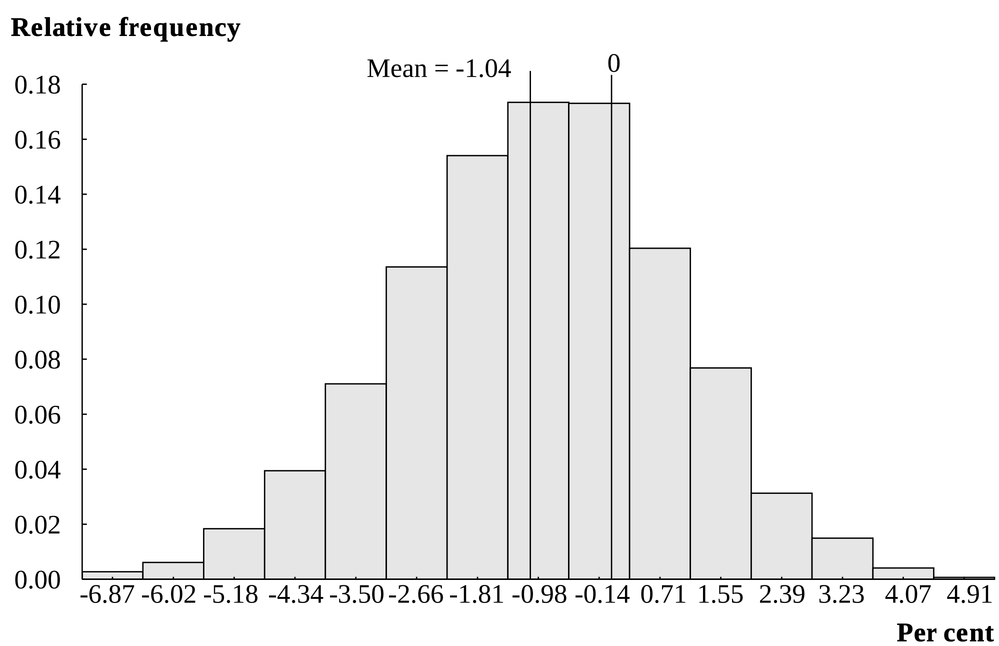


Figure 2
Frequency distribution for inflation in 3,000 draws
Case 4: Monetary authority uses non-linear model; the economy is linear.
(Inflation is measured relative to target.)



damage was already done. Moreover, the rejections of symmetry for Case 2 are not so prevalent or strong that one could be confident of rejecting misspecification before serious trouble had arisen several times. Recall that one trial in our experiment is the conceptual equivalent of the entire sample of postwar data.

Table 4
Information on skewness of inflation in the trials

Case		Skewness statistic (average of 60 trials)	Symmetry test Rejections	Per cent
1	$E(L\|L)$	-0.02	4/60	6.7
2	$E(L\|N)$	0.56	21/60	35.0
3	$E(N\|N)$	0.11	4/60	6.7
4	$E(N\|L)$	-0.09	4/60	6.7

We now compare the results for skewness in the Case 2 simulations with the properties of the historical data. The historical skewness statistic is 1.02 for the consumer price index (CPI) and 1.05 for the gross domestic product (GDP) deflator, in both cases measured using non-overlapping, 4-quarter rates of increase from 1953Q4 to 1991Q4. Based on standard tests, symmetry is rejected statistically for both series at the 2 per cent significance level. Both show strong skewness towards periodic bouts of relatively high inflation. The statistic generated by our simulations of Case 2—a non-linear world being controlled by a monetary authority under the assumption of linearity—is 0.56 on average over the 60 trials (with a standard deviation of 0.42) and 0.75 for the pooled sample of all 3,000 observations. Our Case 2 produces a distribution of inflation with properties not unlike the historical data.[34]

While it could be argued that the postwar world has been characterized by an unusual "trial" with quite atypically inflationary shocks, and while it is theoretically possible that the skewness in the data was actually caused by

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34. If we compute the skewness statistics using the overlapping quarterly measures of 4-quarter rates of increase from 1953Q1 to 1991Q4, we find 0.75 for the CPI and 0.95 for the GDP deflator. Both statistics would result in rejection of symmetry at the 5 per cent significance level.

changes in the policy goals of the Bank of Canada towards episodic tolerance of bouts of inflation, we think that it is worth considering seriously the alternative explanation—that there is asymmetry in the dynamic linkage between the state of excess demand and inflation, which was not taken into account fully in historical policy decisions.

