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# The Bank of Canada 2015 Retailer Survey on the Cost of Payment Methods: Estimation of the Total Private Cost for Large Businesses

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# Abstract

The Bank of Canada 2015 Retailer Survey on the Cost of Payment Methods faced low response rates and outliers in sample data for two of its retailer strata: chains and large independent businesses. This technical report investigates whether it is appropriate to combine these two strata to produce more accurate estimates of the total private cost to large businesses of the main payment methods. It uses two approaches to compute the total cost. First, a sample-based approach assumes consistency of some sample ratios and calibrates the sample to the known population totals of auxiliary variables. Second, a model-based approach uses outlier-robust estimation methods. The results show that, unlike for payments by cash and debit cards, there is relatively little difference between the approaches in determining the cost of payment by credit card. The model-based approach is recommended because it does not allocate the retailer's total costs in advance between fixed and variable costs, and it uses outlier-robust estimation methods.

*Bank topic: Econometric and statistical methods*
*JEL code: C12; C83*

# Résumé

L'enquête réalisée en 2015 par la Banque du Canada sur les coûts des différents modes de paiement pour les détaillants présente des taux de réponse faibles ainsi que des valeurs aberrantes dans deux strates de l'échantillon, à savoir les chaînes de magasins et les grandes entreprises indépendantes. Ce rapport technique examine s'il est utile de combiner ces deux strates pour produire des estimations plus précises du coût individuel total des principaux modes de paiement pour les grandes entreprises. Le coût total est calculé selon deux approches. La première, qui est fondée sur l'utilisation d'échantillons, suppose la convergence de certains ratios de l'échantillon et fait correspondre par calage les estimations aux totaux de variables auxiliaires connues pour la population. La seconde, qui est fondée sur des modèles, utilise des méthodes d'estimation robustes à la présence des valeurs aberrantes dans l'échantillon. Les résultats montrent que la différence entre les deux approches est relativement faible pour les coûts de paiement par carte de crédit, ce qui n'est pas le cas pour les coûts de paiement en espèces et par carte de débit. L'approche fondée sur des modèles est recommandée, car elle n'effectue pas de répartition a priori des coûts totaux entre les coûts fixes et les coûts variables pour les détaillants, et utilise des méthodes d'estimation robustes à la présence des valeurs aberrantes dans l'échantillon.

*Sujet : Méthodes économétriques et statistiques*
*Code JEL : C12; C83*

# 1 Introduction

In 2015, the Bank of Canada conducted the Retailer Survey on the Cost of Payment Methods (RCPM survey) to estimate the total private and social costs of various payment methods. The target population was Canadian merchants that accepted payments from consumers at the point of sale in 2014. To account for economies of scale, the RCPM survey frame is partitioned into two sets of locations, the first containing independent single locations, and the second consisting of clusters of chain locations. Each cluster is represented by its headquarters (HQ), which is expected to provide information for the whole chain. The set of independent single locations is divided into three strata according to the number of employees: the single location stratum A (SLA) for businesses with fewer than 5 employees, the single location stratum B (SLB) for businesses with 5 to 49 employees, and the single location stratum C (SLC) for businesses with 50 or more employees.

The RCPM survey is a voluntary survey with a significant number of outliers in sample data and a low response rate: around 8.0 per cent for the SLC and 4.5 per cent for the HQs. Given these low rates, this report investigates whether one can combine the SLC and HQ strata to increase the sample size and therefore reduce the variance of the estimates. However, the combination should not introduce bias. To assess this, outlier-robust Wald and Fisher tests for regression parameters were performed, and the results show that, at the 5 per cent level, the combination does not increase the risk of bias.

This technical report focuses on the total private costs of cash, debit card and credit card payments for the combined SLC and HQ strata, called “large businesses” and denoted by LB. Given that sampling weights for the HQs are not available (see Welte 2017), one option is to use a sample-based approach by assuming consistency of some sample ratios and calibrating the sample to the known population totals of auxiliary variables. An alternative is to use an outlier-robust model-based approach.

The results show that the difference between the two approaches in estimating the cost of paying by credit cards is relatively small. However, in estimating the costs of cash and debit cards, the discrepancies between the two approaches are quite large. One possible explanation is that unlike the model-based approach, the sample-based method makes use of the prior allocation of the retailer’s total costs between fixed and variables costs. For that reason, the recommendation is to use the model-based approach for calculating the total private costs of LB. In addition, the model-based approach uses robust estimation methods to deals with the issue of outliers in the sample.

The rest of the technical report is organized as follows: Section 2 presents the methodology and the results of combining the SLC and HQ strata. Section 3 presents the total private cost framework. Section 4 details the methodology used for the estimation of the total private costs. It proceeds in two steps, the first addressing the sample-based approach and the second, the model-based approach. Section 5 presents the results of the total cost calculation, and Section 6 concludes. A flow chart summary of the LB analysis is provided in the appendix.

# 2 Combining the SLC and HQ Strata

The methodology outlined here determines whether the SLC and HQ strata can be combined within the framework of the model-based approach. The main assumption (A1) is as follows:

**Assumption A1.** A merchant's costs are generated by a linear regression model (1) for both the SLC and HQ populations:

$$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + \epsilon_{ij}, \quad i = 1, \ldots, n. \tag{1}$$

Here, $i$ refers to the merchant, $j$ is the payment instrument, $(f_j, a_j, b_j)^\top$ is the regression coefficient, $z_{ij}$ is the merchant's volume of transactions with $j$ available from the sample, $v_{ij}$ is the merchant's value of transactions with $j$ available from the sample, $y_{ij}$ is the observed merchant's cost available from the sample, and $n$ is the sample size. For a given $j$, $\epsilon_{ij}$ are assumed to be independent but not necessarily identically distributed random vectors, such that: $E\left(\epsilon_{ij} \middle| z_{ij}, v_{ij}\right) = 0$ and $E\left(\epsilon_{ij}^2 \middle| z_{ij}, v_{ij}\right) = \sigma_j^2 g_j(z_{ij}, v_{ij})$, where $g_j$ is a known real bivariate function and $\sigma_j^2$ is unknown.

The objective is to test the validity of assumption A1. In other words, it is to test two elements: first, that the linear model specification holds, and second, that SLC and HQ have the same regression coefficients. Indeed, the homogeneity test of regression coefficients should be performed on a valid model.

## 2.1 Linear model specification test

The test performed here is White's model specification test. White (1980) states that "the null hypothesis maintains not only that the errors are homoskedastic, but also that they are independent of the regressors, *and* that the model is correctly specified":

$$
\begin{aligned}
&H_0 : E\left\{(1, z_{ij}, v_{ij})^\top \epsilon_{ij}\right\} = (0,0,0)^\top \quad \text{and} \quad g_j(z_{i_1 j}, v_{i_1 j}) = g_j(z_{i_2 j}, v_{i_2 j}) \quad \forall i_1, i_2 \\
&vs. \\
&H_1 : E\left\{(1, z_{ij}, v_{ij})^\top \epsilon_{ij}\right\} \neq (0,0,0)^\top \quad \text{or} \quad \exists i_1, i_2 \quad \text{such as} \quad g_j(z_{i_1 j}, v_{i_1 j}) \neq g_j(z_{i_2 j}, v_{i_2 j}).
\end{aligned}
$$

Denote by $\chi^2$ the test statistics. Under $H_0$, $\chi^2$ follows a chi square with five degrees of freedom. For details, see theorem 2 in White (1980). If model (1) is assumed to be correctly specified, then the rejection of the null hypothesis may reasonably be attributed to heteroscedasticity. The advantage of the White test is that if the null hypothesis is not rejected, it implies that there is no evidence against the mean and variance specification of model (1). In this case a robust test of equality of regression coefficients should be conducted to support the validity of the second element of assumption A1.

## 2.2 Robust test of equality of regression coefficients

The test performed examines the homogeneity of the SLC and HQ strata. Assuming linear specification holds, this homogeneity refers to the equality of the regression coefficients of the underlying super-population model. Denote by $\left(f_j^{SLC}, a_j^{SLC}, b_j^{SLC}\right)^\top$ and $\left(f_j^{HQ}, a_j^{HQ}, b_j^{HQ}\right)^\top$ the regression coefficients for SLC and HQ, respectively. The objective is to test whether or not these regression coefficients are equal, that is,

$$
\begin{aligned}
&H_0 : \left(f_j^{SLC}, a_j^{SLC}, b_j^{SLC}\right)^\top = \left(f_j^{HQ}, a_j^{HQ}, b_j^{HQ}\right)^\top \\
&vs. \\
&H_1 : \left(f_j^{SLC}, a_j^{SLC}, b_j^{SLC}\right)^\top \neq \left(f_j^{HQ}, a_j^{HQ}, b_j^{HQ}\right)^\top .
\end{aligned}
\tag{2}
$$

To achieve this, a robust MM-regression introduced by Yohai (1987) is performed on the model in expression (3) using the pooled SLC and HQ samples:

$$
y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j} I_i + c_{1j} I_i z_{ij} + c_{2j} I_i v_{ij} + \epsilon_{ij}, \quad i = 1, \ldots, n, \tag{3}
$$

where $I_i = 1$ if $i \in HQ$ and $I_i = 0$ if $i \in SLC.$

Test problem (2) is equivalent to test (4) given below:

$$
H_0 : (c_{0j}, c_{1j}, c_{2j})^\top = (0,0,0)^\top \quad vs. \quad H_1 : (c_{0j}, c_{1j}, c_{2j})^\top \neq (0,0,0)^\top . \tag{4}
$$

Denote by $\tau$ the test statistics of the robust version of the Fisher test. Asymptotically, $\tau \sim \lambda\chi_3^2$, where $\lambda$ is a standardization factor. (Refer to Hampel et al. 1986, 346 for details.) The second test used is a robust version of the Wald test denoted by $R_n^2$. It follows asymptotically a $\chi_3^2$. (See Hampel et al. 1986, 364 for more details.) The non-rejection of the null hypothesis supports the validity of assumption A1. Both the White test and the equality of regression coefficient test are applied to the LB sample data.

## 2.3 Description of the LB merchant data

The SLC and HQ samples sent for the collection contain 1,389 and 830 units, respectively. Once the nonresponses and out-of-scope responses are removed, and the units are assigned to their observed size stratum, the survey respondents are distributed as follows: 114 units and 36 units belong to the SLC and the HQ strata, respectively. In addition, 30 units in the sample were stratum jumpers from independent single locations to branches without HQs. These units represent 19 HQs; therefore, the HQ sample size is increased by 19, and, accordingly, the SLC's population size is reduced by 545, corresponding to the sum of the weights of these 30 units. Ultimately, the population sizes are 2,210 for SLC and 6,913 for HQ (Welte 2017).

Those missing HQs are imputed by assuming that all branches in the chain share the same cost structure (discount rate, etc.). Consequently, the branch sample means are multiplied by the number of operating branches of these chains to obtain the HQ value of each item, while accounting for the fact that some branches operate more than one store. Moreover, missing values for some items are imputed using the nearest-neighbour imputation method within sales and economic activities classes. Overall, the final sample contains 114 SLC units and 55 HQs, including the 19 imputed from the 30 branches. The HQ sample contains 33 retail chains, which cover 40.7 per cent of the total Canadian retail sales.

Table 1 reports the mean and median of the LB total sales and number of employees in 2014. The mean is higher than the median for both strata, as well as for both size variables. That suggests the data are right-skewed, which is common in business surveys. Figure 1 confirms that the distribution of the total sales is right-skewed for the SLC, although this is not the case for HQs. Rather, there appears to be a mixture of three distributions, one of which may be that of the SLC. In the next subsection, analytical tests are performed to assess whether the merchant costs for SLC and HQ are generated by the same model up to a variance structure depending on their volume and the value of transactions.

## 2.4 Model validation

The model diagnostics are performed independently for all payment methods using model (3). Analytical validation is first performed using the White (1980) test for specification and heteroscedasticity. Then, a second step of model validation is done with residual graphs to address the limits of the White test in the presence of the outliers (Alih and Ong 2015).

