International Risk Cycles

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Abstract

This paper presents a two-country real business cycle model with two novel features: (1) exogenous shocks to worldwide uncertainty, (2) heterogeneous exposures to the world aggregate shock. We show that these two features lead to significant progress in tackling internationational finance puzzles. When world risk increases, investment decreases and a worldwide recession follows, even in the absence of technology shocks. Capital pulls out of the riskier country, which experiences the largest recession. Both stock markets tank and the high interest rate currency depreciates. The model thus provides a rationale for the existence of international equity and currency risk premia, and links them to macroeconomic dynamics. Empirically, we measure the response of macroeconomic aggregates and exchange rates to an aggregate volatility shock, and find support for the model's mechanism.

Keywords: business cycles, time-varying risk premia, disasters, jumps, international macroeconomics, Backus-Smith puzzle, exchange rate volatility, UIP puzzle, carry trade.

JEL: E32, E44, G12.

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1 Introduction

Dynamic stochastic general equilibrium models used in international macroeconomics are inconsistent with basic features of asset prices and exchange rates, such as their volatility, their correlation across countries, and their correlation with macroeconomic aggregates. Moreover, they imply very small equity and currency risk premia, in contrast to the large average excess returns which are well documented in the empirical finance literature. This paper tries to make progress in these dimensions by proposing a parsimonious model in the real business cycle tradition of Backus, Kehoe and Kydland (1992).

Our model is based on two simple assumptions: (1) aggregate risk varies over time; (2) countries differ in their exposures to aggregate risk. Backus, Foresi and Telmer (2001) and Lustig, Roussanov and Verdelhan (2009) show that these two assumptions are needed to account for several well-established features of the data, such as the failure of the uncovered interest rate parity condition (UIP) or the high average return to a "carry trade" strategy – i.e, borrowing in low interest rate countries and lending in high interest rate countries. We embody their insights in a two-country model with production.

Building on the work of Rietz (1988), Barro (2006), Gabaix (2008), and Gourio (2010), we propose an international real business cycle (RBC) model with a small, stochastically time-varying risk of economic "disaster". Variations in disaster risk lead not only to variations in risk premia and asset prices, but also in macroeconomic quantities. An increase in the probability of disaster leads to a decline of investment and output, because higher uncertainty makes it less attractive to invest. Demand for precautionary savings increase, leading the yield on risk-free assets to fall, while spreads on risky securities increase. These business cycle dynamics occur with no change in total factor productivity (TFP). In our model, the two countries have different riskiness, and the recession and the decline in stock prices are stronger in the more risky country. Capital flows out of the risky economy, as investors seek safety. Finally, the behavior of the real exchange rate turns out to depend on the market structure. In a complete market model, the more risky country currency appreciates, while in a model with incomplete markets and trade costs the more risky currency depreciates. Hence, the exposure of the exchange rate to disasters can rationalize the carry trade risk premium.

To make the results transparent, we consider two variants of the model. The first variant focuses on a limiting case where trade costs are infinite and thus countries do not trade. Here, we assume that financial markets are complete; it implies that the exchange rate is the ratio of the home and foreign pricing kernels. The second variant introduces convex trade costs, allowing for international capital flows. For tractability, we assume that one country is large, while the other country is small and thus takes prices as given. The exchange rate is then determined by the direction of capital flows and trade costs. In the second variant, we also assume that markets are incomplete: the small country has only access to one domestic currency-denominated bond.

Overall, we show that the two simple assumptions above are sufficient to generate a large number of predictions consistent with the data. The first variant of the model replicates (i) the volatility of the real exchange rate; (ii) macroeconomic aggregates are more correlated than can be accounted solely by TFP shocks; (iii) asset returns are more correlated across countries than output or dividends; (iii) the correlation of relative consumption growth and the exchange rate is significantly less than unity (the Backus and Smith (1993) anomaly); (iv) the failure of UIP and the high average returns to a carry trade strategy; (v) the time-varying correlations of exchange rates and stock returns, which are low in normal times and high during crises (i.e., when the probability of disaster is high). Our model is also consistent with basic business cycle and asset pricing facts in the US, such as the mean and volatility of equity and risk-free returns, the predictability of returns, and the correlation of asset prices and macroeconomic aggregates such as investment or output. While our model does make significant progress in tackling the asset pricing anomalies of the RBC model, it is subject to the usual criticisms regarding the labor market anomalies in the RBC model: we do not make progress in this dimension.

The second variant of the model, which endogenizes capital flows, is quantitatively less successful. While our mechanism substantially increases the volatility of exchange rates compared to a model with only TFP shocks, exchange rates are still too smooth. In both models, the unconditional correlations of the exchange rate with equity returns and consumption growth, while lower than in most real business cycle models, are still higher than in the data.

Crucial to our success is the use of recursive preferences, as introduced by Epstein and Zin (1989): risk-aversion is higher than the inverse of the intertemporal elasticity of substitution (IES), so that agents favor early resolution of uncertainty. Such preferences allow to decouple consumption from the market price of risk. As a result, periods with large amounts of uncertainty (when the probability of disaster increases) are “bad” times, even though consumption might initially rise. Intuitively, investors care not only about current but also future utility. When risk increases, future (continuation) utility is low, both because of a low future mean and because of higher uncertainty. This low future utility increases the marginal utility of wealth immediately. In the (first) model, high (low) interest rate currencies thus tend to depreciate (appreciate) in bad times, as they do in the data. Since investments in high interest rate currencies pay badly in bad times, investors expect positive carry trade excess returns. Should such depreciations happen in good times (e.g when marginal utility is low), currency carry trades would be good hedges to aggregate shocks and would not offer positive expected excess returns.

The main limitation of our approach is a tension between matching the average carry trade return and the volatility of quantities and prices. One the one hand, our model (at least the first variant), like any complete market model, implies that the countries which are more risky in terms of their exchange rates are less risky in terms of their domestic “fundamentals” such as consumption, output, or stock prices. Specifically, a country that is more exposed to aggregate shocks will have a larger decline in its “fundamentals”when a shock hits, but its exchange rate will appreciate. This is a direct consequence of risk-sharing, as shown by Backus and Smith (1993). Countries which suffer large shocks have high marginal utility and an hypothetical social planner would thus want to allocate them more goods from abroad. The reason this does not occur must be that it is very costly to transfer goods from the less to the more risky country. In other terms, goods in the more risky country are relatively expensive, and its exchange rate is high. On the other hand, high interest rate currencies tend to depreciate in bad times while low interest rate currencies tend to appreciate. This simple behavior is at the heart of any risk-based explanation of carry trade excess returns. The tension is that the more risky countries in terms of fundamentals have low interest rates because of larger precautionary savings. Hence, in our model, the more risky countries in terms of fundamentals have low interest rates and are the funding currencies of the carry trade, while the less risky countries are the high interest rate countries. While surprising, this negative relation between fundamental risk and currency risk may be consistent with bond, currency and equity markets. Verdelhan (2010) finds that low interest rate currencies offer high average equity excess returns in local currencies, consistent with large risk premia to compensate for large fundamental risk. We also show that a prototypical funding country, Japan, is indeed more risky in terms of fundamentals than a prototypical investing country, New Zealand. However, and here lies the tension, in the data, low interest rate countries have lower volatility of both quantities and equity prices, while our model predicts the opposite. We discuss some possible solutions to this conundrum later.

We test our model's empirical predictions by measuring the response of quantities and prices to volatility shocks. We measure volatility using the monthly standard deviation of realized daily equity returns. This measurement is consistent with the theory, since in our model, increases in the probability of disaster are associated with high equity return volatility. Two clear results emerge. We find that there is substantial co-movement of volatility worldwide. This is consistent with the model because we assume that disasters are common across countries. We focus on a group of six countries (Belgium, Canada, France, Japan, United Kingdom, and United States) for which monthly volatility series are available over the last forty years. Following the model's logic, we use the average standard deviation across these countries as a measure of global uncertainty. We find that when global volatility increases, industrial production and GDP fall, while unemployment rises. The impact of uncertainty is negative for all countries. These results generalize Bloom's (2009) empirical findings for the US, to a large set of OECD countries and different measures of volatilities. We also document some suggestive patterns regarding the response of exchange rates and current accounts to a volatility shock.

Overall, our model suggests a novel interpretation of the international 2007-2009 crisis as an increase in the perceived probability of a disaster. There is ample evidence that investors around the world feared a Great Depression scenario in Fall 2008. In the model, such an increase in perceived risk leads, in itself, to a large recession, and to large declines in stock markets. Countries that are more risky – i.e the ones which would be more affected should the disaster actually hit – are more affected by the increase in the probability of disaster, leading to larger declines in output, investment, stock prices, and interest rates. Capital flows out of the more risky countries, consistent with the patterns observed in Eastern Europe during the crisis. Finally, in a complete market model, the more risky currencies appreciate, while in a model with incomplete markets, the more risky currencies may depreciate.

We now rapidly review the literature on disaster risk. Pioneered by Rietz (1988) and Barro (2006), this class of models has received much attention recently in the macroeconomics and finance literature.[1] These models can potentially reproduce the dynamics of risk premia, provided that there is some time-variation in the quantity or price of disaster risk. In this paper, we do so by assuming that the disaster probability is time-varying as in Gabaix (2008) and Gourio (2009). The papers closest to ours are Farhi and Gabaix (2008) and ongoing work by Gourinchas, Rey and Govillot (2010). Farhi and Gabaix (2008) are the first to show that a disaster risk model can reproduce the UIP puzzle. Gourinchas et al. (2010) document that the US provides insurance to the rest of the world, especially in times of global stress, and show that a simple disaster risk model accounts for the large collapse in US net foreign assets. Both papers consider endowment economies with power utility and no trade. In this paper, we study production economies with Epstein and Zin (1989) preferences and international trade.

The risk of an economic disaster may be a strictly rational expectation, but it can also be interpreted as a potentially biased, or excessively volatile, belief. For instance, during the recent

[1] Ait-Sahalia, Wang and Yared (2001), Backus, Chernov and Martin (2009), Barro and Ursua (2009), Bates (2009), Bollerslev and Todorov (2009), Gabaix (2008), Gourio (2008, 2009), Julliard and Ghosh (2008), Liu, Pan and Wang (2005), Martin (2008), Pan (2002), Santa-Clara and Yan (2009) and Wachter (2008) all study disaster risk on equity and bond markets. Barro and Ursua (2008) build an international database of consumption disasters in the twentieth century. Barro (2009) studies the welfare costs of rare disasters. Jurek (2008) and Farhi, Fraiberger, Gabaix, Ranciere and Verdelhan (2009) confront the disaster risk model to currency option data. Nakamura, Steinsson, Barro and Ursua (2010) provide evidence that some disasters are global while others are local.

financial crisis, many commentators, including well-known macroeconomists, have highlighted the possibility that the U.S. economy might fall into another Great Depression. Our model studies the macroeconomic effects of a time-varying belief for international economics. This simple modeling device captures the idea that aggregate uncertainty is sometimes high: people sometimes worry about the possibility of a deep recession. It also captures the idea that there are some asset price changes that are not obviously related to current or future TFP, i.e. "bubbles", "animal spirits", and which in turn affect the macroeconomy.

The rest of the paper is organized as follows. Section I documents some stylized facts on currency, bond and equity markets that motivate the key assumptions of our model. Section II presents a two-country model without international trade, and shows that this model can replicate the dynamics of macroeconomic quantities, asset prices, and exchange rates. Section III explores the role of capital flows by considering a small open economy that trades with the rest of the world. Section IV links the model to the data and studies the impact of global uncertainty on macroeconomic variables. A separate appendix presents data sources, additional data statistics, our computational methods for the models of section II and of section III, and some additional results and robustness checks for these models.

2 What Do We Learn From International Asset Prices?

International real business cycle models tend to ignore asset prices. Yet, stylized facts on currency, bond and equity markets impose some strong necessary conditions on the pricing kernels of any model in international economics. In this section, we rapidly review these necessary conditions and how they guide our modeling choices.

Large risk premia in currency and equity markets We start with a rapid review of stylized facts on international equity, bond and currency markets. We focus on what happened in OECD countries over the post-Bretton Woods sample. We report additional information on our data set in Appendix A.

It is well-known that the average return on US equities is large. Even after taking into account the on-going Great Recession, the MSCI index of US stocks offers an annual average return over the last forty years of 3.4% above the short-term interest rate. This large average excess return is not unique to the US economy. Over the same period, the average excess return over a wide group of OECD countries is 3.2%. These excess returns are volatile but they imply Sharpe ratios (defined as annualized average excess returns divided by their annualized standard deviations) of 0.22 for the U.S. and 0.15 for the other OECD countries respectively. These Sharpe ratios are large, but actually lower than their equivalent on century-long time series of US stock returns.

Currency markets also offer large risk premia. Following Lustig and Verdelhan (2007), we sort countries into portfolios based on interest rates in order to study carry trade returns. The first (last) portfolio contains the currencies with the lowest (highest) interest rates. According to the standard uncovered interest rate parity (UIP) condition, changes in exchange rates should equal the corresponding interest rate differential between home and foreign countries. In this case, currency excess returns are zero. A large body of empirical work, however, starting with Hansen and Hodrick (1980) and Fama (1984), reports violations of UIP in the time series. Table 1 reports violations of UIP in the cross-section. We report average currency excess returns rx from the perspective of the US investor. These excess returns are equal to differences between foreign and US short term interest rates ($i^{\star} - i^{US}$) and the corresponding changes in exchange rates (Δs). Currency carry trades correspond to simple investment strategies that borrow in low interest rate currencies (e.g short the first portfolio) and that invest in high interest rate currencies (e.g long the last portfolio). This high-minus-low strategy delivers an excess return of above 6% per annum and a Sharpe ratio of 0.64, even higher than the Sharpe ratios on equity markets.

These average large excess returns on equity and currency markets imply necessary conditions on domestic and foreign pricing kernels.

Time-varying higher moments Backus et al. (2001) show that, if markets are complete, pricing kernels must have time-varying higher moments in order to replicate deviations from the UIP condition. The argument is as follows. Write the Euler equations that characterizes the return

Table 1: Portfolios - Countries Sorted on Nominal Interest Rates

Portfolio	1	2	3	4	5
			Changes in Exchange Rates: Δs^j		
$Mean$	-1.65	-1.98	-0.66	-0.14	8.62
Std	10.87	9.78	9.77	10.34	13.64
			Interest Rate Differences: $i^j - i^{US}$		
$Mean$	-2.12	0.23	1.63	3.33	14.72
Std	2.16	1.93	2.02	2.51	10.65
			Currency Excess Returns: rx^j		
$Mean$	-0.47	2.22	2.29	3.48	6.10
Std	11.63	10.65	10.30	11.00	11.39
SR	-0.04	0.21	0.22	0.32	0.54
			High-minus-Low Currency Excess Returns: $rx^j - rx^1$		
$Mean$		2.69	2.76	3.95	6.57
Std		5.62	7.55	7.31	10.24
SR		0.48	0.37	0.54	0.64
			Real Interest Rates: $r^{f,j}$		
$Mean$	0.63	1.75	2.24	3.14	5.34
Std	2.27	2.80	2.57	2.66	5.10
			Inflation Rates: π^j		
$Mean$	2.74	4.10	5.15	5.74	15.21
Std	2.60	2.97	3.15	2.88	8.87
			Equity Excess Returns: $r^{e,j} - i^j$		
$Mean$	4.87	5.46	6.65	2.84	-2.39
Std	18.17	19.79	20.30	21.10	28.92
			Equity Return Volatilities: $\sigma(r^{e,j})$		
$Mean$	14.99	15.85	17.31	17.00	23.66
Std	6.65	7.16	5.63	6.00	10.44

Notes: This table reports, for each portfolio j, the average change in log spot exchange rates Δs^j, the average interest rate difference $f^j - s^j$, the average log excess return rx^j and the average return on the long short strategy $rx^j - rx^1$. The last four panels reports average real interest rates, average inflation rates, average foreign equity excess returns in local currencies, along with the standard deviations of these foreign equity returns. The portfolios are constructed by sorting currencies into five groups at time t based on short-term nominal interest rates at the end of period $t-1$. The first portfolio contains currencies with the lowest interest rates. The last portfolio contains currencies with the highest interest rates. Data are monthly. We focus on OECD countries. The sample starts in January 1971 and ends in December 2009.

