Reducing Opaqueness in Over-the-Counter Markets

Zhuo Zhong\textsuperscript{1}

Department of Economics, Cornell University

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Abstract

This paper evaluates if having a centralized market can reduce opaqueness in an over-the-counter market. I show that a competitive centralized market incentivizes dealers in the over-the-counter market to reduce opaqueness, whereas a noncompetitive centralized market does not. The competition between the competitive centralized market and the over-the-counter market forces dealers in the latter to reduce opaqueness. With the noncompetitive centralized market, opportunities for collusion incentivize dealers to increase opaqueness. Specifically, the natural monopoly market maker arising in the noncompetitive centralized market coordinates his spread according to dealers’ spreads. The coordination implies that opaqueness benefits both the dealers and the monopoly market maker. To demonstrate these arguments, I extend Rust and Hall (2003) by characterizing opaqueness with Knightian uncertainty. Based on the model, I show that with the competitive centralized market, opaqueness decreases dealers’ profits; whereas with the noncompetitive centralized market, I find the inverse relation.

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1. Introduction

Over-the-counter markets (OTC) are often opaque, as they fail to disclose information regarding the trades publicly. In the 2008-2009 financial crisis, opaqueness in OTC derivative markets complicated the price discovery, thereby deterring investors from trading. The lack of trading reduced market liquidity, which further complicated the price discovery and lead to another round of liquidity deterioration. Having experienced this detrimental impact of opaqueness, in the post crisis era policy makers universally call for reforms to reduce opaqueness in OTC markets.\(^2\) One of the ongoing reformations is to trade standard OTC products in centralized markets.\(^3\) This can lead to the coexistence of centralized and OTC trading. How will this coexistence affect market making and trading in OTC markets? Furthermore, as dealers can benefit from opaqueness (see Madhavan (1995) and Yin (2005)), will having centralized markets incentivize dealers to reduce opaqueness as it is supposed to? These questions are important to understand financial reformations that attempt to increase transparency in OTC markets. To the best of my knowledge, this paper is the first attempt to address these questions.

In this paper, I develop a model where a centralized market operates simultaneously with an opaque OTC market. In the centralized market, a finite number of market makers compete for order flows by posting bid-ask spreads. In a competitive centralized market, the winning market maker sets his spread to deter potential entrance of other market makers. Whereas in a noncompetitive centralized market, the entrance threat of other market makers is not credible. I find that whether the centralized market is competitive or not generates different impacts on the OTC market. While a competitive centralized market causes dealers’ profits to decrease under greater opaqueness, a noncompetitive centralized market leads to the opposite. The contradicting results are due to the change in the relation between the centralized market and the OTC market. Specifically, when the competitive centralized market becomes noncompetitive, opportunities for cooperation arise and replace the competition relation between these two markets. Based on these findings, I suggest that regulators should adopt market structures that boost competition among market makers, e.g., the electronic limit order book, as the primary industrial organization for the centralized market.


\(^3\) In the United States, the Dodd-Frank act requires to trade standard swaps in “swap execution facilities”, where multiple participants can trade on publicly available prices made by other participants. And in Europe, the MiFID II sets out the requirement for trading derivatives on organized venues, the “organized trading facilities”.
The model developed in this paper extends Spulber (1996) and Rust and Hall (2003) by incorporating opaqueness in the OTC market. In the benchmark model, I analyze an economy that consists of the OTC market only. I show that greater opaqueness leads to larger bid-ask spreads in the OTC market. This result implies that reducing opaqueness will decrease trading costs, thereby increasing market efficiency. However, the welfare analysis indicates that dealers oppose reducing opaqueness because of smaller profits. The driving force behind these results is that opaqueness makes traders’ outside options ambiguous, and hence, reduces the value of search. Thus traders search less. The insufficient search leads to increases in traders’ trading costs. Since traders’ losses are dealers’ gains, dealers profit from opaqueness.

To explore the impact of centralized trading, I extend the benchmark model to include an additional market -- a centralized market. When the centralized market is competitive, the bid-ask spread depends on other market makers’ transaction costs, and hence, is independent of OTC trading. As a result, the centralized market attracts traders who want to avoid trading ambiguously in the OTC market. Under greater opaqueness, dealers lose their customers to the centralized market which decreases their profits.

However, the noncompetitive centralized market overturns the above relation between dealers’ profits and opaqueness. The natural monopoly in the centralized market adjusts its bid-ask spread along with changes in dealers’ bid-ask spreads. Specifically, the bid-ask spread in the noncompetitive centralized market is positively correlated with the bid-ask spreads in the OTC market. This dependence implies that dealers and the monopoly can collude to increase trading costs so as to profit from opaqueness.

In addition, I explore how opaqueness affects the ability to survive of the centralized market and the OTC market, respectively. I find that greater opaqueness increases the centralized market’s ability to survive regardless of the competitiveness in it. However, opaqueness is not the key determinant of the OTC market’s viability. The comparison between transaction costs in the OTC market and the centralized market (both competitive and noncompetitive) determines if the OTC market will be eliminated under the entry of the centralized market. In short, when the centralized market has substantial lower transaction costs than the OTC market, the latter cannot survive in the equilibrium.

In this paper, I represent opaqueness in OTC markets by Knightian uncertainty. Knightian uncertainty describes the situation when the odds of future states are unknown. Knightian uncertainty assumes that the decision maker has a set of priors rather than a unique prior. Thus, the degree of
Knightian uncertainty can be measured by the size of the set of priors. Past studies have shown Knightian uncertainty could arise if the decision maker has vague information (Ellsberg (1961)); or if the decision maker has insufficient knowledge (Easley and O’Hara (2009), (2010a), and (2010b)); or if the decision maker has adopted incorrect models (Hansen and Sargent (2001)). Since opaqueness means that some trading information (e.g. quotes, prices, and order flows) is unavailable or unreliable, traders in OTC markets only have vague information, and hence, face Knightian uncertainty.

I adopt the search model to describe trading in OTC markets for the following reasons. Firstly, most OTC markets are dealership markets. In dealership markets, trades are conducted through bilateral negotiations with traders on one side of the trades, and dealers on the other side. As terms of bilateral trades are not public, traders have to search among dealers for price information. Hence, the search model in which economic agents seek out the optimal deal captures the bilateral trading mechanism. Secondly, Yin (2005) shows that search costs are crucial in analyzing fragmented markets, which include OTC markets, even for infinitesimal amount. This is because the friction created by search significantly changes price behaviors between fragmented markets and centralized markets.

Dealers in my model adjust their bid-ask spreads to maximize their profits under inventory constraints. Hence, my model falls into the inventory-based market microstructure models (e.g. Amihud and Mendelson (1980), and Ho and Stoll (1981)), which is different from the information-based market microstructure models (e.g. Milgrom and Glosten (1985), and Easley and O’Hara (1987)). Traders in my model are “liquidity traders” rather than “informed traders”.

In the next section, I review the related literature. In Section 3, I set out the benchmark model, where only an OTC market operates in the economy. In Section 4, I extend the benchmark model to include a centralized market. In Section 5, I discuss the empirical implications of my model and conclude.

2. Related Literature

Since OTC markets are typical examples of fragmented markets, my paper adds on the market fragmentation and transparency literature. Unlike most of the past studies treating fragmented markets as

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4 Spulber (1996) and Rust and Hall (2003) use the same framework to describe trading in dealership markets, of which OTC markets are special examples. Duffie et al. (2005), (2007), and Lagos and Rocheteau (2009) adopt a different search framework to model OTC trading. I will discuss the differences in Section 2.
completely opaque (e.g., Biais (1993), Madhavan (1995), Pagano and Roell (1996), de Frutos and Manzano (2002), and Yin (2005)), I allow for different degrees of opaqueness in fragmented markets. In addition, while those past studies only compare equilibrium outcomes between fragmented markets and centralized markets, I study the relation between these two markets when both operate in the economy. With these two distinct features, my model generates novel results that show the impact of opaqueness on equilibrium depends on whether there exists centralized markets, and whether those centralized markets are competitive. Though Gehrig (1993), Rust and Hall (2003) also study the equilibrium when fragmented markets operate simultaneously with centralized markets, they do not consider varying degrees of opaqueness in fragmented markets. In fact, my model generalizes the work by Rust and Hall (2003).

My paper also belongs to the growing literature on ambiguity or Knightian uncertainty in market microstructure research. In a series of papers, Easley and O’Hara (2009), Easley and O’Hara (2010a), and Easley and O’Hara (2010b) show how Knightian uncertainty can affect market trading, and how certain designs of the market microstructure can lessen Knightian uncertainty, and hence, increases market participation and liquidity. My study on Knightian uncertainty in OTC markets complements theirs. While they focus on the exchange trading that only takes place in centralized markets, I examine the OTC trading that takes place in fragmented markets. Chen and Zhong (2011) also study Knightian uncertainty in OTC markets. However, they do not analyze the establishment of centralized markets in the economy, which distinguishes my model from theirs.