### 2.4.1 Results of the White test for specification and heteroscedasticity

Table 3 presents the results of the White test for specification and heteroscedasticity when the variance structure of model (3) is assumed to be homoscedastic. The null hypothesis is not rejected for all payment methods at the 5 per cent level. However, these results should be taken with caution because the average covariance matrix for the specification test is singular, which violates assumption $2(b)$ of the White test. This is why the degree of freedom of the test statistics in Table 3 changes from one payment instrument to another. In this case, it is natural to consider and examine the validity of a potential heteroscedastic model. The modelling of the variance structure follows White (1980) by including the predictor, square predictor and cross product in the robust MM-regression (5):

$$\begin{aligned}\hat{\epsilon}_{iR}^2 &= \alpha_{0j} + \alpha_{1j} z_{ij} + \alpha_{2j} v_{ij} + \alpha_{3j} I_i + \alpha_{4j} I_i z_{ij} + \alpha_{5j} I_i v_{ij} \\ &\quad + \alpha_{6j} z_{ij}^2 + \alpha_{7j} v_{ij}^2 + \alpha_{8j} z_{ij} v_{ij} + \alpha_{9j} I_i z_{ij}^2 + \alpha_{10j} I_i v_{ij}^2 \\ &\quad + \alpha_{11j} I_i z_{ij} v_{ij}, \quad i = 1, \ldots, n. \end{aligned} \tag{5}$$

The square and cross product, as well as the variables $I_i z_{ij}$ and $I_i v_{ij}$, cause linear dependency in regression (5). Therefore, the linear function in expression (6) is used for the modelling of the variance structure. If the model is validated, this allows the difference in variance structures between SLC and HQ:

$$\hat{\epsilon}_{iR}^2 = \alpha_{0j} + \alpha_{1j} z_{ij} + \alpha_{2j} v_{ij} + \alpha_{3j} I_i. \tag{6}$$

The model in expression (6) is fitted using robust MM-regression, and the negative variance estimates are replaced by the smallest positive one. Beaumont and Bocci (2009) and Beaumont, Haziza and Bocci (2011), among others, also replace negative unit variance estimates by the smallest positive in the context of variance estimation under imputation in the presence of quantitative auxiliary variables. For the cash payment method, up to 26 per cent of the units have a negative variance, with 11 per cent for debit cards and only 3 per cent for credit cards. Given the high percentage of negative variances for cash, and the relatively high percentage for debit cards, some variables or combinations of variables are removed from the model in expression (6), and a residual analysis of the heteroscedastic models is performed. Ultimately, the following ad hoc variance structures are retained for comparison with the homoscedastic model: (i) for the cash payment method, $g_{cash}\left(z_{icash}, v_{icash}, I_i\right) = \alpha_{cash} z_{icash} + v_{icash}$, where $\alpha_{cash} = 9.3$ is the sample average cash transaction value; (ii) for the debit card payment method, the model in expression (6), without the constant and the variable $I_i$, is fitted. The variance structures described above and retained for comparison with the homoscedastic model are given in expressions (7), (8) and (9):

$$j = \text{cash}: \quad g_j(z_{ij}, v_{ij}, I_i) = 9.3 z_{ij} + v_{ij}; \tag{7}$$

$$j = \text{debit}: \quad g_j(z_{ij}, v_{ij}, I_i) = 14.1097 z_{ij} + 1.0836 v_{ij}; \tag{8}$$

$$j = \text{credit}: \quad g_j(z_{ij}, v_{ij}, I_i) = 1240800000 - 38.0696 z_{ij} + 0.1439 v_{ij} + 1135300000 I_i. \tag{9}$$

Table 3 presents the results of the White test of specification for the heteroscedastic model. The null hypothesis is not rejected for all payment methods at the 5 per cent level. These results should again be taken with caution because the average covariance matrix for the specification test is also singular. Moreover, the transformation to the heteroscedastic model does not eliminate the effect of outliers, which limits the scope of the White test. Given the limitations of the model diagnostic using the White test, in the next section a second round of model validation is conducted with the residual graphs.

### 2.4.2 Model validation with the residual graphs

The graphical method to validate the model is conducted for both cases: when the variance structure of model (3) is homoscedastic and when it is heteroscedastic. In the latter case, the residual graphs presented here concern the transformed homoscedastic model given below:

$$\tilde{y}_{ij} = \tilde{\mathbf{x}}_{ij}^{\top} \boldsymbol{\beta}_j + \tilde{\epsilon}_{ij}, \tag{10}$$

where $\quad \tilde{y}_{ij} = y_{ij}/g_{ij}, \quad \tilde{\mathbf{x}} = \left(1, z_{ij}, v_{ij}, I_i, I_i z_{ij}, I_i v_{ij}\right)^{\top}/g_{ij}, \quad \tilde{\epsilon}_{ij} = \epsilon_{ij}/g_{ij}$
and $\quad g_{ij} = g_j\left(z_{ij}, v_{ij}, I_i\right).$

For the cash payment instrument, Figure 2 presents the plots of the standardized residuals and the square standardized residuals versus the predicted values, when large fitted values and the absolute standardized residuals greater than 4 are removed. About 22 per cent and 20 per cent of outliers are deleted for the homoscedastic and the heteroscedastic models, respectively. Plots (a) and (b) correspond to the standardized residuals and the square standardized residuals, respectively, for the case where the errors are homoscedastic; similarly, plots (c) and (d) correspond to the standardized residuals and the square standardized residuals, respectively, for the case where the errors are heteroscedastic. The curve in blue is a non-parametric smooth line, namely the LOWESS (locally weighted scatter plot smoothing), which helps to show the potential relationship between the residuals or square residuals and the predicted values. A careful observation of Figure 2 leads to two results: On the one hand, plots (a) and (b) show that there is no evidence of a pattern in the distribution of the residuals or the square residuals against the predicted values. However, the LOWESS curve is not a straight line parallel to the abscissa axis. On the other hand, plots (c) and (d) show a better distribution of the residuals with a flatter LOWESS curve than in the previous plots (a) and (b), respectively.

For the debit card payment instrument, Figure 3 presents the plots of the standardized residuals and square standardized residuals versus the predicted values, when large fitted values and the absolute standardized residuals greater than 4 are removed. About 29 per cent of the outliers are deleted for both the homoscedastic and heteroscedastic models. However, for plots (a) and (b), the majority of the observations are still confined to a small part of the graph toward the smallest predicted values. Therefore, the patterns clearly observed in the two plots with the LOWESS appear to be driven by the outliers. Unlike in plots (a) and (b), there is no evidence of a tendency in plots (c) and (d) in Figure 3. The LOWESS curves for plots (c) and (d) are straight lines approximately parallel to the abscissa axis, which seems to validate the heteroscedastic model.

For the credit card payment instrument, Figure 4 presents the plots of the standardized residuals and square standardized residuals versus the predicted values, when large predicted values and the absolute standardized residuals greater than 4 are removed. About 28 per cent of outliers are deleted for both the homoscedastic and the heteroscedastic models. The patterns of plots (c) and (d) are similar to those of plots (a) and (b), respectively; in addition, their LOWESS curves are both flat. For this reason, the homoscedastic model is used for simplicity in the further analysis of the credit card payment method.

Ultimately, the analytical and residual graph model validation leads to the following variance structures for each payment method:

$$j = \text{cash}: \quad g_j(z_{ij}, v_{ij}) = 9.3z_{ij} + v_{ij}; \tag{11}$$

$$j = \text{debit}: \quad g_j(z_{ij}, v_{ij}) = 14.1097z_{ij} + 1.0836v_{ij}; \tag{12}$$

$$j = \text{credit}: \quad g_j(z_{ij}, v_{ij}) = 1. \tag{13}$$

### 2.5 Results of the test of equality of regression coefficients

Fisher's non-robust test at a 5 per cent significance level does not reject the null hypothesis for credit cards, yet rejects it for the cash and debit card payment methods (see Table 4). However, it is well known that this ANOVA-type test is highly sensitive to the presence of outliers in the data (Markatou and He 1994; Salibian-Barrera, Van Aelst and Yohai 2016). Robust versions of the linear regression test have been developed (see Hampel et al. 1986, Markatou and He 1994 and Salibian-Barrera, Van Aelst and Yohai 2016, among others), and Table 4 provides results for the robust versions of the Wald and Fisher tests, which are available in the SAS 9.4 software. At the 5 per cent significance level, the null hypothesis is consistently not rejected for both tests and all payment methods. That is, the tests suggest that there is not enough evidence to reject the assertion that SLC and HQ have the same regression coefficient under linear model (1). Therefore, they can be combined to provide reliable estimates of the total and the marginal costs of cash and card transactions.

## 3 Total Private Cost Framework

A merchant's private costs include both the resources consumed and the fees paid to financial institutions and infrastructures. In the literature on the cost of payment instruments, the merchant's cost function is generally assumed to be linear in the number and value of transactions. The European Commission Directorate-General for Competition (European Commission 2015) also uses a linear cost function for its analysis. The RCPM survey follows the same approach. This naturally leads to a linear regression super-population model linking the costs as well as the number and value of transactions for each merchant.

In the rest of this report, references to the population are denoted by capital letters, while references to the units are denoted by lowercase letters. For each merchant, the linear cost function assumption leads to the following expression:

$$y_{ij} = f_{ij} + a_{ij} z_{ij} + b_{ij} v_{ij}, \quad i = 1, \ldots, N. \tag{14}$$

Here, $i$ refers to the merchant, $j$ is the payment instrument, $f_{ij}$ is the merchant's fixed costs, $a_{ij}$ is the cost incurred each time a transaction with $j$ takes place, $b_{ij}$ is the cost incurred per dollar of sales with $j$, $y_{ij}$ is the merchant's total cost, $z_{ij}$ is the merchant's volume of transactions with $j$, $v_{ij}$ is the merchant's value of transactions with $j$, and $N$ is the LB population size. The parameter of interest is the total population's private cost for the payment instrument $j$ :

$$\begin{aligned} Y_j &= \sum_{i=1}^{N} y_{ij} \\ &= \sum_{i=1}^{N} f_{ij} + \Big(\sum_{i=1}^{N} h_{ij} a_{ij}\Big)\Big(\sum_{i=1}^{N} z_{ij}\Big) + \Big(\sum_{i=1}^{N} k_{ij} b_{ij}\Big)\Big(\sum_{i=1}^{N} v_{ij}\Big), \end{aligned} \tag{15}$$

where $h_{ij} = \frac{z_{ij}}{\sum_{i=1}^{N} z_{ij}}$, and $k_{ij} = \frac{v_{ij}}{\sum_{i=1}^{N} v_{ij}}$.

Expression (15) can also be written as

$$Y_j = F_j + A_j Z_j + B_j V_j, \tag{16}$$

where $Z_j = \sum_{i=1}^{N} z_{ij}$ is the total population volume of transactions, $V_j = \sum_{i=1}^{N} v_{ij}$ is the total population value of transactions, $F_j$, is the total population fixed cost, and $A_j$ and $B_j$ are population marginal costs for payment instrument $j$:

$$F_j = \sum_{i=1}^{N} F_{ij}, \qquad A_j = \sum_{i=1}^{N} h_{ij} a_{ij}, \qquad B_j = \sum_{i=1}^{N} k_{ij} b_{ij}. \tag{17}$$

The main objective of the RCPM survey is to estimate the total cost $Y_j$; however, the estimation of its components is another of the objectives, and, in this technical report, this is achieved by estimating the fixed costs $F_j$ and the marginal costs $A_j$ and $B_j$. This allows the allocation of the total costs between fixed costs ($F_j$), transaction-related variable costs ($A_j Z_j$) and value-related variable costs ($B_j V_j$).

# 4 Total Cost Estimation Methods

This section presents sample-based and model-based approaches for the estimation of the total costs. Both methods estimate marginal costs at the first step and use these estimates to compute the total costs at the second step.

## 4.1 Sample-based approach

Four main assumptions are made for the sample-based approach:

**Assumption SB1.** For each cost item, a prior allocation between fixed costs ($f_{ij}$), transaction-related variable costs ($a_{ij} z_{ij}$) and value-related variable costs ($b_{ij} v_{ij}$) is available. Therefore, these costs are assumed to be known for each merchant.