$R^{\star}_{t+1}$ on any foreign asset. We use a superscript $\star$ to denote a foreign variable. M and $M^{\star}$ denote the pricing kernels of the domestic and foreign investors, and Q is the real exchange rate in U.S.

good per foreign good:

$$
\begin{aligned}
E_t\left[M^\star_{t+1}R^\star_{t+1}\right] &= 1, \\
E_t\left[M_{t+1}\frac{Q_{t+1}}{Q_t}R^\star_{t+1}\right] &= 1.
\end{aligned}
$$

If markets are complete, the pricing kernel is unique and thus the change in real exchange rate is equal to:

$$
\frac{Q_{t+1}}{Q_t} = \frac{M^\star_{t+1}}{M_{t+1}}. \tag{2.1}
$$

If market are incomplete, consider the projection of M on the space of traded assets. The reasoning that follows applies there too. Backus et al. (2001) show that the log currency risk premia are equal to the half differences in the higher conditional moments of the log pricing kernels:[2]

$$
E_t(r^e_{t+1}) = \sum_{j=2}^{\infty}\frac{\kappa_j}{j!} - \sum_{j=2}^{\infty}\frac{\kappa^\star_j}{j!},
$$

where κ_j denotes the cumulant of order j of the log pricing kernel. As UIP tests and Table show, currency excess returns are predictable using interest rate differences. As a result, expected currency excess returns vary over time. The equation above implies that higher moments of the pricing kernels must also vary over time.

[2] To make this point transparent, let us consider the special case of log-normal pricing kernels and returns. Then, risk-free rates are:

$$
r_t = -\log E_t M_{t+1} = -E_t m_{t+1} - \frac{1}{2}Var_t(m_{t+1}), \text{ and } r^\star_t = -\log E_t M^\star_{t+1} = -E_t m^\star_{t+1} - \frac{1}{2}Var_t(m^\star_{t+1}).
$$

The expected change in the exchange rate is:

$$
E_t(\Delta q_{t+1}) = -E_t(m_{t+1}) + E_t(m^\star_{t+1}) = -r^\star_t + r_t - \frac{1}{2}Var_t(m^\star_{t+1}) + \frac{1}{2}Var_t(m_{t+1}).
$$

As a result, the expected log currency excess return is equal to:

$$
E_t(r^e_{t+1}) = r^\star_t - r_t + E_t(\Delta q_{t+1}) = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m^\star_{t+1}).
$$

High correlation of pricing kernels In order to replicate both the equity premium and the exchange rate volatility, Brandt, Cochrane and Santa-Clara (2006) show that pricing kernels must be highly correlated. To see this point, let us start again from equation 2.1 which defines the exchange rate in complete markets. It implies that the variance of real exchange rate changes is equal to:

$$\sigma^2(\Delta q) = \sigma^2(m) + \sigma^2(m^\star) - 2\rho(m, m^\star)\sigma(m)\sigma(m^\star).$$

In order to fit the equity premium, we know that the variance of the stochastic discount factor has to be high (Mehra and Prescott (1985) and Hansen and Jagannathan (1991)). This condition does not depend on preferences. A simple example makes this point obvious: if pricing kernels are log-normals, the maximum Sharpe ratio is equal to the standard deviation of the log pricing kernel.

But if $\rho(m, m^\star)$ is small, then the variance of the exchange rate is approximately twice the variance of the stochastic discount factor. Building on this observation, Brandt et al. (2006) show that the actual real exchange rate is much smoother than the theoretical one implied by asset pricing models, unless pricing kernels are highly correlated across countries.

Note that power utility would not deliver this feature. The correlation among consumption growth shocks across countries is low. Power utility thus implies a low correlation of stochastic discount factors. High risk aversion produces high equity risk premia, but exchange rates are then too volatile (and risk-free rates too high).

Moreover, the cross-country correlation of stock prices is much greater than the correlation of fundamentals (for example, dividends). Colacito and Croce (2008) address this puzzle in a Bansal and Yaron (2004)'s long run risk model. They assume that the time-varying means of consumption growth rates are perfectly correlated across countries. Our paper offers a different interpretation. Here, the cross-country correlation of stock prices is much greater than the correlation of fundamentals because asset prices are driven by the disaster probability which is common across countries. Yet countries differ in terms of fundamentals, both in normal times and in times of disaster.

Common component and heterogenous loadings Lustig et al. (2009) show that a necessary condition for a model to replicate the empirical evidence on currency risk premia is the existence of a common component in stochastic discount factors across countries and some heterogeneity in the loadings on this common component. This necessary condition derives from the clear factor structure of the exchange rates in the currency portfolios above. Table 16 in the separate Appendix reports such a principal component decomposition. The first principal component is close to the mean of these exchange rate series; it corresponds to the dollar component of all the exchange rate series (expressed in foreign currency per US dollar). The second principal component is the most interesting: Lustig et al. (2009) show that it accounts for the cross-section of currency excess returns. High interest rate currencies offer high excess returns on average because they load more on this common component.

This paper Disasters are non-gaussian, large and negative jumps that happen rarely. By introducing disasters in an otherwise standard real business cycle framework, we thus modify the higher moments of the pricing kernels. Moreover, by assuming that the probability of disasters varies trough time, we allow for time-variation in these higher moments, following the call of Backus et al. (2001).

In this paper, disasters affect all countries at the same time. When the disaster probability increases, it does so for all countries. Hence, disasters create a source of global risk. They ensure that pricing kernels have a common component and are thus highly correlated, following the call of Brandt et al. (2006).

Countries differ, however, in how severely they are affected should a disaster occur. This introduces a source of heterogeneity across countries. The heterogeneity in terms of response to a common shock satisfies the necessary conditions uncovered by Lustig et al. (2009).

3 A two-country model without international trade

In this section, we consider a two-country, one-good international real business cycle model. Following recent studies (e.g. Alvarez, Atkeson and Kehoe (2009), Colacito and Croce (2008), and Verdelhan (2010)), we assume that asset markets are complete, but frictions in goods markets completely prevent trade in goods. Hence, in equilibrium, there is no risk-sharing. Intuitively, this can be viewed as the limiting case when trade costs go to infinity, or when home bias is very large. As a result, we can solve for the allocation of each country separately. We then define the exchange rate as the ratio of the foreign and domestic stochastic discount factors, i.e. the shadow relative value of the good in the two countries. The setup without trade is a natural starting point, because solving the model with trade and with recursive utility is difficult, and it is unlikely that the model with trade can match the data if the model without trade does not. For instance, the volatility of the exchange rate is necessarily lower in the model with trade.

3.1 Model Setup

The model follows Gourio (2009) who developed a closed-economy real business cycle model with time-varying risk of disaster. The reader is referred to that paper for more details and discussion of possible variations on the setup.

Domestic economy In the home country, a representative consumer maximizes a recursive utility function,

$$V_t = \left(\left(C_t^{\upsilon} (1 - N_t)^{1-\upsilon} \right)^{1-\gamma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}.$$

Here υ reflects the preference for consumption C_t as opposed to leisure $1-N_t$, γ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and θ measures risk aversion towards static gambles over the bundle. The risk aversion over consumption is $\upsilon\theta$ (Swanson (2009)).

There is a representative firm, which produces output using a standard Cobb-Douglas production function:

$$Y_t = K_t^\alpha \left(z_t N_t\right)^{1-\alpha},$$

where K_t is the capital stock and z_t denotes the total factor productivity (TFP), to be described below. The firm accumulates capital subject to adjustment costs:

$$K_{t+1} = \left((1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t\right)(1 - x_{t+1}b_k),$$

where ϕ is an increasing and concave function, whose curvature captures adjustment costs. The dummy variable x_{t+1} is 1 if a disaster hits at time $t+1$ (with probability p_t) and 0 otherwise (with probability $1-p_t$). The parameter b_k represents the capital destruction following a disaster.

The assumption that a disaster reduces the capital stock requires some discussion. The simplest interpretation is that disasters are wars which physically destroy capital, but there are alternative interpretations. For instance, b_k could reflect expropriation of capital holders (if the capital is taken away and then not used as effectively as before), or it could be a "technological revolution" that makes a large share of the capital worthless. It could also be that even though physical capital is not literally destroyed, some intangible capital (such as matches between firms, employees, and customers) is lost. Finally, one can imagine a situation where the demand for some types of goods falls sharply, rendering worthless the factories producing these goods. The key point is that this assumption is important to make capital risky. From this standpoint, it is a fairly sensible assumption.[3]

Since there is no trade, the ressource constraint is simply $C_t + I_t = Y_t$. Total factor productivity (TFP) follows a unit root process, and is affected by standard "small normal shocks" ε_{t+1} as well as disasters:

$$\log z_{t+1} = \log z_t + \mu + \sigma\varepsilon_{t+1} + x_{t+1}\log(1 - b_{tfp}),$$

where ε_{t+1} is *i.i.d.*, normally distributed with zero mean and unit variance $N(0,1)$, μ is the drift of TFP, σ is the standard deviation of gaussian shocks, and b_{tfp} is the reduction in TFP following

[3] One possibility is to make capital endogenously risky by assuming large adjustment costs; in this case a large negative shock to TFP reduces investment and hence makes the price of capital – marginal Q – fall significantly.

a disaster. For instance, if $b_{tfp} = .2$, then TFP falls by 20% following a disaster. The probability of disaster p_t follows an AR(1) in log:

$$\log(p_{t+1}) = \rho \log(p_t) + \mu_p + \sigma_p \varepsilon_{p,t+1},$$

where $\varepsilon_{p,t+1}$ is *i.i.d.* normally distributed with zero mean and unit variance $N(0,1)$.[4]

Foreign economy The two countries are perfectly symmetric in terms of preferences and technology, hence we have similar equations for the foreign country. We denote with a star the quantities and prices of the foreign country. Hence, the foreign country is characterized by the following set of equations that mirror the domestic economy:

$$\begin{aligned}
V_t^\star &= \left(\left(C_t^{\star\upsilon} (1 - N_t^\star)^{1-\upsilon} \right)^{1-\gamma} + \beta E_t \left(V_{t+1}^{\star 1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \\
Y_t^\star &= K_t^{\star\alpha} \left(z_t^\star N_t^\star \right)^{1-\alpha}, \\
K_{t+1}^\star &= \left((1-\delta) K_t^\star + \phi \left(\frac{I_t^\star}{K_t^\star} \right) K_t^\star \right) (1 - x_{t+1} b_k^\star), \\
\log z_{t+1}^\star &= \log z_t^\star + \mu + \sigma^\star \varepsilon_{t+1}^\star + x_{t+1} \log(1 - b_{tfp}^\star).
\end{aligned}$$

The disaster is perfectly correlated across the two countries: the same x_{t+1} (indicator of disaster realization), p_{t+1} (probability of disaster) and $\varepsilon_{p,t+1}$ (innovation to the log probability of disaster) apply to the two countries. The two countries differ only in their riskiness parameters b_k and b_{tfp}.[5] We think of this simple assumption as capturing the fact that countries have different risk, perhaps due to different industry compositions or different financial structures (Burnside and Tabova (2009)).

We allow the normally distributed shocks ε_t and ε_t^* to be contemporaneously correlated, con-

[4]The probability of disaster needs to lie between 0 and 1. The AR(1) specification does not ensure that constraint. When we solve our model, however, we approximate the AR(1) process with a Markov chain, whose support does lie between 0 and 1.

[5]An interesting extension of the model is to make the riskiness of each country, i.e. the parameters b_k, b_{tfp}, themselves stochastic. The identity of the more risky country would change over time, and shocks to b_k, b_{tfp} would also create additional dynamics.

sistent with our data. Compared to a standard IRBC model, our model adds a common source of shocks through the probability of disaster shock $\varepsilon_{p,t+1}$. Overall, the model has four shocks: the usual TFP shocks at home and abroad ε_{t+1} and ε^*_{t+1}, as well as the realization of disaster x_{t+1}, and the shock to the probability of disaster $\varepsilon_{p,t+1}$.

Pricing kernels In this model, the stochastic discount factor in the home country is

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\gamma)} \left(\frac{V_{t+1}}{E_t\left(V_{t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta},$$

and similarly in the foreign economy. We define the real exchange rate Q_t in units of domestic goods per foreign good, i.e. 1 foreign good $= Q_t$ home goods, so that a higher Q_t reflects a foreign appreciation and a home depreciation. As shown in the previous section, under complete markets the exchange rate must satisfy

$$\frac{Q_{t+1}}{Q_t} = \frac{M^\star_{t+1}}{M_{t+1}}. \tag{3.1}$$

We price three assets in each country. First, we consider zero-net-supply risk-free claims, which pay one unit of goods in each country with certainty next period. The price of this risk-free asset is $P^{rf}_t = E_t\left(M_{t+1}\right)$ in the home country and $P^{\star rf}_t = E_t\left(M^\star_{t+1}\right)$ in the foreign country, with corresponding yields $r^f_t = 1/P^{rf}_t - 1$ and $r^{\star f}_t = 1/P^{\star rf}_t - 1$.

Identifying a short-term government bond with this risk-free asset may be a reasonable assumption for safe countries, but is probably not realistic for risky countries. This leads us, following Barro (2006), to identify short-term government bonds as assets that are subject to default risk during disasters.[6] Assuming that the government bond pays off one unit of the goods if no disaster happens, and $\chi < 1$ unit if the disaster happens (i.e. χ is the recovery rate on government bonds), the price of this asset is thus $P^{gov}_t = E_t\left(M_{t,t+1}(1 - x_{t+1} + x_{t+1}\chi)\right)$ in the home country, and similarly in foreign; the corresponding yields are denoted r_t and $r^\star_t$.

Last, and following Abel (1999) among others, we consider an "equity" asset, which we define

[6]Empirically, default often takes the form of high rates of inflation, which reduce the real value of nominal government debt.

as an asset that pays out $D_t = Y_t^{\lambda}$ in the home country, and similarly in the foreign country. The parameter λ captures the financial and operating leverage of corporate firms.[7]

3.2 Calibration

The parameters that we use are listed in Table 2. The period is one quarter. A first group of parameters $(\alpha, \delta, \upsilon, \mu, \sigma, \eta)$ follow the real business cycle literature (Cooley and Prescott (1995)). The functional form for the adjustment cost function follows Jermann (1998): $\Phi(x) = a_1 \frac{x^{1-\eta}}{1-\eta} + a_2$, where a_1 and a_2 are set such that the steady-state is independent of η and the marginal Q is one in the steady-state.

A second group of parameters pertains to the modeling of disasters, and borrows from Barro (2006) and Gourio (2009). We assume that the probability of disaster is on average 1.7% per year, or 0.425% per quarter. We assume that the disaster size in the home country is $b_k = b_{tfp} = .43$, and is lower in the foreign country: $b_k = b_{tfp} = .35$.[8] Hence, we assume that the foreign country is the less risky one. The persistence of the log probability of disaster is 0.92, and the unconditional standard deviation is 1.85. These figures are picked to replicate approximately the mean and volatility of equity returns.

Following Abel (1999) and Barro (2006), the leverage parameters λ is set to 2. We set risk aversion θ equal to 6, and an intertemporal elasticity of substitution of consumption (IES) of 2.

Last, we assume that the correlation of TFP shocks ε_t and ε_t^* is 0.3, consistent with our data. On top of this benchmark calibration, we also present results from the model with no disasters (i.e. the basic RBC model), and the model where both countries are equally risky (i.e., $b_k = b_k^*$ and $b_{tfp} = b_{tfp}^*$), to illustrate the key mechanisms of the model.

