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Stressed but not Helpless: Strategic Behavior of Banks Under Adverse Market Conditions

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Abstract

We model bank management actions in severe stress test conditions using a game-theoretical framework. Banks update their balance sheets to strategically maximize risk-adjusted returns to shareholders given three regulatory constraints and feedback effects related to fire sales, interactions of loan supply and demand, and deteriorating funding conditions. The framework allows us to study the role of strategic behaviors in amplifying or mitigating adverse macrofinancial shocks in a banking system and the role of macroprudential policies in the mitigation of systemic risk. In a macro-consistent stress testing application, we show that a trade-off can arise between banking stability (solvency) and macroeconomic stability (lending) and test whether the release of a countercyclical capital buffer can reduce systemic risk.

Topics: Central bank research, Economic models, Financial institutions, Financial stability, Financial system regulation and policies

JEL codes: C63; C72; G21

1. Introduction

Stress test models are an established method to assess the resilience of banks to adverse economic conditions. Their significance in detecting vulnerable banks and assessing how those vulnerable banks impact the financial system flourished after the eruption of the 2007–09 financial crisis (BCBS (2018), Anderson et al. (2018)). As argued in the stress testing literature, one of the critical challenges in developing these models is to account for bank responses to adverse market conditions and the associated systemic risk and feedback effects.[1] The empirical evidence on banks' active reactions to stress urges central banks to develop stress test models in this direction.[2] Notably, remedial actions and feedback effects matter for the magnitude and distribution of stress test losses.[3] State-of-the-art stress test models capture some second-round effects of a stress scenario (e.g., MacDonald & Traclet (2018)). However, these models typically neglect strategic interactions between banks and are based on mechanistic rules. They are either completely or mostly built using the agent-based approach to bank behaviors (e.g., ECB (2017), Dent et al. (2016)). Otherwise, modeling balance sheet structures under stressed market conditions is difficult because of the complexity of bank balance sheets and interactions of multiple regulatory standards.[4]. We propose an optimization model that addresses some of these issues.[5]

The aim of this paper is to model the management actions of banks under stressful market conditions and to study the resulting feedback effects in the banking system, i.e., how decisions of one bank influence the decisions of other banks. We can then assess whether those decisions mitigate or amplify the initial shocks. In the model, banks adjust their balance sheets in the interests of their shareholders. Banks are considered to be risk averse and subject to regulatory leverage, risk-weighted capital, and liquidity constraints. The management actions generate externalities such as fire sales, changes in credit supply, and funding conditions. We argue that the proposed behavioral approach to stress testing is novel and superior in capturing system-wide feedback effects to the approaches that either rely on static balance sheets or apply mechanistic behavioral rules.

First, the proposed optimization of bank utility functions, building upon optimal portfolio choice theory of Markowitz (1952), provides a consistent framework for modeling asset and liability management.

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[1]See Kapinos et al. (2015), BCBS (2015), ECB (2017), Hałaj & Henry (2017) or Anderson et al. (2018).
[2]See Boyd & Nicoló (2005), Jimenez et al. (2012).
[3]For example, Busch et al. (2017) compare the results of a stress test conducted under a static and prescribed dynamic balance sheet assumption for German banks; Brinkhoff et al. (2018) report results of a survey of banks and insurers about their responses to adverse macrofinancial conditions and quantify them in terms of changes in solvency ratios.
[4]See Schuermann (2014) and Cetorelli & Goldberg (2016)
[5]The optimization approach to bank asset and liability management dates back to Kusy & Ziemba (1986). This strand of research uses stochastic programming from operations research (Consigli & Dempster (1998); Klaassen (1998); Robert & Weissensteiner (2011)). Bank balance sheet problems were tackled with approaches of optimal portfolio choice with transaction costs (see Davis & Norman (1990); Hilberink & Rogers (2002)) or the robust optimization tools, e.g., Gülpinar & Pachamanova (2013). However, they are computationally intensive.

Our model delivers predictions fast and efficiently; the framework is flexible to deal with granular balance sheets and management actions. This allows us to model the trade-offs that arise when a bank simultaneously chooses optimal levels of profitability, riskiness, liquidity, and solvency. In comparison, these trade-offs are often ignored in complex agent-based models, where each bank sells assets and repays liabilities to sequentially satisfy regulatory constraints. At the other end of the spectrum, models assuming representative agents with only a handful of asset and liability categories do not reflect the complexity of the financial system and thus cannot be applied to detailed balance sheets to understand risk drivers and detect pockets of risk (e.g., Greenwood et al. (2015)).

Second, we capture the interactions of banks with a broader market, i.e., the impact of bank actions on lending conditions, asset prices, and funding costs. The optimal response of each bank to financial shocks is conditional on the strategies of other banks that may be foreseeable, especially in a concentrated banking system like in Canada, or in case of long stress horizon. The actions of banks generate supply shocks to credit growth. These shocks may be caused by regulatory constraints (see Berger & Udell (1994) for the intuition), adjustments to banks' solvency positions (Adelino & Ferreira 2016), or profitability considerations. Changes in the loan supply may impact the profitability of new loans owned by banks. Profitability of loans depends not only on the risk scenario and macroprudential requirements but also on the competition in the credit market (see Basten (2020) for empirical evidence). A similar story holds for securities. Prices of securities depend on the transacted volumes. This dependence matters for fire-sale losses. Aware of the solvency-liquidity nexus (Pierret 2015), we consider three types of feedback effects associated with funding costs, i.e., those related to the leverage ratio of the borrowing bank, severity of stress test scenarios, and overall riskiness of the banking industry.

Third, we capture preemptive actions of banks to stress conditions prescribed in stress test scenarios. By applying tools from game theory, we attempt to narrow the gap between stylized microeconomic banking models and the complex management practices adopted by banks. Ceteris paribus, changes in the macro environment or market conditions can trigger bank reactions even if regulatory constraints are not binding. Banks can react in advance to a buildup of risks in their balance sheets, to higher uncertainty, and to deteriorating funding conditions. These preemptive reactions may amplify even some moderate shocks. In comparison, models with heuristic behavioral rules can rarely detect the early buildup of systemic risk or require ad hoc assumptions about the preemptive behaviors of banks.

Fourth, we establish the existence and uniqueness of the solution for each given scenario while preserving the richness of information that banks use to decide on their actions. We account for various characteristics of loans and securities, interest rates, credit profiles, maturity structures, regulatory parameters consistent with Basel III guidelines, and scenario-sensitive risk weights. With the minimum theoretical assumptions in place, we formulate the balance sheet problem as a one-stage game between banks with quadratic utility functions. The simple formulation of the game allows for theoretical predictions about the impact of stress scenarios and regulation on banks' behaviors under stress. Moreover, the proposed formulation of the model results in a low computational complexity, which facilitates comparison of the results for different scenarios and different levels of balance sheet granularity. The explicit solution allows us to use the equilibrium conditions to estimate some unobserved parameters of the model, such as sensitivity of asset prices to transacted volumes.

Our approach is similar to some simpler models of optimized dynamic balance sheets of banks, e.g., Hałaj (2013) or Hałaj (2016). In addition, we account for price elasticities in the loan market, explicitly modeling fire sales and including the relationship between solvency and funding costs in the optimal portfolio choice problem. This makes the framework suitable for detailed scenario-based analysis. For instance, macrofinancial scenarios comprehensively capturing key financial risks (e.g., see Anand et al. (2014), ECB (2017)) can be used to study deleveraging pressure in the economy, liquidity dislocations due to strategic responses of banks, and the forced liquidation of assets and its market-wide impact following financial shocks or elevated market uncertainty. From a policy perspective, regulatory requirements and their role in mitigating or amplifying financial shocks can be analyzed.

Notably, accounting for macroeconomic feedback effects in stress tests may require connecting the results of individual banks' stress tests with a semi-structural macroeconomic model (Krznar & Matheson 2017) or DSGE models (ECB 2017). In our paper, we take a more micro-founded approach that allows for more richness in behavioral and regulatory characteristics. The game theory approach is not new to stress testing modeling. For example, Fique (2017) applies a global game model to quantify the rollover risk and changes in funding costs caused by changes in a bank's solvency level. However, our model connects banks through all components of their balance sheets.

The main metrics used in the model to assess the impact of stress on the banking system are bank solvency and liquidity positions after financial shocks and bank management actions. By comparing the outcomes of the model executed with and without the management actions, we can disentangle the contribution to bank losses of strategic behaviors under financial distress. We illustrate the dynamics of the model in two ways. We start with a stylized example of bank balance sheets to distill the effects of strategic behaviors under stress. Next, we apply the model to assess the impact of the domestic systemically important banks' (D-SIBs) management actions under a consistent macrofinancial stress test scenario. It is similar in severity to the one designed by the International Monetary Fund (IMF) in the context of the 2019 Financial System Assessment Program.[6]

Several findings are particularly interesting. First, we show that there is a trade-off between banking stability and macroeconomic stability, should the largest Canadian banks reduce lending to businesses. Our results indicate that macroprudential buffers can mitigate the impact of shocks on lending activities; however, the changes in lending would be marginal. This happens because the capital ratios of a majority of D-SIBs stay above the required limit even after including the prudential buffer. The largest Canadian banks have become well capitalized over the previous decade. As such, erosion of the banks' capital even during the most severe stress still leaves most of them compliant with the regulation. The roots of the credit crunch phenomenon that we observe in the model lie in the strategic behaviors of banks. To preserve profitability, banks react to a severe shock by asset substitution and deleveraging, endogenously reducing the supply of loans to risky borrowers and, consequently, maintaining capital adequacy.

Second, strategic interactions lead to heterogeneous outcomes in terms of liquidity and capital ratios. Banks value the profitability, risk, and liquidity aspects of their portfolios differently. Heterogeneity in the business models of banks matters for how liquid and profitable they are after these banks respond to stress. We also observe the role of policy interaction: binding regulatory ratios change the feasible set of actions for banks and make them reconsider their portfolios. Therefore, the proposed framework may be useful for prudential authorities with access to detailed financial data to test vulnerabilities in the banking system and the efficiency of the complex Basel III regulatory framework.[7]

Third, in the empirical exercise we find that competition in the banking sector in Canada plays a small role in the decisions of banks under stress. We explain this with the high concentration of the market power that D-SIBs maintain in Canada.

The rest of the paper is organized as follows: in section 2 we present a brief overview of the model. Section 3 introduces the banking system and formulates the incentives of each bank. Section 4 defines equilibrium and establishes the existence and uniqueness of it. Section 5 presents numerical examples to illustrate sensitivity of the results to different stress test scenarios and discusses how feedback effects can amplify the initial shocks. Section 6 applies the model to a consistent stress test scenario. Section 7 concludes.

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[6]Although the scenario is similar in spirit to the 2019 Financial System Assessment Program of the IMF for Canada, the stress test results are not directly comparable because of differences in data, assumptions, and modeling choices. For the application of the current Bank of Canada stress test tools, in the context of the same scenario, see Gaa et al. (2019).

[7]BIS, 'Basel III: international regulatory framework for banks.'

2. The model in a nutshell

The model is a one period game played at $t = 0$ with payoffs being delivered at $t = 1$. At the beginning, i.e., at $t = 0$, banks face a macro scenario. The scenario determines random distribution of asset returns and tightness of the funding conditions. During this period, all banks simultaneously decide to adjust their balance sheets.

The objective of each bank is to maximize its risk-adjusted expected return on capital. The decision of each bank impacts the payoffs of other banks through current asset prices and funding costs. In particular, pecuniary externalities arise as securities are not perfectly liquid, demand for loans is inelastic, and pricing is competitive. In addition, funding costs depend on banks' leverage.

Banks are aware of the impact that their actions have on asset prices and funding costs. Thus, banks act strategically, and the model can be interpreted as a static game with complementarities.[8]

We assume two separate setups: when banks are subject to regulations at the time of portfolio adjustments and, as a benchmark, when they are not. If the regulations are in place, banks are required to stay above the prespecified leverage ratio, risk-weighted capital ratio, and liquidity coverage ratio.

At $t = 1$, assets and liabilities generate random return, which transforms into profits and losses impacting capital level of banks. At the end of the game, debt obligations are repaid to the funding providers and non-compliant banks are determined.

The one-period model can be applied in a multi-period setup. In the stress test applications presented in the paper, one-period models are run sequentially. Specifically, the output of the model in a given period, i.e., optimised balance sheet structures, is used to parameterize the banking system in the subsequent period.

3. Setup of the banking system

We first describe the incentives of one bank b assuming strategies of other banks $(1, ..., b-1, b+1, ..., N_b)$ are given. We will later use the derived optimal responses of banks to find the Nash equilibrium.

3.1. **Status quo balance sheets.**

At the beginning of the first period, the bank observes a status quo balance sheet. It is convenient to think of the status quo balance sheet as the balance sheet last observed in the data before a bank changes its exposures or as the balance sheet that grows at the same rate as the whole industry. Some assets of the balance sheet mature and require replacement.

[8] We take this game-theoretical modeling approach because the banking sector in Canada is highly concentrated, so the banks are less likely to be price takers and more likely to relate their strategies to the performance of others. We also assume a complete information setup because we model events that can take place within multiple weeks or even months, and thus it is reasonable to assume that banks are aware of each other's balance sheets.

We separate all assets into controlled and uncontrolled categories. If the maturing assets are uncontrolled, the bank replenishes them to the status quo volume. If the maturing assets are controlled, the bank may adjust them to a different volume. The separation into controlled and uncontrolled asset categories allows us to distinguish between asset classes in a competitive market from asset classes that are residual in the asset and liability management process or grow organically. In this way, some assets in the balance sheets impact profits and regulatory constraints of banks even if we do not model a market for them (e.g., assets related to property, plant, and equipment or any intangible assets). It also allows us to decrease the complexity of the optimization, if necessary, by moving certain categories from controlled to uncontrolled.

The status quo asset portfolio of the bank is composed of

- N_x classes of controlled loans with status quo amounts $a_b^{L,x} = (a_{b,1}^{L,x}, ..., a_{b,N_x}^{L,x})$,
- N classes of uncontrolled loans with status quo amounts $a_b^{L} = (a_{b,1}^{L}, ..., a_{b,N}^{L})$,
- M_x classes of controlled securities with status quo amounts $a_b^{S,x} = (a_{b,1}^{S,x}, ..., a_{b,M_x}^{S,x})$.[9]

Initially, the bank is funded by capital e_b and four types of liabilities l_b:

- retail deposits in amount l_b^r,
- unsecured wholesale funding in amount l_b^{uns},
- secured funding in amount l_b^{coll},
- other liabilities in amount l_b^{oth}.[10]

Bank b strategically updates its balance sheet by choosing a vector of nominal changes $x_b = (x_b^L, x_b^S)$ relative to the status quo. The strategies of other banks $x_{-b} = [x_1, ..., x_{b-1}, x_{b+1}, ..., x_{N_b}]$ are currently taken as given. Following the balance sheet revision, banks' controlled exposures either decrease or increase by x_b, while uncontrolled exposures stay at the status quo level:

$$\begin{aligned} a_b^{S,x} &\longrightarrow a_b^{S,x} + x_b^S, \\ a_b^{L,x} &\longrightarrow a_b^{L,x} + x_b^L, \\ a_b^{L} &\longrightarrow a_b^{L}. \end{aligned}$$

Strategic changes in the asset side of balance sheet and maturing debt obligations may require the bank to either repay some debt or raise new funding. For simplicity, we assume that bank b raises new funding according to the exogenous weights $w_b = (w_b^r, w_b^{uncoll}, w_b^{coll}, w_b^{oth})$. By setting those weights

[9] The securities may include detailed categories of cash equivalents, government securities, bonds, mortgage-backed securities, reverse repo, and others.

[10] More funding classes can be added to the model. We focus on these liabilities as the most simplistic setup necessary to capture differences in possible run-off rates that may occur when funding providers respond to stress. A split between collateralized and uncollateralized funding captures the heterogeneity in the funding cost sensitivities of these categories.

independently of the bank's optimal choice of assets, we may only partly match maturity of assets and liabilities. The bank expects the following changes in liabilities when planning for asset changes:[11]

$$
\begin{aligned}
l_b^r &\longrightarrow l_b^r + w_b^r 1'_{N_x+M_x} x_b \\
l_b^{uncoll} &\longrightarrow l_b^{uncoll} + w_b^{uncoll} 1'_{N_x+M_x} x_b \\
l_b^{coll} &\longrightarrow l_b^{coll} + w_b^{coll} 1'_{N_x+M_x} x_b \\
l_b^{oth} &\longrightarrow l_b^{oth} + w_b^{oth} 1'_{N_x+M_x} x_b
\end{aligned}
$$

Therefore, the heterogeneity of funding weights across banks contributes to the heterogeneity of marginal funding costs and, as a result, leads to different balance sheet choices. As will become apparent, heterogeneity in funding sources w_b also creates heterogeneity in regulatory constraints.

In the model, banks cannot raise new equity or restructure debt. Changes in capital at $t = 2$ can only be due to incurred losses and retained earnings.[12] It is possible that strategic adjustments in the asset portfolios of other banks lead to excessive supply or demand for securities and initiate market price changes. This creates mark-to-market losses and, as a result, requires the bank to raise funding beyond what the bank was planning for. In this case, we assume that the bank closes the funding gap at the risk-free rate r_b^f, which can be thought of as the overnight central bank rate or the minimum expense rate of all funding sources used by bank b.

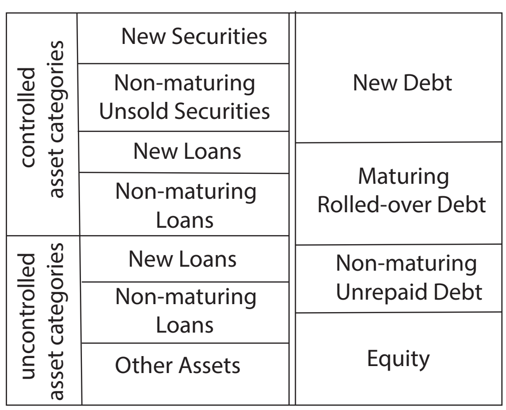

Figure 1. Balance sheet of a bank

Figure 1 presents an illustrative balance sheet structure of a bank after the changes. As is clear from the figure, even zero changes to the balance sheet structure of a bank make the performance of this bank

[11]In the paper, we accept the following mathematical notation: 1_n is a column vector of size n consisting of unit entries, and $1_{n,m}$ is a matrix of size (n, m) consisting of the unit entries.