My work also adds to search models that characterize OTC markets. Both the search formulation and the price determination in my model are different from Duffie, Garleanu, and Pedersen (2005), and Duffie, Garleanu, and Pedersen (2007). In their papers, both traders and dealers search, whereas in my paper, only traders search, and they search through unknown price distributions. In their papers, prices are set through negotiations, whereas in my paper, dealers decide the price. The differences in price determinations imply that their prices are trade prices, whereas mine are quotes. My study is in line with Zhu (2011) who studies quotes in opaque OTC markets by examining dealers quoting strategies under contact-order uncertainty, but with a different model. More importantly, while Zhu (2011) focuses only on OTC markets, I focus on the interaction between OTC markets and the centralized market.

My search model generalizes the work by Spulber (1996), and Rust and Hall (2003) with Knightian uncertainty in search processes. Since incorporating Knightian uncertainty implicitly adds
another search cost in their models, my model converges to theirs, when there is no Knightian uncertainty.

3. The Search Equilibrium in an OTC Market

In this section, I extend the search model in Spulber (1996) to use as the benchmark. In the benchmark model, only the OTC market operates. The economy consists of a continuum of buying traders (buyers) and a continuum of selling traders (sellers). Traders’ types depend on their valuations of the asset. I denote \( v^B \) as the buyer’s valuation and \( v^S \) as the seller’s valuation, and I assume both \( v^B \) and \( v^S \) follow the uniform distribution over \([0, 1]\). This assumption summarizes the heterogeneity of the traders. I do not intend to explore this heterogeneity, since the paper focuses on the trading structure in not the asset valuation. The economy also consists of a continuum of dealers. Dealers connect the buys and sells in the OTC market, and they are heterogeneous in their transaction costs. Denoting \( k \) as the dealer’s transaction cost, I assume \( k \) follows the uniform distribution over \([k, 1]\), where \( k \) denotes the transaction cost for the most efficient dealer. The heterogeneity among transaction costs reflects that dealers adopt different technologies or use different pricing models.

Traders engage in the sequential search process due to fragmentation in the OTC market. Furthermore, they search with Knightian uncertainty due to opaqueness in the OTC market. That is, traders’ prior knowledge is a set of distributions over dealers’ quotes. This set of priors available to traders is an \( \epsilon \)-contamination of historical distributions over bid and ask prices. In particular, for any given \( \epsilon \), buyers have the following set of priors,

\[
P^B(\epsilon) \equiv \{(1 - \epsilon)P_a + \epsilon\mu : \mu \in M\},
\]

where \( P_a \) is the historical distribution of ask prices and \( M \) is the set of all probability measures on the Borel set of real numbers. Sellers have the following set of priors,

\[
P^S(\epsilon) \equiv \{(1 - \epsilon)P_b + \epsilon\mu : \mu \in M\},
\]

where \( P_b \) is the historical distribution of bid prices. In either \( P^B(\epsilon) \) or \( P^S(\epsilon) \), when \( \epsilon \) is zero, the set implodes to a unique prior, which indicates no Knightian uncertainty, and as \( \epsilon \) grows, the degree of

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5 Chen and Zhong (2011) uses the same model to investigate the pre-trade transparency in OTC markets.

6 The \( \epsilon \)-contamination refers to the procedure of introducing a set of priors by contaminating a single hypothetical prior with an \( \epsilon \) probability ball around it. To be more specific, the set of priors \( P(\epsilon) \) is

\[
P(\epsilon) \equiv ((1 - \epsilon)P_0 + \epsilon\mu : \mu \in M),
\]

where \( P_0 \) is the hypothetical prior and \( \mu \) is any probability distribution in the relevant space.
Knightian uncertainty increases. As Knightian uncertainty represents opaqueness in the OTC market, $\epsilon$ becomes the measure of opaqueness in the OTC market. Larger $\epsilon$ indicates greater opaqueness. In addition, the cores of $\epsilon$-contamination sets imply that traders construct their priors by adding noises to the distributions of historical prices. Thus, how noisy traders’ priors are depends on how opaque the OTC market is.

Admittedly, I am making an implicit assumption to equate the $\epsilon$-contamination to Knightian uncertainty, since the former is a special case of the latter. However, I believe implications obtained with the $\epsilon$-contamination would still hold under a more general specification of Knightian uncertainty.

3.1. Traders' Decisions

For any given $\epsilon$, a buyer maximizes his minimal expected future payoff,

$$\min \left\{ \int l(a)dP : P \in P^B(\epsilon) \right\},$$  \hspace{1cm} (3)

where $l(a)$ is the discounted future payoff. More specifically, $l(a) = \beta^t(v^B - a)$ if he trades at time $t$, or zero otherwise. That is,

$$l(a) = \begin{cases} \beta^t(v^B - a), & \text{if he trades at time } t; \\ 0, & \text{otherwise}. \end{cases}$$  \hspace{1cm} (4)

In the above, $\beta$ is the discount factor.

By Schmeidler (1989), the buyer’s objective function equals the Choquet integral of discounted future payoff $l(a)$ with respect to a convex probability capacity $\theta_a$, which means

$$\min \left\{ \int l(a)dP : P \in P^B(\epsilon) \right\} = \int l(a)d\theta_a.$$  \hspace{1cm} (5)

$\theta_a$ is, for any given measurable set $E$,

$$\theta_a(E) = \begin{cases} (1 - \epsilon)P_a(E), & \text{if } E \neq \Omega; \\ 1, & \text{if } E = \Omega. \end{cases}$$  \hspace{1cm} (6)

And $\Omega$ represents all asks.

As shown in Nishimura and Ozaki (2004), the Bellman equation associated with the above problem is,

$$V^B(a, v^B) = \max \left\{ 0, v^B - a, \beta \int V^B(\hat{a}, v^B)d\theta_a \right\}.$$  \hspace{1cm} (7)

In eq. (7), $V^B(a)$ is the value function for the buyer who has an ask offer $a$ at hand. And $\hat{a}$ is his next randomly received ask if he continues to search. $V^B(a)$ reflects the choices that the buyer has: (i) do nothing; (ii) accept the dealer's ask; (iii) reject the ask price and continue to search. Obviously, if the
buyer has \( v^B \leq a \) (\( a \) is the lower bound of the ask prices offered by dealers), he will never trade or search.\(^7\) When \( v^B > a \), the optimal search strategy for the buyer is to accept any ask greater than his reservation buying price. The reservation buying price \( r^B(v^B) \) is the solution of the following equation,

\[
v^B = r^B(v^B) + \frac{\beta}{1 - \beta} \int_a^{r^B(v^B)} \theta_a \{ a \leq a \} \, da.
\]

According to Nishimura and Ozaki (2004), eq. (8) is equal to the following equation,

\[
v^B = r^B(v^B) + \frac{\beta(1 - \epsilon)}{1 - \beta} \int_a^{r^B(v^B)} P_a \{ a \leq a \} \, da.
\]

Applying the Implicit Function Theorem, I can show that \( r^B(v^B) \) is a strictly increasing function of \( v^B \) on the interval \((v^B, 1]\). The lower bound of the interval \( v^B \) denotes to the valuation of the marginal buyer, whose gain from trading is zero, i.e., \( v^B = a = r^B(v^B) \).

Similarly, I derive the seller’s reservation price, which is the solution of the following equation

\[
v^S = r^S(v^S) - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_{r^S(v^S)}^{\bar{b}} P_b \{ b \geq b \} \, db.
\]

And \( r^S(v^S) \) strictly increases on the interval \([0, \bar{v}^S] \). \( \bar{v}^S \) denotes to the valuation of the marginal seller, whose gain from trading is zero, i.e., \( \bar{v}^S = \bar{b} = r^S(\bar{v}^S) \).

3.2. Dealers’ Decisions

Since \( v^B \) is uniformly distributed on \((v^B, 1]\) and \( r^B(v^B) \) is monotone on the interval \((v^B, 1]\), by change of variables, the density of the reservation buying prices is

\[
f^B(r^B) = \frac{1 - \beta + P_a(r^B)(1 - \epsilon)\beta}{(1 - \beta)(1 - v^B)}.
\]

Analogously, the density of the reservation selling prices is

\[
f^S(r^S) = \frac{1 - \beta + (1 - P_b(r^S))(1 - \epsilon)\beta}{(1 - \beta)r^S}.
\]

A dealer posts stationary bid and ask to maximize his expected discounted profits. Meantime, the dealer has to maintain his inventory position, which means that his expected demand shall equal to his

\(^7\) For technical reasons, I assume that when a trader is indifferent between trading in the market or not, he chooses not to trade. That is, when a buyer has valuation \( v^B = a \) he quits, and when a seller has valuation \( v^S = \bar{b} \) he quits. \( \bar{b} \) is the upper bound of bids offered by dealers.
expected supply.\(^8\)

As \(N\) is the total mass of dealers operating in the market, \(\frac{1 - \nu^B}{N} f^B(r^B)\) represents the density of buyers for every dealer. The number of buyers, who have reservation price \(r^B\), visiting the dealer is as follows: 1 at date-0, \(P_a^0[\alpha \geq r^B]\) at date-1, \(P_a^2[\alpha \geq r^B]\) at date-2, ..., \(P_a^t[\alpha \geq r^B]\) at date-\(t\). If the dealer sets the ask to \(\alpha\), then the market demand at time \(t\) is

\[
D_t(\alpha) = \frac{1 - \nu^B}{N} \int_a^{\overline{\alpha}} P_a^t[\alpha \geq r^B] f^B(r^B) dr^B
\]

\[
= \frac{1}{N} \int_a^{\pi (1 - F_a(r^B))^t (1 - \beta + F_a(r^B)(1 - \epsilon)\beta)} dr^B,
\]

where \(\overline{\alpha}\) is the upper bound of asks in the OTC market.