Our calibration of disasters builds on the earlier work of Barro (2006) and Barro and Ursua

[7] The model-based definition of equity is the asset that pays out the model dividends $Y_t - w_t N_t - I_t$. Like in most business cycle models, however, these model dividends are significantly less volatile and procyclical than the dividends in the data. As a result, even though the model equity risk premium is positive, the price of a claim to dividends is not volatile enough in the model, compared to the data. This motivates our introduction of leverage, which is standard in the asset pricing literature (see for instance Bansal and Yaron (2004), Wachter (2008)).

[8] The Solow residual is $z^{1-\alpha}$, so the downward shift in the home production function during disasters is actually 31% in the home country and 25% in the foreign country.

(2008) who document – using panel data on consumption and output – that large declines in economic activity are fairly frequent. The recent crisis also illustrates some large declines in consumption or GDP: for instance, real consumption in Iceland was expected to drop by 7.1% in 2008 and 24.1% in 2009, according to the official government forecast of January 2009. According to the IMF World economic outlook published in April 2009, output in Germany, Ireland, Ukraine, Japan, Latvia, Singapore, Taiwan, were expected to contract by respectively 5.6%, 8.0%, 8.0%, 6.2%, 12.0%, 10.0%, 7.5% in 2009 *alone*. Second, it is also possible to change the calibration, by increasing risk aversion while reducing the size or probability of disasters; this yields nearly identical implications.

Our value for the IES is larger than the standard estimates of Hall (1988). However, recent empirical evidence suggests that higher values are empirically plausible (see Bansal and Yaron (2004), Guvenen (2006), Mulligan (2004), Vissing-Jørgensen (2002)). A high IES generates the sensible comparative statics that higher expected growth leads to higher asset prices, and higher uncertainty leads to lower asset prices. Finally, note that in the model, the correlation of consumption growth and the risk-free rate is low, and the standard Hall (1988) regressions are significantly biased towards zero.

3.3 Impulse response functions

Figure 1 presents the impulse response functions to an increase to the probability of disaster. To save space, we report in the separate appendix the impulse response functions to a standard TFP shock, and to a disaster realization. Within each country, an increase in disaster risk leads to a decline of investment, output and employment, and on impact an increase in consumption, as in Gourio (2009). As discussed in that paper, a shock to disaster risk is analytically equivalent (for quantities) to a preference shock: agents become more impatient and decide to invest less since the risk-adjusted return on capital is lower.[9] As a result, there is less incentive to produce and to

[9] This result relies on the intertemporal elasticity of substitution (IES) being greater than unity, so that agents save less when the (risk-adjusted) return is lower. If the IES is smaller than unity, the wealth effect prevails, and agents save more when the return is lower.

Table 2: Calibration Parameters

		Closed		Open	
Parameter	Symbol	Home	Foreign	Small	Large
Capital share	α	.34	.34	.34	.34
Depreciation rate	δ	.02	.02	.02	.02
Share of consumption in utility	υ	.30	.30	.30	.30
Discount factor	β	.994	.994	.994	.994
Adjustment cost curvature	η	.15	.15	.15	.15
Trend growth of TFP	μ	.0025	.0025	.0025	.0025
Standard deviation of ordinary TFP shock	σ	.01	.01	.01	.01
IES	$1/\gamma$	2	2	2	2
Risk aversion over consumption-leisure bundle	θ	6	6	6	6
Drop in TFP in case of disaster	b_{tfp}	.43	.35	.43	.35
Capital destruction in case of disaster	b_k	.43	.35	.43	.35
Foreign debt default in case of disaster	b_{debt}	-	-	.43	-
Recovery rate for domestic bonds in case of disaster	χ	.83	.86	-	-
Non-stochastic steady state of foreign asset	$\underline{b}$	-	-	-0.5	-
Trade cost	κ	-	-	2	-
Portfolio adjustment cost	χ	-	-	0.001	-
Persistence of log(p)	ρ_p	.92	.92	.92	.92
Unconditional std. dev. of log(p)	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.85	1.85	1.85	1.85
Leverage	λ	2	2	2	2

Notes: This table reports the parameters used in our simulations of closed economies ("Closed") and the small open economy model ("Open"). Home and foreign countries differ only with respect to the impact of a world shock on the stock of capital, debt and total factor productivity.

work. The two countries are affected by the same shock: its magnitude, however, is larger in the country where disaster risk is larger, leading to larger recessions. Equity prices drop because of an increase in the discount rate, and the risk-free rate falls as demand for precautionary savings rises. Last, the exchange rate of the more risky country appreciates. Since this country is consuming less, it must be that goods are more expensive there. This is a general feature of models with complete markets, which imply perfect risk-sharing: intuitively, a shock has no income effect but only a substitution effect.

3.4 Quantitative results

This section illustrates the effect of shocks to the world-wide disaster probability by comparing the implications for quantities, asset prices and exchange rates of three models: (i) a standard real business cycle model with only TFP shocks; (ii) a model with time-varying risk of disaster, when both countries are equally risky; (iii) our benchmark model with time-varying risk of disaster, when the home country is more risky.[10]

The separate appendix details our numerical solution method. Given our interest in time-varying risk premia, we cannot log-linearize the model, and we resort to a fairly standard discrete dynamic programming approach.

Quantities Panel II of Table 3 reports the quantity implications of the different models as well as the data. The first row reports the results for the standard RBC model (i.e. no disaster risk). Since there is no trade and countries are perfectly symmetric, the quantity dynamics are exactly the same in the two countries. The quantity patterns within each country reflect the usual RBC results: consumption is smoother than GDP, investment is more volatile than GDP, and employment is volatile, but less than in the data. The cross-country correlation of consumption, employment, investment and output are all equal to the assumed cross-correlation of TFP shocks, i.e. 0.3, since

[10]Note that the model with positive, but constant probability of disaster, has very similar implications to the model with only TFP shocks. The only differences regard the mean returns on equity, on the risk-free asset, and on the carry trade: the equity premium is higher, the mean risk-free rate is lower, and the mean carry trade excess return is larger. All volatilities and correlations, however, are exactly as in the RBC model, since TFP shocks are the only source of fluctuations (at least in samples without disasters).

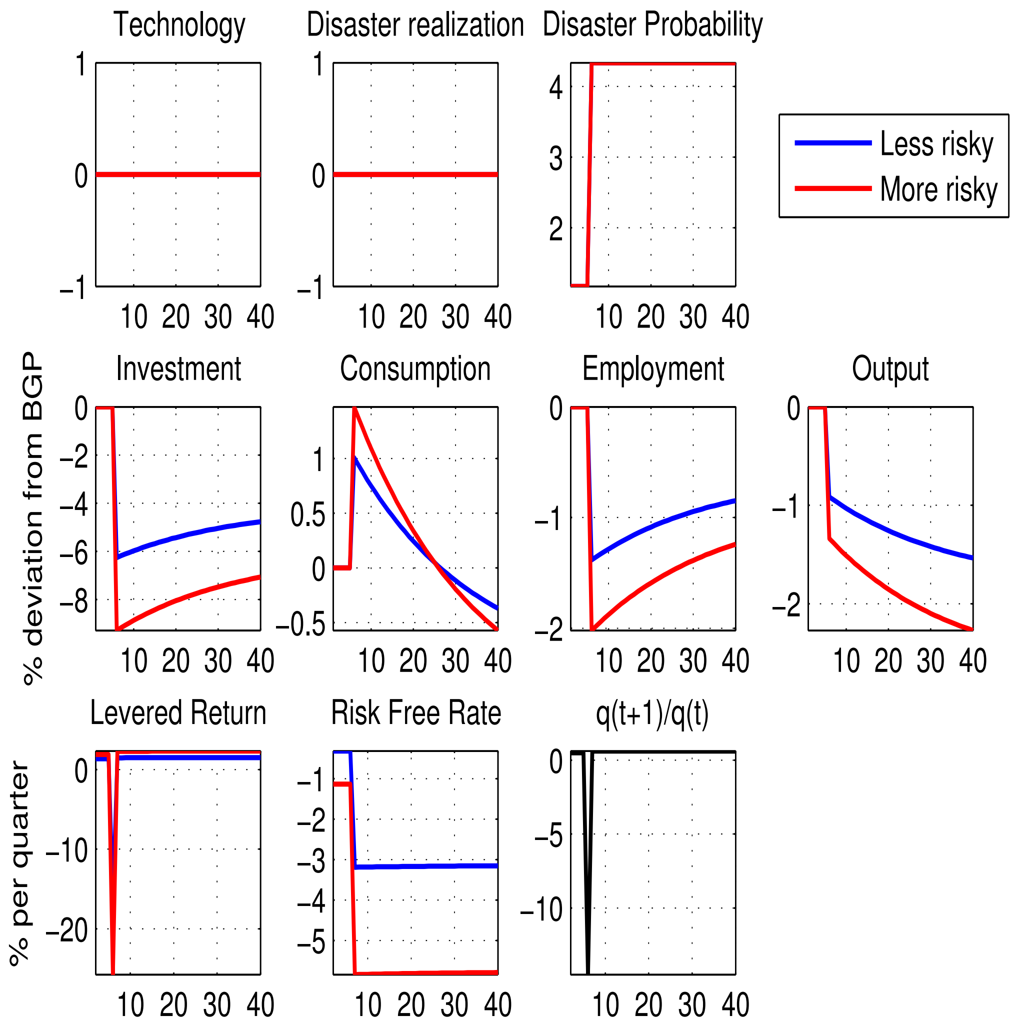

Figure 1: **Impulse Response Functions – Closed Economy – Disaster Probability Shock**: This figure presents the impulse response functions of different macroeconomic and financial variables to an increase to the probability of disaster.

there is no endogenous interaction between the two countries and TFP shocks are the only source of fluctuations.

Turning to the model with both TFP shocks and disaster risk shocks, we see that when both countries are equally risky (row 2), the additional shock raises the volatility of all series, but especially investment and employment. The risk shock is common across countries, hence it increases the cross-country correlation of quantities, especially investment and employment. Our model, however, still implies that the cross-country correlation of consumption should be greater than that of output, a puzzle noted at least since Backus et al. (1992). Finally, when one country is less risky (row 3), the same mechanism applies, but to a lower extent for the less risky country.

All the moments are calculated by simulating the model in samples without disasters; see below for a discussion of the effect of calculating them in samples with disasters. We compare these simulated moments to their counterparts in the data. We consider two comparison sets. The first set focuses on the US versus the rest of the world. For the domestic economy, we report statistics on the US. For the foreign economy, we report averages obtained over all OECD countries, excluding the US. This set of moments is standard, but it assumes that the US is always the riskier economy (with the higher share of capital destroyed in case of disaster) and thus a low interest rate country. This description fits well the US economy during the recent mortgage crisis, which originated in and affected most the US banking sector. Yet, it seems far-fetched to always equate the US to the riskier economy. We thus consider a second set of moments that focus on the high versus low interest rate countries. To do so, we sort countries on their interest rate levels and build 5 portfolios as in Table 1. For the domestic economy, we report statistics for countries in the first portfolio (e.g, low interest rate countries). For the foreign economy, we report statistics for countries in the last portfolio (e.g, high interest rate countries).

Asset prices Table 4 reports the mean and volatility of the risk-free rate and the equity return in both countries, as well as the cross-country correlation of risk-free rates and equity returns. Without disaster risk, and given our low risk aversion, the model predicts tiny risk premia within each country, and equity returns are not volatile enough. Furthermore, the correlation of returns equals the correlation of TFP shocks. With high and time-varying risk of disaster, the model predicts the right order of magnitude for the mean and volatility of both equity and risk-free returns. As discussed in Gourio (2009), the model can also replicate the within-country correlation of quantities (such as output or investment) and asset prices or expected returns. Moreover, returns are more correlated across countries since they are all driven by the common fear of

Table 3: Business Cycle Statistics — Closed Economies

	Standard Deviations								Cross-country Correlations			
	$\sigma(\Delta c)$	$\sigma(\Delta i)$	$\sigma(\Delta n)$	$\sigma(\Delta y)$	$\sigma(\Delta c^\star)$	$\sigma(\Delta i^\star)$	$\sigma(\Delta n^\star)$	$\sigma(\Delta y^\star)$	$(\Delta c, \Delta c^\star)$	$(\Delta i, \Delta i^\star)$	$(\Delta n, \Delta n^\star)$	$(\Delta y, \Delta y^\star)$
Panel I: Data												
US vs ROW	0.68	2.52	0.64	0.85	1.18	4.93	0.73	1.23	0.16	0.23	0.24	0.28
Low vs High	0.61	3.31	0.44	0.72	1.14	4.03	0.87	1.05	0.22	-0.01	0.16	0.31
Panel II: Model												
RBC	0.53	1.49	0.19	0.78	0.53	1.49	0.19	0.78	0.30	0.30	0.30	0.30
Disaster	0.62	2.63	0.47	0.83	0.62	2.63	0.47	0.83	0.49	0.77	0.89	0.38
Benchmark	0.62	2.61	0.47	0.83	0.57	2.05	0.35	0.81	0.43	0.68	0.83	0.35

Notes: This table reports the standard deviations of log differences in consumption, investment, labor and output, along with the cross-country correlation of these variables. Panel I reports moments from the actual data. We use series from the OECD database, available on Datastream. Data are quarterly. The maximum sample period is 1970.I–2009.IV, but sample windows vary across countries. Additional information is available in Appendix A. The first line focuses on the US versus the rest of the world. For the domestic economy, we report statistics on the US. For the foreign economy, we report averages obtained over all OECD countries, excluding the US. The second line focuses on high versus low interest rate countries. We sort countries on their interest rate levels and build 5 portfolios as in Table 1. For the domestic economy, we report statistics for countries in the first portfolio (low interest rate countries). For the foreign economy, we report statistics for countries in the last portfolio (high interest rate countries). Panel II is constructed by simulating the model, assuming no disasters are actually realized (see the appendix for simulations which include disaster realizations). We consider three variants of the model: (1) a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky; (3) our benchmark model with time-varying risk of disaster, where the domestic country is more risky.

Table 4: Asset Prices — Closed Economies

	Averages				Standard Deviations				Cross-country Corr.	
	$E(r^e)$	$E(r^f)$	$E(r^{e,\star})$	$E(r^{f,\star})$	$\sigma(r^e)$	$\sigma(r^f)$	$\sigma(r^{e,\star})$	$\sigma(r^{f,\star})$	$(r^e, r^{e,\star})$	$(r^f, r^{f,\star})$
					Panel I: Data					
US vs ROW	1.17	0.32	1.27	0.88	7.87	1.14	12.78	2.08	0.54	0.29
Low vs High	1.20	0.18	1.73	1.35	7.70	0.59	12.85	1.57	0.72	0.47
					Panel II: Model					
RBC	0.84	0.81	0.84	0.81	1.63	0.05	1.63	0.05	0.30	0.30
Disaster	1.93	0.16	1.93	0.16	7.75	1.68	7.75	1.68	0.97	1.00
Benchmark	1.93	0.17	1.38	0.43	7.69	1.65	4.64	1.04	0.93	1.00

Notes: This table reports the averages and standard deviations of log equity returns and log risk-free rates, along with the cross-country correlation of these variables. Panel I reports moments from the actual data. We use series from the IMF and MSCI databases, available on Datastream. Data are quarterly. The maximum sample period is 1970.I–2009.IV, but sample windows vary across countries. Additional information is available in Appendix A. The first line focuses on the US versus the rest of the world. For the domestic economy, we report statistics on the US. For the foreign economy, we report averages obtained over all OECD countries, excluding the US. The second line focuses on high versus low interest rate countries. We sort countries on their interest rate levels and build 5 portfolios as in Table 1. For the domestic economy, we report statistics for countries in the first portfolio (low interest rate countries). For the foreign economy, we report statistics for countries in the last portfolio (high interest rate countries). Panel II is constructed by simulating the model, assuming no disasters are actually realized (see the appendix for simulations which include disaster realizations). We consider three variants of the model: (1) a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky; (3) our benchmark model with time-varying risk of disaster, where the domestic country is more risky.

disaster. Because risk has a powerful effect on returns, their correlation is significantly higher than the correlation of "fundamentals" such as output or investment. This rationalizes an observation which is commonly viewed as puzzling – the "excess comovement" of stock markets.

