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Mei Dong[1] and Janet Hua Jiang[2]

[1]Currency Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
meidong@bankofcanada.ca

[2]Funds Management and Banking Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
and
University of Manitoba
jjiang@bankofcanada.ca

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Abstract

We study price posting with undirected search in a search-theoretic monetary model with divisible money and divisible goods. *Ex ante* homogeneous buyers experience match specific preference shocks in bilateral trades. The shocks follow a continuous distribution and the realization of the shocks is private information. We show that generically there exists a unique price posting monetary equilibrium. In equilibrium, each seller posts a continuous pricing schedule that exhibits quantity discounts. Buyers spend only when they have high enough preferences. As their preferences are higher, they spend more till they become cash constrained. Since inflation reduces the future purchasing power of money and the value of retaining money, buyers tend to spend their money faster in response to higher inflation. In particular, more buyers choose to spend money and buyers spend on average a higher fraction of their money. The model naturally captures the hot potato effect of inflation along both the intensive margin and the extensive margin.

JEL classification: D82, D83, E31
Bank classification: Inflation and prices; Economic models

Résumé

Les auteures étudient le comportement d'acheteurs qui ne peuvent observer les prix affichés qu'après leur appariement à un vendeur, dans le cadre d'un modèle théorique de prospection monétaire caractérisé par la divisibilité de la monnaie et des biens. Les préférences des acheteurs, considérés a priori comme homogènes, font l'objet de chocs durant les rencontres bilatérales avec le vendeur. Ces chocs suivent une loi de distribution continue et leur réalisation constitue une information connue des seuls acheteurs. Les auteures montrent qu'il existe de façon générale un équilibre monétaire unique pour les prix affichés. À l'équilibre, chaque vendeur affiche une série continue de prix assortis de rabais sur les quantités. Les acheteurs ne dépensent que si leurs préférences sont suffisamment fortes, et plus leurs préférences s'élèvent, plus ils consomment, jusqu'à épuisement de leurs fonds. Puisque l'inflation diminue le pouvoir d'achat futur ainsi que l'intérêt qu'il y a à conserver de l'argent, les acheteurs sont portés à dépenser plus rapidement lorsque l'inflation augmente. En particulier, un plus grand nombre d'acheteurs décident de dépenser et ils dépensent en moyenne une part accrue de la monnaie en leur possession. Le modèle parvient à recréer l'effet de délestage causé par la hausse de l'inflation, tant dans sa dimension intensive que dans sa dimension extensive.

Classification JEL : D82, D83, E31
Classification de la Banque : Inflation et prix; Modèles économiques

1 Introduction

We build a search theoretic monetary model where sellers post prices and buyers engage in undirected search. The framework is based on Lagos and Wright (2005) and Rocheteau and Wright (2005). Search is undirected in the sense that buyers observe the posted prices only after being matched with sellers. Price posting with undirected search (hereafter price posting) is an important pricing mechanism to study because it captures the characteristics of many daily exchanges.[1] In many occasions, buyers randomly enter a store, read the price labels and decide whether or not to make a purchase.

In the search theoretic monetary literature, it has been a challenge to generate equilibria with valued fiat money under price posting. With a positive nominal interest rate, when buyers hold money, they make a costly ex ante investment. The existence of monetary equilibria hinges critically on the condition that buyers extract some trading surplus during monetary exchange. In a typical monetary model with price posting, sellers propose the terms of trade to extract the entire trading surplus. Monetary equilibria unravel as a result.

To generate monetary equilibria under price posting, we introduce private information about match specific preference shocks that affect buyers' marginal utility of consumption. Private information about preferences prevents the seller from extracting all the trading surplus and restores monetary equilibria. We show that when the preference shock follows a continuous distribution, there exists a unique monetary equilibrium with a continuum of prices. The model provides a useful framework to study the seller's pricing decision, the buyer's spending pattern, and the effects of inflation on these behaviors.

In equilibrium, the seller posts a non-linear pricing schedule that exhibits quantity discounts: lower unit prices for purchases of larger sizes. Quantity discounts are frequently observed in practice. The traditional explanation is that the unit cost of producing and/or selling a larger quantity is lower. In our model, quantity discounts exist even when the unit cost is constant, and private information about preferences is the key to generating quantity discounts.

After the preference shock is realized, buyers spend money only when they have high enough preferences. When preferences are higher, buyers increase spending till they are cash constrained. The model provides an intuitive explanation for the "hot potato" effect of inflation, i.e., inflation induces buyers to spend money faster. Buyers with low preferences retain all or part of their money for future spending. As inflation rises, the benefit of waiting for future purchases diminishes. Therefore, less buyers choose to retain all of their money, which means more buyers start spending money. Among those who retain part of their money before, higher inflation induces them to purchase more goods and spend a higher fraction of their money.

[1] We use price posting to refer to price posting with undirected search in the rest of the paper. When price posting is combined with directed search and trade is bilateral, it is often labelled as competitive search in the literature.

There have been earlier attempts to study price posting within the search theoretic monetary framework. As in our paper, the key to the existence of a monetary equilibrium under price posting is private information about match specific preference shocks. Jafarey and Masters (2003) and Curtis and Wright (2004) study price posting using the indivisible money model of Trejos and Wright (1995). In Curtis and Wright (2004), there are multiple (≥ 2) realizations of the preference shock and in equilibrium, sellers post at most two prices. Jafarey and Masters (2003) allow the preference shock to follow a uniform distribution. Interestingly, there is a single price posted in equilibrium. More recently, Ennis (2008) extends price posting to a divisible money framework as in Lagos and Wright (2005). He specifies a distribution of preference shocks that takes two values, and shows that sellers post a single price in equilibrium.

These earlier models are characterized by a simple pricing schedule and spending pattern and as a consequence, are not suitable for analyzing how inflation affects pricing and spending decisions. In particular, there exists "the law of two prices" as emphasized by Curtis and Wright (2004). Furthermore, a common feature shared by all the previous papers is that buyers always spend all their money once they are matched with sellers. Our paper shows that "the law of two prices" breaks down in a model with divisible money and a continuous distribution of preference shocks. The model generates richer (and more realistic) pricing schedule and spending pattern, and is suitable to study the effects of inflation on these behaviors.

The hot potato effect has been examined in several recent papers. In Lagos and Rocheteau (2005), inflation may increase the buyer's trading surplus. As a result, buyers increase search intensity and spend money faster. The explanation, however, is not robust and applies only when the terms of trade are determined by competitive search and when inflation is at low levels. Ennis (2009) assumes that sellers have more opportunities to rebalance money holdings than buyers. Inflation induces buyers to search harder for sellers to off-load their money. Liu *et al.* (forthcoming) consider free entry by buyers in monetary exchange. Inflation reduces buyers' trading surplus, so fewer buyers choose to enter, which increases the matching probability for those who do enter. In their model, buyers are able to spend faster simply because they have more opportunities to trade, not because they actively try to get rid of their money. In addition, the result is sensitive to the assumption of free entry by buyers. The result is reversed if we consider free entry by sellers. Nosal (forthcoming) assumes that accepting a current trade reduces (exogenously) the probability of future trading. Inflation reduces the value of future trading and buyers are more likely to accept current trades.

Compared with the existing literature, our explanation is the closest to the narrative description of the hot potato effect of inflation.[2] The hot potato effect is robust and exists at

[2] Here is a narrative description of the hot potato effect of inflation by Keynes (1924): "In Vienna, during the period of collapse ... [it] became a seasonable witticism to allege that a prudent man at a cafe ordering a bock of beer should order a second bock at the same time, even at the expense of drinking it tepid, lest

all levels of inflation. A novel result from the model is that it captures the hot potato effect along both the intensive margin (those who spent before spend more) and the extensive margins (those who did not spend before start spending).

The rest of the paper proceeds as follows. Section 2 describes the environment. In Section 3, we characterize the monetary equilibrium when the preference shock follows a uniform distribution. Section 4 examines the effects of inflation and rationalizes the hot potato effect of inflation. In Section 5, the model is extended to allow for a more general continuous distribution of preference shocks. We discuss some related work in Section 6 and Section 7 concludes. The technical proofs of Lemma 1 and results in Section 5 are provided in the Appendix.

2 Environment

The model is based on Rocheteau and Wright (2005). Time is discrete and runs from 0 to ∞. A decentralized market (DM) and a centralized market (CM) open sequentially in each period. The discount factor between two periods is $0 < \beta < 1$. There are two permanent types of agents: buyers and sellers distinguished by their roles in the DM. There is one nonstorable good in each market: a CM good and a DM good.

The CM is a centrally located competitive spot market. In the CM, all agents can consume or produce the CM good x. The utility of consuming x units of the CM good is x. If $x < 0$, it means that the agent produces and incurs disutility.

In the DM, agents are anonymous. Buyers and sellers are randomly matched and the matching function is such that one buyer meets one seller with probability 1.[3] Buyers are those who want to consume but cannot produce. Sellers can produce but do not want to consume. This generates a lack of double coincidence of wants problem, which together with anonymity, makes money essential as the medium of exchange.[4] For a seller, the disutility of producing q units of the DM good is $c(q)$ with $c(0) = 0$, $c' > 0$ and $c'' \geq 0$. By consuming q units of the DM good, a buyer's utility is $eu(q)$,where $e \geq 0$ is a preference parameter that determines the buyer's marginal utility of consumption. The function $u(q)$ satisfies $u(0) = 0$, $u' > 0 > u''$ and $u'(0) = \infty$. All buyers are *ex ante* identical before being matched with sellers. *Ex post*, however, they are subject to match specific preference shocks and become heterogeneous during their matches with sellers. The realization of e follows a uniform distribution on the interval $[0, 1]$. Buyers hold private information about the realization of e. It is straightforward that $eu'(q_e^*) = c'(q_e^*)$ characterizes the first-best

the price should rise meanwhile."