Figures 3 and 4 show the pooled samples of outcomes for inflation in Cases 1 and 3, where the monetary authority uses the correct model. We have nothing more to say about these results. Note the symmetry of the outcomes and the much smaller dispersions, especially relative to Case 2.

We have shown that an incorrect assumption of linearity leads to upward bias, greater variability and skewness in the outcome for inflation. When inflation becomes entrenched in expectations it becomes harder to control in the face of shocks and harder to bring back to the target level. A consequence is that the monetary authority has to react more strongly once an error has been made than it would have if it had correctly perceived the inflationary potential of the situation in the first place. This tends to occur with greater frequency in Case 2 and leads to the lower average level and greater variability in output that we documented above. In addition, however, the fact that inflation tends to break away periodically creates, for the same reason, a tendency towards severe recessions.

Figures 5 to 8 show the pooled sample of outcomes for output, relative to the deterministic steady state, for each of our four cases. In Figures 5 and 6, we show the two cases where the monetary authority thinks the world is linear. Note the sharp skewness for output in Case 2. Whereas in Case 1 the results are symmetric and the probability of observing an output gap of less than -4 per cent is less than 0.03, in Case 2 the same probability is over 0.15. It is interesting to compare this result with Case 3, where the world is correctly modelled as non-linear (Figure 7). There is a tendency to skewness in Case 3, but the variability of the cycle is much lower. As a result, output gaps, relative to the deterministic mean, of less than -4 per cent

Figure 3
Frequency distribution for inflation in 3,000 draws
Case 1: Monetary authority uses linear model; the economy is linear.
(Inflation is measured relative to target.)

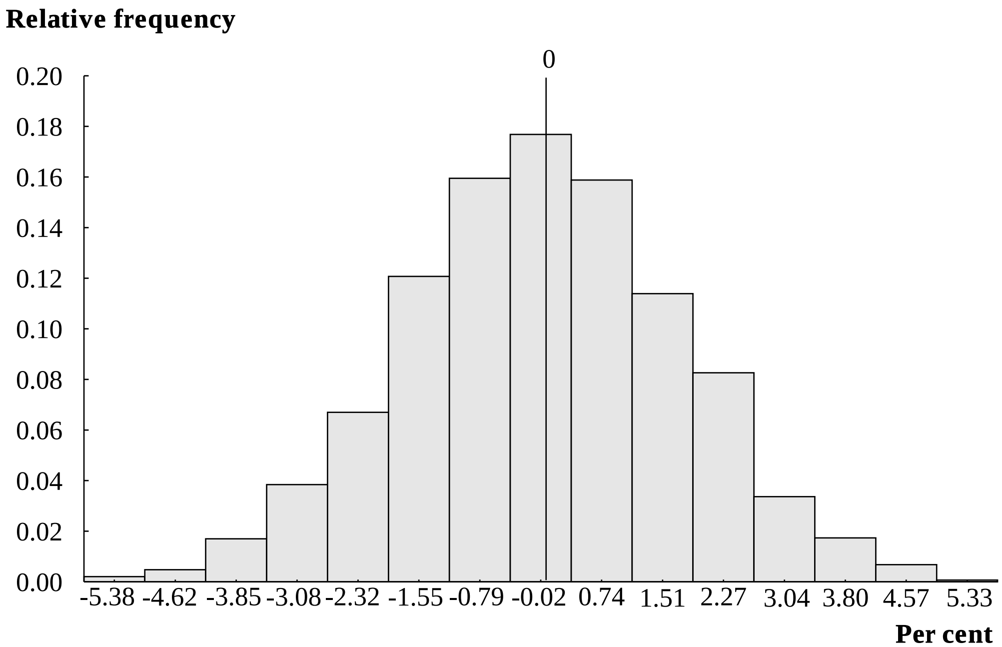


Figure 4
Frequency distribution for inflation in 3,000 draws
Case 3: Monetary authority uses non-linear model; the economy is non-linear.
(Inflation is measured relative to target.)