Exchange rates and carry trade returns Table 5 shows how the exchange rate behaves in our model. Even in the model with only TFP shocks, the exchange rate has a significant volatility, since consumption growth rates are only weakly correlated. Adding disaster risk does not change this result if the two countries have the same exposure to disaster risk, since it is a common shock. By definition, the exchange rate is not affected by shocks that affect both countries' marginal utilities equally. When the risk exposures of the two countries are different, the exchange rate volatility rises, because of the additional shock that affects the countries' marginal utilities differently, and this volatility becomes close to the one in the data.

Turning to the correlation of exchange rates with macroeconomic aggregates or financial prices, we see that in the basic RBC model the exchange rate of a country appreciates when its output or consumption goes down, or when its equity return goes down. In particular, as noted since Backus and Smith (1993) and Kollmann (1995), the correlation between changes in exchange rates and relative consumption growth rate equals one.[11] This result holds exactly with power utility; while we have Epstein-Zin utility, the basic disaster risk model still generates a very strong correlation of 0.9967 when there are only TFP shocks.

Our benchmark model, in contrast, leads to a weaker correlation of exchange rates with output growth (0.61) and especially consumption growth (0.39). This is because shocks to disaster probability generate a negative correlation between the country's exchange rate and its consumption growth. From a technical standpoint, the delinking between consumption and marginal utility is created by recursive preferences, i.e. a preference for early resolution of uncertainty, combined with shocks to uncertainty. The model, however, still implies a too strong correlation of equity returns and exchange rates.

We now turn to the riskiness of exchange rates, a key motivation of our study. Empirically, a "carry trade" strategy (borrowing in the low interest rate country, and lending in the high interest rate country, taking on the exchange rate risk) generates significant averate excess returns. This strategy generates a log excess return equal to:

$$r^e_{t+1} = r^\star_t - r_t + \Delta q_{t+1},$$

[11] Backus and Smith (1993) note that in complete markets and with power utility, the change in the real exchange rate is equal to the relative consumption growth in two countries times the risk-aversion coefficient, thus implying a perfect correlation between the consumption growth and real exchange rate variations. Yet, in the data, Backus and Smith (1993) find that the actual correlation between exchange rate changes and consumption growth rates is low and often negative. Chari, Kehoe and McGrattan (2002), Corsetti, Dedola and Leduc (2008) and Benigno and Thoenissen (2008) confirm their findings.

Table 5: Real Exchange Rates — Closed Economies

	$E(\Delta q)$	$\sigma(\Delta q)$	$(\Delta q, r^e - r^{e,\star})$	$(\Delta q, r^f - r^{f,\star})$	$(\Delta q, \Delta c - \Delta c^\star)$	$(\Delta q, \Delta y - \Delta y^\star)$
				Panel I: Data		
US vs ROW	-0.26	5.95	-0.15	-0.01	0.07	0.03
Low vs High	2.65	4.90	-0.01	-0.08	0.17	-0.04
				Panel II: Model		
RBC	0.03	2.33	1.00	.02	1.00	1.00
Disaster	0.03	2.31	1.00	.02	1.00	1.00
Benchmark	0.53	4.33	.98	-.24	.39	.61

Notes: This table reports the averages and standard deviations of changes in log real exchange rates, along with the cross-country correlation of changes in log real exchange rates with cross-country differences in real equity returns, real risk-free rates, real consumption growth and real output growth. Panel I reports moments from the actual data. We use series from the IMF, OECD and MSCI databases, available on Datastream. Data are quarterly. The maximum sample period is 1970.I–2009.IV, but sample windows vary across countries. Additional information is available in Appendix A. The first line focuses on the US versus the rest of the world. For the domestic economy, we report statistics on the US. For the foreign economy, we report averages obtained over all OECD countries, excluding the US. The second line focuses on high versus low interest rate countries. We sort countries on their interest rate levels and build 5 portfolios as in Table 1. For the domestic economy, we report statistics for countries in the first portfolio (low interest rate countries). For the foreign economy, we report statistics for countries in the last portfolio (high interest rate countries). Panel II uses simulated series. We consider three models: (1) a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky; (3) our benchmark model with time-varying risk of disaster, where the domestic country is more risky.

where a $\star$ denotes the high interest rate country. The return to this carry trade strategy can be calculated using either risk-free assets, or government bonds, which are exposed to disaster risk. In our model, the high interest rate country is the country with the lowest fundamental risk – the lowest b: precautionary savings are small, hence the interest rate is high. If a disaster occurs, this country's currency depreciate, since its marginal utility falls by a smaller amount. Table 6 reports the mean and volatility of this currency excess return, as well as the risk of this strategy, which according to the consumption CAPM, can be measured as the covariance with consumption growth, or according to the CAPM, as the covariance with equity returns.

In the standard RBC model, the carry trade strategy offers a small positive excess return, because a domestic consumer investing abroad receives a higher than expected return when her home currency depreciates, i.e. in good times when her marginal utility is low. The carry trade average excess return is a risk premium for taking on more TFP shocks. This risk premium is small, because risk aversion is low and shocks are small. Note the relatively high correlation of the carry trade return with domestic consumption growth: the consumption CAPM provides a good explanation for the expected return on the carry trade. (This correlation is significantly less than one because the exchange rate is also driven by some foreign shocks which have a low correlation with domestic consumption growth shocks.)

With disaster risk and equal exposures to disaster risk, the exchange rate does not respond to disasters or disaster risk, and hence the carry trade return is the same as in the model with only TFP shocks.

Last, in our benchmark model, with heterogeneous exposures to disaster risk, the carry trade generates a significant excess return, which is not fully captured by its correlation with consumption growth. Carry trades pay off badly both when disasters hit and when disaster risks rise. Because on impact, increases in disaster risk coincide with increases in consumption, the consumption CAPM mismeasures the risk of the carry trade. Note, however, that the market CAPM does not suffer from this measurement problem: the carry trade return is strongly correlated with the market return.

Finally, we note that our model reproduces the negative slope in the traditional uncovered interest rate parity tests:

$$\Delta q_{t+1} = \alpha_{UIP} + \beta_{UIP} \left(r_t - r_t^* \right) + \varepsilon_{t+1},$$

where we find a slope close to -1.5, consistent with the data. In contrast, the RBC model, where risk-neutrality holds almost perfectly, generates a slope of 1. This success is due to variations in the probability of disaster, which is the main variable driving the interest rate differential: when p is high, the risk-free rate of the more risky country is lower, due to heightened precautionary savings, and the exchange rate risk premium is high. As a result, the foreign currency tends to appreciate to compensate for the higher risk. Note that we reproduce the forward discount puzzle in an economy without frictions. Additional frictions (like information heterogeneity or infrequent portfolio rebalancing) could strengthen deviations from UIP and add interesting dynamics to exchange rates (see Bacchetta and van Wincoop (2010) and Bacchetta and van Wincoop (2006) for the impact of such frictions on exchange rates).

An interesting implication of our model, which is likely to be shared by many models which replicate the carry trade excess return, is that the exchange rate is somewhat forecastable. In particular, $E_t \left(\frac{Q_{t+1}}{Q_t} \right) > 1$, i.e. the less risky country's exchange rate tend to appreciate on average. This follows because the exchange rate of the country that is relatively less affected by disasters turns out to be the *most* risky currency: it depreciates during disasters or when the probability of disaster rises. In practice however, this mean appreciation may be hard to tease apart from ongoing shocks, and especially if the identify of the more risky country changes over time.

Conditional correlations While unconditional correlations between carry trade returns or exchange rates and macroeconomic aggregates or financial returns are usually small, there is compelling evidence that exchange rates become highly correlated with macroeconomic aggregates or financial returns in times of crises. Lustig and Verdelhan (2008) document that the correlation of carry trade and US stock market returns is low on average but increases sharply by 50 to 90 basis points during financial crisis and recessions in the post-Bretton Woods sample. For exam-

Table 6: Carry Trade Excess Returns — Closed Economies

	$E(rx_{t+1})$	$\sigma(rx_{t+1})$	$(rx_{t+1}, \Delta c)$	$(rx_{t+1}, \Delta c^{\star})$	(rx_{t+1}, r^{m}_{t+1})	$(rx_{t+1}, r^{m\star}_{t+1})$	β_{UIP}
				Panel I: Data			
Low vs High	1.62	4.78	-0.01	0.19	-0.00	-0.02	-0.30
				Panel II: Model			
RBC	0.03	2.32	0.59	-0.59	0.59	-0.59	1.00
Disaster	0.03	2.30	0.50	-0.50	0.12	-0.13	0.97
Benchmark	0.70	4.50	-0.18	-0.60	0.89	0.69	-1.47

Notes: This table reports the averages and standard deviations of carry trade excess returns, along with the cross-country correlation of these excess returns with real consumption growth and real stock market returns. The last column reports the UIP slope coefficient. Panel I reports moments from the actual data. We use series from the IMF, OECD and MSCI databases, available on Datastream. Data are quarterly. The maximum sample period is 1970.I–2009.IV, but sample windows vary across countries. Additional information is available in Appendix A. We focuse on high versus low interest rate countries. We sort countries on their interest rate levels and build 5 portfolios as in Table 1. For the domestic economy, we report statistics for countries in the first portfolio (low interest rate countries). For the foreign economy, we report statistics for countries in the last portfolio (high interest rate countries). Panel II uses simulated series. We consider three models: (1) a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky; (3) our benchmark model with time-varying risk of disaster, where the domestic country is more risky.

ple, this correlation increased to 0.65 during the US subprime mortgage crisis (July 2007 to June 2009). This section shows that our model reproduces these time-varying correlations. A simple way to capture periods of crises, both in the model and in the data, is to look at subsamples where volatility is high or low.[12]

Table 7 reports statistics by splitting the sample into the periods when volatility is higher than the 75th percentile or below the 25th percentile. When volatility is high, risk is high, the carry trade return is more volatile (6.99 vs. 2.40), and is more correlated with equity returns (0.95 vs. 0.62 when volatility is low), and the exchange rate becomes more strongly correlated with the stock market. The carry trade mean return should be higher when volatility is higher: as risk is higher, risk premia should be higher (1.46 vs. 0.42). Finally, the correlation of the exchange rate with output turns from negative to positive when volatility is high.

Quantity risk versus currency risk It is important to recognize that risk has different meanings in terms of quantities or currencies. The more risky countries in terms of their domestic fundamentals such as consumption, output, or stock prices, are less risky in terms of their exchange rate. This is a direct implication of the definition of exchange rates in complete markets (cf equation 3.1). This definition implies that the exchange rate appreciates when a country has a particularly low marginal utility. Hence, the funding currencies (with low interest rates) of carry trade returns are the currencies with high domestic risk (and thus large precautionary savings). The high-interest rate currencies, which are risky from the perspective of the carry trade since they depreciate in bad times, actually are the currencies of the less risky countries in terms of quantities.

This difference between quantity and currency risk extends to equity excess returns. As Verdelhan (2010), we report in Table 1 that equity excess returns are indeed lower in the high interest rate countries.

[12] In the data, the VIX index is a commonly used measure of volatility and times of crisis. VIX is the Chicago Board of Exchange implied volatility measure, which is calculated based on exchange-traded options on the S&P500 stock market index. In our model, we compute the conditional standard deviation of the market return, under the risk-neutral measure. It does not, however, make a large difference to our results if we use the conditional standard deviation of market returns (under the physical measure, i.e. the return volatility).

Table 7: High vs Low Volatility — Closed Economies

	High Vol.		Low Vol.	
	(rx_{t+1}, r^e_{t+1})	$(rx_{t+1}, r^{e\star}_{t+1})$	(rx_{t+1}, r^e_{t+1})	$(rx_{t+1}, r^{e\star}_{t+1})$
RBC	0.61	-0.61	0.61	-0.61
Disaster	0.07	-0.07	0.46	-0.46
Benchmark	0.95	0.87	0.62	-0.38
	$(rx_{t+1}, \Delta \log c)$	$(rx_{t+1}, \Delta \log c^\star)$	$(rx_{t+1}, \Delta \log c)$	$(rx_{t+1}, \Delta \log c^\star)$
RBC	0.60	-0.60	0.61	-0.61
Disaster	0.40	-0.40	0.59	-0.59
Benchmark	-0.56	-0.72	0.54	-0.57
	$(rx_{t+1}, \Delta \log y)$	$(rx_{t+1}, \Delta \log y^\star)$	$(rx_{t+1}, \Delta \log y)$	$(rx_{t+1}, \Delta \log y^\star)$
RBC	0.03	2.18	0.02	2.15
Disaster	0.02	2.30	0.02	2.28
Benchmark	1.87	7.31	0.43	2.40
	$E(rx_{t+1})$	$\sigma(rx_{t+1})$	$E(rx_{t+1})$	$\sigma(rx_{t+1})$
RBC	0.03	2.18	0.02	2.15
Disaster	0.02	2.30	0.02	2.28
Benchmark	1.47	7.03	0.42	2.40
	$(\Delta q, \Delta \log y)$	$(\Delta q, \Delta \log y^\star)$	$(\Delta q, \Delta \log y)$	$(\Delta q, \Delta \log y^\star)$
RBC	-0.61	0.61	-0.61	0.61
Disaster	-0.50	0.50	-0.59	0.59
Benchmark	0.19	0.69	-0.55	0.58
	$(\Delta q, r^e_{t+1})$	$(\Delta q, r^{e,\star}_{t+1})$	$(\Delta q, r^e_{t+1})$	$(\Delta q, r^{e,\star}_{t+1})$
RBC	0.61	-0.61	0.61	-0.61
Disaster	0.07	-0.07	0.46	-0.46
Benchmark	0.94	0.87	0.62	-0.38

Notes: This table first reports the correlation of carry trade excess returns (rx_{t+1}) with real – domestic or foreign – stock market returns (r^e_{t+1}, first panel), consumption growth ($\Delta \log c$, second panel), output growth ($\Delta \log y$, third panel) when volatility is high (left panel) or low (right panel). This table then reports the mean and standard deviation of carry trade excess returns (r^e_{t+1}, fourth panel), again when volatility is high (left panel) or low (right panel). Finally, this table reports the correlation of changes in real exchange rates (Δq) with output growth and real stock market returns (fifth and sixth panels). We consider three models: (1) a standard real business cycle model with only TFP shocks; (2) a model with time-varying risk of disaster, when both countries are equally risky; (3) our benchmark model with time-varying risk of disaster, where the domestic country is more risky.

3.5 Robustness

In this section, we discuss several extensions of our model.

Samples including disasters We report in the appendix the same statistics discussed in this section, but calculated in long samples which include disaster realizations. Computing the statistics in that way does not affect our main results. Disaster realizations create additional volatility in macroeconomic quantities (except for employment which is unaffected by disasters in our calibration), and because disasters are worldwide they significantly increase the comovement of quantities. The mean return on equity is reduced by about 30bp per quarter, or 1.2% per year, in the more risky country, but the equity premium remains about 5% per year: sample selection does not drive our results. Similarly, the mean excess carry trade return is reduced slightly from 70bp to 59bp per quarter. The equity return is more volatile (8.67% per quarter vs. 7.69% per quarter in samples without disasters), and is the exchange rate is slightly more volatile as well, since it moves sharply when a disaster occurs. Finally, the correlation of the carry trade return with consumption in the home country turn from negative (-0.18) to positive (0.27), reflecting better the underlying risk of the carry trade.

