Information insensitive securities: the benefits of Central Counterparties*

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Abstract

This paper combines the idea that securities should be information insensitive in order to be liquid, with the idea that an infrastructure which performs clearing of trades, and offers additional services to the counterparties, facilitates securities to be liquid.

In a recent paper, Gorton et al.[3] argue that securities that serve as a transaction medium should be the least information-sensitive, and derive sufficient conditions for such security to be debt. Also, a few recent papers[1] emphasize the role of a Central Counterparty (CCP) in internalizing externalities that stem from the opacity of over-the-counter (OTC) transactions and the role of a CCP in providing the participants with insurance and cost effective clearing services.

This paper focuses on the latter role: it develops a framework very similar to Gorton et al.[3], modified to analyze some of the functions that a CCP performs. It shows that for any type of security, regardless of whether it is debt or equity, clearing transactions through a CCP reduces the extent to which securities are information-sensitive. Two functions of a CCP are key: multilateral netting and the existence of a default fund. By reducing the exposure of each counterparty to one another (or to the CCP), both netting and the default fund reduce the incentives of traders to acquire information about the payoffs of the security they are trading.

A role of CCPs that has been identified by policy makers as fostering liquidity and stability of OTC transactions, is to perform margin calls to adjust the available collateral posted for each participant's net position, following the marking to market of securities. In this framework, however, the perception that margin calls foster liquidity is incorrect: traders have even more incentives to acquire information about the securities' payoff so that fewer transactions, which would be welfare improving, are carried out.

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[1]Acharya and Bisin [1], Duffie and Zhu [5] and Koeppl and Monnet [9] among others.

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1 Introduction

Some of the concerns that drove massive policy interventions during the $2007-2010$ financial crisis were related to a *dry up*[2] in markets where the securities that were traded and used as collateral for transactions, were perceived as highly liquid. One explanation for such dry up is the existence of a high degree of private information in financial markets, especially over-the-counter (OTC) markets[3], that made them prone to adverse selection. It is well known[4] that the fear of adverse selection on either side of a transaction may impair trade: if the seller knows that he's giving away something worth more than he's being paid for, he will want the price to be higher. If the buyer is wary that the seller knows more than he does, he may rather wait to learn more before trading. Therefore, markets with adverse selection may suddenly become illiquid. We refer to liquidity as the ability to trade quickly without moving the price: this paper investigates what this means in the context of financial transactions where adverse selection may arise endogenously through the (costly) acquisition of private information by one counterparty. This paper also studies the effect of the provision of clearing and insurance services to the counterparties in financial transactions, on the liquidity of the securities they trade.

Historically, liquid securities are securities that function as money because they have the property of being immune from adverse selection when trading: even when it is feasible for agents to produce private information about the securities' payoffs, no agent finds it

[2] See [**?**], [**?**], [**?**].
[3] Gorton and Metrick [8], [7].
[4] Akerlof [2] seminal work and related research.

profitable so that no adverse selection arises. This property has been described in [**?**],[3] as *information insensitivity*: there is immediacy of trade as the counterparties can trade without being taken advantage of.

In the monetary history of the United States there are several examples of securities that were used as money: during the banking panics of the 19^{th} century the banks themselves developed sophisticated mechanisms to preserve information insensitivity of their deposits. They set up private bank clearing houses that would stop the publication of individual bank accounting information[5] and issued clearinghouse loan certificates directly to the public, as *joint* liabilities of the clearinghouse members. Private banks were essentially acting as a single institution, responsible for each other's obligation during a panic and issuing circulating currency (the loan certificate). Because they were obligations of *all* the clearinghouse members, loan certificates were insensitive to information about a specific bank.

Other examples of securities used as money are those traded in the repurchase agreement (*repo*) market, which grew dramatically in the United States after the changes in contracting conventions introduced in the early 1980s[6]. In a repo, the lender deposits/lends money, earning interest overnight (at the *repo rate*), and receives securities in exchange as collateral to back up the loan. A special feature of a repo is that the lender takes delivery of the securities posted as collateral, thus having them in his possesion and being entitled to use them as collateral in another unrelated transaction. The securities used as collateral are typically very liquid and circulate, as money, to guarantee other financial transactions[7]: a key feature that makes them very liquid is the exemption from the application of the

[5] Which banks were required by the clearinghouse to publish in the newspapers. See [**?**].
[6] See [**?**]
[7] See [**?**],[**?**].

automatic stay of bankruptcy law[8], introduced in 1984, which distinguished repos from secured loans. Creditors could thus sell the securities underlying a repo promptly in the event of the borrower's default. Therefore the securities that collateralized a repo were insensitive to information about a specific repo counterparty.

As in for clearinghouses' loan certificates and repos, liquidity of a security requires some degree of symmetry of information among trading partners, which may be easiest to achieve when everyone is ignorant. Gorton et al.[3] work focus on this idea. They study an environment where trading securities is desirable from a welfare perspective and where one counterparty of a transaction can choose to acquire private information and show that an optimal contract between the parties is characterized by minimizing the value of information acquisition. Under some conditions no adverse selection arises and the efficient allocation can be implemented.

This paper provides a framework similar to Gorton et al.[3] and shows that some features of a mechanism that will be called a CCP, help minimizing the value of acquiring information for any type of security. The crucial features of such mechanism are multilateral netting and the provision of insurance to counterparties in a transaction, through margins and default fund.

The main idea is the following: as in Gorton et al.[3] the value of acquiring information to a buyer of an asset is the expected payoff in states of the world where the actual payoff is smaller than the price he paid for the security. Analogously, the value of acquiring information to a seller of an asset is given by the expected payments from the assets in states of the world where the actual payments exceeds the price he received for the asset.

[8] Congress enacted the Bankruptcy Amendments and Federal Judgeship Act of 1984, 42 exempting from application of the automatic stay repos on Treasury and federal agency securities, bank certificates of deposit, and bankers acceptances. See [**?**].

Therefore a seller (a buyer) has incentive to acquire private information when the payoff from agreeing to trade with the same amount of information as his counterparty, does not compensate for the expected forgone payments he would have otherwise received holding the asset (the price he has to pay to acquire the asset). In this context, the provision of services that are valuable to agents when they trade, raises the payoff to trading without acquiring private information in any state of the world, thus making the asset itself more information insensitive. This paper refers to a CCP[9] as a set of clearing and insurance services that are valuable to the counterparties when agreeing to trade: multilateral netting of positions and insurance.

Multilateral netting[10] is an arrangement among three or more parties to net their obligations: it is arithmetically achieved by summing each participants bilateral net positions with the other participants to arrive at a multilateral net position. When such netting is conducted through a CCP that is legally substituted as the buyer to every seller and the seller to every buyer, the multilateral net position represents the bilateral net position between each participant and the CCP. There are however other financial institutions that provide netting services to traders in OTC markets who do not clear their transactions through a CCP: one example is TriOptima with its TriReduce product[11].

[9]In the model there is nothing more than the provision of clearing and insurance services that the CCP does: we are not assuming that the CCP has superior abilities in overcoming incentive constraints relative to agents in a bilateral trade, rather we focus on the specific features of a service provided by the CCP and its effect on the equilibrium terms of trade in the bilateral contract between the counterparties. Therefore any other institution that provides such services will also deliver the same welfare implications as what we define as a CCP. In fact, the paper refers to the results in each proposition as a feature of the specific service (netting or insurance) analyzed.

[10]This is the official definition provided by the Bank for International Settlement in its *Glossary of terms used in payments and settlement systems* http://www.bis.org/publ/cpss00b.pdf.

[11]Because TriOptima is not a CCP, each counterparty remains liable for its obligations towards all its counterparties as those are not legally assumed by TriOptima. Therefore the netting product that TriOptima offers, tri-reduce, is referred to as providing *compression* of trades, but apart from not legally substituting the counterparties in their obligations, with this product TriOptima offers a services analogous to multilateral netting. Because this paper does not focus on the function of a CCP as substituting the

In an environment where an optimal contract[12] between two parties requires obligations to be collateralized and where posting collateral has an opportunity cost, the larger the collateral requirement the larger the cost of agreeing to trade. Therefore any arrangement that permits the counterparties to achieve the desired allocation from trading with a smaller amount of collateral to post, is a valuable arrangement for trading because it raises the payoff to a trade.

Multilateral netting, by cumulating each individual position into a single net exposure, reduces the expected payment gross of collateral of the individual transactions considered separately. The smaller the net exposure, the smaller the total cost of posting collateral in order to trade, the smaller the incentive to acquire private information before agreeing to trade. In this sense multilateral netting makes the securities traded more information insensitive.

Similarly, in an environment where collateral provides insurance against counterparty risk, any arrangement that involves insurance is valuable to traders. Any bilateral transactions may involve collateral posting for insurance purposes, as part of the optimal contract between traders: we refer to this benchmark as an economy with margins[13]. The collateral set aside guarantees a minimum level of consumption/payment even in states of the world where a counterparty exogenously defaults[14].

counterparties in a transactions (a process called *novation*), then in the framework we set up, the service offered by a compression of trades is exactly what is modeled as multilateral netting. See http://www.trioptima.com/services/triReduce.html

[12] We refer to an optimal contract as the allocation/redistribution of initial resources between traders, resulting from the solution to those traders' decision problem.