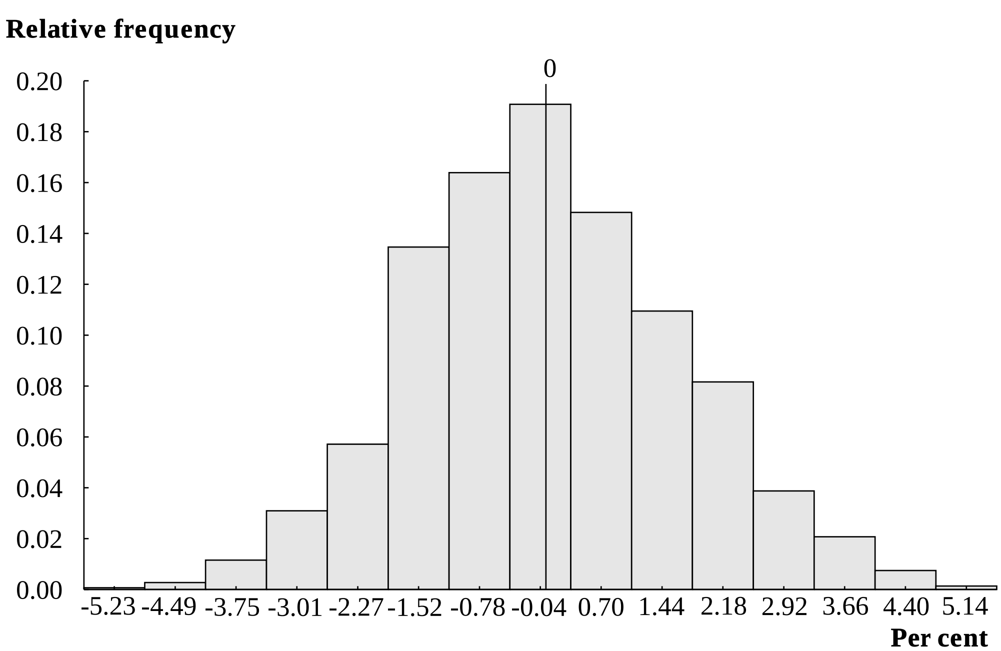


Figure 5
Frequency distribution for output in 3,000 draws
Case 1: Monetary authority uses linear model; the economy is linear.
(Output is measured relative to the deterministic steady state.)