**Assumption SB2.** The multivariate vectors $(f_j, a_{ij}, b_{ij})^\top$ are assumed to be independent random variables with mean $(f_j, a_j, b_j)^\top$ and of the variance-covariance matrix given in expression (18):

$$Var\left\{(f_j, a_{ij}, b_{ij})^\top\right\} = \begin{pmatrix} \sigma_f^2 & 0 & 0 \\ 0 & \frac{\sigma_a^2}{z_{ij}} & 0 \\ 0 & 0 & \frac{\sigma_b^2}{z_{ij}}. \end{pmatrix}, \quad i = 1, \ldots, N. \tag{18}$$

**Assumption SB3.** The sampling design is non-informative with respect to the underlying multivariate model. That is, the underlying multivariate model holds for the sample and the non-sample part of the population.

**Assumption SB4.** A vector $3 \times 1$ of three ratios converge in probability to the vector $(1, 1, 1)^\top$ under the multivariate model when both the sample and the population size tend to infinity. These three ratios are (i) a sample fixed-cost share over the population fixed-cost share; (ii) the sample marginal transaction-related variable costs over the population marginal transaction-related variable costs; and (iii) the sample value-related variable costs over the population marginal value-related variable costs.

Denote by $\hat{P}_{j_{sb}}$ the share of the fixed costs derived from the sample and by $\left(\hat{A}_{j_{sb}}, \hat{B}_{j_{sb}}\right)^\top$ the vector of marginal volume and value cost estimates; the expressions of these estimates are given by

$$\hat{P}_{j_{sb}} = \frac{\sum_{i=1}^{n} f_{ij}}{\sum_{i=1}^{n} y_{ij}}, \qquad \hat{A}_{j_{sb}} = \frac{\sum_{i=1}^{n} a_{ij} z_{ij}}{\sum_{i=1}^{n} z_{ij}}, \qquad \hat{B}_{j_{sb}} = \frac{\sum_{i=1}^{n} b_{ij} v_{ij}}{\sum_{i=1}^{n} v_{ij}},$$

where $n$ is the sample size. Therefore, the total population estimator of the variable component of the population total cost is given by $\widehat{TVC}_j = \hat{A}_{j_{sb}} \hat{Z}_j^{LB} + \hat{B}_{j_{sb}} \hat{V}_j^{LB}$, and the population estimator of the fixed cost is then derived by $\hat{F}_{j_{sb}} = \frac{\hat{P}_{j_{sb}}}{1-\hat{P}_{j_{sb}}} \left\{\hat{A}_{j_{sb}} \hat{Z}_j^{LB} + \hat{B}_{j_{sb}} \hat{V}_j^{LB}\right\}$, where

$$\hat{Z}_j^{LB} = Z_j - \hat{Z}_j^{SLAB} \quad \text{and} \quad \hat{V}_j^{LB} = V_j - \hat{V}_j^{SLAB}. \tag{19}$$

Here $(Z_j, V_j)^\top$ is the vector of total Canadian volume and value of transactions available from Bank of Canada internal estimates (Nield 2017), and $\left(\hat{Z}_j^{SLAB}, \hat{V}_j^{SLAB}\right)^\top$ is the vector of the total volume and value of transaction estimates of the SLA and SLB populations (see Chen and Shen 2017 for details).

The validity of these assumptions is beyond the scope of this technical report. However, the sample-based approach allows for the calculation of an ad hoc estimator of the total costs, followed by a comparison of this approach with the results of the model-based approach. Denote by $\widehat{Y}_{j_{sb}}$ the sample-based estimator of the population total cost. Replacing $(F_j, A_j, B_j)^\top$ by $\left(\hat{F}_{j_{sb}}, \hat{A}_{j_{sb}}, \hat{B}_{j_{sb}}\right)^\top$ in expression (16) leads to the following expression of the sample-based population total cost estimator:

$$\widehat{Y}_{j_{sb}} \quad = \quad \frac{1}{1-\hat{P}_{j_{sb}}}\left\{\hat{A}_{j_{sb}}\hat{Z}_j^{LB} + \hat{B}_{j_{sb}}\hat{V}_j^{LB}\right\}. \tag{20}$$

## 4.2 Model-based approach

The model-based approach assumes that there is a stochastic structure from an artificially infinite population (also called a super-population model; see Royall 1970), which generates the values of the finite population. Brewer (1963) and Royall (1970) use the model-based approach to propose the best linear unbiased predictor (BLUP) of the finite population total under the linear regression super-population model. Such an approach supposes that the variable of interest, $y_{ij}$, is random. To make this possible, $f_{ij}$, $a_{ij}$ and $b_{ij}$ are assumed to be independent random variables with mean $f_j$, $a_j$ and $b_j$, respectively. Specifically, $f_{ij}$ is supposed to have a constant variance, whereas the variances of $a_{ij}$ and $b_{ij}$ are functions of $z_{ij}$ and $v_{ij}$, respectively. This may also be written as

$$f_{ij} = f_j + \epsilon_{ij}^f, \qquad a_{ij} = a_j + \epsilon_{ij}^a, \qquad b_{ij} = b_j + \epsilon_{ij}^b, \qquad i = 1, \ldots, N, \tag{21}$$

where

$$E\left(\epsilon_{ij}^f\right) = 0; \quad E\left(\epsilon_{ij}^a\right) = 0; \quad E\left(\epsilon_{ij}^b\right) = 0 \quad \forall i, j;$$

$$E\left(\epsilon_{i_1j}^f\epsilon_{i_2j}^f\right) = 0; \quad E\left(\epsilon_{i_1j}^a\epsilon_{i_2j}^a\right) = 0; \quad E\left(\epsilon_{i_1j}^b\epsilon_{i_2j}^b\right) = 0 \quad \forall i_1 \neq i_2 \quad \forall j;$$

$$E\left(\epsilon_{ij}^f\epsilon_{ij}^f\right) = \sigma_f^2; \quad E\left(\epsilon_{ij}^a\epsilon_{ij}^a\right) = \sigma_a^2\left(z_{ij}\right); \quad E\left(\epsilon_{ij}^b\epsilon_{ij}^b\right) = \sigma_b^2\left(v_{ij}\right) \forall i, j;$$

$$E\left(\epsilon_{ij}^f\epsilon_{ij}^a\right) = 0; \quad E\left(\epsilon_{ij}^f\epsilon_{ij}^b\right) = 0; \quad E\left(\epsilon_{ij}^a\epsilon_{ij}^b\right) = 0 \quad \forall i, j.$$

Replacing $f_{ij}$, $a_{ij}$ and $b_{ij}$ by their expression in the linear cost function (14) leads to a working model exactly equal to model (1) given in assumption A1 with $\epsilon_{ij} = \epsilon_{ij}^f + \epsilon_{ij}^a z_{ij} + \epsilon_{ij}^b v_{ij}$. If $\sigma_a^2\left(z_{ij}\right) \propto 1/z_{ij}^2$ and $\sigma_b^2\left(z_{ij}\right) \propto 1/v_{ij}^2$, then the working model is homoscedastic; and if $\sigma_f^2 = 0$ and $\sigma_a^2\left(z_{ij}\right) \propto 1/z_{ij}$ and $\sigma_b^2\left(z_{ij}\right) \propto 1/v_{ij}$, then the underlying model is heteroscedastic with the variance structure given in expression (6). Another assumption of the model-based approach is the availability of the LB vector $\left(Z_j^{LB}, V_j^{LB}\right)^\top$ of total population volume and value of transactions. In this technical report, $\left(Z_j^{LB}, V_j^{LB}\right)^\top$ is not known and is estimated by the vector $\left(\hat{Z}_j^{LB}, \hat{V}_j^{LB}\right)^\top$ where components are given in expression (19). The BLUP of the population total cost under model (1) is given by

$$\hat{Y}_{jBLUP} = \sum_{i=1}^{n} y_{ij} + (N-n)\,\hat{f}_j + \hat{a}_j \left(\hat{Z}_j^{LB} - \sum_{i=1}^{n} z_{ij}\right) + \hat{b}_j \left(\hat{V}_j^{LB} - \sum_{i=1}^{n} v_{ij}\right), \qquad (22)$$

where $\hat{\boldsymbol{\beta}}_j = \left(\hat{f}_j, \hat{a}_j, \hat{b}_j\right)^\top$ is the best linear unbiased estimator (BLUE) of $\boldsymbol{\beta}_j = (f_j, a_j, b_j)^\top$. Given that the variance of the error term is a linear combination of the auxiliary variables, the BLUP in expression (22) is exactly equal to the projection estimator in (23), given below:

$$\hat{Y}_{jPROJ} = N\hat{f}_j + \hat{a}_j\hat{Z}_j^{LB} + \hat{b}_j\hat{V}_j^{LB}. \qquad (23)$$

Due to the presence of outliers in the sample, the non-robust estimator in (22) is unstable. Therefore, the robust version of (22) is obtained by replacing the BLUE of the regression coefficient by its robust MM-estimator, denoted by $\hat{\boldsymbol{\beta}}_{jR} = \left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}\right)^\top$ (see Huber 1981 and Yohai 1987 for details about M-estimation and MM-estimation procedures, respectively). The expression of the proposed robust BLUP estimator is given by

$$\hat{Y}_{jRBLUP} = \sum_{i=1}^{n} y_{ij} + (N-n)\,\hat{f}_{jR} + \hat{a}_{jR} \left(\hat{Z}_j^{LB} - \sum_{i=1}^{n} z_{ij}\right) + \hat{b}_{jR} \left(\hat{V}_j^{LB} - \sum_{i=1}^{n} v_{ij}\right). \qquad (24)$$

Similarly, the robust projection estimator is derived and its expression is given in expression (25):

$$\hat{Y}_{jRPROJ} = N\hat{f}_{jR} + \hat{a}_{jR}\hat{Z}_j^{LB} + \hat{b}_{jR}\hat{V}_j^{LB}. \qquad (25)$$

Note that, contrary to the BLUP in (22), the robust BLUP in (24) is not equal to the robust projection estimator in (25). Like most robust estimators, expressions (24) and (25) might have a higher bias. Indeed, they inherit MM-estimator robustness properties and thus the related bias. Chambers (1986) introduces a bias-corrected robust finite population estimator, which makes a trade-off between the bias and the variance through an appropriate chosen tuning constant. Beaumont, Haziza and Ruiz-Gazen (2013) develop an alternate version based on the concept of conditional bias. The advantage of the conditional bias relative to the ChamberS-estimator is the tuning constant. Indeed the choice of the tuning constant for the ChamberS-estimator is an open question, whereas Beaumont, Haziza and Ruiz-Gazen (2013) propose an adaptive method of choosing the tuning constant that is easy to implement and leads to a good trade-off between the bias and the variance. The implementation of the bias-corrected estimators is beyond the scope of this technical report and will be studied in future policy research work.

# 5 Results of the Sample-Based Versus Model-Based Approaches

Table 5 presents the results of the parameter estimates of the sample-based and model-based approaches. For the latter, the BLUE and the robust BLUE are given as well as their standard errors in parentheses. It is clear that for each payment method, the estimates are different regardless of the approach or the estimation method used. In general, robust BLUE estimates are higher than the estimates based on the sample. It should be noted that these results are not directly comparable because the models and/or the estimation methods are different. However, due to the presence of outliers in the sample, it is recommended to use the results derived from the robust MM-estimation method, based on the models validated in Section 2.4.