By an analogous derivation, the date-\(t\) supply associated with the bid price \(b\) is,

\[
S_t(b) = \frac{1}{N} \int_b^{\beta (r^S)(1 - \beta + (1 - F_b(r^S))(1 - \epsilon)\beta)} dr^S,
\]

where \(\beta\) is the lower bound of bids in OTC markets.

Given demand \(D_t(\alpha)\) and supply \(S_t(b)\), the dealer's objective is

\[
\max_{a,b} \sum_{t=0}^{\infty} \beta^t (aD_t(a) - (b + k)S_t(b)),
\]

subjects to

\[
D_t(\alpha) = S_t(b).
\]

3.3. The Stationary Search Equilibrium

Proposition 1 describes the stationary search equilibrium in the OTC market, in which traders maximizing their minimum expected payoffs, and dealers maximizing their expected profits.

Proposition 1 [The Benchmark Equilibrium]

For any given \(\epsilon\), there exists a continuously differentiable symmetric equilibrium pricing policy, \(a(k), b(k)\), with \(a(k)\) increasing and \(b(k)\) decreasing in \(k\) for all \(k \leq k^*\), where \(k^*\) denotes the marginal dealer whose profit margin and trading volume are zeros. The pricing policy functions satisfy

\(^8\) Spulber (1996), Rust and Hall (2003), Duffie et al. (2005) and (2007), and Lagos and Rocheteau (2009) use the same assumption in their search models.
\[ a(k) = e^{-\int k^* \gamma(z) dz} \left( \frac{k^* + 1}{2} + \int_k^{k^*} \left( -\frac{1}{4} + \frac{1 + z}{2} Y(z) \right) e^{\int_z^{k^*} \gamma(u) du} dz \right), \] (17)
\[ b(k) = 1 - a(k), \] (18)
\[ k^* = a(k^*) - b(k^*), \] (19)

where
\[ Y(z) = -\frac{\beta}{2(k^* - k)} \left( \frac{1}{1 - \beta(k^* - z)} - \frac{1 - \epsilon}{1 - \beta + \frac{z - k}{k^* - k} (1 - \epsilon)\beta} \right), \] (20)

and \( k^* \) is the solution of the following equation
\[ 1 = \frac{k^* + 1}{2} + \frac{\beta(1 - \epsilon)}{1 - \beta} \left( \frac{k^* + 1}{2} - \frac{1}{k^* - k} \int_k^{k^*} a(k) dk \right). \] (21)

In the stationary equilibrium, the historical distributions of prices coincide with the equilibrium distributions of prices. The equilibrium obtained above is similar to the rational expectations equilibrium conceptually as the equilibrium prices confirm traders’ set of priors. But in terms of the equilibrium outcomes, my equilibrium is different from the rational expectations equilibrium. In my equilibrium, traders’ predictions on prices systematically deviate from the equilibrium prices, whereas in the rational expectations equilibrium, traders’ predictions on prices are self-fulfilling. These systematic deviations in my equilibrium depend on opaqueness in the OTC market. When the OTC market is fully transparent, the equilibrium becomes the rational expectations equilibrium obtained in Spulber (1996).

### 3.4. Comparative Statics

Obviously, the price system in Proposition 1 is non-linear. Therefore, the analytical solution for the price system does not exist, in general. Due to the lack of analytical solution, I show the comparative statics numerically. Setting \( \beta = 0.9 \) and \( \underline{k} = 0.005 \), I solve the equilibrium with \( \epsilon \) ranges from 0 to 0.5.\(^9\)

[Insert Figure 1 Here]

Figure 1 shows that the average bid-ask spread in the OTC market increases as \( \epsilon \) increases. As \( \epsilon \) represents the degree of opaqueness in the OTC market, the increasing \( \epsilon \) indicates greater opaqueness in the OTC market. Thus, Figure 1 shows that when the OTC market gets more opaque, the average bid-ask spread increases.

\(^9\) I obtain similar results with other assigned parameter values.
Figure 2 illustrates how changes in $\epsilon$ alter the demography in the economy. The left panel of Figure 2 shows that the total mass of dealers in the OTC market increases as $\epsilon$ increases, whereas the right panel of Figure 2 shows the total mass of traders in the OTC market decrease as $\epsilon$ increases. This means that the impact of opaqueness on the OTC market has two folds. On one hand, greater opaqueness encourages the participation of dealers; on the other hand, greater opaqueness discourages the participation of traders.

To decompose comparative results in Figure 1 and Figure 2, I compare the ask prices when the OTC market is transparent (that is, when $\epsilon = 0$) with the ask prices when the OTC market is opaque (that is, when $\epsilon > 0$). Figure 3 shows the results from this comparison. In the right panel of Figure 3, I find that the cumulative density function of asks shifts toward the right when $\epsilon > 0$. The shift means that buyers are more likely to receive higher asks from dealers when the OTC market is opaque, i.e., $\epsilon > 0$. Since in the equilibrium, $b(k) = 1 - a(k)$, the dealer who increases his ask also decreases his bid. Hence, when the OTC market is opaque, all operating dealers’ bid-ask spreads become larger. Consequently, the increasing bid-ask spreads discourage traders to trade, since their trading costs increase.

The force driving results in Figure 3 is the decrease of the search value in the opaque OTC market. In Figure 4, I compare traders’ reservation values between different opaqueness regimes in the OTC market. I show that buyers’ reservation buying prices are higher, and sellers’ reservation selling prices are lower, when $\epsilon > 0$. That is, buyers are willing to buy at higher prices and sellers are willing to sell at lower prices, when the OTC market is opaque. These results imply that the search value is lower in the opaque OTC market. The OTC market opaqueness increases traders’ uncertainty on outside options. As traders become uncertain about their outside options, they are willing to accept worse offers. As a result of that, bid-ask spreads increase with the degree of opaqueness.

3.5. The Welfare Analysis

I define the gains from trade as the sum of traders’ surplus

\[ \text{Uncertainty here refers to Knightian uncertainty.} \]

10
\begin{align}
\int_{v^B}^{1} (v^B - r^B(v^B))dv^B + \int_{0}^{v^S} (r^S(v^S) - v^S)dv^S.
\end{align} \tag{22}

[Insert Figure 5 Here]

Figure 5 shows how changes in $\epsilon$ change traders’ total surplus and dealers’ total profits. The left panel of Figure 5 shows that as $\epsilon$ increases, traders’ surplus decreases. This means that the gains from trade decrease under greater opaqueness. While traders suffer from opaqueness, dealers benefit from opaqueness. In the right panel of Figure 5, I show dealers’ total profits increase as $\epsilon$ increases. The result that dealers are better off in the opaque OTC market is consistent with Madhavan (1995) and Yin (2005).

The welfare analysis indicates that though reducing opaqueness decreases trading costs and increases market efficiency, it harms the dealers in the OTC market. As a result, dealers who are vital in connecting buys and sells in the OTC market, oppose to reduce opaqueness. Can we align dealers’ interests with traders to reduce opaqueness?

4. Stationary Search Equilibria with a Centralized Market

In this section, I show that having a competitive centralized market to compete with the OTC market will provide incentives for dealers to reduce opaqueness of the latter. The model extends the work by Rust and Hall (2003)

The centralized market is a trading venue. On the venue, there are $m$ dealers with transaction costs $K_1, K_2, \ldots, K_m$. To differentiate them from dealers in the OTC market, I denote dealers in the centralized market as “market makers.” Market makers post publicly available asks and bids. In addition, I assume all market makers adopt inventory constraints, that is, demand shall equal to supply.

4.1. Traders’ Decisions under the Existence of the Centralized Market

Traders have an additional option that is to trade in the centralized market. This additional option changes traders’ trading decisions. Specifically, a buyer who has not yet chosen to search has three options: (i) do nothing; (ii) buy a unit of asset in the centralized market at price $a_c$; (iii) search for a better price in the OTC market. Hence, the buyer’s value function before he searches is
\[
W^B(a_c, v^B) = \max\left\{0, v^B - a_c, \beta \int V^B(\hat{a}, a_c, v^B) \, d\theta_a\right\},
\]

where \(V^B(\hat{a}, a_c, v^B)\) denotes the value function for the buyer when he searches in the OTC market and \(\hat{a}\) is the next random ask received. Once the buyer starts to search in the OTC market, he has the fourth option of accepting the current ask, \(a\). The buyer’s value function when he searches is

\[
V^B(a, a_c, v^B) = \max\left\{0, v^B - a, v^B - a_c, \beta \int V^B(\hat{a}, a_c, v^B) \, d\theta_a\right\}.
\]

Similarly, seller’s value function before he searches is

\[
W^S(b_c, v^S) = \max\left\{0, b_c - v^S, \beta \int V^S(\hat{b}, b_c, v^S) \, d\theta_b\right\},
\]

where \(V^S(\hat{b}, b_c, v^S)\) denotes the value function for the seller if he decides to search in the OTC market and \(\hat{b}\) is the next random bid received. And the seller’s value function when he searches is

\[
V^S(b, b_c, v^S) = \max\left\{0, b - v^S, b - v^S, \beta \int V^S(\hat{b}, b_c, v^S) \, d\theta_b\right\},
\]

where \(b\) is the current bid.