[13] Generally, margin is the term for collateral used to secure an obligation, either realised or potential. In securities markets, the collateral deposited by a customer to secure a loan from a broker to purchase shares. In a CCP, the deposit of collateral to guarantee performance on an obligation or cover potential market movements on unsettled transactions is sometimes referred to as margin. See *Glossary of terms used in payments and settlement systems*, http://www.bis.org/publ/cpss00b.pdf.

[14] This paper abstracts from moral hazard considerations: collateral is useful purely for insurance purposes. There is no strategic default in the model, so that there is no role for collateral to play in order to

In this benchmark economy traders bear the cost of posting collateral only if they agree to trade, thus reducing the payoff to trading. Also, the collateral needs to be sufficient to compensate for a specific counterparty's risk of default. So two components of the economy with margin are relevant for their effect on the terms of trade: the impact of the margin on incentives to trade and the amount of collateral needed to insure against a specific idiosyncratic risk of default.

Besides margins, CCPs use another risk management tool: a default or guarantee fund, which is a loss sharing agreement between participants in a CCP regarding the allocation of any loss arising when one or more participants fail to fulfil their obligation. The arrangement stipulates how the loss will be shared among the parties[15].

As with margins, two features of a default fund are key for their effect on the terms of trade: by *de facto* mutualizing losses, a default fund pools idiosyncratic risks, thus allowing traders to achieve the same level of insurance as margins do with a lower collateral requirement. Also, typically the contribution to a default fund is independent of a specific individual transaction: members of a CCP pay their contributions because by being members they take advantage of certain services, among which the transfer of counterparty risk from the bilateral counterparty to the CCP (*novation*). Therefore the contribution to the default fund is akin to collateral that needs to be posted regardless of whether a specific trade is agreed upon or not: relative to its effect on incentives to trade or not to trade, it affects the payoffs to both strategies in exactly the same way. In this sense we say that a default fund does not distort traders' strategies and incentives to agree to trade.

Both margins and default fund increase the expected payoff to a buyer of a security in states of the world where the seller is not performing: insurance is desirable. However, a

discipline incentives to default.

[15]See *Glossary of terms used in payments and settlement systems*, http://www.bis.org/publ/cpss00b.pdf.

default fund reduces the incentives of a buyer to acquire information more than margins do through i) a more efficient risk management, due to idiosyncratic risk pooling, and ii) a non distortionary effect on agents' strategies. In this sense then a default fund is a Pareto superior risk management tool in an economy where information insensitivity is desirable[16].

Interestingly, margin calls[17] that are increasing functions of a participant's net exposure have the opposite effect: they increase incentives to acquire information in states of the world where such incentives were already present, and introduce such incentives in states of the world where they were not present.

2 Model

The economy is populated by two agents, A and B, who live for one period. There are two consumption goods: x and ω. Agent A is endowed with a stochastic amount $\tilde{x}$ of good x distributed according to a distribution function $F(x)$, with support $X = [\underline{x}, \overline{x}]$, which for the remainder of this section is simply assumed to be defined as follows:

$$\tilde{x} = \begin{cases} x_L & \text{w.p.} \quad p_L \\ x_H & \text{w.p.} \quad p_H = (1 - p_L) \end{cases} \tag{1}$$

Agent B is endowed with $\overline{\omega}$ units of good ω.

Preferences of both agents are represented by utility functions: $\mathcal{U}^A(c^A_\omega, c^A_x) = c^A_\omega + E_x c^A_x$, $\mathcal{U}^B(c^B_\omega, c^B_x) = E_x U^B(c^B_x) + c^B_\omega$, where c^i_j denotes consumption of good j by agent i.

[16] This paper abstracts from moral hazard considerations: it does not study whether and how the existence of a loss sharing agreement, as a default fund, affects the optimal risk profile traders choose.

[17] i.e. adjustments to collateral requirement according to the market value of the securities posted as collateral for a transaction. See http://www.bis.org/publ/cpss00b.pdf.

Let $U^B : \mathbb{R}^+ \to \mathbb{R}^+$ satisfy the following assumption:

Assumption 1

$$U^B \in C^2, U^B(0) = 0, U'^B > 0, U''^B < 0, U'^B(0) = +\infty, U'^B(c_x) > 1, \forall c_x \in C$$

where C denotes the set of feasible c_x given the realization x.

Assuming $U'^B(c_x) > 1, \forall c_x \in C$ guarantees that the marginal utility of consuming a unit of good x is always larger for agent B than for agent A.

Agent A dies with probability $\lambda \in (0, 1)$ before consumption takes place.

There are two technologies available to agent A and use good x as input, a productive stochastic technology ($\tilde{y}(\cdot)$) and a storage technology ($y(\cdot)$):

$$\tilde{y}(x) = \begin{cases} \rho x \quad \rho > 1 & \text{if agent A is alive} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$y(x) = x \tag{3}$$

The timing for the consumption and investment decisions and shocks' realizations is described in Figure 1: at the beginning of the period agents A and B are endowed with $\tilde{x}$ and $\overline{\omega}$. Once uncertainty on $\tilde{x}$ is resolved, agent A has access to a productive technology (2) and a storage technology (3). After the investment into each technology is made, the uncertainty on agent A's survival is resolved and at the end of the period the productive and storage technologies produce output (where κ denotes the amount of storage).

Lemma 1 *Under assumption 1 and because $\lambda > 0$, any Pareto Optimal allocation is always such that any physical resources of good x available in the economy are allocated for*

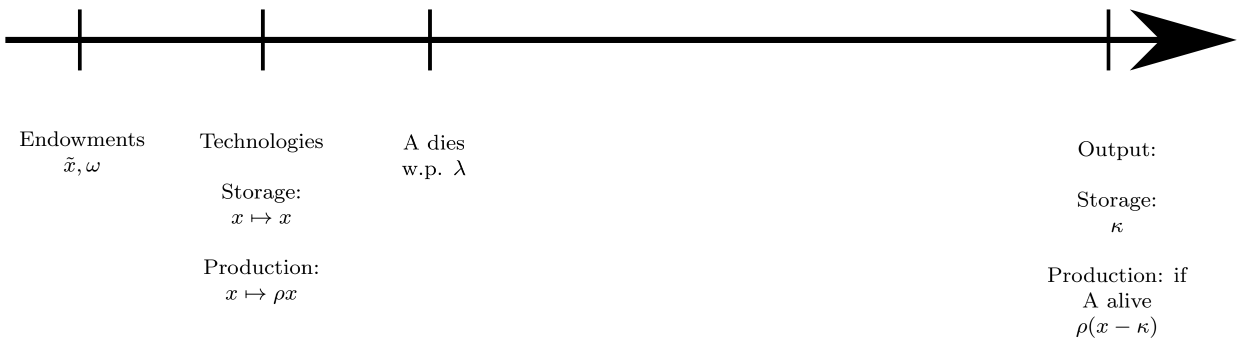

FIGURE 1: Environment

consumption to agent B and the consumption allocation of good ω is indeterminate[18]*. Also, the input into the storage technology is strictly positive.*

Proof. The social planner's problem is:

$$\max_{\{c_\omega^A, c_x^A, c_\omega^B, c_x^B, \bar{c}_\omega^B, \bar{c}_x^B, \kappa_x\}} \quad \{(1-\lambda)E_x\mathcal{U}^A(c_\omega^A, c_x^A) + (1-\lambda)E_x\mathcal{U}^B(c_\omega^B, c_x^B) + \lambda E_x\mathcal{U}^B(\bar{c}_\omega^B, \bar{c}_x^B)\}$$

$$s.t. \quad c_\omega^A + c_\omega^B \leq \overline{\omega}$$

$$\bar{c}_\omega^B \leq \overline{\omega}$$

$$c_x^A + c_x^B \leq \tilde{y}(x-\kappa_x) + y(\kappa_x), \quad \forall x \tag{4}$$

$$\bar{c}_x^B \leq y(\kappa_x) = \kappa_x \leq x, \quad \forall x \tag{5}$$

where κ_x denotes the amount of good x stored and $\bar{c}_\omega^B, \bar{c}_x^B$ the amounts of good ω and x respectively that are consumed by agent B if agent A dies. Using (2) and (3) constraints (4), (5) are simply: $c_x^A + c_x^B \leq \rho(x-\kappa_x) + \kappa_x$ and $\bar{c}_x^B \leq \kappa_x$. Since $\mathcal{U}'^{\mathcal{B}}(c_x^B) = U'^B(c_x^B) > 1 = \mathcal{U}'^{\mathcal{A}}(c_x^A)$ $\forall x$ then a solution to the social planner's problem $(\{c_\omega^{A*}, c_x^{A*}, c_\omega^{B*}, c_x^{B*}, \kappa_x^*\}_{x\in X})$ is $c_x^{B*} = x$. Also, assumption 1 and $\lambda > 0$ guarantee that $\kappa_x^* > 0$ because $U'^B(0) = +\infty$. In particular[19] if $\lambda > \frac{\rho-1}{\rho}$ then $\kappa_x^* = x$. Otherwise if $\exists \bar{b} > 1$ such that $U'^B(c) < \bar{b}$ $\forall c \geq \underline{c}, \underline{c} \in$

[18] Unless the Pareto weight on agent B is low enough.