Table 6 shows the results of the total private cost estimates for the sample-based approach, the BLUP, the robust projection and the robust BLUP estimator. For the cash and debit card payment methods, the robust projection and the robust BLUP are similar, with the absolute relative difference being less than 10 per cent. However, both of these estimates are at least 25 per cent higher than those in the sample-based approach. For cash and debit cards, the discrepancies between the approaches may be due to the fact that the sample-based method makes use of the prior allocation of the merchants' total costs between fixed and variable costs. The results of the credit card payment method are similar in both robust methods, with an absolute relative difference of less than 1 per cent. However, the difference between the sample-based and the robust model-based approaches is about 10 per cent. The difference observed for the non-robust BLUP is due to the large estimate of the fixed costs (see Table 5). Table 6 also shows that for both approaches, about 60 per cent of the total private costs incurred by retailers are made up of credit card payments. Finally, the table shows that the total private costs incurred by LB for all payment methods represent about 0.21 per cent of the gross domestic product (GDP) for the sample-based approach and about 0.25 per cent for the robust estimator in the model-based approach.

Adding up the results from SLA and SLB leads to an overall total private cost of payment methods to GDP of about 0.51 per cent for all Canadian retailers in 2014 (Kosse et al. 2017). Schmiedel, Kostova and Ruttenberg (2012) provide a ratio of 0.44 per cent for a set of 13

European countries in the reference year 2009. The difference is apparent as the comparison should take into account the temporal dimension. Indeed, the payment landscape is continually changing. Arango et al. (2012) show that, unlike cash transactions, credit card transactions have been growing as a share of the overall value of payments over the last two decades. In addition, data from both Payments Canada (Tompkins 2015) and CANSIM Table 380-0063 (Statistics Canada 2016) show that the value of credit card transactions is rising faster than nominal GDP. In fact, the total Canadian value of credit card transactions grew by 39.8 per cent between 2009 and 2014, while the nominal GDP grew by 23.9 per cent in the same period. This suggests that the ratio of overall Canadian total retail private costs to GDP (0.51 per cent) may be comparable to that of the European countries if the growing share of credit card transactions and related costs are taken into account.

Figure 5 presents the allocation of the total costs between fixed, transaction-related and value-related variable costs for both the sample-based approach and the robust projection estimator. For the sample-based approach, the allocation is the merchants' prior allocation of their total costs. For the credit card payment method, the fixed-cost share is less for the sample based-approach while the variable-cost shares are approximately similar. For the cash and the debit card payment methods, however, the share of the fixed costs is higher for the sample-based approach than it is for the model-based approach. The main difference concerns the debit card payment instrument, where the share of the value-related variable costs is 46 per cent for the robust projection compared with only 1 per cent for the sample-based approach estimator. This may be due to non-robustness properties of the estimators used, as well as the fact that the sample-based method makes use of the prior allocation of the merchants' total costs between fixed and variable costs.

At the current stage, the model-based approach is recommended for calculating the total private cost. Indeed, there are at least three advantages to this option. The first is that no assumption is made about the allocation of the cost item between fixed and variable costs. The second is the robust method used to deal with the presence of outliers. The third advantage, and not least, is that the model has been validated by both the White test of specification and the residual graphs analysis.

# 6 Conclusion

The results of the test of equality of regression coefficients show that the SLC and HQ strata are homogeneous. Therefore, they can be combined to provide reliable estimates of both the total and marginal costs of cash and card transactions. However, given the relatively small sample size, the power of the test is weak. In particular, Salibian-Barrera, Van Aelst and Yohai (2016) show with simulations that the robust Wald test should be avoided for a small sample, because it is not reliable in terms of either level or power. They suggest bootstrapping the test statistics using the fast and robust bootstrap for small samples.

For the credit card payment method, the results of the total LB private cost calculations are similar for both sample-based and robust model-based approaches. The difference observed for the cash and debit card payment methods might be due to the fact that, unlike the MM-estimation used for the model-based method, the sample-based approach is not robust to the presence of outliers. Therefore, the proposed robust model-based method should be used for further analysis.

This technical report is subject to some caveats. The sample size is relatively small (between 151 and 166, depending on the payment method), and the number of outliers is very large, with an estimated outlier proportion of about 16 per cent, using a cut-off of 3 for the normalized residuals in the robust MM-procedure. Thus, despite the use of high-breakdown-point outlier-resistant methods, the estimators used remain unstable. This can lead to misleading results for the homogeneity test.

For the cleaning and editing of the raw data from the field, a minimum number of variables and criteria are set to identify candidate records for the editing and imputation. This selective editing is based on the errors that have a significant impact on the total survey error (Hidiroglou and Berthelot 1986; Granquist and Kovar 1997). Then cleaning is done for records that verify these criteria.

Noted that larger predicted values may refer to economies of scale, which may be captured by adding a square term in the regression. Thus, further model specification could be done for the purpose of efficiency analysis of the payment methods.

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Table 1: Sample mean and median of total sales and number of employees in 2014

| | SLC | | HQ | |
|---|---|---|---|---|
| | Total sales (millions) | Number of employees | Total sales (millions) | Number of employees |
| Mean | 5.6 | 97 | 18,700 | 12,519 |
| Median | 1.3 | 76 | 30 | 750 |
| Sample size | 114 | 114 | 55 | 55 |
| Missing values | 10 | 1 | 0 | 1 |

Note: Unweighted means and medians are computed.

Table 2: White test of first-and second-moment specifications for the model with homoscedastic variance structure

| Payment instrument | Degree of freedom | Test statistics | P-value | Number of observations used |
|---|---|---|---|---|
| Cash | 13 | 13.35 | 0.4214 | 166 |
| Debit | 13 | 18.90 | 0.1262 | 151 |
| Credit | 11 | 5.36 | 0.9123 | 155 |

Note: The test statistics follow a $\chi^2_{k(k+1)/2}$, where $k = 6$ is the number of predictors in regression (3) plus the constant. The average covariance matrix for the test is singular. In addition, because of the redundancies occurring for a large number of units in the sample, the degree of freedom is automatically adjusted by the SAS 9.4 software; this explains the difference in the degree of freedom between payment methods.

Table 3: White test of first-and second-moment specifications for the model with heteroscedastic variance structure (6)

| Payment instrument | Degree of freedom | Test statistics | P-value | Number of observations used |
|---|---|---|---|---|
| Cash | 12 | 15.87 | 0.1972 | 166 |
| Debit | 12 | 8.73 | 0.7261 | 151 |
| Credit | 10 | 9.30 | 0.5043 | 155 |

Note: The test statistics follows a $\chi^2_{k(k+1)/2}$, where $k = 6$ is the number of predictors in regression (10). The average covariance matrix for the test is singular. In addition, because of the redundancies occurring for a large number of units in the sample, the degree of freedom is automatically adjusted by the SAS 9.4 software; this explains the difference in the degree of freedom between payment methods.

Table 4: Linear test of equality of regression coefficients

| Payment instrument | Degree of freedom | Test statistics | Lambda | P-value |
|---|---|---|---|---|
| | | Non-robust Fisher test | | |
| Cash | (3,160) | 19.9200 | | <0.0001 |
| Debit | (3,145) | 8.8200 | | <0.0001 |
| Credit | (3,149) | 0.5500 | | 0.6510 |
| | | Robust version of Fisher test | | |
| Cash | 3 | 0.2338 | 0.9529 | 0.9700 |
| Debit | 3 | 0.0000 | 0.9529 | 1.0000 |
| Credit | 3 | 0.4402 | 0.9529 | 0.9272 |
| | | Robust version of Wald test | | |
| Cash | 3 | 2.7416 | | 0.4332 |
| Debit | 3 | 2.5108 | | 0.4733 |
| Credit | 3 | 3.9227 | | 0.2699 |

Note: The non-robust Fisher test has an $F$ distribution, with $(3, n-3)$ degrees of freedom, where $n$ is the number of units used for each payment method. Lambda $(\lambda)$ is a standardization factor of the test statistics $\tau$. Asymptotically, $\tau \sim \lambda\chi^2_3$ (refer to Hampel et al. 1986, p. 346 for details). The test statistics of the robust Wald test follows asymptotically a $\chi^2_3$ (see Hampel et al. 1986, p. 364 for more details). The robust tests computed are those available in the SAS 9.4 software.

Table 5: Fixed-cost mean and marginal-cost estimate results

| Model parameters | | Cash | Debit | Credit |
|---|---|---|---|---|
| | | Sample-based fixed-cost mean and marginal-cost estimates | | |
| Fixed-cost mean | $(f_j)$ | 17,084 | 13,831 | 13,989 |
| Marginal volume cost | $(a_j)$ | 0.0613 | 0.1531 | 0.1535 |
| Marginal value cost | $(b_j)$ | 0.0041 | 0.0000 | 0.0190 |
| | | Model-based non-robust fixed-cost mean and marginal-cost estimates | | |
| Fixed-cost mean | $(f_j)$ | $20,365_{(2,615)}$ | $134_{(381)}$ | $253,821_{(292,495)}$ |
| Marginal volume cost | $(a_j)$ | $0.0365_{(0.0534)}$ | $0.1131_{(0.0195)}$ | $0.0618_{(0.0442)}$ |
| Marginal value cost | $(b_j)$ | $0.0092_{(0.0058)}$ | $0.0026_{(0.0007)}$ | $0.0207_{(0.0010)}$ |
| | | Model-based robust fixed-cost mean and marginal-cost estimates | | |
| Fixed-cost mean | $(f_j)$ | $8,752_{(286)}$ | $4,665_{(593)}$ | $38,599_{(5,901)}$ |
| Marginal volume cost | $(a_j)$ | $0.0897_{(0.0099)}$ | $0.1434_{(0.0126)}$ | $0.1265_{(0.0028)}$ |
| Marginal value cost | $(b_j)$ | $0.0067_{(0.0007)}$ | $0.0031_{(0.0004)}$ | $0.0196_{(0.0001)}$ |
| Number of observations used | | 166 | 151 | 155 |

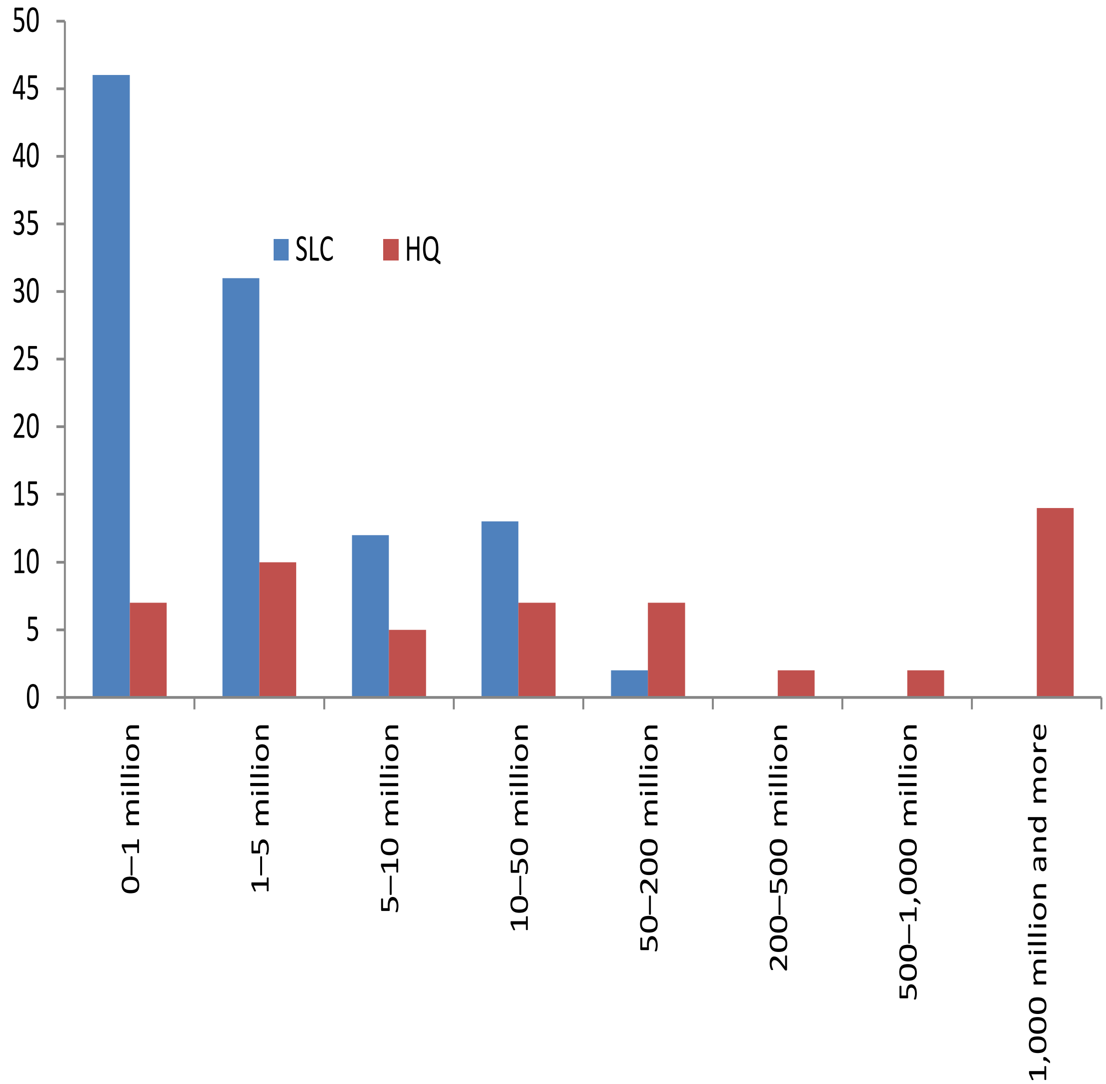
Note: $f_j, a_j$ and $b_j$ are regression coefficients of the model in expression (1): $y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + \epsilon_{ij}$, with $E\left(\epsilon_{ij} \middle| z_{ij}, v_{ij}\right) = 0$, $E\left(\epsilon_{ij}^2 \middle| z_{ij}, v_{ij}\right) = \sigma_j^2 g_j(z_{ij}, v_{ij})$, $\sigma_j^2$ is unknown, and $g_j(z_{ij}, v_{ij})$ is given by expressions $g_j(z_{ij}, v_{ij}) = 9.3z_{ij} + v_{ij}$ for the cash, $g_j(z_{ij}, v_{ij}) = 14.1097z_{ij} + 1.0836v_{ij}$ for the debit cards and $g_j(z_{ij}, v_{ij}) = 1$ for the credit cards. The standard errors estimates are in brackets.