From buyers and sellers’ value functions when they search (eq. (24) and eq. (26)), if all dealers’ asks are lower than the centralized market’s ask, and if all bids are higher than the centralized market’s bid, then in the equilibrium no trader will trade in the centralized market. On the other side, if no dealer can offer ask lower than the centralized market’s ask, and if no dealer can bid higher than the centralized market’s bid, then in the equilibrium all traders will trade in the centralized market. The intermediate stage is when some but not all dealers are able to offer lower asks and higher bids than the centralized market, then some traders will trade in the centralized market and some will trade in the OTC market in the equilibrium. I start the analysis from the intermediate stage equilibrium, since the other two are extreme cases of the intermediate stage equilibrium.

4.2. The Equilibrium in which OTC Markets Coexist with the Centralized Market

Traders’ Decisions

As discussed in the above, no dealer can survive by posting an ask \(a\) higher than the ask from the centralized market \(a_c\). Thus, \(a_c\) is the upper bound of asks in the OTC market. Let \(a\) be the lower bound of asks in the OTC market. Buyer’s value function when he searches (eq. (24)) implies that any buyer
with $v^B \leq a$ will never trade. Hence, $a$ determines the marginal buyer. That is, $a = v^B$ where $v^B$ is the marginal buyer's valuation of the asset.

For the buyer whose reservation value equals to $a_c$ when he searches, let $\tilde{v}^B$ be his valuation of the asset. **Proposition 2** describes buyers' optimal strategies in choosing which market to trade in.

**Proposition 2**

A buyer's optimal strategy depending on his type $v^B$ is as follows:

- if $v^B \in [\tilde{v}^B, 1]$, then it is optimal for the buyer to bypass the OTC market and purchase the asset immediately from the centralized market at the ask price $a_c$;
- if $v^B \in (\tilde{v}^B, v^B)$, then it is optimal for the buyer to trade in the OTC market;
- if $v^B \in [0, \tilde{v}^B]$, then it is not optimal for the buyer to trade in the centralized market nor in the OTC market.

When $v^B \in (\tilde{v}^B, v^B)$, the buyer's optimal search strategy is a reservation price policy with the reservation price implicitly defined as

$$v^B = r^B(v^B) + \frac{\beta(1 - \epsilon)}{1 - \beta} \int_{a}^{r^B(v^B)} P_a[a \leq \tilde{a}]d\tilde{a}. \quad (27)$$

By Implicit Function Theorem, I can show that $r^B(v^B)$ is monotone on $(\tilde{v}^B, v^B)$. Thus, eq. (27) implies

$$\tilde{v}^B = a_c + \frac{\beta(1 - \epsilon)}{1 - \beta} \int_{a}^{a_c} P_a[a \leq \tilde{a}]d\tilde{a}. \quad (28)$$

Likewise, I have **Proposition 3** describe sellers' optimal strategies in choosing which market to trade in. In **Proposition 3**, $\tilde{v}^S$ denotes the seller with reservation value equals to the centralized market's bid, $b_c$, and $\tilde{S}$ is the marginal seller whose gain from trading is zero.

**Proposition 3**

A seller's optimal strategy depending on his type $v^S$ is as follows:

- if $v^S \in [\tilde{v}^S, 1]$, then it is not optimal for the seller to trade in the centralized market nor in the OTC market;
- if $v^S \in (\tilde{v}^S, \tilde{v}^S)$, then it is optimal for the seller to trade in the OTC market;
if \( \nu^S \in [0, \nu^S] \), then it is optimal for the seller to bypass the OTC market and sell the asset immediately in the centralized market at the bid price \( b_c \).

From the above, when \( \nu^S \in (\nu^S, \bar{\nu}^S) \), the seller's optimal search strategy is a reservation price policy, and the reservation price is implicitly defined as the follows,

\[
\nu^S = r^\nu(\nu^S) - \beta(1 - \epsilon) \frac{1}{1 - \beta} \int_{r^\nu(\nu^S)}^B p_b[b \geq \hat{b}] dB.
\]

Similarly, I can show that \( r^\nu(\nu^S) \) is a strictly increasing function of \( \nu^S \) on the interval \( (\nu^S, \bar{\nu}^S) \) by Implicit Function Theorem. Thus, \( \nu^S \) is defined as

\[
\nu^S = b_c - \beta(1 - \epsilon) \frac{1}{1 - \beta} \int_{b_c}^P p_b[b \geq \hat{b}] dB.
\]

**Dealers’ Decisions**

With an analogous derivation in Section 3, I have the demand and supply for a dealer at time \( t \),

\[
D^D_t(a) = \frac{1}{N^D} \int_a^{1 - F_a(r^B)} \frac{(1 - F_a(r^B))^t}{1 - \beta} \left(1 - \beta + F_a(r^B)\right)(1 - \epsilon)\beta dr^B,
\]

\[
S^D_t(b) = \frac{1}{N^D} \int_b^{b\nu_t(\nu^S)} \frac{F_b^t(r^S)}{1 - \beta} \left(1 - \beta + (1 - F_b(r^S))\right)(1 - \epsilon)\beta dr^S,
\]

in which \( N^D \) is the total mass of the surviving dealers.

With the constraint of keeping demand equal to supply, a dealer is trying to maximize his expected discount profits. That is,

\[
\max_{a, b} \sum_{t=0}^\infty \beta^t \left( a D^D_t(a) - (b + k) S^D_t(b) \right),
\]

subject to

\[
D^D_t(a) = S^D_t(b).
\]

**Market Makers’ Decisions and the Competitiveness of the Centralized Market**

All market makers post their asks and bids in the centralized market. And all asks and bids are public. This means that any market maker can observe prices set by others. Since market makers
compete for order flows from traders, the most efficient market maker (who has the least transaction cost) will charge a bid-ask spread that is less or equal to the next most efficient market maker’s transaction cost to become the monopoly in the centralized market. Thus, denoting \( K_{(2)} \) as the second order statistic of \( \{K_1, K_2, ..., K_m\} \), the bid-ask spread in the centralized market satisfies the following condition,

\[
a_c - b_c \leq K_{(2)}. \tag{35}
\]

From Proposition 2, market demand for the centralized market is,

\[
D^C(a_c) = 1 - v^b(a, a_c) = 1 - a_c - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_a^{a_c} P_a[a < \hat{a}] d\hat{a}. \tag{36}
\]

And from Proposition 3, market supply for the centralized market is,

\[
S^C(b_c) = v^s(b, \beta) = b_c - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_{b_c}^{\beta} P_b[b > \hat{b}] d\hat{b}. \tag{37}
\]

The monopoly market maker chooses \( a_c \) and \( b_c \) to maximize his expected discounted profits. That is,

\[
\max_{a_c, b_c} a_c D^C(a_c) - (b_c + K_{(1)}) S^C(b_c), \tag{38}
\]

subject to

\[
D^C(a_c) = S^C(b_c), \tag{39}
\]

\[
a_c - b_c \leq K_{(2)}, \tag{40}
\]

where \( K_{(1)} \), the monopoly market maker’s transaction cost, is the first order statistic of \( \{K_1, K_2, ..., K_m\} \).

Two sets of solutions can arise when I solve the above maximization problem. The first set is the corner solution, which is when the inequality (40) binds. This indicates that the centralized market is competitive. The monopoly market maker has to post a bid-ask spread equaling to \( K_{(2)} \) to deter the entry of other market makers. The second set is the interior solution, which is when the inequality (40) unbinds. This indicates that the centralized market is not competitive. The most efficient market maker becomes the natural monopoly whose action does not depend on other market makers. Unlike the corner solution where the bid-ask spread equals to \( K_{(2)} \), the bid-ask spread in this case depends on \( K_{(1)} \). Hence, given \( k, \epsilon \), and \( \beta \), whether monopoly market maker’s profit maximization is the corner solution or the interior solution depends on \( K_{(1)} \) and \( K_{(2)} \).

Stationary Search Equilibria
Given that two sets of solutions can emerge in the monopoly market maker’s profit maximization problem, two equilibria emerge correspondingly.

**Proposition 4** characterizes the equilibrium when the monopoly market maker’s profit maximization generates the corner solution.