[19] After substituting the constraints in the objective function, since it is strictly increasing in all the

$(0, \underline{x}]$ and $\lambda < \frac{\rho-1}{\bar{b}+\rho-1}$ then $\kappa_x^* \in (0, x), \forall x$. ■

2.1 Equilibrium

In this economy events unfold according to the following timing, also described in Figure 2:

1. At the beginning of the period Nature draws a realization x of $\tilde{x}$ which is not publicly observable
2. agents A and B meet; B makes a TIOLI offer to A.
3. A can run a technology to privately learn x at a cost γ (paid in utils) and:
   - if γ is paid, then A accepts or rejects TIOLI offer based on x
   - otherwise A accepts or rejects TIOLI offer without information about x
4. settlement and consumption take place: there is full commitment

Definition 1 *A TIOLI offer is a contract $\{T_x, s(x), \kappa_x^A\}_{x \in X}$:*

- *T_x is a transfer from B to A of good ω, which could depend on x. The transfer*[20] *is feasible if $T_x \leq \omega \ \forall x$*

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arguments, we obtain:

$$-(1-\lambda)f(x) + (1-\lambda)f(x)U^{'B}(c_x^B) \geq 0 \quad (6)$$
$$-(1-\lambda)f(x)(\rho-1) + \lambda f(x)U^{'B}(\kappa_x) \geq 0 \quad (7)$$

where $f(x)$ denotes the density function associated with $F(x)$. From (7), if $\lambda > \frac{\rho-1}{\rho}$ then $\kappa_x^* = x$ because by assumption 1 $U^{'B}(c) > 1, \forall c \in C$, which implies $\lambda U^{'B}(\kappa_x) - (1-\lambda)(\rho-1) > \lambda - (1-\lambda)(\rho-1)$. Otherwise if $\lambda U^{'B}(c) < \bar{b}, \forall c \geq \underline{c}$ then $\lambda < \frac{\rho-1}{\bar{b}+\rho-1}$ implies $\lambda U^{'B}(c) - (1-\lambda)(\rho-1) < 0$. Because by assumption 1 $U^{'B}(0) = +\infty$ and $U^{'B}(\cdot)$ is continuous then $\exists c^* \in (0, c), \forall c \geq \underline{c} : -(1-\lambda)(\rho-1) + \lambda U^{'B}(c^*) = 0$.

[20] More formally, a transfer is a function $T : X \mapsto \mathcal{T}$ where $\mathcal{T} = \{t \in \mathbb{R}_+ : (x \in X) \Rightarrow t \in [0, \omega]\}$. With a little abuse of notation we denote it simply by T_x.

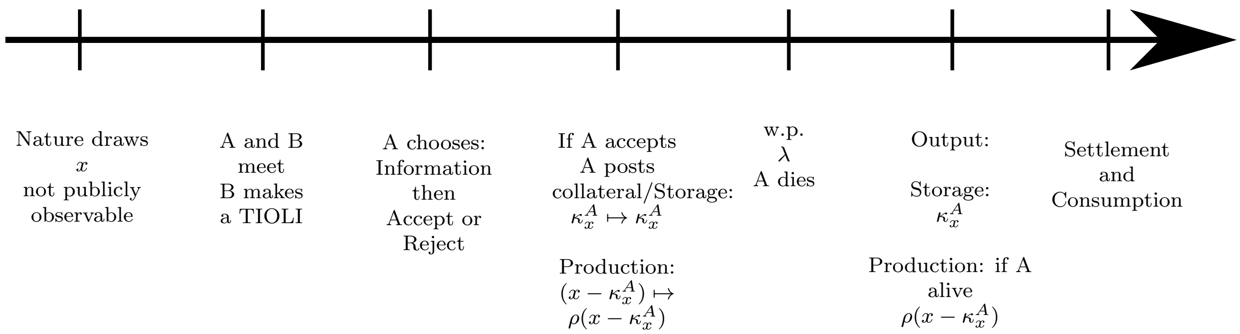

FIGURE 2: Timing

- *$s(x)$ is a transfer from A to B of good x: it is a weakly increasing function*[21] *(or security) which is feasible if* $s(x) \in [0, \rho(x-\kappa_x^A)+\kappa_x^A]$
- *κ_x^A is the input of good x into the storage technology*[22]*: it is feasible if* $\kappa_x^A \in [0, x]$

After observing the TIOLI offer he receives from agent B, agent A decides whether to acquire information about x or not, and whether to accept or not possibly conditional on having learned the realization of x. The decision problem of agent B is to choose between a TIOLI offer that induces information acquisition by agent A and one that does not. Since running the information acquisition technology is inefficient in a Pareto sense[23] and since we are interested in characterizing the contract that can implement a Pareto efficient allocation[24], then the following analysis focuses on the characterization of a TIOLI offer that induces no information acquisition by agent A. Therefore the decision problem of agent

[21] As also assumed in [3]. More formally, a security is a function $s: X \mapsto \mathcal{Y}$ where $\mathcal{Y} = \{y \in \mathbb{R}_+ : (x \in X) \Rightarrow y \in [0, \rho(x-\kappa_x^A)]\}$.

[22] The input in the storage technology is a function $\kappa^A : X \mapsto \mathcal{K}$, with $\mathcal{K} = \{k \in \mathbb{R}_+ : (x \in X) \Rightarrow k \in [0, x]\}$. With a little abuse of notation we denote it simply by κ_x^A.

[23] because it is wasteful

[24] In a separate set of notes, available at http://sites.google.com/site/carapellaf/research, it is shown that for different types of securities there are different partitions of the state space where the information acquisition contract is preferred and offered to agent A by agent B, and where the contract that induces no information acquisition is instead chosen.

B is to maximize his expected utility subject to agent A accepting the offer[25] (participation constraint) and not acquiring information (incentive constraint):

$$
\begin{aligned}
(P1) \quad \max_{\{s(x),T_x,\kappa_x^A\}} \quad & (1-\lambda)[E_x U^B(s(x)) + \overline{\omega} - E_x T_x] + \lambda[E_x U^B(\kappa_x^A) + \overline{\omega}] && (8)\\
s.t. \quad & (1-\lambda)E_x[T_x + \rho(x - k_x^A) + k_x^A - s(x)] \geq (1-\lambda)E_x \rho x && (9)\\
& \Pr\Big(x : T_x - k_x^A(\rho-1) - s(x) < 0\Big)(1-\lambda)\Big[k_x^A(\rho-1) + s(x) - T_x\Big] \leq \gamma && (10)
\end{aligned}
$$

With probability $(1-\lambda)$ agent A survives: the output from the productive technology (2) is produced and the contract is settled, so that agent B receives $s(x)$ from agent A and pays him T_x units of good ω. Also, agent B can still consume the units of good ω left over from his endowment $\overline{\omega}$ net of paying T_x to agent A.

With probability λ agent A dies: no output from the productive technology (2) is produced, and the only units of good x available in the economy are the ones produced by the storage technology (3). So that agent B receives κ_x^A and does not pay agent A since he is no longer alive and gets no utility out of consumption. Also, agent B can still consume his endowment $\overline{\omega}$.

Constraint (9) is the participation constraint for agent A: the expected payoff from accepting without acquiring information weakly dominates the expected payoff from rejecting without acquiring information. The left hand side is the payoff to agent A from consuming the amount of good ω that is transferred to him via T_x and the amount of good x that is produced with the productive technology (2) net of the amount he was required to store (κ_x^A) and net of the amount he was required to transfer to agent B, $s(x)$; agent A also gets his collateral κ_x^A back (output from the storage technology (3)). The right hand

[25] feasibility constraints have been already substituted in.

side is the payoff to agent A from consuming the amount of good x that is realized. Agent A accepts the contract when the left hand side exceeds the right hand side, assuming no information is acquired[26].

Constraint (10) is the incentive constraint for agent A: the expected payoff to agent A from accepting without acquiring information weakly dominates the expected payoff from acquiring information. Agent A prefers to accept without acquiring information[27] when the cost of acquiring information (γ) exceeds the expected loss from having to pay agent B more than what he receives from agent B in states of the world where he would have preferred not to trade, if he had known the realization of x ($s(x) - T_x + k_x^A(\rho - 1)$ for a given x). Such payment includes foregone production on the collateral stored.

In order to understand the relevance of information insensitivity in this environment, consider a simple example where $F(x)$ is assumed to be defined by (1) and compare two different benchmark economies: one with full information about the realization of x ($\gamma = 0$) and one with costly information acquisition about the realization of x ($\gamma > 0$).