[3] The model is abstract from matching friction merely for simplicity. Allowing for a general matching function will not change our main results.

[4] Research on micro-founded monetary theory studies the frictions that make money essential. The consensus in the literature (see Kocherlakota 1998a, b) is that money is essential when three frictions exist: lack of double coincidence of wants, lack of commitment and private information about individual trading histories. Here we capture the last two frictions by "anonymity".

allocation q_e^* for $e \in [0,1]$.

The terms of trade in the DM are determined by price posting with undirected search. Before a buyer and a seller meet, the seller posts the terms of trade which consist of a menu of price-quantity pairs, and the buyer does not observe the posting. Once they are matched, the buyer sees the posted terms of trade and decides which price-quantity pair to take from the menu. As in other papers that study price posting, buyers may choose not to trade at all after they are matched.

Fiat money is supplied by the monetary authority. Money supply M_t grows at a constant rate $\gamma \geq \beta$ so that $M_t = \gamma M_{t-1}$. New money is used to finance a lump-sum transfer to buyers at the beginning of each CM. Let $T_t = (\gamma - 1)M_{t-1}$ be the amount of nominal transfer to each buyer.

3 Price Posting Equilibrium

Throughout this paper, we assume that money balances are observable. To solve the equilibrium, we first analyze choices in the CM and then move back to consider choices in the DM.

3.1 Decision Making in the CM

In the CM, agents rebalance their money holdings by trading money for the CM good, x, or vice versa. We first consider a buyer's problem. Let $W^b(m)$ denote the buyer's value function while entering the CM with m units of money. Let $V^b(\hat{m}_+)$ be the value function for the buyer in the DM of the next period, where $\hat{m}_+$ is the buyer's choice of money holding to enter the DM. We have

$$\begin{aligned} W^b(m) &= \max_{x,\hat{m}_+} \left[x + \beta V^b(\hat{m}_+) \right] \\ \text{s.t. } x &= \phi(m + T - \hat{m}_+), \end{aligned}$$

where ϕ is the value of money in the CM. Defining $z = \phi m$, $\tau = \phi T$ and $\hat{z}_+ = \phi_+ \hat{m}_+$, we can rewrite the buyer's problem in real terms as

$$W^b(z) = \max_{\hat{z}_+} \left[z + \tau - \frac{\phi}{\phi_+}\hat{z}_+ + \beta V^b\left(\hat{z}_+\right) \right].$$

Note that due to quasilinear preferences, the choice of $\hat{z}_+$ is independent of z and $W^b(z)$ is linear in z with $dW^b(z)/dz = 1$. The first-order condition is

$$\beta \frac{dV^b\left(\hat{z}_+\right)}{d\hat{z}_+} = \frac{\phi}{\phi_+} \text{ if } \hat{z}_+ > 0.$$

For a seller with z units of real money balance upon entering the CM, let $W^s(z)$ be his value function. It is a standard result that when the nominal interest rate is positive, the seller spends all the money accumulated in the previous DM on x and carries 0 money balance to the following DM. The seller's value function is given by $W^s(z) = z + \beta V^s(0)$. Note that $dW^s(z)/dz = 1$.

3.2 Decision Making in the DM

In the DM, the seller posts a menu of price-quantity pairs (q_e, z_e) for all $e \in [0,1]$. We assume that the seller can observe $\hat{z}$, the buyer's money balance while entering into the DM, so that the seller's posting may depend on $\hat{z}$. The matching function is such that each buyer meets a seller with probability 1. Upon matching, the match specific preference shock is realized and is the buyer's private information. After seeing the seller's posting, the buyer decides whether to trade or not, and if he decides to trade, which (q_e, z_e) to take.

The seller's choice of pricing schedule is in essence a mechanism design problem, where the seller is the principal and the buyer is the agent with private information (about his preferences). We can apply the revelation principle to formulate the seller's problem as follows: taking the buyer's money balance, $\hat{z}$, and the distribution of preference shocks as given, choose $(q_e, z_e)_{e\in[0,1]}$ to maximize his DM value function. Formally,

$$V^s(0) = \max_{\{q_e, z_e\}_{e\in[0,1]}} \int_0^1 [-c(q_e) + W^s(z_e)]de \tag{1}$$

$$\text{s.t.} \begin{cases} q_e \geq 0, & \text{(NC)} \\ z_e \leq \hat{z}, & \text{(CC)} \\ eu(q_e) - z_e \geq eu(q_{e'}) - z_{e'} \text{ for all } e, e' \in [0,1], & \text{(IC)} \\ eu(q_e) - z_e \geq 0. & \text{(PC)} \end{cases}$$

The four constraints are the non-negativity constraints (NC), the cash constraints (CC), the incentive constraints (IC) and the participation constraints (PC), respectively.[5] The formulation of the ICs and the PCs uses the property that $W^b(z)$ is linear in z.

In the following, we simplify the seller's problem (1) in several steps. First, note that we can ignore the PCs for all $e > 0$ because they are implied by the PC for $e = 0$ and the ICs. In addition, the PC binds for buyers with $e = 0$. Second, we use the result from Mas-Collell, Winston and Green (1995, Proposition 23.D.1, page 888) to find the necessary and sufficient conditions to guarantee that the ICs are satisfied. Then we replace the ICs with these conditions and convert the seller's problem into an optimal control problem.[6]

[5] It may not be obvious at first glance why the seller should take the buyer's cash constraint into consideration: he may include some (q, z) pairs with $z > \hat{z}$ to help to align the buyer's incentives. However, a second thought makes it clear that such choices will not be effective. Pairs with $z > \hat{z}$ are beyond the means of the buyer and will be disregarded by the buyer, and therefore, will be irrelevant for the purpose of incentive alignment. As a result, when the seller posts terms of trade, only pairs with $z \leq \hat{z}$ will be included.

[6] A similar technique is also adopted in Faig and Jerez (2006) and Thomas (2002). In Faig and Jerez,

Let $v_e \equiv eu(q_e) - z_e$ denote the buyer's *ex post* trading surplus when the realization of the preference shock is e. The ICs are equivalent to:

$$\begin{aligned} dq_e/de &\geq 0, \\ dv_e/de &= u(q_e). \end{aligned}$$

Using v_e as the state variable and q_e as the control variable, we can restate the seller's problem as the following[7]

$$\max_{\{q_e, v_e\}_{e\in[0,1]}} \int_0^1 [-c(q_e) + eu(q_e) - v_e]de$$

$$\text{s.t.} \begin{cases} q_e \geq 0, & \text{(NC)} \\ eu(q_e) - v_e \leq \hat{z}, & \text{(CC)} \\ dq_e/de \geq 0, & \text{(IC1)} \\ dv_e/de = u(q_e), & \text{(IC2)} \\ v_0 = 0. & \text{(PC)} \end{cases}$$

The solution to the seller's problem is a schedule of $(q_e, z_e)_{e\in[0,1]}$ as a function of $\hat{z}$.

Define $\bar{z}_1$ as[8]

$$\bar{z} = \frac{1}{2}\left[u(q_1^*) + c(q_1^*)\right] \tag{2}$$

where q_1^* solves $u'(q_1) = c'(q_1)$. Lemma 1 fully characterizes the seller's optimal pricing schedule as a function of $\hat{z}$.

Lemma 1 *For any given $\hat{z} \leq \bar{z}$, the optimal solution $[q_e(\hat{z}), z_e(\hat{z}), v_e(\hat{z})]$ for all $e \in [0,1]$ to the seller's pricing problem is unique and is characterized by:*

(i) For $e \in [0, e_0]$, $q_e = z_e = v_e = 0$;

(ii) For $e \in [e_0, \hat{e}]$,

$$\begin{aligned} q_e &: (2e - 2\hat{e} + \hat{e}^2)u'(q_e) = c'(q_e), && (3) \\ v_e &: v_e = \left(e - \hat{e} + \frac{\hat{e}^2}{2}\right)u(q_e) - \frac{1}{2}c(q_e), \\ z_e &: z_e = \left(\hat{e} - \frac{\hat{e}^2}{2}\right)u(q_e) + \frac{1}{2}c(q_e); && (4) \end{aligned}$$

the technique is used to solve the seller's problem in a competitive search model where buyers have private information about the realization of preference shocks. Thomas uses the technique to solve a static decision problem of seller facing a consumer with unknown demand and a budget constraint.

[7] Note that the objective function of the seller is rewritten by using the property that $W^s(z)$ is linear in z.

[8] As discussed in the proof of Lemma 1 in the Appendix, $\bar{z}$ is the amount of real cash balances charged to a buyer who has the highest realization of the preference shock and buys the efficient amount. If buyers hold more than $\bar{z}$ real cash balances, they will not be cash constrained in any realization of e. It will be clear later in this section that with a positive nominal interest rate, buyers will choose $\hat{z} < \bar{z}$.