Small sample bias Our statistics are computed by averaging across 100 very long samples (50,000 periods each). The motivation for running simulations with very long samples is that one important statistic that we report, the slope of the standard UIP regression, appears to be affected by an important small sample bias. If one simulates samples of length 200 or 300 periods, all of our statistics are the same, except for this regression slope, which is -2.31 in the RBC model and -2.52 in our benchmark. In contrast, in long samples the slope is 1.00 the RBC model and -1.47 in our benchmark.[13]

Recursive utility vs. expected utility Recursive utility allows for a clear separation between the coefficient of relative risk aversion (θ) and the elasticity of intertemporal substitution (IES

[13] Alvarez et al. (2009) also report an important small sample bias.

$1/\sigma$). This section discusses how our results are affected if we change either the risk aversion coefficient, or the IES, to make them equal. In this case the model reverts back to the familiar case of expected utility preferences.

In our model, risk aversion determines both the magnitude of risk premia and the size of the response of quantities to a disaster risk shock (since the importance of disaster risk depends on risk aversion). Hence, solving the model with expected utility, with same IES but a lower risk aversion ($\theta = .5$ so that $\theta = \sigma$) yields the same qualitative effects, but they are much reduced in size. For instance, the equity premium is 47bp per quarter (176bp in the benchmark), and the cross-correlation of investment is 0.51 (0.68 in the benchmark). One possibility would be to adjust the calibration by increasing disaster risk to make the effects bigger.

In contrast, changing the IES leads to a qualitative change in behavior of the model. If the IES is less than unity, the model implies that an increase in disaster risk leads to a boom of investment, employment and output. Besides this counterintuitive effect, the model also does not generate volatile exchange rates, the average carry trade return, the failure of UIP or the Backus-Smith anomaly, since increase in the probability of disaster now lead consumption to go down, and hence to be correlated with the exchange rate. We conclude that an IES above unity is critical for our results.

Idiosyncratic disaster risk We focus in the paper on worldwide disasters, which affect all countries simultaneously; as we discussed in section 2, this is required to account for the documented properties of the carry trade. Of course, there are also "idiosyncratic disaster", i.e. large negative shocks which occur in only one country. In our model, if a country has a large disaster, its output, consumption, investment, and stock prices drop sharply, while the interest rate and employment are unaffected, and the exchange rate appreciates (assuming complete markets). An increase in disaster risk, on the other hand, would lead to a decline in investment, employment, output, stock prices, and the interest rate, and an increase in consumption and the exchange rate on impact. If capital was mobile across countries, capital would flow out of the now-more risky country.

Different dynamics of disasters In our calibration, we use a very simple modeling of disasters: they are instantaneous, permanent, and the capital destruction equals the TFP destruction. However, one may want to model disasters as graduals, or transitory, and one may want to assume that capital destruction or TFP destruction plays a more prominent role. The reader is refereed to Gourio (2009) for a discussion of these different assumptions.

4 Endogenous capital flows: a small open economy model

The previous section shows that a parsimonious model that combines (1) time-varying world disaster risk, (2) heterogeneous exposures to disaster risk, and (3) recursive utility can account for a large number of stylized facts about macroeconomic quantities, asset returns, and exchange rates. This model, however, assumes that no exchange of goods actually takes place, so that countries are unable to share risk, smooth consumption, or increase output through trade. In this section, we consider a small open economy model with international trade and capital flows. We thus merge our earlier approach with the standard international real business cycle literature (Backus et al. (1992); Baxter (1995), Baxter and Crucini (1995)). It allows us to ascertain the effect of shocks to disaster risk on the current account and capital flows. Moreover, quantities, asset prices and exchange rates now reflect the potential capital reallocation across countries.

Our approach, following a large literature in finance, relies on the preference for early resolution of uncertainty introduced by recursive preferences. It turns out that solving a two-country model with recursive preferences and production is technically quite challenging. As a result, in this section we focus on a limiting case: one country is assumed to be so large that its interaction with the rest of the world is negligible, while the other country is small and takes prices as given. This is essentially a standard “small open economy” assumption.[14]

[14]Previously, we solved a two-country model with expected utility preferences. However, since a shock to the probability of disaster leads to an initial increase in consumption on impact, states with high probability of disaster are “good states” which carry a negative risk premium. This makes it difficult to match the data.

4.1 Model Setup

We assume that there is a large economy and a small economy. From the standpoint of the large economy, trade and capital flows are negligible, hence we solve the allocation in that country assuming that it is closed, as in the previous section. We then solve the problem of the small economy, which takes the world interest rate as given. The small economy can lend or borrow in world financial markets, using a standard debt contract. Yet, it might default during disasters. Following Samuelson (1954), we introduce trade costs that generate a time-varing wedge between the value of domestic goods and foreign goods, i.e. a volatile real exchange rate.[15]

4.1.1 The Large Economy

As in the previous section, the equilibrium,of the large economy is characterized by the following system of equations that describes preferences, the production function, the law of motion of physical capital, the feasibility constraint, and the law of motion of productivity and disaster probability:

$$
\begin{aligned}
V_t &= \left(\left(C_t^{\upsilon}(1-N_t)^{1-\upsilon}\right)^{1-\gamma} + \beta E_t \left(V_{t+1}^{1-\theta}\right)^{\frac{1-\gamma}{1-\theta}}\right)^{\frac{1}{1-\gamma}}, \\
Y_t &= K_t^{\alpha}\left(z_t N_t\right)^{1-\alpha}, \\
K_{t+1} &= \left((1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t\right)(1-x_{t+1}b_k), \\
Y_t &= C_t + I_t, \\
\log z_{t+1} &= \log z_t + \mu + \sigma\varepsilon_{t+1} + x_{t+1}\log(1-b_{tfp}), \\
\log p_{t+1} &= \rho \log p_t + \mu_p + \varepsilon_{p,t+1}.
\end{aligned}
$$

This equilibrium can be represented recursively in terms of two state variables, the detrended

[15] Proportional (iceberg-like) shipping costs were first proposed by Samuelson (1954), and then used among many others by Dumas (1992), Sercu, Uppal and Hulle (1995), Sercu and Uppal (2003), Obstfeld and Rogoff (2000), and Fitzgerald (2008) to study real exchange rates. None of these papers tackle the equity and forward premium puzzles. Hollifield and Uppal (1997) show that proportional trade costs are not enough to reproduce the forward premium puzzle when agents are characterized by power utility, not even at extreme levels of constant risk aversion or high trading costs.

capital $k_t = K_t/z_t$ and the probability of disaster p_t. Hence, we can solve for all quantities as functions of these state variables, e.g. $c(k_t, p_t) = C_t/z_t$, $i(k_t, p_t) = I_t/z_t$, etc., and similarly we can calculate asset prices, given the stochastic discount factor $M_{t+1} = M(k_t, p_t, \varepsilon_{t+1}, x_{t+1}, p_{t+1})$.

4.1.2 The Small Open Economy

We denote the prices and quantities of the small open economy with a star ($\star$). As in Section 3, the small open economy (SOE) and the large economy (LE) differ in their riskiness parameter b_k, b_{tfp}. As before, we assume that the disaster is a worldwide event, thus perfectly correlated across countries. The only foreign asset accessible to the small economy is a defaultable bond B_t. When B_t is positive, the SOE has positive net foreign assets. from abroad. This asset pays out goods in the SOE (i.e. it is denominated in its own currency), so that it does not expose the SOE to exchange rate risk premia.[16] We assume that the SOE defaults by an amount b_{debt} following a disaster. We will consider different possible values for b_{debt}.

Shipping costs To model exchange rates, we introduce convex shipping costs. We assume that the cost of trading a net flow of goods z from the small open economy to the large economy is given by $\kappa\left(\frac{z}{Y_t^\star}\right) Y_t^\star$, where $Y_t^\star$ is the GDP of the small open economy, and the function κ captures trade costs. With this formulation, shipping costs are homogeneous of degree one in $(z, Y_t^\star)$. Thus they remain equal to a constant fraction of GDP as the economy grows. We assume that κ is a convex function, satisfying $\kappa(0) = 0$, $\kappa''(s) > 0$ for all s, $\kappa'(s) < 0$ for $s < 0$, and $\kappa'(s) > 0$ for $s > 0$. In our simulations, we focus on the following simple quadratic example: $\kappa(s) = \frac{\kappa}{2}s^2$. We hence depart from the standard iceberg assumption – this assumption implies that, conditional on countries trading, the real exchange rate is constant. The convexity we introduce may be driven by the underlying heterogeneity of goods traded: countries first trade the goods with the lowest shipping costs (or the largest gains to trade), and then switch to goods with higher shipping costs.

[16] Alternatively, we could assume that the defaultable asset is labeled in the foreign currency instead of the home currency. This would modify its risk properties by changing the hedging possibilities that are available to the small open economy. More generally, it would be interesting to solve this small open economy model assuming complete markets. It seems difficult, however, to solve that model numerically.

This smooth shipping cost function leads to a simple relation between net exports over GDP and the real exchange rate. Consider the problem of a trader who buys goods in the small open economy, and sell them in the large economy. The real exchange rate Q_t is defined in terms of goods of the large economy per good of the small economy, i.e. 1 SOE good is worth Q_t LE goods. The shipping company's profit for bringing x goods from the SOE to the LE is equal to:

$$x\left(\frac{1}{Q_t}-1\right)-\kappa\left(\frac{x}{Y_t^\star}\right)Y_t^\star.$$

The first term is the revenue received: in the large economy, one good is worth $\frac{1}{Q_t}$ SOE good. The second term is the trade cost. In this expression, each trader takes total output $Y_t^\star$ as given. The first order condition with respect to x yields:

$$\frac{1}{Q_t}=1+\kappa'\left(\frac{x}{Y_t^\star}\right).$$

This expression must hold for the equilibrium net exports $x = NX_t^\star$. A higher value of the real exchange rate Q_t corresponds to an appreciation of the SOE's currency. This appreciation is associated with a decrease of net exports out of the SOE, as in the data.

Maximization problem The small open economy's problem is to maximize the utility of the representative agent by choosing consumption, employment, investment in home capital, and investment in foreign asset (or debt). Mathematically, the equations describing preferences, production, capital accumulation and the evolution of TFP are as in the previous section:

$$\begin{aligned}
V_t^\star &= \left((C_t^{\star\upsilon}(1-N_t^\star)^{1-\upsilon})^{1-\gamma}+\beta E_t\left(V_{t+1}^{\star 1-\theta}\right)^{\frac{1-\gamma}{1-\theta}}\right)^{\frac{1}{1-\gamma}},\\
Y_t^\star &= K_t^{\star\alpha}\left(z_t^\star N_t^\star\right)^{1-\alpha},\\
\log z_{t+1}^\star &= \log z_{t+1}^\star+\mu+\sigma\varepsilon_{t+1}^\star+x_{t+1}\log(1-b_{tfp}^\star),\\
K_{t+1}^\star &= \left((1-\delta)K_t^\star+\phi\left(\frac{I_t^\star}{K_t^\star}\right)K_t^\star\right)(1-b_k^\star x_{t+1}).
\end{aligned}$$

But the resource constraint now allows consumption and investment to be financed using foreign borrowing or lending:

$$
\begin{aligned}
C_t^\star + I_t^\star + p_t^B \widetilde{B}_{t+1} &= B_t^\star + Y_t^\star, \\
B_{t+1}^\star &= \widetilde{B}_{t+1}(1 - x_{t+1} b_{debt}).
\end{aligned}
$$

Here p_t^B is the price of a one-period bond that pays out 1 unit of good in the SOE next period if no disaster happens, and $1 - b_{debt}$ good if the disaster happens. This bond price is determined by the large economy's investors and thus satisfy:

$$
p_t^B = E_t \left(M_{t+1} \frac{Q_{t+1}}{Q_t} (1 - x_{t+1} b_{debt}) \right). \tag{4.1}
$$

This Euler equation simply comes from the first-order condition of an investor in the large economy. If she invests 1 LE good in this bond, she obtains $\frac{1}{Q_t}$ goods in the SOE, hence she buys $\frac{1}{Q_t p_t^B}$ bonds. Each bond then pays out $(1 - x_{t+1} b_{debt})$ SOE good, i.e. $Q_{t+1}(1 - x_{t+1} b_{debt})$ LE goods. Hence, her return is equal to:

$$
R_{t+1}^c = \frac{Q_{t+1}(1 - x_{t+1} b_{debt})}{p_t^B Q_t},
$$

and it satisfies the Euler equation $E_t \left(M_{t+1} R_{t+1}^c \right) = 1$.

Importantly, the representative agent in the small open economy behaves competitively – it does not take into account his impact on exchange rates when making consumption or investment decisions. As is standard, we view the representative agent as standing in for a large number of individuals who do not affect prices.

We assume that the SOE borrows and lends in its own currency. It implies that the large economy borrows and lends in foreign currency. This is clearly a simplification for two reasons. First, Tille (2004) and Lane and Shambaugh (2010) report that US external assets are mostly denominated in foreign currencies while US external liabilities are in dollars. Second, the choice of currency used to borrow or lend is likely linked to the currency used to trade. We do not study here the optimal currency denomination of trade (see Bacchetta and van Wincoop (2005) for relevant evidence and partial and general equilibrium models of this choice). We do not study investments in foreign equity. Yet, since the SOE can borrow from or lend to the large economy, while also investing in its own physical capital, our model implies some portfolio choices. It thus relates to a recent stream of papers on international portfolio allocation problem (see Evans and Hnatkovska (2005), Engel and Matsumoto (2009), Devereux and Sutherland (2010, forthcoming), and Coeurdacier, Kollmann and Martin (2010), for recent contributions on this question).

Finally, we define net exports and current accounts. Note that we do not include shipping costs in the SOE's resource constraint; we assume that they are paid in the large economy or are negligible. We define net exports as

$$NX_t^\star \stackrel{def}{=} Y_t^\star - C_t^\star - I_t^\star.$$

The current account tracks the accumulation of net foreign assets:

$$CA_t \stackrel{def}{=} B_{t+1}^\star - B_t^\star = \frac{B_t^\star + NX_t^\star}{(1 - x_{t+1} b_{debt}) p_t^B} - B_t^\star = B_t^\star \left(\frac{1}{(1 - x_{t+1} b_{debt}) p_t^B} - 1 \right) + \frac{NX_t^\star}{(1 - x_{t+1} b_{debt}) p_t^B}.$$

This definition reflects the usual accumulation equation for net foreign assets, as long as no disaster takes place. But it does not take into account many important features of the US data (like redemptions): see Lane and Milesi-Ferretti (2001) for a detailed presentation of US net foreign assets. Note that the current account is affected by the revaluation effect emphasized by Gourinchas and Rey: a shock which affects the price of debt will lead to a jump in the current account, even if net exports do not change. (Similarly a disaster which leads to default will also lead to a jump in the current account.)

Bellman equation It is useful, both for conceptual clarity and for setting up our numerical algorithm, to derive a recursive formulation of this problem. While we could in theory consider separately firms and households, it is slightly easier in practice to solve a single maximization problem.