[26] which is guaranteed by the incentive constraint

[27] The expected payoff from accepting without acquiring information is simply the left hand side of constraint (9). The expected payoff from acquiring information is given by the payoff from accepting (rejecting) to trade, weighted by the probability that the realization of x is such that the TIOLI offer is accepted (rejected), net of paying the cost γ. If we let $\underline{X} = \{x : T_x - k_x^A(\rho - 1) - s(x) \geq 0\}$ and $\overline{X} = \{x : T_x - k_x^A(\rho - 1) - s(x) < 0\}$, it is simply:

$$(1-\lambda)\Big\{\int_{\underline{X}} [T_x + \rho x - k_x^A(\rho - 1) - s(x)]dF(x) + \rho \int_{\overline{X}} x dF(x)\Big\} - \gamma$$

Then the incentive constraint is:

$$(1-\lambda)E_x[T_x + \rho(x - k_x^A) + k_x^A - s(x)] \geq (1-\lambda)\Big\{\int_{\underline{X}} [T_x + \rho x - k_x^A(\rho - 1) - s(x)]dF(x) + \rho \int_{\overline{X}} x dF(x)\Big\} - \gamma$$

which can be simply rearranged to yield (10).

2.1.1 Full information ($\gamma = 0$)

With full information there is no incentive constraint and the relevant participation constraint is:

$$(\overline{\omega} \geq) T_x \geq \kappa_x^A(\rho - 1) + s(x), \;\; \forall x \tag{11}$$

Then the following result is straightforward:

Lemma 2 *Under full information a Pareto optimal allocation is implemented in this economy if and only if*

$$\overline{\omega} \;\; \geq \;\; T_{x_H} \geq \kappa_H^A(\rho - 1) + s(x_H) \tag{12}$$

$$\overline{\omega} \;\; \geq \;\; T_{x_L} \geq \kappa_L^A(\rho - 1) + s(x_L) \tag{13}$$

where $T_{x_i} = T(x_i)$. *Notice that agent B's participation constraint is always satisfied, by assumption 1.*

With $x_H > x_L$ then by definition 1 $s(x_H) \geq s(x_L)$, and by proposition 2 $\kappa_H^A \geq \kappa_L^A$: therefore (12) is simply:

$$\overline{\omega} \geq (T_{x_H} \geq) s(x_H) + \kappa_H^A(\rho - 1) \tag{14}$$

2.1.2 Costly information acquisition ($\gamma > 0$)

With costly information acquisition, constraints (9) and (10) are[28]:

$$
\begin{aligned}
E_x[T_x - \kappa_x^A(\rho - 1) - s(x)] &\geq 0 \qquad (16)\\
p_H[s(x_H) + \kappa_H^A(\rho - 1) - T_H] &\leq \gamma \qquad (17)
\end{aligned}
$$

which can be rearranged as:

$$
\begin{aligned}
T_H &\geq s(x_H) + \kappa_H^A(\rho - 1) - \frac{p_L}{p_H}[T_L - \kappa_L^A(\rho - 1) - s(x_L)]\\
T_H &\geq s(x_H) + \kappa_H^A(\rho - 1) - \frac{\gamma}{p_H}
\end{aligned}
$$

Then the following result is straightforward:

Lemma 3 *Under full information a Pareto optimal allocation is implemented in this economy if and only if*

$$\omega \geq T_H \geq \overline{T} = s(x_H) + \kappa_H^A(\rho - 1) - \min\left(\frac{p_L}{p_H}[T_L - \kappa_L^A(\rho - 1) - s(x_L)], \frac{\gamma}{p_H}\right) \qquad (18)$$

where $T_{x_i} = T(x_i)$. Notice that agent B's participation constraint is always satisfied, by

[28] Lemma 4 shows that it is either $x_H \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A\prime}\}}$ or $x_L \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A\prime}\}}$ for the case of a distribution function as defined in (1). If $\overline{\omega}$ is large enough then the relevant constraint (10) is then either (17) or:

$$T_L \geq s(x_L) + \kappa_L^A(\rho - 1) - \frac{\gamma}{p_L} = \underline{T_L} \qquad (15)$$

since only the expected value of the transfer T_x matters to agent B. If the relevant constraint is (15) however, and since by definition 1 $s(x_H) \geq s(x_L)$ and by proposition 2 $\kappa_H^A \geq \kappa_L^A$, then for a contract to satisfy (16) we need $T_H > \underline{T_H} = \kappa_H^A(\rho - 1) + s(x_H) + \frac{p_L}{p_H}[(\rho - 1)\kappa_L^A + s(x_L) - T_L] \geq \kappa_L^A(\rho - 1) + s(x_L) + \frac{p_L}{p_H}[(\rho - 1)\kappa_L^A + s(x_L) - T_L]$. However, since $x_L \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A\prime}\}}$ then it must be that $T_L < \kappa_L^A(\rho - 1) + s(x_L)$ so that $\underline{T_H} > \underline{T_L}$ as defined by the right hand side of (15). This implies that in this economy a Pareto optimal allocation can be implemented only with a tighter lower bound on $\overline{\omega}$ than would be needed with the same transfers $\{s(x), \kappa_x^A\}_{x \in X}$ but with different transfers T_H, T_L so that the relevant incentive constraint is (17).

assumption 1.

Also notice that $\overline{T} < s(x_H) + \kappa_H^A(\rho - 1)$, where, by (14), the right hand side is the lower bound on agent B's endowment $\overline{\omega}$ to implement a Pareto optimal allocation under full information. Therefore, when information is costly to acquire, there is a larger set of economies $\mathcal{E}_{\overline{\omega}}$ where a Pareto optimal allocation can be implemented, where $\mathcal{E}_{\overline{\omega}} = \{\overline{\omega}, \rho, x_H, x_L, p_H, p_L : \kappa_{x_H}^A(\rho - 1) + s(x_H) > \overline{\omega} \geq \overline{T}\}$. In this sense information insensitivity is desirable. The following results characterize the solution to problem $P1$:

Lemma 4 *Let $\{T'_x, s'(x), \kappa_x^{A'}\}_{x \in X}$ denote a solution to problem $P1$. Then there exists at least one $\widehat{x} \in X$ such that $\widehat{x} \notin X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}} = \{x \in X : \Pr(T'_x - \kappa_x^{A'}(\rho - 1) - s'(x) < 0) > 0\}$.*

Proof. By contradiction. Suppose a contract $\{T'_x, s'(x), \kappa_x^{A'}\}_{x \in X}$ solves problem $P1$ and is such that $\forall x \in X, x \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}}$. Then it must be that $\forall x \in X : T_x - k_x^A(\rho - 1) - s(x) < 0$. This implies that the participation constraint (9) will necessarily be violated. Therefore the contract $\{T'_x, s'(x), \kappa_x^{A'}\}_{x \in X}$ cannot be a solution to $P1$. ■

Proposition 1 *Any feasible contract that insures agent B completely against default risk*[29] *is such that $\kappa_x^A = x, \forall x$.*

Proof. It follows from feasibility: (4) and (5) imply that $\overline{c}_x^B = c_x^B$ if and only if $c_x^A = 0$ and $y(\kappa_x) = x, \forall x$. ■

Proposition 2 *Under assumption 1 any solution to problem $P1$ is such that:*

a. $\kappa_x^A > 0, \forall x$

b. κ_x^A *is strictly increasing in $s(x)$:*

[29] That is the probability that agent A dies, λ.

$$s(x_H) > s(x_L) \Rightarrow \kappa^A_{x_H} > \kappa^A_{x_L} \text{ and } s(x_H) = s(x_L) \Leftrightarrow \kappa^A_{x_H} = \kappa^A_{x_L}$$

Proof.

a. It follows from assumption 1 and the fact that $\lambda > 0$.

b. By contradiction: let $\{T'_x, s'(x), \kappa^{A'}_x\}_{x\in X}$ denote a solution to problem $P1$.

$b.1$ suppose first that $\{T'_x, s'(x), \kappa^{A'}_x\}_{x\in X}$ is such that $s'(x_H) > s'(x_L)$ and $\kappa^{A'}_H \leq \kappa^{A'}_L$. Then $\forall\epsilon > 0$ consider the following alternative contract:

$$\widehat{s}(x_H) = s'(x_H) - \epsilon(\rho - 1) \tag{19}$$

$$\widehat{\kappa}^A_H = \kappa^{A'}_H + \epsilon \tag{20}$$

$$\widehat{s}(x_L) = s'(x_L) + \frac{p_H}{p_L}\epsilon(\rho - 1) \tag{21}$$

$$\widehat{\kappa}^A_L = \kappa^{A'}_L - \frac{p_H}{p_L}\epsilon \tag{22}$$

$$\widehat{T}_H = T'_H \tag{23}$$

$$\widehat{T}_L = T'_L \tag{24}$$

It is easily seen that this contract is still feasible: $\widehat{\kappa}^A_H \leq x_H$ since $\kappa^{A'}_H \leq \kappa^{A'}_L$ by contradiction assumption, which yields $\kappa^{A'}_H \leq \kappa^{A'}_L \leq x_L < x_H$ when combined with the resource constraint (5); $\widehat{s}(x_L) \leq [\rho x_L - (\rho-1)\widehat{\kappa}^A_L]$ since $\widehat{\kappa}^A_L < \kappa^{A'}_L$. Now it is left to show that the contract $\{\widehat{s}(x), \widehat{\kappa}^A_x, \widehat{T}_x\}_{x\in X}$ is still in the constraint set of problem $P1$ and yields a strictly higher value to the objective function. Consider first the participation constraint (9):

$$p_H(T'_H - \kappa^{A'}_H(\rho - 1) - s'(x_H)) + p_L(T'_L - \kappa^{A'}_L(\rho - 1) - s'(x_L)) \geq 0 \tag{25}$$

