Intermediary Market Power and Capital Constraints

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Acknowledgements

Abstract

We examine how intermediary capitalization affects asset prices in a framework that allows for intermediary market power. We introduce a model in which capital-constrained intermediaries buy or trade an asset in an imperfectly competitive market, and we show that weaker capital constraints lead to both higher prices and intermediary markups. In exchange markets, this results in reduced market liquidity, while in primary markets it leads to higher auction revenues at an implicit cost of larger price distortion. Using data from Canadian Treasury auctions, we demonstrate how our framework can quantify these effects by linking asset demand to individual intermediaries' balance sheet information.

Topics: Financial institutions; Market structure and pricing
JEL codes: G12, G18, G20, D40, D44, L10

Résumé

Nous examinons comment les capitaux détenus par des intermédiaires influent sur les prix des actifs dans un contexte où ces acteurs ont un pouvoir de marché. Nous présentons un modèle dans lequel des intermédiaires ayant des contraintes de capitaux achètent ou négocient un actif sur un marché de concurrence imparfaite. Nous montrons que de faibles contraintes de capitaux engendrent une hausse des prix ainsi que des marges bénéficiaires plus grandes pour les intermédiaires. Sur les marchés boursiers, cette situation entraîne une baisse de la liquidité, tandis que sur les marchés primaires, elle occasionne une augmentation des revenus des adjudications au coût implicite d'une distorsion plus grande des prix. Au moyen de données tirées des adjudications de titres du Trésor canadien, nous démontrons comment notre modèle réussit à quantifier ces effets en reliant la demande d'actifs aux bilans des intermédiaires.

Sujets : Institutions financières, Structure de marché et fixation des prix
Codes JEL : G12, G18, G20, D40, D44, L10

1 Introduction

What moves asset prices is one of the oldest questions in finance. The intermediary asset pricing literature suggests that the prices of many assets depend not only on the preferences of households, but also on the equity capitalization of financial intermediaries, called dealers (e.g., Gromb and Vayanos (2002); Brunnermeier and Pedersen (2009); He and Krishnamurthy (2012, 2013); Brunnermeier and Sannikov (2014)). In this literature, dealers typically face funding or capital constraints and execute trades in perfectly competitive markets. In practice, however, dealers enjoy market power—as documented for various trade settings, including Treasury, repo, foreign exchange, mortgage-backed securities, and equity securities lending markets (e.g., Hortaçsu et al. (2018); Wallen (2022); Allen and Wittwer (2023); An and Song (2023); Chen et al. (2023); Huber (2023); Pinter and Üslü (2023)).

Our contribution is to study how dealer capitalization affects asset prices and markups and to quantify the effect in a framework that allows for dealer market power (as in Wilson (1979); Klemperer and Meyer (1989); Kyle (1989); Vives (2011); Rostek and Weretka (2012); Rostek and Yoon (2021); among others). We introduce a model in which capital-constrained dealers buy (or trade) assets in an imperfectly competitive market, and we estimate it with data on Canadian Treasury auctions.[1]

In the model, presented in Section 2, dealers compete to buy (or trade) multiple units of an asset of uncertain supply that pays out an uncertain return in the future. They are risk averse and subject to a capital constraint, which depends on the auction outcome. In addition, dealers may have private information about their own balance sheet. The market clears via one out of two auction formats, which represent different financial markets, including primary auctions and exchanges. In the benchmark model, dealers submit decreasing demand functions that specify how much they are willing to pay for different units of the asset; the

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[1]Capital requirements aim to strengthen the risk management of banks and avoid the build-up of systemic risks. Our analysis does not incorporate how these risks change when relaxing constraints.

market clears at the price at which aggregate dealer demand meets supply, and each dealer wins the amount they asked for at that price (uniform price auction). In the extended model, winning dealers pay the prices they bid (discriminatory price auction).

Solving for an equilibrium in this environment is challenging because point-wise maximization—a common approach in the literature—does not work when bidders face outcome-dependent constraints. Instead, we must consider all feasible demand functions. By doing so, we derive necessary conditions for symmetric Bayesian Nash Equilibria (hereinafter referred to as equilibria). Moreover, we establish that there is no linear equilibrium when bidders have private information but derive a unique symmetric linear equilibrium for auctions in which bidders face common uncertainty about supply.

Our model highlights two effects of relaxing capital constraints. On the one hand, the market price increases. This is due to the fact that as the shadow costs of the capital constraint decrease, it becomes cheaper for risk-averse dealers to purchase larger quantities of the asset. On the other hand, dealers exert greater influence on the market price, deviating it further from the price that would result if the market was perfectly competitive. This means that the price distortion due to market power increases in primary markets and reduces market liquidity in exchange markets. The effect is absent in models with perfect competition and intuitively stems from the increased flexibility of dealers to manipulate market outcomes to their advantage when they face weaker constraints.

To demonstrate how to quantify these effects with our framework, we use data on Canadian Treasury auctions, presented in Section 3. The data combine bidding information on all Canadian government bond auctions between January 2019 and February 2022 with balance sheet information of the eight largest dealers and trade-level information from the secondary market. We observe all winning and losing bids and can identify each bidder thanks to unique identifiers. In addition, we see the quarterly Basel III Leverage Ratio (LR) of each dealer at the company holding level (following He et al. (2017)). The LR is the ratio between a bank's capital and its total leverage exposure (which measures a bank's total assets in ad-

dition to some items that aren't on the balance sheet: for instance, derivatives). It must be above a regulatory threshold and is considered to constitute a relevant capital constraint when trading government bonds (CGFS (2016)). Lastly, we gather data on all secondary market trades conducted by dealers. This allows us to measure how volatile returns are for dealers who purchase bonds at auction and subsequently sell them in the secondary market.

In Section 4, we estimate the two key parameters of the model: dealer risk aversion and the shadow costs of the capital constraint. To accomplish this, we employ estimation techniques from the auctions literature (introduced by Guerre et al. (2000); Hortaçsu and McAdams (2010); Kastl (2011)) to estimate each bidder's willingness to pay at a discrete number of points. Then we fit the model-implied functional form for the willingness to pay through these points. Finally, we take advantage of a temporary exemption of domestic government bonds from the LR during the COVID-19 pandemic to identify the degree of dealer risk aversion and their shadow costs of the capital constraint by analyzing how the willingness to pay varies around the policy change.

We find that dealers are moderately risk averse and face sizable shadow costs of the capital constraint. In fact, the median cost (of 3.5%) is as high as the typical margin a dealer charges their clients (i.e., the median difference between the price at which a dealer buys a bond at auction and the price at which she sells this bond in the secondary market). This suggests that dealers barely break even and might explain why so many dealers have left the market (as documented by Allen et al. (2023)).

A back-of-the-envelope calculation tells us that the market yield decreases and the yield distortion due to bid shading increases by 3.4 basis points (bps) when the shadow cost of the capital constraint decreases by 1%. This highlights that relaxing capital constraints leads to a reduction in bond yields, which overall increases auction revenues, at an implicit cost of larger yield distortion due to market power. When the interest rate level is high, these effects can be economically meaningful. In our sample period, where rates are low, however, the effects are small. This suggests that the Canadian regulator did not face a quantitatively

meaningful trade-off when deciding whether to relax or tighten capital constraints during the COVID-19 pandemic.

To conclude our study, in Section 5 we draw a closer connection to the intermediary asset pricing literature by extending our analysis to study how intermediary market power affects whether commonly considered intermediary frictions (such as moral hazard or capital constraints) matter for asset prices. We show that the price effect of these frictions depends on the degree of market power. Hopefully, this motivates future research that can analyze the implications of intermediary market power in a macroeconomic model of intermediary asset pricing and empirical research to assess the degree of competition in different financial markets.

Related literature. By analyzing how capital constraints affect asset prices when dealers have market power, we contribute to five distinct strands of the literature.

The paper's topic fits into an ample intermediary asset pricing literature that examines the impact of dealer capitalization (or leverage) on asset price behavior due to constraints on debt (e.g., Brunnermeier and Pedersen (2009)), or constraints on equity (e.g., He and Krishnamurthy (2013, 2012); Brunnermeier and Sannikov (2014)). Given our focus on banks, we follow He et al. (2017) and rely on equity constraints. The key difference relative to these (macroeconomic) models is that we zoom in on the market in which intermediaries interact and allow dealers to impact prices as a result of market power.[2]

The market clears via a multi-unit auction following Wilson (1979), Kyle (1989), and Klemperer and Meyer (1989), and more recent contributions including Vayanos (1999), Vives (2011), Gârleanu and Pedersen (2013), Rostek and Weretka (2012, 2015), Malamud and

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[2]Our extended model, presented in Section 5, is more similar to the intermediary asset pricing literature, which abstracts from market power with a few recent exceptions (e.g. Corbae and D'Erasmo (2021); Jamilov (2021); Wang et al. (2022)). These papers introduce monopolistic or Cournot competition of banks vis-à-vis firms or consumers, while we analyze market power in a trade setting. Therefore, our insights, especially those on the linkage between intermediary market power and capital constraints, are fundamentally different from those found in this literature.

Rostek (2017), Du and Zhu (2017), Kyle et al. (2017), Bergemann et al. (2021), Wittwer (2021), Rostek and Yoon (2021), and Zhang (2022). Our innovation in this literature is introducing bidder constraints that are dependent on the auction outcome. While we focus on capital constraints, our methods to characterize equilibria generalize to auctions with other types of constraints, such as budget constraints.

Our empirical analysis adds to an ample and growing literature on the relation between intermediary costs or constraints and asset prices (e.g., Adrian and Shin (2010), Adrian et al. (2014), He et al. (2017, 2022), Haddad and Muir (2021), Du et al. (2018, 2023a,b), and Siriwardane et al. (2022)). Most existing studies use market-level data, such as cross-sectional returns of different asset classes, and rely on proxy variables to capture intermediary costs, such as the VIX, or aggregate capital holdings. We zoom in on one market in which we can establish a direct relationship between dealer capitalization and asset demand. Further, we estimate the shadow costs of the capital constraint, while existing studies tend to provide lower bounds, and the dealer's degree of risk aversion. For this, we construct our own volatility measure using secondary market trade data.

For estimation, we adopt techniques from the literature on multi-unit auctions, developed by Guerre et al. (2000), Hortaçsu and McAdams (2010), and Kastl (2011) and extended by Hortaçsu and Kastl (2012) and Allen et al. (2020, 2023). This literature commonly assumes that financial institutions are risk-neutral, with the exception of Gupta and Lamba (2017), who exogenously choose a risk-aversion parameter to simulate their model. However, the assumption of risk neutrality stands in contrast to the related (market microstructure) literature which builds on Kyle (1989) and assumes that financial institutions have preferences with constant absolute risk aversion (CARA). We follow this literature and impose CARA preferences to circumvent the impossibility result by Guerre et al. (2009) that one cannot non-parametrically identify risk aversion (in first-price auctions).