[12]Static equity is a common assumption in stress test exercises, because banks are less likely to raise new equity or change dividend policy during stressful events (e.g., ECB (2017)).

conditional on the strategies of others. This happens because the bank is required to update maturing assets and liabilities at the new prices. Moreover, strategies of other banks impact profitability and valuation of some balance sheet categories. The next two sections focus in detail on the externalities that take place.

3.2. **Profitability of securities and mark-to-market accounting.** In period $t = 0$, bank b replenishes some fraction of matured securities a_b^S and adjusts the status quo volumes by amount x_b^S. If the balance sheets of all banks are identical to the ones in the status quo, replenishing of previously held securities does not create market impact. Otherwise, each bank is subject to pecuniary cost (aka fire-sales cost) applied to mark-to-market security holdings. Without banks taking action, we assume that market prices are the same as in the status quo. If banks altogether adjust their exposure by $x_b^S + x_{-b}^S 1_{N_b-1}$, the current market prices change by percentage

$$\alpha^S(x_b^S + x_{-b}^S 1_{N_b-1}),$$

where α^S is the (M_x, M_x) diagonal matrix of price-response sensitivities with elements of α^S being positive, meaning more sales lead to more negative price changes, and more purchases lead to more positive price changes.

A share of the securities holdings that is mark-to-market adjusts in value accordingly. If the initial holding were $\mu_b^S a_b^S$, the post-sales holdings became $(1 + \alpha^S(x_b^S + x_{-b}^S 1_{N_b-1}))\mu_b^S a_b^S$. [13] Moreover, current transactions of the bank took place at the new prices. As such, the bank is required to transact securities in the amount of $x_b^S - \alpha^S(x_b + x_{-b})\mu_b^S a_b^S$ to reach total exposure of $x_b^S + a_b^S$. For instance, if the market price for a given security i has increased relative to the status quo, the bank is required to purchase securities in the amount of less than x_b^i to end up with the asset exposure of $a_b^i + x_b^i$. This corresponds to the capital losses of

$$loss_b^S = -\alpha_b^S(x_b + x_{-b})\mu^S a_b^S.$$

Short-term changes in market prices at $t = 0$ also impact returns expected to be paid on the new and remaining securities at $t = 1$. This feedback effect is incorporated in the model to capture two typical market dynamics. First, short-term impact on asset prices may reflect pure liquidity consideration; thus the price is expected to rebounce in the future period. Second, excessive fire sales or asset purchases may lead to changes in macroeconomic expectations of other market participants and create momentum contributing to longer-term price recovery or decline. Third, changes in returns may naturally arise for fixed income securities due to reverse relationships of price and yield. In this paper, we average the two effects by using the first-order approximation for the relationship between current market prices

[13] We keep the fraction of mark-to-market securities of bank b in the diagonal matrix form μ_b^S for the convenience of notation.

and expected rates of returns. In particular, without externalities in place, the bank expects to receive per dollar return r_b^S on holdings a_b^S. The actual returns $r_b^{S,x}$ are also subject to random shock ϵ_b^S with variance $\mathbb{V}(\epsilon_b^S) > 0$. We do not impose any additional distributional assumptions on ϵ_b^S and allow for non-zero correlations between the returns on securities. With externalities being applied, the vector of asset returns of bank b is adjusted to the market momentum in proportion to the price adjustment:

$$r_b^{S,x} = r_b^S - \beta^S \alpha^S (x_b^S + x_{-b}^S 1_{N_b-1}) + \epsilon_b^S,$$

where β^S is a matrix of recovery rates, complementary to the matrix of price persistence $1 - \beta^S$.

We can now define the income generated by the portfolio of securities as income from the updated securities minus the losses from sales and purchases:

$$I_b^S = r_b^{S,x'} (x_b^S + a_b^{S,x}) - loss_b^S.$$

Thus, income from holding securities takes the following quadratic form:

$$I_b^S = (r_b^S + \epsilon_b^S)(x_b^S + a_b^{S,x}) - (x_b^S + x_{-b}^S 1_{N_b-1})' \alpha^S (\beta^S x_b^S + (\beta^S - \mu_b^S) a_b^{S,x}).$$

How to interpret the parameter β^S? Suppose an illustrative bank holds $1,000$ dollars of bonds. For simplicity, assume 10% of this portfolio is not mark-to-market and half of it matures and needs to be updated to maintain the same maturity profile. Next, imagine that at $t = 0$ the bank decides to sell 500 dollars from its marketable part due to strategic response to stress. This would result in the final exposure of 500 on the balance sheet without considering mark-to-market adjustment. Assume the depressed market price falls to 96 cents per dollar after the sale. So mark-to-market securities are reduced immediately by the amount

$$loss_b^S = 0.04 \times 900 = 36.$$

By the time the bank extracts profits from the portfolio, the market price will adjust further. If the market will not update its macroeconomic and market-based expectation, the price will eventually return to pre-stress over time, so the corresponding income rate will roughly adjust by

$$\beta^S \times \alpha_b^S (x_b^S + x_{-b}^S 1_{N_b-1}) = (.90 + .10 \times 0.5) \times 0.04 = 0.038.$$

This means that the fire-sale loss vanishes and the bank makes profit by updating bond portfolio at smaller prices during fire sales. In this example, $\beta^S = 0.95$.

In a stress scenario, however, it is possible that the fire sales will lead to longer-term pricing impacts. Alternatively, it could be assumed that market price recovers 90%, and the corresponding income rate will adjust by $\beta^S = (.80 \times .90 + .10 \times 0.5)$. This example illustrates why a stress testing exercise should be time sensitive independent of the model being used. While fire sales may temporarily decrease the bank's capital, they may not persist through the whole scenario. Our model allows banks to monitor their short-term regulatory ratios while achieving longer-term objectives.

3.3. **Profitability of loans and price competition.** Banks renew uncontrolled loan exposures to the same volumes as in the status quo. Fractions $\mu_b^{L,x}$ of controlled loans $a_b^{L,x}$ and fractions μ_b^L of uncontrolled loans a_b^L mature before the management decisions take place. Similar to the case of securities, banks update controlled exposures by taking a combination of the four actions: issuing new loans, changing the composition of loans, redirecting funds between loans and securities, and shedding the unused funds to repay liabilities.

The realization of loan returns at $t = 1$ is random. Expected returns and volatilities are likely to be different for new and old loans due to differences in maturity and macro conditions of when they are issued. The bank expects return $r_b^{L,c,new}$ on controlled new loans, $r_b^{L,u,new}$ on uncontrolled new loans, $r_b^{L,c,old}$ on controlled non-maturing loans, and $r_b^{L,u,old}$ on uncontrolled non-maturing loans. The expected rates of return account for expected contractual rates and fees minus the expected credit losses.[14] We aggregate all volatility of returns into the corresponding error terms $\epsilon_b^{L,c,new}$, $\epsilon_b^{L,u,new}$, $\epsilon_b^{L,c,old}$, and $\epsilon_b^{L,u,old}$. These errors capture variability in contractual rates, fees, and default losses. We do not impose any distributional assumptions on the loan performance, apart from their variance being strictly positive, and allow for non-zero correlations between the errors.

In addition, banks may impose externalities on each other through loan origination. If, altogether, banks acquire additional market shares $x_b^L + x_{-b}^L 1_{N_b-1}$, they must offer a discount on the loan rate in the amount of

$$\alpha^L(x_b^L + x_{-b}^L 1_{N_b-1}),$$

where α^L is (N_x, N_x) positive diagonal matrix of price-response sensitivities for loans. This assumption is consistent with the logic that banks issue most profitable loans first, so higher loan origination increases competition between banks for the most valued customers. Besides, the search for new customers is costly.[15] Thus, the combined expected return for new controlled loans is defined as

$$r_b^{L,x} = r_b^{L,c,new} - \alpha^L(x_b^L + x_{-b}^L 1_{N_b-1}).$$

[14]In reality, banks decide on the large categories of their balance sheet before the individual loans are issued, which may create additional uncertainty about the contractual rates and fees at the time when the decisions are made. The model is flexible to accommodate this uncertainty.

[15]For feasible empirical applications of the model, we assume that banks are equally powerful in gaining new market share, so the same sensitivity is applied independently of which bank is expanding.

Notably, there is no impact on the credit growth for uncontrolled and non-maturing loans, because these loan exposures are kept unchanged from the status quo.

In summary, bank b receives the following income from the loan portfolio:

$$
\begin{aligned}
I_b^L &= (r_b^{L,c,old} + \epsilon_b^{L,c,old})'(1-\mu_b^{L,x})a_b^{L,x} \\
&+ (r_b^{L,c,new} - (x_b^L + x_{-b}^L 1_{N_b-1})\alpha^L + \epsilon_b^{L,c,new})'(\mu_b^{L,x} a_b^{L,x} + x_b^L) \\
&+ (r_b^{L,u,old} + \epsilon_b^{L,u,old})'(1-\mu_b^L)a_b^L \\
&+ (r_b^{L,u,new} + \epsilon_b^{L,u,new})'\mu_b^L a_b^L.
\end{aligned}
$$

3.4. **Regulatory ratios.** Changes in banks' balance sheets lead to the corresponding changes in leverage, liquidity coverage, and capital ratios. In the stress test models used in the literature, banks either do not need to comply with certain regulatory constraints or have less flexibility in satisfying these constraints. Due to these assumptions, simulated behaviors of banks often lead to a violation of regulatory requirements for some banks, in which it is said that these banks fail the stress test. To stay consistent with the previous methodologies, we first focus on unconstrained strategies of banks. While dealing with the unconstrained model, we still measure regulatory ratios at the beginning and at the end of the game to see if the banks' actions lead to violations of regulatory constraints. Further in the paper, we adopt a more realistic assumption by requiring banks to comply with the requirements when deciding on the management actions. In both of these setups, we will use the formulas for regulatory ratios as defined below.

We model regulatory requirements consistently with the Basel III framework. The measurements are taken when the banks are experiencing their largest capital and liquidity reductions, that is, before asset prices recover following the fire sales.

(1) *Risk-weighted capital ratio*

A bank initially faces the status quo capital ratio of τ_b^{CAP} and corresponding risk-weighted assets RWA_b. Let ω_b be the risk weights of controlled assets $a_b^x = [a_b^{L,x}, a_b^{S,x}]$ and ω_b^a be the combined risk weights for controlled and uncontrolled assets a_b. Both status quo capital ratio and risk weights are bank specific and sensitive to the scenario. Risk weights are not updated with the balance sheet.[16]

[16]This assumption means that banks keep the same risk profile within each category. If constant risk weights is not a realistic assumption in a given application, we recommend using more narrow asset categories.

Once the bank decides to proceed with the asset changes, its risk-weighted capital ratio changes accordingly:

$$\tau_b^{\text{CAP}} = \frac{e_b}{\text{RWA}_b} \longrightarrow \frac{e_b - loss_b^S}{\text{RWA}_b + \omega_b' x_b}. \tag{1}$$

Thus, the bank falls within the limits of the capital requirement $\bar{\tau}^{\text{CAP}}$ if and only if the behavioral adjustment in the risk-weighted assets is within reasonable bounds, meaning

$$\Phi_b^{\text{CAP}} = \frac{1}{e_b}\omega_b' x_b + \frac{1}{\tau^{CAP} e_b} loss_b^S - \left(\frac{1}{\bar{\tau}^{\text{CAP}}} - \frac{1}{\tau_b^{\text{CAP}}}\right) \leq 0. \tag{2}$$

Overall, gap Φ_b^{CAP} indicates bank's solvency risk relative to the existing amount of capital.[17]

(2) *Leverage ratio*

The bank initially faces the status quo leverage ratio of τ_b^{LEV}. In the process of balance sheet changes, the bank adjusts its leverage ratio accordingly:

$$\tau_b^{\text{LEV}} = \frac{e_b}{1'_{N_x+N+M_x} a_b} \longrightarrow \frac{e_b - loss_b^S}{1'_{N_x+N+M_x} a_b + 1'_{N_x+M_x} x_b}.$$

Thus, the bank falls within the limits of the leverage requirement $\bar{\tau}^{\text{LEV}}$ if and only if the behavioral adjustment in the balance sheet size is within reasonable bounds, meaning

$$\Phi_b^{\text{LEV}} = \frac{1}{e_b} 1'_{N_x+M_x} x_b + \frac{1}{\tau^{LEV} e_b} loss_b^S - \left(\frac{1}{\bar{\tau}^{\text{LEV}}} - \frac{1}{\tau_b^{\text{LEV}}}\right) \leq 0. \tag{3}$$

Overall, gap Φ_b^{LEV} indicates the bank's indebtedness status.

(3) *Liquidity ratio*

The bank initially faces the status quo liquidity ratio of τ_b^{LIQ}. The liquidity ratio is designed to measure the ability of the bank to withstand short-term withdrawals with available cash resources and asset sales. Let ϕ^a be a vector of regulatory weights measuring the potential of assets to generate cash within one month. They define a pool of high-quality liquid assets, HQLA_b. Also, within a one-month period, the bank expects that deposits and other liabilities will be withdrawn or will mature according to run-off rates ϕ^l. They define a volume of liquidity outflows LO_b. Therefore, balance sheet changes lead to the following transformation of the liquidity ratio:

$$\tau_b^{\text{LIQ}} = \frac{\text{HQLA}_b}{\text{LO}_b} \longrightarrow \frac{\text{HQLA}_b + \phi^{a'} x_b}{\text{LO}_b + \phi^{l'} \text{diag}(w_b) 1_{4,N_x+M_x} x_b + loss_b^S}.$$

[17] Alternatively, banks may target a specific level of capital. We can partly address such an assumption by setting the capital constraint to the targeted level.

The bank falls within the limits of the liquidity requirement $\bar{\tau}^{\text{LIQ}}$ if and only if the behavioral adjustment in the liquidity balances is within reasonable bounds, meaning

$$\Phi_b^{\text{LIQ}} = \frac{1}{\phi^{a\prime} a_b}\left(\phi^{l\prime}\operatorname{diag}(w_b)1_{4,N_x+M_x} - \frac{1}{\bar{\tau}^{\text{LIQ}}}\phi^{a\prime}\right)x_b + \frac{1}{\phi^{a\prime} a_b} loss_b^S - \left(\frac{1}{\bar{\tau}^{\text{LIQ}}} - \frac{1}{\tau_b^{\text{LIQ}}}\right). \leq 0 \tag{4}$$

Overall, gap Φ_b^{LIQ} indicates the level of the bank's liquidity risk.

3.5. **Funding cost.** Investors that provide funding to banks are sensitive to the counterparty and sector-specific credit risk. We assume that regulatory leverage gap Φ_b^{LEV} serves as a good proxy for the solvency position of bank b.[18] Furthermore, if the industry is on average unbalanced with respect to the regulatory ratios, meaning $\frac{1}{N_b-1}\Phi_{-b}^{LEV}1_{N_b-1}$ is high, higher margins will be imposed on all borrowing banks. We model changes in marginal funding costs of uncollateralized funding as a proportional change to the average between a bank's own leverage gap and sector-specific gap with sensitivity c_f. The marginal funding cost of uncollateralized funding is defined in a similar way with sensitivity $c_f + c_f^{coll}$.

In conclusion, changes from status quo rate c_b^0 to strategy-dependent rate are the following:

$$c_b = c_b^0 + C_f\left(x_b\frac{1}{e_b} + \frac{1}{N_b-1}x_{-b}e_{-b}^{inv}\right), \tag{5}$$

where sensitivity of funding cost to the bank's own strategy x_b is captured by matrix

$$C_f = [c_f \; c_f \; (c_f + c_f^{coll}) \; c_f]' 1'_{N_x+M_x}. \tag{6}$$

In the equation above, e_{-b}^{inv} is the vector of inverse equity exposures of all banks except b.

Assuming that all banks are funded by investors with the same behavioral responses, we assume that matrix C_f contains equal solvency-based sensitivity c_f for all funding types and additional collateral-based sensitivity. This homogeneity assumption can be relaxed to some degree that still allows for the identification using historical data. One can define a richer set of funding sensitivities if, for example, banks have relationships of different strength with their liquidity providers or different funding providers evaluate solvency risk differently.

We are now ready to find the absolute changes in the bank's funding cost. At the beginning of the game, some share of funding matures and requires replacement. Depending on the choice of the asset structure, the bank may roll over maturing funding $\nu_b l_b$ in full or partial amounts, obtain new funding, or repay funding before maturity. In this process, the bank alternates sources of funding using exogenous

[18] In periods of financial stress, funding providers may request that borrowing banks update them on details of balance sheet portfolios to evaluate not only banks' overall indebtedness but also risk-weighted assets and liquidity buffers. We do not account for this option to keep the derivations simple. However, the model can be easily extended to account for capital gap Φ_b^{CAP} and liquidity gap Φ_b^{LIQ} in the decisions of funding providers.

funding allocation weights w_b.[19] Therefore, the management actions of the bank generate new volumes for the existing funding categories

$$l_b + w_b 1'_{N_x+M_x} x_b \tag{7}$$

and yield overall funding costs

$$l'_b(1-\nu_b)c^0_b + (l'_b\nu_b + x'_b 1_{N_x+M_x,4} w_b)c_b + r^f_b loss^S_b. \tag{8}$$

Similar to the interest income, we express the total interest expense of the bank in the matrix form

$$FundCost_b = l'{}_b c^0_b + \frac{1}{N_b-1} l'_b \nu_b C_f x_{-b} e^{inv}_{-b} - r^f_b a^{S,x'}_b \mu^S_b \alpha^S x^S_{-b} 1_{N_b-1}$$
$$+ \left(c^{0'}_b w_b 1_{4,N_x+M_x} + \frac{1}{e_b} l'_b \nu_b C_f + \frac{1}{N_b-1} e^{inv'}_b x_{-b}{}' C_f{}' w_b 1_{4,N_x+M_x} - r^f_b [0_{N_x} \ (a^{S,x}_b)' \mu^S_b \alpha^S] \right) x_b$$
$$+ \frac{1}{e_b}(c_f + c^{coll}_f w^{coll}_b) x'_b 1_{N_x+M_x,N_x+M_x} x_b.$$

Non-interest income. In addition, banks receive non-interest income defined as the component of the net profit not related to the interest income, credit risk, or fees and commissions. For simplicity, I^{nint}_b is assumed to be independent of other asset returns and proportional to the bank's total assets:

$$I^{nint}_b = z^x_b (1'_{N_x+M_x} x_b + 1'_{N_x+M_x+N} a_b),$$

with the non-interest margin being a random variable centered around z_b,

$$z^x_b = z_b + \epsilon^z_b,$$

with positive volatility $\mathbb{V}(\epsilon^z_b) > 0$.

