Banks, Credit Market Frictions, and Business Cycles

by Ali Dib
Banks, Credit Market Frictions, and Business Cycles

by

Ali Dib

International Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
adib@bankofcanada.ca
Acknowledgements

For their comments and discussions, I am grateful to Ron Alquist, Ricardo Caballero, Lawrence Christiano, Carlos de Resende, Brigitte Desroches, Andrea Gerali, Sharon Kozicki, Robert Lafrance, Philipp Maier, Federico Mandelman, Virginia Queijo von Heideken, Julio Rotemberg, Francisco Ruge-Murcia, Eric Santor, Lawrence Schembri, Jack Selody, Moez Souissi, Skander Van den Heuvel, seminar participants at the Bank of Canada, MIT, the IMF, Federal Reserve Bank of Richmond, Federal Reserve Bank of Atlanta, Reserve Bank of Australia, the European Central Bank, Sveriges Riksbank, the Deutsche Bundesbank, Australian National University, University of Ottawa, Dalhousie University, and participants at the BIS/ECB workshop on “Monetary Policy and Financial Stability,” the Bank of Canada/IMF workshop on “Economic Modelling and the Financial Crisis,” the 2009 NBER/Federal Reserve Bank of Philadelphia workshop on “Methods and Applications for the DSGE Models,” the CEA, and the CEF.
Abstract

The author proposes a micro-founded framework that incorporates an active banking sector into a dynamic stochastic general-equilibrium model with a financial accelerator. He evaluates the role of the banking sector in the transmission and propagation of the real effects of aggregate shocks, and assesses the importance of financial shocks in U.S. business cycle fluctuations. The banking sector consists of two types of profit-maximizing banks that offer different banking services and transact in an interbank market. Loans are produced using interbank borrowing and bank capital subject to a regulatory capital requirement. Banks have monopoly power, set nominal deposit and prime lending rates, choose their leverage ratio and their portfolio composition, and can endogenously default on a fraction of their interbank borrowing. Because it is costly to raise capital to satisfy the regulatory capital requirement, the banking sector attenuates the real effects of financial shocks, reduces macroeconomic volatilities, and helps stabilize the economy. The model also includes two unconventional monetary policies (quantitative and qualitative easing) that reduce the negative impacts of financial crises.

JEL classification: E32, E44, G1
Bank classification: Economic models; Business fluctuations and cycles; Credit and credit aggregates; Financial stability

Résumé

L’auteur propose un cadre aux fondements microéconomiques qui intègre un secteur bancaire actif dans un modèle d’équilibre général dynamique et stochastique avec accélérateur financier. Il évalue le rôle du secteur bancaire dans la transmission et la propagation des effets réels des chocs globaux et estime l’importance des chocs financiers dans les fluctuations économiques aux États-Unis. Le secteur bancaire est constitué de deux catégories d’établissements qui offrent des services bancaires différents et réalisent des opérations sur un marché interbancaire en cherchant à maximiser leurs profits. Les prêts sont financés par des emprunts interbancaires et des fonds propres, sous réserve du respect d’exigences réglementaires en matière de fonds propres bancaires. Les banques sont en situation de monopole, fixent les taux nominaux applicables aux dépôts et les taux préférentiels nominaux applicables aux prêts, déterminent elles-mêmes leur ratio de levier et la composition de leur portefeuille et peuvent manquer de façon endogène à leurs obligations à l’égard d’une partie de leurs emprunts interbancaires. Étant donné que la mobilisation de fonds propres pour satisfaire aux exigences réglementaires comporte des coûts, le secteur bancaire atténue les effets réels des chocs financiers, réduit les volatilités macroéconomiques et aide à stabiliser l’économie. Le modèle intègre également deux mesures de politique monétaire non traditionnelles (l’assouplissement quantitatif et l’assouplissement direct du crédit) qui diminuent les effets négatifs des crises financières.

Classification JEL : E32, E44, G1
Classification de la Banque : Modèles économiques; Cycles et fluctuations économiques; Crédit et agrégats du crédit; Stabilité financière
1. Introduction

In light of the recent financial crisis, real-financial linkages have become the focus of attention of an increasing number of papers that aim to develop dynamic stochastic general-equilibrium (DSGE) models with financial frictions in both the demand and supply side of credit markets. Such models provide a better understanding of the role of the banking sector in macroeconomic fluctuations, by incorporating a structural framework to examine banks' behaviour in the transmission and propagation of aggregate shocks, and to assess the importance of financial shocks, originating in the banking sector, as a source of the business cycles. Before the financial crisis, the banking sector was largely ignored in most DSGE models.\footnote{Some recent exceptions are Goodfriend and McCallum (2007); Markovic (2006); and Van den Heuvel (2009).} Moreover, in the literature, financial frictions are usually modelled only on the demand side of credit markets, using either the Bernanke, Gertler, and Gilchrist (1999, BGG hereafter) financial accelerator mechanism or the Iacoviello (2005) framework.\footnote{For example, Carlstrom and Fuerst (1997); Cespedes, Chang, and Velasco (2004); Faia and Monacelli (2007); Christensen and Dib (2008); De Graeve (2008); Queijo von Heideken (2009). Nevertheless, Christiano, Eichenbaum, and Evans (2005) totally abstract from the financial frictions in their model.} This demand-side focus was influenced by the Modigliani–Miller theorem, which states that the capital structure of banks is irrelevant for their lending decisions.

This paper proposes a micro-founded framework that incorporates an active banking sector, which includes an interbank market and banks subject to a capital requirement, into a DSGE model with a financial accelerator à la BGG.\footnote{This framework is fully micro-founded in the sense that all banks maximize profits and take optimal decisions under different constraints.} Unlike previous studies that incorporate bank capital to solve the moral hazard problem between households and banks, this paper introduces bank capital to satisfy the capital requirement exogenously imposed by regulators.\footnote{Examples of these studies are Holmstrom and Tirole (1997); Gertler and Kiyotaki (2010); Goodfriend and McCallum (2007); Markovic (2006); Zhang (2009); Meh and Moran (2010).} It is a precondition for banks to operate and provide loans to entrepreneurs. This requirement causes bank capital to attenuate the real effects of aggregate shocks, rather than act as an amplification mechanism as in previous studies. For instance, in the event of a positive shock, an increase in borrowing demand by entrepreneurs forces banks to increase their leverage ratio and/or bank capital holdings to be able to extend their loan supply. A higher leverage ratio and/or higher
bank capital imply higher marginal costs of raising bank capital and, thus, higher marginal costs of producing loans. Banks transfer these additional costs to entrepreneurs by charging a higher lending rate. This increases external financing costs and erodes part of the initial demand from entrepreneurs for loans to finance new investment. Consequently, the increase in investment and output will be smaller in the presence of the capital requirement.\footnote{Van den Heuvel (2008) studies the welfare cost of bank capital requirement in a DSGE model.}

This paper is related to the following studies: Goodhart, Sunirand, and Tsomocos (2006); Cúrdia and Woodford (2009); de Walque, Pierrard, and Rouabah (2010); Christiano, Motto and Rostagno (2010); Gerali et al. (2010); Gertler and Karadi (2010); Gertler and Kiyotaki (2010). Our model is a DSGE model for a closed economy based on BGG. The key innovation is formally modelling an active banking sector that includes an interbank market and imposes a regulatory capital requirement. The model incorporates an optimizing banking sector with two types of monopolistically competitive banks: “savings banks” and “lending banks.” Banks supply different banking services and transact in the interbank market.\footnote{The two different banks are necessary to generate heterogeneity, which in turn leads to an interbank market where different banks can interact.} Savings banks are financial intermediaries that are net lenders (creditors) in the interbank market, whereas lending banks are net borrowers (debtors). Banks have monopoly power when setting nominal deposit and prime lending rates, but subject to quadratic adjustment costs.

Savings banks collect deposits from workers, set nominal deposit rates, and optimally choose the composition of their portfolio (composed of government bonds and risky interbank lending). Lending banks borrow from savings banks in the interbank market and raise bank capital (equity) from households (shareholders) in the financial market to satisfy the capital requirement.

In addition, lending banks optimally choose their leverage ratio; that is, the ratio of loans to bank capital subject to the maximum leverage ratio imposed by regulators. We assume that banks that hold capital in excess of the required level receive convex gains.\footnote{The cost of bank capital depends on the bank’s capital position. If banks hold excess bank capital, the marginal cost of raising equity in the financial market will be lower, since banks are well capitalized.} This implies that variations in the banks’ leverage ratio directly affect the marginal cost of raising bank capital. Therefore, movements in the banks’ leverage ratio may have substantial effects on business cycle fluctuations, as pointed out by Fostel and Geanakoplos (2008) and Geanakoplos (2009). Following Goodhart, Sunirand, and Tsomocos (2006), we assume endogenous strategic defaults
on interbank borrowing. Also, as in Gertler and Kiyotaki (2010), lending banks’ managers can divert a fraction of bank capital received from shareholders for their own benefit. Nevertheless, when defaulting on interbank borrowing or diverting a fraction of bank capital, lending banks must pay convex penalties in the next period. Finally, to introduce unconventional (quantitative and qualitative) monetary policies in the model, we assume that the lending banks can receive, if needed, injections of money from the central bank and/or swap a fraction of their loans (risky assets) for government bonds.\(^8\)

In this framework, variations in a bank’s balance sheet can affect credit supply conditions and, thus, the real economy through the following channels: (1) variations in the marginal cost of raising bank capital; (2) the optimal choice of the banks’ leverage ratio subject to the capital requirement; (3) monopoly power in setting nominal deposit and lending interest rates with nominal rigidities, implying time-varying interest rate spreads over business cycles;\(^9\) (4) the optimal allocation of the banks’ portfolios between interbank lending (risky assets) and risk-free asset holdings; and (5) the default risk channels that arise from the endogenous default on interbank borrowing and diversion of a fraction of bank capital.

The economy is subject to supply and demand shocks, financial shocks (riskiness and the financial intermediation process), and unconventional monetary policy shocks. The supply and demand shocks are similar to those commonly used in the literature; however, financial shocks require some explanation. Riskiness shocks are modelled as shocks to the elasticity of the risk premium that affect the external finance costs of entrepreneurs. They are meant to represent shocks to the standard deviation of the entrepreneurial distribution, as in Christiano, Motto, and Rostagno (2010); shocks to agency costs paid by lending banks to monitor the entrepreneurs’ output; and/or shocks to the entrepreneurs’ default threshold.\(^10\) Shocks to the financial intermediation process are exogenous events that affect the credit supply of lending banks. They may represent technological advances or disruptions in the intermediation process.

---

8 Quantitative monetary easing, which is associated with newly created money, expands banks’ balance sheets; qualitative monetary easing (swapping banks’ assets for government bonds) changes only banks’ assets compositions.

9 See Curdia and Woodford (2009) for the importance of time-varying spreads on monetary policy.

10 As shown in BGG, the elasticity of the external finance premium to the entrepreneurs’ leverage ratio depends on the standard deviation of the entrepreneurial distribution, the agency cost parameter, and entrepreneurs’ default threshold.
or approximate perceived changes in creditworthiness. Finally, unconventional monetary policy shocks include quantitative and qualitative monetary easing shocks used by the central banks to provide liquidity to the banking system and to ease financial conditions during times of financial crises.