Table 6: Total cost estimates: Sample-based versus model-based approach

| | Cash | Debit | Credit | Total |
|---|---|---|---|---|
| | Sample-based approach estimator | | | |
| Total cost (CAD millions) | 819 | 628 | 2,682 | 4,129 |
| Percentage of GDP (%) | 0.042 | 0.032 | 0.136 | 0.209 |
| Payment method share (%) | 19.8 | 15.2 | 65.0 | 100.0 |
| | Non-robust BLUP estimator | | | |
| Total cost (CAD millions) | 1,158 | 731 | 4,911 | 6,800 |
| Percentage of GDP (%) | 0.059 | 0.037 | 0.249 | 0.345 |
| Payment method share (%) | 17.0 | 10.8 | 72.2 | 100.0 |
| | Robust projection estimator | | | |
| Total cost (CAD millions) | 1,107 | 937 | 2,928 | 4,972 |
| Percentage of GDP (%) | 0.056 | 0.047 | 0.148 | 0.252 |
| Payment method share (%) | 22.3 | 18.8 | 58.9 | 100.0 |
| | Robust BLUP estimator | | | |
| Total cost (CAD millions) | 1,031 | 885 | 2,954 | 4,869 |
| Percentage of GDP (%) | 0.052 | 0.045 | 0.150 | 0.247 |
| Payment method share (%) | 21.2 | 18.2 | 60.7 | 100.0 |

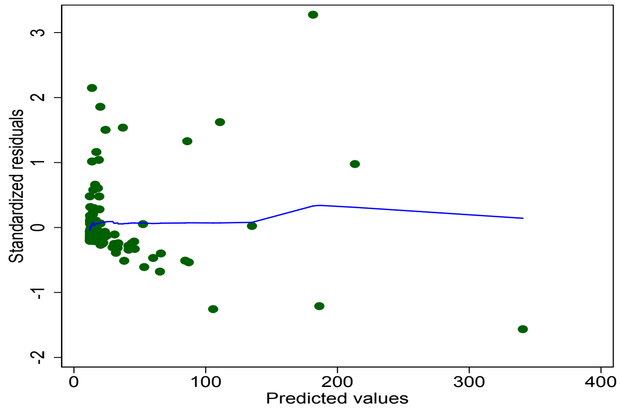
Note: GDP is taken from Statistics Canada CANSIM Table 380-0064, year 2014.

Figure 1: Sample distribution of the LB' total sales in the RCPM survey

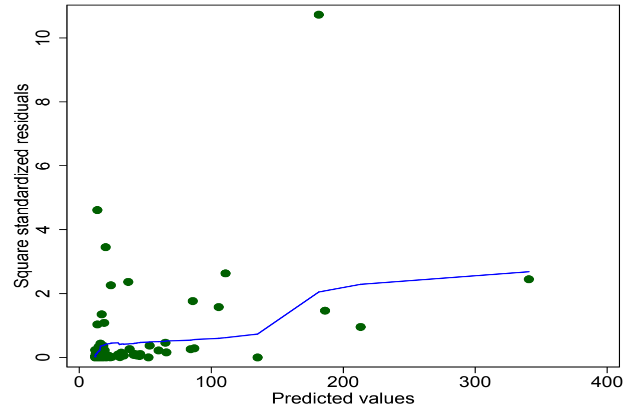



Note: The y-axis represents the number of merchants in the sample and the x-axis represents Canadian dollars.

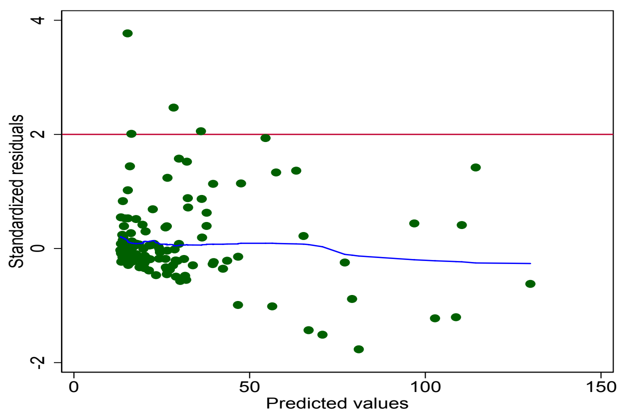
Figure 2: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the cash payment method where outliers are deleted from the sample



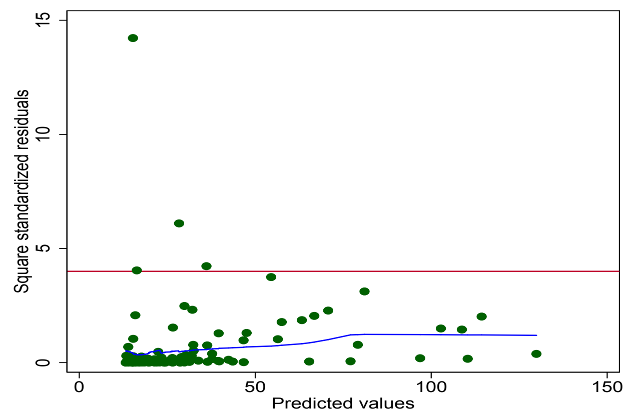

(a) Cash: homoscedastic residuals




(b) Cash: homoscedastic square residuals




(c) Cash: heteroscedastic residuals




(d) Cash: heteroscedastic square residuals

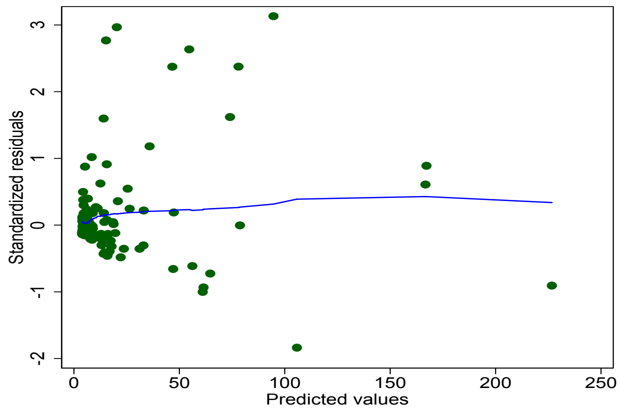
Note: The standardized residual is given by

$$\hat{e}_{ij} = \frac{y_{ij} - \hat{f}_{jR} - \hat{a}_{jR} z_{ij} - \hat{b}_{jR} v_{ij} - \hat{c}_{0jR} I_i - \hat{c}_{1jR} I_i z_{ij} - \hat{c}_{1jR} I_i v_{ij}}{\hat{\sigma}_{jR} g_j(z_{ij}, v_{ij}, I_i)},$$

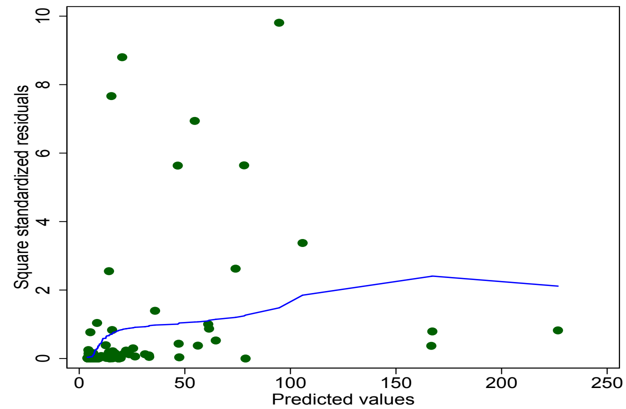
where $\left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}, \hat{c}_{0jR}, \hat{c}_{1jR}, \hat{c}_{2jR}\right)^{\top}$ is the MM-robust estimate of the regression coefficient $(f_j, a_j, b_j, c_{0j}, c_{1j}, c_{2j})^{\top}$, for the model in expression (3) given by
$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j} I_i + c_{1j} I_i z_{ij} + c_{2j} I_i v_{ij} + \epsilon_{ij}$,
$\hat{\sigma}_{jR}$ is the robust scale estimate and $g_j(z_{ij}, v_{ij}, I_i) = 9.3 z_{ij} + v_{ij}$ or $g_j(z_{ij}, v_{ij}, I_i) = 1$ for the heteroscedastic and the homoscedastic model, respectively.
The S-estimator is used for the initial estimates for the MM-regression where the Yohai function is used for both the S and the MM-estimators. The parameters of the Yohai function (available in SAS 9.4 software) are $k_0 = 0.7405$ and $k_1 = 0.8679$ for the S and the MM-estimators, respectively. These values correspond to the 25 per cent breakdown value and an efficiency of 0.85 for the MM-estimator.
About 22 per cent and 20 per cent of large predicted values or absolute standardized residuals greater than 4 are deleted for the homoscedastic and the heteroscedastic models, respectively.

The predicted values in plots (a) and (b) are in thousands of CAD, while those in plots (c) and (d) have no units.