This approach is similar to a handful of papers that estimate risk aversion in auctions for procurement, timber, and other non-financial goods (e.g., Campo et al. (2011); Bolotnyy

and Vasserman (2023); Häfner (2023); Luo and Takahashi (2023)). Of these papers, only Häfner (2023) considers multi-unit auctions and is therefore closest to our work. The auction approach complements the common macroeconomic practice of calibrating risk aversion for households using Euler equations. Since the risk aversion of intermediaries plays a crucial role in intermediary asset pricing models, our estimates can provide valuable input for calibrating these models.

2   Model

Our goal is to study how prices and markups change when capital constraints are relaxed or tightened and dealers have market power.

Definition 1. *A markup is the difference between the price at which the market would clear if it was perfectly competitive and the price at which it clears under imperfect competition; or equivalently, the difference between the yield at which the market clears under imperfect competition versus perfect competition.*

In our benchmark, we model market clearing via a uniform price auction, in which winning bidders pay the market clearing price. Here the markup increases in price impact—a common object of interest. In Appendix A we derive analogous results for discriminatory price auctions, in which winning bidders pay their own bids. In order to remain consistent with the prevailing theoretical literature, we opt for the uniform price auction as our benchmark despite our empirical application involving discriminatory price auctions.

The market may be one-sided, meaning that bidders buy but not sell, or double-sided, so that bidders buy and sell. In practice, some primary markets, for instance in the U.S., clear via one-sided uniform price auctions, while others, for instance in Canada, clear via one-sided discriminatory price auctions. Trading on an exchange can be approximated via a double-sided uniform price auction, where packages of limit orders form demand schedules (e.g., Kyle (1989)).

In order to facilitate the comparison with the empirical analysis, we present our framework using a one-sided market but explain how to adjust it to represent a double-sided market. Proofs are in Appendix E. Random variables are highlighted in **bold**.

2.1 Players, preferences, and constraints

There are $N > 2$ dealers who compete for units of an asset in an auction. When there are finitely many dealers, each one has some market power in that it can impact the market clearing price. When $N \to \infty$ each dealer is a price-taker, and the market is perfectly competitive.

Total supply $\boldsymbol{A}$ is random; it is drawn from some continuous distribution with support $(0, \overline{A}]$ where $\overline{A} \in \mathbb{R}^+$ and has a strictly positive density. In our empirical application, supply is random because dealers don't know the issuance size when they compete. In other settings, the supply might be random due to noise traders.

Each dealer i holds portfolio, z_i, in inventory, which was acquired at price $\psi \in \mathbb{R}^+$, and (equity) capital, E_i, on their balance sheet. For convenience, we summarize both balance-sheet items in one variable, respectively, but note that both may consist of multiple subitems. For example, inventory may include a variety of security types. In this case, price ψ represents the average per-unit price of a security in inventory; it may be a function of other model primitives: for example, the distribution of asset supply, or the number of dealers.

Both the asset supplied at auction and inventory generate an unknown (gross) return in the future. One unit of the asset pays a return of $\boldsymbol{R}$, while one unit of the inventory gives $\boldsymbol{R_z}$. In our empirical application, where the asset is a government bond, $\boldsymbol{R}$ represents the price obtained from selling the bond post-auction, which is unknown at the time of the auction. Following the related literature, we assume that returns are jointly Normally distributed,

$$\begin{pmatrix} \boldsymbol{R} \\ \boldsymbol{R_z} \end{pmatrix} \sim N \left(\begin{pmatrix} \mu \\ \mu_z \end{pmatrix}, \begin{pmatrix} \sigma^2 & \iota\sigma\sigma_z \\ \iota\sigma\sigma_z & \sigma_z^2 \end{pmatrix} \right) \text{ with } \mu, \mu_z, \sigma, \sigma_z > 0, \text{ and } \iota \in [-1, 1]. \quad (1)$$

The inventory and capital positions are part of a multi-dimensional signal, $\boldsymbol{\theta_i}$, that each dealer observes before bidding. The signal, $\boldsymbol{\theta_i}$, is either the private information of dealer i or commonly known by all dealers. When the signal is private, it is drawn independently across dealers from some continuous distribution on bounded support and strictly positive density. In this case, dealers face private and aggregate uncertainty when bidding. When the signal is observed by all dealers, we assume that all dealers are identical: $E_i = E, z_i = z$ with $E \in \mathbb{R}^+$ and $z \in \mathbb{R}$. We introduce the framework for the more general case with private information. Without private information, we simply omit θ_i in all expressions.

Given signal θ_i, each dealer submits a decreasing (inverse) demand schedule: $p_i(\cdot, \theta_i) : \mathbb{R}^+ \to \mathbb{R}^+$, which specifies how many units of the asset, a, the dealer seeks to buy for price $p_i(a, \theta_i)$. We denote the inverse by $a_i(p, \theta_i) = p_i^{-1}(p, \theta_i)$, if it exists. In a double-sided market, such as an exchange, the demand schedule represents demand net of supply.[3]

To develop the theory, we assume that demand functions are twice continuous and strictly decreasing, and we denote the set of functions with that property by $\mathcal{B}$. Working with continuous demand functions is common in the related theory literature in order to achieve tractability, even though in practice demand functions are often discrete. For example, bidders must submit step functions in most Treasury auctions. Therefore, we also provide equilibrium conditions for step functions in Appendix A.

Once all dealers have submitted their demand curves, the auction clears at the price, P^c, such that aggregate demand meets total supply:

$$P^c : \sum_i a_i(P^c, \theta_i) = A. \tag{2}$$

Each dealer pays the market clearing price, $P^c = p_i(a_i^c, \theta_i)$, for the amount won, $a_i^c = a_i(P^c, \theta_i)$ at that price. To highlight equilibria, we refer to the equilibrium market clearing

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[3]Our model of the exchange market abstracts from strategic investors who don't face capital constraints. This reflects the fact that it is common for non-dealers to invest via dealers or brokers on exchanges, rather than directly.

price by P^* and the winning amount by a_i^*.

Each dealer chooses their demand function to maximize their expected CARA utility from earning wealth, $\omega_i(\boldsymbol{a_i^c}, \boldsymbol{P^c})$, that is generated at market clearing:

$$U(p_i(\cdot, \theta_i)) = \mathbb{E}\left[1 - \exp\left(-\rho\omega_i(\boldsymbol{a_i^c}, \boldsymbol{P^c})\right)\big|\theta_i\right]. \tag{3}$$

Parameter $\rho > 0$ measures the dealer's degree of risk aversion. Future wealth, $\omega_i(\boldsymbol{a_i^c}, \boldsymbol{P^c})$, is equal to the assets' payoffs net of the prices paid:

$$\omega_i(\boldsymbol{a_i^c}, \boldsymbol{P^c}) = (\boldsymbol{R} - \boldsymbol{P^c})\boldsymbol{a_i^c} + (\boldsymbol{R_z} - \psi)z_i. \tag{4}$$

Motivated by the Basel III requirement that states that banks must hold sufficient equity capital, E_i, relative to their total balance sheet exposure, $[\boldsymbol{P^c}\boldsymbol{a_i^c} + \psi z_i]$, each dealer faces capital constraint,

$$\kappa\mathbb{E}[\boldsymbol{P^c}\boldsymbol{a_i^c} + \psi z_i|\theta_i] \leq E_i. \tag{5}$$

Threshold $\kappa > 0$ is commonly known. Alternatively, the threshold might be dealer-specific. Then it would simply be part of a dealer's type, θ_i. Further, the auctioned asset could be weighted by a different threshold than the inventory, as would be the case under a risk-weighted capital constraint. Throughout the paper, we denote the Lagrange multiplier of this constraint by $\lambda_i \geq 0$ and refer to $\lambda_i\kappa$ as the shadow cost of the capital constraint.

When bidding, the dealer does not yet know where the auction will clear and therefore takes an expectation of the capital constraint. This timing assumption is motivated by the fact that dealers are forward-looking and that capital positions are regularly monitored via different regulatory forms.[4] As a consequence, dealers need to demonstrate sufficient capital

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[4]In addition to capital being in the numerator of the LR constraint, which is reported end-of-quarter, it plays an important role in the construction of liquidity metrics such as the liquidity coverage ratio and net cumulative cash flow, which are monitored at higher frequencies. Inventories also show up in H4, as shown in Figure 2a. For details see: `https://www.osfi-bsif.gc.ca/Eng/fi-if/rtn-rlv/fr-rf/dti-id/Pages/LCR_Ret_Ins_Jan_2015.aspx`; `https://www.osfi-bsif.gc.ca/Eng/fi-if/rtn-rlv/fr-rf/dti-id/Pages/nccf2023_dft.aspx`, accessed on 09/02/2023.

coverage on average, in addition to the end of each quarter. The auction does not lead to adjustments in the nominal value of the dealer's inventory portfolio. In our empirical application, this assumption reflects the fact that a dealer's portfolio comprises a range of on- and off-balance sheet items whose market values are marginally influenced, if at all, by the issuance of government bonds via regular Treasury auctions.

As an alternative to the capital constraint, we could assume that each dealer faces a balance sheet cost that depends on the nominal amount of the asset that they hold on their balance sheet post-auction. If the marginal balance sheet cost is constant, and depends on private information to the dealer, this specification is essentially equivalent to assuming that dealers face capital constraint (5).

2.2 Equilibria

We focus on symmetric equilibria since dealers are ex-ante identical.

Definition 2. *A symmetric equilibrium is a collection of demand functions $p^*(\cdot, \theta_i)$ that for each dealer, and almost every θ_i, maximizes expected surplus (3) subject to capital constraint (5). An equilibrium is linear if $\frac{\partial p^*(a, \theta_i)}{\partial a}$ is constant at all a.*

To derive equilibrium conditions, take the perspective of dealer i with information θ_i, and assume that all other dealers $j \neq i$ play the symmetric equilibrium strategy, $p^*(\cdot, \theta_j)$. Dealer i chooses a best response, $p_i(\cdot, \theta_i)$, that maximizes their expected surplus (3). Given that returns are Normally distributed, as specified in expression (1), the maximization problem is equivalent to:

$$\max_{p_i(\cdot, \theta_i) \in \mathcal{B}} \mathbb{E}\left[V(\boldsymbol{a_i^c}, \theta_i) - p_i(\boldsymbol{a_i^c}, \theta_i)\boldsymbol{a_i^c} - \psi z_i | \theta_i\right] \tag{6}$$

$$\text{with } V(\boldsymbol{a_i^c}, \theta_i) = \mu \boldsymbol{a_i^c} + \mu_z z_i - \frac{\rho \sigma^2}{2}[\boldsymbol{a_i^c}]^2 - \frac{\rho \sigma_z^2}{2}[z_i]^2 - \rho \iota \sigma \sigma_z \boldsymbol{a_i^c} z_i \tag{7}$$

$$\text{subject to capital constraint: } \kappa \mathbb{E}[p_i(\boldsymbol{a_i^c}, \theta_i)\boldsymbol{a_i^c} + \psi z_i | \theta_i] \leq E_i, \tag{8}$$

$$\text{and market clearing: } \boldsymbol{a_i^c} = \boldsymbol{A} - \sum_{j \neq i} a^*(p_i(\boldsymbol{a_i^c}, \theta_i), \boldsymbol{\theta_j}), \tag{9}$$

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as well as natural boundary conditions.[5] The dealer's best response must coincide with the strategy chosen by other dealers with the same information, θ_i, in a symmetric equilibrium.