If the original contract satisfies (9) then also the alternative contract $\{\widehat{s}(x), \widehat{\kappa}^A_x, \widehat{T}_x\}_{x\in X}$ does:

$$\begin{aligned} p_H(\widehat{T}_H - \widehat{\kappa}^A_H(\rho-1) - \widehat{s}(x_H)) + p_L(\widehat{T}_L - \widehat{\kappa}^A_L(\rho-1) - \widehat{s}(x_L)) &\geq 0 \\ p_H[T'_H - \kappa^{A'}_H(\rho-1) + \epsilon(\rho-1) - s'(x_H) - \epsilon(\rho-1)] + & \\ p_L[T'_L - \kappa^{A'}_L(\rho-1) + \frac{p_H}{p_L}\epsilon(\rho-1) - s'(x_L) - \frac{p_H}{p_L}\epsilon(\rho-1)] &\geq 0 \end{aligned}$$

Consider now the incentive constraint (10). By lemma 4 either $x_L \in X^{IC}_{\{T'_x, s'(x), \kappa^{A'}_x\}}$ or $x_H \in X^{IC}_{\{T'_x, s'(x), \kappa^{A'}_x\}}$ but not both. Without loss of generality[30] suppose $x_H \in X^{IC}_{\{T'_x, s'(x), \kappa^{A'}_x\}}$. Then the only perturbations to the original contract that are relevant to the incentive constraint (10) are (19) and (20). By construction the incentive constraint (10) is unaffected because:

$$\begin{aligned} \widehat{T}_H - \widehat{\kappa}^A_H(\rho-1) - \widehat{s}(x_H) &= T'_H - \kappa^{A'}_H(\rho-1) - \epsilon(\rho-1) - s'(x_H) + \epsilon(\rho-1) \\ &= T'_H - \kappa^{A'}_H(\rho-1) - s'(x_H) \end{aligned}$$

Therefore:

$$\begin{aligned} \Pr\left(\widehat{T}_H - \widehat{\kappa}^A_H(\rho-1) - \widehat{s}(x_H) < 0\right) &= \Pr\left(T'_H - \kappa^{A'}_H(\rho-1) - s'(x_H)) < 0\right) \\ \widehat{s}(x_H) + \widehat{\kappa}^A_H(\rho-1) - \widehat{T}_H &= s'(x_H)) + \kappa^{A'}_H(\rho-1) - T'_H \end{aligned}$$

Therefore the alternative contract defined by (19)-(24) is still in the constraint set to problem $P1$. Now it remains to argue that it achieves a higher value of

[30] Otherwise, if $x_L \in X^{IC}_{\{T'_x, s'(x), \kappa^{A'}_x\}}$ the argument can be reproduced just by relabeling the relevant terms of the contract.

the objective function:

$$\begin{aligned}
\frac{\partial \mathcal{U}^B}{\partial \epsilon} &= (1-\lambda)(\rho-1)[p_L \frac{p_H}{p_L} U^{B'}(s'_L) - p_H U^{B'}(s'_H)] + \lambda[p_H U^{B'}(\kappa_H^{A'}) - p_L \frac{p_H}{p_L} U^{B'}(\kappa_L^{A'})] \\
&= (1-\lambda)(\rho-1)p_H[U^{B'}(s'_L) - U^{B'}(s'_H)] + \lambda p_H[U^{B'}(\kappa_H^{A'}) - U^{B'}(\kappa_L^{A'})] \qquad (26) \\
&> 0 \qquad (27)
\end{aligned}$$

where the last inequality follows from $s'_L < s'_H$ and $\kappa_H^{A'} \leq \kappa_L^{A'}$.

$b.2$ For the if part suppose that $\{T'_x, s'(x), \kappa_x^{A'}\}_{x \in X}$ is such that $s'(x_H) = s'(x_L)$ and $\kappa_H^{A'} < \kappa_L^{A'}$. Then $\forall \epsilon > 0$ consider the alternative contract defined by (19)-(24). As showed above, this contract is still feasible and is still in the constraint set of problem $P1$. To show that it is a profitable deviation from the original contract, notice that in (26):

$$\begin{aligned}
s'(x_H) = s'(x_L) &\Rightarrow U^{B'}(s'_L) - U^{B'}(s'_H) = 0 \\
\kappa_H^{A'} < \kappa_L^{A'} &\Rightarrow U^{B'}(\kappa_H^{A'}) - U^{B'}(\kappa_L^{A'}) > 0
\end{aligned}$$

So that (27) still holds. Therefore the original contract cannot be a solution to $P1$. The case where $s'(x_H) = s'(x_L)$ and $\kappa_H^{A'} > \kappa_L^{A'}$ is proven by the same argument, with some relabeling.

For the only if part suppose that $\{T'_x, s'(x), \kappa_x^{A'}\}_{x \in X}$ is such that $\kappa_H^{A'} = \kappa_L^{A'}$ and $s'(x_H) > s'(x_L)$. Then $\forall \epsilon > 0$ consider the alternative contract defined by (19)-(24). As showed above, this contract is still feasible and is still in the constraint set of problem $P1$; it is also a profitable deviation because

$$\begin{aligned}
s'(x_H) > s'(x_L) &\Rightarrow U^{B'}(s'_L) - U^{B'}(s'_H) > 0 \\
\kappa_H^{A'} = \kappa_L^{A'} &\Rightarrow U^{B'}(\kappa_H^{A'}) - U^{B'}(\kappa_L^{A'}) = 0
\end{aligned}$$

So that (27) still holds. The case where $\kappa_H^{A'} = \kappa_L^{A'}$ and $s'(x_H) < s'(x_L)$ is proven by the same argument, with some relabeling.

■

Using the results in proposition 2 it is easy to show that if the endowment of agent B is large enough then the Pareto optimal allocation is implemented by the contract that solves $P1$.

Proposition 3 *If $\overline{\omega} \geq \rho x_H$ then for any function U^B that satisfies assumption 1 any solution to problem $P1$ is Pareto optimal.*

Proof. Let a solution to problem $P1$ be denoted $\{T'_x, s'(x), \kappa_x^{A'}\}_{x\in X}$. Lemma 4 implies that it is either $x_H \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}}$ or $x_L \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}}$ but not both: suppose that $x_H \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}}$ then constraint (10) is simply:

$$T'_H \geq \kappa_H^{A'}(\rho - 1) + s'(x_H) - \frac{\gamma}{p_H(\rho - 1)} \tag{28}$$

Constraint (9) is:

$$T'_H \geq \kappa_H^{A'}(\rho - 1) + s'(x_H) - \frac{p_L}{p_H}[T'_L - s'_L - \kappa_L^{A'}(\rho - 1)] \tag{29}$$

so that an upper bound on the right hand side of both constraints is ρx_H and if $\overline{\omega} \geq \rho x_H$ then both constraints are satisfied. The same argument goes through if $x_L \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}}$: the only difference being that the incentive constraint will place a lower bound on T'_L: $T'_L \geq \kappa_L^{A'}(\rho - 1) + s'(x_L) - \frac{\gamma}{p_L(\rho-1)}$. ■

Notice that $s'(x) = \rho x - (\rho - 1)\kappa_x^{A'}, \forall x \in X$: only if for some $x' \in X$ there exists no feasible $T_{x'}$ such that the participation and incentive constraints (9) and (10) are satisfied, then $s'(x') < \rho x' - (\rho - 1)\kappa_{x'}^{A'}$. This is because assumption 1 guarantees that the marginal utility that B gets from

and consider the incentive constraint (10) first: by proposition 2 κ_x^A is strictly increasing in x, so $\kappa_H^A(\rho - 1) + s(x_H) > \kappa_L^A(\rho - 1) + s(x_L)$. By lemma 4 it must be that either $x_H \in X^{IC}_{\{T'_x, s'(x), \kappa_x^{A'}\}}$ or $T_L - (\kappa_L^A(\rho - 1) + s(x_L))$ but not both: therefore unless a solution to $P1$ is such $T_H > \kappa_H^A(\rho - 1) + s(x_H)$

Proposition 3 provides a framework for thinking about the welfare implications of any solution to problem $P1$: from proposition 2 we know that some collateral is part of the optimal contract from the point of view of agent B. Whether for the contract that agent B offers to agent A it is feasible to implement a Pareto optimal allocation or not, depends on how rich agent B is: if his endowment of good ω ($\overline{\omega}$) is large enough then he is able to get agent A to accept the contract without acquiring information. Such allocation involves insurance provision to agent B against counterparty default via posting collateral κ_x^A. From a Pareto optimaliy perspective collateral posting (investment into the storage technology (3)) is desirable. Therefore collateral is a necessary feature for any contract to feasibly implement a Pareto optimal allocation: in the next sections we will analyze how different mechanisms affect the solution to problem $P1$, and a necessary feature for those mechanisms to feasibly implement a Pareto optimal allocation, is for them to involve some collateral posting. We will analyze the welfare implications of each of these mechanisms: we will say that if a mechanism induces a change to the model that enlarges (shrinks) the partition of the state space for which proposition 3 holds, then such mechanism is welfare improving (reducing) in this economy. We will also identify each mechanism with a specific arrangement we observe in clearing and settlement of financial transactions through CCPs. So that each mechanism is a function that CCPs play.