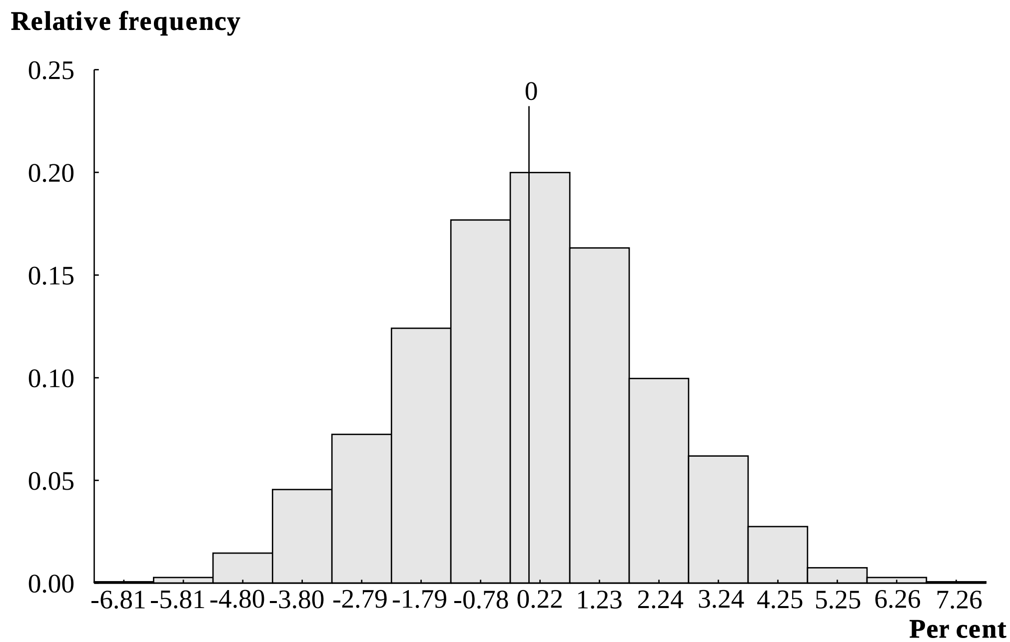


Figure 6
Frequency distribution for output in 3,000 draws
Case 2: Monetary authority uses linear model; the economy is non-linear.
(Output is measured relative to the deterministic steady state.)

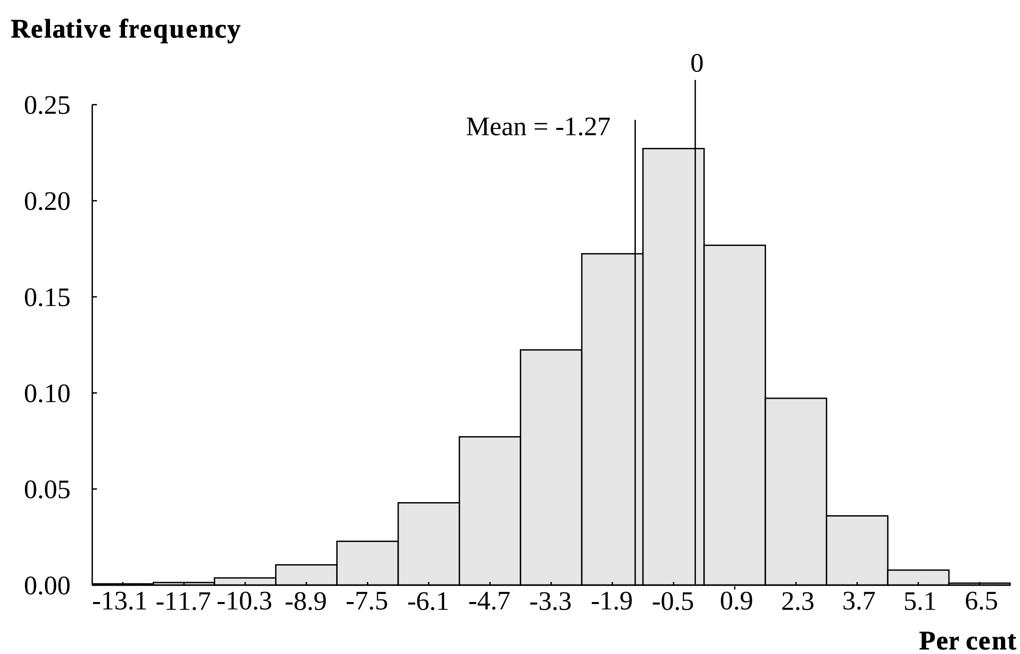


Figure 7
Frequency distribution for output in 3,000 draws
Case 3: Monetary authority uses non-linear model; the economy is non-linear.
(Output is measured relative to the deterministic steady state.)