**Proposition 4 [The Corner Equilibrium]**

For any given \( \varepsilon \), there exists a continuously differentiable symmetric equilibrium pricing policy, \( a(k) \) and \( b(k) \), with \( a(k) \) increasing and \( b(k) \) decreasing in \( k \) for all \( k \leq k < K(2) \). The pricing policy functions satisfy,

\[
 a(k) = e^{-\int_k^{K(2)} Y(z) dz} \left( \frac{K(2) + 1}{2} \right) + \int_{K_d}^{K(2)} \left( -\frac{1}{4} + \frac{1 + z}{2} Y(z) \right) e^{\int_z^{K(2)} Y(u) du} dz, \tag{41}
\]

\[
 b(k) = 1 - a(k), \tag{42}
\]

where

\[
 Y(z) = \frac{\beta}{2(K(2) - k)} \left( \frac{1}{1 - \frac{\beta(K(2) - z)}{K(2) - k}} - \frac{1 - \varepsilon}{1 - \beta + \frac{z - k}{K(2) - k}(1 - \varepsilon)\beta} \right), \tag{43}
\]

The centralized market’s prices are

\[
 a_c = \frac{K(2) + 1}{2}, \tag{44}
\]

\[
 b_c = 1 - a_c. \tag{45}
\]

**Proposition 5** characterizes the equilibrium where the monopoly’s profit maximization generates the interior solution.

**Proposition 5 [The Interior Equilibrium]**

For any given \( \varepsilon \), there exists a continuously differentiable symmetric equilibrium pricing policy, \( a(k) \) and \( b(k) \), with \( a(k) \) increasing and \( b(k) \) decreasing in \( k \) for all \( k \leq k < k^{**} \), where \( k^{**} \) denotes the marginal dealer whose profit and trading volume are zeros. The pricing policy functions satisfy,
\[ a(k) = e^{-\int_{K}^{k} Y(z) dz} \left( \frac{k^{**} + 1}{2} \right. \]

\[ + \int_{k_d}^{k^{**}} \left( -\frac{1}{4} + \frac{(1+z)}{2} Y(z) \right) e^{\int_{k}^{k^{**}} Y(u) du} dz \), \]

\[ b(k) = 1 - a(k), \]

\[ k^{**} = a(k^{**}) - b(k^{**}), \]

where

\[ Y(z) = \frac{\beta}{2(k^{**} - \bar{k})} \left( \frac{1}{1 - \beta(k^{**} - z)} - \frac{1 - \epsilon}{1 - \beta + \frac{z - k}{k^{**} - \bar{k}} (1 - \epsilon) \beta} \right) \]

The centralized market’s prices are

\[ a_c = a(k^{**}), \]

\[ b_c = 1 - a_c. \]

\[ k^{**} \text{ is defined as follows} \]

\[ k^{**} = \arg \max_{k \in (k_{K(2)})} \left( \bar{k} - K(1) \right) \left( 1 - \frac{\bar{k} + 1}{2} \right. \]

\[ - \frac{\beta(1 - \epsilon)}{1 - \beta} \left( \frac{\bar{k} + 1}{2} - \frac{1}{\bar{k} - k} \int_{k}^{\bar{k}} a(k) dk \right) \]

in which \( \bar{k} \) represents the monopoly market maker’s bid-ask spread.

To determine which equilibrium will show up, I first solve the interior equilibrium but with out the competitive constraint, i.e., \( a_c - b_c \leq K(2) \). Let \( k_u^{**} \) denote to the solution. If \( k_u^{**} \geq K(2) \), then the corner equilibrium will show up, as other market makers can undercut the monopoly market maker’s unconstraint spread \( k_u^{**} \). If \( k_u^{**} < K(2) \), then the interior equilibrium will show up.

Given \( k, \epsilon, \) and \( \beta \) fixed, \( k_u^{**} \) depends only on \( K(1) \). Thus, the pair of \( K(1) \) and \( K(2) \) determines which equilibrium to emerge. With \( \epsilon = 0.5, k = 0.005, \) and \( \beta = 0.9 \), Figure 6 shows for what pairs of \( K(1) \) and \( K(2) \) that the corner equilibrium will emerge, and for what pairs of \( K(1) \) and \( K(2) \) that the interior equilibrium will emerge.

**Comparative Statics in the Corner Equilibrium**
Setting $\beta = 0.9, k = 0.005, K_{(1)} = 0.29, \text{ and } K_{(2)} = 0.3$, I solve the equilibrium with $\epsilon$ ranges from 0 to 0.5.\textsuperscript{11}

Figure 7 shows that the average bid-ask spread in the OTC market decreases as $\epsilon$ increases. This means that the average spread shrinks, when the OTC market becomes more opaque. This result differs from the finding in Section 3, where I show greater opaqueness enlarges the average spread (see Figure 1).

In Figure 8, I show that how dealers and traders respond to changes in $\epsilon$. The left panel of Figure 8 shows that the total mass of dealers is independent of changes in $\epsilon$, while the right panel of Figure 8 shows that the total mass of traders increases as $\epsilon$ increases. The increase in the total mass of traders with larger $\epsilon$ implies that more traders participate in trading when the OTC market gets more opaque. These results again differ from the finding in Section 3, where greater opaqueness discourages traders to participate.

To understand different results obtained here, I compute the distribution of traders between the OTC market and the centralized market under different degree of opaqueness ($\epsilon$). From the left panel in Figure 9, trades in the OTC market decrease as $\epsilon$ increases, whereas the right panel in Figure 9 shows that trades in the centralized market increase as $\epsilon$ increases. This implies that traders migrate to the centralized market, when the OTC market gets more opaque.

The migration of traders highlights how the existence of the centralized market affects equilibrium outcomes. Analogous to the benchmark model in Section 3, when the OTC market gets more opaque, the value of search decreases which leads to changes in traders’ reservation values (as shown in Figure 10). However, in contrast to the benchmark model, traders here have an additional option — trading in the centralized market. Thus, when the search value decreases, rather than negotiating with dealers under ambiguous outside options, traders choose to trade in the centralized market. Furthermore, Figure 10 shows that high valuation buyers and low valuation sellers suffer the

\textsuperscript{11} Numerical results are robust to parameter choices.
most from greater opaqueness. Hence, most migrants are high valuation buyers and low valuation sellers. The remaining buyers in the OTC market are with low valuations, and the remaining sellers are with high valuations. This forces dealers to lower their asks and increase their bids to accustom to the remaining traders. In addition, dealers also want to attract more trading with smaller bid-ask spreads. Figure 11 verifies these changes in dealers’ asks and bids.

Defining the gains from trade as the sum of traders’ surplus

\[
\int_{0}^{1} (v_B - a_c) d\nu_B + \int_{0}^{\sigma_S} (b_c - \nu^S) d\nu^S \\
+ \int_{\sigma_B}^{\sigma_S} (v_B - r_B(v_B)) d\nu_B + \int_{\sigma^S}^{\nu^S} (r^S(v^S) - \nu^S) d\nu^S,
\]

I show the welfare changes of traders, dealers, and the monopoly market maker in Figure 12.

The left panel of Figure 12 shows that the gains from trade decrease as $\epsilon$ increases. This is because opaqueness makes it more costly to trade. The middle panel of Figure 12 shows that dealers’ total profits decrease as $\epsilon$ increases. This is because dealers have less volume and smaller bid-ask spreads. The right panel of Figure 12 shows the monopoly market maker’s profits increase as $\epsilon$ increases. This is because more traders migrate to the centralized market due to greater opaqueness in the OTC market. These results indicate that when there is a competitive centralized market in the equilibrium, greater opaqueness incur losses not only to trades, but also to dealers. Hence, introducing a competitive centralized market to the economy is an effective approach to incentivize dealers to reduce opaqueness in the OTC market.

**Comparative Statics for the Interior Equilibrium**

Setting $\beta = 0.9, k = 0.005, K_{(1)} = 0.009, \text{ and } K_{(2)} = 0.3$, I solve the equilibrium with $\epsilon$ ranges from 0 to 0.5.\(^{12}\)

Figure 13 shows that how the average bid-ask spread in the OTC market and the spread in the centralized market change with different $\epsilon$. Unlike in the corner equilibrium, the OTC market’s average

\(^{12}\) Numerical results are robust to parameter choices.
spread in the interior equilibrium increases under greater opaqueness. In addition, greater opaqueness also increases the spread in the centralized market.

[Insert Figure 14 Here]

In Figure 14, I show how dealers and traders respond to changes in $\epsilon$. Again, unlike findings in the corner equilibrium, in the interior equilibrium, greater opaqueness leads to less participation from traders, but more participation from dealers.

The different results obtained from the corner equilibrium and the interior equilibrium are due to the impact of opaqueness on the monopoly market maker. In the interior equilibrium, the monopoly market maker does not fear the entry of other market makers, which enables him to charge the unconstrained ask and bid with the spread equals to $k_{u}^{*}$. This spread depends on spreads in the OTC market, and hence, opaqueness in the OTC market. This dependency implicitly offers opportunities for the monopoly to collude with dealers in the OTC market. Under greater opaqueness, conjecturing dealers in the OTC market would enlarge their bid-ask spreads, the monopoly market maker enlarges his bid-ask spread correspondingly. Increasing spreads in both the OTC market and the centralized market discourage traders to participate. Whereas, in the corner equilibrium, the monopoly market maker’s spread is independent of dealers’. This makes the centralized market as a safe haven for traders to avoid opaqueness in the OTC market. As a result, under greater opaqueness, the OTC market loses their market shares to the centralized market. The centralized market is indeed competing with the OTC market.