Proposition 1. *In any symmetric equilibrium, dealer i submits demand function, $p^*(\cdot, \theta_i)$, such that $p^*(a, \theta_i) = p$ for all a, given by*

$$p = \frac{v_i(a)}{1 + \lambda_i \kappa} - shading(a, p | \theta_i), \tag{10}$$

$$where \quad \frac{\partial V(a, \theta_i)}{\partial a} = v_i(a) = \mu - \rho \sigma [\sigma a + \iota \sigma_z z_i] \tag{11}$$

is the dealer's marginal utility from amount a; $\lambda_i \geq 0$ is the Lagrange multiplier of the capital constraint, $\lambda_i [E_i - \kappa \mathbb{E}[p^*(\boldsymbol{a_i^*}, \theta_i) \boldsymbol{a_i^*} + \psi z_i | \theta_i]] = 0$; and $shading(a, p | \theta_i) = -a \left(\frac{\partial G(a, p | \theta_i)}{\partial a} / \frac{\partial G(a, p | \theta_i)}{\partial p} \right)$. Here $G(a, p | \theta_i) = \Pr(\boldsymbol{A} - \sum_{j \neq i} a^*(p^*(a, \theta_i), \boldsymbol{\theta_j})) \leq a | \theta_i)$ is the probability that dealer i, who bids price $p = p^*(a, \theta_i)$, wins less than a at market clearance given that the other dealers play the equilibrium.

Proposition 1 provides necessary equilibrium conditions for uniform price auctions with smooth functions; Proposition 3 in Appendix A outlines analogous conditions for discriminatory price auctions and step functions.

In all settings, the dealer bids as if participating in a standard multi-unit auction without capital constraints, where the marginal utility, $v_i(a)$, is discounted by the shadow cost of the capital constraint, $\lambda_i \kappa$. Concretely, the dealer's willingness to pay for amount a is

$$\tilde{v}_i(a) = v_i(a)(1 + \lambda_i \kappa)^{-1}. \tag{12}$$

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[5]Transforming the maximization problem is a common technique in the related literature (e.g., Malamud and Rostek (2017)). To understand the validity of this approach, rewrite (3) as $U(p_i(\cdot, \theta_i)) = \mathbb{E}\left[\mathbb{E}\left[1 - \exp(-\rho \omega_i(\boldsymbol{a_i^c}, p_i(\boldsymbol{a_i^c}, \theta_i)))\right] | \theta_i\right]$, where the first expectation is w.r.t. $\boldsymbol{a_i^c}$ and the second expectation is w.r.t. to the Normally distributed returns, $(\boldsymbol{R} \ \boldsymbol{R_z})$. Now insert $\omega_i(\boldsymbol{a_i^c}, p_i(\boldsymbol{a_i^c}, \theta_i))$ given by (4) and take the expectation w.r.t. $(\boldsymbol{R} \ \boldsymbol{R_z})$ to obtain: $U(p_i(\cdot, \theta_i)) = \mathbb{E}\left[1 - \exp(-\rho\{V(\boldsymbol{a_i^c}, \theta_i) - p_i(\boldsymbol{a_i^c}, \theta_i)\boldsymbol{a_i^c} - \psi z_i\}) | \theta_i\right]$ with $V(\boldsymbol{a_i^c}, \theta_i) = \mu \boldsymbol{a_i^c} + \mu_z z_i - \frac{\rho \sigma^2}{2}[\boldsymbol{a_i^c}]^2 - \frac{\rho \sigma_z^2}{2}[z_i]^2 - \rho \iota \sigma \sigma_z \boldsymbol{a_i^c} z_i$. Given that $1 - \exp(-\rho y)$ is strictly increasing for any $y \in \mathbb{R}$, maximizing $U(p_i(\cdot, \theta_i))$ is equivalent to maximizing $\mathbb{E}\left[V(\boldsymbol{a_i^c}, \theta_i) - p_i(\boldsymbol{a_i^c}, \theta_i)\boldsymbol{a_i^c} - \psi z_i | \theta_i\right]$. This remains true when adding capital constraint (5).

The marginal utility, defined in equation (11), decreases as the amount of the asset increases. For the initial unit of the asset, a dealer who holds no inventory obtains the asset's per-unit return μ. For subsequent units the marginal benefit diminishes, depending on the variance of the asset's return, σ^2, and the dealer's degree of risk aversion, ρ. When holding an inventory position, the per-unit return of the asset is reduced, unless it is uncorrelated with the inventory's return.

The shadow cost of the constraint, $\lambda_i \kappa$, is strictly positive when the capital constraint is binding, and zero otherwise. Notably, the Lagrange multiplier, λ_i, is a function of the dealer's private information, θ_i, since it relies on the dealer's expectations of winning, which, in turn, is influenced by the dealer's bidding behavior that is shaped by θ_i.

As in auctions without constraint, dealers shade their willingness to pay to minimize payments, unless the market is perfectly competitive. The shading factors depend on the rules of the auction (as explained, for instance, by Clark et al. (2021)). In uniform price auctions, shading is pinned down by the trade-off between the expected marginal utility and the expected loss due to price impact, explained in more detail below.

Deriving sufficiency conditions under which the equilibrium of Proposition 3 exists, and deriving an explicit functional form of equilibrium demand, is challenging. This is the case even for uniform price auctions with smooth demand functions. The reason is that the slope in the dealer's willingness to pay changes randomly in the dealer's private information—as if the dealer had private information about her effective degree of risk aversion, $\rho(1 + \lambda_i \kappa)^{-1}$. With random slopes there is no linear equilibrium when dealers have market power.

To see why this is the case, let all dealers other than dealer i submit a linear demand curve. The necessary equilibrium conditions imply that dealer i's best response is linear if and only if observing price realization p does not update their belief about the other dealers' constraints. However, even if we assume that this holds for all dealers, the market clearing price is a function of the Lagrange multipliers, and thus private information, of all dealers. Therefore, the dealer would update their belief when observing the price—a contradiction.

Corollary 1. *(i) There is no linear equilibrium when dealers have private information and face capital constraints, unless the market is perfectly competitive. (ii) Under perfect competition, i.e., when $N \to \infty$, dealer i submits her willingness to pay, $\tilde{v}_i(a)$.*

Corollary 1 *(i)* implies that the common tools of the related literature—which almost exclusively focuses on linear equilibria to obtain tractability—do not apply. For instance, it is not possible to solve for a dealer's best response by point-wise maximization. Maximizing over demand functions, we can, however, prove equilibrium existence and provide an explicit characterization of equilibrium demand when abstracting from private information.[6]

Proposition 2. *Let all dealers share the same information with inventory position $z \in \mathbb{R}$, and equity capital $E > 0$. There exists a unique symmetric linear equilibrium in which each dealer submits*

$$p^*(a) = \frac{1}{1 + \lambda\kappa}\left(\mu - \rho\sigma^2\left(\frac{N-1}{N-2}\right)a - \rho\iota\sigma\sigma_z z\right), \quad with \tag{13}$$

$$\lambda = \begin{cases} 0 & if \ \frac{E}{\kappa} \geq B + \psi z \\ \frac{B}{E - \kappa\psi z} - \frac{1}{\kappa} > 0 & if \ \frac{E}{\kappa} < B + \psi z \end{cases} \quad where \ B = (\mu - \rho\iota\sigma\sigma_z z)\mathbb{E}\left[\frac{A}{N}\right] - \rho\sigma^2\left(\frac{N-1}{N-2}\right)\mathbb{E}\left[\left(\frac{A}{N}\right)^2\right].$$

Intuitively, an equilibrium bid $p^*(a)$ for amount a equalizes the marginal utility (which is the analogue to $v_i(a)$ in Proposition 1), with the marginal payment:

$$\mu - \rho\sigma[\sigma a + \iota\sigma_z z] = (1 + \lambda\kappa)[p^*(a) + a\Lambda]. \tag{14}$$

Here $\Lambda = \frac{1}{N-2}\frac{\rho\sigma^2}{1+\lambda\kappa}$ measures the dealer's price impact. Λ is known as Kyles' lambda and is 0 when the market is perfectly competitive. The inverse of the price impact is a common measure of liquidity in exchange markets (e.g., Vayanos and Wang (2013); Malamud and

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[6]Alternatively, we could solve for an equilibrium when dealers are asymmetric, for instance, due to different inventory positions, but face no uncertainty. This equilibrium is analogous to Proposition 2, but is not unique—a common feature in uniform price auctions (Klemperer and Meyer (1989)).

Rostek (2017)).

The marginal payment has several components and depends on the shadow cost of the capital constraint, $\lambda\kappa$, and the dealer's price impact, Λ. When the constraint is not binding ($\lambda = 0$) and dealers are price-takers ($\Lambda = 0$), the marginal payment is just the price that the dealer has to pay for amount a. When the constraint binds ($\lambda > 0$) and dealers are price-takers ($\Lambda_i = 0$), the marginal payment is the price they have to pay plus a shadow cost that comes from the capital constraint, which is similar to an ad valorem tax. When dealers face a binding capital constraint ($\lambda > 0$) and have market power ($\Lambda \neq 0$), Λa measures by how much a dealer's choice impacts the effective price. Not only does this depend on their risk aversion and the number of players in the market, it also depends on the shadow cost of the capital constraint.

To illustrate how to solve for an equilibrium in uniform price auctions in which bidders face outcome-dependent constraints, we sketch the proof of Proposition 2. A reader who is not interested in technical details may skip ahead to Section 2.3.