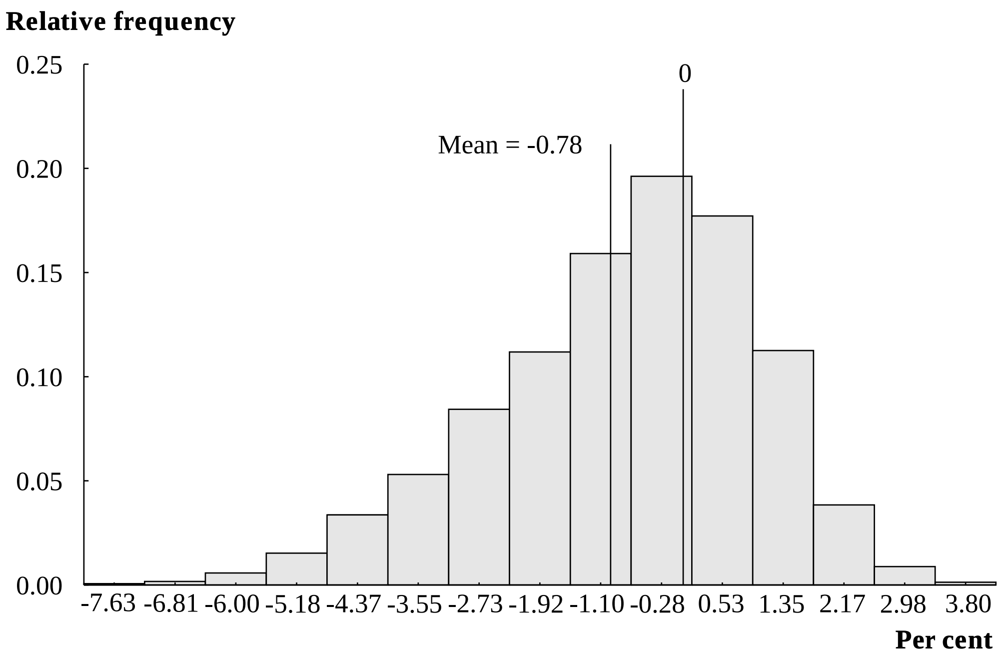
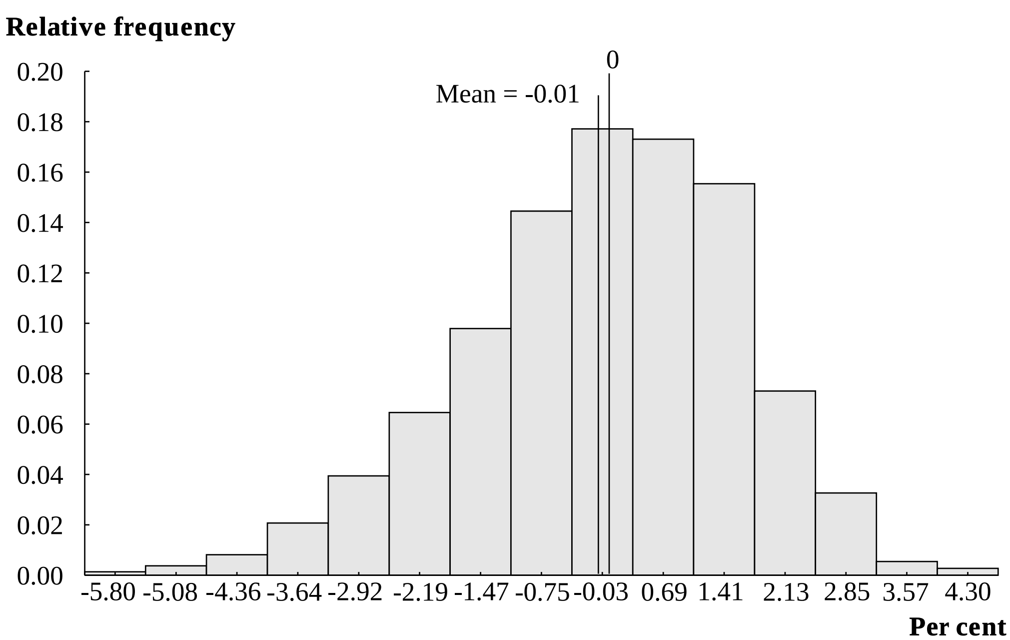


Figure 8
Frequency distribution for output in 3,000 draws
Case 4: Monetary authority uses non-linear model; the economy is linear.
(Output is measured relative to the deterministic steady state.)



occur with only about 5 per cent probability.[35] It is mainly the failure to recognize the non-linearity that leads to the tendency to sharp recessions.