Incentives of banks. Each bank faces trade-offs between profitability, riskiness, solvency, and liquidity. We schematically illustrate the feedback effects in Figure 2. Utility function is a natural way for the bank to resolve such trade-offs. When deciding on the adjustments, the bank maximizes its risk-adjusted return on equity:

$$u_b(x_b, x_{-b}) = \mathbb{E}\left(\frac{e_b + NetInc_b(x_b, x_{-b})}{e_b}\right) - \gamma \mathbb{V}\left(\frac{e_b + NetInc_b(x_b, x_{-b})}{e_b}\right), \tag{9}$$

where $\mathbb{E}$ and $\mathbb{V}$ are expectation and variance operators and the net income is defined as the composition of revenues from holding loans and securities and non-interest income, minus the funding costs:

$$NetInc_b = I^L_b + I^S_b + I^{nint}_b - FundCost_b. \tag{10}$$

[19]We use a notation where ν_b and w_b denote diagonalized matrices of weights.

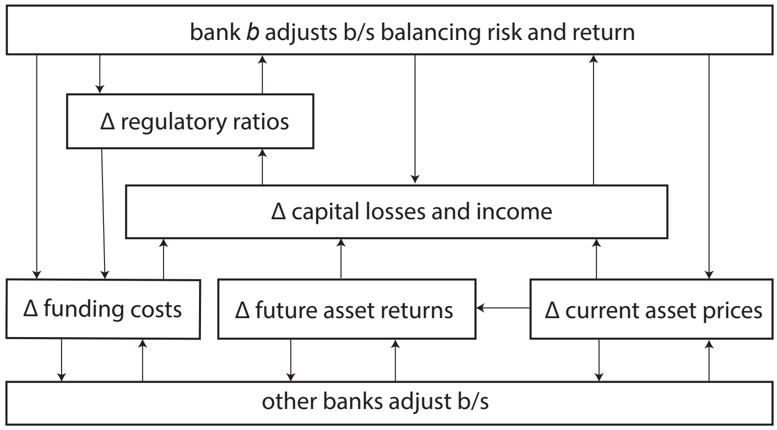

Figure 2. Endogenous dependence between the bank's strategies and outcomes

We assume that the utility function of banks jointly measures the well-being of shareholders, debt holders, and managers. We do not model any conflicts within the bank holding company and take the dividend policy during the stressful times as exogenous.

4. Equilibrium

4.1. **Equilibrium notion for unconstrained banks.**

Each bank b strategically chooses balance sheet changes x_b to maximize utility (9). In the game with unconstrained banks, the feasible set of strategies is defined as those x_b that satisfy the inventory condition:

(0) *Inventory constraints*

Maturing loans and securities can only be sold up to the outstanding volumes.[20]

$$x_b \quad \geq \quad - \begin{bmatrix} \mu_b^L a_b^{L,x} \\ a_b^{S,x} \end{bmatrix} \tag{11}$$

A strategic choice of each bank is impacted by the strategies of others, which makes the model a game. The notion of equilibrium of the game is standard.

Definition 4.1. The feasible strategies of banks constitute Nash equilibrium $(x_1, ..., x_{N_b})$ if a deviation by any bank $b \in \{1, ..., N_b\}$ from x_b to another feasible strategy $\tilde{x}_b$ makes this bank weakly worse off:

$$u_b(\tilde{x}_b, x_{-b}) \leq u_b(x_b, x_{-b}).$$

[20] We assume that banks do not liquidate loans prematurely. This assumption can be relaxed to allow for more flexibility in management actions. We also assume that a bank is interested in liquidating only mark-to-market securities.

Best-response strategy for regulatory unconstrained bank. Before finding the equilibrium of the game, we first discuss the optimal strategy of each bank given the strategies of others.

The equilibrium is described by a quadratic N_b-player game with strategies defined on a convex set. We restrict the bank's actions to a compact set of strategies $x_b \in [\underline{x}, \bar{x}]$. Given that this interval is sufficiently wide, this assumption is weak in our framework because banks cannot liquidate more than they hold on balance sheets and, given a fixed amount of capital, they can invest only up to the binding leverage constraint. In the unconstrained case, this assumption can also be made non-restrictive by choosing a wider interval of values.

Then the best response strategy of the bank can be found by solving a quadratic optimization problem specified below. It is convenient to present the results of the theorem with the use of the simplifying notation for the unexpected income component generated by the status quo balance sheet:

$$\epsilon_b^a = (\epsilon_b^{L,c,o}(1_{N_x} - \mu_b^x) + \epsilon_b^{L,c,n}\mu_b^x)a_b^{L,x} + (\epsilon_b^{L,u,o}(1_N - \mu_b) + \epsilon_b^{L,u,n}\mu_b)a_b^L + \epsilon_b^S a_b^S.$$

Theorem 4.2. *Best response $x_b^* \in [\underline{x}, \bar{x}]$ of regulatory unconstrained bank b given strategies of other banks $x_{-b} = [x_1, ..., x_{b-1}, x_{b+1}, ..., x_{N_b}]'$ can be found as a solution of the quadratic optimization problem:*

$$x_b^*(x_{-b}) = \underset{x_b \in [\underline{x}, \bar{x}]}{\operatorname{argmin}} \frac{1}{2} x_b' Q_b x_b + x_b' \left(m_b^0 + \sum_{k \neq b} m_{bk}' x_k \right),$$

subject to the inventory constraint (11), where the linear impact of the balance sheet adjustment is defined as

$$\begin{aligned} m_b^0 &= -\begin{bmatrix} r_b^{L,c,new} - \alpha^L \mu_b^{L,x} a_b^{L,x} \\ r_b^S - \alpha^S \left(\beta^S - \mu_b^S(1 - r_b^f)\right) a_b^{S,x} \end{bmatrix} \\ &+ \left[c_b^{0\prime} w_b + \frac{1}{e_b}(c_f w_b' \nu + c_f^{coll} w_b^{coll} \nu_b^{coll}) 1'_{N_x+N+M_x} a_b - z_b \right] 1_{N_x+M_x} \\ &+ \frac{2\gamma}{e_b} \begin{bmatrix} \mathbb{C}ov(\epsilon_b^{L,c,n}, \epsilon_b^a) \\ \mathbb{C}ov(\epsilon_b^S, \epsilon_b^a) \end{bmatrix} \end{aligned}$$

(12)

and the side effect of bank k's strategy is captured by

$$\begin{aligned} m_{bk}' &= \begin{bmatrix} \alpha^L & 0 \\ 0 & \alpha^S \beta^S \end{bmatrix} \\ &+ \frac{1}{N_b - 1} \frac{1}{e_k} (c_f + c_f^{coll} w_k^{coll}) 1_{N_x+M_x, N_x+M_x}. \end{aligned}$$

(13)

The second-order impact of bank b's balance sheet adjustments is defined by the matrix

$$
\begin{aligned}
Q_b &= 2\begin{bmatrix}\alpha^L & 0\\ 0 & \alpha^S\beta^S\end{bmatrix} + \frac{2}{e_b}(c_f + c_f^{coll} w_b^{coll})1_{N_x+M_x,N_x+M_x}\\
&+ \frac{2\gamma}{e_b}\begin{bmatrix}\mathbb{V}\left(\epsilon_b^{L,c,new'}\right) & \mathbb{C}ov\left(\epsilon_b^{L,c,new'},\epsilon_b^{S'}\right)\\ \mathbb{C}ov\left(\epsilon_b^{L,c,new'},\epsilon_b^{S'}\right) & \mathbb{V}\left(\epsilon_b^{S}\right)\end{bmatrix}\\
&+ \frac{2\gamma}{e_b}\mathbb{V}(\epsilon_b^z)1_{N_x+M_x,N_x+M_x}. \qquad (14)
\end{aligned}
$$

Proof. The theorem is given without proof, as it follows immediately from the setup of the optimization problem of one bank. □

The solution of the single-bank optimization problem is well defined, meaning it exists and is unique.

Corollary 4.3. *The bank's best-response asset structure x_b^* in the unconstrained banking game is defined by equation*

$$x_b^* = -Q_b^{-1}m_b^0 - \sum_{k\neq b} Q_b^{-1}m'_{bk}x_k. \qquad (15)$$

Proof. The set of feasible strategies is convex, non-empty, and compact. Thus, strict convexity of the optimization function will guarantee existence and uniqueness. It is sufficient to show that matrix Q_b is positive definite for the optimization function to be strictly convex. This follows from the assumptions on externalities being non-negative: $\alpha^L \geq 0, \alpha^S\beta^S \geq 0, c_f + c_f^{coll}\omega_b^{coll} \geq 0$ for all b. In addition, variance-covariance matrices of $\epsilon_b^{L,c,new}$ and ϵ_b^S are positive semi-definite, and error terms ϵ_b^z have a positive variance. All these guarantee that matrix Q_b is positive definite as a sum of positive definite and positive semi-definite matrices. As such, the first-order conditions are necessary and sufficient, and one can define the inverse matrix of Q_b. □

If the inventory condition is binding for some asset categories, constrained solution x_b^* can be found by solving a Kuhn-Tucker optimization problem. We prove that inventory constraints would change the best-response balance sheet as if the bank experiences additional marginal cost λ_b^{IC}:

Corollary 4.4. *Bank's best-response asset structure x_b^* in the banking game with inventory constraints (11) is defined by equation*

$$x_b^* = -Q_b^{-1}(m_b^0 - \lambda_b^{IC}) - \sum_{k\neq b} Q_b^{-1}m'_{bk}x_k \qquad (16)$$

such that the multiplier λ_b^{IC} *is defined by*

$$\lambda_b^{IC} = \min\left(Q_b(-[\mu_b^L a_b^{L,x} \;\; a_b^{S,x}]' + m_b^0) + \sum_{k\neq b} m'_{bk}x_k, 0_{N_x+M_x}\right). \tag{17}$$

Proof. This result follows directly from the first-order conditions combined with the complementary slackness conditions, which require that either $\lambda_{b,i}^{IC} = 0$ for loan i ($\lambda_{b,j}^{IC} = 0$ for security j) or $\lambda_{b,i}^{IC}$ to be positive and defined from the boundary condition: $x_{b,i}^*(\lambda_{b,i}^{IC}) = \mu_{b,i}^L a_{b,i}^{L,x}$ (the condition for security j is $x_{b,j}^*(\lambda_{b,j}^{IC}) = a_{b,j}^{S,x}$).

The first-order conditions are sufficient because matrix Q_b is positive definite, constraints are defined by linear (convex) functions, and the set of admissible strategies is non-empty. □

In reality, the inventory constraint is most likely to bind if a bank has to keep non-maturing loans on the books for the category that is not attractive due to low profitability or high risk. Then, according to Corollary 4.4, the existence of such loans on the books will impact the overall balance sheet choices of the bank, as resulting profitability and risk of the whole balance sheet may not be aligned with the bank's risk sensitivity γ.

4.2. **Equilibrium strategies for regulatory unconstrained banks.** We now formulate the first equilibrium result. For this purpose, we define matrix $\bar{Q}$, which captures second-round impacts and externalities of strategic responses:

$$\bar{Q} = \begin{bmatrix} Q_1 & m'_{12} & \vdots & m'_{1,N^b} \\ m'_{21} & Q_2 & \vdots & m'_{2,N^b} \\ \vdots & \vdots & \vdots & \vdots \\ m'_{N^b,1} & m'_{N^b,2} & \vdots & Q_{N^b} \end{bmatrix} \tag{18}$$

Theorem 4.5. *If matrix* $\bar{Q}$ *is non-singular, the equilibrium of the unconstrained banking game exists and is unique with the optimal strategies being*

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N^b} \end{bmatrix} = -\bar{Q}^{-1} \begin{bmatrix} m_1^0 \\ m_2^0 \\ \vdots \\ m_{N_b}^0 \end{bmatrix} \tag{19}$$

Proof. This follows directly from the best-response strategies of banks in the unconstrained game. □

A similar result takes place for banks bounded by inventory constraints.

Theorem 4.6. *If $\bar{Q}$ is non-singular, the equilibrium of the banking game with inventory constraints (11) exists and is unique. The corresponding optimal strategies are*

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N^b} \end{bmatrix} = -\bar{Q}^{-1} \begin{bmatrix} m_1^0 - \lambda_1^{IC} \\ m_2^0 - \lambda_2^{IC} \\ \vdots \\ m_{N_b}^0 - \lambda_{N_b}^{IC} \end{bmatrix}, \tag{20}$$

where matrix $\bar{Q}$ is defined in equation (18) and multipliers λ^{IC} in equation (17).

Proof. This follows directly from the best response strategies of banks subject to the inventory constraints. □

For illustrative purposes, we provide sufficient condition on the balance sheets of banks that guarantee the existence and uniqueness of the equilibrium. This condition requires the largest bank in the model not to be very different in equity size from the rest of the banks. The six largest banks of Canada offer a perfect illustration of the condition below.

Proposition 4.7. *Matrix $\bar{Q}$ is positive definite (and non-singular) if the inverse value of the largest equity in the system, defined as equity of bank $b^* = argmax(e_1, ..., e_{N_b})$, is at least $\frac{c_f}{3(c_f + c_f^{coll})}$ times larger than the average inverse equity of the rest of the banks:*

$$\frac{1}{e_{b^*}} > \frac{c_f}{3(c_f + c_f^{coll})} \sum_{k \neq b^*} \frac{1}{(N_b - 1)} \frac{1}{e_k} > 0.$$

Proof. The proof is in Appendix C. □

4.3. **Optimal portfolio.** In some cases, for instance for bench-marking purposes, it may be convenient to pick the equilibrium balance sheets as status quo balance sheets.

Corollary 4.8. *Banks keep their balance sheets unchanged if and only if*

$$m_1^0 = ... = m_{N_b}^0 = 0_{N_x + M_x}. \tag{21}$$

Thus, the composition of the status quo balance sheets not requiring adjustments is such that for each bank b, loan categories i, j, and security category k, the equilibrium exposures are related as follows:

$$\frac{a_{b,i}^{L,x}}{a_{b,j}^{L,x}} = \frac{r_{b,i}^{L,c,new} - \frac{2\gamma}{e_b} \mathbb{C}ov(\epsilon_{b,i}^{L,c,n}, \epsilon_b^a)}{r_{b,j}^{L,c,new} - \frac{2\gamma}{e_b} \mathbb{C}ov(\epsilon_{b,j}^{L,c,n}, \epsilon_b^a)} \frac{\alpha_k^L \mu_{b,j}^{L,x}}{\alpha_i^L \mu_{b,i}^{L,x}}$$

$$\frac{a_{b,i}^{L,x}}{a_{b,k}^{S}} = \frac{r_{b,i}^{L,c,new} - \frac{2\gamma}{e_b}\mathbb{C}ov(\epsilon_{b,i}^{L,c,n}, \epsilon_b^a)}{r_{b,k}^S - \frac{2\gamma}{e_b}\mathbb{C}ov(\epsilon_{b,k}^S, \epsilon_b^a)} \frac{\alpha_k^S(\beta^S - \mu_{b,k}^S(1+r_b^f))}{\alpha_i^L \mu_{b,i}^{L,x}}.$$

The intuition behind the equations above is that banks limit their exposure to assets that are highly correlated with the existing portfolio and traded on very competitive markets unless the assets generate a sufficient rate of return.

4.4. **Equilibrium notion for regulatory constrained banks.**

Best response strategies of banks. When banks are conditioned by regulatory constraints, their best response strategies can be found as a solution to a Kuhn-Tacker optimization problem subject to inequality constraints (1), (2), (3) and complementary slackness conditions. In this representation, the Lagrangian multipliers λ_b^{LEV}, λ_b^{CAP}, and λ_b^{LIQ} express the shadow costs that a bank's shareholders face when the bank binds the regulatory requirements.

Theorem 4.9. *If matrix $\bar{Q}$ is positive definite, the equilibrium of the banking game with regulatory constraints (1)-(3) exists and is unique.*

Proof. The set of feasible strategies is convex, non-empty, and compact. According to Rosen (1965), it is sufficient to show that matrix $G(x)+G'(x)$ is negative definite for the equilibrium to exist and be unique, where G is Jacobian of

$$g(x) = \begin{bmatrix} \cdots \\ -Q_b x_b - m_b^0 - \sum_{k\neq b} m'_{bk} x_k. \\ \cdots \end{bmatrix}$$

In our case, $G = -\bar{Q}$. As such, it is sufficient condition that matrix $\bar{Q}$ is positive definite for the equilibrium to exist and be unique. □

As in the unconstrained case, condition in Proposition 4.7 is sufficient for $\bar{Q}$ to be positive definite.

The constrained game requires us to use numerical approximation methods to solve the equilibrium. Even though the solution exists and is unique, there is no closed-form solution available to handle both strategies and the shadow cost parameters λ_b^{LEV}, λ_b^{CAP}, and λ_b^{LIQ}. We borrow an approach from a class of Generalized Nash Equilibrium algorithms, described, for instance, in Facchinei & Kanzow (2010). It is based on a Gauss-Seidel sequence of best-response functions. Validity of such methods follows from the next result of Rosen (1965) for the concave N_b-person games:

Corollary 4.10. *The equilibrium of the constrained banking game can be found as the fixed point of function $F(x)$ defined as*

$$F(x) = \max_{y\in S^x} \rho(x,y),$$

where for two strategy vectors x, y defined on the set of feasible strategies $S^x \subset R^{(N+M)\times N_b}$ function, function ρ is defined as

$$\rho(x, y) = \sum_b u_b(x_1, ..., x_{b-1}, y_b, x_{b+1}, ..., x_{N_b}).$$

The constrained banking game provides additional insights about the impact of macroprudential regulation on the profits of banks. The shadow costs are interdependent, meaning that the impact of each regulatory constraint on the balance sheet changes can be reduced or magnified by other constraints. This observation adds to the discussion about the prudential capital buffers, which are imposed on banks to prevent reduction in the loan origination during stressful events. The reduction in lending is an admissible strategy of banks to restructure their balance sheets. Clearly, banks with binding liquidity and leverage ratios will reduce lending differently than banks without these constraints. Thus, a release of the prudential buffer should be conditional on the overall performance of banks. Moreover, the magnitude of shadow costs λ^{CAP}, λ^{LEV}, and λ^{LIQ} can serve as an indicator for the immediate costs that regulation imposes on banks for the sake of financial stability. The next theoretical section and empirical exercise provide more intuition about the impact of buffer release on banks' lending.

5. Mechanics of the model through lenses of a simplified example

We consider a simplified fictitious financial market to illustrate the dependence between the regulatory constraints, strategies of banks, and externalities they impose on each other's profits and financial ratios.