Section 3 uses the results from propositions 2-3 to show that multilateral netting enlarges the set of economies where a Pareto optimal allocation can be implemented. Such welfare implications of multilateral netting are a corollary to proposition 2.

Section 4 uses the results from propositions 1-3 to show that when compared to margin requirements, a default fund enlarges the set of economies where a Pareto optimal allocation can be implemented. Such welfare implications follow from proposition 1.

3 Multilateral Netting

Multilateral netting is the agreed offsetting of positions or obligations among three or more trading partners. More formally, multilateral netting is defined by the Bank for International Settlements[31] for central counterparties as arithmetically achieved by summing each participant's bilateral net positions with the other participants, to arrive at a multilateral net position. The central counterparty is legally substituted as the buyer to every seller and the seller to every buyer so that the multilateral net position represents the bilateral net position between each participant and the central counterparty.

As an example, Figure 3 shows the flow of payments (or delivery of a good) that each of three trading partners owes the others: agent A owes 5 units of the good to agent C, who owes 1 unit of the good to agent B, who, in turn, owes 2 units of the good to agent A. Suppose that each financial transaction involving payments from one agent to another requires some collateral posting as a function of the payment itself[32] and let collateral requirements be denoted respectively $\kappa(5), \kappa(1), \kappa(2)$. When the position of each participant is netted out with the positions of the other participants through a central

[31] See Glossary of terms used in payments and settlement systems, http://www.bis.org/publ/cpss00b.pdf.
[32] As proposition 2 shows.

agent like a CCP, then the net position and collateral requirements for each agent are shown in Figure 4: agent A owes the CCP 3 units of the good, and collateral $\kappa(3)$, agent B owes the CCP 1 units of the good, and collateral $\kappa(1)$ and the CCP owes agent C 4 units of the good. Because agent C is a net creditor of the CCP then no collateral is required for him to post. Multilateral netting thus reduces the amount of collateral required in

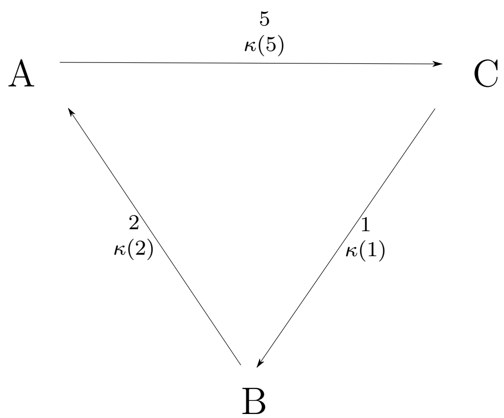

FIGURE 3: Bilateral Clearing

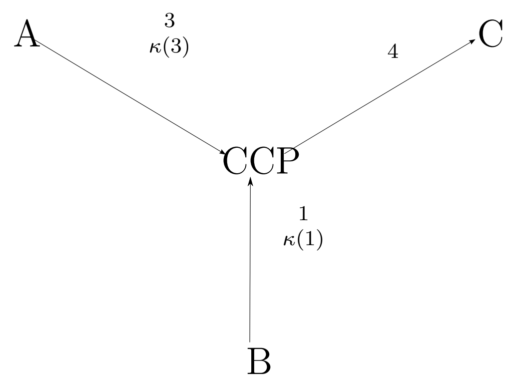

FIGURE 4: Central Clearing with Multilateral Netting

an economy in order to implement a given allocation (flow of payments stemming from financial transactions). To the extent that collateral is costly to post because of forgone investment opportunities that agents would otherwise take advantage of, then multilateral netting allows more productive projects to be undertaken: in the model described in section 2 such productive project is the productive technology (2).

Although this section identifies multilateral netting as a function that CCPs play, the results shown are not restricted to CCPs only but apply to any entity that provides multilateral netting.

In order to formally model multilateral netting, the environment of section 2 is slightly modified to allow for a third agent,S[33], to trade with agent A and B[34]. Agent A is endowed with a stochastic amount $\tilde{x}$ of good x as in section 2 and with an amount $\overline{\nu}$ of good ν. Agent B is endowed with an amount $\overline{\omega}$ of good ω, as in section 2, and agent S is endowed with a stochastic amount $\tilde{y}$ of good y, which is independent of $\tilde{x}$.

Preferences are represented by utility functions: $\mathcal{U}^A(c_\omega^A, c_x^A, c_y^A, c_\nu^A) = c_\omega^A + E_x c_x^A + E_y c_y^A + c_\nu^A$; $\mathcal{U}^B(c_\omega^B, c_x^B, c_y^B) = E_{x,y} U^B(c_x^B + c_y^B) + c_\omega^B$, and $\mathcal{U}^S(c_\nu^S, c_y^S) = \alpha c_\nu^S + E_y c_y^S$ with $\alpha > 1$; c_j^i denotes consumption of good j by agent i and $U^B : \mathbb{R}^+ \to \mathbb{R}^+$ satisfies assumption 1 as in section 2.

Agent A has access to two technologies which use goods x or ν as inputs: a productive stochastic technology $(\tilde{y}(z), z = x, \nu)$ and a storage technology $(y(z), z = x, \nu)$:

$$\tilde{y}(z) = \begin{cases} \rho z \quad \rho > 1 & \text{if agent A is alive} \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

$$y(z) = z \tag{31}$$

The timing for the consumption/investment decisions and shocks' realizations is described in Figure 5: at the beginning of the period Nature draws a realization x of $\tilde{x}$ and y of $\tilde{y}$ which are not publicly observable. Then agents A and B meet, B makes a TIOLI offer to A, and agents A and S meet, S makes a TIOLI offer to A. Once he receives the TIOLI offer, agent A can run a technology to privately learn x at a cost γ (paid in utils) and if γ is paid, then A accepts or rejects TIOLI offer based on x. Otherwise A accepts or rejects TIOLI offer without information about x. Similarly A can run the information acquisition technology to privately learn y and then decide whether to accept or reject the TIOLI offer made by agent S. At the end of the period settlement and consumption take place: there

[33]who will be the seller of a security.
[34]who will be the buyer of a security.

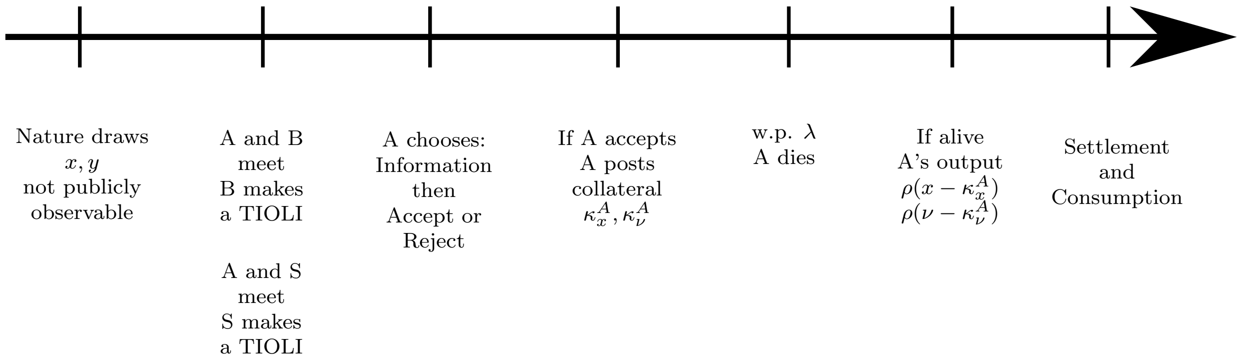

FIGURE 5: Timing

is full commitment. Notice that it is assumed that agent B and S do not meet[35].

With bilateral clearing no multilateral netting is feasible[36]. Therefore the decision problem of agent B is the same as $P1$. The TIOLI offer that agent S makes to agent A does not change constraints in $P1$ because it is entirely exogenous to any decision variable of agent B. Both the left and right hand side of constraints (9) and (10) are affected in the same way, so that agent A's payoffs from trading with agent S cancels out in both constraints. Agent A and B trade, clear and settle bilaterally as in the equilibrium characterized in section 2. Agent A and S clear and settle bilaterally: with analogous notation to section 2 agent S delivers $s_s(y)$ units of good y to agent A and gets some good ν in exchange, with the terms of trade determined by the TIOLI that solves his decision problem, which is analogous to agent B's decision problem $P1$, apart from relabeling.

Suppose now that transactions are cleared centrally through a CCP which performs

[35]This assumption guarantees that a simple trading pattern arises in equilibrium: agents B and S trading with agent A. Alternatively a different trading pattern may arise with agent B trading ω good with agent A against ν good, so that the ν good can be used by agent B to trade with agent S against good y: in this trading patter however, agent A would not face an information acquisition problem when deciding whether to accept or reject the offer made by agent B. Because the focus of this paper is to study the effect on information insensitivity of a contract of different mechanisms (or functions CCPs play), then we prefer to focus on the simple trading pattern where both agents B and S want to trade with agent A.