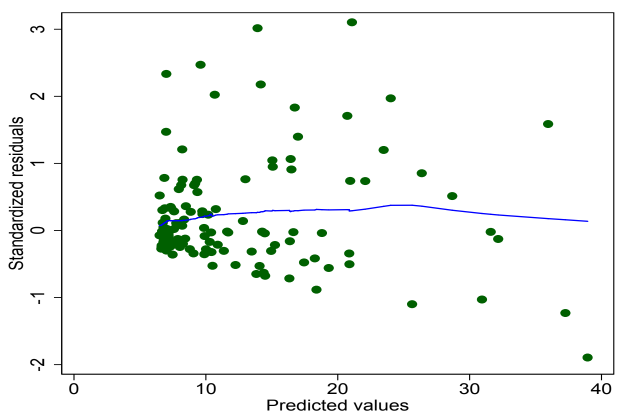
Figure 3: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the debit payment method where outliers are deleted from the sample



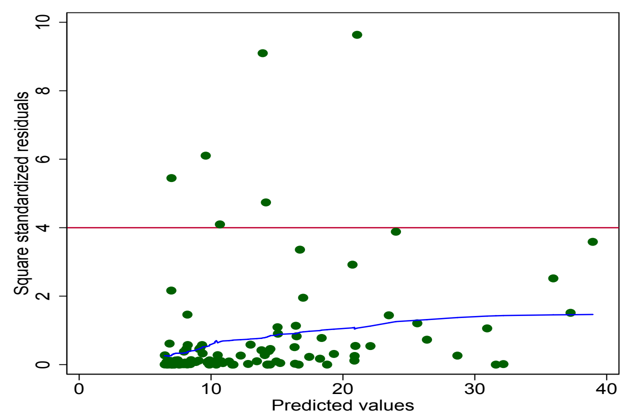

(a) Debit: homoscedastic residuals




(b) Debit: homoscedastic square residuals




(c) Debit: heteroscedastic residuals




(d) Debit: heteroscedastic square residuals

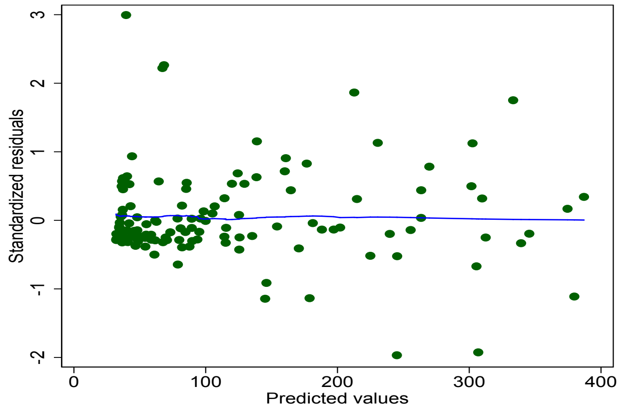
Note: The standardized residual is given by

$$\hat{e}_{ij} = \frac{y_{ij} - \hat{f}_{jR} - \hat{a}_{jR} z_{ij} - \hat{b}_{jR} v_{ij} - \hat{c}_{0jR} I_i - \hat{c}_{1jR} I_i z_{ij} - \hat{c}_{1jR} I_i v_{ij}}{\hat{\sigma}_{jR} g_j(z_{ij}, v_{ij}, I_i)},$$

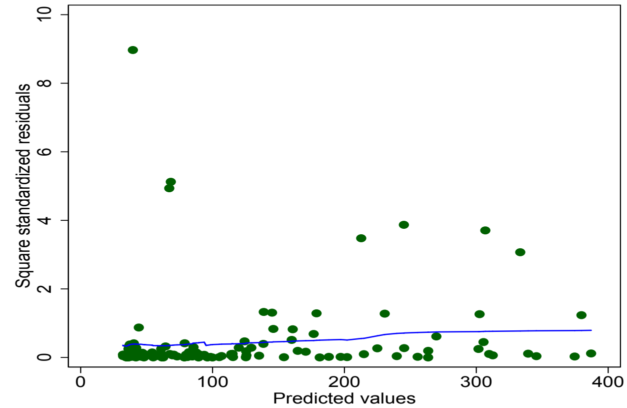
where $\left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}, \hat{c}_{0jR}, \hat{c}_{1jR}, \hat{c}_{2jR}\right)^\top$ is the MM-robust estimate of the regression coefficient $(f_j, a_j, b_j, c_{0j}, c_{1j}, c_{2j})^\top$, for the model in expression (3) given by
$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j} I_i + c_{1j} I_i z_{ij} + c_{2j} I_i v_{ij} + \epsilon_{ij}$,
$\hat{\sigma}_{jR}$ is the robust scale estimate and $g_j(z_{ij}, v_{ij}, I_i) = 14.1097 z_{ij} + 1.0836 v_{ij}$ or $g_j(z_{ij}, v_{ij}, I_i) = 1$ for the heteroscedastic and the homoscedastic model, respectively.
The S-estimator is used for the initial estimates for the MM-regression where the Yohai function is used for both the S and the MM-estimators. The parameters of the Yohai function (available in SAS 9.4 software) are $k_0 = 0.7405$ and $k_1 = 0.8679$ for the S and the MM-estimators, respectively. These values correspond to the 25 per cent breakdown value and an efficiency of 0.85 for the MM-estimator.
About 29 per cent of large predicted values or absolute standardized residuals greater than 4 are deleted for both the homoscedastic and the heteroscedastic models, respectively.

The predicted values in plots (a) and (b) are in thousands of CAD, while those in plots (c) and (d) have no units.

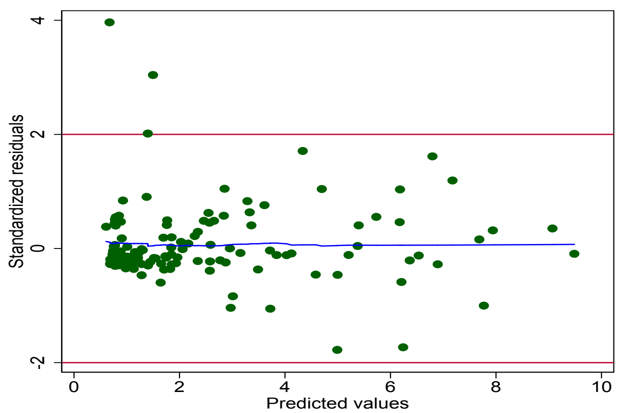
Figure 4: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the credit payment method where outliers are deleted from the sample



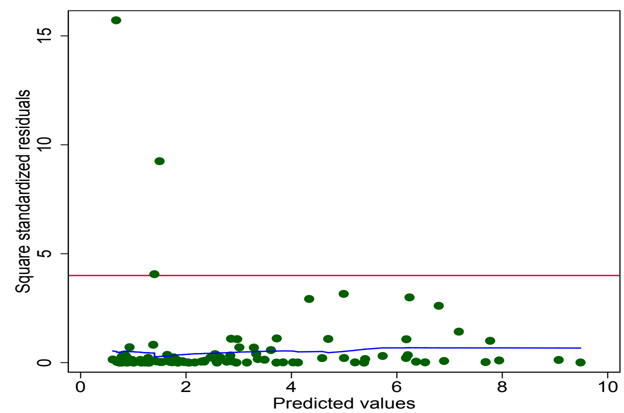

(a) Credit: homoscedastic residuals




(b) Credit: homoscedastic square residuals




(c) Credit: heteroscedastic residuals




(d) Credit: heteroscedastic square residuals

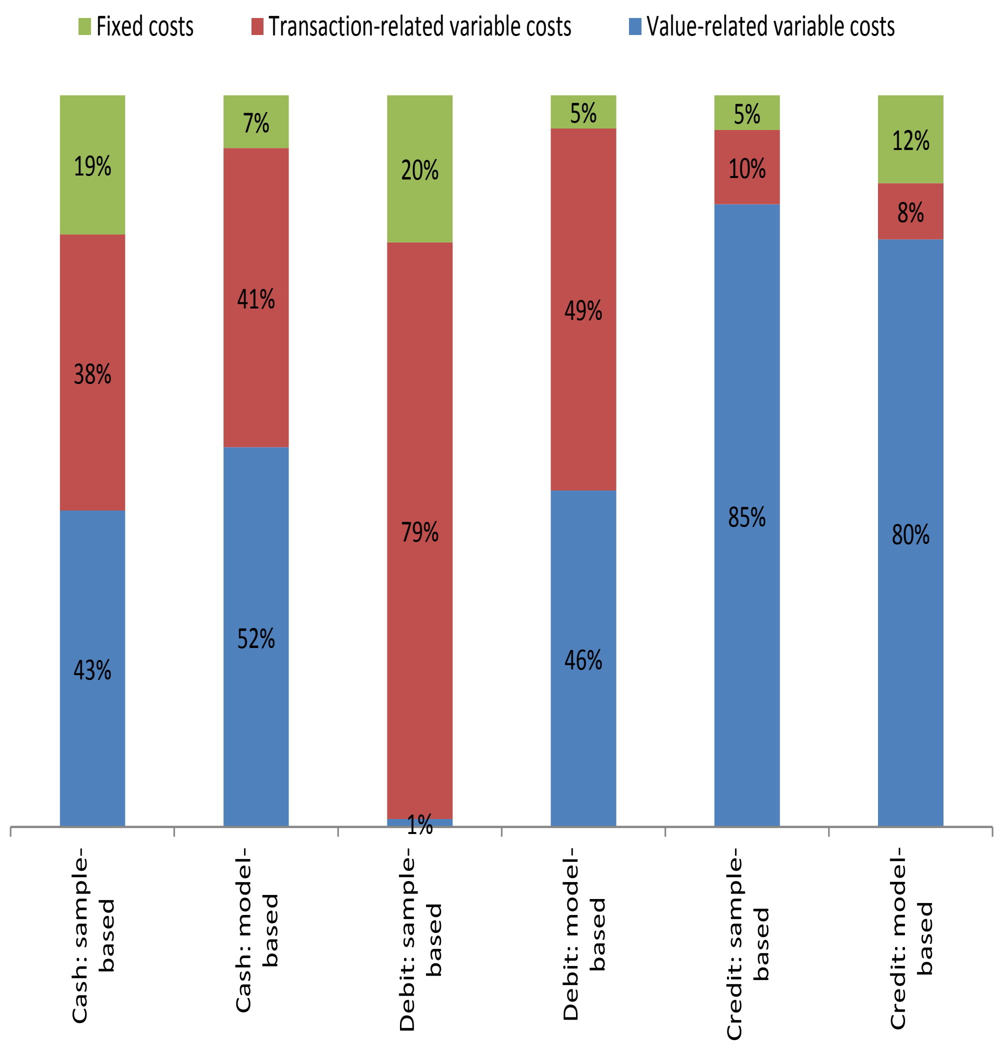
Note: The standardized residual is given by

$$\hat{e}_{ij} = \frac{y_{ij} - \hat{f}_{jR} - \hat{a}_{jR} z_{ij} - \hat{b}_{jR} v_{ij} - \hat{c}_{0jR} I_i - \hat{c}_{1jR} I_i z_{ij} - \hat{c}_{1jR} I_i v_{ij}}{\hat{\sigma}_{jR} g_j(z_{ij}, v_{ij}, I_i)},$$

where $\left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}, \hat{c}_{0jR}, \hat{c}_{1jR}, \hat{c}_{2jR}\right)^{\top}$ is the MM-robust estimate of the regression coefficient $(f_j, a_j, b_j, c_{0j}, c_{1j}, c_{2j})^{\top}$, for the model in expression (3) given by
$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j} I_i + c_{1j} I_i z_{ij} + c_{2j} I_i v_{ij} + \epsilon_{ij}$,
$\hat{\sigma}_{jR}$ is the robust scale estimate and $g_j(z_{ij}, v_{ij}, I_i) = 1240800000 - 38.0696 z_{ij} + 0.1439 v_{ij} + 1135300000 I_i$ or $g_j(z_{ij}, v_{ij}, I_i) = 1$ for the heteroscedastic and the homoscedastic model, respectively.
The S-estimator is used for the initial estimates for the MM-regression where the Yohai function is used for both the S and the MM-estimators. The parameters of the Yohai function (available in SAS 9.4 software) are $k_0 = 0.7405$ and $k_1 = 0.8679$ for the S and the MM-estimators, respectively. These values correspond to the 25 per cent breakdown value and an efficiency of 0.85 for the MM-estimator.
About 28 per cent of large predicted values or absolute standardized residuals greater than 4 are deleted for both the homoscedastic and the heteroscedastic models, respectively.

The predicted values in plots (a) and (b) are in thousands of CAD, while those in plots (c) and (d) have no units.