The market is composed of two identical banks $b \in \{1, 2\}$. The initial structure of each balance sheet and its parameters are displayed in Table 1. Each bank holds two types of loans, L0 and L1, and one type of securities S0. In expectations, loans L0 are more profitable but more risky than L1. The assets are funded with secured funding F0 and equity E0. In addition, the banks generate non-interest income from other activities, with expected marginal income rate z being equal to 0.1% of balance sheet size. All assets are controlled in the model. We assume that securities S0 are liquid and less profitable than loans L0 and L1. Banks keep S0 mostly to manage their liquidity flow in consistency with their internal liquidity requirement, which happens to be aligned with the LCR regulation.

We assume $c_f^{coll} = 0.001$, which means that 10 units of additional debt funding increases funding costs by 10 bps. We calibrate the remaining price sensitivities α^L, α^S such that the equilibrium balance sheets are equivalent to the initial balance sheets. Thus, results of Corollary 4.7 are applied. This implies sensitivity $\alpha^S = 3.3 \times 10^{-5}$ for the security class and sensitivities $\alpha_0^L = 1.2 \times 10^{-4}$ and $\alpha_1^L = 2.6 \times 10^{-4}$ for two loan classes. The magnitudes of sensitivities can be interpreted as follows. Liquidation of 25 units of S0, which is 10% of each bank's securities holdings, translates into 8.25 bps change in expected return on S0. Similarly, a 10% increase in the loan exposures L0 and L1, which is equivalent to 37.5 units each, implies a decrease in expected returns: 45 bps and 97.5 bps correspondingly.

item	category	a/units	r/c_0	σ	d	lgd	ω
$S0$	Government bonds	250	0.004	0.002		20	20
$L0$	Corporate loans	375	0.010	0.005	0.01	40	75
$L1$	Mortgages loans	375	0.010	0.002	0.01	40	50
F0	Deposits	950	0.001	0.000			
e	Capital	50					
I^{nint}	Net non-interest income	10	0.001	0.001			

item	category	ϕ^l/ϕ^a	α^S	μ	α^L	c_f^{coll}
$S0$	Government bonds	75	3.3E-05	2		
$L0$	Corporate loans	0		5	1.2E-04	
$L1$	Mortgages loans	0		10	2.6E-04	
$F0$	Deposits	20		40		0.001

Table 1. Structure of the simplified balance sheet and parameters
Note: a – volume; r – expected rate of return for S0, L0 and L1; c_0 – funding cost for F0; σ – standard deviation of return; d – default probability; lgd – loss-given default; ω – capital risk weight; ϕ^a – regulatory liquidity weights equal to 1-haircut for S0, L0, and L1; ϕ^l – run-off rate for F0; μ - maturity (in quarters); α^S – sensitivity of returns to transacted volumes; α^L – sensitivity of returns on loans to aggregate changes in loan supply; c_f^{coll} – sensitivity of secured funding costs to changes in banks' leverage ratios.

In this setup, we conduct four experiments to illustrate the mechanics of the model and its policy relevance. We shock some of the parameters and look at the reoptimized structures of the balance sheets and financial ratios. First, we shock the expected return of loans L0 equally for the two banks. Second, we assume a credit loss impacting L0 loans before the game starts, which erodes the capital of banks. In this setup, we investigate the role of prudential capital buffers in mitigating the results of the shock. Third, we shock only one bank with a credit loss to see how the competition on the market influences the decisions of banks. Finally, we study implications of banks getting more risk averse under adverse market conditions.

5.1. **Stressed expected returns.** Stressed interest income is a typical way a stress scenario translates into balance sheet parameters. We consider two levels of severity: a decline of expected return on one loan type, L0, by -20bps and by -40bps.

In those scenarios, banks are able to keep capital ratios almost unchanged by reducing their lending L0 (see Table 12). Moreover, banks substitute less-profitable loans with other asset classes (L0 and S0), which become relatively more profitable.

Competition between banks helps to soften the impact to the real economy. In '-40bps' shock scenario, banks decrease lending by -2.5% in Nash equilibrium but would decrease the same exposure by an additional 1.3% if they responded in isolation, i.e., not considering the other bank's response to stress. The competition also influences assets which are not directly affected by the initial shock. In particular, loans L1 and securities S0 increase less than in the absence of the competition. Therefore, the game has a stabilizing effect on a bank's balance sheet and on loan supply.

5.2. **Credit losses and a role of prudential buffers.** Stress scenarios usually have a profound impact via credit risk. Loan losses diminish capital ratios impacting capital adequacy, which may lead to deleveraging and secondary effects, such as fire sales or increased funding costs. What if banks were to defend their capital adequacy through management actions? Will the shock be mitigated completely? As illustrated in the example below, responses of banks to stress may result in restored capital positions but less lending. Reduction in lending further impacts the real economy and in the long run imposes additional losses on the financial system. In this paper, we do not focus on these spiraling impacts but rather quantify if a capital buffer can prevent immense reduction in lending under stress. In this way, we come close to evaluating the effectiveness of the counter-cyclical capital (CCyB) buffer conditional on a given scenario.[21] [22]

We consider two scenarios. A milder scenario assumes +3pp increase in default probabilities of loans L0, and a more severe one, a +6pp increase. The elevated default probabilities translate into lower expected returns on the respective asset class. In addition, to reflect changes in the initial conditions of the model, we adjust capital risk weight for L0 loans in the same proportion as the Basel III regulatory risk weights would adjust due to updates in the probabilities of default (see Appendix D for the applied formulas). In a nutshell, the regulatory formulas for risk weights quantify impacts of default probabilities and other loan characteristics such as loss-given defaults, maturity, and loan type. Finally, we impose a total capital limit of 8.5%, with 0.5% being a removable capital buffer.

The results of the stress test are such that only in the severe scenario do banks reach the capital constraint as a result of management actions (see Table 7). In particular, in the milder case, banks let the ratios fall by 0.9 pps and do not materially adjust their balance sheet compositions. The externalities that banks impose on each other are not substantial. In the severe scenario, banks operate at the margin of the capital requirement. To stay the most profitable and compliant at the same time, banks deleverage loans with the highest risk weight (L0) by 7.2%, also divesting from other asset classes.

Assuming that 50bps of the capital ratio is a prudential buffer, we look at the actions of banks in case the buffer is released. The released buffer serves as intended, being an additional loss absorption capacity. Banks let the capital fall by 2.0 pps. The additional capacity allows banks to keep profitable

[21] In Canada, the counter-cyclical buffer is called the domestic stability buffer.

[22] Our paper neither addresses the timing aspects of the CCyB nor quantifies the precision error that regulators may face when composing a realistic stress scenario.

loan exposures L0 and L1, with only slight adjustment of securities S0 and their LCR ratios. This demonstrates how a release of the capital buffer can be effective, meaning that the banks continue to moderately support real economy in the stressful times, which would not be possible otherwise.

5.3. **Idiosyncratic credit shock.** The way that strategic interactions of banks may play out in times of stress can best be observed when the shock hits only a subset of banks. It is relevant in the policy-relevant stress test exercises with stress usually unevenly distributed across the system. An idiosyncratic shock hitting only one bank is a stylized scenario that allows us to see the role of the market in absorbing the consequences of this banks' management actions.

Similar to subsection 5.2, we apply loan L0 loss, but this time only to the first bank. Idiosyncratic credit losses may occur if a bank has a tendency to acquire more risky loans in a given market segment. An asymmetric distribution of losses across banks usually occurs in a plausible stress test scenario used by regulators and market overseers.

Bank 1 responds in a similar way to the case of a common loan loss shock considered in subsection 5.2, mostly by reducing its exposure to L0 (see Table 8). The second bank acquires a part of the market share in loan L0 type and provides a cushion for the loan supply in this segment of the market. Additionally, it increases exposure in L1 and S0, letting its capital ratio drop by 20 bps. Its response to the shock to bank 1 is magnified if the regulatory requirements become binding for bank 1.

The changes of the second bank's balance sheet are almost entirely due to the strategic interaction in the equilibrium. Changes reported for optimal vs. initial and optimal vs. no game setup are almost identical. Clearly, bank 2 does not have incentive to change its balance sheet structure unless it expects bank 1 to respond to the shock prescribed in the stress scenario. The feedback effect of banks' strategies through interaction on the lending market is an important mechanism that plays out in the comprehensive macro-financial stress scenario.

5.4. **Risk aversion.** An expected stress scenario may not only change risk and return parameters of banks' balance sheets but also alter their appetite for risk. To understand how this can influence banks' management actions, we apply the credit risk scenario used in subsection 5.2, but now with the increased risk aversion parameter $\gamma = 10$ to compare with the baseline case of $\gamma = 2$. The results are shown in Table 11.

Based on the results, we make three observations: on lending activities, liquidity buffers, and the strategic component of management actions. First, we find that more risk-averse banks decrease their L0 exposure to a greater degree. Notably, the change is more pronounced in the case of a less severe stress scenario: changes in L0 are -1.9% for $\gamma = 10$ and -0.1% for $\gamma = 2$ in the scenario with +3pp increase in the default probability. This happens because the less severe stress is driven by risk and return considerations rather than capital constraints that have to be factored into management actions in the more severe stress scenario. Second, with higher sensitivity to risk, banks further reduce leverage in the equilibrium by keeping fewer securities S0 (e.g. by -4.0% comparing with -2.9% in the baseline case of the risk aversion). Third, more risk aversion activates strategic interaction between banks, which does not take place in case of $\gamma = 2$. Specifically, conditioning balance sheet changes on the balance sheets of peers mitigates the decline in lending L0 and in securities holdings S0. The mitigation is larger under the less severe scenario since banks have more flexibility to adjust balance sheets without the binding constraints. Moreover, in the '+6pp' case, the positive impact of the interactions is partly offset by the negative externality in segment L1, where banks decide to cut lending by an additional -0.3%. Finally, in the cases with more risk-averse banks, strategic interaction leads to a buildup of LCR buffers (by 0.2pp and 0.5pp in the less and more severe scenarios, respectively). We conclude that risk aversion may have a significant and diverse impact on banks' behaviors under stress.

6. Application of the model using a consistent stress scenario

To assess the role of banks' actions under stress and externalities they may have on each other in real markets, we apply a consistent stress test scenario to the six Canadian D-SIBs.[23] The Canadian banking system is highly concentrated, with D-SIBs accounting for about 90% of total assets of deposit-taking institutions. Thus, responses of the six banks may have a profound impact on the economy overall. We consider a scenario similar in dynamics and magnitude to the one applied in the Financial Sector Assessment Program (FSAP) exercise conducted by the IMF in 2019 for the Canadian economy. Some specifics of the scenario are given in the sections that follow.[24]

6.1. **Translation of the consistent scenario to the parameters of the model.**

6.1.1. *Stress horizon of the macro-financial scenario.* The IMF designed a stress test scenario for Canada consistent across the key macro-financial variables of the economy.[25] The consistency is introduced by a specific narrative. The initial trigger is a disruption in international trade and global production chains, followed by disorderly financial market adjustments (IMF 2019). Tightening global financial conditions then set off global housing market and credit cycle downturns. These external shocks result in a sharp housing market correction in Canada, along with significant financial stress and large currency

[23]The banks in the sample are the Bank of Montreal, Bank of Nova Scotia, Canadian Imperial Bank of Commerce, National Bank of Canada, Royal Bank of Canada, and Toronto-Dominion Bank.

[24]Although our scenario is similar to that of the IMF, the two exercises differ along many dimensions, such as granularity of data, satellite models, and calibration assumptions. Thus, direct comparison of the quantitative results is not performed.

[25]See Adrian et al. (2020) for the overview of the IMF stress testing approach.

depreciation. Consequently, market rates snap back. We consider five quarters of the first two most stressful years of the scenario, i.e., from 2019Q1 to 2020Q1, with period 2018Q4 included as benchmark. The scenario corresponds to a real GDP decline of -6.3% within two years from the end of 2018 (see Table 2). The scenario is also characterized by increased yields and housing price correction.

		Historical stress periods		Pre-stress periods		Adverse scenario	
		1991	2009	2017	2018	2019	2020
Canada							
	Real GDP growth (annual rate)	-3.4	-4.0	3.0	2.1	-3.1	-4.2
	10-year government bond yield	9.8	3.0	1.9	2.4	3.7	3.2
	3-month government T-bill yield	10.0	0.2	0.7	1.4	3.5	3.0
	House price (2017=100)			100	104	82	66
	Equity price (2017=100)			100	103	61	70
Unites States							
	Real GDP growth (annual rate)	-0.9	-3.0	2.2	2.9	-2.4	-2.9
	10-year government bond yield	8.0	2.8	2.3	2.9	3.7	3.2
	House price (2017=100)			100	107	103	103
	Equity price (2017=100)			100	113	75	84

Table 2. Key macro-financial variables in the consistent stress scenario

We apply the model on a quarterly basis. Balance sheets are adjusted from period to period conditional on the equilibrium outcomes in one-period games. This means that banks experience additional shocks in the latter quarters of the stress test due to cumulative losses carried over through time. The impact of banks' balance sheet actions and corresponding externalities in the first quarters are also reflected in the following quarters.

To run the stress test exercise, we fit the model to the historical data from 2015Q3 to 2018Q4 to estimate unknown coefficients and use additional data sets as early as 1991Q1 to calibrate credit and income shocks consistently with the macro variables in the scenario.

6.1.2. *Portfolio of assets and funding sources.* Banks hold assets and funding categories grouped as presented in Table 3. With this level of balance sheet granularity, we capture essential heterogeneity in capital risk weights and profit margins.[26] To calibrate the balance sheet exposures, we use financial returns of banks to the Office of the Superintendent of Financial Institutions (OSFI).[27]

We calibrate balance sheets on a level that is more detailed than that used in the model (as in Table 3) to better understand the economic intuition behind the heterogeneous predictions of the model across banks, to more precisely calibrate risk weights and liquidity weights, and to model credit risk at a more granular product level (the detailed balance sheet includes nine categories of repo exposures, nine

[26]Individual balance sheets are not reported to keep the confidentiality of data.
[27]Balance sheet exposures are available in the Balance Sheet (M4) return. We refer to OSFI returns Non-mortgage Loans (A2) and Loans in Arrears (N3) for the detailed composition of loan portfolios of banks, and the Net Cumulative Cash Flow (NCCF) return and Securities (B2) return for the detailed securities holdings.

categories of business loans, and seven categories of fixed-income securities). This leads to the bank-specific rates of return, probabilities of default, regulatory weights, and volatility indexes of assets and liabilities. For clarity, we only report the aggregated model results.

Summary statistics about the pre-stress exposures of D-SIBs are reported in Table 13. According to the pre-stress data, the six banks are diversified in terms of their size, with total assets distributed between 0.6 and 1.5 trillion CAD. Securities constitute around 40% of banks' holdings; deposits constitute around 44% of funding exposures. The average CET1 capital ratio of D-SIBs was 11.7% before the stress (minimum was 11.3%), which is well above the regulatory minimum of 8%, including the 2.5% capital conservation buffer and 1% surcharge for D-SIBs. In the last quarter preceding the outset of the stress test, the banks were subject to the counter-cyclical buffer of 1.5%. In 2019, the buffer was changed to 1.75% and eventually was released to 1% in 2020.

Assets/liabilities/equity	Item	Broad category	Model category (if different)
a	S0	securities	cash, government debt, deposits with federal deposit-taking institutions
a	S1	securities	equities
a	S2	securities	corporate debt
a	S3	securities	reverse repo
a	L0	corporate loans	corporate non-financial loans
a	L1	corporate loans	financial loans
a	L2	consumer loans	credit card loans
a	L3	mortgages	mortgages uninsured
a	L4	mortgages	mortgages insured
a	L5	mortgages	non-residential mortgages
a	L6	other loans	loans to governments and public loans
a	L7	consumer loans	lease receivables
l	F0	deposits	
l	F1	wholesale	wholesale unsecured
l	F2	wholesale	wholesale secured
l	F3	other funding	
e	e_0	equity	

Table 3. Categories of balance sheet

6.1.3. *Credit risk.* We use modified non-performing loan (NPL) ratios for modeling default probabilities of loans. The approach with NPL ratios is a common practice in stress testing, applied in the Bank of Canada's Framework for Risk Identification and Assessment and in stress tests of other jurisdictions (e.g., ECB (2017)). In this paper, projections of NPLs for each bank and each loan category are based on a panel error-correction model described in the stress testing technical report of MacDonald & Traclet (2018). The NPL ratios are then transformed to the default probabilities consistent with the aggregated levels reported by banks in the Basel Capital Adequacy Reporting (BCAR) statements. Similar to the rates of return and funding costs, computations of default rates are done at the product level and later aggregated into the categories presented in the paper. The aggregated projected default probabilities are shown in Figure 4.[28] According to the projections, loans are heavily impacted by the stress scenario, with the average default rates of corporate loans increasing from 1.74% to 8.97% annually, default rates of mortgages from 1.02% to 3.27%, and default rates of other retail loans from 1.59% to 9.21%.

The impact of credit losses is twofold. First, banks expect credit losses consistent with the scenario when deciding on optimal strategy: marginal credit losses are accounted for in the calculations of expected returns. Credit losses are computed as a product of exposures and their default probabilities, loss-given default, and utilization rates.[29] Second, we apply aggregated credit losses to the equity of each bank at the beginning of the next period, so that the model can be used in the repetitive way.[30]

6.1.4. *Calibrating expectations of asset returns and funding costs.* We use quarterly regulatory data from 1999Q1 to 2018Q4 to model expectations of the rates of returns and marginal funding costs given the macro scenario.[31] Return rates are the sums of interest income rates and relevant fees. As such, these rates should be perceived as effective rates and not contractual ones. Effective rates are less variable across time due to hedging, diversification, and other strategies of banks aimed at stabilizing their returns. Returns on loans also exclude marginal credit losses calculated as the product of default rates, losses given default, and utilization rates (to account for credit lines, if necessary). We use regression analysis to predict the expected rates of return, variances of returns, and marginal funding costs.[32] Similarly, funding costs are calculated as the ratios of interest expenses and funding exposures. Because of data restrictions, both new and non-maturing loans are assumed to deliver the same expected return.[33] We refer the reader to Table 13 for the summary statistics of the pre-stress asset returns, marginal funding costs, and volatilities of returns.