[36]Multilateral netting is feasible when all the transactions that can be netted are submitted for clearing to a central agent (or algorithm) who can calculate the net position for each trader.

multilateral netting: because the CCP replaces the original counterparties in the existing contractual obligations[37], then agent A pays $s(x)-s_s(y)$ units of good x to the CCP, which transfers to agent B the amount of good x received by agent A. Agent S pays $s_s(y)$ units of good y to the CCP which transfers them to agent B. Since agent B is indifferent between consuming good x and y, then he is indifferent between being transferred by agent A the amount of good x required by the contract that solves $P1$ (i.e. $s(x)$) or only an amount of good x equal to the net payment agent A owes the rest of the economy (i.e. $s(x) - s_s(y)$), and being transferred the remainder in units of good y by agent S (i.e. $s(y)$). Therefore with multilateral netting agent A's obligation is to pay $s(x) - s_s(y)$ units of good x: with probability λ agent A dies, in which case no output from the productive technology (2) is produced and the only units of good x available in the economy that can be delivered to agent B, are the units that were invested in the storage technology (3). Agent B, however, still receives $s_s(y)$ units of good y from agent S, because all transactions are multilaterally netted and agent A is not responsible for delivering agent S's obligation. Therefore the consumption of good x that agent B may wish to be insured for, $s(x) - s_s(y)$, is smaller than the amount of consumption he would want to be insured for with bilateral clearing, $s(x)$. In this model insuring consumption of good x is achieved through collateral posting (i.e. investment into the storage technology (3)).

Definition 2 *With multilateral netting (e.g. performed by a CCP), a TIOLI offer by agent B to agent A is a contract* $\{T_x, s(x), \kappa^A_{xy}\}_{x\in X} : \{T_x, s(x)\}_{x\in X}$ *are as in definition 1 and* κ^A_{xy} *denotes the amount of good x that agent A is required to store when $s_s(y)$ units of good y are paid by agent S (e.g. to the CCP) according to his TIOLI offer to agent A.*

[37] By a process that is referred to as novation (see http://www.bis.org/publ/cpss00b.pdf.)

With central clearing and multilateral netting then, agent B's decision problem is:

$$(P2) \max_{\{s(x),T_x,\kappa_x^A\}} \quad (1-\lambda)[E_x U^B(s(x)) + \overline{\omega} - E_x T_x] + \lambda[E_{x,y} U^B(\kappa_{x,y}^A) + \overline{\omega}] \tag{32}$$

$$s.t. \quad (1-\lambda)E_{x,y}[T_x + \rho(x - \kappa_{x,y}^A) + \kappa_{x,y}^A - s(x)] \geq (1-\lambda)E_x \rho x \tag{33}$$

$$\Pr\Big((1-\lambda)[T_x - s(x) - \rho E_{y|x}\kappa_{x,y}^A] < 0\Big)(1-\lambda)\Big[s(x) + \rho E_{y|x}\kappa_{x,y}^A - T_x\Big] \leq \gamma \tag{34}$$

The following result is then straightforward:

Proposition 4 *Under assumption 1 and if* $s_s(y) > 0, \forall y \in Y$ *then* $E_{y|x}\kappa_{x,y}^A < \kappa_x^A$.

Proof. It follows directly from proposition 2 and $s(x) - s_s(y) < s(x)$ because of multilateral netting. ■

Proposition 4 is relevant for the welfare comparison of an economy with bilateral clearing versus an economy with central clearing and multilateral netting. By reducing the need of collateral that supports the same trading flows and level of consumption, multilateral netting relaxes both the participation and incentive constraints of problem $P2$. In fact the left hand side of (33) is larger than the left hand side of (9); similarly the right hand side of (34) is smaller than the right hand side of (10) because:

$$Pr\Big([T_x - s(x) - \rho E_{y|x}\kappa_{x,y}^A] < 0\Big) \quad < \quad \Pr\Big([T_x - s(x) - \rho E_x \kappa_x^A] < 0\Big) \tag{35}$$

$$\Big[s(x) + \rho E_{y|x}\kappa_{x,y}^A - T_x\Big] \quad < \quad \Big[s(x) + \rho E_x \kappa_x^A - T_x\Big] \tag{36}$$

Therefore with central clearing and multilateral netting it is feasible to implement a Pareto optimal allocation in a larger set of economies: the reason is that by providing a service that is valuable to trading partners, multilateral netting raises the payoff to accepting the TIOLI offer without acquiring information relative to the expected payoff of acquiring information and then accepting or rejecting based on the observed value of x. In this sense, multilateral netting makes the security $s(x)$ more information insensitive. Agent A has a smaller incentive to acquire information about the actual realization of x because he's better off with the allocation he is offered: the smaller collateral requirement allows him to invest a larger amount of his endowment of good x in the productive technology (2) and therefore to consume more good x when the technology matures at the end of the period. This is formalized by the incentive constraint being relaxed. In this sense multilateral netting is welfare improving: it affects the terms of trade between agent B and A in a way that allows a Pareto efficient allocation to be implemented in a larger set of economies $\mathcal{E}_{\overline{\omega}}$, indexed by $\overline{\omega}$ among other primitives of the model.

4 Counterparty risk management through a Default Fund

This section completely abstracts from multilateral netting and focuses on a different function that CCPs have been emphasized to play both by policy makers and academics: insurance provision to the participants through efficient counterparty risk management. The goal of this section is to study margin requirements and default fund as counterparty risk management tools CCPs use, and compare them relative to the information insensitivity of the security that agents trade bilaterally and submit to the CCP for central clearing.

In the model of section 2 a margin requirement is simply the collateral that agent A posts. The contract that solves $P1$ provides agent B with counterparty risk insurance, as chosen by agent B himself: he requires agent A to post collateral κ_x^A (i.e. to invest κ_x^A into the storage technology (3)).

We will not assume any intrinsic ability of the CCP to choose a possibly Pareto superior collateral requirement relative to what agent B does when making a TIOLI. Whether a CCP plays other economic roles which allow it to provide participants with a more efficient margining scheme, is out of the scope of this paper. In fact, in the model of section 2 the collateral requirement κ_x^A may well be efficient, depending on how rich agent B is (how large $\overline{\omega}$ is). In what follows, a margin requirement by a CCP is the investment into the storage technology (3), as κ_x^A is[38]. Therefore when we think of an economy with margin requirements we think of the benchmark equilibrium of section 2: we will compare such equilibrium with the equilibrium in an economy with a default fund.

Notice that we will focus on a mechanism that resembles what a default fund is in practice, and identify it with a CCP since it is one of the functions that CCPs perform, or services that CCPs provide. However, nothing in this analysis is peculiar exclusively to a CCP: any insurance provision that is carried out similarly to the mechanism here defined as a default fund in a CCP, will have the same effect on the equilibrium terms of trade between agents and on the resulting allocation.

A default or guarantee fund in a CCP is designed to cover excess losses in a default event that occurs under *extreme but plausible* market conditions[39] agreed between the CCP. Default funds are typically made up of contributions from both clearing participants and the CCP, and unlike margins, default funds operate on a pooled basis, which means non-defaulting clearing participants may be required to share any losses due to a default of another clearing participant.

[38]The Bank for International Settlement defines margins in a CCP as collateral to secure an obligation. See Glossary of terms used in Payment and Settlement Systems, http://www.bis.org/publ/cpss00b.pdf.

[39]The rationale for a default fund differs from the rationale for margin requirement: the latter is intended to cover the range of potential default losses under normal market conditions. However, the margin pledged by a defaulting clearing participant may not be sufficient to cover losses if the default occurs under extreme market conditions. In this situation, losses in excess of the defaulting clearing participants margin are covered by the default fund.

In order to model the pooling feature of a default fund, the environment of section 2 is slightly modified: the economy is populated by a continuum $[0,1]$ of agents of types A and B respectively, whose endowments, preferences and access to technologies are the same as in section 2. Let $\tilde{x}$ be *iid* across type A agents and assume that each type A meets a type B and always trades bilaterally.

A default fund scheme is defined as follows: it requires a contribution in the amount τ_x^A of good x to be invested in the storage technology (3) by every agent A. The contribution to the fund is made regardless of whether the TIOLI by agent B is accepted or rejected[40] and the fund pays out every time the counterparty has no goods to pay for his obligations.

Notice that in this environment a social planner constrained only by feasibility, would insure both against variance of $\tilde{x}$ and default risk[41]. By pooling risks across all A agents, the default fund scheme is capable of implementing such efficient allocation, whereas margins alone cannot feasibly do so because they are investment in storage within the pair rather than across A-B agents pairs. Therefore, for the purposes of comparing the default fund's effect on the equilibrium terms of trade and on the information insensitivity of the securities $s(x)$, relative to that of margins, we provide an example of default fund scheme that insures only against default risk λ.

Let $\tilde{s}(x)$ denote consumption of good x for B agents whose A defaulted and let $\tilde{s}(x) = s(x)$. Given $\tilde{s}(x)$, design the contribution to the default fund so that it provides full counterparty risk insurance:

$$\tau_{\tilde{s}}^A = \lambda E_x \tilde{s}(x) \tag{37}$$

[40] Any commitment issues are assumed away.

[41] That is to say the probability λ that agent A dies.