<u>Proof of Proposition 2:</u> We guess that there is a symmetric linear equilibrium, $a^G(p) = \alpha - \beta p$ with $\alpha, \beta > 0$, and assume that all dealers other than dealer i play this equilibrium guess. Dealer i takes the behavior of her competitors as given and chooses points on the residual supply curve $RS_i(p) = \boldsymbol{A} - \sum_{j \neq i} a_j^G(p)$, which shifts randomly only in parallel. This implies that, for every price p on every demand function that the dealer may submit, there is a unique (random) point at which the residual supply curve intercepts the quantity axis: $\boldsymbol{Z} = \boldsymbol{A} - (N - 1)\alpha$. Therefore we can maximize over bidding functions, $b(\cdot)$, that map from realizations of Z to prices, and be certain that for every $b(\cdot)$ there is a unique demand functions, $p(\cdot)$. Imposing market clearance, by inserting $RS(b(Z), Z) = Z + (N - 1)\beta b(Z)$ into the objective function, the dealer's maximization problem—the analogue of problem (6)—reads as follows:

$$\max_{b(\cdot) \in \mathcal{B}} \mathbb{E}[V(RS(b(\boldsymbol{Z}), \boldsymbol{Z})) - b(\boldsymbol{Z})RS(b(\boldsymbol{Z}), \boldsymbol{Z}) - \psi z] \text{ s.t.: } \kappa\mathbb{E}[b(\boldsymbol{Z})RS(b(\boldsymbol{Z}), \boldsymbol{Z}) + \psi z] \leq E.$$

Abbreviating $b(\cdot)$ by b with derivative b', this problem is equivalent to $\max_{b \in \mathcal{B}} I(b)$ subject to $L(b) \geq 0$, with $I(b) = \int_{\underline{Z}}^{\overline{Z}} F(b, Z)\phi(Z)dZ$, where $F(b, Z) = V(RS(b, Z), Z) - bRS(b, Z) - \psi z$, and $L(b) = E - \kappa\psi z - \int_{\underline{Z}}^{\overline{Z}} H(b, Z)\phi(Z)dZ$, where $H(b, Z) = \kappa bRS(b, Z)$. Here $\phi(Z)$ is the density function of $\mathbf{Z}$ which has support $[\underline{Z}, \overline{Z}]$.

With this, function b^* is optimal if $L(b^*) \geq 0, \lambda L(b^*) = 0, \lambda \geq 0$, $\frac{\partial(F + \lambda H)}{\partial b} - \frac{d}{dZ}\left(\frac{\partial(F + \lambda H)}{\partial b'}\right)$ evaluated at the optimum is 0 for all Z:

$$\mu - \rho\sigma[\sigma RS(b^*, Z) + \iota\sigma_z z] = (1 + \lambda\kappa)\left[b^* + RS(b^*, Z)\left(\frac{\partial RS(b^*, Z)}{\partial b}\right)^{-1}\right], \qquad (15)$$

and the natural boundary conditions are satisfied. This is always the case, because $F + \lambda H$ is independent of b'. Note that condition (15) is equivalent to condition (14).

Importantly, density function $\phi(Z)$ does not depend on b, which would not be the case had we maximized over demand functions, $p(\cdot)$, directly. This feature enables us to show that the function b^* that fulfills the necessary conditions is indeed optimal. Given that $F(b, Z)$ and $K(b, Z) = F(b, Z) + \lambda H(b, Z)$ are for any Z, and $\lambda \geq 0$, strictly concave as functions of b, we know that $K(b, Z) - K(b^*, Z) < \frac{\partial K(b, Z)}{\partial b}(b - b^*) \leq 0$ for any b and any Z. Multiplying both sides with $\phi(Z)$ and integrating, we see that $\int_{\mathbb{R}} K(b, Z)\phi(Z)dZ < \int_{\mathbb{R}} K(b^*, Z)\phi(Z)dZ$, and similarly for $F(b, Z)$.

From here it is straightforward to solve for an equilibrium and show that it is unique within the class of symmetric linear equilibria. For this we rely on the property that function $b^*(\cdot)$ implies a unique demand function $p^*(\cdot)$. Then we match coefficients of the dealer's best reply in (15) with the equilibrium guess and show that these coefficients are unique. In this equilibrium, each dealer wins $\frac{A}{N}$, the market clears at $P^* = \frac{1}{1 + \lambda\kappa}\left(\mu - \rho\sigma^2\left(\frac{N-1}{N-2}\right)\frac{A}{N} - \rho\iota\sigma\sigma_z z\right)$, and depending on the exogenous parameters of the model, the capital constraint either binds $(\lambda > 0)$ or not $(\lambda = 0)$. $\qquad \square$

2.3 How capital constraints affect prices and markups

The main prediction of the model is about what happens when the capital constraint is relaxed, for instance, because the minimal capital threshold decreases. We examine three effects: the effect on price, price impact (and consequently market liquidity), and markup. While all of these effects are derived from our own model and offer novel insights, we particularly emphasize the effects on price impact and markup. These effects are absent in the existing literature, which assumes perfectly competitive markets.

Corollary 2. *Let $P^*(0)$ and $P^*(\Lambda)$ denote the market prices under perfect and imperfect competition, respectively, and consider a relaxation of capital constraints which decreases the shadow cost of the constraint for all dealers.*

(i) When dealers face only aggregate uncertainty, demand $p^*(\cdot)$ of each dealer i becomes steeper, and market price $P^*(\Lambda)$ increases. The price impact $\Lambda = \frac{1}{N-2}\frac{\rho\sigma^2}{1+\lambda\kappa}$ of each dealer i, and the markup, $P^*(0) - P^*(\Lambda) = \Lambda\frac{A}{N}$, increase, while market liquidity, $1/\Lambda$, decreases.

(ii) When dealers have private information, a dealer's demand $\tilde{v}_i(\cdot)$ becomes steeper, and the market price $P^*(0)$ increases if the market is perfectly competitive. When at least some dealers shade their bids due to market power, the effects depend on the distribution of signals and supply, and the number of competing dealers.

Figure 1 illustrates two types of effects from relaxing capital constraints. The first type is non-strategic. Since the effective price, $(1 + \lambda\kappa)p$, decreases, it becomes cheaper for the dealer to buy larger amounts. The dealer's willingness to pay shifts upward and becomes steeper.[7] As a result, the market price would increase if the market was perfectly competitive, unless supply adjusts. This prediction is in line with He and Krishnamurthy (2012, 2013) and Brunnermeier and Sannikov (2014). In their models, a positive shock to a dealer's net

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[7]The willingness to pay becomes steeper rather than just shifting in parallel because the capital constraint depends on the nominal, not the real, value of the amount the dealer wins at market clearance. If the constraint was in real values, the willingness to pay would only shift in parallel.

worth, i.e., equity capital, increases its risk-bearing capacity, which leads to higher asset prices. In our model, risk aversion is constant.

The second type of effect is strategic and is absent in existing models that feature perfect competition. Dealers have higher price impact, and thus enjoy more market power, when their constraints are relaxed. Higher market power leads to lower market liquidity, and stronger bid shading, resulting in a larger markup. This pushes down the market price relative to the price that would arise in a perfectly competitive market. When dealers face only aggregate uncertainty, the non-strategic price effect dominates the strategic price effect, so that the market clearing price increases when constraints are relaxed.

To understand these effects, it helps to go through how the dealer determines her best-response in a simplified environment with complete information. In this case, she trades against a known residual supply curve, $RS_i(p) = A - \sum_{j \neq i} a_j(p)$, and chooses the point on that curve that maximizes her own surplus. If the other dealers submit flatter demand curves, the residual supply curve is flatter. This implies that the dealer impacts the market clearing price more strongly by her own demand—moving along a flat residual supply curve changes the price more strongly than moving along a steep residual supply curve. The dealer's price impact and the markup increase, while liquidity decreases.

A similar logic applies when dealers face uncertainty. The only difference is that the dealer now trades against a residual supply, which randomly shifts in parallel. When there is private information, the residual supply curve moves randomly in arbitrary ways. Therefore, we cannot make a clear prediction.

This highlights that the effects depend on whether dealers have access to private information when bidding, and the degree of competition. When dealers face only aggregate uncertainty or the market is perfectly competitive, we can derive by how much the price, price impact, and markup change in response to a change in the shadow cost of the capital constraint, $\lambda \kappa$.

Figure 1: Non-strategic and strategic effect when capital constraints are relaxed

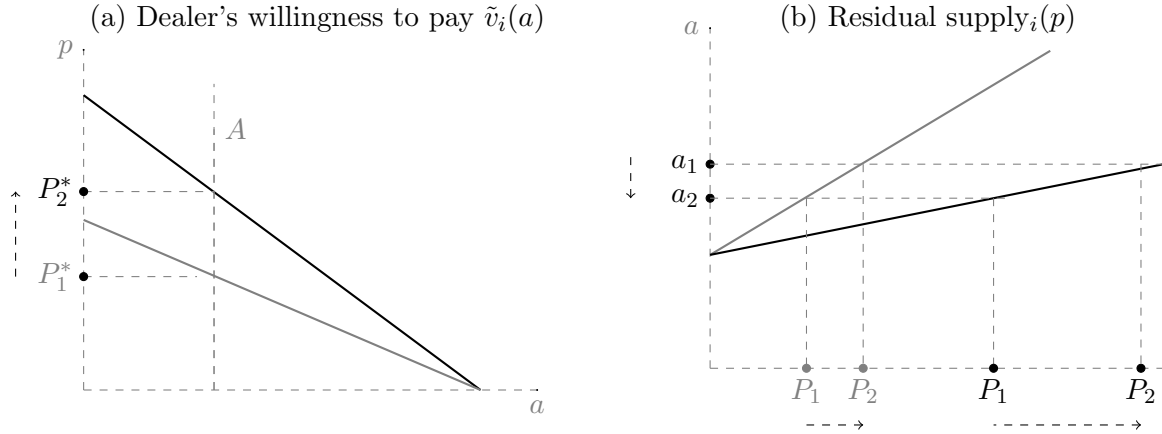(a) Dealer's willingness to pay $\tilde{v}_i(a)$ (b) Residual supply$_i(p)$

Figure 1 illustrates the change in the dealer's willingness to pay and her residual supply curve, conditional on one realization of supply, when capital constraints are relaxed in (a) and (b), respectively, for the case without private uncertainty and zero-inventories ($z = 0$). Initial curves are in gray; curves with relaxed constraints are in black. In (a) we see how relaxing constraints increases the market clearing price, P^*, when supply is fixed in a perfectly competitive market. In (b) we see the increase in the price impact, which measures by how much the clearing price changes, $P_2 - P_1$, when the dealer marginally changes her demand from a_1 to a_2.

Corollary 3. *(i) When dealers have market power but no private information, a 1% decrease in the shadow cost of the capital constraint, $\lambda\kappa$, leads to an increase in the market price, the price impact, and the markup equal to $\eta = \left|\frac{1}{1+\lambda\kappa} - 1\right|$ %. (ii) When the market is perfectly competitive so that the markup and price impact are zero, the market price increases by η when dealers don't have private information, and by $\eta^\infty = \left|\frac{1}{1+\mathbb{E}[\lambda_i]\kappa} - 1\right|$ % when they do.*

Summarizing, our model helps explain how capital constraints affect asset prices, price impact (and, with that, liquidity), and markups. When dealers have no private information, their demand becomes steeper, and the price, price impact, and markup increase when capital constraints are relaxed. For primary markets, this highlights that relaxing capital constraints increases auction revenues at an implicit cost of larger price distortion. In the context of exchange markets, higher markups indicate reduced market liquidity. In the presence of private information, it becomes an empirical question whether, and to what extent, capital constraints affect demand, prices, price impact, liquidity, and markups.