Figure 5: Fixed versus variable cost allocation




Note: The sample-based and robust projection model-based approach estimators are given by

$$\hat{Y}_{j_{sb}} = \left(\hat{A}_{j_{sb}}\hat{Z}_j^{LB} + \hat{B}_{j_{sb}}\hat{V}_j^{LB}\right) / \left(1 - \hat{P}_{j_{sb}}\right)$$

and

$$\hat{Y}_{j_{RPROJ}} = N\hat{f}_{jR} + \hat{a}_{jR}\hat{Z}_j^{LB} + \hat{b}_{jR}\hat{V}_j^{LB},$$

respectively. Then, the transaction-related variable-cost shares are given by $\hat{\nu}^z_{jsb} = \hat{A}_{j_{sb}}\hat{Z}_j^{LB}/\hat{Y}_{j_{sb}}$ and $\hat{\nu}^z_{jRPROJ} = \hat{a}_{jR}\hat{Z}_j^{LB}/\hat{Y}_{j_{jRPROJ}}$ for the sample-based and the robust projection model-based approaches, respectively. Similarly, the value-related variable-cost shares are given by $\hat{\nu}^v_{jsb} = \hat{B}_{j_{sb}}\hat{V}_j^{LB}/\hat{Y}_{j_{sb}}$ and $\hat{\nu}^v_{jRPROJ} = \hat{b}_{jR}\hat{V}_j^{LB}/\hat{Y}_{j_{jRPROJ}}$ for the sample-based and the robust projection model-based approaches, respectively. Ultimately, the fixed-cost shares are derived by $\hat{\nu}^f_{jsb} = 1 - \hat{\nu}^z_{jsb} - \hat{\nu}^v_{jsb}$ and $\hat{\nu}^f_{jRPROJ} = 1 - \hat{\nu}^z_{jRPROJ} - \hat{\nu}^v_{jRPROJ}$ for the sample-based and the robust projection model-based approaches, respectively.

# Appendix

- Table A1: Variables used for the calculation of the merchants' costs

- Figure A1: Road map to compute total private cost estimates for the large businesses

- Figure A2: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the cash payment method where the full sample is used

- Figure A3: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the debit card payment method where the full sample is used

- Figure A4: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the credit card payment method where the full sample is used

Table A1: Variables used for the calculation of the merchants' costs

| | **Cash** | **Debit cards** | **Credit cards** |
|---|---|---|---|
| [1] | Number of transactions | Number of transactions | Number of transactions |
| [2] | Value of transactions | Value of transactions | Value of transactions |
| [3] | Additional cash withdrawal/ deposit fees | Additional debit card fees to financial institutions | Additional credit card fees to financial institutions |
| [4] | Time spent on cash activities | Time spent on debit card activities | Time spent on credit card activities |
| [5] | Foregone interest on cash balances | Acquirer fees | Acquirer fees |
| [6] | Front-office costs | Front-office costs | Front-office costs |
| [7] | Business account costs | Business account costs | Business account costs |
| [8] | Number of cash registers owned | Number of card terminals owned | Number of card terminals owned |
| [9] | Number of cash registers rented | Number of card terminals rented | Number of card terminals rented |
| [10] | Cost of cash registers rented | Cost of card terminals rented | Cost of card terminals rented |
| [11] | Number of authentication devices owned | | |
| [12] | Number of authentication devices rented | | |
| [13] | Cost of authentication devices rented | | |
| [14] | Number of other equipment owned | | |
| [15] | Number of other equipment rented | | |
| [16] | Cost of other equipment rented | | |
| [17] | Fees to specialized cash transportation company | | |
| [18] | Insurance | Insurance | Insurance |
| [19] | Fraud loss | Fraud loss | Fraud loss |
| [20] | Fraud time | Fraud time | Fraud time |

Note: For each payment method, the number in the first column at the left of the table indicates the end of the previous variable, if any, and the start of a new variable.
Some variables such as the bank account costs provide aggregate information for all payment methods. In this case, auxiliary information is used to allocate the cost to each payment method.

Figure A1: Flow chart summary of the analysis

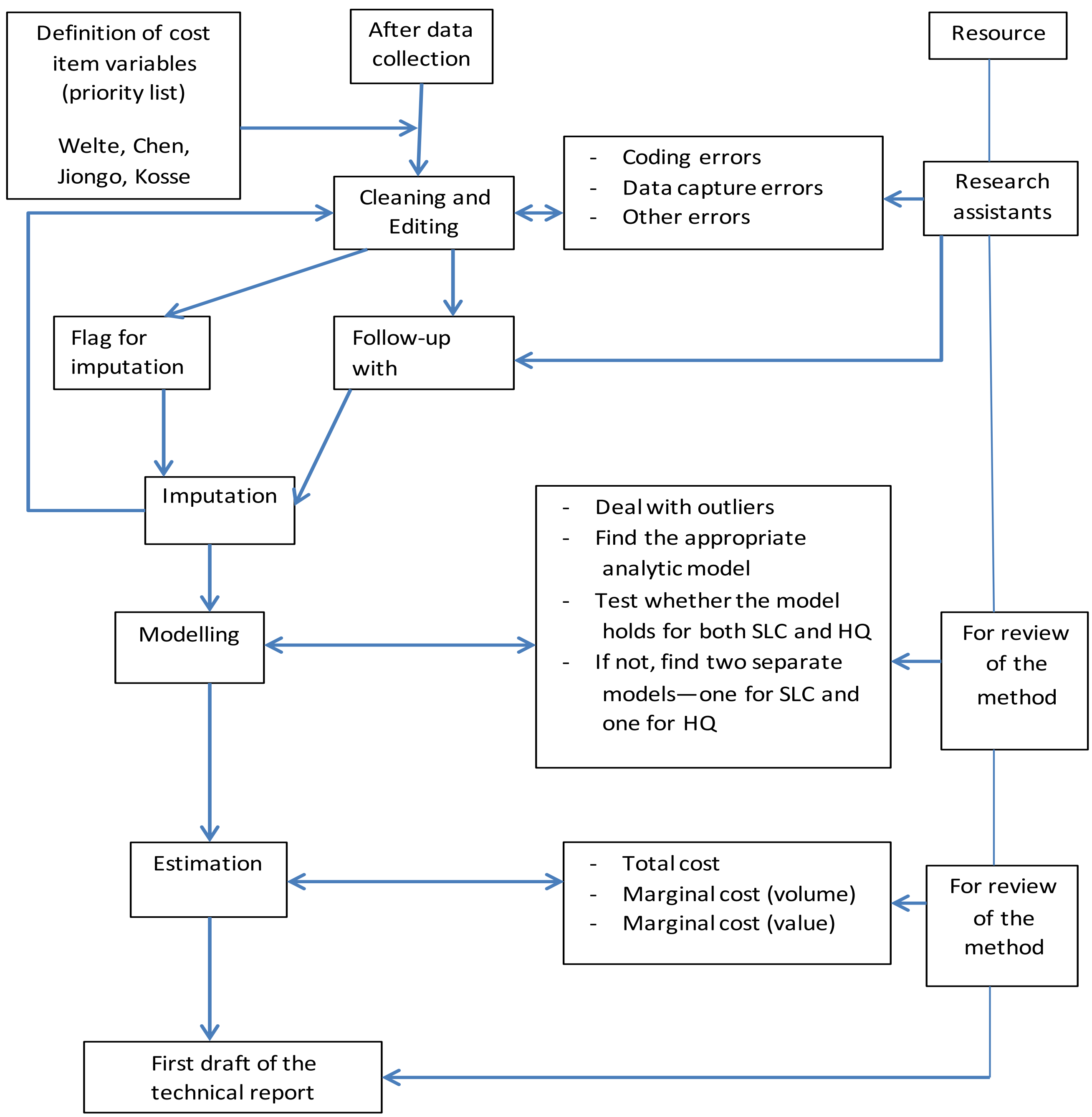
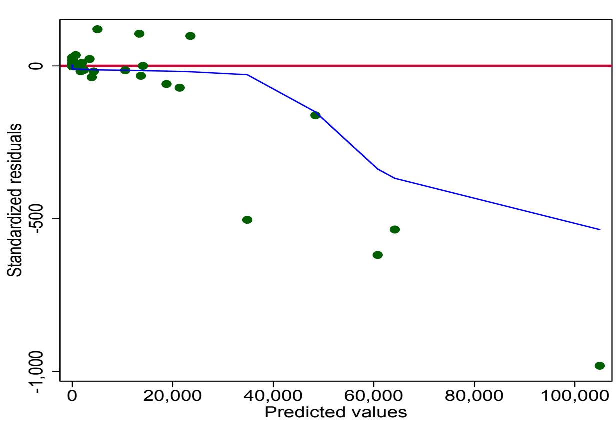
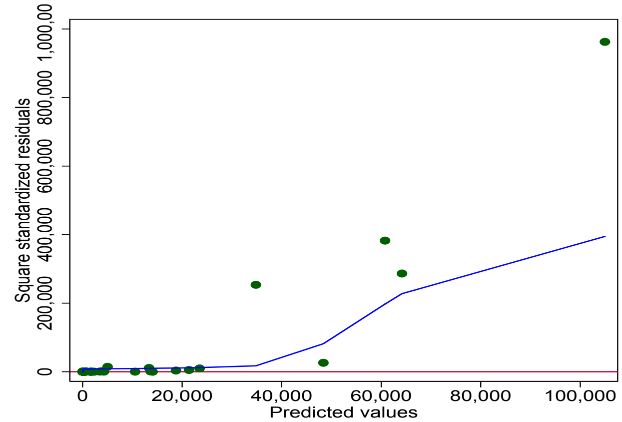


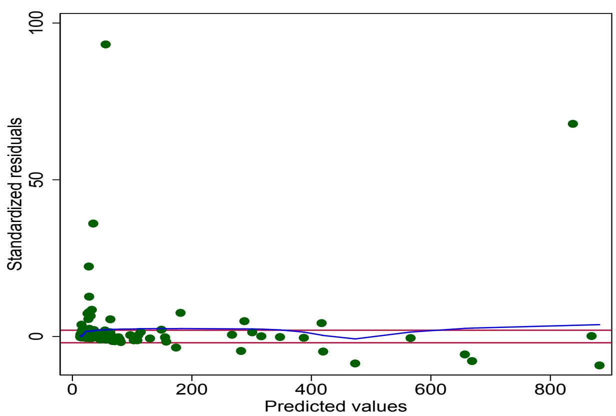
Figure A2: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the cash payment method where the full sample is used



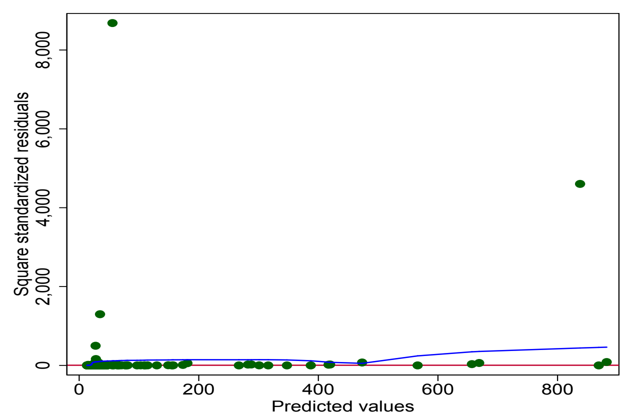

(a) Cash: homoscedastic residuals




(b) Cash: homoscedastic square residuals




(c) Cash: heteroscedastic residuals



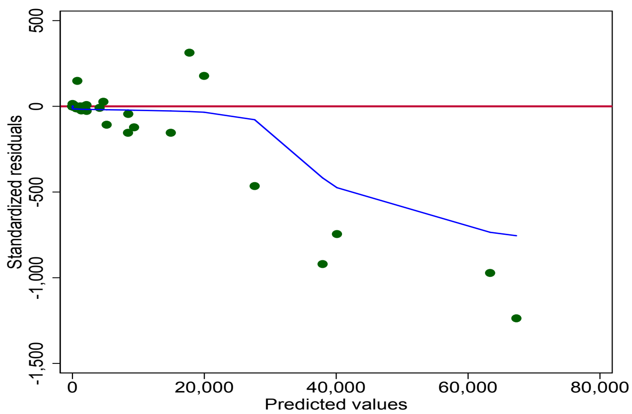

(d) Cash: heteroscedastic square residuals

Note: The standardized residual is given by

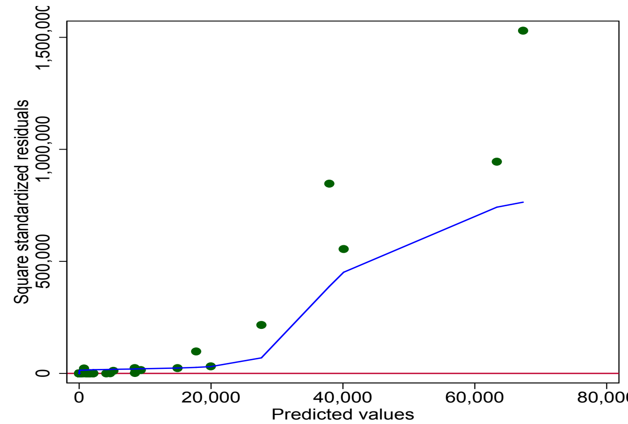
$$\hat{e}_{ij} = \frac{y_{ij} - \hat{f}_{jR} - \hat{a}_{jR} z_{ij} - \hat{b}_{jR} v_{ij} - \hat{c}_{0jR} I_i - \hat{c}_{1jR} I_i z_{ij} - \hat{c}_{1jR} I_i v_{ij}}{\hat{\sigma}_{jR} g_j(z_{ij}, v_{ij}, I_i)},$$