Taking as given the bond price function p^B, which depends on all the state variables (of both the large and the small economies), the utility of the SOE is

$$
\begin{aligned}
W^\star(K^\star, z^\star, p, B^\star, k) &= \max_{C^\star, I^\star, N^\star, \widetilde{B}^\star} \left\{ \begin{array}{c} \left(C^{\star\upsilon}(1-N^\star)^{1-\upsilon}\right)^{1-\gamma} \\ +\beta\left(E_{p',\varepsilon',\varepsilon^{\star\prime},x'} W^\star(K^{\star\prime}, z^{\star\prime}, p', B^{\star\prime}, k')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\}, \\
\text{subject to} &: \\
C^\star + I^\star + p^B(K^\star, z^\star, p, B^\star, k)\widetilde{B}^\star &\leq B^\star + z^{\star 1-\alpha} K^{\star\alpha} N^{\star 1-\alpha}, \\
K^{\star\prime} &= \left((1-\delta)K^\star + \phi\left(\frac{I^\star}{K^\star}\right)K^\star\right)(1 - b_k^\star x'), \\
\log z^{\star\prime} &= \log z^\star + \mu + \sigma\varepsilon^{\star\prime} + x'\log(1 - b_{tfp}^\star), \\
B^{\star\prime} &= \widetilde{B^\star}(1 - x' b_{debt}), \\
k' &= H(k, p, x', \varepsilon').
\end{aligned}
$$

Here k denotes the large economy's detrended capital stock ($k = K/z$): it affects the large economy's SDF and hence the bond price p^B. We use H to denote its law of motion, given by the solution of the large economy's maximization problem, i.e. the solution of the closed economy model. Following Schmitt-Grohé and Uribe (2003), we consider a "portfolio adjustment" cost $-\frac{\psi}{2}\left(b^\star - \underline{b}\right)^2$, where b is the detrended debt $b = B/z$, that we subtract from the resource constraint. Such portfolio adjustment cost is often used to pin down the steady-state level of debt when solving models with linear approximations, and to make the level of debt stationary in order to generate finite second moments. In our case, we solve the model with nonlinear methods and the "steady-state" level of debt is also determined by a hedging demand. We thus use a small portfolio adjustment costs simply to facilitate numerical convergence. The magnitude of ψ affects the volatility of the current account relative to GDP.

To solve this maximization problem, we take first-order conditions with respect to $c^\star, i^\star, N^\star$, and $\widetilde{b}^\star$, and combine them with envelope conditions. This gives a system of five equations that characterizes the equilibrium of the economy. We then solve this system of five functional equations, by approximating the policy functions using Chebychev polynomials. We detail our solution method in a separate appendix, and here we simply list these equations. We rescale all the SOE variables by the level of technology z^* in the SOE, e.g. $c^* = C^*/z^*$, etc.; moreover note that the bond pricing function can be written as $p^B(k^\star, k, p, b^\star)$ according to equation 4.1.

We start with three key Euler equations. For the representative investor in the SOE, the Euler equation that defines the return on physical capital is the standard con

$$\begin{aligned} 1 &= E_{p',\varepsilon^{\star\prime},x'}\left[M^\star(k^\star, k, p, b^\star, x', p', \varepsilon^{\star\prime})R_I^*\right], \\ \text{where } R_I^\star &= \phi'\left(\frac{i^\star}{k^\star}\right)\left(\alpha k^{\star\prime\alpha-1}N^{\star\prime 1-\alpha} + \frac{1-\delta+\phi\left(\frac{i^{\star\prime}}{k^{\star\prime}}\right) - \phi'\left(\frac{i^{\star\prime}}{k^{\star\prime}}\right)\frac{i^{\star\prime}}{k^{\star\prime}}}{\phi'\left(\frac{i^{\star\prime}}{k^{\star\prime}}\right)}\right)(1-b_k^\star x'). \end{aligned}$$

$R_I^\star$ denotes the return on capital. If there are no adjustment costs, $R_I^\star$ simplifies to:

$$\left(\alpha k^{\star\prime\alpha-1}N^{\star\prime 1-\alpha} + 1 - \delta\right)(1-b_k^\star x'),$$

where the first term is the standard marginal product of capital and the second term reflects capital destruction during disasters.

The return on the foreign asset satisfies two Euler equations since it must be priced from the perspective of investors in both the large and small economies. Let us first define the return of the foreign asset from the perspective of the SOE's investor:

$$R_b = \frac{(1-x'b_{debt})}{p^B(k^*, k, p, b^*)}.$$

The two Euler equations are thus:

$$\begin{aligned} E_{p',\varepsilon',x'}\left(M(k,p,x',p',\varepsilon')\frac{1+\kappa'\left(\frac{y^{\star\prime}-c^{\star\prime}-i^{\star\prime}}{y^{\star\prime}}\right)}{1+\kappa'\left(\frac{y^\star-c^\star-i^\star}{y^\star}\right)}\frac{(1-x'b_{debt})}{p^B(k^*,k,p,b^*)}\right) &= 1, \\ E_{p',\varepsilon^{\star\prime},x'}\left(M^\star(k^\star,p,b^\star,x',p',\varepsilon^{\star\prime})\left(1-\psi\left(b^\star-\underline{b}\right)\right)\frac{(1-x'b_{debt})}{p^B(k^*,k,p,b^*)}\right) &= 1. \end{aligned}$$

These two equations jointly determine the price p^B of foreign debt, and net exports. Note that in the absence of disaster risk in debt – $b_{debt} = 0$ – and in the absence of bond adjustment costs, the risk free rate in the small economy, $R_f^\star = 1/E(M^\star)$ is determined by solely by p^B. If there is some disaster risk in debt ($b_{debt} > 0$), the price p^B of the foreign assets contains a premium over the risk free rate in the large country because of the uncertainty about future disaster realization. In addition there p^B also contains a premium for exchange rate risk.

The third equation corresponds to the standard labor market clearing condition: the marginal product of labor equals the wage equals the marginal rate of substitution:

$$\frac{1-\upsilon}{\upsilon}\frac{c^\star(k^\star,p,k,b^\star)}{1-N^\star(k^\star,p,k,b^\star)} = (1-\alpha)k^{\star\alpha}N^\star(k^\star,p,k,b^\star)^{-\alpha}$$

The fourth and fifth equations are the resource constraint and the Bellman equation, which have to hold with equality. With these five equations, we can solve the model. We solve the model numerically using projection methods. We make one further simplifying assumption: to reduce the dimensionality of the problem, we assume that the LE relative capital stock k stays equal to its steady-state value. The appendix details our numerical method. We now describe rapidly our calibration before turning to our simulation results.

4.2 Calibration

Most of the parameters are as in the previous section. In particular, we set the large economy to be exactly as in Section 3. We assume that the small country is more risky in the sense that it faces a greater destruction of resources in case of disasters. The new parameters describe shipping costs (κ and ζ) and portfolio adjustment costs (ψ and $\bar{b}$). Table 2 reports all the parameter values.

We follow Schmitt-Grohé and Uribe (2003) and assume that $\psi = 0.001$ and we set $\bar{b} = -2$. We also set the trade cost parameter κ and equal to 2. As noted earlier, trade costs are quadratic. When net exports represent 10% (20%) of output, trade costs equal 1% (4%) of output. In those cases, trade costs represent one-tenth and around one-sixth of net exports. This trade cost parameter is thus conservative.[17]

[17] Anderson and van Wincoop (2004) provide an extensive survey of the trade cost literature and conclude that total international trade costs, which include transportation costs and border-related trade barriers, represent an ad-valorem tax of about 74%. This total trading cost encompasses border-related trade barriers, which represent a 44% cost and are estimated through direct observation and inferred costs. Transportation costs stricto sensu represent 21%. This value is close to the 25% used in Obstfeld and Rogoff (2000). In this paper, we use quadratic

Finally, we need to fix the relative values of the disasters' impacts on physical capital, TFP and debt. To simplify, we focus here on the simplest case: the three disaster parameters (the capital destruction b_k, the TFP loss b_{tfp} and the debt default parameter b_{debt}) are identical and equal to the values in Table 2. We report in a separate appendix a detailed description of impulse response functions obtained when these parameters differ.

4.3 Impulse Response Functions

The small open economy model of differs markedly from the closed economy model of the previous section. To make this point transparent, we report responses both to a shock to the disaster probability, and to the realization itself of disaster. Let us start with the latter.

The effect of a disaster We start with the effects of disasters presented in Figure 2. Compared to the closed economy case studied in Section 3, the effect of a disaster now also depends on the debt default parameter b_{debt}. If the small open economy has negative (positive) net foreign assets b^*, and if b_{debt} is larger than b_k, the country benefits from a positive (negative) wealth effect, as the debt is reduced relative to the wealth of the country. As explained above, here we focus on the knife-edge case where $b_k = b_{tfp} = b_{debt}$. In this case, the response of the SOE can be characterized analytically: consumption, investment, output, debt, the capital stock, and productivity all fall by the same amount, and hours do not change. Hence, the reaction of the small economy to disasters is very similar to the case of a closed economy. Because the disaster size is larger in the SOE, all the quantities fall by more than in the large economy. Employment does not react because the wealth effect is exactly offset by the substitution effect of a lower wage. Because debt is also reduced by the same amount, the debt-output ratio does not change. Consequently net exports and exchange rates do not respond to disasters.

To prove this result, simply notice the law of motion for detrended capital and foreign asset

instead of proportional costs. This modeling choice has two advantages: it avoids kinks in the solution due to non linearities and, as already noted, it allows exchange rates to fluctuate even when countries trade.

holdings:

$$k^{\star\prime} = \left((1-\delta)k^{\star} + \phi\left(\frac{i^{\star}}{k^{\star}}\right)k^{\star}\right)\frac{(1-b_k^{\star}x')}{(1-b_{tfp}^{\star})e^{\mu+\sigma\varepsilon^{\star\prime}}},$$

$$b^{\star\prime} = \widetilde{b}^{\star}\frac{(1-x'b_{debt})}{(1-x'b_{tfp}^{\star})e^{\mu+\sigma\varepsilon^{\star\prime}}},$$

Obviously, the realization of a disaster will not affect the evolution of detrended capital and foreign assets as long as $b_k = b_{tfp} = b_{debt}$. Hence, the state variables are not affected by a disaster realization, which implies that detrended aggregates are not affected either. For instance, detrended consumption is $c_t = c(k_t^*, b_t^*, p_t)$, which is unaffected since k_t^* and b_t^* are unaffected, and hence $C_t = c_t^* z_t^*$ simply falls according to z_t^*. Note that the absence of response of net exports and exchange rates is specific to our assumption that all disaster parameters are equal ($b_k = b_{tfp} = b_{debt}$). As we show in the separate appendix, exchange rates respond to disasters if these parameters are not all equal.

The effect of an increase in the probability of disaster We turn now to the effects of disaster probability shocks reported in Figure 3. A shock to the probability of disaster now affects the economy through two effects: a change in the riskiness of the home technology, and a change in the interest rate at which the SOE borrows or lends. This can lead to endogenous changes in the net foreign asset position (i.e, capital reallocation) depending on the riskiness of the foreign asset relative to capital. The change in the interest rate also implies wealth and substitution effects. The wealth effect depends on level of debt/asset holdings. The substitution effect implies that a rise in the interest rate generally lowers consumption and investment, but increases employment and hence output. For this reason, our model echoes the findings of Neumeyer and Perri (2005). They report that many emerging economies face large changes in interest rates over time. They also document the large impact that these exogenous changes have on a small countries business cycles. In our model the change in the interest rate is partly a change in the risk-free rate, and partly a change in the risk premium. Moreover, in our model this shock to interest rate is also correlated with a change in the riskiness of domestic investment.

An increase in the probability of disaster – for the case where all disaster parameters are identical – decreases investment and increases consumption. Investment into physical capital becomes more risky and therefore less attractive, i.e the risk-adjusted return is reduced. The reduction of investment is larger in the small economy than in the large economy because more physical capital would be destroyed in the small economy should a disaster materialize. Consumption-savings choices depend on the IES because of offsetting income and substitution effects. If the IES is larger than one (the case we focus on), the substitution effect dominates and investment is reduced. Consumption rises when the disaster probability increases because investment falls more than output. The combination of the fall in output with the rise in consumption and the reduced fall in investment imply that net exports rise and hence the exchange rate depreciates.

A key element in the model lies in the price of the foreign debt q^B. In contrast to much of the existing literature, this price is not exogenously fixed but determined by investors of the large economy. Their marginal utility is less affected by disasters than in autarky. As a result, interest rates fall less than they would in autarky. Hence a disaster probability shock is equivalent to a rise in interest rates if compared to a closed economy with the same parameters.

An increase in the probability of disaster in the small open economy model can then be viewed as a combination of a shock to the probability of disaster in a closed economy model with a change to the SOE's interest rate. Recall that a rise in the probability of disaster in a closed economy - as described above in Section 3 - leads to fall in investment, employment and output as well as a rise in consumption. An increase in the interest rate in a closed economy makes saving more attractive. Consumption consequently drops and agents work more. The rise in employment leads to an increase in output. These two effects explain why consumption in the small open economy rises and employment and output initially fall less than in the large economy - despite being more risky.

The spread between the domestic risk free rate and the yield on the defaultable foreign asset rises. This is because the expected probability of disaster and hence the default probability rises. The spread reflects the default probability and a risk premium because risk-averse agents try to avoid exposure to disaster risk. Given the assumption that the large economy prices this debt, the risk premium is determined by the marginal utility of investors in the large economy.

Overall, compared to the case of a closed economy with the same parameters (b_k and b_{tfp}) consumption rises less, investment falls more, and employment and output fall less. This is because the increase in disaster probability - as in the closed economy - is combined with a relative rise in interest rates. With respect to welfare, the small country is slightly better off than a closed economy because it has access to foreign assets - which is priced by the less risky large economy.

Finally, note that net exports rise as the small economy accumulates assets abroad, which are safer. This rise in net exports is made possible by a depreciation of the currency. These two patterns are consistent with the experience of emerging markets, which are often subject to "sudden stops" when international investors become more cautious and decide to reduce total risk exposure: the current account goes up and the exchange rate sharply depreciates (as in Eastern Europe in Fall 2008).

4.4 Quantitative Analysis of the Model

To further study the model, we report in this section statistics calculated by simulating the model. Our benchmark case is again the simplest model where all disaster parameters in the small open economy are the same. To illustrate the effect of time-varying risk, we compare our benchmark model to a model with no disaster risk, and which is thus driven entirely by TFP shocks.

As robustness checks, we also report results from three variants: (i) a model where debt default in case of disaster is smaller ($b_{debt} = 0.4$) than capital and technology destruction, (ii) a model where capital and debt destructions in disaster are smaller than technology destruction ($b_k = b_{debt} = 0.4$) and finally, (iii) a model where capital destruction and debt default in disaster are larger than technology destruction ($b_{tfp} = 0.4$).