3   Institutional setting and data

We illustrate how to use our framework to empirically analyze how prices and markups change when capital constraints change, using data on Canadian Treasury auctions. These auctions utilize the discriminatory price format, where price impact is not well defined. Hence, we exclude price impact (and market liquidity) from now onward.

Market players.   There are eight deposit-taking primary dealers in Canada who are federally regulated and therefore face the Basel III constraint.[8] They dominate the Canadian Treasury market and intermediate the vast majority of the daily trade volume in government bonds. More broadly, these banks dominate the Canadian banking sector and hold over 90% of the sector's assets.

Primary dealers have a responsibility, as market-makers, to buy bonds from the government and trade them with investors, brokers, or one another to provide liquidity in the secondary market. They hold a substantial amount of bonds on their own balance sheets (see Appendix Figure A1). In exchange, primary dealers enjoy benefits, including privileged access to liquidity facilities and overnight repurchase operations at the central bank.

Treasury auctions.   Governments issue bonds of different maturities in the primary market via regularly held uniform price or discriminatory price auctions. In Canada, auctions are discriminatory price. Each bidder submits a step function with at most $K = 7$ steps, which specifies how much a bidder offers to pay for specific amounts of the asset for sale. Auctions take place several days a week. Anyone may participate, but the largest eight dealers purchase the majority of the Treasury supply.

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[8]In total there are eleven primary dealers. One of these dealers is provincially regulated, and two are private securities dealers. They face different capital regulation than the eight dealers we study. We therefore do not observe any balance sheet information for these players. Technically, two of the eight banks have multiple dealers. For example, the Bank of Montreal has two dealers (Bank of Montreal and BMO Nesbitt Burns) who attend different Treasury auctions and therefore do not compete or share information within an auction. We treat them as one dealer.

Capital constraints. According to a survey among market participants, the Basel III LR represents the most relevant capital constraint when trading government bonds (CGFS (2016)). This regulatory requirement came into effect in September 2014 to reduce systematic risk—a benefit which we do not consider in this paper. We focus on the cost side of the constraint, which was emphasized by Duffie (2018), He et al. (2022), and others.

Formally, the LR measures a bank's Tier 1 capital relative to its total leverage exposure, and must be at least 3%:

$$\text{LR}_{iq} = \frac{\text{regulatory Tier 1 capital of bank } i \text{ in quarter } q}{\text{total leverage exposure of } i \text{ in } q}.$$

Tier 1 capital consists primarily of common stock and disclosed reserves (or retained earnings), but may also include non-redeemable non-cumulative preferred stock; the leverage exposure includes the total notional value of all cash and repo transactions of all securities, including government bonds, regardless of which securities are used as collateral (for more details, see OSFI (2023)).

In reality, banks refrain from getting close to the Basel III threshold.[9] One reason for this is that each institution faces an additional supervisory LR threshold that reflects the underlying risk of the bank's operations. Another reason is that banks tend to hold sufficient conservation buffer for Tier 1 capital so as to avoid punishment in the form of restricted distributions (including dividends and share buybacks, discretionary payments and bonus payments to staff). Through the lens of the model, this implies that the relevant threshold of the capital constraint is unobservable to the econometrician.

Regulatory change. To separately identify shadow costs of the capital constraint and risk aversion, we rely on a regulatory change that temporarily eliminated the capital constraint for Treasuries. When dealers failed to absorb the extraordinary supply of government bonds

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[9]Barth et al. (2005) and others document that bank capital is substantially above the regulatory minimum in countries other than Canada.

Figure 2: The effect of the exemption on Treasury positions and the LR

(a) Aggregated positions in Treasuries
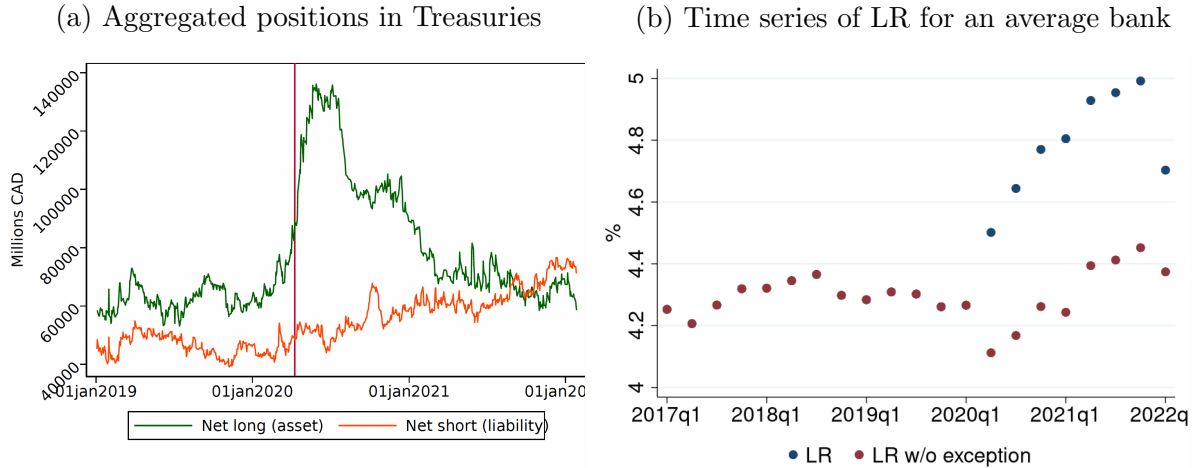
(b) Time series of LR for an average bank

Figure 2a shows the aggregated amount of Canadian government bonds that the biggest six Canadian banks hold in long (in green) and short (in red) positions in millions of C$ from January 2019 until February 2022. The vertical line is April 9, 2020, when the exemption period began. Figure 2b shows the time series of the actual LR (in %) of an average bank in blue. In red is the counterfactual LR that the average bank would have had in absence of the exemption. In 2022q1, the LR does not get back to its original level, partially because central bank reserves are still exempted.

in March 2020, government bonds, central bank reserves, and sovereign-issued securities that qualify as high-quality liquid assets (HQLA) were temporarily exempted from the LR constraint—starting on April 9, 2020.[10] As a result, the LR spiked upward, moving away from the constraint (see Figure 2b). The exemption of government bonds and HQLA ended on December 31, 2021, while reserves continued to be excluded.

Data. We combine multiple data sources. First, we obtain bidding data of all regular government bond auctions between January 1, 2019, and February 1, 2022, from the Bank of Canada. We see how much is issued of which security, and the maturity category, of

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[10]Exposures related to the US Government Payment Protection Program (PPP), which are minor in the case of Canadian banks, were also temporarily exempted. The announcement to start the exemption period is available at: `www.osfi-bsif.gc.ca/Eng/fi-if/in-ai/Pages/20200409-dti-let.aspx`; the one to end it is here: `www.osfi-bsif.gc.ca/Eng/fi-if/in-ai/Pages/lrfbunwd.aspx`, both accessed on 09/02/2023.

which there are five (2Y, 3Y, 5Y, 10Y and 30Y). We also observe who bids (identified by a legal entity identifier) and all winning and losing bids at auction closure. For consistency, we restrict attention to bids of the eight dealers who are deposit-taking throughout most of the paper.

Second, we collect balance sheet information for these eight dealers at the company holding level. Specifically, we obtain the quarterly LRs of each dealer from 2015q1 until 2022q1 from the Leverage Requirements Return. In addition, we obtain the daily aggregated long and short positions in government bonds of the six largest dealers from the Collateral and Pledging Report (H4). Finally, we collect information on who holds government bonds— banks versus other investor types—from the National Accounts (Statistics Canada).

Third, we gather information on the volatility of the return, i.e., the price, that a dealer expects to obtain from selling government bonds in the secondary market. For this we leverage the fact that dealers start selling bonds that are about to be issued at auction when the tender call opens, which happens one week before the auction closes. This means that dealers already observe the distribution of prices at which they can sell a particular bond, which gives them a precise idea about the return volatility. To also observe this price distribution, we obtain prices (and yields) of essentially all trades with Canadian government bonds from January 1, 2019, until February 1, 2022. These data are collected by the Industry Regulatory Organization of Canada in the Debt Securities Transaction Reporting System and are made available for research with a time lag.

Fourth, we collect the Implied Volatility Index for Canadian Treasuries over the same time period. This index measures the expected volatility of the market over the next 30 days and is based on option prices on short-term interest rate futures (Chang and Feunou (2014)). It is similar to the Merrill Lynch Option Volatility Estimate (MOVE) for U.S. Treasuries.

Summary statistics. An overview of the main variables is presented in Table 1. Note that for our empirical findings, we express bond values in yields-to-maturity rather than

Table 1: Summary statistics

	Mean	Median	Std	Min	Max
Supply (in bn C$)	4.12	4.00	1.23	1.40	7.00
Average bid yield (in %)	1.04	1.09	0.58	0.20	2.18
Years to maturity	8.40	5.03	9.62	2.0	32.62
Number of (deposit-taking) dealers	8	8	0	8	8
Number of steps in demand curve	4.80	5	1.43	1	7
Maximal amount demanded (in % of supply)	7.28	6.25	3.84	0.07	35
Amount dealer won (in % of supply)	6.55	4.98	6.22	0	44.22
Quarterly LR (in %)	4.41	4.36	0.28	-	-
Return volatility (normalized)	1	0.76	0.90	0	7.93

Table 1 shows the average, median, standard deviation, minimum, and maximum of key variables in our sample. Our auction data goes from January 1, 2019, until February 1, 2022, and counts 176 bond auctions. The LR data goes from 2015q1 until 2022q1. The min and max LR are empty because we cannot disclose this information.

prices. This makes the value of bonds that have different maturities and coupon payments more comparable.

In line with this convention, we compute the auction-specific return volatilities as standard deviation of yields (expressed in %) at which a dealer sells a bond that is to be auctioned during the week preceeding the auction. To avoid our estimates being driven by the absolute magnitude of the volatility, we normalize the return volatility by its average. Figure 3 shows that the resulting return volatility is similar, yet not identical, to the Implied Volatility Index for Canadian Treasuries (in %).

4   Quantification

To quantify by how much the yield and markups change when capital constraints are relaxed and tightened, we adjust the benchmark model to better fit the data-generating process.