(iii) For $e \in [\hat{e}, 1]$,

$$\begin{aligned} q_e &: q_e = \hat{q} \text{ where } \hat{e}^2 u'(\hat{q}) = c'(\hat{q}), & (5)\\ z_e &: z_e = \hat{z}, & (6)\\ v_e &: v_e = eu(\hat{q}) - \hat{z}; \end{aligned}$$

(iv) e_0 *and* $\hat{e}$ *are given by*

$$\hat{z} = (\hat{e} - \frac{\hat{e}^2}{2})u(\hat{q}) + \frac{1}{2}c(\hat{q}) \quad (7)$$

$$e_0 = \hat{e} - \frac{\hat{e}^2}{2} \quad (8)$$

For $\hat{z} > \bar{z}$, *(7) is replaced by* $\hat{e} = 1$.

Proof. See the Appendix ■

3.3 Monetary Equilibrium

Given the seller's optimal pricing schedule in the DM, we are now ready to derive the buyer's demand for money in the CM. As the buyer knows how (q_e, z_e, v_e) depends on $\hat{z}$, the buyer's value function in the DM is

$$\begin{aligned} V^b(\hat{z}) &= \int_0^1 [eu(q_e(\hat{z})) + \hat{z} - z_e(\hat{z})]de + W^b(0) \\ &= S(\hat{z}) + \hat{z} + W^b(0), \end{aligned}$$

where $S(\hat{z}) \equiv \int_0^1 v_e(\hat{z})de$ is the buyer's expected trading surplus in the DM. In the CM, the buyer's choice of $\hat{z}_+$ satisfies

$$\frac{\phi}{\phi_+} = \beta \frac{\partial V^b(\hat{z}_+)}{\partial \hat{z}_+} = \beta \left[S'(\hat{z}_+) + 1 \right].$$

We will focus on the steady state equilibrium where $\hat{z}$ is constant and $\phi/\phi_+ = \gamma$. The equilibrium $\hat{z}$ is given by

$$\frac{\gamma}{\beta} - 1 = S'(\hat{z}). \quad (9)$$

The nominal interest rate is determined by the Fisher equation, $i = \gamma/\beta - 1$. As long as $\gamma > \beta$ or the nominal interest rate is positive, the cash constraint will bind for buyers with high enough preferences, so the optimal $\hat{z}$ cannot be greater than $\bar{z}$. The buyer's choice of $\hat{z}$ from (9), together with the seller's optimal pricing schedule, completes the description of the monetary equilibrium under price posting.

Definition 1 *In the steady state, a price posting monetary equilibrium is a list of* $[q_e(\hat{z}), z_e(\hat{z})]$

for all $e \in [0,1]$ and $\hat{z}$ such that [1] $[q_e(\hat{z}), z_e(\hat{z})]$ maximizes $V^s(0)$ given $\hat{z}$ (characterized by (3),(4),(5),(6), (7) and (8)) ; and [2] $\hat{z}$ maximizes $W^b(z)$ (characterized by (9)).

Proposition 1 *There exists a unique monetary equilibrium for generic values of γ.*

Proof. We first show that knowing how (q_e, z_e) depends on $\hat{z}$, buyers choose a unique $\hat{z}$ in the CM. Differentiating $S(\hat{z})$,

$$S'(\hat{z}) = \frac{1}{2}(1-\hat{e}^2)u'(\hat{q})\frac{d\hat{q}}{d\hat{z}} - \frac{1}{2}(1-\hat{e})\left[\hat{e}^2 u(\hat{q}) - c(\hat{q})\right]\frac{d\hat{e}}{d\hat{z}} - (1-\hat{e}),$$

where $d\hat{q}/d\hat{z}$ and $d\hat{e}/d\hat{z}$ are solved by differentiating (5) and (7) with respect to $\hat{z}$. Note that $S'(\hat{z})$ does not directly depend on i. Following the proof in Wright (2008), $\hat{z}$ is unique for generic values of γ. From Lemma 1, a unique $\hat{z}$ leads to a unique pricing schedule. In addition, one can show that $S'(0) = \infty$. Hence, a monetary equilibrium exists and is unique for generic values of γ.■

Jean et al. (2010) show that the existence of multiple equilibria is quite robust in monetary models under price posting. They also point out that multiplicity of monetary equilibria depends on the assumption of the timing. Multiple equilibria are more likely to occur when buyers and sellers move simultaneously. In our model, buyers move first by choosing their money holdings. Sellers then post the terms of trade after observing the buyer's choice of money balances. The timing leads to a unique monetary equilibrium, which is consistent with the discussion in Jean et al. (2010).

In price posting equilibrium, all buyers choose the same $\hat{z}$ in the CM. As long as $\gamma < \infty$, we have $\hat{z} > 0$, and $\hat{e}, e_0$ and $(\hat{e} - e_0)$ are all positive. A continuum of prices is observed when preference shocks follow a uniform distribution. The number of prices observed in equilibrium is measured by $\hat{e} - e_0$. This is in contrast to Ennis (2008), where only a single price is observed with a two-point distribution of preference shocks. This is also different from Jafarey and Masters (2003), and Curtis and Wright (2004), where at most two prices are observed in equilibrium because money is indivisible. The "law of two prices" proposed in Curtis and Wright (2004) cannot be generalized to models with divisible money and a continuous distribution of preference shocks. Compared with the previous papers on price posting, our model generates richer pricing schedule (on the seller's side) and spending pattern (on the buyer's side).

In equilibrium, buyers with $e \in [0, e_0]$ choose not to spend money and consume nothing.[9] Buyers in this group do not value consumption much. As a result, it is better for them to hold on to their money balances and wait for future consumption opportunities. For buyers with $e \in [\hat{e}, 1]$, the cash constraint binds and buyers exhaust their money holdings during exchanges. Buyers with $e \in [e_0, \hat{e}]$ spend part of their money holdings and as e increases,

[9] We model hetergeneous preferences differently from Thomas (2002). In Thomas (2002), preference heterogeneity enters linearly into the utility function and every buyer chooses to spend some money no matter how low the buyer's preference is.

q_e and z_e both increase. The (real) unit price charged by the seller depends on the size of the purchase as follows:

$$\frac{z_e}{q_e} = \left(\hat{e} - \frac{\hat{e}^2}{2}\right)\frac{u(q_e)}{q_e} + \frac{1}{2}\frac{c(q_e)}{q_e} \text{ for } e \in [e_0, \hat{e}]. \tag{10}$$

The next proposition establishes that larger quantities are associated with lower per unit prices when the cost function is linear.

Proposition 2 *Quantity discounts: when* $c(q) = q$, $d(z_e/q_e)/dq_e < 0$ *for* $e \in [e_0, \hat{e}]$.

The proof follows directly from (10). This result appears to be consistent with the pricing strategy in reality. One common practice used by sellers is to offer quantity discounts, i.e., lower unit prices for larger purchases. The traditional explanation for quantity discounts is the economies of scale. Sellers charge low unit prices for large quantities because the unit cost for production and/or sale of larger quantities is lower. Here, quantity discounts are driven by private information about the buyer's preference and exist even if the unit cost is constant. To see this more clearly, consider the case where the sellers knows the buyer's preferences. In this situation, the seller offers (q_e, z_e) to maximize $-c(q_e) + z_e$ subject to $eu(q_e) - z_e = 0$. The solution to the problem is characterized by $eu'(q_e) = c'(q_e)$ and $z_e = eu(q_e)$. In the absence of private information, $z_e/q_e = eu(q_e)/q_e$. There are two factors that determine the unit price: e and $u(q_e)/q_e$. The first factor induces quantity premium: the seller charges a high price if he knows that the buyer likes the good very much. The second factor leads to quantity discounts and is due to diminishing marginal utility. When the seller knows the buyer's preference, the first factor may dominate so that the unit price may increase with e, or equivalently q_e. The existence of private information prevents the seller from exploiting buyers with high preferences and the first factor disappears (note that in (10), the term before $u(q_e)/q_e$ is constant for all e).

The result that private information may induce quantity discounts can also be found in non-monetary models. Examples are Maskin and Riley (1984), Che and Gale (2000), Thomas (2002), and Faig and Jerez (2005). A significant difference of this paper from those papers is that there is an endogenous budget constraint faced by buyers. Furthermore, the budget constraint will be affected by the inflation rate, as we will show in the next section. Another difference is that the study of a general equilibrium model generates testable implications about how aggregate macroeconomic conditions such as inflation affect the degree of quantity discounts. Suppose the cost function is linear and the utility function is CRRA with the RRA coefficient between 0 and 1. One can show that $\frac{d[d(z_e/q_e)/dq_e]}{d\gamma} > 0$, which means that the degree of quantity discounts decreases when inflation increases. That is, inflation leads to more linear pricing. It would be interesting to see if this result is supported by data.

Price posting is also important for the existence of quantity discounts. If we change the trading protocol to buyer's take-it-or-leave-it offers or competitive pricing, linear prices will be observed when the unit cost is constant.[10]

4 Hot Potato Effect of Inflation

In this economy, it is easy to verify that the Friedman rule cannot achieve the first-best allocation due to the hold-up problem. Inflation has a significant effect on the buyer's spending pattern. First, inflation affects the buyer's choice of real money balance or the budget constraint. Second, inflation affects the speed at which buyers spend money.