[Insert Figure 15 Here]

Figure 15 shows the changes in the welfare of traders, dealers, and the monopoly market maker. Similar to the corner equilibrium, traders’ total surplus decreases as $\epsilon$ increases (see the left panel of Figure 15). This is because the impact of opaqueness on traders is the same as before. The middle panel of Figure 15 shows that dealers’ total profits increase as $\epsilon$ increases. And the right panel of Figure 15 shows the monopoly market maker’s profits increase as $\epsilon$ increases. These results indicate that when the centralized market is not competitive, the existence of it in the equilibrium cannot incentivize dealers to reduce opaqueness in the OTC market.

### 4.3. The Equilibrium in which the Centralized Market Fails to Survive
As mentioned previously, the equilibrium in which the OTC market coexists with the centralized market is the intermediate stage equilibrium. It is possible that the centralized market loses all trades to the OTC market, and hence, fails to survive. In this extreme equilibrium, establishing the centralized market is futile to incentivize dealers in the OTC market to reduce opaqueness, since the centralized market will not survive in the equilibrium.

The condition for the extreme equilibrium, in which the centralized market fails to survive, is illustrated by Proposition 6.

**Proposition 6**

A centralized market fails to survive in the equilibrium if and only if $K_{(1)} > k^*$, where $k^*$ is defined in Proposition 1. The equilibrium is the same as in Proposition 1.

Figure 1 shows that $k^*$ increases in $\epsilon$. As $k^*$ represents the upper bound for the centralized market to survive, Figure 1 implies that greater opaqueness in the OTC market makes it easier for the centralized market to survive in the equilibrium. More specifically, when the OTC market has high opaqueness, the centralized market can survive even if market makers in it have high transaction costs.

Proposition 6 suggests that stricter regulations, which can raise transaction costs, would not destruct the centralized market’s viability, if the OTC market is with great opaqueness. This sheds light on the tussle between regulators over the strictness of rules on SEFs. Some policy makers are afraid that stricter rules will impair the viability of SEFs as stricter rules raise transaction costs. According to Proposition 6, if OTC markets on swaps are of great opaqueness, SEFs with stricter rules will still survive in the equilibrium.

Admittedly, the ultimate goal for SEFs is to replace OTC markets on standardized swaps, which is out of the scope of the above analysis. To understand how the centralized market, e.g., a SEF, can replace an OTC market, I analyze the other extreme equilibrium, in which the OTC market fails to survive, in the following section.

### 4.4. The Equilibrium in which the OTC Market Fails to Survive

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13 SEFs are under the regulation of both the CFTC and the SEC. The two agencies have disagreed on rules over SEFs. In short, the industry deems the CFTC’s proposed rules to be stricter than the SEC’s. For example, the CFTC requests swap traders to obtain at least five quotes before they trade, whereas the SEC requests traders to obtain at least one quote.
In this section, I show the condition for the centralized market to replace the OTC market in the equilibrium. The key determinant is the comparison of the transaction cost between the centralized market and the OTC market. However, in one special case, opaqueness of the OTC market also has influence on the takeover.

**The OTC Market Fails to Survive in the Corner Equilibrium**

As shown in Section 4.2, when the centralized market is competitive, the monopoly market maker’s optimal choice is to post his bid-ask spread at the next most efficient market maker’s transaction cost, i.e., \( K_2 \). The equilibrium is the corner equilibrium. The condition that the centralized market replaces the OTC market in the corner equilibrium is described as follows.

**Proposition 7**

In the corner equilibrium, the OTC markets fail to survive if and only if \( k > K_2 \). The equilibrium prices are

\[
a_c = 1 - b_c = \frac{K_2 + 1}{2}. \tag{54}
\]

Proposition 7 implies that for a competitive centralized market to successfully replace the OTC market, market makers’ transaction costs must be lower than dealers’ transaction costs. More precisely, the next most efficient market maker’s transaction cost, which is the spread in the centralized market, shall be lower than the most efficient dealer’s transaction cost.

The above paragraph suggests that to replace the OTC swap market with the SEF, which is a competitive market, market makers in the SEF shall have lower transaction costs than dealers in the OTC swap market. From this perspective, stricter rules on the SEF are not in favor of the takeover, even though they do not impair the SEF’s viability.

As the OTC market fails to survive in the equilibrium, and the monopoly market maker in the centralized market charges a fixed spread to deter the entrance of other market makers, opaqueness in the OTC market does not exert any influence on the equilibrium outcomes. However, opaqueness in the OTC market impacts the condition for the OTC market to survive, when the entrance threat from another market maker is not credible, i.e., the centralized market is not competitive.
The OTC Market Fails to Survive in the Interior Equilibrium

When the centralized market is noncompetitive, the monopoly market maker faces no entrance threat from other market makers. The monopoly market maker posts bid-ask spread that is his interior solution of the profit maximization. The equilibrium obtained is the interior equilibrium. The condition for the centralized market to replace the OTC market in the interior equilibrium is more complex. As shown in Rust and Hall (2003), when the OTC market fails to survive in the interior equilibrium, two pricing strategies can happen for the monopoly market maker in the centralized market. The first one is that the monopoly market maker charges “limited price,” which equals to the lower bound of dealers’ transaction costs to deter the entrance of dealers. The second one is that the monopoly market maker charges unlimited monopoly prices, since the threat of dealers’ entrances is not credible. The equilibrium selection depends on the lower bound of dealers’ transaction costs.

**Proposition 8 [Limited Prices by the Market Maker]**

In the interior equilibrium, the OTC market fails to survive if and only if

\[
(F^\ast_1) \left(1 - \frac{k^* + 1}{2} - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_k^{k^*} a(k)dk \right) 
\leq \frac{(k - K(1))(1 - k)}{2},
\]

where \(k^*\) is defined in Proposition 5. The equilibrium prices are

\[
a_c = 1 - b_c = \frac{k + 1}{2}.
\]

**Proposition 9 [Unlimited Prices by the Market Maker]**

The OTC market fails to survive in the equilibrium if and only if \(\frac{K(1) + 1}{2} < \min\{k_1, K(2)\}\). The equilibrium prices are

\[
a_c = 1 - b_c = \frac{K(1) + 3}{4}.
\]

[Insert Figure 16 Here]

The upper panel of Figure 16 shows the region of the lower bound of dealers’ transaction costs, \(k\), for the OTC market to survive in the interior equilibrium. In addition, the upper panel of Figure 16 also
illustrates the equilibrium selection when the OTC market fails to survive. The lower panel of Figure 16 shows how changes in $\epsilon$ affects the viability of the OTC market. Specifically, the OTC market can survive with higher transaction costs when $\epsilon$ is large. When the OTC market has great opaqueness, the noncompetitive centralized market is less likely to replace it, since the noncompetitive centralized market prefers to keep the OTC market in order to profit from opaqueness in it. However, when transaction costs in the OTC market are substantially larger than transaction costs in the centralized market, the centralized market will find it is more profitable to replace the OTC market (see the upper panel of Figure 16).

The analysis on the viability of the OTC market under the noncompetitive centralized market sheds light on the OTC corporate bond market. In the OTC corporate bond market, banks are major dealers, who provide intermediary services. The “Volker Rule,” which limits the proprietary trading from banks, increases banks transaction costs. As a result, transaction costs in the OTC corporate bond market will be lifted once the “Volker Rule” is enforced. From the upper panel of Figure 16, the lift in transaction costs in the OTC corporate bond market will increase the chance that a noncompetitive centralized market replaces the OTC market on the corporate bond. In fact, the BlackRock has recently planned to launch its “Aladdin” matching platform on trading corporate bonds, which is likely to be a noncompetitive centralized market, to compete with the OTC corporate bond market.\footnote{BlackRock Inc. is planning to launch a trading platform this year that would let the world's largest money manager and its peers bypass Wall Street and trade bonds directly with one another. - Wall Street Journal, Apr 12th, 2012.}

5. Concluding Remarks

My paper suggests that setting up a centralized market can be an efficient way to incentivize dealers to choose for less opaque OTC markets. This is because the migration of order flows to the centralized market under greater opaqueness leads to smaller profits for dealers in OTC markets. However, the centralized market has to be competitive to generate competition pressure on OTC markets. Results obtained in my paper support recent reformations in OTC derivative markets aiming to reduce opaqueness in those markets.

My model provides some empirical implications. Firstly, when an OTC market is the only intermediary in the economy, greater opaqueness in the OTC market increases the average bid-ask spread. Secondly, when a centralized market and an OTC market coexist in the economy, the correlation...
between degrees of opaqueness and prices can be used to test if the centralized market compete or collude with the OTC market. Specifically, the centralized market, which competes with the OTC market, predicts that greater opaqueness will decrease the average bid-ask spread in the OTC market, but will not change the bid-ask spread in the centralized market. Whereas, the centralized market, which colludes with the OTC market, predicts that greater opaqueness increases both the OTC market and the centralized market's bid-ask spreads.