Figure 3: Return volatility

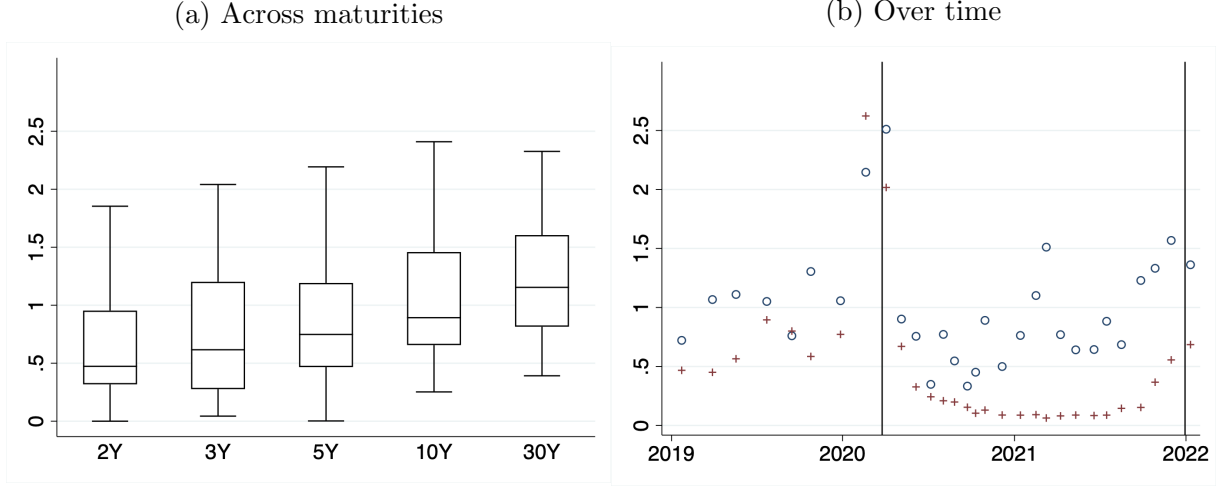(a) Across maturities   (b) Over time

Figure 3a shows the distribution of the normalized return volatility for each maturity category, excluding outliers. Figure 3b shows a binned scatter plot of the return volatility (in circles) and the implied volatility index in % (in pluses) across time. The correlation between these two volatility indices is 0.3. The black lines mark the beginning (09 April 2020) and end (01 January 2022) of the exemption period.

Model adjustments. Consider an auction t that issues a bond of maturity $m=\{2Y, 3Y, 5Y, 10Y, 30Y\}$. In line with the institutional setting, the auction is discriminatory price, and bidders submit step functions, i.e., sets of $K \geq 1$ quantity-price tuples, $\{a_k, p_k\}_{k=1}^K$. Thus, a dealer's equilibrium demand satisfies the condition of Proposition 3 (ii) in the Appendix.

As predicted by our theory, a dealer who draws iid private information θ_{ti} from some auction-specific distribution on the day of auction t is willing to offer

$$\tilde{v}_{tik} = f_t(\theta_{ti}) - \beta_{ti}\sigma_t^2 a_{tik}, \text{ with } \beta_{ti} = \frac{\rho_m}{1 + \lambda\kappa_{ti}} \tag{16}$$

for amount a_{tik}, where $f_t(\cdot)$ is some continuous function, for instance, $\mu_t - \rho_m \iota \sigma_t \sigma_z z_{ti}$.[11] Parameter $\rho_m \geq 0$ measures the degree of risk aversion for a bond with maturity m. Thus,

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[11] Above we have provided one micro-foundation for this willingness to pay. For estimation, it is sufficient to assume that willingness to pay is given by (16). This implies, for example, that we don't necessarily have to assume that returns are normally distributed.

we allow, but do not impose, risk aversion to vary in the bond's maturity to reflect the fact that longer bonds may be riskier to hold than shorter bonds. Parameters $\lambda \kappa_{ti} \geq 0$ represent the shadow costs of the capital constraint. Note that we estimate the product of the Lagrange multiplier of the constraint and the capital threshold to avoid having to specify a capital threshold. To highlight this, we relabel the shadow costs $\lambda \kappa_{ti}$ instead of $\lambda_{ti} \kappa$.

Identification and estimation. To identify the shadow costs of the capital constraint, $\lambda \kappa_{ti}$, and the dealer's degree of risk aversion, ρ_m, we proceed in two steps.

First, given bids in the auction, we back out how much dealers are truly willing to pay at each step k they submit, under the assumption that all bidders are rational and play the equilibrium. For this, we need to estimate the distribution of the market clearing price, $\boldsymbol{P_t^*}$, from the perspective of each dealer, θ_{ti}. To do this, we adopt the resampling procedure introduced by Allen et al. (2023), who build on Hortaçsu and Kastl (2012) and Hortaçsu and McAdams (2010).

This resampling procedure takes institutional details of Canadian Treasury auctions, which are omitted in our theoretic model, into account, so as to obtain an unbiased estimate of the price distribution. For example, it adjusts for the fact that there are not only dealers, but also customers who bid via dealers. Importantly, these details only affect the way we estimate the price distribution, but not the equilibrium condition itself for a given price distribution. For example, a dealer's information set, θ_{ti}, includes a customer's bid if the dealer observed a customer's bid before bidding (see Hortaçsu and Kastl (2012)).

Once we know all elements of the equilibrium condition, we can solve for the unique value, $\tilde{v}_{tik}$, that rationalizes the observed bid in each auction t of each dealer i at each submitted step k (Kastl (2011)).

Second, we fit the model-implied functional form of the dealer's willingness to pay (16) and leverage the policy change to separately identify the degree of risk aversion and shadow costs, under the assumption that risk aversion (per maturity class) is constant around the

policy change.[12] Using value functions with at least two steps, which represent 99% of all functions, we estimate parameters $\{\rho_m, \lambda\kappa_{ti}\}$ for all m, t, i, by fitting

$$\tilde{v}_{tik} = \zeta_{ti} - \sum_{it} \beta_{ti}\mathbb{I}(dealer = i)\mathbb{I}(auction = t)\sigma_t^2 a_{tik} + \epsilon_{tik} \qquad (17)$$

with $\beta_{ti} = \rho_m(1 + \lambda\kappa_{ti})^{-1}$ such that $\lambda\kappa_{ti} = 0$ for all dealers i and auctions t when Treasuries are exempt, $\lambda\kappa_{ti} \geq 0$ otherwise, and $\rho_m \geq 0$. Here ζ_{ti} is a dealer-auction fixed effect, σ_t^2 is the return volatility plotted in Figure (3), and ϵ_{tik} represents finite sample measurement error in the values, $\tilde{v}_{tik}$.

We estimate two separate sets of parameters, one for when the exemption period started in 2020, and one when it ended in 2021. Given that capital requirements must be fulfilled quarterly, we use data from auctions that took place within one quarter around each policy change, i.e., 2020q1–2020q2 and 2021q4–2022q1.

We express bid and values in percentages of yields, and quantities in percentages of auction supply to avoid that changes in the supply, which increased substantially during the COVID-19 pandemic, affect our estimates. Therefore, β_{ti} measures by how many percentage points the dealer's willingness to pay decreases when demand increases by 1% of auctions supply in an auction with average return volatility ($\sigma_t^2 = 1$).

Estimation findings. Before identifying our parameters of interest, we analyze the slope coefficients, β_{ti}, from regression (17), when estimated without constraints on shadow costs or risk aversion, and using data from all auctions in our sample. We do this using estimated values and observed bids.

Our theory predicts that the willingness to pay, expressed in equation (16), becomes steeper when Treasuries are exempt if constraints bind. In that case, slope coefficients, β_{ti}, should be larger during the exemption period than in regular times. If competition is

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[12]An alternative would be to set risk aversion constant across maturities—this is rejected by the data, as shown in Figure 6.

sufficiently strong or dealers face little private information, we expect this to hold also for bidding functions. Figure 4 shows that both conjectures are true in our data. The median slope of the willingness to pay during the exemption period is lower than that of submitted bidding functions because bid shading decreases in quantity, as shown in Figure 5a.

Next, we separate the degree of risk aversion from the shadow costs by estimating regression (17) with constraints using data of auctions around the policy changes. We find that risk aversion is relatively low for all bond types with no clear pattern with respect to maturity (see Figure 6). The median degree of risk aversion is 0.006. This implies that a typical dealer is willing to pay 0.6 bps less for 1% more of the auction supply in an auction with average return volatility. If dealers were risk neutral, their willingness to pay would be perfectly flat.[13]

In comparison, the existing auction literature estimates risk aversion of similar, yet typically larger, magnitudes in non-financial settings, but given CARA preferences. Most papers consider single-unit auctions. For instance, Bolotnyy and Vasserman (2023) estimate a median degree of risk aversion of firms in procurement auctions to be 0.08. One exception is Häfner (2023), who analyzes discriminatory price auctions for Swiss tariff-rate quotas. He finds that the majority of bidders exhibit a risk aversion parameter of 0.007.

Shadow costs, which are shown in Figure 7, vary substantially across dealers and auctions, reaching higher values in 2020, when dealers struggled to absorb excess supply of Treasuries onto their balance sheets, than in 2022 when markets had calmed down. The long tail in the distribution of shadow costs suggests that there are some auctions in which some dealers expect to take losses. The median shadow cost is 3.5%.

Our cost estimates are in the range of existing estimates found in other markets using different data and different methodologies. For instance, Du et al. (2018) use the overnight

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[13]One way of quantifying the degree of risk aversion is to compute the certainty equivalent. We refrain from doing so because it is not straightforward in our case. The reason is that there are two layers of uncertainty. The first layer comes from the fact that the asset's return is random. The second layer comes from the fact that the auction outcome is uncertain.

Figure 4: Slope coefficients of estimated value and observed bids
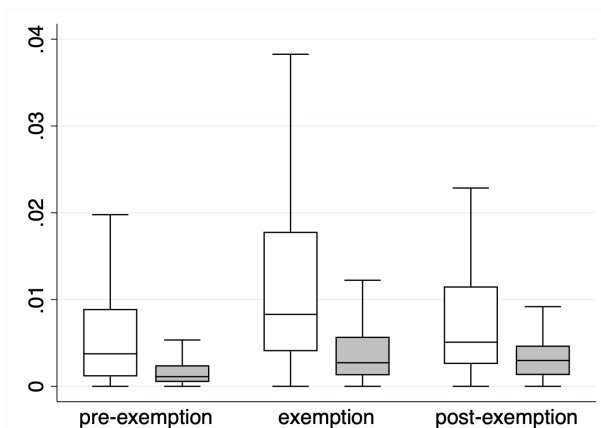
The white box plots of Figure 4 show the distribution of the estimated slopes coefficient of the dealers' willingness to pay in auction t of regression (17) without imposing restrictions on ρ or $\lambda\kappa_{ti}$ for three time periods: before the exemption of Treasuries from the LR (2019q1–2020q1), during the exemption period (2020q1–2021q4), and after the exemption (2022q1). The gray box plots show the analogue when using bids instead of estimated values. Dealer values and bids are in %; quantities are in % of auction supply.

spread between the interest rates on excess reserves paid by the Federal Reserve and the Fed Funds as proxy for the shadow costs of bank's balance sheets, which is a couple of basis points. Adrian et al. (2014) fit an augmented Fama-French factor model using quarterly balance sheet data from U.S. security broker-dealers from 1968q1 to 2009q4 and U.S. stock returns. They compute a price of leverage (which proxies for the funding constraint of Brunnermeier and Pedersen (2009), among others) of roughly 10% per year.