We use two types of regressions to link returns and costs to the macro scenario. The vector error correction model is applied for finding expected income rates, fees, and funding costs; the GARCH(1,1) model is applied for the corresponding variances. We will now focus on the income rates; regressions for fees and costs are similar. The expected rates are explained by the following long-term variables: rates

[28]These *de facto* point-in-time (PIT) default probabilities drive the changes in the through-the-cycle (TTC) default probabilities used in the risk-weighted capital ratio formula and, consequently, banks' solvency constraints (see Appendix D). Specifically, the TTC default probabilities evolve according to changes in the PIT default probabilities scaled down by the duration of the economic cycle.
[29]Remaining parameters are extracted from BCAR returns and adjusted to downturn values to reflect the severity of the stress scenario.
[30]We assume that banks' provisioning for losses is non-strategic and identical to the pre-stress period.
[31]Rates and costs are calibrated using an Income Statement (P3) and the corresponding asset volumes.
[32]Marginal fees and costs that tend to be acyclic and insensitive to the scenario are projected to have the same value throughout the whole stress test period.
[33]Nevertheless, we calibrate maturity rates of loans using A4 OSFI return on new and existing lending to limit the ability of banks to sell non-maturing loans and to quantify capital risk weights.

for 3-month Treasury bills in the USA and Canada, and rates for 5-year government bonds in the USA and Canada. The short-term variations in rates are explained by the following variables:[34]

(1) AR (1) lag,
(2) Changes in real GDP growth in Canada (for mortgage-related assets),
(3) Changes in rate of 5-year government bonds in Canada,
(4) Changes in rate of 3-month Treasury bills in Canada,
(5) Changes in house price growth in Canada (for mortgage-related assets),
(6) Changes in rate of 5-year government bonds in the USA,
(7) Changes in rate of 3-month Treasury bills in the USA.

Regressors in the GARCH equations for variances include variables (3)-(4) and (6)-(7).

Regressions are run for each bank separately to account for differences in their asset portfolios. The projections are done for the narrowest asset categories, which are later aggregated into the categories used for the model.

Combining default rates with the projections of rates, we find that the average return rates of corporate loans after accounting for credit losses decreased from 4.01% to 1.82% annually, mortgage rates from 2.84% to 2.38%, and default rates of other retail loans from 9.1% to 6.8%. This decline mostly reflects stressed credit losses. Interest income rates remain more stable across time and behave in consistency with the hump-shaped pattern of the government interest rates fixed by the scenario. Initial rates of returns are given in Table 13. The projections are shown in Figure 4.

6.1.5. *Calibration of price and cost sensitivities.* One of the contributions of our paper is accounting for the impact of competition on a bank's balance sheet formation. Its quantification depends on price and cost sensitivities that banks are assumed to factor into their decision process. Our modeling approach allows us to estimate the sensitivities empirically. The model can be fully calibrated beginning in 2015Q3. To tackle the data limitation, we apply a two-step procedure. First, we estimate the funding cost sensitivities c_f and c_f^{coll} using longer time series. Second, we use the estimates of c_f and c_f^{coll} to calibrate price sensitivities for loans α^L and securities α^S. Under the assumption that the banks are not regulatory constrained during the estimation period, we use equilibrium condition (19) to derive the structural equations for the sensitivities given observed balance sheet changes (x_b^L, x_b^S) for each period.

[34]While performing sensitivity analysis, we also used alternative independent variables, such as stock market indicators, exchange rates, inflation rate, and unemployment rate. This led to either multi-collinearity problems or poor model performance. Moreover, to accommodate for data limitations and time-series properties of the data, we restrict the number of lags in the regression to one.

Corollary 6.1. *In equilibrium of Theorem 18, the following relationships hold for a given time t, bank b, loan category i, and security category j:*

$$\frac{r^{L,x}_{b,i,t} - c^{'}_{b,t}w_{b,t} + z^{x}_{b,t} - V^{L}_{b,i,t} - \Delta mc_{b,t}}{\mu^{L,x}_{b,i,t}a^{L,x}_{b,i,t} + x^{L}_{b,i,t}} = \alpha^{L}_{i} + \varepsilon^{L}_{b,i,t} \tag{22}$$

$$\frac{r^{S,x}_{b,j,t} - c^{'}_{b,t}w_{b,t} + z^{x}_{b,t} - V^{S}_{b,j,t} - \Delta mc_{b,t}}{\left(\beta^{S} - (1-r^{f}_{b})\mu^{S,x}_{b,j,t}\right)a^{S,x}_{b,j,t} + \beta^{S}x^{S}_{b,j,t}} = \alpha^{S}_{j} + \varepsilon^{S}_{b,j,t}, \tag{23}$$

with errors centered at zero $E[\varepsilon^{L}_{b,i,t}] = 0$ and $E[\varepsilon^{S}_{b,j,t}] = 0$, the funding cost impact being

$$\Delta mc_{b,t} = C^{\prime}_{f}(w_{b,t}\nu_{b,t}1^{\prime}_{N_x+N+M_x}a_{b,t} + w_{b,t}1^{\prime}_{N_x+M_x}x_{b,t})\frac{1}{e_{b,t}}$$

and the volatility matrix being

$$\begin{bmatrix} V^{L}_{b,t} \\ V^{S}_{b,t} \end{bmatrix} = \frac{2\gamma}{e_{b,t}}\left(\begin{bmatrix} \mathbb{V}(\epsilon^{L,c,new'}_{b,t}) & Cov(\epsilon^{L,c,new'}_{b,t}, \epsilon^{S'}_{b,t}) \\ Cov(\epsilon^{L,c,new'}_{b,t}, \epsilon^{S'}_{b,t}) & \mathbb{V}(\epsilon^{S'}_{b,t}) \end{bmatrix} + \mathbb{V}(\epsilon^{z}_{b,t})\right)x_{b,t} + \frac{2\gamma}{e_{b,t}}\begin{bmatrix} \mathbb{C}ov(\epsilon^{L,c,n}_{b,t}, \epsilon^{a}_{b,t}) \\ \mathbb{C}ov(\epsilon^{S}_{b,t}, \epsilon^{a}_{b,t}) \end{bmatrix}.$$

Proof. See Appendix B for details. □

The intuition behind equations (22) and (23) is that sensitivities explain why the assets are purchased (or sold) below or above the level of marginal surplus that they provide. The observed marginal surpluses are presented at the numerator of the left-hand side of equations and equal to the observed marginal income rates $r^{L,c,new}_{b,j,t}$ and $r^{S,x}_{b,j,t}$ and average non-interest income rate $z^{x}_{b,t}$, less average funding cost $c^{'}_{b,t}w_{b,t}$, risk premia of the assets ($V^{L}_{b,t}$ and $V^{S}_{b,t}$), and funding costs associated with deleveraging $\Delta mc_{b,t}$. The denominator of each ratio is the volume of the assets being transacted in a given period. In equilibrium, the ratio of marginal surplus and the transaction volumes is the same across banks, so the measurement error term on the right-hand side is expected to be centered around zero. This intuition is in line with our initial definition of sensitivities as measures of competitiveness and externalities that banks impose on each other when trying to change their market shares.

Because we expect market forces to be more or less the same within the time span we consider, α^{L} and α^{S} during a stress scenario are kept constant. We also assume they have been constant in the pre-stress data and estimate the regressions implied by (22) and (23).

Finally, it is helpful to explain the nature of the error terms of (22) and (23). The error terms appear naturally in the equilibrium conditions because the banks make their rebalancing decisions based on expected returns $r^{L}_{b,j,t}$ and $r^{S}_{b,j,t}$, whereas what we observe in the data are realized rates $r^{L,x}_{b,j,t}$ and $r^{S,x}_{b,j,t}$, which also exclude adjustment costs (e.g., marginal fire-sales costs and costs associated with market share expansion; see Section 3 for details). These error terms also include differences between the expected marginal non-interest income $z_{b,t}$ and the realized one $z^x_{b,t}$. This makes the errors to be bank-, asset-, and time-specific.

	Dependent variable:					
	diff(repo rate)			diff(total funding rate)		
	(1)	(2)	(3)	(4)	(5)	(6)
diff(lev)	0.0001***	0.0001**	0.0001***	0.00001***	0.00001*	0.00001**
	(0.00004)	(0.00004)	(0.00004)	(0.0000005)	(0.000001)	(0.00001)
diff(tbill_3m)		0.218***			0.028***	
		(0.049)			(0.007)	
diff(govbond_5y)			−0.068***			−0.008**
			(0.024)			(0.003)
Observations	240	240	240	240	240	240
R^2	0.049	0.124	0.080	0.037	0.105	0.064
Adjusted R^2	0.025	0.097	0.052	0.012	0.078	0.035
F Statistic	12.129***	16.385***	10.090***	8.997***	13.666***	7.874***

Note: $^{*}p<0.10$; $^{**}p<0.05$; $^{***}p<0.01$

Table 4. Sensitivity of funding costs to changes in leverage ratios
Note: quarterly data for 2009 Q1-2018 Q4; 'repo rate' – marginal funding costs on repurchase agreements; 'total funding rate' – average rate of expenses, i.e., expenses divided by total interest-bearing funding sources; 'lev' – average leverage ratio across bank of interest and the industry computed as in formula (5).

We begin by estimating funding elasticities using a subset of balance sheet and income data available since 2009Q1. The estimation using narrow data is possible since in the model we assume that the marginal changes in the funding costs depend only on the aggregate changes of banks' balance sheets for which a longer time series exists. We use relationship (5) to set up the regressions for funding elasticities. For the dependent variables, we use inter-temporal changes in the marginal funding cost for the first regression and inter-temporal changes in the marginal funding cost for secured funding for the second regression. For explanatory variables of interest, we use equally weighted value between the bank's leverage ratio and the industry's leverage ratio. The results of the estimation are given in Table 4. According to the results, estimates for both c_f and c_f^{coll} are statistically significant and meaningful in terms of signs. The regression outcomes are also robust to inclusion of explanatory variables controlling for general developments in macro conditions, such as rates of three-month Treasury bills and five-year government bonds.

In the second step of the calibration, we find elasticities by using fitted estimates of c_f and c_f^{coll}. In general, the data support the hypothesis of sensitivities being stable across time and similar across banks. The summary statistics and results of the estimations are reported in Table 5. Without additional variables being included, the estimates are identical to the mean observed values of sensitivities. For a robustness check, we run additional regressions with macro variables as regressors. The intention of this is to verify that no variable is missing in explaining the balance sheet decision of banks and that elasticities are not cyclic across time. The results support our hypothesis that the left-hand-side parts of (22) and (23) are not sensitive to the business cycle. This provides evidence for validity of linear price impacts in our sample. We will use the mean elasticities as a benchmark in our stress scenario. Further in the paper, we conduct supporting analysis by randomly sampling elasticities from the estimated distribution and running the stress test model to verify that our stress test results are robust.

6.2. **Calibration of regulatory requirements.** The model provides optimal balance sheet structure independently of how the income will be utilized: through dividends or retained earnings. However, for the transition between periods, it is required to specify how the income is reinvested. In the benchmark environment, we assume banks are keeping dividends constant at pre-stress data. This assumption can be defended with two arguments. First, an increase in a dividends ratio above the pre-stress level is either unlikely or requires more sophisticated reasoning, given the severity of capital losses. Second, a decrease in dividends does not only help to maintain the capital buffer but also sends a negative signal to investors, which may further put pressure on the bank's funding costs. As such, more analysis would be needed to evaluate this trade-off. Last but not least, in the Canadian context, there were only rare episodes of D-SIBs using their dividends to mitigate stress. In addition, we incorporate an assumption about static loan loss provision. This allows us to see the maximum impact of stress on a bank's capital ratios and attribute it to the scenario.

If capital ratios fall sufficiently low, the capital conservation requirements will limit distributions of dividends in accordance with BCBS and OSFI standards (see Table 6). For example, a D-SIB with a CET1 capital ratio in the range of 5.375% to 6.25%, in 2019, would be required to maintain the equivalent of 80% of its earnings in the subsequent quarter.

To model leverage requirements of the Canadian banks, we need to work with a different measure of capital than CET1 capital, i.e., with Tier 1 capital, and to incorporate some off-balance sheet (OBS) exposures into the measure of the balance sheet size. To capture OSFI definition of the leverage ratio, we include static net derivative exposures, securities financing transaction (SFT) exposures, and other OBS items into the exposure measure of the balance sheet size. Likewise, we assume that changes in Tier 1 capital come solely from the changes in CET 1 capital.

Finally, we calibrate weights for LCR and RWA based on Basel III guidelines. First, we use standardized liquidity haircuts. Second, we allow capital risk weights to adjust with the scenario. Non-stress risk weights are calibrated based on the regulatory reports of banks. Changes in risk weights, depending on the scenario, are calculated based on Basel III IRB formulas given stressed bank-specific default probabilities, losses given default, and maturities structure of exposures (see Appendix D for details).

6.2.1. *Inter-temporal adjustments.* The model is run quarter by quarter. Theoretically, optimal balance sheets at time t are used as the starting-point balance sheets for the subsequent quarter $t+1$, with the previously described assumptions on dividend distribution, income, and credit losses. We assume that the bank assets grow proportionally with the income applied after taxes and dividends. Similarly, bank equity changes by the amount of non-distributed income $RetEarn_{b,t}$:

$$RetEarn_{b,t} = NetInc_{b,t}(1 - tax_b) - Dividends_b.$$

Liabilities adjust proportionally, with total debt being the difference between updated assets and capital. Pre-stress regulatory ratios are updated consistently with the dynamics of exposures and their liquidity and credit risk characteristics.

6.3. **Results of the stress test exercise.** Based on the consistent stress scenario exercise, we obtain five policy-relevant results: (1) asset substitution and reduction in lending, (2) heterogeneity of banks' responses, (3) compliance with regulatory ratios, (4) release of capital buffers under stress, and (5) minimum role of strategic behaviors for lending activity.

(1) Asset substitution and reduction in lending. Banks shrink and rebalance their portfolios along the horizon of the stress test (Figure 4, panel A). They reduce exposure to business loans (denoted L0) by 26.37%, to uninsured and insured mortgage loans (L3 and L4) by 5.18% and 4.26% correspondingly, and shrink their consumer loans (L7) by 33.95%. As these categories are the largest on the balance sheets, their reduction has the most significant impact on regulatory ratios and profitability. Banks unequally rebalance their portfolio, with the biggest reduction falling onto the business loans.

Mortgages are reduced less than business and consumer loans for two reasons. First, the credit quality of mortgages is not as severely damaged by the scenario. Second, mortgages are longer-term loans, typically with relatively low credit loss rates, especially because a large fraction of mortgages are insured in Canada.

Banks also reduce other types of exposures: financial loans (L1) by 18.42%, their credit card (L2) exposures by 10.25%, and their non-residential mortgages (L5) by 38.5%; however, these changes are small in magnitude.

Adjustment in securities portfolios is relatively small compared to business loans. Banks liquidate government bonds, corporate bonds, and repo in the range of 10-13%, and even marginally increase holdings of equities, as the impact of the scenario on these instruments is not as severe.[35]

[35]A drop in the average equity price in the scenario does not correspond one to one to the income that banks make by holding equities. This is due to hedging, appreciation of foreign securities, and steady dividends that banks accumulate over time.

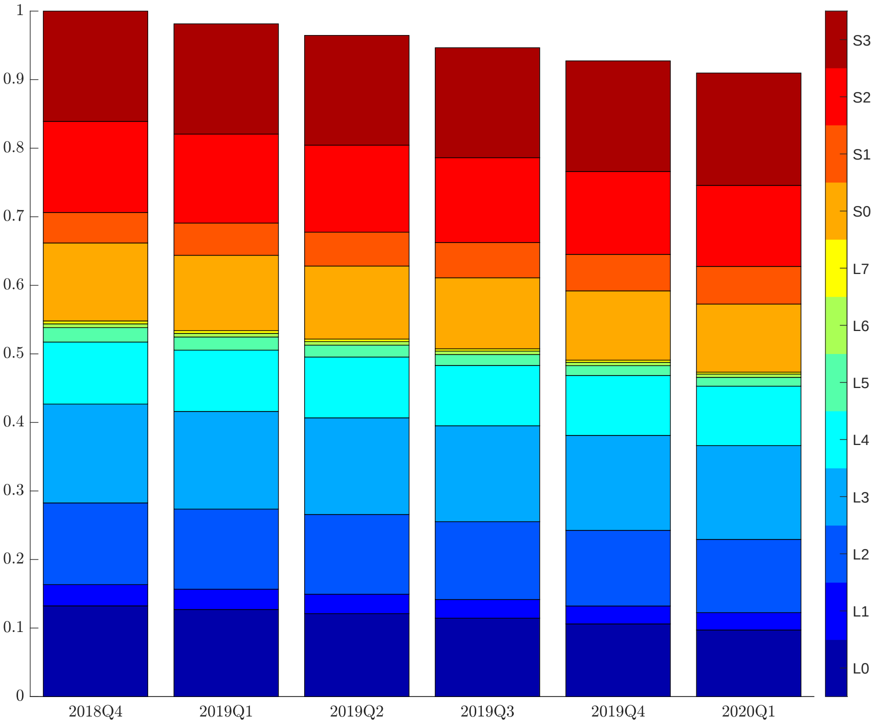

Figure 3. Changes in the composition and growth of balance sheets
Note: Assets of D-SIBs as % to the initial pre-stress holdings.

(2) Heterogeneity of banks' responses. Banks respond differently to the same stress scenario. The contrast comes from two channels: the quality of the assets and banks' business models. First, banks' assets within the same category vary in expected profitability, credit quality, and volatility of generated profits. Because banks' default and income rates respond differently to the macro variables in the historical data, the scenario projections related to loan quality are also different across banks. This is caused by dissimilarities in business models and geography to which banks are exposed. The same is true for securities: banks have different strategies regarding hedging of the market, the interest rate, and currency risk. They also keep heterogeneous cushions of liquid assets and are exposed to Canadian vs. foreign securities to various degrees. Moreover, higher marginal funding costs and lower initial capital ratios make it more beneficial for some banks to adjust their portfolios.