Proposition 3 *The effects of inflation on* $(\hat{z}, e_0, \hat{e})$*:* $d\hat{z}/d\gamma < 0$, $de_0/d\gamma < 0$, *and* $d\hat{e}/d\gamma < 0$.

Proof. Consider $\hat{z} = \bar{z}$. As $\hat{e} = 1$, it is straightforward that $S'(\bar{z}) = 0$. The optimal $\hat{z}$ satisfies $S'(\hat{z}) + 1 - \frac{\gamma}{\beta} = 0$. Since we focus on $\hat{z} \leq \bar{z}$, it implies that $S'(\bar{z}) + 1 - \frac{\gamma}{\beta} < 0$ for $\gamma > \beta$ and $\gamma \to \beta$ from above. As $S'(0) = \infty$, we know that $S'(0) + 1 - \frac{\gamma}{\beta} = \infty$. For $\hat{z} \in [0, \bar{z}]$, a generically unique optimal $\hat{z}$ implies that $S''(\hat{z}) < 0$. From $S'(\hat{z}) + 1 - \frac{\gamma}{\beta} = 0$, $\frac{d\hat{z}}{d\gamma} = \frac{1}{\beta S''(\hat{z})} < 0$.

Using (5), one can find $\frac{d\hat{q}}{d\gamma}$ in terms of $\frac{d\hat{e}}{d\gamma}$, which is substituted into (7) to get $\frac{d\hat{e}}{d\gamma} < 0$. It follows from (8) that $\frac{de_0}{d\gamma} < 0$ as well.■

Proposition 4 *The effects of inflation on* $(q_e, z_e, z_e/\hat{z})$ *for* $e \in [e_0, \hat{e}]$*:* $dq_e/d\gamma > 0$, $dz_e/d\gamma > 0$, *and* $d(z_e/\hat{z})/d\gamma > 0$ *if* $u''' \geq 0$ *and* $c''' \leq 0$.

Proof. As $d\hat{e}/d\gamma < 0$, proving $dq_e/d\gamma > 0$ and $dz_e/d\gamma > 0$ is equivalent to proving $dq_e/d\hat{e} < 0$ and $dz_e/d\hat{e} < 0$. We do this by proving $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$ and $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$, or $q_e(e;\hat{e})$ and $z_e(e;\hat{e})$ are steeper for lower values of $\hat{e}$.

We first show that $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$. Use (3) that characterizes q_e for $e \in [e_0, \hat{e}]$ to calculate dq_e/de as (to simplify notations, we omit the arguments of the functions $u(\cdot)$ and $c(\cdot)$):

$$\frac{dq_e}{de} = \frac{2u'^2}{c''u' - c'u''} > 0$$

The term $\frac{(u')^2}{c''u'-c'u''}$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$:

$$\begin{aligned}
\frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{d\hat{e}} &= \frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{dq_e}\frac{dq_e}{d\hat{e}} \\
&= \frac{2u'(c''u'-c'u'') - u'^3c''' + u'^2c'u'''}{(c''u'-c'u'')^2}\frac{dq_e}{d\hat{e}} \\
&< 0 \text{ if } c''' < 0 \text{ and } u''' > 0.
\end{aligned}$$

[10] It can be shown that when buyers have private information about their preferences, quantity discounts can also be observed under price posting with directed search (or competitive search) when the unit cost is constant.

Now we show $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$. Recall that $dz_e/de = eu'(q_e)dq_e/de$, so

$$\frac{dz_e}{de} = \frac{eu'^3}{c''u' - c'u''} > 0.$$

The term $\frac{eu'^3}{c''u'-c'u''}$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$ because

$$\begin{aligned} & \frac{d\left(\frac{eu'^3}{c''u'-c'u''}\right)}{dq_e} \\ = \quad & e\frac{3u'^2(c''u' - c'u'') - u'^4c''' + u'^3c'u'''}{(c''u' - c'u'')^2} \\ > \quad & 0 \text{ if } c''' < 0 \text{ and } u''' > 0. \end{aligned}$$

As $dz_e/d\gamma > 0$ and $d\hat{z}/d\gamma < 0$, $d(z_e/\hat{z})/d\gamma > 0$.■

Here, we have the standard result that inflation decreases the demand for real money balances and makes the cash constraint more stringent. Besides that, the model provides an intuitive explanation about the "hot potato" effect of inflation, which means that buyers spend money faster when inflation is higher. In price posting equilibrium, buyers with higher preferences spend more while those with lower preferences spend less and retain some money for future exchanges. Inflation reduces the future purchasing power of money and the benefit of retaining money. Buyers respond by buying more goods and speeding up spending. The model can capture the hot potato effect along both the extensive and intensive margins. As shown in Proposition 3, inflation reduces e_0, implying that more buyers start spending money. Proposition 4 implies that for those who spend money, higher inflation induces them to spend more money, captured by higher q_e and $z_e/\hat{z}$. It naturally follows that the aggregate speed of money spending defined as $\int_e \frac{z_e}{\hat{z}} de$ increases with inflation. Refer to Figure 1 for a graphical illustration of the hot potato effect of inflation.

There are several recent attempts to rationalize the hot potato effect of inflation. Lagos and Rocheteau (2005) endogenize search intensity, and they show that inflation may increase a buyer's trading surplus. As a result, buyers may search more intensively which increases the probability of spending. The problem is that the result holds only for low inflation rates and a particular pricing mechanism – competitive search. Bargaining cannot deliver similar results because inflation monotonically reduces the buyer's surplus in a match and thus the search intensity.

Ennis (2009) assumes that sellers have more opportunities to rebalance money holdings than buyers (i.e., they have more frequent access to the centralized market than buyers), inflation makes buyers search harder to find sellers to off-load their money.

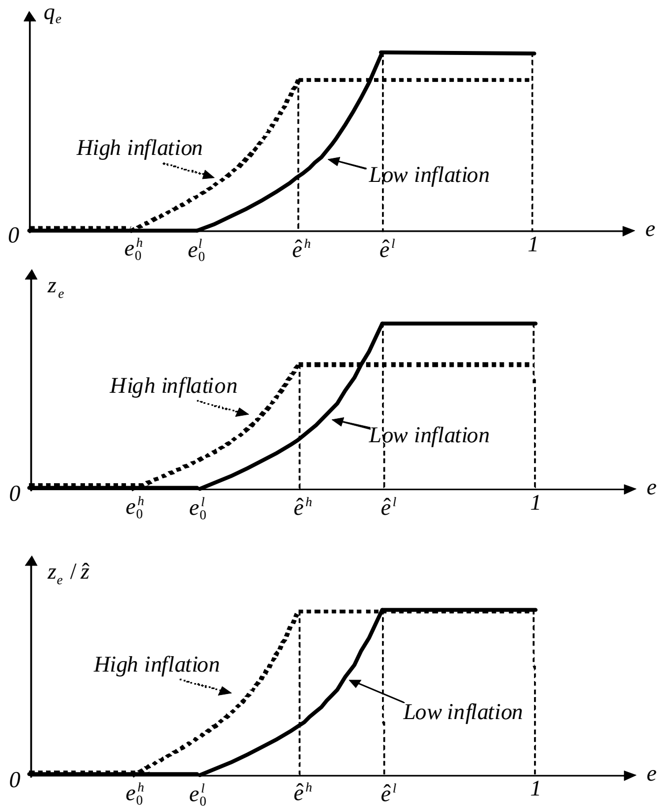

Figure 1: Hot Potato Effect of Inflation

More recently, Liu *et al.* (forthcoming) assume that buyers pay the entry fee to enter the DM. Inflation reduces buyers' trading surplus, so fewer buyers choose to enter. For those who do enter, the matching probability becomes higher. Buyers are able to spend faster simply because they have more opportunities to trade, not because they actively try to get rid of their money. Furthermore, the result that the matching probability increases with inflation depends on whether the free entry is assumed by buyers or sellers. For example, in Rocheteau and Wright (2005), sellers pay the entry fee and inflation monotonically reduces buyers' trading probability. To explain the hot potato effect of inflation, it is not clear that one should assume the entry fee is paid by buyers rather than by sellers.

In Nosal (forthcoming), it is assumed that accepting a current trade involves an exogenous cost that lowers the probability of future trading. Inflation reduces the value of future trading and people are more likely to accept a current trade. Our paper and Nosal (forthcoming) both emphasize that buyers spend faster in fear that money will lose purchasing power in the future due to inflation. We provide a more integrated model and do not rely on assuming the exogenous reduction of future trading probability as a consequence of accepting a current trade. In our model, the cost of buying more today is less money to spend in the future.

Compared with the existing literature, the mechanism in our model is closest to the narrative description of the hot potato effect: people rush to buy more and spend money faster in fear that the value of the money left in their pocket will depreciate quickly. The result that inflation raises the quantity of purchases for some buyers is a rare result in monetary models.[11] Our model also generates a robust hot potato effect that exists at all levels of inflation. We provide an intuitive and integrated explanation without exogenous assumptions about who pays the entry fee, who has better access to the centralized market, or whether current trade reduces the probability of future trading. Finally, our model captures the hot potato effect along both the extensive margin and the intensive margin.