The model is calibrated to the U.S. economy and used to evaluate the role of the banking sector in the transmission and propagation of the real effects of aggregate shocks, to assess the importance of financial shocks in the U.S. business cycle fluctuations, and to examine the potential role of unconventional monetary policies in offsetting the negative impacts of financial crises.

The model is successful in reproducing most of the salient features of the U.S. economy: key macroeconomic volatilities, autocorrelations, and correlations with output. Importantly, the impulse responses of key macro variables to different shocks show that, under the capital requirement, the active banking sector attenuates the real effects of aggregate shocks, particularly financial shocks, and thus helps stabilize the economy. Moreover, the dynamic effects of financial shocks originating in the banking sector have substantial impacts on the U.S. business cycles, and could be a substantial source of macroeconomic fluctuations. We also find that bank leverage is procyclical, indicating that banks are willing to expand more loans during booms and tend to restrict their supply of credit during recessions.

This paper is organized as follows. In section 2, we describe the model. In section 3, we discuss the parameter calibration. In section 4, we report and discuss the empirical results. Section 5 offers some conclusions.

2. The Model

The model has two types of households (workers and bankers) that differ in their degrees of risk aversion and in the access to financial markets. The banking sector consists of two types of heterogeneous monopolistically competitive banks. We call them “savings” and “lending” banks, to indicate that they offer different banking services but interact in an interbank market. As in BGG, the production sector consists of entrepreneurs, capital producers, and retailers.

11Examples of shocks to the financial intermediation process are advances in financial engineering, credit rationing, and highly sophisticated methods for sharing risk.
Finally, there is a central bank and a government.

2.1 Households

2.1.1 Workers

Workers derive utility from total consumption, $C^w_t$, and leisure, $1 - H_t$, where $H_t$ denotes hours worked. The workers’ preferences are described by the following expected utility function:

$$V^w_0 = E_0 \sum_{t=0}^{\infty} \beta^w_t u (C^w_t, H_t).$$  \hfill (1)

The single-period utility is

$$u(\cdot) = \frac{e_t}{1 - \gamma_w} \left( \frac{C^w_t}{(C^w_{t-1})^\varphi} \right)^{1-\gamma_w} \frac{\eta(1 - H_t)^{1-\varsigma}}{1 - \varsigma},$$  \hfill (2)

where $\varphi \in (0, 1)$ is a habit formation parameter, $\gamma_w$ is a positive parameter denoting the workers’ risk aversion, and $\varsigma$ is the inverse of the Frisch wage elasticity of labour supply. The parameter $\eta$ measures the weight on leisure in the utility function. $e_t$ is a preference shock that is common to workers and bankers, and follows an AR(1) process.

The representative worker enters period $t$ with $D_{t-1}$ units of real deposits in savings banks. Deposits pay the gross nominal interest rate $R^D_t$ set by savings banks. During period $t$, workers supply labour to the entrepreneurs, for which they receive real labour payment $W_tH_t$, ($W_t$ is the economy-wide real wage). Furthermore, they receive dividend payments, $\Pi^R_t$, from retail firms, as well as a lump-sum transfer from the monetary authority, $T_t$, and they pay lump-sum taxes to the government, $\tilde{T}^w_t$. Workers allocate their funds to private consumption and real deposits, $D_t$. Their budget constraint in real terms is

$$C^w_t + D_t \leq W_tH_t + \frac{R^D_{t-1}D_{t-1}}{\pi_t} + \Pi^R_t + T_t - \tilde{T}^w_t,$$  \hfill (3)

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. A representative worker chooses $C^w_t$, $H_t$, and $D_t$ to maximize the expected lifetime utility, equation (1), subject to the single-period utility function, equation (2), and the budget constraint, equation (3). The first-order conditions of this optimization problem are reported in Appendix A.
2.1.2 Bankers

Bankers are the owners of the two types of banks, from which they receive profits. They consume, save in government bonds, and accumulate bank capital supplied to lending banks. Their preferences depend only on consumption and are given by

\[ V_0^b = E_0 \sum_{t=0}^{\infty} \beta^t u \left( C_t^b \right). \]  

(4)

The single-period utility function is

\[ u(\cdot) = \frac{e_t}{1 - \gamma_b} \left( \frac{C_t^b}{(C_{t-1}^b)^{\gamma_b}} \right)^{1-\gamma_b}, \]  

(5)

where \( \gamma_b \) is a positive structural parameter denoting bankers’ risk aversion.

Bankers enter period \( t \) with \( (1 - \delta_{t-1})Z_{t-1} \) units of the stock of bank capital, whose price is \( Q_t^Z \) in period \( t \), where \( \delta_{t-1}^Z \) is the fraction of bank capital diverted by lending bank managers in \( t - 1 \), and \( Z_t \) is the volume of bank equity (shares) held by bankers. Bank capital pays a contingent nominal return rate (dividend), \( R_t^Z \). Bankers also enter period \( t \) with \( B_{t-1} \) units of government bonds that pay the risk-free nominal interest rate \( R_t \). During period \( t \), bankers receive profit payments, \( \Pi_{t}^{sb} \) and \( \Pi_{t}^{lb} \), from savings and lending banks, and pay lump-sum taxes to government, \( \tilde{T}_t^b \). They allocate these funds to consumption, \( C_t^b \), government bonds, \( B_t \), and bank capital, \( Z_t \). We assume that bankers pay costs when adjusting their stock of bank capital across periods.\(^\text{12}\) Formally, the adjustment costs are given by

\[ Adj_t^Z = \frac{\chi_Z}{2} \left( \frac{Z_t}{Z_{t-1}} - 1 \right)^2 Q_t^Z Z_t, \]  

(6)

where \( \chi_Z \) is a positive parameter determining the bank capital adjustment costs. The bankers’ budget constraint, in real terms, is

\[ C_t^b + Q_t^Z Z_t + B_t = \frac{R_{t-1}B_{t-1}}{\pi_t} + \frac{R_t^Z (1 - \delta^Z_{t-1})Q_t^Z Z_{t-1}}{\pi_t} - Adj_t^Z + \Pi_{t}^{sb} + \Pi_{t}^{lb} - \tilde{T}_t^b. \]  

(7)

\(^\text{12}\)We interpret these adjustment costs as costs paid to brokers, or the costs of collecting information about the banks’ balance sheet.
A representative banker chooses $C_t^b$, $B_t$, and $Z_t$ in order to maximize the expected lifetime utility equation (4), subject to equations (5)–(7). The first-order conditions for this optimization problem are:

$$e_t \left( \frac{C_t^b}{C_t^{b-1}} \right)^{1-\gamma_b} - \beta_b \varphi E_t \left[ e_{t+1} \left( \frac{C_{t+1}^b}{C_t^b} \right)^{1-\gamma_b} \right] = C_t^b \lambda_t^b; \quad (8)$$

$$\frac{\lambda_t^b}{R_t} = \beta_b E_t \left[ \frac{\lambda_t^{b+1}}{\pi_{t+1}} \right]; \quad (9)$$

$$\beta_b E_t \left\{ \frac{\lambda_{t+1}^w Q_{t+1}^Z}{\pi_{t+1}} \left[ (1 - \delta_t^Z) R_t^{Z_{t+1}} + \chi Z \left( \frac{Z_{t+1}}{Z_t} - 1 \right) \left( \frac{Z_{t+1}}{Z_t} \right)^2 \pi_{t+1} \right] \right\} = \lambda_t^w Q_t^Z \left[ 1 + \chi Z \left( \frac{Z_t}{Z_{t-1}} - 1 \right) \frac{Z_t}{Z_{t-1}} \right]; \quad (10)$$

where $\lambda_t^b$ is the Lagrangian multiplier associated with the bankers' budget constraint.

Equation (8) determines the marginal utility of the bankers' consumption. Equation (9) relates the marginal rate of substitution to the real risk-free rate. Finally, equation (10) corresponds to the optimal dynamic evolution of the stock of bank capital. Combining conditions (9) and (10) yields the following condition relating the expected return on bank capital, $E_t R_t^{Z_{t+1}}$, to the risk-free rate, $R_t$, the diversion on bank capital, $\delta_t^Z$, and the current costs/future gains of adjusting the stock of bank capital:

$$E_t \left\{ \frac{Q_{t+1}^Z}{Q_t^Z} \left[ (1 - \delta_t^Z) R_t^{Z_{t+1}} + \chi Z \left( \frac{Z_{t+1}}{Z_t} - 1 \right) \left( \frac{Z_{t+1}}{Z_t} \right)^2 \pi_{t+1} \right] \right\} = R_t \left[ 1 + \chi Z \left( \frac{Z_t}{Z_{t-1}} - 1 \right) \frac{Z_t}{Z_{t-1}} \right]. \quad (11)$$

This condition leads to three channels, through which changes in bank capital affect the real economy. The first is the price expectation channel, which arises from expectations of capital gains or losses from holding bank capital shares, due to expected changes in the price of bank capital $E_t \left[ Q_{t+1}^Z/Q_t^Z \right]$. The second is the adjustment cost channel, which is a result of asymmetric information between bankers and banks. The presence of adjustment costs is necessary to reflect asymmetric information, and the adjustment costs are interpreted as costs to enter into the bank capital market. The third channel is the diversion risk channel that arises from the existence of the possibility that banks' managers divert a fraction $\delta_t^Z$ of bank capital.
repayment to their own benefits. Therefore, movements in bank capital, caused by macroeco-
nomic fluctuations, have direct impacts on bank capital accumulation and, consequently, on 
credit supply conditions.

2.2 Banking sector

The banking sector consists of two types of heterogeneous profit-maximizing banks: savings 
and lending banks.

2.2.1 Savings banks

Savings banks refer to all financial intermediaries that are net creditors (lenders) in the inter-
bank market. There is a continuum of monopolistically competitive, profit-maximizing savings 
banks indexed by \( j \in (0, 1) \). Each \( j \) bank collects fully insured deposits from workers and pays 
a deposit interest rate \( R_{Dj,t} \), which is optimally set as a markdown over the marginal return of 
its assets. In addition, it optimally allocates a fraction \( s_{j,t} \) of deposits to lending in the inter-
bank market and uses the remaining fraction \( 1 - s_{j,t} \) to purchase government bonds. Thus, the 
\( j^{th} \) savings bank’s portfolio is composed of interbank lending \( D_{IBj,t} = s_{j,t}D_{j,t} \) and government 
bonds \( B_{sbj,t} = (1 - s_{j,t})D_{j,t} \). Interbank lending pays a gross nominal interbank rate \( R_{IBt} \) and 
is subject to a probability \( \delta_{Dt} \) that lending banks default on their interbank borrowing. The 
interbank rate is endogenously determined to clear the interbank market. Table 1 shows the 
balance sheet of the \( j^{th} \) savings bank.