The intensity of asset substitution depends on the specific characteristics of each portfolio. Composition of the balance sheets and correlation across returns and credit losses determine whether a bank

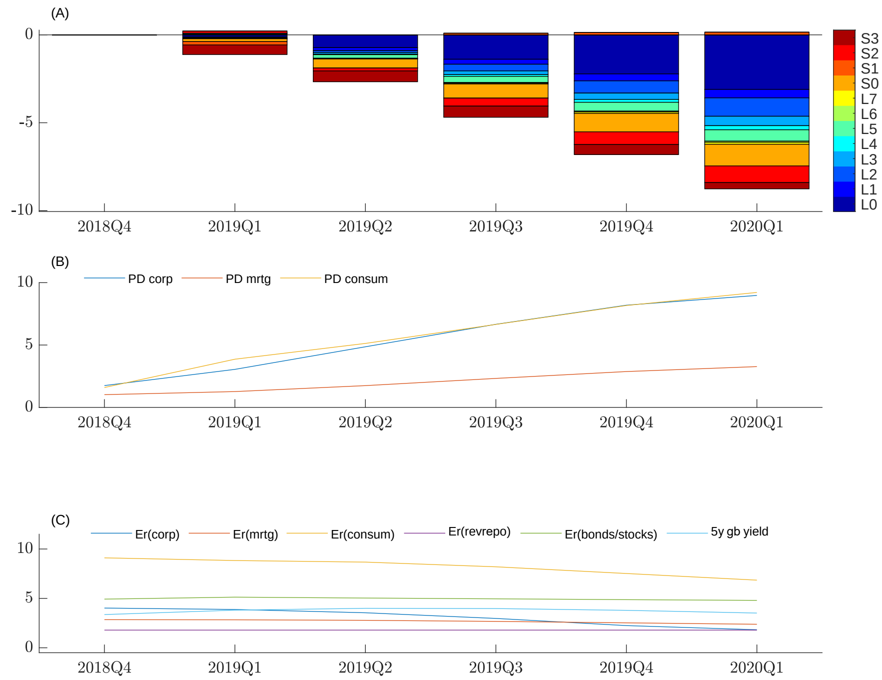

Figure 4. Changes in balance sheets and scenario highlights
Note: y-axis – panel A) change in asset composition due to the risky scenario; panel B) default probability for business loans, retail mortgage loans, and consumer loans; panel C) expected rate of return for corresponding assets and rate of government bonds with 5-year maturity (all numbers in %).

can substitute one asset category for another. Finally, while we do not model off-balance sheet exposures directly, the degree to which a bank uses them still propagates through income-generating process. Similarly, some banks earn income from the subsidiaries, which is applied to the balance sheet through distributed income and losses without being allocated to any on-balance sheet category in particular.

The heterogeneity in banks' responses to shocks is evident from regulatory ratios in Figure 5. Some banks increase their capital and liquidity ratios relative to the pre-stress period, while others fall below the pre-stress level and even hit the regulatory constraints. Banks that are exposed to deteriorating loan categories more than others face larger losses. Among those banks that face similar losses, some choose to keep impacted loans on their balance sheets hoping for future income growth, while others reduce their balance sheets substantially, increasing leverage ratio. Reduction in balance sheets also leads to a decrease in risk-weighted assets that boosts the capital ratios.

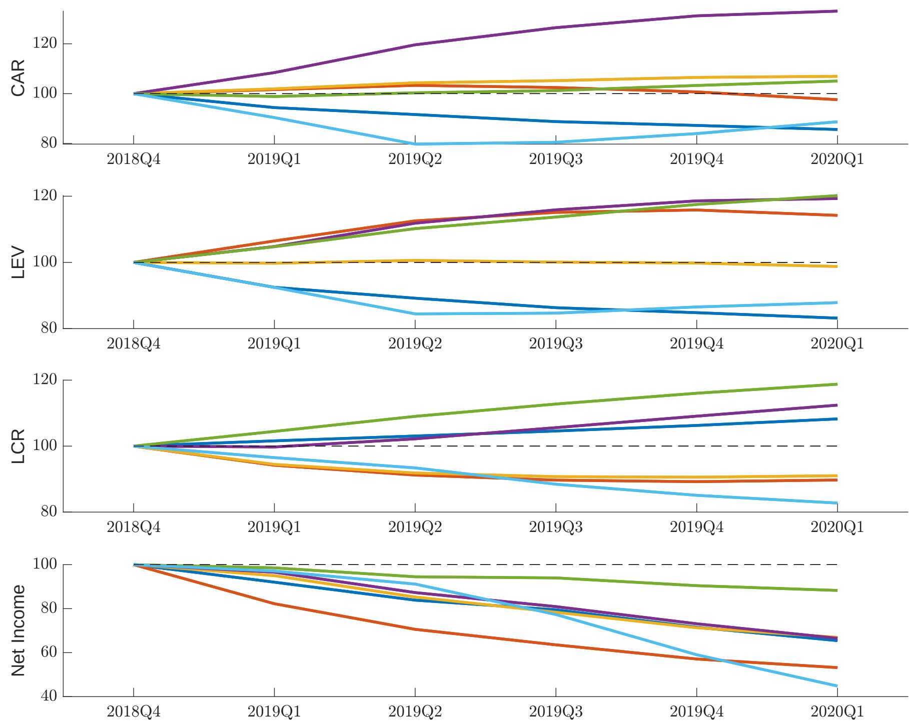

Figure 5. Evolution of regulatory ratios in the equilibrium
Note: x-axes: periods in stress horizon; y-axes: LCR – liquidity coverage ratio τ_b^{LIQ}, LEV – leverage ratio τ_b^{LEV}, CAR – capital adequacy ratio τ_b^{CAP}; units are 100× (regulatory ratio at time t/ regulatory ratio in 2018 Q4); each line represents one of the D-SIBs.

The heterogeneity in the model-based regulatory ratios also occurs because the model assumes no reputation risk for banks that stay compliant but fall below their pre-stress levels. To relax this assumption, we try to impose higher limits on regulatory ratios to see whether banks can reach them by giving up on profitability.

(3) Compliance with regulatory ratios. Assets of the banking system shrink by 10% (Figure 4, panel A). The impact of the stress scenario seems especially large when compared to the historical data prior to the hypothetical scenario. Balance sheets of banks grew annually on average 7.8% in 2015–2018 due to high profitability that also contributed to a build-up of capital. Decrease in business lending primarily helped banks to divest capital-intensive assets. As a result, all banks met their regulatory limit of 8%. This includes 4.5% requirement for CET1 capital, 2.5% for capital conservation buffer, 1% for the D-SIB surcharge, assuming release of 1.75% of the Domestic Stability Buffer (DSB). The banks also stayed

above the leverage limit, while two banks came close to reaching it. D-SIBs stayed compliant in terms of their liquidity, with two banks almost reaching 100%. The correct way to interpret these results is the following. In the interest of future profitability, the banks restricted themselves to some assets to a higher degree than the capital was impacted by the losses. This kept the capital and leverage ratios higher than they would be in the static balance sheet model. Also, in the interest of profitability, some banks liquidate proportionally more liquid than illiquid assets, leaving them eventually less liquid than pre-stress.

There is a possibility that banks could find an allocation of assets that result in higher regulatory ratios but lower expected income (adjusted for risk). To understand whether banks had difficulty keeping their regulatory ratios afloat, we run experiments assuming an alternative calibration of the limits. We find that only one out of six banks is able to maintain the pre-stress levels of capital, leverage, and liquidity simultaneously. The rest of the banks have difficulties reaching liquidity or capital targets. Second, we confirm that all banks are able to maintain their liquidity ratios above the pre-stress level while complying with the regulation.

To illustrate how D-SIBs can maintain their initial ratios, we run the model assuming the capital ratio being constrained at 9.75%, leverage ratio limit being at 3.75%, and liquidity ratio being at 110%. This calibration assumes no release of the counter-cyclical (DSB) buffer. The intention of the buffer being released is to return lending to the domestic real economy. As such, we expect higher reduction in lending in the alternative calibration. We also set leverage and liquidity limits closer to the pre-stress values to test whether banks can reach such levels in equilibrium. As illustrated in Figure 6, the outcome of the model with these increased regulatory limits is overall similar. All banks are able to stay within the limits, with one bank coming close to binding 9.75% capital ratio, two banks reaching 3.75% leverage ratio starting 2019 Q2, and two banks dropping their liquidity ratios to 110% beginning 2019 Q4.

(4) Release of capital buffers under stress: small impact on lending. An alternative calibration we described above (with capital ratio limit being 9.75%, leverage ratio limit being at 3.75%, and liquidity ratio limit at 110%) is helpful to understand the intuition behind the credit reduction in the model. We find that for this calibration, reduction in lending is of similar magnitude, as in the case of lower targets for ratios. For instance, reduction in business lending increases from 26.37% to 26.84% (see Figure 6). Similarly, reduction in lending gets slightly less severe for mortgages: from 5.18% to 5.25% and 4.26% to 4.29%; consumer loans are reduced roughly in the same way: from 10.25% to 10.43%.

We next see whether a release of the counter-cyclical capital buffer, with leverage and liquidity ratios staying at the regulatory minimum, plays an important role to mitigate stress for the Canadian banks. As such, we assume that D-SIBs still need to comply with the minimum of 9.75% capital ratio. We find

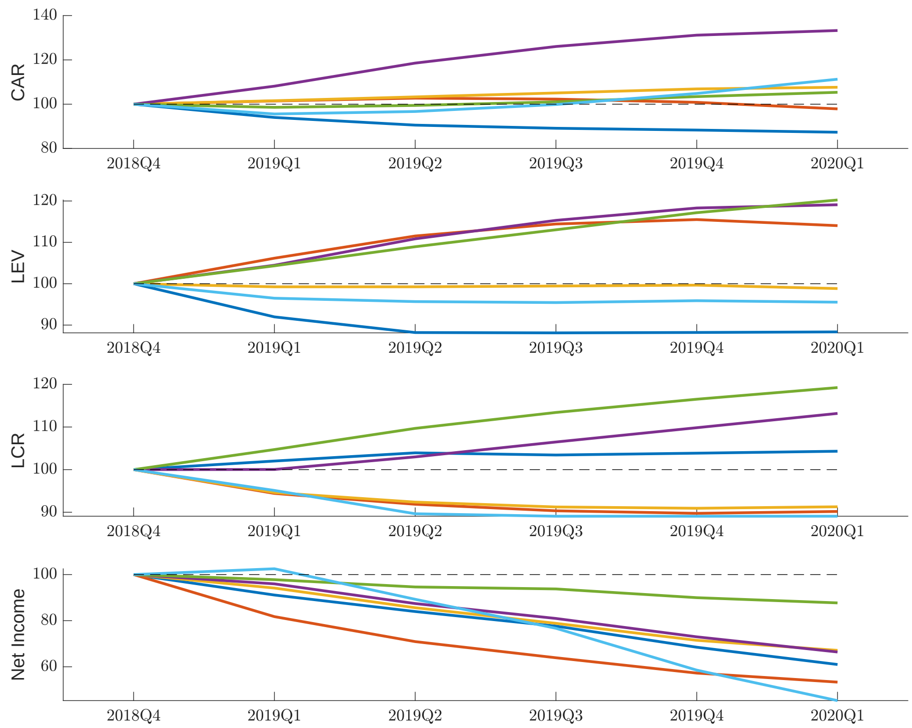

Figure 6. Evolution of regulatory ratios in the equilibrium with higher liquidity and leverage ratios and Domestic Stability Buffer not being released
Note: x-axes: periods in stress horizon; y-axes: LCR – liquidity coverage ratio τ_b^{LIQ}, LEV – leverage ratio τ_b^{LEV}, CAR – capital adequacy ratio τ_b^{CAP}; units are 100×(regulatory ratio at time t/regulatory ratio in 2018 Q4); each line represents one of the D-SIBs.

similar predictions to the previous calibration. For instance, business loans get reduced by 26.6%, in comparison to 26.84% in the calibration with all ratios being higher and 26.37% in the calibration with all ratio limits being at the regulatory required.

To summarize, the counter-cyclical buffer is effective in our scenario because its presence leads to higher capital levels of banks and lower lending to the real economy. However, the key differences in the lending behavior in stress vs. no stress environments come from the behavioral responses of banks and not from the binding capital ratios.

This prediction is in line with the simplifying example we presented in Section (5). In that example, the release of capital buffers played an important role in supporting lending whenever banks were capital constrained. Similarly, in the consistent stress scenario, only a few banks reach the regulatory minimum, and the behavior of most of the banks is not restricted by the capital constraints. This result, however,

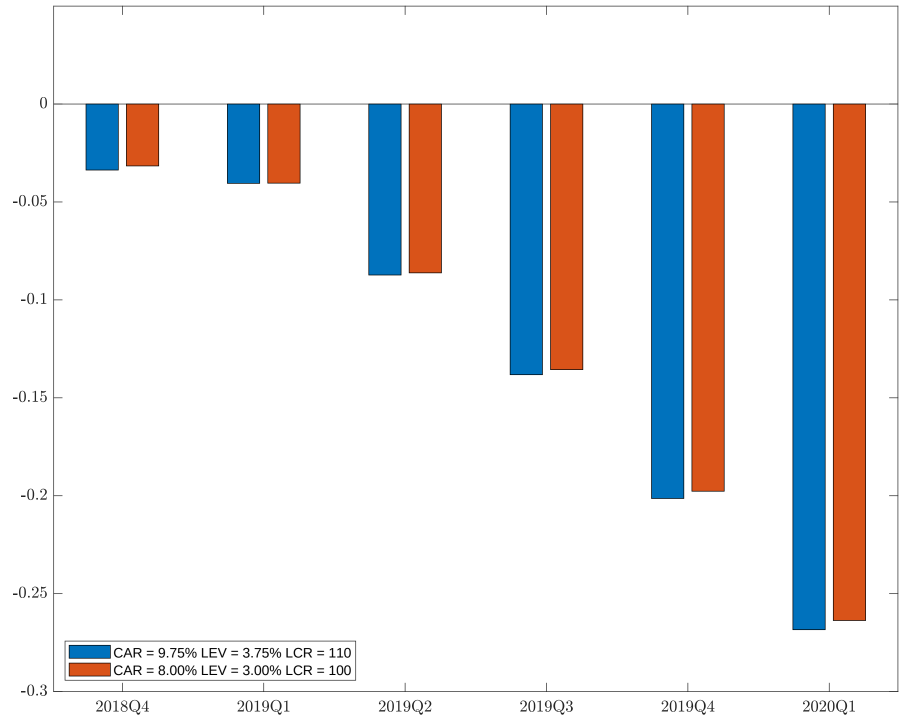

Figure 7. Evolution of business loans in the stress test scenario depending on whether domestic stability buffer is applied and higher leverage and liquidity ratios are targeted
Note: x-axes: periods in stress horizon; y-axes: aggregate change of business loans relative to the aggregate volume of business loan portfolio as of 2018Q4.

does not mean that the scenario is not severe. Rather, it is the introduction of the Basel III reforms that contributed to a substantial increase of capital buffers kept by Canadian D-SIBs and, consequently, to a significant increase of the loss absorption capacity.

(5) Minimum role of strategic behaviors for lending activity. We compare the baseline model that allows banks to internalize price externalities with an alternative version of the model where price-taking banks do not consider other banks' management actions. The results indicate that the competition among banks does not impact their behavior during stress to a high degree. For instance, business lending would shrink only 0.10% less if banks acted in isolation and the income rates were not subject to the externalities (see Figure 8). We explain it with the fact that the Canadian banking sector is highly concentrated; as such, marginal changes in the market shares of D-SIBs are unlikely to be very costly to them.

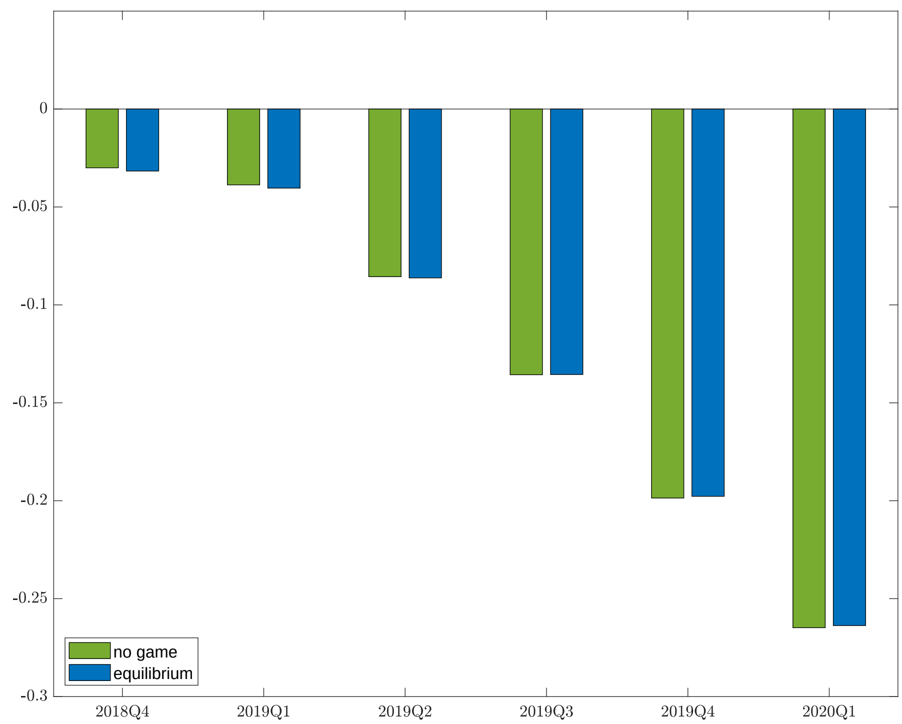

Figure 8. Evolution of business loans in the stress test scenario
Note: x-axes: periods in stress horizon; y-axes: aggregate change of business loans relative to the aggregate volume of business loan portfolio observed in 2018Q4; 'no game': banks reoptimize, not considering that there is an impact of their decisions on returns from assets; 'Nash equilibrium': reoptimization with calibrated price impact (baseline model).

6.4. **Sensitivity analysis: calibration of elasticities.** Few additional comments need to be made regarding the interpretation of our results. The quantitative model predictions are dependent on the elasticities α^S and α^L. Because the price elasticities are estimated using non-stress data, it would be beneficial to conduct some sensitivity analysis to account for possible unobserved effects of competition. For robustness checks, we run the model multiple times for different plausible values of sensitivity parameters. In particular, we do Monte Carlo draws using normal distribution with mean and standard deviation produced by the sample. The results are given in Figures 10, 11, 12, and 13. We conclude that our qualitative findings about the bank's responses to stress are robust to the selection of alternative price sensitivity parameters α^L and α^S (see Figure 10).

7. Conclusions

We have built a model of banks' strategic behavior under stress using a concept of risk-adjusted profitability as a criterion for selecting the optimal structure of assets in the balance sheets. The model accounts for regulatory constraints imposed on solvency and liquidity conditions of banks. The model explicitly delivers equilibrium balance sheets, which happen to be unique in most empirical cases. This allows us to fit the model to historical data and estimate unobserved parameters of the model. We apply the framework to a granular structure of banks' assets and liabilities in the Canadian banking system to test their resilience given an unlikely but plausible risk scenario. Consequently, we are able to derive optimal investment strategies using the historical data and in the simulated environment. In the stress exercise that we conduct, we illustrated the importance of management actions of banks when assessing the systemic risk implications of stress conditions. As shown, banks transfer systemic risk back to the real economy by reducing lending to the most stressed sectors of the economy. Release of the counter-cyclical capital buffer mitigates the contagious impact on real economy but to a small degree, given that most banks are not capital constraints. Banks are also able to stay at or above the regulatory required leverage and liquidity limits. The impact of stress is heterogenously reflected in banks' regulatory ratios, with some banks improving the ratios through a selective liquidation of securities, and with some banks keeping the deteriorating assets on the balance sheet.