5 Extension to More General Distributions

The above analysis assumes that the distribution of preference shocks follows a uniform distribution. In this section, we extend the model by allowing preference shocks to follow a more general continuous distribution. Let $f(e)$ and $F(e)$ denote the p.d.f. and the c.d.f. of the distribution, respectively. Without loss of generality, we still focus on $e \in [0, 1]$. We assume that $ef(e)$ is increasing in e, i.e., $f(e) + ef'(e) > 0$.[12]

5.1 Seller's Price Posting Decision

The choice problems for buyers and sellers in each market remain the same as before. We first solve for the seller's optimal price-posting problem, taking $\hat{z}$ as given. Similar to the case with a uniform distribution, define $\bar{z}$ as

$$\bar{z} = u(q_1^*) - \int_{e_0}^{1} u(q_x)dx$$

where q_1^* is defined as before and $\left(e_0, q_{x\in[e_0,1]}\right)$solves

$$\begin{aligned} e_0 f(e_0) + F(e_0) &= 1, \\ \left[e - \frac{1 - F(e)}{f(e)}\right] u'(q_x) &= c'(q_x). \end{aligned}$$

We characterize the solution to the seller's problem in Lemma 2.

Lemma 2 *For any given $\hat{z} \leq \bar{z}$, the optimal solution (q_e, z_e, v_e) for all $e \in [0, 1]$ to the seller's price-posting problem is unique and is characterized by*

[11] Exceptions are Peterson and Shi (2004) and Faig and Jerez (2006). Both papers feature buyers having private information about match specific preference shocks. Inflation induces some buyers to purchase more goods. Perterson and Shi (2004) use the large household model of money and the terms of trade are determined by the buyer's take-it-or-leave-it offer. In Faig and Jerez (2006), the terms of trade in the DM are determined by competitive search.

[12] Alternatively, if the distribution has a monotonic hazard rate, i.e., $\frac{f(e)}{1-F(e)}$ increases e, all the results in this section go through.

(i) For $e \in [0, e_0]$, $q_e = z_e = v_e = 0$;

(ii) For $e \in [e_0, \hat{e}]$,

$$\begin{aligned} q_e &: \left[e - \frac{1 - F(e) - \Sigma_{\hat{e}}}{f(e)} \right] u'(q_e) = c'(q_e), && (11) \\ \Sigma_{\hat{e}} &: \Sigma_{\hat{e}} = \frac{[1 - F(\hat{e})]^2}{\hat{e} f(\hat{e}) + 1 - F(\hat{e})}, \\ z_e &: z_e = eu(q_e) - v_e, \\ v_e &: v_e = \int_0^e u(q_e) dF(e); \end{aligned}$$

(iii) For $e \in [\hat{e}, 1]$,

$$\begin{aligned} q_e &: q_e = \hat{q} \textit{ where } \left[\hat{e} - \frac{1 - F(\hat{e}) - \Sigma_{\hat{e}}}{f(\hat{e})} \right] u'(\hat{q}) = c'(\hat{q}), && (12) \\ z_e &: z_e = \hat{z}, \\ v_e &: v_e = eu(\hat{q}) - \hat{z}; \end{aligned}$$

(iv) e_0 *and* $\hat{e}$ *are given by*

$$\hat{e} u(\hat{q}) - \int_{e_0}^{\hat{e}} u(q_x) dx = \hat{z}, \tag{13}$$

$$e_0 f(e_0) + F(e_0) - 1 + \Sigma_{\hat{e}} = 0. \tag{14}$$

For $\hat{z} > \bar{z}$, *(13) is replaced by* $\hat{e} = 1$.

Proof. See the Appendix.■

With a general continuous distribution, the pricing schedule posted by sellers is again to extract buyers' private information about their preference shocks. Low-preference buyers optimally choose to hold on to their money balances and wait for future consumption opportunities. The endogenous extensive margin effect still exists. Buyers with preferences higher than e_0 but below $\hat{e}$ spend part of their money balances and consume more as e increases. Buyers with $e \in [\hat{e}, 1]$ spend all their money. Similar comparative statics results also apply.

Proposition 5 *The seller's optimal pricing schedule has the following properties:* $d\hat{e}/d\hat{z} > 0$, $d\hat{q}/d\hat{z} > 0$, $de_0/d\hat{z} > 0$ *and* $dq_e/d\hat{z} < 0$ *for* $e \in [e_0, \hat{e}]$.

Proof. See the appendix.■

Proposition 6 *Quantity discounts: when* $c(q) = q$, $d(z_e/q_e)/dq_e < 0$ *for* $e \in [e_0, \hat{e}]$.

Proof. See the appendix.■

5.2 Equilibrium

After deriving the seller's pricing schedule in the DM, we can solve the buyer's demand for money in the CM from (9). The definition of a price posting equilibrium remains the same as in Definition 1. Since $S(\hat{z})$ and $S'(\hat{z})$ do not directly depend on i, a unique monetary equilibrium exists for generic values of γ. Recall that $ef(e)$ is increasing in e. From (14), one can show that $1 - \Sigma_{\hat{e}} < \hat{e}f(\hat{e}) + F(\hat{e})$ and hence $e_0 < \hat{e}$. There is a continuum of prices observed in equilibrium, which confirms that the law of two prices in Curtis and Wright (2004) cannot be generalized to a divisible money framework with a general continuous distribution.

Proposition 7 *The effects of inflation on* $(\hat{z}, e_0, \hat{e})$*:* $d\hat{z}/d\gamma < 0$*,* $de_0/d\gamma < 0$ *and* $d\hat{e}/d\gamma < 0$*.*

Proof. See the appendix.■

Proposition 8 *The effects of inflation on* $(q_e, z_e, z_e/\hat{z})$ *for* $e \in [e_0, \hat{e}]$*:* $dq_e/d\gamma > 0$*,* $dz_e/d\gamma > 0$*, and* $d(z_e/\hat{z})/d\gamma > 0$ *if* $u''' \geq 0$ *and* $c''' \leq 0$ *and* $f'(e) \leq 0$*.*

Proof. See the appendix.■

Inflation reduces buyers' choice of the real money balance. Inflation also induces more buyers to spend their money. The model with a more general distribution can still capture the hot potato effect along the extensive margin. To show the hot potato effect along the intensive margin, we need to impose the assumption that $f'(e) \leq 0$.

To summarize, most of our results from a uniform distribution of preference shocks remain valid when the distribution of preference shocks is extended to a more general continuous distribution. The price-posting equilibrium captures commonly observed phenomena such as quantity discounts and the hot potato effect of inflation. It also verifies that the divisibility of money matters: the law of two prices found in the indivisible money framework no longer holds when money becomes divisible.

6 Discussion

We study price posting with undirected search in this paper. Price posting is often combined with directed search in the literature. The key difference between undirected search and directed search is that buyers observe the pricing schedule before they choose their money balances and can direct their search to a particular pricing schedule under directed search, whereas buyers have no information of the pricing schedule when they choose their money balances under undirected search.

Faig and Jerez (2006) study private information in a very similar environment except that undirected search is replaced by directed search, which is labelled as competitive search. They show that there exists a continuum of price-quantity pairs in competitive search monetary equilibrium. Despite the similarity, there are some important differences in the properties of these two equilibria.

Theoretically, compared to Faig and Jerez (2006), the price posting equilibrium in our paper endogenously generates an extensive margin due to the existence of ex post participation constraints.[13] Our model predicts that inflation induces more buyers to participate in trading, whereas all buyers always trade in competitive search equilibrium. As a result, inflation affects output and welfare in different ways. Under competitive search, inflation affects the economy through only the intensive margin and the Friedman rule completely removes the inefficiency. Under undirected search, the effect of inflation is more complicated: it hurts the intensive margin, but improves the extensive margin by encouraging more buyers to trade.

Quantitatively, since sellers have the power to propose the pricing schedule, the holdup problem in price posting equilibrium is more severe than in competitive search equilibrium. Using the consumption equivalence measure, we find that the welfare cost of 10% inflation against the Friedman rule is 7.24%.[14] In contrast, the welfare cost of 10% inflation against the Friedman rule in Faig and Jerez (2006) is less than 1%, which further shows that competitive search is a more efficient pricing mechanism.

7 Conclusion

In this paper, we develop a search theoretic monetary model where sellers post prices and search is undirected. It contributes to the literature on price posting by considering a continuous distribution of the match specific preference shocks in a divisible money framework. We show that there exists a unique monetary equilibrium under price posting. Unlike the predictions from the indivisible money framework or the divisible money framework with a two-point distribution of preference shocks, equilibrium is characterized by a continuum of price-quantity pairs and the law of two prices as emphasized in Curtis and Wright (2004) does not hold. Compared with earlier models of price posting, our model generates a richer pricing schedule and spending pattern.

The equilibrium pricing schedule exhibits quantity discounts, a commonly observed practice. The presence of private information about preferences implies that the seller prefers to offer lower unit prices to induce buyers with higher preferences to buy more even if the

[13] It can be shown that v_0 is negative in competitive search equilibrium. One can also interpret the competitive search equilibrium without participation constraints as the following two-part payment scheme. Suppose that market makers can charge all buyers an entry fee of $-v_0$ to the submarkets before the realization of the preference shocks. This ensures that the pricing schedule generates non-negative trading surplus for buyers at the trading stage. Ex ante, all buyers have the incentive to pay the entry fee. Ex post, all buyers choose a price-quantity pair voluntarily with non-negative payoff.