<table>
<thead>
<tr>
<th>Table 1: Savings bank’s balance sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>Interbank lending: ( D_{IBj,t} )</td>
</tr>
<tr>
<td>Government bonds: ( B_{sbj,t} )</td>
</tr>
</tbody>
</table>

Given monopolistic competition and the imperfect substitution between deposits, the \( j^{th} \) 
savings bank faces the following deposit supply function, that is increasing in the relative 
deposit interest rate. As in Gerali et al. (2010), the individual deposit supply is

\[
D_{j,t} = \left( \frac{R_{Dj,t}}{R_{It}} \right)^{\vartheta_D} D_t, \quad (12)
\]
where \( D_{j,t} \) is deposits supplied to bank \( j \), while \( D_t \) denotes total deposits in the economy, and \( \vartheta_D > 1 \) is the elasticity of substitution between different types of deposits.\(^{13}\)

Savings banks face quadratic adjustment costs à la Rotemberg (1982) when adjusting the deposit interest rate:

\[
Ad_{R, j,t}^{D} = \frac{\phi_{RD}}{2} \left( \frac{R_{D,j,t}}{R_{D,j,t-1}} - 1 \right)^2 D_t, \tag{13}
\]

where \( \phi_{RD} > 0 \) is an adjustment cost parameter. These adjustment costs imply an interest rate spread, between deposit and policy rates, that varies over the business cycle. In addition, we assume that savings banks must pay monitoring costs when lending in the interbank market. They incur higher monitoring costs if the share \( s_{j,t} \) of deposits lent in the interbank market deviates from a target level \( \bar{s} \). The individual cost of monitoring interbank lending is

\[
\Delta_{s,j,t}^s = \frac{\chi_s}{2} \left( (s_{j,t} - \bar{s})D_{j,t} \right)^2, \tag{14}
\]

where \( \chi_s \) is a positive parameter determining the steady-state value of the monitoring costs.

Formally, the \( j^{th} \) savings bank’s optimization problem is:

\[
\max_{\{s_{j,t}, R_{D,j,t}\}} E_0 \sum_{t=0}^{\infty} \beta_t^{\lambda_t^b} \left\{ \left[ s_{j,t}R_t^{IB}(1 - \delta_D^t) + (1 - s_{j,t})R_t - R_{D,j,t} \right] D_{j,t} - Ad_{R_D,j,t}^{D} - \Delta_{s,j,t}^s \right\},
\]

subject to equations (12)–(14).\(^{14}\) Since bankers are the owners of banks, the discount factor is the stochastic process \( \beta_t^{\lambda_t^b} \), where \( \lambda_t^b \) denotes the marginal utility of bankers’ consumption. The term \( s_{j,t}R_t^{IB}(1 - \delta_D^t) + (1 - s_{j,t})R_t \) is the gross nominal return of savings banks’ assets.

\(^{13}\)This supply function is derived from the definition of the aggregate supply of deposits, \( D_t \), and the corresponding deposit interest rate, \( R_{D,t} \), in the monopolistic competition framework, as follows:

\[
D_t = \left( \int_0^1 D_{j,t}^{1+\vartheta_D} dj \right)^{\frac{1}{1+\vartheta_D}} \quad \text{and} \quad R_t^{D} = \left( \int_0^1 R_{j,t}^{D,1+\vartheta_D} dj \right)^{\frac{1}{1+\vartheta_D}}, \tag{15}
\]

where \( D_{j,t} \) and \( R_{j,t}^{D} \) are, respectively, the supply and deposit interest rates faced by each savings bank \( j \in (0, 1) \).

\(^{14}\)Savings banks take \( R_t^{IB} \), \( R_t \), and \( \delta_D^t \) as given when maximizing their profits.
In symmetric equilibrium where \( s_{j,t} = s_t \) and \( R^D_{j,t} = R^D_t \) for all \( t > 0 \), the first-order conditions of this optimization problem with respect to \( s_t \) and \( R^D_t \) are:

\[
s_t = \bar{s} + \frac{R^IB_t(1 - \delta^D_t) - R_t}{\chi s_D t}; \tag{15}
\]

\[
\frac{1 + \frac{\partial D}{\partial D}(R^D_t - 1)}{1} = \frac{s_t(R^IB_t - 1)(1 - \delta^D_t) + (1 - s_t)(R_t - 1) - \chi s(s_t - \bar{s})^2 D_t}{1 - \frac{\partial D}{\partial D}(R^D_t - 1) + \frac{\beta \theta D}{\partial D} \left( \frac{R^D_{t+1}}{R^D_{t-1}} - 1 \right) \frac{R^D_{t+1}}{R^D_t}}; \tag{16}
\]

where the interbank rate is given by \( R^IB_t = R_t(1 + \delta^D_t) + \chi s(s_t - \bar{s})D_t \). This rate includes the risk-free rate, which is the opportunity cost of savings banks for not investing total deposits in government bonds. It also compensates savings banks for the default risk they are facing in the interbank market, and covers the average marginal cost of monitoring interbank lending. Consequently, the spread between interbank and policy rates, \( R^IB_t - R_t \), is increasing in the probability of default in the interbank market and in the marginal cost of monitoring interbank lending. In normal times, this spread is constant.\(^{15}\)

Condition (15) describes the share of deposits allocated to interbank lending as decreasing in the probability of default on interbank lending, in the risk-free interest rate, and in the total deposits, while it is increasing in the interbank rate. Note that an increase in \( s_t \) indirectly leads to an expansion in credit supply available in the interbank market. Therefore, the rising riskiness of interbank lending (a higher \( \delta^D_t \)) encourages savings banks to increase their risk-free holdings and to reduce their interbank lending. Condition (16) defines the deposit interest rate as a markdown of the net average return of savings banks’ assets.\(^{16}\)

Therefore, this framework adds two channels through which savings banks’ behaviour affects credit supply conditions. First, optimal allocation of deposits between interbank lending and risk-free asset holding affects directly the availability of loanable funds supplied to firms as credit to finance new investment. Second, nominal stickiness in the deposit rates influences the intertemporal substitution of consumption across periods.\(^{17}\)

\(^{15}\)In the absence of financial stress, variations in \( \delta^D_t \) and \( \chi s_D_t \) are very small.

\(^{16}\)This condition allows us to derive an equation relating \( R^D_t \) to \( R^D_{j,t-1} \), \( R^D_{j,t+1} \), and \( R^IB_t \).

\(^{17}\)Since the marginal rate of substitution equals the deposit rate, the sluggishness in this rate affects the intertemporal substitution between current and future consumption.
2.2.2 Lending banks

Lending banks refer to all net debtor (borrower) banks in the interbank market. There is a continuum of monopolistically competitive lending banks indexed by \( j \in (0, 1) \). Lending banks borrow from savings banks in the interbank market and raise bank capital from bankers to satisfy the capital requirement. We assume that the stock of bank capital \( Z_t \) is valued at capital price \( Q^Z_t \) and held by banks as government bonds that pay the risk-free return rate \( R_t \). In addition, lending banks can receive money from the central bank, which can be interpreted as quantitative monetary easing. Also, if needed, lending banks may swap a fraction of their risky assets (loans to firms) for government bonds from the central bank (qualitative monetary easing). Through these two channels, the central bank can provide liquidity to lending banks in times of financial stress.

Production of loans

To produce loans, \( L_{j,t} \), provided to entrepreneurs, each lending bank \( j \) combines funds received from savings banks in the interbank market, \( D_{j,t}^{IB} \), plus any injection of money, \( m_{j,t} \), with the value of bank capital raised from bankers, \( Q^Z_t Z_{j,t} \), plus any new assets swapped with the central bank, \( x_{j,t} \). We assume that banks use the following Leontief technology to produce loans:

\[
L_{j,t} = \min \{ D_{j,t}^{IB} + m_{j,t}; \kappa_{j,t} \left( Q^Z_t Z_{j,t} + x_{j,t} \right) \} \Gamma_t, \tag{17}
\]

where \( \kappa_{j,t} \leq \bar{\kappa} \) is bank \( j \)’s optimally chosen leverage ratio and \( \bar{\kappa} \) is the maximum leverage ratio imposed by regulators.\(^{18}\) When \( \kappa_{j,t} < \bar{\kappa} \), bank \( j \) holds excess capital. \( \Gamma_t \) is a shock to the financial intermediation process affecting credit supply. It represents exogenous factors affecting loan production and the banks’ balance sheet, such as perceived changes in creditworthiness, technological changes in the intermediation process due to advances in computational finance, and sophisticated methods of sharing risk.\(^{19}\) It is assumed that \( m_t, x_t, \) and \( \Gamma_t \) evolve exogenously, following AR(1) processes.\(^{20}\)

Leontief technology implies perfect complementarity between interbank borrowing and bank capital, and imposes the capital requirement, which attenuates the real effects of different

\(^{18}\)\( \kappa_{j,t} \) is the ratio of the bank’s loans to bank capital, which is the inverse of the required minimum bank capital ratio.

\(^{19}\)Banks may underevaluate (overevaluate) risk during booms (recessions), which affects the loan supply.

\(^{20}\)The steady-state values of \( m_t \) and \( x_t \) are zero, while that of \( \Gamma_t \) is equal to unity.
shocks. For example, following a positive technology shock, the demand for investment and
loans increases. Loan expansion requires a higher bank leverage ratio or the raising of fresh
bank capital in the financial market. These two actions are, however, costly for the lending
banks. Consequently, the marginal cost of producing loans increases and banks raise their
lending rate to entrepreneurs. This, in turn, increases the external financing costs and partly
offsets initial increases in investment demand. Furthermore, with Leontief technology, the
marginal cost of producing loans is simply the weighted sum of the cost of borrowing in the
interbank market and the marginal cost of raising bank capital. Table 2 shows the $j^{th}$ lending
bank’s balance sheet in the period $t$.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans: $L_{j,t} - x_{j,t}$</td>
<td>Interbank borrowing: $D_{j,t}^{IB}$</td>
</tr>
<tr>
<td>Government bonds: $B_{j,t}^{gb}$</td>
<td>Bank capital: $Q^2 Z_{j,t}$</td>
</tr>
<tr>
<td></td>
<td>Central bank’s money injection: $m_{j,t}$</td>
</tr>
<tr>
<td></td>
<td>Other terms: $(\Gamma_t - 1)(D_{j,t}^{IB} + m_{j,t})$</td>
</tr>
</tbody>
</table>

Note that swapping a fraction of loans for government bonds, $x_{j,t}$, simply changes the composi-
tion of banks’ assets, while the injection of money and financial intermediation shocks, $m_{j,t}$
and $\Gamma_t$, imply an expansion or contraction of banks’ balance sheets.\(^\text{21}\)

**The optimization problem**

The lending bank $j$ optimally sets the prime lending rate, $R_{L,j,t}$, as a markup over the marginal
cost of producing loans, and faces quadratic costs when adjusting the prime lending rate. These
adjustment costs are modelled à la Rotemberg (1982):

\[
A d_{j,t}^{R_L} = \frac{\phi_{R_L}}{2} \left( \frac{R_{L,j,t}}{R_{L,j,t-1}} - 1 \right)^2 L_t,
\]

\(^\text{21}\)The term $(\Gamma_t - 1)(D_{j,t}^{IB} + m_{j,t})$ represents the off-balance-sheet operations. It is used to balance the lending
banks’ balance sheet in the presence of financial intermediation shocks.
where $\phi_{RL} > 0$ is an adjustment cost parameter. When lending to entrepreneurs, the $j^{th}$ lending bank faces the following demand function for loans:

$$L_{j,t} = \left( \frac{R_{j,t}^L}{R_t^L} \right)^{-\vartheta_L} L_t,$$

where $\vartheta_L > 1$ is the elasticity of substitution between different types of provided loans.\(^{22}\)