Given a contract $\{T_x, s(x), \kappa_x^A\}_{x\in X}$, a default fund contribution scheme is feasible if $x_L > \tau_{x,s}^A$. As an example consider a contract such that the security $s(x)$ agent B buys from agent A, is a complete transfer of ownership of the random endowment of good x in every state of the world net of what has been required to invest into storage: $s^O(x) = \rho(x - \tau_x^A)$. Then denote the contribution to a default fund scheme that provides full counterparty risk insurance, as in (37), by:

$$\tau^{A*} = \frac{\lambda\rho}{1+\rho} E_x(x)$$

Any other security $s(x)$ different from $s^O(x)$, is such that $s(x) \leq s^O(x)$ to the extent that it is part of a feasible contract $\{T_x, s(x), \kappa_x^A\}_{x\in X}$ as in definition 1. Therefore if a default fund contribution scheme is feasible for $\tilde{s}(x) = s^O(x)$ then it is also feasible for any other feasible security $s(x)$.

For the remainder of the section assume that x_L satisfies the following assumption:

Assumption 2

$$x_L > \tau^{A*}$$

Also, recall that the contribution to the default fund scheme is made regardless of whether a specific transaction occurs or not in the period[42]. Then, given a feasible contract

[42] That is to say regardless of the TIOLI is accepted or rejected.

$\{T_x, s(x), \kappa_x^A\}_{x \in X}$ the decision problem of agent B is:

$$(P3) \max_{\{s(x),T_x\}} \quad E_x U^B(s(x)) + \overline{\omega} - E_x(1-\lambda)T_x \tag{38}$$

$$s.t. \quad (1-\lambda)E_x[T_x + \rho(x - \tau_s^A) - s(x)] \geq (1-\lambda)E_x\rho(x - \tau_s^A) \tag{39}$$

$$\Pr\Big(T_x - s(x) < 0\Big)(1-\lambda)\Big[s(x) - T_x\Big] \leq \gamma \tag{40}$$

Proposition 5 *Given a security $s(x)$, let $\widehat{\tau}_s^A = \lambda E_x s(x)$ denote any default fund scheme that satisfies (37) (i.e. provides full counterparty insurance) and satisfies assumption 2. Then the set of Pareto optimal allocations that can be achieved with margins can also be achieved with a default fund. Furthermore, the set of Pareto optimal allocations that can be achieved with a default fund is strictly larger than those achieved with margins.*

Proof. Because $\widehat{\tau}_s^A$ satisfies (37) then it provides full counterparty insurance. In an economy where margins are used to insure against counterparty risk, a full insurance allocation can be achieved only if $\kappa_x^A = x, \forall x$, by proposition 1. For any $\epsilon > 0$ such that assumption 2 is still satisfied, consider the following alternative default fund scheme:

$$\tau^{A'} = \widehat{\tau}^A + \epsilon \tag{41}$$

Then such default fund scheme is feasible, because $\tau^{A'} \leq x, \forall x \in X$, and it achieves full counterparty insurance because $\tau^{A'} > \widehat{\tau}^A$. At settlement, this default fund scheme is such that it has to pay the obligations of the λ A agents who died, which amount to $\lambda E_x s(x)$ units of good x. The resources the fund has at settlement are obtained from the contributions $\tau^{A'}$ of all agent of type A, which amount to $\lambda(E_x(x) + \epsilon)$ units of good x.

Then this default fund scheme is feasible, achieves full counterparty insurance and has extra resources $\delta = \frac{\lambda}{1-\lambda}\epsilon > 0$ that can be rebated to the agents of type A who accept agent B's TIOLI and did not die. This results in constraints in problem $P3$ being relaxed: in fact $\forall \delta > 0$ the participation and incentive constraints (39) and (40) become:

$$E_x[T + \delta - s(x)] \geq 0 \tag{42}$$

$$\Pr(T + \delta - s(x) < 0)(1-\lambda)(s(x) - T - \delta) \leq \gamma \tag{43}$$

Therefore any Pareto optimal allocation that can be achieved with margins can also be achieved with a default fund.

Furthermore, because the contribution to the default fund scheme is independent of a specific financial transaction then it does not affect the constraint set in problem $P3$ because it cancels out from both sides of (39) and (40). By comparing constraints in $P3$ with constraints in $P1$ where counterparty insurance is provided by margin requirements, it is easy to see that (39) is slacker than (33) because proposition 2 implies that $\kappa_x^A > 0$; similarly (40) is slacker than (34). Therefore the set of Pareto optimal allocations that can be achieved with a default fund is strictly larger than those achieved with margins. ■

Proposition 5 shows that by pooling idiosyncratic risk, a default fund is capable of providing any level of counterparty insurance that margins provide, with fewer resources. To the extent that the resources needed to provide insurance have an outside option that is valuable to traders, reducing their need raises the payoff from trading and reduces the incentive to acquire information. In this sense a default fund makes the securities underlying each transactions more information insensitive than margins do. In this economy it is desirable for securities to be information insensitive, because trading them results in a

Pareto optimal allocation being implementable in a larger set of economies. Therefore a default fund is a more efficient counterparty risk management tool for CCPs relative to margins[43].

Also, because the default fund is prepaid by each participant, then it acts as a lump sum tax in this environment, because it does not distort agents' incentives and strategies. A margin requirement instead is linked to the specific obligation in a given transaction: therefore it acts as a wedge on agents incentives both to participate to trade and to accept the TIOLI without acquiring information. In this sense, a default fund makes the traded security more information insensitive than margin requirements. This implies that a Pareto optimal allocation can be implemented in a larger set of economies: a default fund scheme is welfare improving relative to margins.

5 Comments

The model described in this section can be interpreted as a stylized representation of several types of financial transactions that are carried out both on an exchange or over the counter (OTC): one of the most natural applications is collateralized lending, such as a repo for instance. The borrower wants to borrow against his asset but has to commit to post as collateral part of it. In the United States the Government Securities Division of the Fixed Income Clearing corporation (FICC/GSD) provides clearing, netting, risk management and settlement for a variety of US Government securities, repos and reverse repos: it acts as a CCP also in members' loss allocation procedure collecting mark-to-market margin payments and contributions to a clearing fund deposit. Another very natural application which recalls the origins of CCPs, is futures transactions: in the early 1900s, both in Europe

[43]These results abstract from any moral hazard consideration, which is not modeled explicitly.

and in the United States CCPs emerged associated to coffee and grain exchanges. Nowadays in the United States CME (Chicago Mercantile Exchange) Clearing provides central counterparty clearing and settlement services for exchange traded futures and options contracts traded on the Chicago Board of Trade (CBT). The extent to which collateralized lending and futures contracts seem the most natural applications of this model is simply due to the specific features of the model where there is a delay between contracting and settlement (futures) or where a loan is collateralized by an asset (repos). However if we interpret collateral as haircut, then the link between the model and a wide variety of financial transactions is more apparent: several types of swap contracts (e.g.interest rate, credit default), options, or simply bonds and money market instruments (The Options Clearing Corporation (OCC), ICE Trust US LLC, the National Securities Clearing Corporation (NSCC) the Depository Trust and Clearing Corporation (DTCC) are the relevant US central counterparties). In fact, all the results in the paper apply to any type of security, as simple as debt or equity and as elaborate as any derivative contract.

6 Conclusions

This paper constructs a model where trading goods/securities is beneficial: in this model information insensitivity is desirable because it allows trades to occur *easily* and the resulting allocation is a Pareto improvement over the original allocation. Within this framework, the paper shows that CCPs can enhance the information insensitivity[44] of the securities they clear by relaxing incentive constraints through:

- insurance provision

[44]Which in this model is a form of liquidity

- saving on collateral

Relative to the provision of insurance, this paper shows that a default fund relaxes constraints further than margin, and it does so by risk pooling and by its prepaid feature which does not distort agents' incentives and strategies.

Therefore, in any economy where securities need to be liquid to decentralize a Pareto optimal allocation, then CCPs are welfare enhancing, because they provide further liquidity and ease of trading

References

[1] V. Acharya and A. Bisin. Counterparty risk externality: Centralized vs over-the-counter markets. *Mimeo New York University*, June 2010.

[2] G. A. Akerlof. The market for lemons: Qualitative uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84 (3):488–500, September 1970.

[3] T. V. Dang, G. Gorton, and B. Holmstrom. Financial crisis and the optimality od debt for liquidity provision. *Mimeo Yale University*, August 2010.

[4] J. Davidson. *Stochastic Limit Theory.* Oxford University Press, 1994.

[5] D. Duffie and H. Zhu. Does a central clearing counterparty reduce counterparty risk? *Mimeo GSB Stanford University*, July 2010.

[6] G. Fusai and A. Ronconi. *Implementing Models in Quantitative Finance: Methods Cases.* Springer Finance.

[7] G. Gorton and A. Metrick. Regulating the shadow banking system. *forthcoming in Brookings Papers on Economic Activity*, November 2010.

[8] G. Gorton and A. Metrick. Securitized banking and the run on repo. *Yale ICF Working Paper No. 09-14*, 2010.

[9] T. V. Koeppl and C. Monnet. The emergence and future of central counterparties. *Federal Reserve Bank of Philadelphia, Working Paper 10-30*, September 2010.