[14] To estimate the welfare cost of inflation, we modify the model to allow buyers have quasilinear preferences as in Lagos and Wright (2005) to be consistent with other papers on the welfare cost of inflation in this literature. We also use the nonlinear least square following Lagos and Wright (2005) to find the parameter values.

unit cost of production is constant.

In terms of the buyer's spending pattern, the model provides a natural explanation of the hot potato effect of inflation: buyers spend money faster as inflation rises. In the presence of preference shocks, buyers start spending only if they have high enough preferences, and they spend more when they have higher preferences till they are cash constrained. Those with low preferences choose to retain some money for future spending. When inflation is higher, fearing that their money will lose value quickly if they wait, buyers speed up spending. Among existing theories on the hot potato effect, our explanation resembles most closely the narrative description of the phenomenon. We are also able to capture the hot potato effect along both the extensive margin (more buyers start to spend) and the intensive margin (those who spent before spend more) at all levels of inflation.

A Appendix

The appendix includes proofs for the seller's optimal pricing schedule when the shock follows a uniform distribution, and for all the results in section 5 where preference shocks follow more general continuous distributions.

A.1 Proof of Lemma 1: Seller's Optimal Price Posting with a Uniform Distribution

Proof. We first disregard the IC1 $dq_e/de \geq 0$ (and we will impose it later) and find the condition that characterize the optimal choice of (q_e, z_e). The Hamiltonian of the optimal control problem is

$$H = [-c(q_e) + eu(q_e) - v_e] + \lambda_e u(q_e),$$

where λ_e is the co-state variable. The Lagrangian is

$$\begin{aligned}\mathcal{L} = &-c(q_e) + eu(q_e) - v_e + \lambda_e u(q_e)\\ &+ \mu_e[\hat{z} - eu(q_e) + v_e] + \theta_e q_e,\end{aligned}$$

where μ_e and θ_e are the Lagrangian multipliers associated with the CC and the NC, respectively. The first-order conditions are

$$\begin{aligned}
q_e &: (e + \lambda_e - \mu_e e)u'(q_e) = c'(q_e) - \theta_e, && (15)\\
v_e &: -1 + \mu_e = -\frac{d\lambda_e}{de}, && (16)\\
\lambda_e &: u(q_e) = \frac{dv_e}{de}, && (17)\\
\mu_e &: \hat{z} - eu(q_e) + v_e \geq 0, \text{ if } > 0, \text{ then } \mu_e = 0, && (18)\\
\theta_e &: q_e \geq 0, \text{ if } > 0, \text{ then } \theta_e = 0. && (19)
\end{aligned}$$

The transversality condition is $\lambda_1 v_1 = 0$. In a monetary equilibrium, $v_1 > 0$ and $\lambda_1 = 0$. Integrating $d\lambda_x/dx$ over the interval $[e, 1]$, we have

$$\begin{aligned}
\int_e^1 \frac{d\lambda_x}{dx}dx &= \int_e^1 (1 - \mu_x)dx\\
&\to \lambda_1 - \lambda_e = 1 - e - \int_e^1 \mu_x dx\\
&\to \lambda_e = (e-1) + \Sigma_e \text{ with } \Sigma_e \equiv \int_e^1 \mu_x dx,
\end{aligned}$$

where the last step uses the transversality condition. Substituting λ_e into (15),

$$[(2e-1) + \Sigma_e - \mu_e e]u'(q_e) = c'(q_e) - \theta_e. \quad (20)$$

Now we impose the constraint $dq_e/de \geq 0$ (and as a result $dz_e/de \geq 0$). Given this, we can consider the seller's problem in three regions of e divided by $0 \leq e_0 \leq \hat{e} \leq 1$.

Case (a). When e is small ($e \leq e_0$), no exchange occurs or $q_e = z_e = v_e = 0$. NC binds and CC is loose.

Case (b). For intermediate values of e ($e_0 \leq e \leq \hat{e}$), neither NC nor CC binds, i.e., $\mu_e = \theta_e = 0$. In this case, (20) reduces to[15]

$$[(2e-1) + \Sigma_{\hat{e}}]u'(q_e) = c'(q_e), \tag{21}$$

which can be used to solve q_e. Since $u'' < 0 \leq c''$, the solution to (21) satisfies $dq_e/de > 0$. Therefore, we can use (21) to characterize the solution of q_e for $e_0 \leq e \leq \hat{e}$.

Case (c). When e is high ($e \geq \hat{e}$), the buyer is charged all his money holding, or $z_e = \hat{z}$ and $q_e = \hat{q}$. We can solve $\hat{q}$ from

$$[(2\hat{e}-1) + \Sigma_{\hat{e}}]u'(\hat{q}) = c'(\hat{q}). \tag{22}$$

In the next step, we will find the term $\Sigma_{\hat{e}}$ in (21) and (22) as a function of $\hat{e}$. Since $q_e = \hat{q}$ for all $e > \hat{e}$, $\hat{q}$ also solves

$$[2e - 1 + \Sigma_e - \mu_e e]\, u'(\hat{q}) = c'(\hat{q}). \tag{23}$$

Combining (22) and (23), and using $\mu_e = -\frac{d\Sigma_e}{de}$, we have a differential equation of Σ_e,

$$2\hat{e} + \Sigma_{\hat{e}} = 2e + \Sigma_e + e\frac{d\Sigma_e}{de}. \tag{24}$$

Together with the end-point conditions $\mu_{\hat{e}} = 0$ and $\Sigma_1 = 0$, (24) gives rises to the following result[16]

$$\begin{aligned} \Sigma_e &= \left[(1-e) + \hat{e}^2\left(1 - \frac{1}{e}\right)\right], \\ \Sigma_{\hat{e}} &= (1-\hat{e})^2, \\ \mu_e &= \left[1 - (\frac{\hat{e}}{e})^2\right]. \end{aligned} \tag{25}$$

The final step to complete the solution to the seller's problem is to determine e_0 and $\hat{e}$ as functions of $\hat{z}$. Note that e_0 can be expressed as a function of $\hat{e}$ and is determined by

$$[(2e_0 - 1) + \Sigma_{\hat{e}}]u'(0) = c'(0) \text{ or } e_0 = \hat{e} - \frac{\hat{e}^2}{2}.$$

As a result, it suffices to find $\hat{e}$ as a function of $\hat{z}$. From (21) and the definition of v_e, $\hat{e}$ is

[15] Notice that since $\mu_e = 0$ for $e \in [0, \hat{e}]$, we have $\Sigma_e = \Sigma_{\hat{e}}$ for $e \in [0, \hat{e}]$.

[16] We solve (24) by a guess-and-verify method. We guess that $\mu_e = 1 + k/e^2$ and find that $k = -\hat{e}^2$.

solved from[17]

$$\hat{z} = (\hat{e} - \frac{\hat{e}^2}{2})u(\hat{q}) + \frac{1}{2}c(\hat{q}).$$

In general, it is possible that $\hat{e}$ takes the corner solution $\hat{e} = 1$ (so that no buyer is cash constrained) when $\hat{z}$ is large enough. This situation occurs when $\hat{z} \geq \bar{z}$ with $\bar{z}$ given by (2). ■

A.2 Proof of Lemma 2: Seller's Optimal Price Posting

Proof. The seller's problem is

$$\max_{q_e, v_e} \int_0^1 [-c(q_e) + eu(q_e) - v_e] dF(e)$$

$$\text{s.t.} \begin{cases} q_e \geq 0; & \text{(NC)} \\ eu(q_e) - v_e \leq \hat{z}; & \text{(CC)} \\ dq_e/de \geq 0; & \text{(IC1)} \\ dv_e/de = u(q_e); & \text{(IC2)} \\ v_0 = 0. & \text{(PC)} \end{cases}$$

for all $e \in [0, 1]$.

To proceed, we first ignore the constraint $dq_e/de \geq 0$. We will impose the constraint later to find the solution. The Hamiltonian of the optimal control problem is $H = [-c(q_e) + eu(q_e) - v_e]f(e) + \lambda_e u(q_e)$ where λ_e is the co-state variable. The Lagrangian is

$$\begin{aligned} \mathcal{L} &= f(e)[eu(q_e) - c(q_e) - v_e] + \lambda_e u(q_e) \\ &\quad + \mu_e[\hat{z} - eu(q_e) + v_e] \\ &\quad + \theta_e q_e, \end{aligned}$$

where μ_e and θ_e are the Lagrangian multipliers associated with the cash constraint and the

[17] Integrating both sides of (21) with respect to q_e over $[0, \hat{e}]$ gives $\int_0^{\hat{e}} [2e - 1 + \Sigma_{\hat{e}}] u'(q_e) dq_e = \int_0^{\hat{e}} c'(q_e) dq_e$ or $\int_0^{\hat{e}} 2eu'(q_e) dq_e - (1 - \Sigma_{\hat{e}}) u(\hat{q}) = c(\hat{q})$. Integration by parts on the first term gives $2\left[\hat{e}u(\hat{q}) - \int_0^{\hat{e}} u(q_e) de\right] - (1 - \Sigma_{\hat{e}}) u(\hat{q}) = c(\hat{q})$. Combining this with $v_e = \int_0^{\hat{e}} u(q_e) de = \hat{e}u(\hat{q}) - \hat{z}$ and (25), we have the equation implicitly defining $\hat{e}(\hat{z})$.