The $j^{th}$ lending bank optimally chooses its leverage ratio $\kappa_{j,t}$, subject to the maximum leverage ratio imposed by the regulators, $\bar{\kappa}$. We assume that having a lower leverage ratio than the maximum required level entails convex gains for the bank. Changes in the optimally chosen leverage ratio directly affect the marginal cost of raising bank capital and, thus, the marginal costs of producing loans. The quadratic gains for bank $j$ are modelled using:

$$\Delta \kappa_{j,t} = \frac{\chi_{\kappa}}{2} \left( \frac{\bar{\kappa} - \kappa_{j,t}}{\bar{\kappa}} \right)^2 (\kappa_{j,t}^2 - 1) \pi_t Z_t Z_{j,t},$$

where $\chi_{\kappa}$ is a positive parameter. Note that when $\kappa_{j,t} = \bar{\kappa}$, the bank actual and required leverage ratios are equal, thus $\Delta \kappa_{j,t} = 0$. In contrast, when $\kappa_{j,t} < \bar{\kappa}$, banks maintain excess bank capital, which reduces the costs of raising bank capital.\(^{23}\)

Furthermore, following Goodhart, Sunirand, and Tsomacos (2006), we allow lending banks to optimally default on a fraction of their interbank borrowing, $\delta^D_{j,t} > 0$. In addition, lending banks’ managers can divert a fraction, $\delta^Z_{j,t}$, of bank capital to their own benefit. The default on interbank lending can be strategic or mandatory (when banks cannot afford to repay their debt). Nonetheless, it is costly for banks to default on the interbank borrowing or divert a fraction of bank capital. In this case, banks must pay convex penalties in the next period. The $j^{th}$ bank’s penalties are given by:

$$\Delta D_{j,t} = \frac{\chi_{\Delta D}}{2} \left( \frac{\delta^D_{j,t-1} D^IB_{j,t-1}}{\pi_t} \right)^2 R_{t-1}^IB$$

\(^{22}\)This demand function is derived from the definition of the aggregate demand for loans, $L_t$, and the corresponding prime lending rate, $R_t^L$, in the monopolistic competition framework, as follows:

$$L_t = \left( \int_0^1 L_{j,t}^{1-\vartheta_L} dj \right) \frac{1}{\vartheta_L}$$

and $R_t^L = \left( \int_0^1 R_{j,t}^{1-\vartheta_L} dj \right)^{-\frac{1}{\vartheta_L}}$, where $L_{j,t}$ and $R_{j,t}^L$ are demand for loans and the lending rate faced by each lending bank $j \in (0, 1)$.\(^{23}\)Equation (27) shows the relation between the marginal cost of loans and the cost of raising bank capital.
and
\[
\Delta Z_{j,t} = \frac{\chi_\delta^2}{2} \left( \frac{\zeta_{j,t-1}^2 Q_{j,t-1}^2 Z_{j,t-1}}{\pi_t} \right)^2 R_{t}^Z, \tag{22}
\]
where \(\chi_\delta^D\) and \(\chi_\delta^Z\) are positive parameters determining the steady-state values of \(\Delta D_t\) and \(\Delta Z_t\), respectively.

Specifically, the \(j^{th}\) lending bank’s optimization problem is to choose \(R_{L,j,t}, \kappa_{j,t}, \delta_{j,t}^D, \text{ and } \delta_{j,t}^Z\) to maximize its profit, and is given by

\[
\max_{\{R_{L,j,t}, \kappa_{j,t}, \delta_{j,t}^D, \delta_{j,t}^Z\}} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \{ R_{L,j,t} L_{j,t} - (1 - \delta_{j,t}^D) R_{t}^B D_{j,t}^B - [(1 - \delta_{j,t}^Z) R_{t+1}^Z - R_t] Q_t^Z Z_{j,t} \}
\]
\[
- Ad_{j,t}^L \Delta D_{j,t} - \Delta Z_{j,t} - R_t m_{j,t} - (R_{L,j,t} - R_t) x_{j,t} \},
\]
subject to equations (17)–(22). The discount factor is given by the stochastic process \(\beta^t \lambda_t\), where \(\lambda_t\) denotes the marginal utility of consumption of bankers—the owners of the lending banks. Note that the term \([(1 - \delta_{j,t}^Z) R_{t+1}^Z - R_t] Q_t^Z Z_{j,t}\) represents the marginal costs of holding one unit of bank capital to satisfy the capital requirement. The marginal cost depends on the payment of the non-diverted fraction of bank capital net of the return from holding bank capital as government bonds. The terms \(R_t m_{j,t}\) and \((R_{L,j,t} - R_t) x_{j,t}\) are the total costs of money injections, received from the central bank, and the costs of swapping a fraction of loans for government bonds.

In symmetric equilibrium, where \(R_{L,j,t} = R_{L,t}, \kappa_{j,t} = \kappa_t, \delta_{j,t}^D = \delta_t^D\), and \(\delta_{j,t}^Z = \delta_t^Z\), for all \(t > 0\), the first-order conditions of this optimization problem are:

\[
R_{L,t} = 1 + \frac{\partial L}{\partial L} (\zeta_t - 1) - \frac{\partial R_{L,t}^L}{\partial L} - \left( \frac{R_{L,t}}{R_{L,t-1}} - 1 \right) \frac{R_{L,t}}{R_{L,t-1}}
\]
\[
+ \beta^t \phi R_{L,t} \left( \frac{R_{L,t}}{R_{L,t-1}} - 1 \right) \frac{R_{L,t}}{R_{L,t-1}} \right] E_t ; \tag{23}
\]
\[
\kappa_t = \bar{\kappa} \left( 1 - \frac{\bar{\kappa} \tau_t (R_{L,t} - 1)}{\chi_\delta^Z Q_t^Z Z_t} \right) ; \tag{24}
\]
\[
\delta_t^D = E_t \left[ \frac{R_t \tau_{t+1}}{\chi_\delta D_{B,t}^B} \right] ; \tag{25}
\]
\[
\delta_t^Z = E_t \left[ \frac{R_t \tau_{t+1}}{\chi_\delta Q_t^Z Z_t} \right] , \tag{26}
\]
where $\zeta_t > 1$ is the marginal cost of producing loans and given by
\[
\zeta_t = \Gamma_t^{-1} \left[ R_t^{IB} + \left( E_t R_{t+1}^Z - R_t - (R_t^L - 1) \frac{\bar{\kappa} - \kappa_t}{\bar{\kappa}} \right) \frac{Q_t^Z}{\kappa_t} \right].
\]

In addition, the Leontief technology implies the following implicit demand functions of interbank borrowing and bank capital:
\[
L_t = \Gamma_t (D_t^{IB} + m_t); \quad (28)
\]
\[
L_t = \Gamma_t \kappa_t (Q_t^Z Z_t + x_t). \quad (29)
\]

The pricing equation (23) relates the net lending rate to the net marginal cost of producing loans, and to the current costs and future gains of adjusting the lending rate. Under the flexible interest rate, with $\phi_{RL} = 0$, the lending rate is set simply as a markup over the marginal cost, $\zeta_t$.\textsuperscript{24} Equation (24) shows that the banks’ optimal leverage ratio increases in the maximum imposed leverage ratio, $\bar{\kappa}$, and in the value of bank capital, $Q_t^Z Z_t$. Moreover, it decreases in the interest rate on loans, because a higher lending rate reduces the entrepreneurs’ demand for loans, which implies bank deleveraging. Equation (25) indicates that the default rate on interbank borrowing increases in expected inflation and the policy rate, while it decreases in total interbank borrowing. Equation (26) states that the diversion of bank capital is increasing in expected inflation and the policy rate, while it is decreasing in the value of bank capital. In equations (25) and (26), $\delta_t^D$ and $\delta_t^Z$ increase in expected inflation and the policy rate because, if defaults occur in $t$, higher expected inflation reduces the expected real costs of penalties paid in the next period; a higher policy rate implies a higher discounted value of the present benefits of defaulting or diverting in the current period.

Equation (27) indicates that the marginal cost of producing loans, $\zeta_t$, is the sum of the interbank market rate, $R_t^{IB}$, and the marginal cost of bank capital $(E_t R_{t+1}^Z - R_t - (R_t^L - 1) \frac{\bar{\kappa} - \kappa_t}{\bar{\kappa}} \frac{Q_t^Z}{\kappa_t})$. Note that the chosen bank leverage ratio $\kappa_t$ positively affects $\zeta_t$. The term $(R_t^L - 1)(\bar{\kappa} - \kappa_t)Q_t^Z / \bar{\kappa} > 0$ is the marginal gain of holding bank capital in excess of the required level.\textsuperscript{25}

In equation (27), the marginal cost of bank capital is increasing in the risky return rate and in the leverage ratio, while it is decreasing in the maximum imposed leverage, the risk-free

\textsuperscript{24}In this case, $R_t^L - 1 = \frac{\sigma_t}{\sigma_t - 1} (\zeta_t - 1)$.

\textsuperscript{25}If $\kappa_t = \bar{\kappa}$, then $\zeta_t = \Gamma_t^{-1} \left[ R_t^{IB} + \bar{\kappa}^{-1} Q_t^Z (R_{t+1}^Z - R_t) \right]$. 

15
rate, and the prime lending rate. A higher leverage ratio reduces excess bank capital, which increases the costs of raising bank capital, and thus increases the marginal cost of producing loans. However, a higher $\bar{\kappa}$ relaxes the capital requirement, which allows banks to expand their lending to firms, while maintaining relatively lower bank capital.

2.3 Production sector

2.3.1 Entrepreneurs

As in BGG, entrepreneurs, who manage wholesale-goods-producing firms, are risk neutral and have a finite expected horizon. The probability that an entrepreneur survives until the next period is $\nu$. This assumption ensures that an entrepreneur’s net worth alone is never sufficient to finance new capital acquisitions, and so the entrepreneur must borrow.

At the end of each period, the entrepreneur purchases capital, $K_{t+1}$, to be used in the next period, at the real price $Q_t^K$. Capital acquisition is financed partly by net worth, $N_t$, and the remainder by borrowing $L_t = Q_t^K K_{t+1} - N_t$ from lending banks. The entrepreneur’s demand for capital depends on its expected marginal return and the expected marginal external financing cost $E_t F_{t+1}$, which equals the real interest rate on external (borrowed) funds. Optimization guarantees that

$$E_t F_{t+1} = E_t \left[ r^K_{t+1} + (1 - \delta)Q^K_{t+1} \right],$$

where $\delta$ is the capital depreciation rate. The expected marginal return of capital is given by the right-side terms of (30), where $r^K_{t+1}$ is the marginal productivity of capital at $t + 1$ and $(1 - \delta)Q^K_{t+1}$ is the value of one unit of capital used in $t + 1$.