where $\left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}, \hat{c}_{0jR}, \hat{c}_{1jR}, \hat{c}_{2jR}\right)^\top$ is the MM-robust estimate of the regression coefficient $(f_j, a_j, b_j, c_{0j}, c_{1j}, c_{2j})^\top$, for the model in expression (3) given by
$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j} I_i + c_{1j} I_i z_{ij} + c_{2j} I_i v_{ij} + \epsilon_{ij}$,
$\hat{\sigma}_{jR}$ is the robust scale estimate and $g_j(z_{ij}, v_{ij}, I_i) = 9.3 z_{ij} + v_{ij}$ or $g_j(z_{ij}, v_{ij}, I_i) = 1$ for the heteroscedastic and the homoscedastic model, respectively.
The S-estimator is used for the initial estimates for the MM-regression where the Yohai function is used for both the S and the MM-estimators. The parameters of the Yohai function are $k_0 = 0.7405$ and $k_1 = 0.8679$ for the S and the MM-estimators, respectively. These values correspond to the 25 per cent breakdown value and an efficiency of 0.85 for the MM-estimator.
The predicted values in plots (a) and (b) are in thousands of CAD, while those in plots (c) and (d) have no units.
This graph is similar to Figure 2. The only difference with the latter is that the outliers are not suppressed.

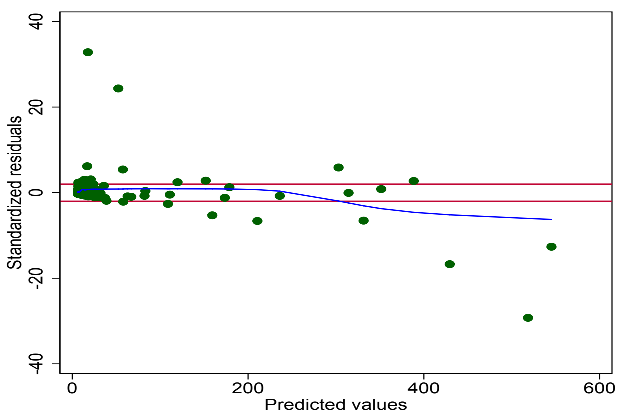
Figure A3: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the debit payment method where the full sample is used



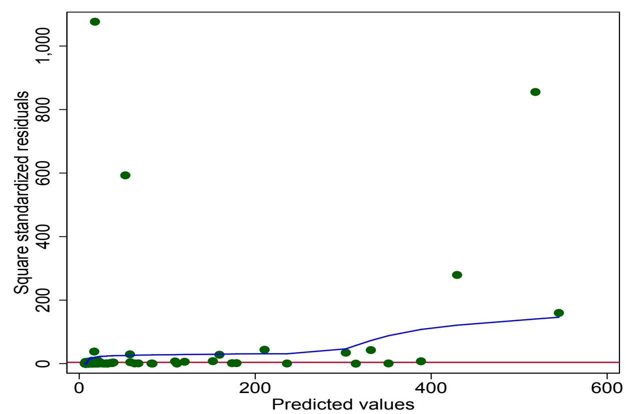

(a) Debit: homoscedastic residuals




(b) Debit: homoscedastic square residuals




(c) Debit: heteroscedastic residuals



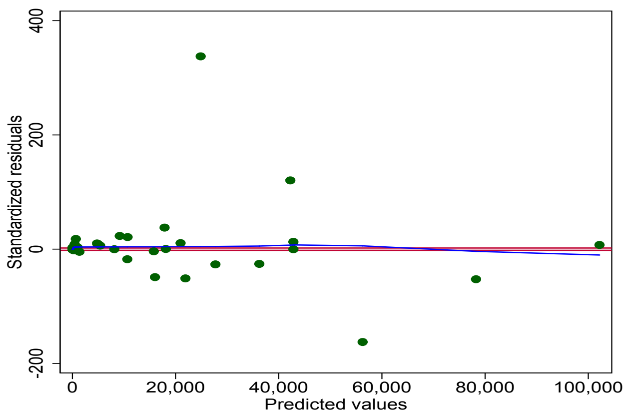

(d) Debit: heteroscedastic square residuals

Note: The standardized residual is given by

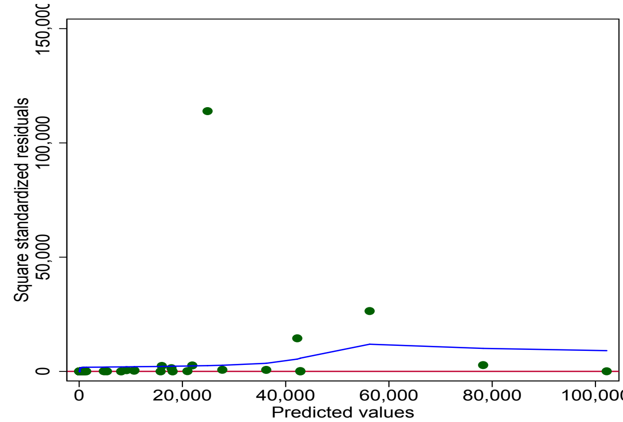
$$\hat{e}_{ij} = \frac{y_{ij} - \hat{f}_{jR} - \hat{a}_{jR} z_{ij} - \hat{b}_{jR} v_{ij} - \hat{c}_{0jR} I_i - \hat{c}_{1jR} I_i z_{ij} - \hat{c}_{1jR} I_i v_{ij}}{\hat{\sigma}_{jR} g_j(z_{ij}, v_{ij}, I_i)},$$

where $\left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}, \hat{c}_{0jR}, \hat{c}_{1jR}, \hat{c}_{2jR}\right)^\top$ is the MM-robust estimate of the regression coefficient $(f_j, a_j, b_j, c_{0j}, c_{1j}, c_{2j})^\top$, for the model in expression (3) given by
$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j} I_i + c_{1j} I_i z_{ij} + c_{2j} I_i v_{ij} + \epsilon_{ij}$,
$\hat{\sigma}_{jR}$ is the robust scale estimate and $g_j(z_{ij}, v_{ij}, I_i) = 14.1097 z_{ij} + 1.0836 v_{ij}$ or $g_j(z_{ij}, v_{ij}, I_i) = 1$ for the heteroscedastic and the homoscedastic model, respectively.
The S-estimator is used for the initial estimates for the MM-regression where the Yohai function is used for both the S and the MM-estimators. The parameters of the Yohai function are $k_0 = 0.7405$ and $k_1 = 0.8679$ for the S and the MM-estimators, respectively. These values correspond to the 25 per cent breakdown value and an efficiency of 0.85 for the MM-estimator.
The predicted values in plots (a) and (b) are in thousands of CAD, while those in plots (c) and (d) have no units.
This graph is similar to Figure 3. The only difference with the latter is that the outliers are not suppressed.

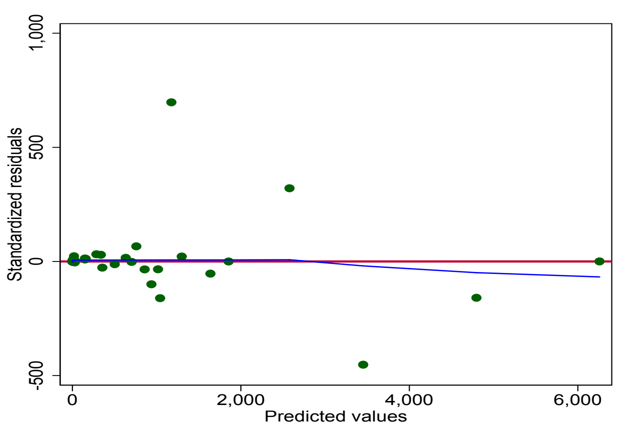
Figure A4: Plots of the standardized residuals and the square standardized residuals versus the predicted values: case of the credit payment method where the full sample is used



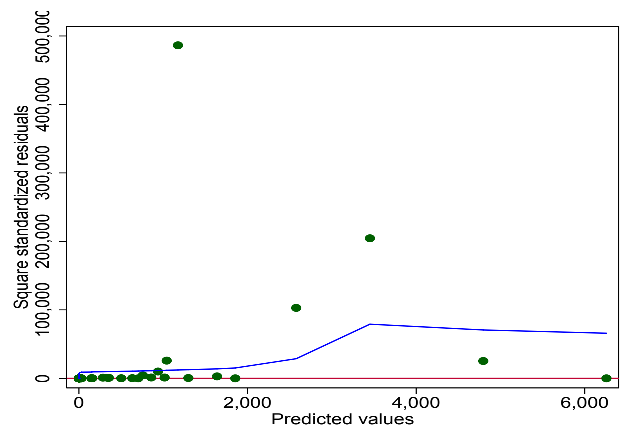

(a) Credit: homoscedastic residuals




(b) Credit: homoscedastic square residuals




(c) Credit: heteroscedastic residuals




(d) Credit: heteroscedastic square residuals

Note: The standardized residual is given by

$$\hat{e}_{ij} = \frac{y_{ij} - \hat{f}_{jR} - \hat{a}_{jR}z_{ij} - \hat{b}_{jR}v_{ij} - \hat{c}_{0jR}I_i - \hat{c}_{1jR}I_i z_{ij} - \hat{c}_{1jR}I_i v_{ij}}{\hat{\sigma}_{jR}g_j(z_{ij}, v_{ij}, I_i)},$$

where $\left(\hat{f}_{jR}, \hat{a}_{jR}, \hat{b}_{jR}, \hat{c}_{0jR}, \hat{c}_{1jR}, \hat{c}_{2jR}\right)^\top$ is the MM-robust estimate of the regression coefficient $(f_j, a_j, b_j, c_{0j}, c_{1j}, c_{2j})^\top$, for the model in expression (3) given by
$y_{ij} = f_j + a_j z_{ij} + b_j v_{ij} + c_{0j}I_i + c_{1j}I_i z_{ij} + c_{2j}I_i v_{ij} + \epsilon_{ij}$,
$\hat{\sigma}_{jR}$ is the robust scale estimate and $g_j(z_{ij}, v_{ij}, I_i) = 1240800000 - 38.0696z_{ij} + 0.1439v_{ij} + 1135300000I_i$ or $g_j(z_{ij}, v_{ij}, I_i) = 1$ for the heteroscedastic and the homoscedastic model, respectively.
The S-estimator is used for the initial estimates for the MM-regression where the Yohai function is used for both the S and the MM-estimators. The parameters of the Yohai function are $k_0 = 0.7405$ and $k_1 = 0.8679$ for the S and the MM-estimators, respectively. These values correspond to the 25 per cent breakdown value and an efficiency of 0.85 for the MM-estimator.
The predicted values in plots (a) and (b) are in thousands of CAD, while those in plots (c) and (d) have no units.
This graph is similar to Figure 4. The only difference with the latter is that the outliers are not suppressed.