NC, respectively. The first-order conditions are

$$\begin{aligned}
q_e &: \; [ef(e) + \lambda_e - \mu_e e]\, u'(q_e) = f(e)c'(q_e) - \theta_e, && (26)\\
v_e &: \; -f(e) + \mu_e = -\frac{d\lambda_e}{de}, && (27)\\
\lambda_e &: \; u(q_e) = \frac{dv_e}{de}, && (28)\\
\mu_e &: \; \hat{z} - eu(q_e) + v_e \geq 0, \text{ if } > 0, \text{ then } \mu_e = 0, && (29)\\
\theta_e &: \; q_e \geq 0, \text{ if } > 0, \text{ then } \theta_e = 0. && (30)
\end{aligned}$$

The transversality condition is $\lambda_1 v_1 = 0$. For monetary equilibrium to exist, $v_1 > 0$ and $\lambda_1 = 0$. Integrating (27) over the interval $[e, 1]$, we have

$$\int_e^1 \frac{d\lambda_x}{dx} dx = F(1) - F(e) - \int_e^1 \mu_x dx$$

and $\lambda_1 - \lambda_e = 1 - F(e) - \Sigma_e$,where $\Sigma_e = \int_e^1 \mu_x dx$. Since we focus on monetary equilibria, it follows from the transversality condition that

$$\lambda_e = F(e) - 1 + \Sigma_e. \tag{31}$$

Plugging (31) into (26), we get

$$[ef(e) + F(e) - 1 + \Sigma_e - \mu_e e]\, u'(q_e) = f(e)c'(q_e) - \theta_e. \tag{32}$$

Since $dq_e/de \geq 0$ (and as a result $dz_e/de \geq 0$), we can discuss the solution to the seller's problem by dividing e into three regions characterized by two threshold values of e, e_0 and $\hat{e}$. We will consider three cases in the following.

Case (a). For $e \leq e_0$, the NC binds and $q_e = z_e = v_e = 0$.

Case (b). For $e \in [e_0, \hat{e}]$, neither NC nor CC binds or $\mu_e = \theta_e = 0$. (32) reduces to[18]

$$[ef(e) + F(e) - 1 + \Sigma_{\hat{e}}]\, u'(q_e) = f(e)c'(q_e) \tag{33}$$

Case (c). For $e \in [\hat{e}, 1]$, CC binds. In this case, $z_e = \hat{z}$, $q_e = \hat{q}$ with $\hat{q}$ solving

$$[\hat{e}f(\hat{e}) + F(\hat{e}) - 1 + \Sigma_{\hat{e}}]\, u'(\hat{q}) = f(\hat{e})c'(\hat{q}) \tag{34}$$

In the next step, we solve for $\Sigma_{\hat{e}}$ as a function of $\hat{e}$. To do this, note that for $e \in (\hat{e}, 1)$, CC binds and $q_e = \hat{q}$ solves

$$[ef(e) + F(e) - 1 + \Sigma_e - \mu_e e]\, u'(\hat{q}) = f(e)c'(\hat{q}) \tag{35}$$

[18]Notice that $\Sigma_e = \Sigma_{\hat{e}}$ for $e \in [0, \hat{e}]$.

Combining (34) and (35), we reach

$$\begin{aligned} & \hat{e}f(\hat{e})f(e) + F(\hat{e})f(e) - f(e) + f(e)\Sigma_{\hat{e}} \\ = \; & ef(e)f(\hat{e}) + F(e)f(\hat{e}) - f(\hat{e}) + f(\hat{e})\Sigma_e + \frac{d\Sigma_e}{de}ef(\hat{e}). \end{aligned} \tag{36}$$

where we use $\mu_e = -\frac{d\Sigma_e}{de}$. Rewriting (36) as

$$\frac{d\Sigma_e}{de} + \Sigma_e \frac{1}{e} = \left[\hat{e} + \frac{F(\hat{e})}{f(\hat{e})} - \frac{1}{f(\hat{e})} + \frac{\Sigma_{\hat{e}}}{f(\hat{e})}\right] \frac{f(e)}{e} - f(e) - \frac{F(e)}{e} + \frac{1}{e} \equiv Q(e),$$

which is a first-order differential equation of Σ_e. Solving the first-order differential equation, we have

$$\Sigma_e = \frac{1}{e} \int eQ(e)de.$$

and

$$e\Sigma_e = \left[\hat{e} + \frac{F(\hat{e})}{f(\hat{e})} - \frac{1}{f(\hat{e})} + \frac{\Sigma_{\hat{e}}}{f(\hat{e})}\right] F(e) - eF(e) + e + C \tag{37}$$

where C is a constant determined by the boundary condition $\Sigma_1 = 0$:

$$C = \frac{1}{f(\hat{e})} - \hat{e} - \frac{F(\hat{e})}{f(\hat{e})} - \frac{\Sigma_{\hat{e}}}{f(\hat{e})}.$$

We can solve $\Sigma_{\hat{e}}$ as a function of $\hat{e}$ by using (37) for $e = \hat{e}$:

$$\Sigma_{\hat{e}}(\hat{e}) = \frac{[1 - F(\hat{e})]^2}{\hat{e}f(\hat{e}) + 1 - F(\hat{e})} = \frac{1 - F(\hat{e})}{\frac{\hat{e}f(\hat{e})}{1 - F(\hat{e})} + 1} < 1 - F(\hat{e}). \tag{38}$$

Since $ef(e)$ increases in e, the numerator of $\Sigma_{\hat{e}}$ increases in $\hat{e}$. It follows that $\Sigma_{\hat{e}}$ decreases in $\hat{e}$.

Finally, we will prove that $dq_e/de > 0$ for $e \in [e_0, \hat{e}]$ to justify that the solution satisfies the constraint $dq_e/de \geq 0$. We rearrange the first-order condition for $q_{e \in [e_0, \hat{e}]}$ as

$$\frac{u'(q_e)}{c'(q_e)} = \frac{f(e)}{ef(e) + F(e) - 1 + \Sigma_{\hat{e}}} = \frac{\frac{1}{e}}{1 + \frac{F(e)-1}{ef(e)} + \frac{\Sigma_{\hat{e}}}{ef(e)}}.$$

As $\frac{1}{e}$ is decreasing in e, we only need to show that $\chi(e) = \frac{F(e)-1}{ef(e)} + \frac{\Sigma_{\hat{e}}}{ef(e)}$ is increasing in e. Since $\Sigma_{\hat{e}} < 1 - F(\hat{e})$ and $f(e) + ef'(e) > 0$,

$$\chi'(e) = \frac{e[f(e)]^2 + [1 - F(e) - \Sigma_{\hat{e}}][f(e) + ef'(e)]}{[ef(e)]^2} > 0.$$

Using the results above, the derivation of Proposition 6 is a straightforward exercise.■

A.3 Proof of Proposition 5: Properties of Seller's Optimal Pricing Schedule

Proof. Let $\varphi(\hat{e}) \equiv \hat{e} - \frac{1-F(\hat{e})-\Sigma_{\hat{e}}}{f(\hat{e})}$, (34) can be rewritten $\varphi(\hat{e}) = \frac{c'(\hat{q})}{u'(\hat{q})}$. Differentiate it with respect to $\hat{e}$ and we have

$$\frac{d\hat{q}}{d\hat{e}} = \frac{\varphi'(\hat{e})(u')^2}{c''u' - c'u''}$$

Use (38) to rearrange $\varphi(\hat{e})$ as

$$\varphi(\hat{e}) = \frac{\hat{e}}{1 + \frac{1-F(\hat{e})}{\hat{e}f(\hat{e})}}.$$

It is easy to show that $\varphi'(\hat{e}) > 0$. If then follows that $d\hat{q}/d\hat{e} > 0$.

Differentiate (14) with respect to $\hat{e}$ and we have

$$\frac{de_0}{d\hat{e}} = -\frac{\frac{d\Sigma_{\hat{e}}}{d\hat{e}}}{2f(e_0) + ef(e_0)} > 0.$$

Differentiating (11) with respect to $\hat{e}$ gives

$$\frac{dq_e}{d\hat{e}} = \frac{\frac{1}{f(e)}\frac{d\Sigma_{\hat{e}}}{d\hat{e}}}{\frac{c''u'-c'u''}{u'^2}} < 0.$$

Differentiate (13) with respect to $\hat{z}$,

$$\hat{e}u'(\hat{q})\frac{d\hat{q}}{d\hat{e}}\frac{d\hat{e}}{d\hat{z}} - \frac{d\hat{e}}{d\hat{z}}\int_{e_0}^{\hat{e}} u'(q_x)\frac{dq_x}{d\hat{e}}dx = 1,$$

from which we derive

$$\frac{d\hat{e}}{d\hat{z}} = \frac{1}{\hat{e}u'(\hat{q})\frac{d\hat{q}}{d\hat{e}} - \int_{e_0}^{\hat{e}} u'(q_x)\frac{dq_x}{d\hat{e}}dx}$$

Since $d\hat{q}/d\hat{e} > 0$, $dq_e/d\hat{e} < 0$ for $e \in [e_0, \hat{e}]$, we have $d\hat{e}/d\hat{z} > 0$. It then follows that $d\hat{q}/d\hat{z} > 0$, $de_0/d\hat{z} > 0$ and $dq_e/d\hat{z} < 0$ for $e \in [e_0, \hat{e}]$. ■