BGG solve a financial contract that maximizes the payoff to the entrepreneur, subject to the lender earning the required rate of return. BGG show that, given the parameter values associated with the cost of monitoring the borrower, the characteristics of the distribution of entrepreneurial returns, and the expected life span of firms, the optimal debt contract implies an external finance premium, $\Psi(\cdot)$, which depends on the entrepreneur’s leverage ratio. The

---

$^{26}$Equation (27) can be written as $\zeta_t = \Gamma_t^{-1} \left[ R_t^{LB} + \frac{E_t R^{Z}_{t+1} - R_t}{\kappa_t} Q_t^Z - (R_t^L - 1) \left( \frac{1}{\nu_t} - \frac{1}{2} \right) Q_t^Z \right]$. Therefore, $\frac{\partial \zeta_t}{\partial \kappa_t} = -\frac{R_t^{Z}_{t+1} - R_t}{\kappa_t} Q_t^Z + \frac{R_t^{L} - 1}{\kappa_t} Q_t^Z > 0$, since $R_t^{Z}_{t+1} - R_t < R_t^{L} - 1$. Also, $\frac{\partial \zeta_t}{\partial \bar{\kappa}} = -\frac{R_t^{L} - 1}{\kappa_t} Q_t^Z < 0$. 

---

16
underlying parameter values determine the elasticity of the external finance premium with respect to firm leverage.

In our framework, the marginal external financing cost is equal to the gross real lending rate plus an external finance premium. Thus, the demand for capital should satisfy the following optimality condition:

\[ E_t F_{t+1} = E_t \left[ \frac{R_t^L}{\pi_{t+1}} \Psi(\cdot) \right], \]  

(31)

where \( E_t \left( \frac{R_t^L}{\pi_{t+1}} \right) \) is an expected real prime lending rate (with \( R_t^L \) set by the lending bank) and the external finance premium is given by

\[ r_p t \equiv \Psi(\cdot) = \Psi \left( \frac{Q^K_t K_{t+1}}{N_t} ; \psi_t \right), \]  

(32)

where \( \Psi'(\cdot) < 0 \) and \( \Psi(1) = 1 \), and \( \psi_t \) represents an aggregate riskiness shock, as in Christiano, Motto, and Rostagno (2010).

The external finance premium, \( \Psi(\cdot) \), depends on the borrower’s equity stake in a project (or, alternatively, the borrower’s leverage ratio). As \( \frac{Q^K_t K_{t+1}}{N_t} \) increases, the borrower increasingly relies on uncollateralized borrowing (higher leverage) to fund the project. Since this raises the incentive to misreport the outcome of the project, the loan becomes riskier, and the cost of borrowing rises.\(^{27}\) Formally, the external finance premium is assumed to have the following functional form:

\[ r_p t \equiv \Psi(\cdot) = \left( \frac{Q^K_t K_{t+1}}{N_t} \right)^{\psi_t}, \]  

(33)

where \( \psi_t \) is a time-varying elasticity of the external finance premium with respect to the entrepreneur’s leverage ratio. Following Christiano, Motto, and Rostagno (2010), we assume that \( \psi_t \), the aggregate riskiness shock, follows an AR(1) process. BGG show that this elasticity depends positively on the standard deviation of the distribution of the entrepreneurs’ idiosyncratic shocks, the agency cost, and the entrepreneurs’ default threshold.\(^{28}\)

Aggregate entrepreneurial net worth evolves according to

\[ N_t = \nu V_t + (1 - \nu) g_t, \]  

(34)

\(^{27}\)When loan riskiness increases, the agency costs rise and the lender’s expected losses increase. A higher external finance premium paid by successful entrepreneurs offsets these higher losses.

\(^{28}\)A positive shock to the standard deviation widens the entrepreneurs’ distribution, and so lending banks are unable to distinguish the quality of the entrepreneurs.
where \( V_t \) denotes the net worth of surviving entrepreneurs net of borrowing costs carried over from the previous period, \( 1 - \nu \) is the share of new entrepreneurs entering the economy, and \( g_t \) is the transfer or “seed money” that new entrepreneurs receive from entrepreneurs who exit.\(^{29}\) \( V_t \) is given by
\[
V_t = \left[ F_t Q_{t-1}^K K_t - E_{t-1} F_t (Q_{t-1}^K K_t - N_{t-1}) \right],
\]
where \( F_t \) is the ex post real return on capital held in \( t \), and
\[
E_{t-1} F_t = E_{t-1} \left[ \frac{R_{t-1}^L}{\pi_t} \Psi \left( \frac{Q_{t-1}^K K_t}{N_{t-1}}, \psi_{t-1} \right) \right]
\]
is the cost of borrowing (the interest rate in the loan contract signed at time \( t - 1 \)). Earnings from operations in this period become the next period’s net worth. In our formulation, borrowers sign a debt contract that specifies a nominal interest rate.\(^{30}\) Loan repayment (in real terms) will then depend on the ex post real interest rate. An unanticipated increase (decrease) in inflation will reduce (increase) the real cost of debt repayment and, therefore, increase (decrease) entrepreneurial net worth.

To produce output \( Y_t \), the entrepreneur uses \( K_t \) units of capital and \( H_t \) units of labour following a constant-returns-to-scale technology:
\[
Y_t \leq A_t K_t^\alpha H_t^{1-\alpha}, \quad \alpha \in (0, 1),
\]
where \( A_t \) is a technology shock common to all entrepreneurs and is assumed to follow a stationary AR(1) process. Each entrepreneur sells his output in a perfectly competitive market for a price that equals the nominal marginal cost. The entrepreneur maximizes profits by choosing \( K_t \) and \( H_t \) subject to the production function (37). See Appendix A for the entrepreneurs’ first-order conditions.

### 2.3.2 Capital producers

Capital producers use a linear technology, subject to an investment-specific shock \( \Upsilon_t \), to produce capital goods \( K_{t+1} \), sold at the end of period \( t \). They use a fraction of final goods purchased from retailers as investment goods, \( I_t \), and the existing capital stock to produce

\(^{29}\)The parameter \( \nu \) affects the persistence of the level of aggregate entrepreneurial net worth.

\(^{30}\)In BGG, the contract is specified in terms of the real interest rate.
new capital. The new capital replaces depreciated capital and adds to the capital stock. The disturbance $\Upsilon_t$ is a shock to the marginal efficiency of investment and is assumed to follow an AR(1) process. Since $I_t$ is expressed in consumption units, $\Upsilon_t$ influences the amount of capital in efficiency units that can be purchased for one unit of consumption. Capital producers are also subject to quadratic investment adjustment costs, specified as $\chi I_t^2 \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t$, where $\chi_I > 0$ is an adjustment cost parameter.

The capital producers’ optimization problem, in real terms, consists of choosing the quantity of investment $I_t$ to maximize their profits, so that:

$$\max_{I_t} E_t \sum_{t=0}^{\infty} \beta^t w^t \left\{ Q^K_t \left[ \Upsilon_t I_t - \frac{\chi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \right] - I_t \right\}. \quad (38)$$

Thus, the optimal condition is

$$\frac{1}{Q^K_t} = \Upsilon_t - \chi_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \beta_w \chi_I E_t \left[ \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \frac{Q^K_{t+1}}{Q^K_t} \frac{\lambda^{w}_{t+1}}{\lambda^{w}_t} \right], \quad (39)$$

which is the standard Tobin’s Q equation that relates the price of capital to the marginal adjustment cost.\(^{31}\)

The quantity and price of capital are determined in the capital market. The entrepreneurial demand curve for capital is obtained from equation (31) and, in Appendix A, equation (A.4), whereas the supply of capital is given by equation (39). The intersection of these curves gives the market-clearing quantity and price of capital. Capital adjustment costs slow down the response of investment to shocks, which directly affects the price of capital. Furthermore, the aggregate capital stock evolves according to

$$K_{t+1} = (1-\delta)K_t + \Upsilon_t I_t - \frac{\chi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t. \quad (40)$$

2.3.3 Retail firms

The retail sector is used to introduce nominal price rigidity into the economy. Retail firms purchase wholesale goods at a price equal to their nominal marginal cost, and differentiate

\(^{31}\)Note that in the absence of investment adjustment costs, the capital price $Q^K_t$ is constant and equals 1. Investment adjustment costs generate capital price variability, which contributes to the volatility of entrepreneurial net worth.
them at no cost. They then sell these differentiated retail goods in a monopolistically competitive market. Following Calvo (1983) and Yun (1996), we assume that each retailer does not reoptimize its selling price unless it receives a random signal. The constant probability of receiving such a signal is \((1 - \phi_p)\); and, with probability \(\phi_p\), the retailer \(j\) must charge the same price as in the preceding period, indexed to the steady-state gross rate of inflation, \(\pi\). At time \(t\), if the retailer \(j\) receives the signal to reoptimize, it chooses a price \(\bar{P}_t(j)\) that maximizes the discounted, expected real total profits for \(l\) periods.

### 2.4 Central bank and government

#### 2.4.1 Central bank

We assume that the central bank adjusts the policy rate, \(R_t\), in response to deviations of inflation, \(\pi_t\), and output, \(Y_t\), from their steady-state values. Thus, monetary policy evolves according to the following Taylor-type policy rule:

\[
\log \left( \frac{R_t}{R} \right) = \varrho_\pi \log \left( \frac{\pi_t}{\pi} \right) + \varrho_Y \log \left( \frac{Y_t}{Y} \right) + \varepsilon_{Rt},
\]

where \(R\), \(\pi\), and \(Y\) are the steady-state values of \(R_t\), \(\pi_t\), and \(Y_t\), respectively, and \(\varepsilon_{Rt}\) is a monetary policy shock normally distributed with zero mean and standard deviation \(\sigma_R\).

During a period of financial stress, the central bank can use unconventional monetary policies: quantitative and/or qualitative monetary easing shocks, \(m_t\) and \(x_t\). Therefore, it can inject money into the banking system and/or swap a fraction of bank loans for government bonds to enhance the lending banks’ capital position.

#### Government

Each period, the government buys a fraction of the final retail good \(G_t\), pays the principal debt from the previous period, and makes interest payments. We assume that the government runs a balanced budget financed with newly contracted debt and lump-sum taxes, \(\bar{T}_t^w + \bar{T}_t^b\). Therefore, the government’s budget constraint is

\[
G_t + \left[ B_{t-1} + B_{t-1}^{sb} + B_{t-1}^{lb} \right] R_{t-1}/\pi_t = B_t + B_t^{sb} + B_t^{lb} + \bar{T}_t^w + \bar{T}_t^b,
\]

where \(B_t^{sb} = (1 - s_t)D_t\) and \(B_t^{lb} = Q_t Z_t + x_t\) are government bonds held by savings and lending banks, respectively. We assume that government spending \(G_t\) follows an AR(1) process.
2.5 Markets clearing

Under Ricardian equivalence, government bonds held by bankers are equal to zero, and so \( B_t = 0 \) in equilibrium. The resource constraint implies that \( Y_t = C_t^w + C_t^b + I_t + G_t + \omega_t \). Total consumption, \( C_t \), is simply the sum of workers’ and bankers’ consumption. Thus, \( C_t = C_t^w + C_t^b \).