The model has several policy implications. First, the paper explores new aspects of the systemic risk transfer not considered previously in the stress testing literature. We highlight the importance of profitability considerations of banks in their lending reduction. As such, we move beyond short-term impacts of stress, such as fire sales, credit losses, and funding cost hikes by complementing them with the longer-term predictions. These are sequential depletion of banks' capital, bank-specific reduction to lending, structural changes in the portfolios of banks, and negative trends in their profitability.

Second, the model can also quantify the role of capital requirements in propagation or reduction of systemic risk in the context of realistic balance sheets. As such, the proposed model can be used to assess the impact of counter-cyclical regulations during stress events.

Third, the model is well suited to consider the interaction of regulatory requirements on financial stability and credit in the context of specific economy. This would contribute to the overall assessment of effectiveness of Basel III reforms and their interdependence in the period of stress.

The model can be improved in several dimensions. First, we work with a linear dependence between asset prices and management actions, which allows us to write down the incentives of banks as a quadratic programming problem. One of the extensions could be to relax this assumption of linearity. For instance, one could embed concave fire-sale prices. In the same way, it is possible to make funding costs conditional not only on the indebtedness of banks but also on their liquidity positions. The key difficulty of such extensions would be to calibrate the unobserved sensitivities using historical data, which would require the utilization of non-linear econometric models.

The second limitation that we would like to address is the partial equilibrium setup of the model. Currently, the severity of lending constraints imposed by banks is not evaluated for the real economy. Thus, there is no feedback loop between the real economy and the financial market. Future work would capture the reverse relationship between lower credit supply and deteriorating financial conditions in the general equilibrium framework.[36] The challenge of doing so would be to keep banks strategic and heterogeneous in each period of the model.

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Appendix A. Tables and figures for stylized example

shock: credit loss on L0, CAR= 8.5				shock: credit loss on L0, CAR= 8.0			
		-3pp	-6pp			-3pp	-6pp
opt vs init	Δ CAR	-1.0	-1.5	opt vs init	Δ CAR	-1.0	-2.0
	Δ LCR	-0.1	4.1		Δ LCR	-0.1	1.2
	Δ L0	-0.1	-12.1		Δ L0	0.1	-3.8
	Δ L1	0.1	-2.8		Δ L1	0.1	-0.8
	Δ S0	-0.1	-2.9		Δ S0	-0.1	-1.0
opt vs nogame	Δ CAR	0.0	0.0	opt vs nogame	Δ CAR	0.0	0.0
	Δ LCR	0.0	0.1		Δ LCR	0.0	0.1
	Δ L0	0.0	0.0		Δ L0	0.0	0.0
	Δ L1	0.0	0.0		Δ L1	0.0	0.0
	Δ S0	0.1	0.1		Δ S0	0.0	0.1

Table 7. Credit loss (stylized simulations)
The table presents results of the simulations where banks adjust their balance sheets after both banks experience an increase of default probabilities for loans L0, which translates into losses to capital and elevated risk weights for outstanding volumes in segment L0. Changes of assets (ΔL0, ΔL1, ΔS0) are in % of the exposures before stress, and financial ratios (ΔCAR, ΔLCR) are in pps. 'opt vs init' – difference between outcomes of the Nash equilibrium and the initial structure of the banking system; 'opt vs nogame' – difference between outcomes of the Nash equilibrium and the outcome of reoptimization of balance sheets in isolation (i.e., without strategic interaction of the management actions).

shocked bank				rest of the system			
		-3pp	-6pp			-3pp	-6pp
opt vs init	Δ CAR	-0.9	-1.5	opt vs init	Δ CAR	0.0	-0.4
	Δ LCR	-0.1	4.2		Δ LCR	0.1	-1.5
	Δ L0	0.0	-12.1		Δ L0	0.2	6.4
	Δ L1	0.1	-2.8		Δ L1	0.1	1.6
	Δ S0	-0.2	-2.7		Δ S0	0.2	2.3
opt vs nogame	Δ CAR	0.0	0.0	opt vs nogame	Δ CAR	0.0	-0.4
	Δ LCR	0.0	0.0		Δ LCR	0.1	-1.5
	Δ L0	-0.1	0.0		Δ L0	0.0	6.2
	Δ L1	0.0	0.0		Δ L1	0.0	1.5
	Δ S0	-0.1	-0.1		Δ S0	0.1	2.2

Table 8. Idiosyncratic credit loss impacting only one bank (stylized simulations)
Note: The table presents results of the simulations where banks adjust their balance sheets after one of the banks experiences an increase of default probabilities for loans L0, which translates into losses to capital and elevated risk weights for outstanding volumes in segment L0. Changes of assets (ΔL0, ΔL1, ΔS0) are in % of the exposures before stress and financial ratios (ΔCAR, ΔLCR) are in pps. 'opt vs init' – the difference between outcomes of the Nash equilibrium and the initial structure of the banking system; 'opt vs nogame' – the difference between outcomes of the Nash equilibrium and the outcome of reoptimization of balance sheets in isolation (i.e., without strategic interaction of the management actions).

shock to both banks			
		+0.001	+0.002
opt vs init	Δ CAR	-1.5	-1.5
	Δ LCR	8.4	11.1
	Δ L0	-13.0	-13.7
	Δ L1	-2.8	-2.8
	Δ S0	3.0	6.9
opt vs nogame	Δ CAR	0.0	0.0
	Δ LCR	0.4	-0.4
	Δ L0	-0.1	-0.1
	Δ L1	0.0	0.0
	Δ S0	0.6	0.9

Table 9. Shock to the sensitivity of funding costs to the leverage ratios: homogeneous system (stylized simulations)
Note: The table presents results of the simulations where banks rebalance their portfolios after sensitivity of funding costs to changes in banks' leverage ratios increase for both banks (two cases: c^f_{coll} increases by +0.001 and +0.002). Changes of assets (ΔL0, ΔL1, ΔS0) are in % of the exposures before stress and financial ratios (ΔCAR, ΔLCR) are in pps. 'opt vs init' – the difference between outcomes of the Nash equilibrium and the initial structure of the banking system; 'opt vs nogame' – the difference between outcomes of the Nash equilibrium and the outcome of reoptimization of balance sheets in isolation (i.e., without strategic interaction of the management actions).

shocked bank				rest of the system			
		+0.001	+0.002			+0.001	+0.002
opt vs init	Δ CAR	-1.5	-1.5	opt vs init	Δ CAR	-1.5	-1.5
	Δ LCR	2.6	-1.8		Δ LCR	4.2	7.0
	Δ L0	-7.8	-11.8		Δ L0	-7.8	-12.7
	Δ L1	-1.9	-2.8		Δ L1	-1.9	-2.8
	Δ S0	0.1	-4.1		Δ S0	0.1	0.9
opt vs nogame	Δ CAR	0.0	0.0	opt vs nogame	Δ CAR	0.0	0.0
	Δ LCR	0.9	1.9		Δ LCR	-0.3	-1.0
	Δ L0	-0.2	-0.4		Δ L0	0.1	0.3
	Δ L1	0.0	0.0		Δ L1	0.0	0.0
	Δ S0	1.2	2.4		Δ S0	-0.5	-1.5

Table 10. Shock to the sensitivity of funding costs to the leverage ratios: heterogeneous system (stylized simulations)
Note: The table presents results of the simulations where banks rebalance their portfolios after sensitivity of funding costs to changes in banks' leverage ratios increase (two cases: c^f_{coll} increases by +0.001 and +0.002) but one of the banks experiences more favorable funding conditions, i.e., it can raise new funding with no costs. Changes of assets (ΔL0, ΔL1, ΔS0) are in % of the exposures before stress and financial ratios (ΔCAR, ΔLCR) are in pps. 'opt vs init' – the difference between outcomes of the Nash equilibrium and the initial structure of the banking system; 'opt vs nogame' – the difference between outcomes of the Nash equilibrium and the outcome of reoptimization of balance sheets in isolation (i.e., without strategic interaction of the management actions).

risk aversion = 2				risk aversion = 10			
		+3pp	+6pp			+3pp	+6pp
opt vs init	Δ CAR	-1.0	-1.5	opt vs init	Δ CAR	-1.0	-1.5
	Δ LCR	-0.1	4.1		Δ LCR	0.0	3.5
	Δ L0	-0.1	-12.1		Δ L0	-1.9	-12.2
	Δ L1	0.0	-2.8		Δ L1	0.2	-2.3
	Δ S0	-0.1	-2.9		Δ S0	-1.7	-4.0
opt vs nogame	Δ CAR	0.0	0.0	opt vs nogame	Δ CAR	-0.1	0.0
	Δ LCR	0.0	0.1		Δ LCR	0.2	0.5
	Δ L0	0.0	0.0		Δ L0	0.9	0.1
	Δ L1	0.0	0.0		Δ L1	-0.1	-0.3
	Δ S0	0.1	0.1		Δ S0	0.8	0.6

Table 11. Shock to the risk aversion of the banks (stylized simulations)
Note: The table presents results of model simulations assuming an increase of the risk aversion parameter from 2 to 10 (i.e., $\gamma = 10$). Banks are assumed to experience a credit loss which stems from an increase of the default probability on loans L0 (two scenarios: +3pp and +6pp increase). Changes of assets (ΔL0, ΔL1, ΔS0) are in % of the exposures before stress and financial ratios (ΔCAR, ΔLCR) are in pps. 'opt vs init' – the difference between outcomes of the Nash equilibrium and the initial structure of the banking system; 'opt vs nogame' – the difference between outcomes of the Nash equilibrium and the outcome of reoptimization of balance sheets in isolation (i.e., without strategic interaction of the management actions).

shock: expected return on L0			
		-20bps	-40bps
opt vs init	Δ CAR	0.1	0.1
	Δ LCR	0.9	1.8
	Δ L0	-1.3	-2.6
	Δ L1	0.1	0.1
	Δ S0	0.7	1.4
opt vs nogame	Δ CAR	0.0	-0.1
	Δ LCR	-0.4	-0.9
	Δ L0	0.6	1.3
	Δ L1	0.0	-0.1
	Δ S0	-0.3	-0.7

Table 12. Shock to expected return (stylized simulations)
The table presents results of the simulations where banks adjust their balance sheets after expected return on loans L0 decreases (two cases: $r^{L0,c,new}$ declines by $-20bps$ and $-40bps$). Changes of assets (ΔL0, ΔL1, ΔS0) are in % of the exposures before stress and financial ratios (ΔCAR, ΔLCR) are in pps. 'opt vs init' – the difference between outcomes of the Nash equilibrium and the initial structure of the banking system; 'opt vs nogame' – the difference between outcomes of the Nash equilibrium and the outcome of reoptimization of balance sheets in isolation (i.e., without strategic interaction of the management actions).

Appendix B. Derivation for fitted sensitivities

We will use the equilibrium condition (19) to derive the regression equation:

$$Q_b x_b = -m_b^0 - \sum_k m_{bk} x_k.$$

After substituting formulas for matrices Q_b, m_b, and m_{bk}, we derive

$$2\begin{bmatrix}\alpha^L & 0\\ 0 & \alpha^S\beta^S\end{bmatrix} x_b + 2(c_f + c_f^{coll} w_b^{coll})\frac{x_b 1_{N_x+M_x}}{e_b} 1_{N_x+M_x}$$

$$+\frac{2\gamma}{e_b}\begin{bmatrix}\mathbb{V}(\epsilon_b^{L,c,new'}) & Cov(\epsilon_b^{L,c,new'}, \epsilon_b^{S'})\\ Cov(\epsilon_b^{L,c,new'}, \epsilon_b^{S'}) & \mathbb{V}(\epsilon_b^{S'})\end{bmatrix} x_b + \frac{2\gamma}{e_b}\mathbb{V}(\epsilon_b^z) x_b$$

$$= \begin{bmatrix} r_b^{L,c,new} - \alpha^L \mu_b^{L,x} a_b^{L,x}\\ r_b^S - \alpha^S(\beta^S - \mu_b^S(1 + r_b^f)) a_b^{S,x}\end{bmatrix}$$

$$-\left[c_b^{0\prime} w_b + \frac{1}{e_b}(c_f w_b' \nu_b + c_f^{coll} w_b^{coll} \nu_b^{coll}) 1'_{N_x+N+M_x} a_b - z_b\right] 1_{4,N_x+M_x}$$

$$-\frac{2\gamma}{e_b}\begin{bmatrix}\mathbb{C}ov(\epsilon_b^{L,c,n}, \epsilon_b^a)\\ \mathbb{C}ov(\epsilon_b^S, \epsilon_b^a)\end{bmatrix}$$

$$-\sum_k \begin{bmatrix}\alpha^L & 0\\ 0 & \alpha^S\beta^S\end{bmatrix} x_k$$

$$-\frac{1}{N_b - 1}\sum_k (c_f + c_f^{coll} w_k^{coll})\frac{x_k 1_{N_x+M_x}}{e_k} 1_{N_x+M_x}.$$

Equilibrium expressions for sensitivities follow directly from the equation above.

Appendix C. Sufficient condition for positive definiteness

We aim to prove that $x'\bar{Q}x > 0$ for non-zero vector $x = (x_1, x_2, ...x_{N^b})$ consisting of arbitrary subvectors $x_i \in R^{N_x+M_x}$.

We introduce simplifying notation for the additive components of Q_b and m_{bk}. First, we define diagonal matrix

$$A = \begin{bmatrix}\alpha^L & 0\\ 0 & \alpha^S\beta^S\end{bmatrix}$$

and corresponding matrix $A^{1/2}$ derived from A by taking square roots of the diagonal elements. Second, we define B_b as a matrix of ones multiplied by a constant term:

$$B_b = \frac{1}{e_b}(c_f + c_f^{coll} w_b^{coll}) 1_{N_x+M_x, N_x+M_x}.$$

Third, we group together the covariance matrix and the non-interest income variance term.

$$V_b = \frac{2\gamma}{e_b} \begin{bmatrix} \mathbb{V}\left(\epsilon_b^{L,c,new'}\right) & \mathbb{C}ov\left(\epsilon_b^{L,c,new'}, \epsilon_b^{S'}\right) \\ \mathbb{C}ov\left(\epsilon_b^{L,c,new'}, \epsilon_b^{S'}\right) & \mathbb{V}\left(\epsilon_b^{S}\right) \end{bmatrix} + \frac{2\gamma}{e_b}\mathbb{V}(\epsilon_b^z) 1_{N_x+M_x, N_x+M_x}$$

With the new notation in place, we represent non-strategic matrices of the utility function of each bank b as

$$Q_b = 2A + 2B_b + V_b$$
$$m'_{bk} = A + \frac{1}{N_b - 1} B_k.$$

Then it is sufficient to prove the positive sign of the quadratic form below:

$$x'\bar{Q}x = x' \begin{bmatrix} 2A & A & \vdots & A \\ A & 2A & \vdots & A \\ \vdots & \vdots & \vdots & \vdots \\ A & A & \vdots & 2A \end{bmatrix} x$$

$$+x' \begin{bmatrix} \frac{3}{2}B_1 - \frac{\sum_{i\neq 1} B_i}{2(N_b-1)} & 0 & \vdots & 0 \\ 0 & \frac{3}{2}B_2 - \frac{\sum_{i\neq 2} B_i}{2(N_b-1)} & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & \frac{3}{2}B_{N_b} - \frac{\sum_{i\neq N_b} B_i}{2(N_b-1)} \end{bmatrix} x$$

$$+x' \begin{bmatrix} \frac{1}{2}B_1 + \frac{\sum_{i\neq 1} B_i}{2(N_b-1)} & \frac{B_2}{N_b-1} & \vdots & \frac{B_{N_b}}{N_b-1} \\ \frac{B_1}{N_b-1} & \frac{1}{2}B_2 + \frac{\sum_{i\neq 2} B_i}{2(N_b-1)} & \vdots & \frac{B_{N_b}}{N_b-1} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{B_1}{N_b-1} & \frac{B_2}{N_b-1} & \vdots & \frac{1}{2}B_{N_b} + \frac{\sum_{i\neq 1} B_{N_b}}{2(N_b-1)} \end{bmatrix} x$$

$$+x' \begin{bmatrix} V_1 & 0 & \vdots & 0 \\ 0 & V_2 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & V_{N_b} \end{bmatrix} x.$$

The first term of $x'\bar{Q}x$ is always positive because it is equal to the sum of squares

$$\sum_i \left(A^{1/2} x_i\right)' \left(A^{1/2} x_i\right) + \left(A^{1/2} \sum_i x_i\right)' \left(A^{1/2} \sum_i x_i\right) > 0.$$

The second term is positive due to the assumption on the diagonal elements. In particular, for arbitrary k-th block matrix on the diagonal

$$\frac{3}{2}x_k'B_kx_k - \frac{1}{2(N_b-1)}x_k'\sum_{i\neq k}B_ix_k \geq \left(\frac{1}{e_{b^*}} - \frac{c_f}{3(c_f+c_f^{coll})}\sum_{k\neq b^*}\frac{1}{(N_b-1)}\frac{1}{e_k}\right)\frac{3(c_f+c_f^{coll})}{2}(x_k'1_{N_x+M_x})^2 \geq 0.$$

To derive relationship above, we used the fact that share of collateralized funding in total funding w_k is a number between zero and one, and $c_f^{coll} < 0$. Equity e^* is defined as the largest equity in the banking system.

The third term is positive as the sum of squares multiplied by constant terms

$$\frac{1}{N_b-1}\sum_k\sum_{i\neq k}\frac{1}{2e_k}(c_f+c_f^{coll}w_k^{coll})(1_{N_x+M_x}'(x_i+x_k))(1_{N_x+M_x}'(x_i+x_k))' \geq 0.$$

The variance-covariance product is also positive given our model assumptions.
Moreover, at least one of the inequalities above is strictly positive if vector x is non-zero. This proves that $x'\bar{Q}x > 0$. Consequently, the theorem of Rosen (1965) is applied.