A.4 Proof of Proposition 6: Quantity Discounts

Proof. For $e \in [e_0, \hat{e}]$, q_e is solved from (11). Integrating both sides from e_0 to e, we have

$$\int_{e_0}^{e}\left[x - \frac{1-F(x)-\Sigma_{\hat{e}}}{f(x)}\right]u'(q_x)dx = \int_{e_0}^{e} c'(q_x)dx$$

$$\rightarrow \left[e - \frac{1-F(e)-\Sigma_{\hat{e}}}{f(e)}\right]u(q_e) - \int_{e_0}^{e} u(q_x)\left\{1 - \frac{-[f(x)]^2 - [1-F(x)-\Sigma_{\hat{e}}]f'(x)}{[f(x)]^2}\right\}dx = c(q_e)$$

$$\rightarrow \int_{e_0}^{e} u(q_x)dx = \left[e - \frac{1-F(e)-\Sigma_{\hat{e}}}{f(e)}\right]u(q_e) - \int_{e_0}^{e} u(q_x)\frac{[f(x)]^2 + [1-F(x)-\Sigma_{\hat{e}}]f'(x)}{[f(x)]^2}dx - c(q_e).$$

The associated money payment is

$$z_e = eu(q_e) - \int_{e_0}^{e} u(q_x)dx$$

The real unit price is

$$\frac{z_e}{q_e} = \frac{eu(q_e) - \int_{e_0}^{e} u(q_x)dx}{q_e}$$

Differentiating z_e/q_e with respect to q_e, we have

$$\begin{aligned}
\frac{\partial\left(\frac{z_e}{q_e}\right)}{\partial q_e} &= \frac{\left[\frac{\partial e}{\partial q_e}u(q_e) + eu'(q_e) - \frac{\partial e}{\partial q_e}u(q_e)\right]q_e - \left[eu(q_e) - \int_{e_0}^{e} u(q_x)dx\right]}{q_e^2} \\
&= \frac{eu'(q_e)q_e - eu(q_e) + \int_{e_0}^{e} u(q_x)dx}{q_e^2} \\
&\propto eu'(q_e)q_e - eu(q_e) + \int_{e_0}^{e} u(q_x)dx \\
&= eu'(q_e)q_e - eu(q_e) + \left[e - \frac{1-F(e)-\Sigma_{\hat{e}}}{f(e)}\right]u(q_e) \\
&\quad - \int_{e_0}^{e} u(q_x)\frac{[f(x)]^2 + [1-F(x)-\Sigma_{\hat{e}}]f'(x)}{[f(x)]^2}dx - c(q_e) \\
&= eu'(q_e)q_e - \frac{1-F(e)-\Sigma_{\hat{e}}}{f(e)}u(q_e) \\
&\quad - \int_{e_0}^{e} u(q_x)\frac{[f(x)]^2 + [1-F(x)-\Sigma_{\hat{e}}]f'(x)}{[f(x)]^2}dx - c(q_e) \\
&= \frac{1-F(e)-\Sigma_{\hat{e}}}{f(e)}\left[u'(q_e)q_e - u(q_e)\right] + c'(q_e)q_e \\
&\quad - \int_{e_0}^{e} u(q_x)\frac{[f(x)]^2 + [1-F(x)-\Sigma_{\hat{e}}]f'(x)}{[f(x)]^2}dx - c(q_e)
\end{aligned}$$

where "$\propto$" represents "have the same sign as".

If $c(q_e)$ is linear, $c'(q_e)q_e = c(q_e)$. We have shown earlier that $\Sigma_{\hat{e}} < 1 - F(e)$ or $1 - F(e) - \Sigma_{\hat{e}} > 0$. Since $u'' < 0$, we have $u'(q_e)q_e - u(q_e) < 0$. To prove the quantity discounts result, we only need to show that $[f(x)]^2 + [1-F(x)-\Sigma_{\hat{e}}]f'(x) > 0$.

$$\begin{aligned}
[f(x)]^2 + [1-F(x)-\Sigma_{\hat{e}}]f'(x) &> 0 \text{ is equivalent to} \\
x\frac{[f(x)]^2}{f(x)} + x\frac{[1-F(x)-\Sigma_{\hat{e}}]f'(x)}{f(x)} &> 0 \text{ or} \\
xf(x) + x\frac{[1-F(x)-\Sigma_{\hat{e}}]f'(x)}{f(x)} &> 0
\end{aligned}$$

Using $x > \frac{1-F(x)-\Sigma_{\hat{e}}}{f(x)} > 0$ for $x > e_0$ (note the first-order condition with respect to q_e), we show that

$$\begin{aligned} & xf(x) + x\frac{[1-F(x)-\Sigma_{\hat{e}}]f'(x)}{f(x)} \\ > \ & 1-F(x)-\Sigma_{\hat{e}} + x\frac{[1-F(x)-\Sigma_{\hat{e}}]f'(x)}{f(x)} \\ = \ & [1-F(x)-\Sigma_{\hat{e}}]\left[1+\frac{xf'(x)}{f(x)}\right] \\ = \ & [1-F(x)-\Sigma_{\hat{e}}]\frac{f(x)+xf'(x)}{f(x)} \\ > \ & 0 \text{ if } xf(x) \text{ increases in } x. \blacksquare \end{aligned}$$

A.5 Proof of Proposition 7: Effects of Inflation on $\hat{z}$

Proof. The proof is similar to the proof of Proposition 4. As $S'(\bar{z}) = 0$, one can show that $S'(\bar{z}) + 1 - \frac{\gamma}{\beta} < 0$ for $\gamma > \beta$ and $\gamma \to \beta$ from above. We again focus on $\hat{z} \in [0, \bar{z}]$. For a generically unique $\hat{z}$, it must be true that $S''(\hat{z}) < 0$. So $\frac{d\hat{z}}{d\gamma} = \frac{1}{\beta S''(\hat{z})} < 0$. The rest of the results follow from Proposition 4. ■

A.6 Proof of Proposition 8: Hot Potato Effect

Proof. As $d\hat{e}/d\gamma < 0$, proving $dq_e/d\gamma > 0$ and $dz_e/d\gamma > 0$ is equivalent to proving $dq_e/d\hat{e} < 0$ and $dz_e/d\hat{e} < 0$. We do this by proving $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$ and $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$, or $q_e(e;\hat{e})$ and $z_e(e;\hat{e})$ are steeper for lower $\hat{e}$.

We first show $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$. Use (11) that characterizes q_e for $e \in [e_0, \hat{e}]$ to calculate dq_e/de as (to simplify notation, we omit the arguments of the functions $u(\cdot)$ and $c(\cdot)$):

$$\frac{dq_e}{de} = \frac{u'^2}{c''u' - c'u''}\left[1 + \frac{f^2 + (1-F-\Sigma_{\hat{e}}]f')}{f^2}\right].$$

The first term $\frac{u'^2}{c''u'-c'u''} > 0$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$:

$$\begin{aligned} \frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{d\hat{e}} &= \frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{dq_e}\frac{dq_e}{d\hat{e}} \\ &\propto \frac{2u'(c''u'-c'u'') - u'^2(c'''u' + c''u'' - c''u'' - c'u''')}{(c''u'-c'u'')^2}\frac{dq_e}{d\hat{e}} \\ &= \frac{2u'(c''u'-c'u'') - u'^3c''' + u'^2c'u'''}{(c''u'-c'u'')^2}\frac{dq_e}{d\hat{e}} \\ &< 0 \text{ if } c''' < 0 \text{ and } u''' > 0. \end{aligned}$$

The second term $1+\frac{f^2+[1-F-\Sigma_{\hat{e}}]f'}{f^2}>0$ decreases in $\hat{e}$ if $f' \leq 0$:

$$\begin{aligned}\frac{d\left[1+\frac{f^2+(1-F-\Sigma_{\hat{e}})f'}{f^2}\right]}{d\hat{e}} &= -\frac{d\Sigma_{\hat{e}}/d\hat{e}}{f^2}f' \\ &\leq 0 \text{ if } f' \leq 0.\end{aligned}$$

Since both the first term and the second term of dq_e/de (are positive and) decrease in $\hat{e}$, $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$.

Now we show $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$. Remember that $z_e = eu'(q_e)dq_e/de$, so

$$\frac{dz_e}{de} = \frac{eu'^3}{c''u' - c'u''}\left[1+\frac{f^2+(1-F-\Sigma_{\hat{e}}f')}{f^2}\right]$$

The first term $\frac{eu'^3}{c''u'-c'u''} > 0$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$:

$$\begin{aligned}&\frac{d\left(\frac{eu'^3}{c''u'-c'u''}\right)}{dq_e} \\ =\ & e\frac{3u'^2(c''u'-c'u'') - u'^3(c'''u'+c''u''-c''u''-c'u''')}{(c''u'-c'u'')^2} \\ =\ & e\frac{3u'^2(c''u'-c'u'') - u'^4c'''+u'^3c'u'''}{(c''u'-c'u'')^2} \\ >\ & 0 \text{ if } c''' < 0 \text{ and } u''' > 0.\end{aligned}$$

The second term is positive and decreases in $\hat{e}$ if $f' \leq 0$. Since both the first term and the second term of dz_e/de (are positive and) decrease in $\hat{e}$, $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$.

As $dz_e/d\gamma > 0$ and $d\hat{z}/d\gamma < 0$, $d(z_e/\hat{z})/d\gamma > 0$.■

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