2.6 Shock processes

Apart from the monetary policy shock, \( \varepsilon_{Rt} \), which is a zero-mean i.i.d. shock with a standard deviation \( \sigma_R \), the other structural shocks follow AR(1) processes:

\[
\log(X_t) = (1 - \rho_X) \log(X) + \rho_X \log(X_{t-1}) + \varepsilon_{Xt},
\]

where \( X_t = \{ A_t, Y_t, \varepsilon_t, G_t, \psi_t, \Gamma_t, x_t, m_t \} \), \( X > 0 \) is the steady-state value of \( X_t \), \( \rho_X \in (-1, 1) \), and \( \varepsilon_{Xt} \) is normally distributed with zero mean and standard deviation \( \sigma_X \).

3. Calibration

We calibrate the model’s parameters to capture the key features of the U.S. economy for the period 1980Q1–2008Q4 using quarterly data. Table 3 reports the calibration values. The steady-state gross domestic inflation rate, \( \pi \), is set equal to 1.0075, which is the historical average in the sample. The discount factors, \( \beta_w \) and \( \beta_b \), are set to 0.9979 and 0.9943 to match the historical averages of nominal deposit and risk-free interest rates, \( R_D \) and \( R_t \) (see Table 4 for the steady-state values of some key variables). The risk-aversion parameters in workers’ and bankers’ utility functions, \( \gamma_w \) and \( \gamma_b \), are set to 3 and 2, respectively, since we assume that workers are more risk averse than bankers. Assuming that workers allocate one third of their time to market activities, we set \( \eta \), the parameter determining the weight of leisure in utility, and \( \varsigma \), the inverse of the elasticity of intertemporal substitution of labour, to 1.013 and 1, respectively. The habit formation parameter, \( \phi \), is set to 0.65, as estimated in Christiano, Motto, and Rostagno (2010).

The capital share in aggregate output production, \( \alpha \), and the capital depreciation rate, \( \delta \), are set to 0.33 and 0.025, respectively. The parameter measuring the degree of monopoly power \( \omega_t \) represents the default penalties minus the gains of excess bank capital holdings.

---

\(^{32}\omega_t \) represents the default penalties minus the gains of excess bank capital holdings.
in the retail-goods market $\theta$ is set to 6, which implies a 20 per cent markup in the steady-state equilibrium. The parameters $\vartheta_D$ and $\vartheta_L$, which measure the degrees of monopoly power of savings and lending banks, are set equal to 1.53 and 4.21, respectively. These values are set to match the historical averages of deposit and prime lending rates, $R^D$ and $R^L$ (see Table 4).

The nominal price rigidity parameter, $\phi_p$, in the Calvo-Yun price contract is set to 0.75, implying that the average price remains unchanged for four quarters. This value is estimated by Christensen and Dib (2008) for the U.S. economy and commonly used in the literature. The parameters of the adjustment costs of deposit and prime lending interest rates, $\phi_{RD}$ and $\phi_{RL}$, are, respectively, set to 40 and 55, to match the standard deviations (volatilities) of the deposit and prime lending rates to those observed in the data.

Monetary policy parameters $\varrho_\pi$ and $\varrho_Y$ are set to values of 1.2 and 0.05, respectively, and these values satisfy the Taylor principle (see Taylor 1993). The standard deviation of the monetary policy shock, $\sigma_R$, is the usually estimated value of 0.006.

The investment and bank capital adjustment cost parameters, $\chi_I$ and $\chi_Z$, are set to 8 and 70, respectively. This is to match the relative volatilities of investment and loans (with respect to output) to those observed in the data. Similarly, the parameter $\chi_s$, which determines the ratio of bank lending to total assets held by the savings banks $s_t$, is set to 0.001, so that the steady-state value of $s_t$ is equal to 0.82, which corresponds to the historical ratio observed in the data.\footnote{In the data, the ratio of total government securities held by banks to their assets, $1 - s$, is 0.18.} The parameter $\chi_\kappa$ is set to 14.45, so that the steady-state value of the bank’s leverage ratio, $\kappa$, is equal to 11.5, which matches the historical average observed in the U.S. data.

Based on the Basel II minimum required bank capital ratio of 8 per cent, we assume that the maximum imposed bank leverage, $\bar{\kappa}$, is 12.5.\footnote{This is because the maximum bank leverage ratio is simply the inverse of the minimum required bank capital ratio, which is 8 per cent in the Basel II Accord.} Similarly, we calibrate $\chi_{\delta D}$ and $\chi_{\delta Z}$, the parameters determining the total costs of banks’ defaults on interbank borrowing and bank capital, at 163.1 and 1078, so that the probability of default in the interbank market and the bank capital diversion are equal to 1 per cent and 1.6 per cent in annual terms (see Table 3).

Following BGG, the steady-state leverage ratio of entrepreneurs, $1 - N/K$, is set to 0.5, matching the historical average. The probability of entrepreneurial survival to the next period,
\( \nu \), is set at 0.9833, while \( \psi \), the steady-state elasticity of the external finance premium, is set at 0.05, the value used by BGG and close to that estimated by Christensen and Dib (2008).\(^{35}\)

We calibrate the shocks’ process parameters using either values in previous studies or estimated values. The parameters of technology, preference, and investment-specific shocks are calibrated using the estimated values in Christensen and Dib (2008). To calibrate the parameters of the government spending process, we use an OLS estimation of government spending in real per capita terms (see Appendix B). The estimated value of \( \rho_G \), the autocorrelation coefficients, is 0.81, while the estimated standard error, \( \sigma_G \), is 0.0166.

To calibrate the parameters of the riskiness shock process \( \psi_t \), we set the autocorrelation coefficient \( \rho_\psi \) to 0.83, the estimated value in Christiano, Motto, and Rostagno (2010), while the standard error \( \sigma_\psi \) is set to 0.05 to match the volatility of the external risk premium to that observed in the data, measured as the difference between Moody’s BAA yield corporate bond yields and the 3-month T-bill rate. We set the autocorrelation coefficient and the standard error of financial intermediation process \( \rho_\Gamma \) and \( \sigma_\Gamma \) to 0.8 and 0.003, respectively. These values are motivated by the potential persistence and low volatility of this financial shock.\(^{36}\) Finally, we set the autocorrelation coefficients of quantitative and qualitative monetary easing shocks, \( \rho_m \) and \( \rho_x \), equal to 0.5, and their standard deviations, \( \sigma_m \) and \( \sigma_x \), to 0.

4. Empirical Results

To assess the role and the importance of banking sector frictions in U.S. business cycle fluctuations, we simulate two alternative models: (1) the above-described model (the baseline model, hereafter) that incorporates both financial frictions in the banking sector and the financial accelerator mechanism, and (2) a model that includes only the financial accelerator mechanism à la BGG (the FA model).\(^{37}\) In addition, as a sensitive analysis exercise, we report the impulse responses of a constrained version of the baseline model without the interest rate rigidity (the

\(^{35}\)Christensen and Dib (2008) estimate \( \psi \) at 0.046 for the U.S. economy.

\(^{36}\)Future work consists of estimating the model’s structural parameters using either a maximum-likelihood procedure, as used in Christensen and Dib (2008), Ireland (2003), and Dib (2003), or a Bayesian approach, as used in Christiano, Motto, and Rostagno (2010), Dib, Mendicino, and Zhang (2008), Elekdag, Justiniano, and Tchakarov (2006), Queijo von Heideken (2009), and others.

\(^{37}\)Note that, besides the external risk premium, the external financing cost depends on the prime lending rate in the baseline model, while it depends on the risk-free rate in the FA model.
4.1 Impulse responses

First, we evaluate the role and implications of banking sector frictions in the transmission and propagation of real effects of standard supply and demand shocks. Second, we analyze the dynamic responses of key macroeconomic variables to financial shocks originating in the banking sector. Figures 1 and 2 show the impulse responses to technology and the monetary policy shocks, respectively. Figures 3 and 4 plot the responses to financial shocks—riskiness and financial intermediation. Finally, Figures 5 and 6 plot the responses to quantitative and qualitative monetary easing shocks. Each response is expressed as the percentage deviation of a variable from its steady-state level.

4.1.1 Responses to technology and monetary policy shocks

Figure 1 plots the responses to a 1 per cent positive technology shock. Following this shock, output, investment, and consumption increase; however, the increase is smaller in the baseline model than in the FA model. In addition, inflation and the policy rate decrease, but the decline is less in the baseline model. In the presence of the banking sector, the expansion of loans to entrepreneurs is subject to the capital requirement. Thus, to expand the loan supply, banks must raise fresh capital in the financial markets, which pushes up the marginal cost of bank capital. Therefore, though the decline in the policy rate reduces the interbank rate, the prime lending rate increases on impact, before gradually falling below its steady-state level. This leads to an increasing spread between the prime lending and policy rates. A higher spread entails higher entrepreneurs’ debt repayment, which erodes the initial increase in their net worth. Thereby, firms’ net worth decreases very slightly in the baseline model, while it increases substantially in the FA model. Consequently, the external finance premium increases persistently in the baseline model, while it falls in the FA model. Because firms’ net worth decreases in the baseline model, firms depend more on borrowing to finance their new capital acquisitions. Therefore, the demand for loans increases persistently in the baseline model, while it increases only temporally in the FA model.

\[\text{For example, Figure 1 shows that, on impact, the policy rate falls in the two models, while the lending rate increases slightly.}\]
Figure 1 shows that, following a positive technology shock, the bank leverage ratio decreases on impact, before moving persistently above its steady-state level. Also, bank capital holding increases persistently, and for a longer term. We note that, following a positive technology shock, the deposit rate declines slightly, while the prime lending rates jump on impact before declining after a quarter. The decrease in the deposit and lending rates is smaller than that in the policy rate because of the costs of adjusting both interest rates, which causes a partial pass-through of policy rate variations to the deposit and prime lending rates. The default rate on interbank borrowing and the diversion rate on bank capital decrease on impact, and are very persistent. Similar results are found in the response of macroeconomic variables to a positive investment-efficiency shock. Figure 7 shows these impulse responses.

Figure 2 shows the responses to an expansionary monetary policy shock; i.e., an exogenous decrease in the policy rate by 100 basis points. In response to this shock, the nominal interest rate drops sharply, while output and investment increase persistently, even for a longer term. The responses of these variables are substantially larger and more persistent in the FA model. In the NOIR model, where interest rates are flexible, the increase in output and investment is smaller than that in the baseline model.

In Figure 2, in the two base-case models with the banking sector, the lending rate increases sharply on impact by 20 basis points, while the policy rate falls by about 100 basis points. This reflects the need of banks to raise their bank capital to satisfy the capital requirement. The higher demand for bank capital, which is costly, increases the marginal cost of producing loans and thus reduces the increase in entrepreneurial net worth. Therefore, net worth rises in the baseline and NOIR model, but by less than in the FA model. This explains the smaller decline in the firms’ risk premium in the models that incorporate the banking sector. In the FA model, the lower funding cost, caused by the decline in the policy rate, stimulates the demand for investment and creates the BGG financial accelerator effects. The capital requirement, therefore, attenuates the real effects of monetary policy shocks. Since firms’ net worth increases slightly in the models with the banking sector, entrepreneurs still need external funds to finance their investment, and so the demand for loans remains almost unchanged, while it decreases substantially in the FA model. Therefore, in the two base-case models, the decrease in the lending rate is significantly smaller than that in the policy rate. This increases the spread
between the lending and the policy rates.