Recessions can, of course, occur in a linear world, the result of response to bad luck with inflation or of unusual negative shocks to output or both. However, the probabilities are balanced in the linear world and one would expect output to be roughly symmetrically distributed about its trend and severe recessions to be quite unusual, as we find in Case 1. We do not think that postwar Canadian experience is like that.

It is difficult to get a clear picture of the historical evidence on skewness of output because we cannot observe potential output. However, to get an idea of the historical facts, we have computed the skewness statistic for the output gap as measured using the standard Hodrick-Prescott filter, applied to the log of output directly, to get a measure of potential output. For the period 1953Q1 to 1992Q2, the statistic based on the third moment is -0.55. Symmetry is rejected at the 1 per cent significance level.[36] The historical data appear to be characterized by skewness towards larger negative output gaps. The statistic generated for the pooled sample of 3,000 draws in our Case 2, where the economy is non-linear but the monetary authority acts on the assumption that it is linear, is -0.54. The correspondence is striking. A number of explanations could be advanced for the skewness in the historical measure and for the severity of recent downturns in economic activity, but it is interesting that a non-linear characterization of the inflation process helps explain such occurrences.

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35. It is important to keep in mind that we have plotted the distribution of output relative to the deterministic mean. For the non-linear model, the mean of the stochastic equilibrium is lower—about 0.8 percentage points lower for Case 3 (Table 2). Relative to this measure of “potential,” the probability of gaps of less than -4 per cent is less than 0.03 in Case 3, similar to the result in Case 1.

36. We did the same exercise using annual data; the skewness statistic is identical, but symmetry is rejected with only 82 per cent confidence. We also computed the skewness statistic for the growth rate of output. The results were similar. For quarterly data the statistic is -0.57 and symmetry is rejected at the 99 per cent confidence level. With annual data, there is still negative skewness; the statistic is -0.40, but symmetry is rejected with only 67 per cent confidence.

5 SUMMARY AND CONCLUSIONS

We have investigated the consequences for policy and policy modelling of the fact that monetary policy must be conducted in an uncertain environment. We have considered two forms of uncertainty: the fact that there are shocks that cannot be known in advance; and the fact that the monetary authority cannot know the true structure of the economy— specifically, the uncertainty as to whether inflation responds symmetrically to excess demand and excess supply.

We describe a simple model, designed and calibrated to reflect a stylized view of how the economy functions and how the policy control process works, at a high level of abstraction. We report stochastic simulations using two versions of the model—one with a linear, the other with a non-linear Phillips curve. We study four cases of combinations of perception and reality with respect to the economy: where the economy is known to be linear, where it is thought to be linear but is actually non-linear, where it is known to be non-linear and, finally, where the economy is thought to be non-linear but is actually linear.

Our results confirm that knowing the true structure of the economy is greatly advantageous in terms of minimizing the variability of output, inflation and the policy instrument around their target or steady-state values. It would have been very surprising if knowledge did not have such beneficial consequences in this exercise.

We would argue, however, that uncertainty about the functioning of the economy is the reality. It is important to seek knowledge about how the economy works; it is equally important to realize that we will never have knowledge in the form of a model that reflects the “true” structure of the economy. We have shown why this fact should have an effect on modelling strategy and on the conduct of monetary policy. We have made the case that when the monetary authority cannot be certain as to the nature of inflation dynamics, as represented here by the form of the Phillips curve, it is prudent to presume that there is an asymmetric form with an inflationary bias. Doing so minimizes the risk of errors that lead to outbursts of inflation followed by relatively severe corrections.

It is the entrenchment of inflationary pressures in expectations that makes inflation especially difficult to wring out of the system in such circumstances. The severe tightening of monetary conditions that is necessary may be seen as the proximate cause of the recession that follows, but our results show that the slings and arrows of the most outrageous fortune (highly inflationary shocks) are far less likely to precipitate severe cyclical downturns than are the cumulative effects of systematic errors in gauging and responding to inflationary tendencies in an economy.