To get a better sense of how large the median shadow cost is in our setting, we compare it to the typical margin charged by a dealer selling a bond. Specifically, we compute the median difference between the bid yields at auction and the average yield obtained from selling one week before or after the auction. This difference is identical to the median shadow cost, suggesting that dealers barely break even in a typical auction. It is not surprising, therefore, that many dealers have exited the Canadian primary market for government debt (see Allen

Figure 5: Shading
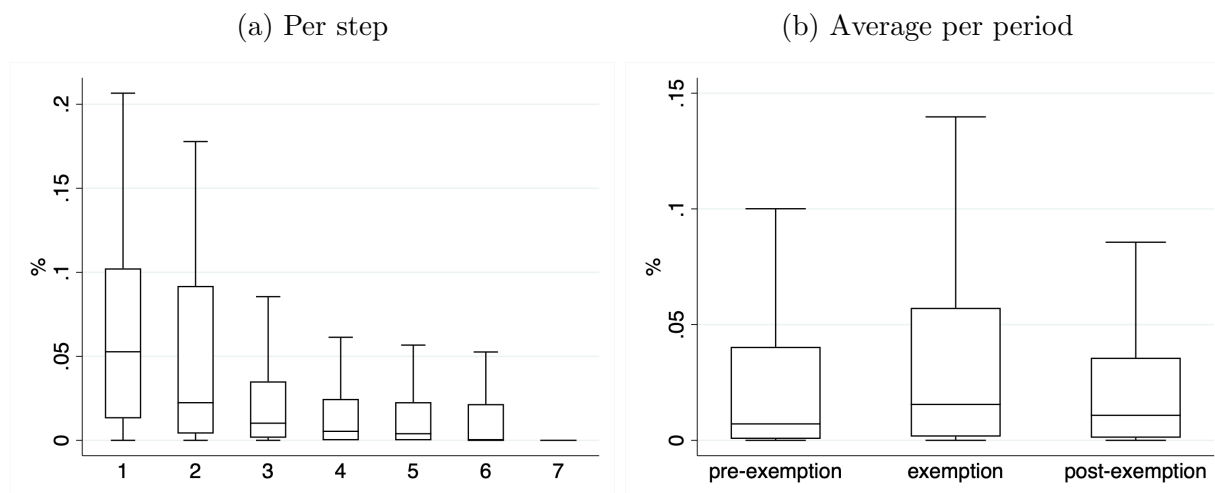
(a) Per step

(b) Average per period

Figure 5a shows box plots of how much dealers shade their bids at each step. It is the difference between the submitted bid and the estimated value, both in percentage. The distribution for each step is taken over dealers and auctions. Shading factors are small in absolute terms, and comparable to those in the literature. Figure 5b shows the distribution of shading across auctions, dealers, and steps in 2020q1 (pre-exemption), 2020q2 and 2021q4 (exemption), and 2022q1 (post-exemption).

et al. (2023)).[14]

Discussion. To separate shadow costs from the degree of risk aversion, we rely on three main identifying assumptions. Here we discuss what happens when they don't hold.

First, we assume that dealers are rational and play the equilibrium strategy of our empirical auction game. This assumption seems reasonable given that dealers are experienced financial institutions trained to participate in these auctions. Despite this, we also estimate the model under the assumption that dealers bid their true willingness to pay plus a random error term. We find that risk aversion is smaller when using bids than when using values

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[14]This finding also suggests that it might be valuable to carry the primary dealer status to generate revenues outside of the Treasury market. For instance, primary dealers have access to central bank liquidity facilities and can use their position in the government bond market to cross-sell to investors other investment products, such as underwriting or trading corporate debt. They are also more likely to attract foreign investors since the primary dealer status signals trustworthiness and stability.
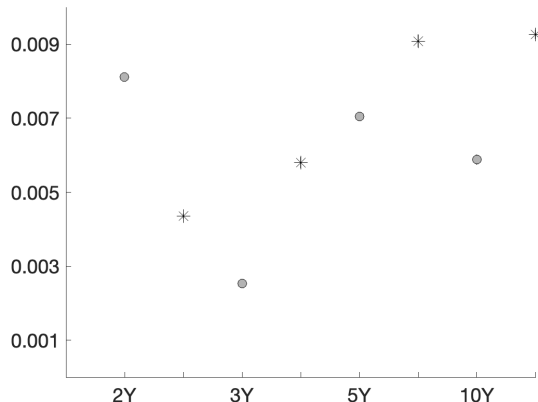
Figure 6: Risk aversion bond type

Figure 6 shows the risk aversion estimates, ρ_m, for $m = \{2Y, 3Y, 5Y, 10Y\}$ around both policy changes in 2020q1–202q2 (in circles) and in 2021q4–2022q1 (in stars). We exclude 30Y bonds because they were not issued in all four quarters we consider. The graph also plots the 95% confidence intervals for 2021q4–2022q1, but given that these intervals are very tight, they are not visible. To compute standard errors and these intervals, we fit equation (17) for each bootstrapped estimate of values. Each coefficient is measured in % of yield relative to % of supply.

because of bid shading (in line with Figure 4). However, the median shadow cost of 3.1% closely aligns with the median obtained from estimates based on willingness to pay.

Second, we assume that a dealer's willingness to pay is given by function (16), but this might not be the case. For example, there could be other balance sheet costs faced by banks apart from those arising from the capital constraint. In such a scenario, what we are actually identifying is the change in the total balance sheet cost as regulations change. To examine this, we could compare the median slope coefficient in Figure 4 across different periods, and would conclude that balance sheet costs were smaller during the exemption period.

Another concern to consider is the potential variation in risk aversion (per maturity) across quarters. This is particularly relevant in 2020, during market turmoil, as opposed to 2021/2022 when markets stabilized and most policies, such as Quantitative Easing, ceased. Therefore, it is reassuring to find shadow cost estimates of similar magnitudes for both periods. However, if we wish to allow for varying degrees of risk aversion across quarters,

Figure 7: Shadow costs
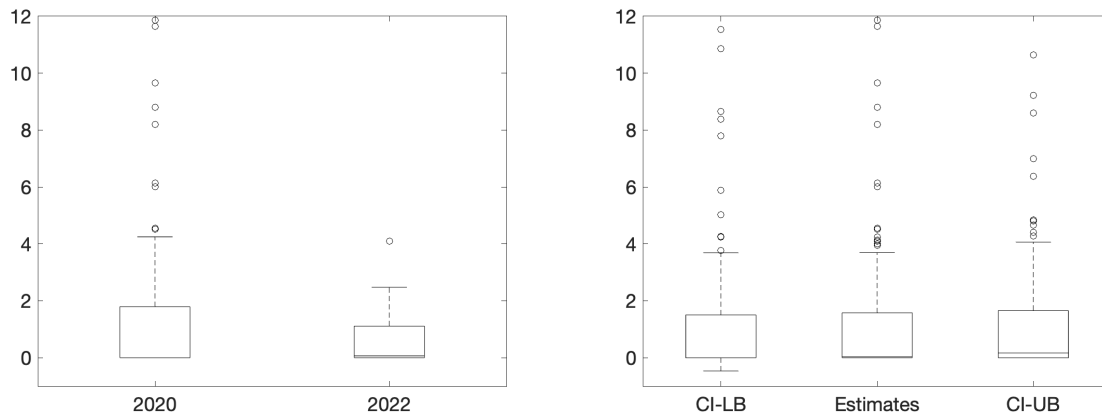
The LHS of Figure 7 shows a distribution of shadow cost point estimates, $\lambda\kappa_{ti}$, over auctions t around the two policy changes in 2020 and 2021-2022, excluding outliers. The RHS shows the distribution of the lower and upper bounds of the 95% confidence intervals for each point estimate (CI-LB and CI-UB, respectively), in addition to the distribution of the point estimates (Estimates) pooled across all auctions, and excluding outliers. The confidence intervals of the shadow costs are bootstrapped, analogous to those of the degree of risk aversion.

regression (17) identifies the change in the dealer's effective risk aversion, β_{ti}, which may depend on shadow costs and other factors in complicated ways.

Third, we assume that we can observe the volatility of the return that a dealer expects to generate from buying bonds at auction and selling them in the secondary market. This assumption seems reasonable as dealers actively trade in anticipation of auctions. However, with finite data, there may be measurement error in the observed volatility variable, which could bias our slope estimates downward. If we didn't observe volatility, we would need to impose more structure on the data, for instance, by taking a stance on the data-generating process of secondary market prices. Further, it would make identifying our parameters of interest more challenging. For more details, see Appendix C.

Counterfactual. We could use the model to precisely quantify by how much the auction yield and markups changed because capital requirements were relaxed (tightened). In practice, this involves computing counterfactual bidding step functions, which is not straightfor-

ward. Only recently has Richert (2023) introduced a numerical method to compute counterfactual bids in multi-unit auctions in which bidders don't face constraints. Even in standard auctions, this method is complex, computationally intense, and requires making assumptions on the distribution of bids, which is endogenous.

As an alternative, we provide a back-of-the-envelope calculation, which leverages our tractable theory of Section 2. In particular, Corollary 3 tells us that the market price (yield) increases (decreases) and the markup increases by $\eta = \left| \frac{1}{1+\lambda\kappa} - 1 \right| \%$ when the shadow cost of the capital constraint decreases by 1%.[15] The statement generalizes to discriminatory price auctions in which dealers face aggregate uncertainty and submit linear demand schedules. This does not fit our empirical setting perfectly. Nevertheless, we can get a rough sense of magnitudes, leveraging the fact that demand functions are approximately linear, as shown in Appendix Table A1.

Using the median shadow cost of the capital constraint (across all auctions and dealers), the η elasticity is about 0.034% or 3.4 bps. To see what this implies for the auction yield and markup, consider the first auction in 2022 after the exemption period ended. This auction cleared at a yield of 1.77%, and the average amount by which a dealer shaded her bid, which approximates the markup due to market power, was roughly 3 bps. Had the exemption not ended—implying a 100% reduction in the shadow cost of the capital constraint—the auction would have cleared at a yield of (1-0.034)1.77% ≈ 1.71%, with a markup of (1+0.034)3 bps ≈ 3.1bps.