The presence of the banking sector implies a significant dampening of the impacts of monetary policy shocks on output, investment, net worth, and loans, since the responses of these variables in the FA model are almost twice as large as in the baseline model, and they persist longer.

Figure 2 also shows that an easing monetary policy shock moves the deposit and prime lending rates in opposite directions: the deposit rate decreases slightly, but persistently, while the prime lending rate rises on impact, before falling below its steady-state value. The bank leverage ratio falls on impact, before increasing one quarter later. The probability of defaulting on interbank borrowing increases after a positive monetary policy shock, while the supply of interbank lending decreases sharply on impact, before dropping persistently below its steady-state level. Also, the impulse responses to a 1 per cent preference and government spending shocks are reported in Figures 8 and 9.

4.1.2 Responses to financial shocks

Figure 3 shows the impulse responses to a 10 per cent increase in the riskiness shock, which is similar to that examined in Christiano, Motto, and Rostagno (2010). This financial shock is interpreted as an exogenous increase in the degree of riskiness in the entrepreneurial sector. It may result from an increase in the standard deviation of the entrepreneurial distribution, and implies that lending banks are unable to distinguish between higher- and lower-risk entrepreneurs. Consequently, they raise the external financing premium to all firms, whatever their leverage positions.

In response to this shock, output, investment, and net worth fall persistently below their steady-state levels in all models. Consumption, however, responds positively, as a result of the wealth effect induced by the higher demand for labour to substitute for declining capital in the production of wholesale goods. In addition, inflation and the policy rate increase in the baseline and NOIR models, while they fall slightly in the FA model.

Note also that the external finance premium rises in response to the riskiness shock, while loans temporarily decline, before jumping above their steady-state levels. Figure 3 shows that the lending banks react to this negative financial shock by increasing their leverage ratio slightly on impact, before persistently reducing it, which implies further accumulation of bank capital
in excess of the required level. Because loans decrease, lending banks reduce their capital holdings in the short term. This reduces the marginal cost of producing loans in the short term and allows firms to gradually reduce investment. Therefore, in the short term, the lending banks are able to reduce the lending rate, despite the increase in the policy rate. This leads to smaller drops in net worth in the two models with the banking sector, compared to that in the FA model. In addition, after this riskiness shock, the defaults on interbank borrowing and the diversion of bank capital increase.

The real impact of the riskiness shocks in the FA model is much larger, implying that the banking sector plays a substantial role in dampening the negative effects of riskiness shocks in the economy. The absence of interest rate rigidity amplifies the dampening effects, since interest rates quickly adjust to reduce the marginal cost of producing loans.

Figure 4 shows the impulse responses to a 1 per cent positive financial intermediation shock. This is a positive shock to “loan production,” leading to rising credit supply without varying the inputs used in the loan production function. Following this shock, loans rise on impact, but fall persistently a few quarters later. At the same time, output, investment, and net worth respond positively to this shock. Nevertheless, inflation and the policy rate decrease sharply. We note also that the bank leverage ratio is procyclical, and the exogenous expansion increases the defaults on interbank borrowing and the diversion of bank capital.

Note that the external finance premium and the deposit and lending rates decline as a result of the shock. The instantaneous decline in the prime lending rate is larger than in the policy rate. This decrease in the spread affects the excess loan supply generated by the positive financial intermediation shock.

4.1.3 Responses to quantitative and qualitative monetary easing shocks

Figure 5 shows the impulse responses to a 1 per cent quantitative monetary easing shock, $m_t$, a positive injection of money into lending banks. This shock gradually increases output, investment, and net worth, while inflation, the policy rate, and the external finance premium decline. Following this shock, the lending banks reduce their prime lending rate to accommodate the impact of this expansionary monetary shock. The shock also causes a substantial decline in excess of bank capital, because banks prefer to rely on cheaper funds from the central bank. This, in turn, reduces the marginal cost of producing loans.
We note that loans increase slightly on impact, but fall persistently two quarters after the shock. This response is explained by the substantial increase in net worth. Firms with sound net worth borrow less to finance their capital acquisitions. Consequently, they reduce their demand for loans. Banks respond to this shock by increasing their leverage ratio and loanable funds, as the fraction of deposits lent out on the interbank market persistently increases.

Interestingly, the default rate on interbank borrowing and the diversion of bank capital increase after this expansionary shock. This reflects the changes in the confidence level of the economic agents with respect to the future riskiness and health of the economy that results from the easing of monetary conditions.

Finally, Figure 6 shows the impulse responses to a 1 per cent positive qualitative monetary easing shock, \( x_t \), in which the central bank swaps a fraction of banks’ loans for government bonds used to enhance the bank capital holdings. This shock affects output and investment only marginally. It leads, however, to higher inflation and policy rates. This shock also reduces the bank leverage ratio and increases both defaults in the economy. Note also that interbank lending increases slightly. Also, the marginal cost of producing loans falls, because of the decline in the cost of raising bank capital following this shock.

Overall, the active banking sector that is subject to the capital requirement, as proposed in this framework, attenuates the real impacts of different shocks. Also, the nominal rigidity of the retail interest rates marginally affects the dynamics of key macroeconomic variables.

### 4.2 Volatility and autocorrelations

In this section, we assess the ability of the baseline model that incorporates the banking frictions to reproduce the salient features of the U.S. business cycles. We consider the model-implied volatilities (standard deviations), relative volatilities, and correlations of output with the key variables of interest. Table 5 reports the standard deviations and relative volatilities of output, investment, consumption, loans, and the external finance premium from the data, and for the two simulated models.\(^\text{39}\) The standard deviations are expressed in percentage terms. The model-implied moments are calculated using all the shocks.

Column 3 in Table 5 shows the standard deviations, relative volatilities, and unconditional

\(^{39}\)In the data, all series are HP-filtered before calculating their standard deviations and unconditional correlations with output.
autocorrelations of the actual data. Columns 4 and 5 report simulations with the baseline and FA models, respectively. Panel A shows that the standard deviation of output is 1.31, investment 6.26, and consumption 1.03. Loans have a standard deviation of 4.60. The external finance premium, however, is considerably less volatile; its standard deviation is only 0.38. Panel B shows that investment and loans are 4.77 and 3.51 times as volatile as output, while consumption and the external finance premium are less volatile than output, with relative volatilities of 0.78 and 0.29, respectively. In Panel C, output, investment, loans, and the external finance premium are highly persistent, with autocorrelation coefficients that are, at least, equal to 0.8; consumption is less so, with a coefficient of 0.73.

The simulation results show that, in the model with an active banking sector, all volatilities are close to those in the data. The FA model, in which the banking sector is absent, overpredicts all the volatilities. This feature is common in standard sticky-price models. The baseline model is also very successful at matching the relative volatility of most of the variables. In contrast, the FA model slightly underpredicts the relative volatilities of consumption and the external finance premium.

Panel C in Table 5 shows the unconditional autocorrelations of the data and of the key variables generated by the two simulated models. In general, both models show larger autocorrelations in output, investment, consumption, and loans than those observed in the data. Both models match the autocorrelation in the external finance premium very well. Interestingly, Table 6 shows that the baseline model is successful in reproducing negative correlations of the external risk premium, and banks’ defaults on interbank borrowing and bank capital with output. Moreover, the model shows that banks’ leverage ratios and the share of interbank lending in total deposits are procyclical (positively correlated with output). Thus, during boom periods, savings banks and lending banks expand their interbank lending and credit supply. This helps to reduce the external finance costs of entrepreneurs and increase investment and output.

5. Conclusion

Following the recent financial crisis, an increasing number of papers have aimed to incorporate an active banking sector into macroeconomic DSGE models. Such models provide a better
understanding of the role of financial intermediation in the transmission and propagation of the real impacts of aggregate shocks, and help evaluate the importance of financial shocks that originate in the banking sector as a source of business cycle fluctuations. This paper contributes to this growing literature by proposing a micro-founded framework to incorporate an active banking sector into DSGE models. Besides the financial accelerator mechanism à la BGG, it introduces financial frictions in the supply side of the credit market using the banks’ balance sheet channel. We assume a banking sector that consists of two types of monopolistically competitive banks that offer different banking services and transact in the interbank market. Banks raise deposits and bank capital (equity) from households. Bank capital is introduced to satisfy the capital requirement imposed by regulators: banks must hold a minimum of bank capital to provide loans to entrepreneurs.

The paper provides a rich and rigorous framework to address monetary and financial stability issues. It allows for policy simulation analysis of factors such as: (1) bank capital regulations; (2) the optimal choice of banks’ leverage ratios; (3) interest rate spreads resulting from the monopoly power of banks when setting deposit and prime lending rates; (4) endogenous bank defaults on interbank borrowing; and (5) the optimal choice of banks’ portfolio compositions.

The key result is that, under the capital requirement, the banking sector dampens the real impacts of different shocks. This, however, contradicts the findings in previous studies that use models with bank capital introduced to solve asymmetric information between households and banks. The model also reproduces the salient features of the U.S. economy: volatilities of key macroeconomic variables and their correlations with output.

The model can be used to address policy and financial stability questions, such as capital requirement regulations, the interaction between monetary policy and financial stability, the impact of macroprudential instruments, and the efficiency versus stability of the banking system. Future work will consist of estimating the model’s structural parameters, incorporating credit to households, and extending the framework to an open-economy model.