In our introductory section, we described briefly some previous work we have done on the possible existence of an asymmetry in the Phillips curve. That work left us with the conclusion that there is an objective empirical case for a non-linear specification. This is important, because one would not want to take a position on a key structural issue just because one could not rule out logically a possible problem; our results make it clear that there are costs to making any mistake.

Our results also indicate, however, that the costs are much greater when the economy is incorrectly presumed to be linear than when it is incorrectly presumed to be non-linear. One would have to be quite sure that the economy was truly linear before it would be prudent to proceed with that assumption in policy decisions and in modelling in support of such decisions. In the face of uncertainty about the structure of the economy, adopting the position that there is a non-linear relationship between excess demand and inflation has the benefit of minimizing the risk of serious cumulative policy errors and, in particular, the tendency towards larger recessions that is fostered by such cumulative errors.

APPENDIX

The Equilibrium Level of Output in a Non-linear, Stochastic Economy

We begin with the non-linear Phillips curve, equation (1), repeated for convenience:

$$\Pi_t = \gamma_1 \cdot \Pi_{t-1} + \gamma_2 \cdot \Pi_t^e + \gamma_3 \cdot (Y_{t-1} - \bar{Y}) + \gamma_4 \cdot Y^*{}_t + \varepsilon_t . \tag{A1}$$

Stochastic equilibrium is a more complex concept than deterministic equilibrium. In a stochastic world, the solution is never precisely the "equilibrium," except by chance, and equilibrium cannot be characterized as an actual fixed point of a process. It is, however, a fixed point in terms of the unconditional expectation of the process. In other words, stochastic equilibrium is defined as the mean value around which the actual values lie and to which they return eventually. We are not interested in worlds where the process becomes dynamically unstable. We assume that the reaction function is such that there is a dynamically stable nominal anchor in the system.

To characterize the properties of the stochastic equilibrium, then, we can start by taking unconditional expectations in equation (A1). Recall that we are considering only worlds with no long-term trade-off between inflation and output, such that $\gamma_1 + \gamma_2 = 1$. A condition of stochastic equilibrium is that, on average, there can be no difference between actual and expected values for inflation, and the unconditional expectations of Π and its lag are the same (and equal to the stochastic mean). Thus, when we take expectations in equation (A1), the inflation terms drop out (the real equilibrium does not depend on the level of inflation chosen in this model) as does the expected value of the shock, and we are left with the condition:

$$E(\gamma_3 \cdot (Y - \bar{Y}) + \gamma_4 \cdot Y^*) = 0\text{, or} \tag{A2}$$

$$E(Y) = \tilde{Y} = \bar{Y} - (\gamma_4 / \gamma_3) E(Y^*) . \tag{A3}$$

Note that since the parameters are both positive, and since the positive gap variable is non-negative, condition (A2) requires that the expected value of the first term be negative. Writing the result in the form of condition (A3), we see immediately that $\tilde{Y}$ lies below $\bar{Y}$. In effect, the distribution of output must be shifted or skewed such that its mean lies below the deterministic steady-state level by just enough to satisfy condition (A2) or (A3). The extent of the shift depends on the degree of non-linearity, as captured in the size of the γ parameters, but it also depends on the properties of the resulting unconditional distribution of output. Let $f(Y)$ represent the unconditional probability density function of outcomes for output, given the model, including the reaction function, and the distributions of the two shocks. We can then rewrite condition (A3) as:

$$\tilde{Y} = \bar{Y} - (\gamma_4/\gamma_3)\int_{\bar{Y}}^{\infty}(Y-\bar{Y})\,f(Y)\,dY\,. \tag{A4}$$

We cannot solve equation (A3) analytically, because the precise $f(Y)$ depends in a complicated manner on the model. However, it is easy to evaluate equation (A4), given an empirical distribution of outcomes to compute the mean shift. One can simply iterate the full process, starting with a guess for $\tilde{Y}$, assembling f(Y) from the trials, re-evaluating $\tilde{Y}$, and then repeating the process until the estimate of $\tilde{Y}$ has converged. This process converges fairly rapidly, but it still requires a lot of computing, since each iteration requires enough trials to assure a reasonable empirical representation of $f(Y)$.

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