There are some limitations in my model. Traders in my model are liquidity traders rather than informed traders. This limits my model to analyze information asymmetry in OTC trading. There are continuum traders and dealers in my model. But, in reality, most OTC trades occurred among finitely many large institutions. While traders in my model can choose which markets to participate (an OTC market or a centralized market), dealers cannot. Dealing with these limitations requires a more complex model which is beyond the scope of this paper. I believe extending the model to resolve these issues can provide more implications on OTC trading.
Appendices

Proof of Proposition 1

Proof:

Let’s denote $D(a) = \sum_t \beta^t D_t(a)$ and $S(b) = \sum_t \beta^t S_t(b)$. Given that $D_t(a)$ is continuous and decreasing function on $[a, \bar{a}]$, it is easy to see that $D(a)$ is continuous and decreasing on $[a, \bar{a}]$. We note that, from the value function of the buyers

$$V^B(a) = \max \left\{ 0, v^B - a, \beta \int V^B(\bar{a}) d\theta \right\}, \quad (A1)$$

$a = v^B$ and $\bar{a} = r^B(1)$. Similarly, we have $S(b)$ continuous and increasing on $[b, \bar{b}]$, in which $b = r^S(0)$ and $\bar{b} = v^S$. As $D_t(a) = S_t(b)$, we have $D(a) = S(b)$. Then, we define the inverse functions $A(q)$ and $B(q)$ mapping from $q$ to prices.

From the inverse function theorem, we have

$$A'(q) = \left( \frac{\partial D}{\partial a} \right)^{-1} \left( -\frac{1 - \beta + F_a(a)(1 - \epsilon)\beta}{N(1 - \beta)(1 - \beta F_a(a))} \right)^{-1}, \quad (A2)$$

$$B'(q) = \left( \frac{\partial S}{\partial b} \right)^{-1} \left( -\frac{1 - \beta + (1 - F_b(b))(1 - \epsilon)\beta}{N(1 - \beta)(1 - \beta F_b(b))} \right)^{-1}. \quad (A3)$$

As $a(k)$ increases in $k$ and $b(k)$ decreases in $k$, we have

$$F_a(a) = P_a[\bar{a} < a] = P_\k [\bar{k} < k] = \frac{k - \bar{k}}{k^* - \bar{k}}, \quad (A4)$$

$$1 - F_b(b) = P_b[\bar{b} < b] = P_\k [\bar{k} < k] = \frac{k - \bar{k}}{k^* - \bar{k}}, \quad (A5)$$

where $k^*$ is the marginal dealer whose profit margin and trading volume are zeros. Thus, the total mass of dealer $N$ equals to $k^* - \bar{k}$.

Plugging $F_a(a)$ and $F_b(b)$ into $A'(q)$ and $B'(q)$ respectively, we obtain

$$A'(q) = -B'(q) = \left( -\frac{1 - \beta + \frac{k - \bar{k}}{k^* - \bar{k}}(1 - \epsilon)\beta}{N(1 - \beta)(1 - \beta \frac{k^* - \bar{k}}{k^* - \bar{k}})} \right)^{-1}. \quad (A6)$$

For the dealer with transaction cost $k$, he chooses $q$ to maximize the expected profit $(A(q) - B(q) - k)q$. The optimality condition implies,

$$A(q) - B(q) - k = (B'(q) - A'(q))q. \quad (A7)$$
Thus, we obtain

$$A(q(k)) - B(q(k)) - k = \frac{2(k^* - k)(1 - \beta)(1 - \frac{\beta(k^* - k)}{k^* - k}) q(k)}{1 - \beta + \frac{k - k}{k^* - k} (1 - \epsilon) \beta}.$$  \hfill (A8)

Substituting $q(k) = D(a(k))$ into equation (A8), we get

$$a(k) - b(k) - k = \frac{2 \left(1 - \frac{\beta(k^* - k)}{k^* - k}\right)}{1 - \beta + \frac{k - k}{k^* - k} (1 - \epsilon) \beta} \int_{a(k)}^{r^B(1)} \frac{1 - \beta + F_a(r^B(1 - \epsilon) \beta)}{1 - \beta (1 - F_a(r^B))} dr^B.$$  \hfill (A9)

Since, for any $k$, $D(a(k)) = S(b(k))$, it implies that $\frac{\partial a}{\partial k} = \frac{\partial b}{\partial k}$. And since $A'(q) = -B'(q)$, we have $\frac{\partial a}{\partial k} = -\frac{\partial b}{\partial k}$. Thus,

$$a(k) + b(k) = C,$$  \hfill (A10)

in which $C$ represents a constant.

From the buyer’s reservation value, we have

$$1 = r^B(1) + \frac{\beta (1 - \epsilon)}{1 - \beta} \int_{a}^{r^B(1)} P_a[a < \hat{a}] d\hat{a}$$

$$= r^B(1) + \frac{\beta (1 - \epsilon)}{1 - \beta} \int_{k}^{k^*} \frac{k - k}{k^* - k} a'(k) dk,$$

where the second equality is obtained by performing a change of variables.

Likewise, we have

$$0 = r^S(0) - \frac{\beta (1 - \epsilon)}{1 - \beta} \int_{r^S(0)}^{\hat{b}} P_b[b > \hat{b}] d\hat{b}$$

$$= r^S(0) + \frac{\beta (1 - \epsilon)}{1 - \beta} \int_{k}^{k^*} \frac{k - k}{k^* - k} b'(k) dk.$$  \hfill (A12)

From the above, it is obvious that $1 = r^B(1) + r^S(0)$. Since $a(k^*) = \bar{a} = r^B(1)$ and $b(k^*) = \bar{b} = r^S(0)$, we have $a(k^*) + b(k^*) = 1$. This implies $C = 1$, and hence,

$$b(k) = 1 - a(k).$$  \hfill (A13)

Plugging the equation (A13) into the optimality condition (equation (A8)) and differentiating with respect to $k$, we arrive at the following differential equation

$$a'(k) - \frac{a(k) \beta X(k)}{2(k^* - k)} = \frac{1}{4} \left(1 + k\right) \beta X(k),$$  \hfill (A14)

where
The solution for the above differential equation is
\[ a(k) = e^{-\int_k^{k^*} Y(z)dz} \left( \frac{k^* + 1}{2} + \int_k^{k^*} \left( -\frac{1}{4} + \frac{1 + z}{2} Y(z) \right) e^{\int_k^{k^*} Y(u)du} dz \right), \] (A16)
where
\[ Y(z) = \frac{\beta X(z)}{2(k^* - k)}. \] (A17)
Thus, equation (A16) determines the equilibrium asks. And the equilibrium bids equal to 1 - a.

To determine the equilibrium \( k^* \), we apply \( k^* = a(k^*) - b(k^*) \) to the buyer’s reservation value \( r^B(1) \) and get
\[ 1 = \frac{k^* + 1}{2} + \frac{\beta (1 - \epsilon) (k^* + 1)}{2} - \frac{1}{k^* - k} \int_k^{k^*} a(k) dk. \] (A18)

**Proof of Proposition 2**

*Proof:*

The buyer follows a reservation pricing strategy, when he searches in the OTC market. This means that
\[ v^B - r^B(v^B) = \beta \int V(\tilde{a}, a_c, v^B) d\theta_a. \] (A19)
Plugging eq. (A19) into the buyer’s value function before he starts to search, eq.(27), I have
\[ W^B(\tilde{a}, v^B) = \max \{ 0, v^B - a_c, v^B - r^B(v^B) \}. \] (A20)
Since \( r^B(v^B) \) increases in \( v^B \), and since \( a_c = r^B(\tilde{v}^B) \), I have for any \( v^B \geq \tilde{v}^B \),
\[ r^B(v^B) \geq a_c \] (A21)
which implies that
\[ v^B - a_c \geq v^B - r^B(v^B). \] (A22)
Whereas, for any \( v^B < \tilde{v}^B \),
\[ r^B(v^B) < a_c \] (A23)
which implies that
\[ v^B - a_c < v^B - r^B(v^B). \] (A24)
Thus, for any buyer with \( v^B \geq \tilde{v}^B \), he is better off to buy the asset in the centralized market. Whereas, for any buyer with \( v^B < \tilde{v}^B \), he is better off to buy the asset in the OTC market.

Moreover, if the buyer has \( v^B \leq \tilde{v}^B = a \), then
$$v^B - a \leq 0, \forall a \in [a, \bar{a}].$$  \tag{A25}

Since $\bar{a} = a_c$, eq. (A25) implies that the buyer loses if he trades either in the OTC market or in the centralized market.

Thus, for any buyer with $v^B \leq \underline{v}^B$, he is better off not to trade in any market.

**Q.E.D**

**Proof of Proposition 3**

**Proof:**

This is similar to the proof of Proposition 2, since sellers and buyers are symmetric.

**Q.E.D**

**Proof of Proposition 4**

**Proof:**

The derivation of the price system in the OTC market is the same as in the proof of Proposition 1 except for the marginal dealer. Specifically, the monopoly market maker charges the bid-ask spread equals to the next most efficient market maker’s transaction cost $K_{(2)}$. Since all surviving dealers have to undercut the bid-ask spread posted by the monopoly market maker, $K_{(2)}$ defines the marginal dealer’s transaction cost. That is,

$$k^{**} = K_{(2)}. \tag{A26}$$

The inventory constraint applied to the monopoly market maker implies that

$$1 - a_c - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_a^{a_c} P_a[a < \tilde{a}] d\tilde{a} = D^c(a_c) = S^c(b_c) \tag{A27}$$

$$= b_c - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_{b_c}^{\tilde{b}} P_b[b > \tilde{b}] d\tilde{b}.$$  

From eq. (A4) and eq. (A5), I have

$$P_a[\tilde{a} < a] = P_b[\tilde{b} > b] = P_k[\tilde{k} < k] = \frac{k - k^{**}}{k^{**} - k}. \tag{A28}$$

Therefore,

$$a_c = 1 - b_c = \frac{K_{(2)} + 1}{2}. \tag{A29}$$

**Q.E.D**

**Proof of Proposition 5**

**Proof:**

Similar to the proof of Proposition 4, the derivation of the price system in the OTC markets is the same as in the proof of Proposition 1 except for the marginal dealer. Since the monopoly market
maker does not fear the entrance of the next most efficient market maker, he posts the bid-ask spread maximizes his expected profits. Consequently, the marginal dealer is no longer the one with transaction cost equals to \( k_{(2)} \), but the one with transaction cost equals to the monopoly market maker’s profit maximizing spread.