Quantities Table 8 reports the implications of the different models on quantities. The first row reports the results for the case where all disaster parameters are identical. We assume as in the previous model that the cross-country correlation of TFP shocks is 0.3. This table is

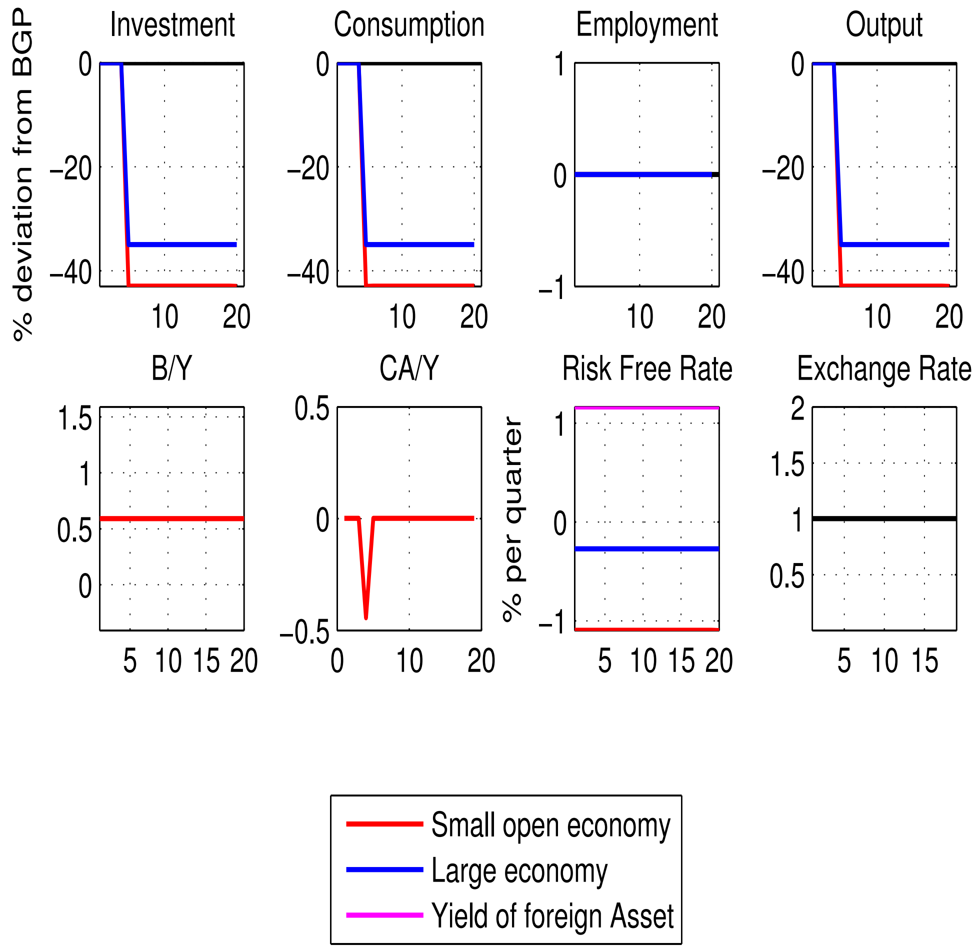

Figure 2: **Impulse Response Functions – Small Open Economy – Disaster Shock**: This figure presents the impulse response functions of different macroeconomic and financial variables to the realization of disaster for the case $b_k = b_{tfp} = b_{debt}$

the counterpart to Table 3 for closed economies. The volatility of consumption growth is close to its empirical counterpart, but investment is too volatile (which may be fixed by introducing capital adjustment costs) and employment too smooth. Moreover, the cross-country correlation of investment growth rates is too high.

We study the impact of different parameters on business cycle statistics. We find that a

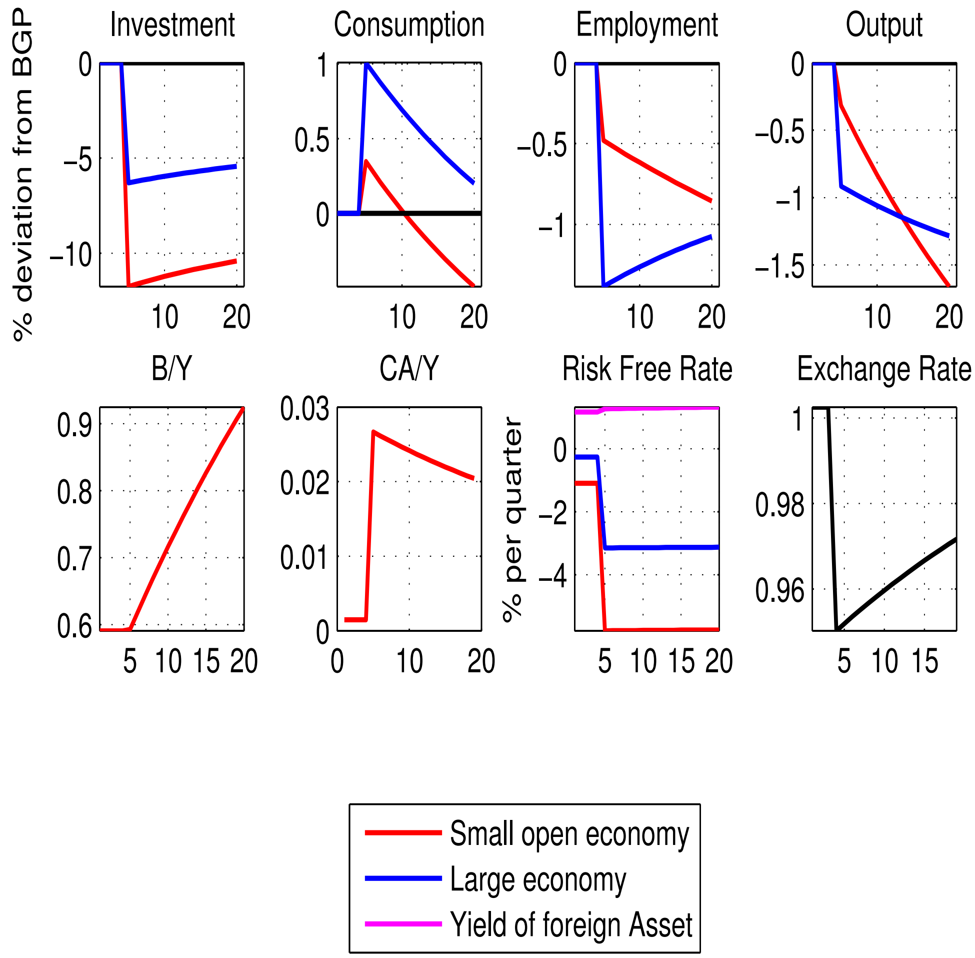

Figure 3: **Impulse Response Functions – Small Open Economy – Disaster Probability Shock**: This figure presents the impulse response functions of different macroeconomic and financial variables to an increase to the probability of disaster for the case $b_k = b_{tfp} = b_{debt}$

reduction in b_{debt} leads to a significantly lower correlation of employment across countries. A reduction in b_k implies that employment is now negatively correlated across countries because shocks to disaster probability lead to opposing responses in this case. Furthermore, because capital is less risky, investment volatility is lower than in the other cases. When capital is relatively more risky, investment volatility is the highest and employment is strongly positively correlated across

Table 8: Business Cycle Statistics — Small Open Economy

	Standard Deviations					Cross-country Correlations			
	$\sigma(\Delta c)$	$\sigma(\Delta i)$	$\sigma(\Delta n)$	$\sigma(\Delta y)$	$\sigma(\Delta \frac{nx}{y})$	$(\Delta c, \Delta c^\star)$	$(\Delta i, \Delta i^\star)$	$(\Delta n, \Delta n^\star)$	$(\Delta y, \Delta y^\star)$
					Benchmark				
$b_k = b_{debt} = b_{tfp}$	0.57	3.10	0.15	0.75	1.10	0.31	0.70	0.48	0.30
					Robustness Checks				
$b_k = b_{tfp} > b_{debt}$	0.57	3.12	0.14	0.75	1.24	0.28	0.71	0.14	0.29
$b_k = b_{debt} < b_{tfp}$	0.57	2.17	0.20	0.76	1.02	0.22	0.63	-0.40	0.26
$b_k = b_{debt} > b_{tfp}$	0.62	3.60	0.33	0.76	0.77	0.40	0.72	0.85	0.34
RBC	0.58	1.52	0.11	0.73	0.35	0.30	0.30	0.30	0.30

Notes: This table reports the standard deviations of log differences in consumption, investment, labor and output, along with the cross-country correlation of these variables. In addition it displays the standard deviation of log difference of net export relative to output. This table is constructed by simulating the model, assuming no disasters are actually realized. We consider five variants of the small open economy model: (1) the simplest model where all disaster parameters are the same in the small open economy, (2) a model where debt default in disaster is smaller than capital and technology destruction, (3) a model where capital and debt destruction in disaster are smaller than technology destruction and finally, (4) a model where capital and debt default in disaster are large than technology destruction, (5) a standard real business cycle model with only TFP shocks.

countries.

Exchange rates and carry trade returns Table 9 shows how exchange rates behave in our model. It is useful to compare it to Table 5 – its counterpart for the closed economy model. The SOE model delivers a negative correlation between changes in exchange rates and interest rate differentials, as in our benchmark closed economy model. As a result, the SOE model delivers deviations from UIP and thus currency excess returns. We study them below.

The SOE loosens the link between changes in exchange rates on the one hand and consumption growth or output growth on the other hand. The correlation between output growth and exchange rates is now close to 0 as in the data. The SOE model, however, does not fully solve the Backus and Smith (1993) puzzle. The correlation between relative consumption growth and exchange rates is equal to 0.3, while it is also close to 0 in the data. The SOE model also loosens the link between exchange rates and equity returns, but not enough compared to the data.

Our current calibration has a major drawback: it delivers exchange rates that are too smooth compared to the data. Note, however, that the standard RBC model produces exchange rates that are five times less volatile than the ones we obtain. Our results thus call for a different form of international trade costs. We use quadratic trade costs for simplicity and ease of comparison with the literature, but a higher curvature in the trade cost function κ appears necessary to match the exchange rate volatility.

Table 10 reports summary statistics on currency carry trade excess returns. It is the counterpart of Table 6 for closed economies. Recall that standard RBC models produce almost no deviations from UIP and zero average currency excess returns. Our SOE model delivers a negative UIP slope coefficient of -0.6, close to the benchmark value in the data. Average currency excess returns, however, are smaller than in the data. They are equal to 1% per annum (0.25% quarterly). In the model, the mechanism of the model is as follows: the carry trade is done using the defaultable bond B. The yield on this bond is larger than the risk-free yield in the large economy, hence the carry trade here consists of borrowing in the large economy, and lending in the small economy. Because the bond B is denominated in the SOE currency, this strategy expose an investor (be it located in the SOE or in the LE) to exchange rate risk. Because the SOE currency depreciates when the probability of disaster rises and marginal utuility is high, this carry trade is a risky strategy, which hence must earn a risk premium.

In the data, carry trade excess returns amount to more than 6% before transaction costs (Table I). But these empirical excess returns correspond to conditional risk premia: portfolios are re-balanced monthly such that investors always lend in the *currently* highest interest rate currencies. In the model, country characteristics are fixed. As a result, the model delivers unconditional risk premia, not conditional ones. In the data, investors are also compensated for investing in currencies with *average* high interest rates, but this compensation is lower. Lustig et al. (2009) estimate that conditional risk premia are twice as large as unconditional risk premia. On a sample of developed countries over the 1995-2010 period, they estimate the former to be – after transaction costs–

Table 9: Real Exchange Rates — Small Open Economy

	$E(\Delta q)$	$\sigma(\Delta q)$	$(\Delta q, r^e - r^{e,\star})$	$(\Delta q, r^f - r^{f,\star})$	$(\Delta q, \Delta c - \Delta c^\star)$	$(\Delta q, \Delta y - \Delta y^\star)$
				Benchmark		
$b_k = b_{debt} = b_{tfp}$	0.00	1.17	0.86	-0.29	0.33	-0.09
				Robustness Checks		
$b_k = b_{tfp} > b_{debt}$	0.00	1.33	0.85	-0.29	0.38	-0.14
$b_k = b_{debt} < b_{tfp}$	0.00	1.08	0.85	-0.28	0.50	-0.20
$b_k = b_{debt} > b_{tfp}$	0.00	0.82	0.76	-0.29	0.14	0.14
RBC	0.00	0.24	0.59	-0.08	0.62	0.54

Notes: This table reports the averages and standard deviations of changes in log real exchange rates, along with the cross-country correlation of changes in log real exchange rates with cross-country differences in real equity returns, real risk-free rates, real consumption growth and real output growth. This table is constructed by simulating the model, assuming no disasters are actually realized. We consider five variants of the small open economy model: (1) the simplest model where all disaster parameters are the same in the small open economy, (2) a model where debt default in disaster is smaller than capital and technology destruction, (3) a model where capital and debt destruction in disaster are smaller than technology destruction and finally, (4) a model where capital and debt default in disaster are large than technology destruction, (5) a standard real business cycle model with only TFP shocks.

equal to 3.8% while the latter is equal to 2.4% per annum. As a result, the model delivers average currency excess returns that are sizable, but twice too small compared to recent data.

5 Impact of Time-Varying Risk on Actual Quantities and Asset Prices

In the previous sections, we have shown that incorporating time-varying risk and heterogeneous exposures to this risk helps the models generate moments closer to the data. In this section, we provide an empirical test of the mechanism by studying the impact of time-varying risk on actual macroeconomic quantities and asset prices. We do so by estimating impulse responses of both quantities and asset prices to a shock to global equity volatility using standard vector

Table 10: Carry Trade Excess Returns — Small Open Economy

	$E(rx_{t+1})$	$\sigma(rx_{t+1})$	$(rx_{t+1}, \Delta c)$	$(rx_{t+1}, \Delta c^{\star})$	(rx_{t+1}, r^{e}_{t+1})	$(rx_{t+1}, r^{e\star}_{t+1})$	β_{UIP}
				Benchmark			
$b_k = b_{debt} = b_{tfp}$	0.62	1.99	-0.01	-0.29	0.82	0.76	-0.57
				Robustness Checks			
$b_k = b_{tfp} > b_{debt}$	0.59	2.02	0.05	-0.31	0.86	0.81	-0.64
$b_k = b_{debt} < b_{tfp}$	0.57	1.82	0.19	-0.28	0.82	0.76	-0.61
$b_k = b_{debt} > b_{tfp}$	0.59	1.75	-0.17	-0.25	0.73	0.68	-0.56
RBC	0.08	0.24	0.96	0.26	0.97	0.28	

Notes: This table reports the averages and standard deviations of carry trade excess returns, along with the cross-country correlation of these excess returns with real consumption growth and real stock market returns. The last column reports the UIP slope coefficient. We consider five variants of the small open economy model: (1) the simplest model where all disaster parameters are the same in the small open economy, (2) a model where debt default in disaster is smaller than capital and technology destruction, (3) a model where capital and debt destruction in disaster are smaller than technology destruction and finally, (4) a model where capital and debt default in disaster are large than technology destruction, (5) a standard real business cycle model with only TFP shocks.

autoregression (VAR). The motivation is that in our model, equity volatility goes up with the probability of disaster.

5.1 Sample

In this section, we focus on a group of six countries for which monthly and quarterly series are readily available over long periods of time: Belgium, Canada, France, Japan, United Kingdom, and United States. These countries constitute our core sample.

To build long time-series of stock market volatility, we import daily MSCI equity returns for all the countries in our sample. We build monthly and quarterly series of stock market volatility by recording standard deviations over calendar months or quarters. In the model, disasters are global. They affect countries differently, but they happen at the same time in all countries. How does this assumption compare to the data? Looking at realized equity return volatilities or option-implied volatilities, our assumption does not seem far-fetched. Option-implied or realized equity volatility series clearly contain a large common component across countries. The first principal component, which is close to the mean of all these series, accounts for more than 40% of total realized variance in a large set of 27 OECD countries over 40 years, and more than 90% of a set of 9 option-implied volatilities.[18] As a result, we take, at each point in time, the mean of our different realized volatility series over the countries in our core sample: this mean constitute our measure of global disaster risk.

5.2 Impact on Quantities

We study the impact of global volatility shocks on monthly and quarterly macroeconomic variables. Our methodology follows Bloom (2009) and focuses on simple VARs. Details about our VARs

[18] Table 15 in Appendix A reports the fraction of total variances explained by the first four principal components of return volatilities. We consider three samples. The first sample comprises all OECD countries except Iceland, Luxembourg, and Slovakia. The second sample focuses on OECD countries for which we have such volatility data for our entire time-window. Countries in this sample are: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, United Kingdom, and United States. The third sample uses option-implied volatilities on equity indices VIX (United States), VDAX (Germany), SMI (Switzerland), CAC (France), BEL (Belgium), AEX (Netherlands), currencies (US Dollar to UK pound, US Dollar to Japanese yen), along with a US bond index (1 month).

are in Appendix B. Briefly, our VAR specification includes the market return, volatility, and macroeconomic time series, and the shock to volatility is assumed to affect only the market return at time 0. (This orthogonalization assumption has little impact on our results.)