Appendix D. Risk weights of capital requirements

A stress test scenario not only impacts the risk and return features of the asset and liability categories in banks' balance sheets, but also restricts the set of available strategies by changing some of the parameters of the capital constraint. This impact -ealizes via risk weights, defined by Basel III capital requirements, BCBS (2017). Risk weights applied to calculate the total risk-weighted assets, and consequently the capital adequacy ratio, are sensitive to changes in some of the risk characteristics of the loans. The sensitivity applies to loan portfolios following the internal risk-based modeling as stipulated in the Basel III guidelines. The risk weights are a function of expected default probabilities, proxies of correlations between PDs at a loan level in the portfolios, loss given defaults, and maturity.

We use the Basel III specifications because they became a standard measure of unexpected credit losses embedded into regulatory as well as risk-management practices. We apply the formulas to calculate the changes in the risk weights during the scenario. Initial risk weights are calibrated with internal credit rating in mind based on the regulatory submissions of banks.

The shape of the function defining the theoretical regulatory risk weights depends on the loan category. Most generally, the differences in functional relationship of risk weights and credit risk parameters hinge on whether loans are in the retail or corporate segment. Within the two segments, further peculiarities in the risk weights are related to loans classifications as follows:

(1) retail: ($\underline{\overline{m}}$) mortgage, ($\underline{\overline{c}}$) consumer (qualified revolving), ($\underline{\overline{o}}$) other retail;

(2) corporate: ($\underline{\overline{r}}$) regulated financial institutions, ($\underline{\overline{u}}$) unregulated financial institutions, ($\underline{\overline{s}}$) small and medium-sized enterprises (SMEs), ($\underline{\overline{h}}$) high-volatility commercial real estate (HVCRE).

The risk weights for sub-portfolios of retail loans differ by a correlation factor R. For default probability d, loss-given default lgd and sub-portfolios $\pi \in \{\underline{\overline{m}}, \underline{\overline{c}}, \underline{\overline{o}}\}$

$$R(d,\pi) = \begin{cases} 0.15 & \text{if } \pi = \underline{\overline{m}} \\ 0.04 & \text{if } \pi = \underline{\overline{c}} \\ 0.03 f_a(d) + 0.16(1 - f_a(d)) & \text{if } \pi = \underline{\overline{o}} \end{cases} \tag{24}$$

where

$$f_a(d) = \frac{1 - \exp(-35d)}{1 - \exp(-35)}.$$

Then, denoting Φ the cumulative density of a standard normal distribution, the risk weight $\omega(d,\pi)$ implied by the IRB formula reads:

$$\omega(d, lgd, \pi) = 12.5 \left(lgd \times \Phi \left(\frac{\Phi^{-1}(d)}{\sqrt{1 - R(d,\pi)}} + \sqrt{\frac{R(d,\pi)}{1 - R(d,\pi)}} \Phi^{-1}(0.999) \right) - lgd \times d \right).$$

Similarly, for the corporate sub-portfolios, the main differentiating feature of the risk weight is the correlation parameter. Maturity μ^ω is another parameter affecting risk weights of corporate exposures. For $\pi \in \{\underline{\overline{r}}, \underline{\overline{u}}, \underline{\overline{s}}, \underline{\overline{h}}\}$,

$$R(d,\pi) = \begin{cases} 0.12 f_a(d) + 0.24(1 - f_a(d)) & \text{if } \pi = \underline{\overline{r}} \\ 1.25(0.12 f(d) + 0.24(1 - f_a(d))) & \text{if } \pi = \underline{\overline{u}} \\ 0.12 f_a(d) + 0.24(1 - f_a(d)) - 0.04(1 - (S_{25} - 5)/45) & \text{if } \pi = \underline{\overline{s}} \\ 0.12 f_a(d) + 0.30(1 - f_a(d)) & \text{if } \pi = \underline{\overline{h}} \end{cases} \tag{25}$$

where

$$f_a(d) = \frac{1 - \exp(-50d)}{1 - \exp(-50)}.$$

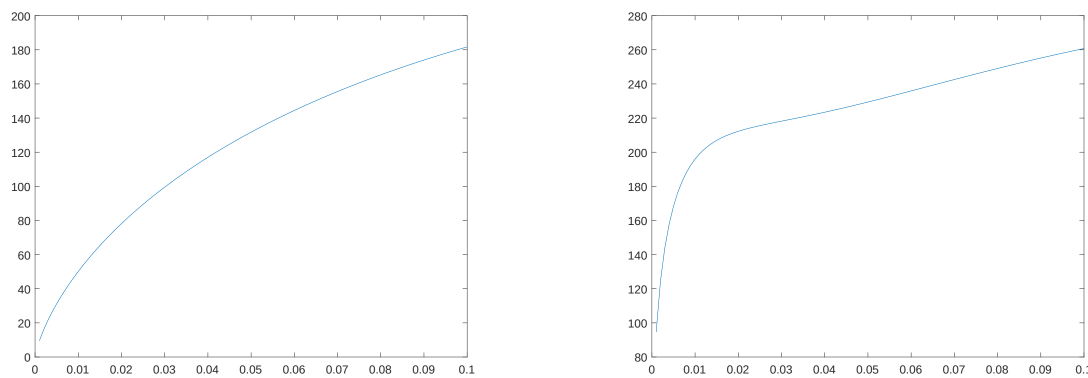
Notably, the risk weight for SMEs factors in a parameter S_{25}, which gauges the annual sales of the SME. We set this parameter to 25, which is half of the maximum annual sale volume that allows a borrower to be classified into the SME sub-portfolio. Consequently, the risk weight parameter can be calculated as follows:

$$\omega(d, lgd, \mu^\omega, \pi) = 12.5 \left(lgd \times \Phi \left(\frac{\Phi^{-1}(d)}{\sqrt{1 - R(d,\pi)}} + \sqrt{\frac{R(d,\pi)}{1 - R(d,\pi)}} \Phi^{-1}(0.999) \right) - lgd \times d \right)$$

$$\times \frac{1 + (\mu^\omega - 2.5) M^{adj}(d)}{1 - 1.5 \mu^{adj}(d)},$$

where $M^{adj}(d) = (0.11852 - 0.05478 \log(d))^2$ is a maturity adjustment.

We illustrate the sensitivity of the risk weights for two sub-portfolios, retail-mortgage and corporate-SME, to changes in default probabilities with a fixed loss given default equal to 40%; see Figure 9. The steepness of the curves declines in probabilities of default translating to more penalizing increases to the risk weights if the changes affect initially small probabilities of default.



(a) Retail, mortgage ($\overline{\underline{m}}$)

(b) Corporate, SMEs ($\underline{\overline{s}}$)

Figure 9. Risk weights as a function of default probability for two categories of IRB loan portfolios
Note: Loss-given default=.40, Maturity=12; x-axis – default probability; y-axis – risk weight

Appendix E. Tables and figures for consistent macro scenario

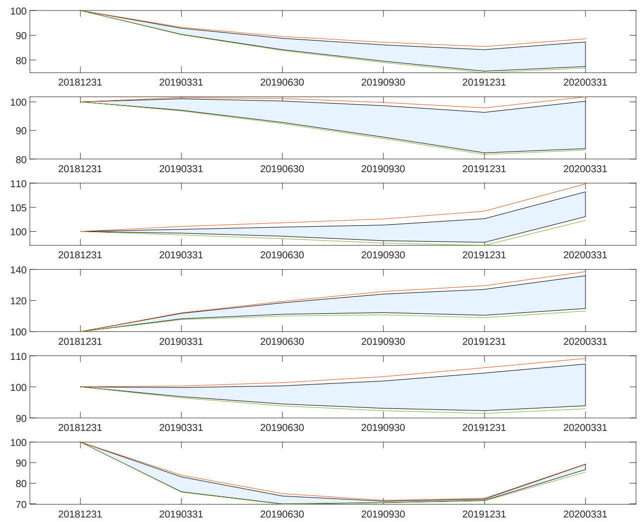


Figure 10. Evolution of the distribution of capital ratio across different values of price sensitivities
Each subplot refers to one bank; y-axis – change of the CAR as a ratio with CAR in 2018Q4 (in%); shaded area – interquartile range with respect to sampled values of α^L and α^S; black line – median; green and red lines – 10% and 90% percentiles

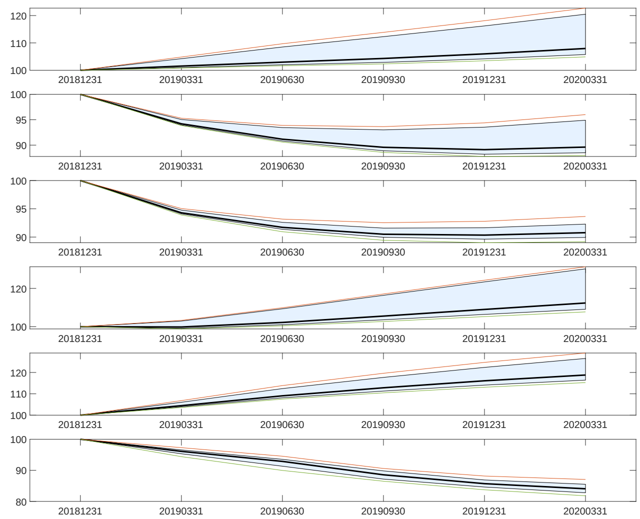


Figure 11. Evolution of the distribution of liquidity ratio across different values of price sensitivities
Each subplot refers to one bank; y-axis – change of the LCR as a ratio with LCR in 2018Q4 (in%); shaded area – interquartile range with respect to sampled values of α^L and α^S; black line – median; green and red lines – 10% and 90% percentiles

	Median	SD	(1): Mean	(2) Asset & Bank FE	(3) FE and Macro
L0	4.56×10^{-7}	3.05×10^{-8}	4.87×10^{-7}	$1.32 \times 10^{-6***}$ (3.40×10^{-7})	$7.14 \times 10^{-6*}$ (3.27×10^{-6})
L1	7.55×10^{-7}	2.41×10^{-7}	1.27×10^{-6}	$2.09 \times 10^{-6} * * *$ (4.06×10^{-7})	$7.91 \times 10^{-6*}$ (3.23×10^{-6})
L2	1.61×10^{-7}	9.99×10^{-8}	1.74×10^{-6}	$2.57 \times 10^{-6***}$ (1.54×10^{-7})	$8.40 \times 10^{-6*}$ (3.40×10^{-6})
L3	5.46×10^{-7}	1.13×10^{-7}	1.02×10^{-6}	$1.86 \times 10^{-6***}$ (4.57×10^{-7})	$7.68 \times 10^{-6*}$ (3.22×10^{-6})
L4	1.99×10^{-6}	1.35×10^{-7}	2.18×10^{-6}	$3.01 \times 10^{-6***}$ (4.42×10^{-7})	$8.83 \times 10^{-6*}$ (3.46×10^{-6})
L5	5.55×10^{-6}	7.51×10^{-7}	7.66×10^{-6}	$8.49 \times 10^{-6**}$ (2.73×10^{-6})	$1.43 \times 10^{-5**}$ (5.45×10^{-6})
L6	2.27×10^{-5}	7.76×10^{-6}	4.47×10^{-5}	$4.57 \times 10^{-5*}$ (1.93×10^{-5})	$5.16 \times 10^{-5*}$ (2.24×10^{-5})
L7	7.61×10^{-6}	9.53×10^{-6}	1.96×10^{-5}	2.03×10^{-5} (1.09×10^{-5})	2.62×10^{-5} (1.41×10^{-5})
S0	5.35×10^{-8}	2.34×10^{-9}	5.78×10^{-8}	$8.92 \times 10^{-7**}$ (3.53×10^{-7})	$6.72 \times 10^{-6*}$ (3.32×10^{-6})
S1	5.65×10^{-7}	1.69×10^{-6}	1.05×10^{-6}	-2.25×10^{-8} (2.61×10^{-6})	$7.68 \times 10^{-6*}$ (3.14×10^{-6})
S2	1.50×10^{-7}	9.15×10^{-9}	1.54×10^{-7}	$1.05 \times 10^{-6***}$ (3.00×10^{-7})	$7.68 \times 10^{-6*}$ (3.02×10^{-6})
S3	5.62×10^{-7}	8.58×10^{-8}	6.64×10^{-7}	$1.97 \times 10^{-6***}$ (4.33×10^{-7})	$7.16 \times 10^{-6*}$ (2.81×10^{-6})
3M T-bill yield	−−	−−	−−	−−	9.90×10^{-6} (8.53×10^{-6})
5Y Gov bond yield	−−	−−	−−	−−	-8.49×10^{-6} (6.73×10^{-6})
Real GDP growth	−−	−−	−−	−−	-2.63×10^{-4} (2.87×10^{-4})
CPI growth	−−	−−	−−	−−	-1.55×10^{-4} (1.94×10^{-4})
Bank Fixed Effects	−−	−−	−−	*Yes*	*Yes*
Observations	993	993	993	993	993
R^2	–	–	0.136	0.1907	0.2081
Adjusted R^2	–	–	0.136	0.1907	0.2081

Note: $^{*}p<0.10$; $^{**}p<0.05$; $^{***}p<0.01$
Robust standard errors are in parentheses.
Errors are clustered at the bank level.

Table 5. Sensitivities of returns to trading volumes of loans (L0-L7) and securities (S0-S3): summary statistics and regression results

CET1 Capital Ratio	Capital Conservation Ratio
4.5% – 5.375%	100%
> 5.375% – 6.250%	80%
> 6.250% – 7.125%	60%
> 7.125% – 8.0%	40%
> 8.0%	0%

Table 6. Minimum capital conservation ratios for D-SIBs at various levels of CET1 ratio

		volume		return		risk		ω		ϕ	β^S	μ	lgd
		mean	std	mean	std	mean	std	mean	std	mean	mean	mean	mean
assets	L0	12.3	5.6	1.0	0.0	0.4	0.1	40.7	8.7	0		4.7	
	L1	25.9	15.9	0.8	0.2	0.8	1.2	10.8	11.0	0		4.9	
	L2	49.8	30.1	4.1	0.9	1.9	0.6	25.4	6.1	0		8.0	
	L3	120.7	60.8	0.7	0.0	0.1	0.0	12.9	3.4	0		14.1	
	L4	75.7	30.8	0.8	0.1	0.2	0.1	0.0	0.0	0		15.0	
	L5	17.6	19.0	0.9	0.1	0.1	0.0	32.9	13.5	0		4.7	
	L6	2.3	4.1	1.2	1.0	2.7	3.1	3.7	2.3	0		8.0	
	L7	3.5	3.2	0.7	0.4	0.3	0.1	3.7	2.3	0		4.7	
	S0	15.8	5.1	0.7	0.2	0.5	0.2	18.6	0	89.1	1.5	1.5	
	S1	18.5	4.8	1.1	0.5	0.7	0.2	106.0	2.0	50.0	1.0	1.0	
	S2	15.9	12.9	1.1	0.3	0.7	0.2	54.2	0	42.1	1.6	1.6	
	S3	13.4	11.8	0.4	0.1	0.1	0.0	2.0	0	45.5	16.4	50.0	
	O0	11.3	1.3	0.0	0.0	0.0	0.0	50.0	0	0			
liabilities	F0	138.8	63.2	0.1	0.0					15.0		16.5	
	F1	142.7	77.6	0.1	0.0					25.0		4.0	
	F2	20.6	12.5	0.4	0.1					13.5		4.0	
	F3	141.5	70.1	0.1	0.0					0			
	E0	36.6	18.2										

Table 13. Descriptive statistics of balance sheet data of Canadian D-SIBs for 2018Q4
Note: 'mean' – average across banks, 'std' – standard deviation across banks; volume – balance sheet volumes as % of total assets; return/cost – expected return on asset categories and cost of funding categories (%); risk of return on asset categories (%); ω – capital risk weight, ϕ^a/ϕ^l – liquidity asset weight and liability run-off rate, μ – maturity in quarters

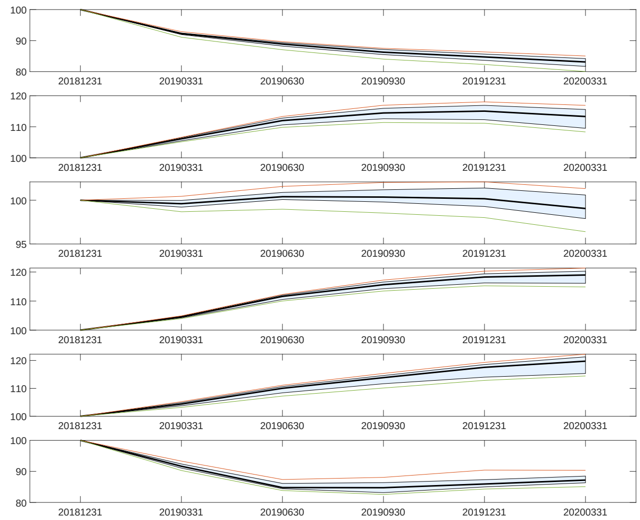


Figure 12. Evolution of the distribution of leverage ratio across different values of price sensitivities

Each subplot refers to one bank; y-axis – change of the leverage ratio as a ratio with LCR in 2018Q4 (in%); shaded area – interquartile range with respect to sampled values of α^L and α^S; black line – median; green and red lines – 10% and 90% percentiles

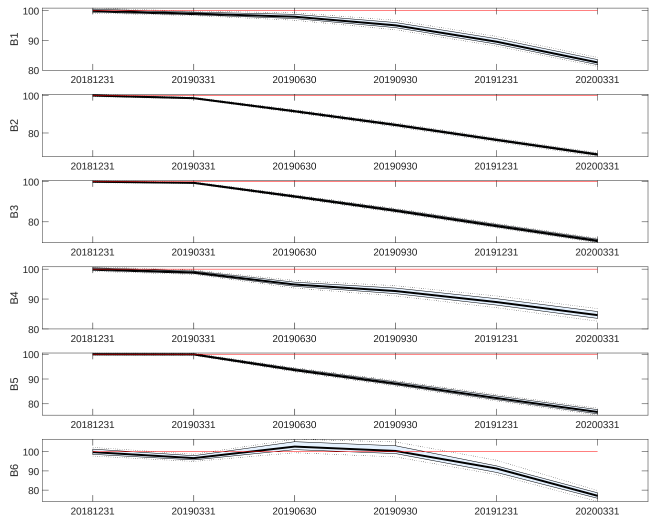


Figure 13. Evolution of the exposure to business loans across different values of price sensitivities
Each subplot refers to one bank; y-axis – change of the leverage ratio as a ratio with LCR in 2018 Q4 (in%); shaded area – interquartile range with respect to sampled values of α^L and α^S; black line – median; green and red lines – 10% and 90% percentiles