Since the monopoly market maker’s inventory constraint implies that \( a_c = 1 - b_c \). Defining \( \tilde{k} \) as the monopoly market maker’s bid-ask spread, I can rewrite the monopoly market maker’s profit maximization as follows,

\[
\max_k (\tilde{k} - K_{(1)}) \left( 1 - \frac{\tilde{k} + 1}{2} - \beta (1 - \epsilon) \left( \frac{\tilde{k} + 1}{2} - \frac{1}{k - k} \int_k^k a(k) dk \right) \right). \tag{A30}
\]

If \( \tilde{k} \geq K_{(2)} \), then the next most efficient market maker will enter and undercut the monopoly market maker’s bid-ask spread. If \( \tilde{k} \leq k \), then no dealers will survive in the equilibrium. Hence, the interior equilibrium would happen, if \( \tilde{k} \in (k, K_{(2)}) \).

Therefore, the marginal dealer \( k^* \) is defined by the \( \tilde{k} \) that maximizes eq. (A30) and is in the interval of \( (k, K_{(2)}) \).

\[Q.E.D\]

**Proof of Proposition 6**

**Proof:**

When all bid-ask spreads in the OTC market in Proposition 1 are smaller than the transaction cost of the most efficient market maker, the establishment of the centralized market is futile. All trades go to the OTC market.

Therefore, the condition for the centralized market to survive is \( K_{(1)} < k^* \).

\[Q.E.D\]

**Proof of Proposition 7**

**Proof:**

In the corner equilibrium, the bid-ask spread in the centralized market equals to \( K_{(2)} \). If no dealer is able to undercut this spread, i.e. \( K_{(2)} < k \), then the OTC market fails to survive in the equilibrium.

\[Q.E.D\]

**Proof of Proposition 8**

**Proof:**
In the interior equilibrium, if no dealer is able to undercut the monopoly market maker’s profit maximizing spread. The monopoly market maker can set spread equal to $k$ to become the only trading intermediary in the economy as long as $k \geq K_{(1)}$. When the monopoly market maker becomes the only trading intermediary, Proposition 2 and Proposition 3 show that the demand in the centralized market is $1 - a_c$, and the supply in the centralized market is $b_c$. The monopoly market maker’s profit maximization becomes

$$\max_{a_c} (2a_c - 1 - K_{(1)}) (1 - a_c).$$

(A31)

The unconstraint optimal choice of $a_c$ is $\frac{K_{(1)}+3}{4}$, and the spread is $\frac{K_{(1)}+1}{2}$.

If $k \leq \frac{K_{(1)}+1}{2}$, then the monopoly market maker cannot set his ask price at the unconstraint optimal choice $\frac{K_{(1)}+3}{4}$. In this case, the quadratic objection function (eq. (A31)) implies that the optimal ask price is $\frac{k+1}{2}$, which generates a spread equals to $k$.

When the monopoly charges limited prices, his profit is $\frac{(k-K_{(1)})(1-k)}{2}$. Hence, the monopoly will charge limited prices to kill dealers in the OTC market if and only if

$$\left(k^* - K_{(1)}\right) \left(1 - \frac{k^* + 1}{2} - \frac{\beta(1 - \epsilon)}{1 - \beta} \int_{k}^{k^*} a(k) \, dk\right) \leq \frac{(k - K_{(1)})(1 - k)}{2}.$$

(A32)

Q.E.D

Proof of Proposition 9

Proof:

From the proof of Proposition 8, if $k > \frac{K_{(1)}+1}{2}$, then the monopoly market maker sets his ask price to the unconstraint profit maximization choice, i.e., $a_c = \frac{K_{(1)}+3}{4}$.

Q.E.D
Figure 1: Comparative statics of the average bid-ask spread in the search equilibrium of Proposition 1 with respect to $\epsilon$. The parameters are $\beta = 0.9$, and $k = 0.005$. 

The Mean of the Spreads

Mean spreads

0.22 0.24 0.26 0.28 0.3 0.32 0.34

0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5

$\epsilon$
Figure 2: Comparative statics of the total mass of dealers and traders in the equilibrium of Proposition 1 with respect to $\epsilon$. The parameters are the same as in Figure 1. The left panel plots the total mass of dealers in the equilibrium, and the right panel plots the total mass of traders in the equilibrium.
Figure 3: Comparing the equilibrium asks between the opaque OTC market ($\varepsilon = 0.5$) and the transparent OTC market ($\varepsilon = 0$). Solid lines illustrate the asks when the OTC market is opaque, while dashed lines show the asks when the OTC market is transparent. The left panel shows the equilibrium asks, and the right panel shows the empirical cumulative density functions of the asks.
Figure 4: Comparing traders’ reservation values between the opaque OTC market ($\epsilon = 0.5$) and the transparent OTC market ($\epsilon = 0$). Solid lines show the reservation values when the OTC market is opaque, and dashed lines show the reservation values when the OTC market is transparent. The left panel illustrates buyers’ reservation buying prices, and the right panel illustrates sellers’ reservation selling prices.
Figure 5: Comparative statics of the welfare in the equilibrium of Proposition 1 with respect to $\epsilon$. Parameters are the same as in Figure 1. The left panel plots traders’ surplus, and the right panel plots dealers’ total profits.
Figure 6: The equilibrium selection when both the OTC market and the centralized market operate in the economy. Parameters are $\epsilon = 0.5, \beta = 0.9$. 

Figure 7: Comparative statics of the average bid-ask spread in the equilibrium of Proposition 4 with respect to $\epsilon$. The parameters are $\beta = 0.9$, $k = 0.005$, $K_{(1)} = 0.29$, and $K_{(2)} = 0.3$. 
Figure 8: Comparative statics of the demographics in the equilibrium of Proposition 4 with respect to $\epsilon$. The parameters are the same as in Figure 7. The left panel plots the total mass of dealers in the equilibrium, and the right panel plots the total mass of traders in the equilibrium.
Figure 9: Comparative statics of traders’ distribution between the OTC market and the centralized market in the equilibrium of Proposition 4 with respect to $\epsilon$. The parameters are the same as in Figure 7. The left panel plots the total mass of traders in the OTC market, and the right panel plots the total mass of traders in the centralized market.
Figure 10: The comparison of traders’ reservation values between the opaque OTC market ($\varepsilon = 0.5$) and the transparent OTC market ($\varepsilon = 0$) in the equilibrium of Proposition 4. Solid lines show the reservation values when the OTC market is opaque, and dashed lines show the reservation values when the OTC market is transparent. The left panel plots buyers’ reservation buying prices, and the right panel plots sellers’ reservation selling prices.
Figure 11: The comparison of the asks and bids between the opaque OTC market ($\varepsilon = 0.5$) and the transparent OTC market ($\varepsilon = 0$) in the equilibrium of Proposition 4. Solid lines are the asks and bids under the opaque OTC market, and dashed lines are the asks and bids under the transparent OTC market. The left part are the asks (the upper plot) and bids (the bottom part), and the right part are empirical cumulative density functions of the asks (the upper plot) and bids (the bottom part).
Figure 12: Comparative statics of the welfare in the equilibrium of Proposition 4 with respect to $\varepsilon$. Parameters are the same as in Figure 7. The left panel plots traders' surplus, and the right panel plots dealers' total profits.
Figure 13: Comparative statics of the average bid-ask spread in the equilibrium of Proposition 5 with respect to $\epsilon$. The parameters are $\beta = 0.9$, $k = 0.005$, $K_{(1)} = 0.009$, and $K_{(2)} = 0.3$. 
Figure 14: Comparative statics of the demographics in the equilibrium of Proposition 5 with respect to $\epsilon$. The parameters are the same as in Figure 13. The left panel plots the total mass of dealers in the equilibrium, and the right panel plots the total mass of traders in the equilibrium.
Figure 15: Comparative statics of the welfare in the equilibrium of Proposition 5 with respect to $\epsilon$. Parameters are the same as in Figure 13. The left panel plots traders’ surplus, and the right panel plots dealers’ total profits.
Figure 16: The upper panel shows the equilibrium selection when the OTC market survives or fails to survive in the economy ($\varepsilon = 0.5$). The lower panel shows the comparative statics of the lower bound of dealers transaction costs, with which the OTC market can survive. Let $K_{(2)} = 1$ to focus on the interior equilibrium. $K_{(1)} = 0.009$ and $\beta = 0.9$. 
References