Figures 4 and 5 report the impulse response functions of industrial production and unemployment rates to a shock on average volatility. In all six countries in our core sample, unemployment rates increase and industrial production indices decrease when volatility increases. Bloom (2009) reports similar results for US employment and industrial production. Note, however, that we are using the *average* volatility, and not the US one as in Bloom (2009). Our results are in line with the model's implications: when the probability of a global disaster increases, a recession ensues with less production and employment, even in the absence of a technology shock. The magnitudes of the responses are economically and statistically significant. A one-standard deviation shock on aggregate volatility implies for example approximately 0.2% decline in industrial production for the US. The 2008 subprime mortgage crisis correspond to a 9 standard deviation shock. All countries present a similar negative effect, but interestingly, even in this limited sample of OECD countries, there is some heterogeneity: for instance, Japan is significantly more affected by a volatility shock than the other countries.

Figures 8 and 9 in the separate Appendix report the same impulse response functions country by country and with standard errors obtained by bootstrapping. For industrial production, all troughs in these six impulse response functions are significantly different from zero. The same result holds for unemployment, except for Belgium where the impulse response function is borderline significant. We obtained similar results using quarterly data for GDP or business fixed investment.

5.3 Impact on the Current Account and Exchange Rates

According to the model, countries that are largely affected by disasters (e.g. low interest rate countries) should see their exchange rate appreciate and current account become more negative (or less positive) when the probability of disaster rise. This mechanism imposes cross-restrictions on the impulse response functions of industrial production, exchange rates and current accounts.

In the model's logic, we expect industrial production in Japan (a low interest rate country) to react more negatively to a volatility shock than industrial production in New Zealand (a high interest rate country). We also expect the Japanese yen to appreciate and the New Zealand dollar to depreciate. Finally, we also expect the Japanese (New Zealand) current account to decrease (increase), as capital flows out of Japan and into New Zealand. This is what we find in the data. Figure 6 reports the impulse-response functions of industrial production, exchange rates, and current accounts to a shock on aggregate volatility. The Japanese economy tanks the most, the yen appreciates and the Japanese current account decreases. The New Zealand economy reacts less to the volatility shocks, the New Zealand dollar depreciates, and the New Zealand current account appreciates. These findings support the model mechanism.

While these results provide substantial support for the model mechanism, some caveats apply. Not all of these impulse response functions are statistically significant: in particular, the sample for New Zealand is short (1988-2008). Also, while these results hold for Japan, New Zealand, and some other countries such as Germany (a low interest rate country) or Australia (a high interest rate country), we have not been able yet to apply them to all countries.

Finally, we note that the model is consistent with recent evidence on currency carry trade excess returns. Lustig et al. (2009) and Menkho, Sarno, Schmeling and Schrimpf (2009) report that carry trade investments pay badly when volatility is high. The former paper uses the option-implied VIX index or a measure of global realized volatility on equity markets, while the latter uses a measure of realized volatility of currency markets. Lustig and Verdelhan (2010) report that carry trade returns are highly correlated to US equity returns when volatility is high. As we have already seen, the model reproduces these two facts.

Verdelhan (2010) reports that high interest rate countries tend to offer low equity excess returns to local investors (expressed in local currencies), while low interest rate countries boast higher equity excess returns. Table 1 reports similar results, except that the median portfolio offers a slightly higher excess return than the first one. The last panel of the same table presents equity return volatilities. The higher the interest rate, the more volatile the equity returns. Again, the

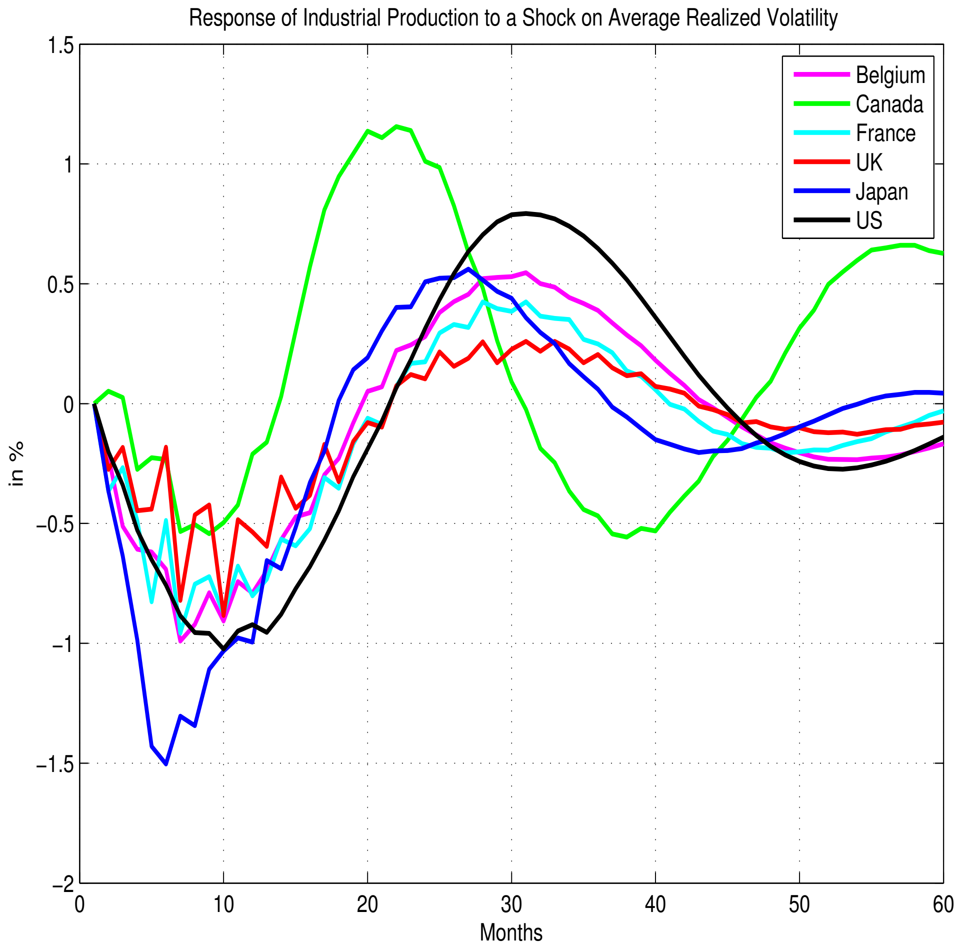

Figure 4: **Response of Industrial Production to a Shock on Average Realized Volatility**: This figure plots the impulse response functions of unemployment rates to a one-standard deviation shock on average volatility in the following countries: Belgium, Canada, France, Japan, United Kingdom, and United States. Data are monthly. The sample is 1970.1 – 2009.12, except for the United Kingdom (1971.1 – 2009.12) and Canada (1995.1 – 2009.12). Volatility measures correspond to the standard deviations of equity returns over calendar months. The average volatility is the mean of these different standard deviations over the same 6 countries. VARs contain the following variables: market returns, volatility, industrial production, consumer price indices and short-term interest rates.

model reproduces these two facts.

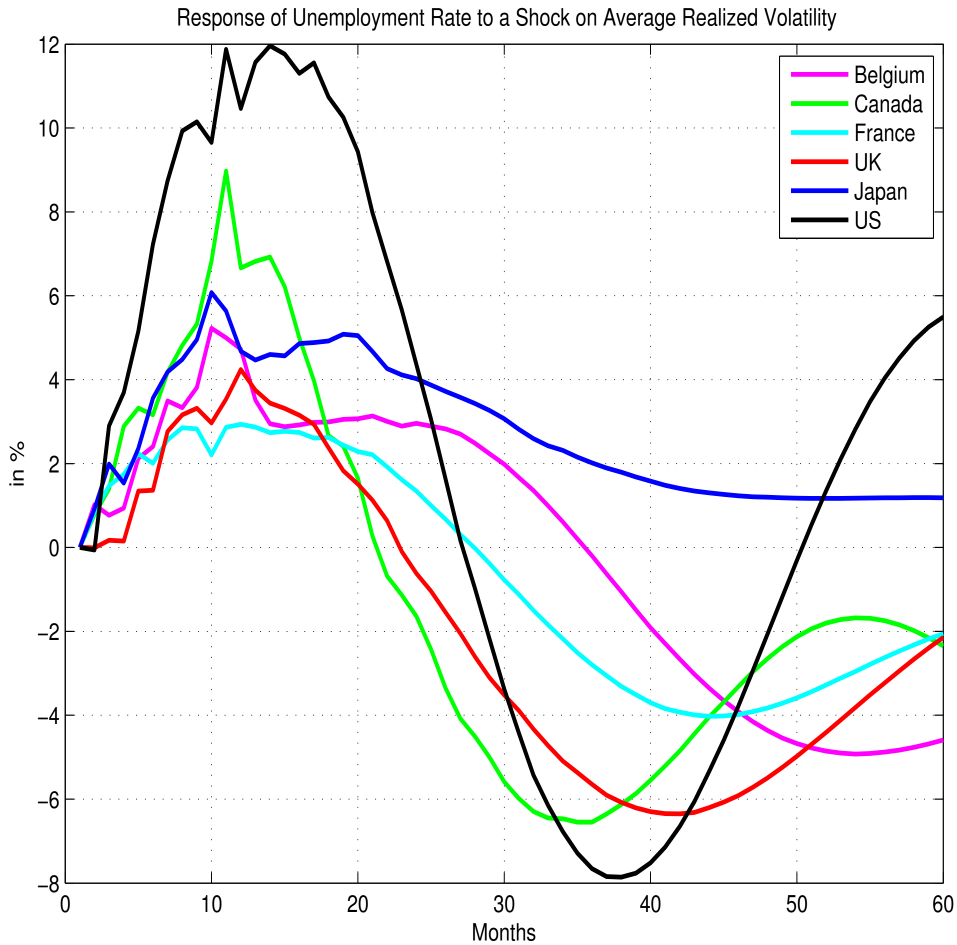

Figure 5: **Response of Unemployment to a Shock on Average Realized Volatility**: This figure plots the impulse response functions of unemployment rates to a one-standard deviation shock on average volatility in the following countries: Belgium, Canada, France, Japan, United Kingdom, and United States. Data are monthly. The sample is 1970.1 – 2009.12, except for France (1978.1 – 2009.12). Volatility measures correspond to the standard deviations of equity returns over calendar months. The average volatility is the mean of these different standard deviations over the same 6 countries. VARs contain the following variables: market returns, volatility, unemployment rates, consumer price indices and short-term interest rates.

6 Conclusion

This paper shows how the combination of time-varying risk,[19] and heterogeneous exposures to that risk, make international business cycle models closer to the data. We provide direct support for

[19]The time-varying aggregate risk comes from changes in the probability of a worldwide disaster. Our results, however, can be generalized to a larger class of stochastic processes. In particular, we may assume that shocks are normally distributed instead of being jumps, provided that risk aversion is high. The key assumption is that aggregate volatility is time-varying. We use disasters because we view them as a plausible, parsimonious explanation of the volatility of asset prices.

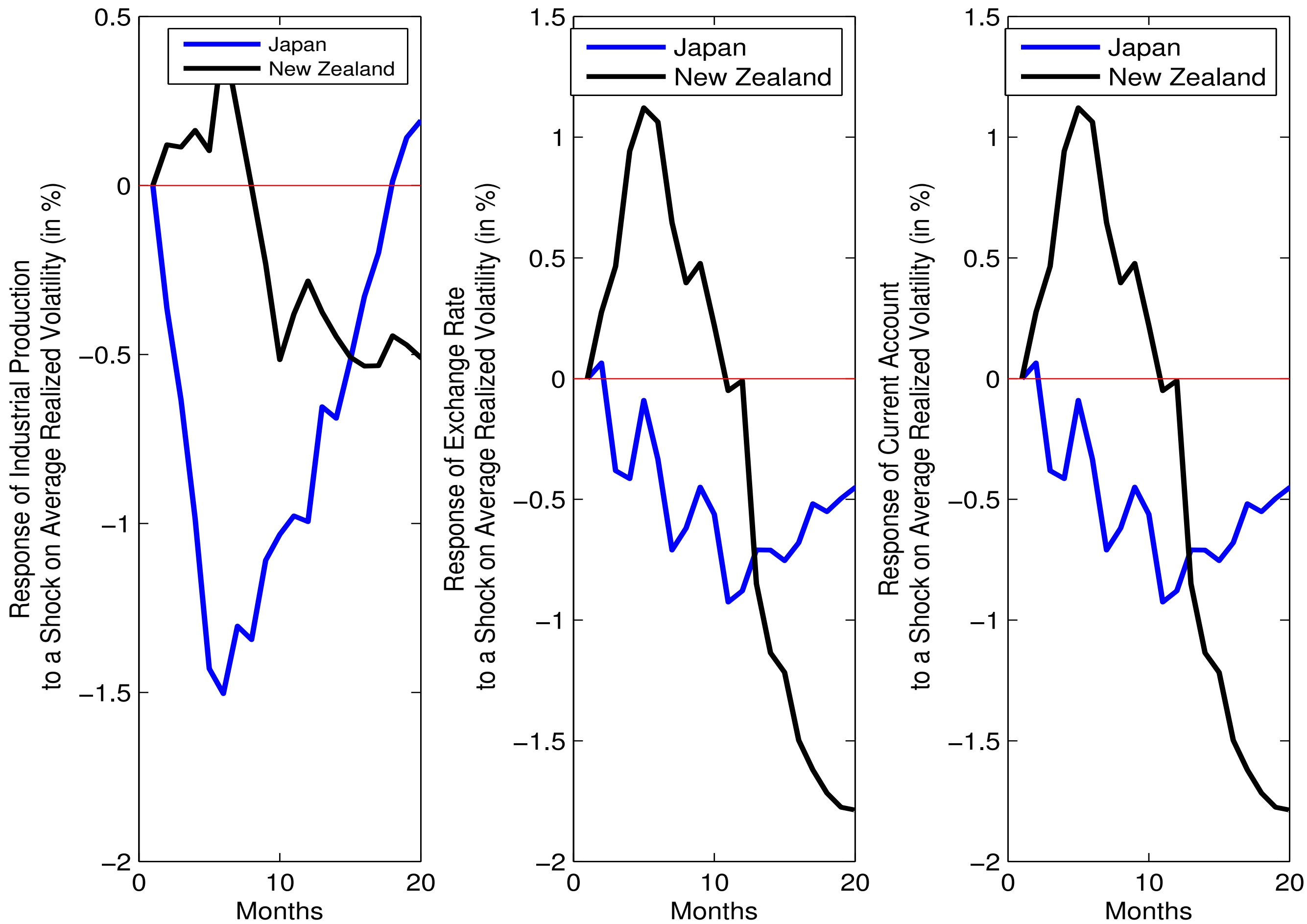

Figure 6: **Responses of Industrial Production, Exchange Rate, and Current Account to a Shock on Average Realized Volatility**: This figure plots the impulse response functions of industrial production, exchange rate, and current account in New Zealand and Japan to a one-standard deviation shock on average volatility in the following countries: Belgium, Canada, France, Japan, United Kingdom, and United States. Data for industrial production and exchange rates are monthly. Data for current accounts are quarterly. The sample is 1970.1 – 2009.12 for Japan and 1988.1 – 2009.12 for New Zealand. Volatility measures correspond to the standard deviations of equity returns over calendar months. The average volatility is the mean of these different standard deviations over these 6 countries. VARs contain the following variables: market returns, volatility, and either industrial production, exchange rate, or current account, along with short-term interest rates.

the model mechanism by showing that volatility shocks have significant effect on macroeconomic aggregates and exchange rates. Overall, our results suggest an interesting interaction between risk premia and international business cycles.

As we noted in the introduction, the main tension in our complete market model is that low interest rate countries are more risky and hence more volatile, while in the data they are more volatile. There are at least two broad possible explanations : first, it may be that the interest rate that we measure is affected by other factors (such as monetary policy or perhaps incomplete markets and other frictions); second, it may be that the high volatility of high interest rate countries is due to additional shocks from which we abstracted in our analysis.

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