---

40 For example, Gertler and Kiyotaki (2010); Gertler and Karadi (2010); Meh and Moran (2010).
References


### Table 3: Parameter Calibration: Baseline Model

<table>
<thead>
<tr>
<th>Preferences</th>
<th>$\beta_w = 0.9979$, $\beta_b = 0.9943$, $\gamma_w = 3$, $\gamma_b = 2$,</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary policy</td>
<td>$\varrho_{\pi} = 1.2$, $\varrho_Y = 0.05$, $\sigma_R = 0.006$,</td>
</tr>
<tr>
<td>Technologies</td>
<td>$\alpha = 0.33$, $\delta = 0.025$,</td>
</tr>
<tr>
<td>Adjustment and default costs</td>
<td>$\chi_I = 8$, $\chi_Z = 70$, $\chi_s = 0.001$, $\chi_\kappa = 14.45$, $\chi_{\delta D} = 163.1$, $\chi_{\delta Z} = 1078$,</td>
</tr>
<tr>
<td>Nominal rigidities</td>
<td>$\phi_p = 0.75$, $\phi_{RD} = 40$, $\phi_{RL} = 55$,</td>
</tr>
<tr>
<td>Financial sector</td>
<td>$\nu = 0.9833$, $\psi = 0.05$, $K/N = 2$, $\bar{\kappa} = 11.5$,</td>
</tr>
<tr>
<td>Exogenous processes</td>
<td>$A = 1$, $\rho_A = 0.8$, $\sigma_A = 0.009$, $\Upsilon = 1$, $\rho_\Upsilon = 0.7$, $\sigma_\Upsilon = 0.033$,</td>
</tr>
<tr>
<td></td>
<td>$\epsilon = 1$, $\rho_\epsilon = 0.8$, $\sigma_\epsilon = 0.0073$,</td>
</tr>
<tr>
<td></td>
<td>$G/Y = 0.17$, $\rho_G = 0.60$, $\sigma_G = 0.0166$,</td>
</tr>
<tr>
<td></td>
<td>$\psi = 0.05$, $\rho_\psi = 0.83$, $\sigma_\psi = 0.050$,</td>
</tr>
<tr>
<td></td>
<td>$\Gamma = 1$, $\rho_\Gamma = 0.8$, $\sigma_\Gamma = 0.003$,</td>
</tr>
<tr>
<td></td>
<td>$m = 0$, $\rho_m = 0.5$, $\sigma_m = 0.00$,</td>
</tr>
<tr>
<td></td>
<td>$x = 0$, $\rho_x = 0.5$, $\sigma_x = 0.00$</td>
</tr>
</tbody>
</table>
Table 4: Steady-State Values and Ratios: Baseline Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>A. Steady-state values</strong></td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>inflation</td>
<td>1.0075</td>
</tr>
<tr>
<td>$R$</td>
<td>policy rate</td>
<td>1.0141</td>
</tr>
<tr>
<td>$R^D$</td>
<td>deposit rate</td>
<td>1.0097</td>
</tr>
<tr>
<td>$R^L$</td>
<td>prime lending</td>
<td>1.0220</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>external finance premium</td>
<td>1.0027</td>
</tr>
<tr>
<td>$S$</td>
<td>fraction of interbank lending</td>
<td>0.82</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>bank leverage ratio</td>
<td>11.5</td>
</tr>
<tr>
<td>$\delta^D$</td>
<td>default on interbank borrowing</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\delta^Z$</td>
<td>diversion on bank capital</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td><strong>B. Steady-state ratios</strong></td>
<td></td>
</tr>
<tr>
<td>$C/Y$</td>
<td>consumption to output</td>
<td>0.661</td>
</tr>
<tr>
<td>$C^w/Y$</td>
<td>workers’ consumption to output</td>
<td>0.624</td>
</tr>
<tr>
<td>$C^b/Y$</td>
<td>bankers’ consumption to output</td>
<td>0.037</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>investment to output</td>
<td>0.16</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>government spending to output</td>
<td>0.17</td>
</tr>
<tr>
<td>$K/Y$</td>
<td>capital stock to output</td>
<td>6.753</td>
</tr>
<tr>
<td>$Z/Y$</td>
<td>bank capital to output</td>
<td>0.294</td>
</tr>
<tr>
<td>$\Pi^S/Y$</td>
<td>savings bank profit to output</td>
<td>0.015</td>
</tr>
<tr>
<td>$\Pi^L/Y$</td>
<td>lending bank profit to output</td>
<td>0.02</td>
</tr>
<tr>
<td>$K/N$</td>
<td>capital to entrepreneurs’ net worth</td>
<td>2</td>
</tr>
</tbody>
</table>
### Table 5: Standard Deviations and Relative Volatilities: (Data 1980Q1–2008Q4)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
<th>Data</th>
<th>Baseline</th>
<th>FA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>A. Standard deviations (in %)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_t$</td>
<td>output</td>
<td>1.31</td>
<td>1.48</td>
<td>2.60</td>
</tr>
<tr>
<td>$I_t$</td>
<td>investment</td>
<td>6.26</td>
<td>7.27</td>
<td>12.28</td>
</tr>
<tr>
<td>$C_t$</td>
<td>consumption</td>
<td>1.03</td>
<td>1.27</td>
<td>1.87</td>
</tr>
<tr>
<td>$L_t$</td>
<td>loans</td>
<td>4.60</td>
<td>4.73</td>
<td>9.00</td>
</tr>
<tr>
<td>$rp_t$</td>
<td>external finance premium</td>
<td>0.38</td>
<td>0.43</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td><strong>B. Relative volatilities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_t$</td>
<td>output</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$I_t$</td>
<td>investment</td>
<td>4.77</td>
<td>4.91</td>
<td>4.72</td>
</tr>
<tr>
<td>$C_t$</td>
<td>consumption</td>
<td>0.78</td>
<td>0.86</td>
<td>0.72</td>
</tr>
<tr>
<td>$L_t$</td>
<td>loans</td>
<td>3.51</td>
<td>3.20</td>
<td>3.46</td>
</tr>
<tr>
<td>$rp_t$</td>
<td>external finance premium</td>
<td>0.29</td>
<td>0.29</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td><strong>C. Autocorrelations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_t$</td>
<td>output</td>
<td>0.81</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>$I_t$</td>
<td>investment</td>
<td>0.80</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>$C_t$</td>
<td>consumption</td>
<td>0.73</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>$L_t$</td>
<td>loans</td>
<td>0.93</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$rp_t$</td>
<td>external finance premium</td>
<td>0.81</td>
<td>0.88</td>
<td>0.90</td>
</tr>
</tbody>
</table>

### Table 6: Correlations with Output (Data 1980Q1–2008Q4)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
<th>Data</th>
<th>Baseline</th>
<th>FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>output</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$I_t$</td>
<td>investment</td>
<td>0.90</td>
<td>0.78</td>
<td>0.87</td>
</tr>
<tr>
<td>$C_t$</td>
<td>consumption</td>
<td>0.85</td>
<td>0.51</td>
<td>0.63</td>
</tr>
<tr>
<td>$L_t$</td>
<td>loans</td>
<td>0.30</td>
<td>0.34</td>
<td>0.39</td>
</tr>
<tr>
<td>$rp_t$</td>
<td>external finance premium</td>
<td>-0.30</td>
<td>-0.25</td>
<td>-0.35</td>
</tr>
<tr>
<td>$\kappa_t$</td>
<td>bank leverage ratio</td>
<td>+</td>
<td>0.48</td>
<td>.</td>
</tr>
<tr>
<td>$\delta_t^D$</td>
<td>default on interbank borrowing</td>
<td>+</td>
<td>-0.36</td>
<td>.</td>
</tr>
<tr>
<td>$\delta_t^Z$</td>
<td>default on bank capital</td>
<td>+</td>
<td>-0.21</td>
<td>.</td>
</tr>
</tbody>
</table>
Figure 1: Responses to a 1 Per Cent Positive Technology Shock

Figure 2: Responses to Monetary Policy Shocks
Figure 3: Responses to Riskiness Shocks

Figure 4: Responses to a 1 Per Cent Financial Intermediation Shock
Figure 5: Responses to a 1 Per Cent Quantitative Monetary Easing Shock

Figure 6: Responses to a 1 Per Cent Qualitative Monetary Easing Shock
Figure 7: Responses to Investment-Efficiency Shocks

Figure 8: Responses to Preference Shocks
Figure 9: Responses to Government Spending Shocks
Appendix A: First-Order Conditions

A.1. Workers’ first-order conditions

The first-order conditions of the workers’ optimization problem are:

\[ e_t \left( \frac{C^w_t}{(C^w_{t-1})^\varphi} \right)^{1-\gamma_w} - \beta_w \varphi E_t \left[ e_{t+1} \left( \frac{C^w_{t+1}}{(C^w_t)^\varphi} \right)^{1-\gamma_w} \right] = C^w_t \lambda^w_t; \quad (A.1) \]

\[ \frac{\eta}{(1 - H_t)^\zeta} = \lambda^w_t W_t; \quad (A.2) \]

\[ \frac{\lambda^w_t}{R^d_t} = \beta_w E_t \left( \frac{\lambda^w_{t+1}}{\pi_{t+1}} \right), \quad (A.3) \]

where \( \lambda^w_t \) is the Lagrangian multiplier associated with the budget constraint.

A.2. Entrepreneurs’ first-order conditions

The first-order conditions of the entrepreneurs’ optimization problem are:

\[ r^K_t = \alpha \xi_t \frac{Y_t}{K_t}; \quad (A.4) \]

\[ W_t = (1 - \alpha) \xi_t \frac{Y_t}{H_t}; \quad (A.5) \]

\[ Y_t = A_t K^{\alpha} H^{1-\alpha}_t, \quad (A.6) \]

where \( \xi_t > 0 \) is the real marginal cost.

A.3. The retailer’s optimization problem

The retailer’s optimization problem is

\[ \max_{\{\bar{P}(j)\}} \mathbb{E}_0 \left[ \sum_{l=0}^{\infty} (\beta_w \phi_p)^l \lambda^w_{t+l} \Pi^R_{t+l}(j) \right], \quad (A.7) \]

subject to the demand function\(^{41}\)

\[ Y_{t+l}(j) = \left( \frac{\bar{P}(j)}{P_{t+l}} \right)^{-\theta} Y_{t+l}, \quad (A.8) \]

\(^{41}\)This demand function is derived from the definition of aggregate demand as the composite of individual final output (retail) goods and the corresponding price index in the monopolistic competition framework, as follows:

\[ Y_{t+l} = \left( \int_0^1 Y_{t+l}(j)^{\frac{\alpha - 1}{\alpha}} dj \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad P_{t+l} = \left( \int_0^1 P_{t+l}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \]

where \( Y_{t+l}(j) \) and \( P_{t+l}(j) \) are the demand and price faced by each individual retailer \( j \in (0, 1) \).
where the retailer’s nominal profit function is

$$\Pi_{t+1}(j) = \left( \pi\tilde{P}_t(j) - P_t\xi_{t+1} \right) Y_{t+1}(j)/P_{t+1}. \quad (A.9)$$

The first-order condition for $\tilde{P}_t(j)$ is

$$\tilde{P}_t(j) = \frac{\theta}{\theta - 1} E_t \sum_{l=0}^{\infty} (\beta w \phi_p)^l \lambda_{t+l}^w Y_{t+l}(j) \xi_{t+l}. \quad (A.10)$$

The aggregate price is

$$P_t^{1-\theta} = \phi_p (\pi P_{t-1})^{1-\theta} + (1 - \phi_p) \tilde{P}_t^{1-\theta}. \quad (A.11)$$

These lead to the following equation:

$$\hat{\pi}_t = \beta w E_t \hat{\pi}_{t+1} + \frac{(1 - \beta w \phi_p)(1 - \phi_p)}{\phi_p} \hat{\xi}_t, \quad (A.12)$$

where $\xi_t$ is the real marginal cost, and variables with hats are log deviations from the steady-state values (such as $\hat{\pi}_t = \log(\pi_t/\pi)$).
Appendix B: Data

1. Loans are measured by Commercial and Industrial Loans of all Commercial Banks (BUS-LOANS), quarterly and seasonally adjusted;

2. The external finance premium is measured by the difference between Moody’s BAA corporate bond yields and the 3-Month Treasury Bill (TB3MS);

3. Inflation is measured by quarterly changes in the GDP deflator (Δ log(\(GDP_D\)));

4. The prime lending rate is measured by the Bank Prime Loan Rate (MPRIME);

5. The monetary policy rate is measured by the 3-Month Treasury Bill (TB3MS);

6. The deposit rate is measured by the weighted average of the rates received on the interest-bearing assets included in M2 (M2OWN);

7. The real money stock is measured by the real M2 money stock per capita;

8. Output is measured by real GDP per capita;

9. Total Consumption is measured by Personal Consumption Expenditures (PCEC);

10. Investment is measured by Gross Private Domestic Investment (GPDI);

11. Government spending is measured by output minus consumption and investment (GDP - PCEC- GPDI).