This approximation suggests that the Canadian regulator did not face a quantitatively meaningful trade-off when deciding whether to relax or tighten capital constraints—in addition to the way the LR affects trading in the secondary market and concerns about systematic risk. Relaxing capital constraints decreased yields and increased markups by small amounts.

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[15]This statement hinges on the assumption that volatility is independent of $\lambda\kappa$. In practice, this might not always be the case (e.g., Du et al. (2023a)). Then our calculation neglects the indirect effect that a change in $\lambda\kappa$ has on the price and the markup via a change in volatility.

This is in line with the insignificant change in bid shading we observe when the policy changed, as shown in Figure 5b.

Robustness. We conduct a series of robustness checks in Appendix D. We explain what happens when we rely on different volatility indices, when expressing quantities in absolute terms rather than in percentages of supply, and when including different samples of bidding functions in the estimation.

5   Implications for intermediary asset pricing

The main focus of this paper is on analyzing the effect of changing capital constraints on the price of an asset and markups that arise due to dealer market power. To draw a closer connection to the intermediary asset pricing literature and inspire future research, we extend our formal analysis in Appendix B to study how intermediary market power affects whether and how commonly considered intermediary frictions (moral hazard or capital constraints) matter for asset prices. Here we only briefly mention the main takeaways.

Our first finding highlights that intermediary market power matters for asset pricing in the presence of moral hazard (see Corollary 4). From the existing literature, we know that we can eliminate moral hazard frictions in some cases by hiring a manager who is paid a sufficiently large fraction of the asset's return to incentivize non-shirking. This is no longer possible when the market is imperfectly competitive. Intuitively, one instrument cannot eliminate two types of frictions.

Our second finding highlights that capital constraints affect the asset price differently depending on the degree of competition (see Corollary 5). Consider a competitive market in which many dealers compete for the asset. If the market becomes less competitive because the number of dealers decreases, the asset price moves further away from the price that would arise without capital constraints. The reason is that each dealer wins more of the asset when fewer of them compete. This increases total exposure and tightens the capital

constraint. The opposite is true when a weakly competitive market with few dealers becomes less competitive because the number of dealers decreases. Now, even though each dealer wins more, the less competitive auction clears at a sufficiently low price. The price effect dominates the quantity effect and relaxes the constraint.

Taken together, these findings underline that it matters to take imperfect competition into account when analyzing how intermediary frictions affect asset prices, and motivate future research to assess the different degrees of competition across asset markets.

6   Conclusion

This paper studies if and how the capitalization of dealers affects asset prices when dealers have market power. We introduce a model to show that weaker capital requirements lead dealers to demand more of the asset at higher prices but also higher markups. We illustrate how to estimate the model with data on Canadian Treasury auctions. Our findings highlight that weaker capital requirements reduce the funding cost of debt but increase market power and reduce market liquidity.

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ONLINE APPENDIX

Intermediary Market Power and Capital Constraints

by Jason Allen and Milena Wittwer

Appendix A provides equilibrium results for discriminatory price auctions. Appendix B generalizes the model to draw implications for the intermediary asset pricing literature. Appendix C explains why it is useful to observe return volatility. Appendix D presents our robustness analysis. Proofs are in Appendix E.

A   Discriminatory price auctions and step functions

Here we adjust our benchmark model to the case of discriminatory price auctions, in which bidders pay the prices they offered to pay for all units won, rather than the market clearing price. We consider two settings: one in which dealers submit continuous demand functions, as in our benchmark model, and one in which dealers must submit step functions, as in Kastl (2012). We use the same notation for continuous demand curves, $p_i(\cdot, \theta_i)$, the probability that a dealer who bids price $p = p(a, \theta_i)$ wins less than a at market clearance given that other dealers play the equilibrium demand $a^*(\cdot, \theta_j)$, $G(a, p|\theta_i) = \Pr(\boldsymbol{A} - \sum_{j \neq i} a^*(p(a, \theta_i), \boldsymbol{\theta_j}) \leq a|\theta_i)$, and the Lagrange multiplier, λ_i, even though all of these are auction-format specific.

Proposition 3. *(i) In any symmetric equilibrium with continuous demand curves, dealer i submits demand functions, $p^*(\cdot, \theta_i)$, such that $p^*(a, \theta_i) = p$ for all a, given by*

$$p = \frac{v_i(a)}{1 + \lambda_i \kappa} - shading(a, p|\theta_i), \tag{18}$$

where $v_i(a)$ and $\lambda_i \geq 0$ are as in Proposition 1, and $shading(a, p|\theta_i) = -\frac{1 - G(a, p|\theta_i)}{\frac{\partial G(a, p|\theta_i)}{\partial p}}$.

(ii) When demand curves are step functions $\{a_k, p_k\}_{k=1}^{K_i}$, the equilibrium function satisfies

$$p_k = \frac{v_i(a)}{1 + \lambda_i \kappa} - \frac{\Pr(p_{k+1} \geq \boldsymbol{P^*}|\theta_i)}{\Pr(p_k > \boldsymbol{P^*} > p_{k+1}|\theta_i)} \tag{19}$$

at every step but the last one; at the last step, the dealer bids truthfully.

When dealers only face aggregate uncertainty and submit continuous demand functions, we can solve for a symmetric equilibrium, under the (additional) assumption that supply $\boldsymbol{A}$ follows a Generalized Pareto distribution with CDF: $1 - (1 + \xi\frac{A}{\nu N})^{-\frac{1}{\xi}}$, with $\xi < \frac{N-1}{N}$ and $\nu = -\xi\frac{\overline{A}}{N}$.

Proposition 4. *Let all dealers share the same information with inventory position $z \in \mathbb{R}$, and equity capital $E > 0$. There exists a symmetric linear equilibrium in which each dealer submits*

$$p^*(a) = \frac{1}{1 + \lambda\kappa}\left(\bar{\mu} - \rho\sigma^2\left(\frac{N-1}{N(1-\xi)-1}\right)a\right), \quad with \tag{20}$$

$$\lambda = \begin{cases} 0 & if \ \frac{E}{\kappa} \geq B + \kappa\psi z \\ \frac{B}{E - \kappa\psi z} - \frac{1}{\kappa} > 0 & if \ \frac{E}{\kappa} < B + \kappa\psi z, \end{cases} \tag{21}$$

where $\bar{\mu} = \mu - \rho\sigma\left(\left(\frac{N}{N(1-\xi)-1}\right)\sigma\nu + \iota\sigma_z z\right)$,
and $B = \left(\frac{\mu N(1-\xi) - \mu - \rho\sigma(N\nu\sigma - \iota\sigma_z z + \iota\sigma_z(1-\xi)Nz)}{N(1-\xi)-1}\right)\mathbb{E}\left[\frac{\boldsymbol{A}}{N}\right] - \left(\frac{(N-1)\rho\sigma^2}{N(1-\xi)-1}\right)\mathbb{E}\left[\left(\frac{\boldsymbol{A}}{N}\right)^2\right].$

Propositions $3(i)$ and 4 are analogues to Propositions 1 and 2. Therefore Corollaries 2 and 3 generalize to discriminatory price auctions, with the exception of the statements about price impact and market liquidity, which are not clearly defined in discriminatory price auctions.

B Intermediary asset pricing implications

Here we study whether the degree of competition between intermediaries affects the way intermediary frictions—specifically, moral hazard and capital constraints—affect asset prices. For this we rely on a simple version of the He and Krishnamurthy (2012, 2013) models,

presented in He and Krishnamurthy (2018), that builds on Holstrom and Tirole (1997). Our contribution is to introduce imperfect competition in the asset market by relying on insights from the literature on auctions and market microstructure. We consider the simplest auction environment without private signals and known supply. It is straightforward to introduce supply uncertainty as in Proposition 2.

Model with moral hazard. The economy runs for three periods, $t = 0, 1, 2$. There is one risky asset of aggregate supply A that pays out a return $\boldsymbol{R} \sim N(\mu, \sigma^2)$ per unit, and a numeraire (cash). The return is unknown to all agents in all periods but the last one.

There are $2 < N < \infty$ banks, indexed by I. Each bank serves a unit mass of households (H) who never consider switching banks, i.e., there are fixed households-bank pairs. In addition, each bank has a trading desk i who is responsible for trading the risky asset.

Households cannot directly invest in the asset market, but must invest via a bank. For this, households and their bank contract with trading desk i (of the bank that serves the households) who invests in the risky asset on the households' behalf.

Banks and households have CARA preferences, that is, holding wealth ω_j generates the following utility for an agent of type $j \in \{H, I\}$:

$$u_j(\omega_j) = 1 - \exp\left(-\rho_j \omega_j\right), \tag{22}$$

with risk aversion $\rho_j > 0$. A trading desk and its bank share the same utility function. Wealth comes from buying and holding the asset. For instance, if agent j gets a_j at price p, the wealth is $\omega_j = a_j(\boldsymbol{R} - p)$.

The sequence of events is as follows: In period 0, each households-bank pair chooses what fraction ϕ_i of the total wealth (that will be generated from investing in the asset) will be paid to trading desk i in period 2.[16] The contract is chosen to maximize their joint

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[16]We could let the pair choose an affine contract parametrized by (K_i, ϕ_i), where ϕ_i is the linear share of the return generated by the investment that is paid to the trading desk, and K_i is a management fee that is paid to the trading desk independent of the return. In the case of CARA

expected utility obtained at the end of the game. Alternatively, you may think of a market designer who chooses ϕ_i's to maximize expected welfare of the economy subject to incentive constraints. In period 1, all N trading desks compete in a uniform price auction to buy a_i of the risky asset, submitting continuous and strictly decreasing demand functions: $a_i(\cdot)$. Each trading desk may decide to shirk or exert effort; $s_i \in \{0, 1\}$, where s_i=1 is shirking. When a trading desk chooses to shirk, the wealth of its bank falls by Δ, but the trading desk gains a private benefit of b. In period 2, the asset pays its return, and all transactions take place.

Proposition 5. *Define $m = \frac{\Delta}{b} - 1 \geq 0$. There exists an equilibrium in which $\phi_i = \phi = \frac{1}{1+m}$, and the clearing price is*

$$P^* = \mu - \left(\frac{\rho_I \sigma^2}{1+m}\right)\left(\frac{N-1}{N-2}\right)\frac{A}{N}. \tag{23}$$

Trading desk i buys amount $\frac{A}{N}$, its bank obtains $\phi\frac{A}{N}$, and each mass of households receives $(1-\phi)\frac{A}{N}$.

In this equilibrium, the market clearing price has the familiar functional form of a uniform price auction with N bidders (here trading desks). $m = \frac{\Delta}{b} - 1$ is the maximum amount of dollars that households can invest (per dollar that the trading desk purchases) so that the trading desk exerts effort in the auction. If the moral hazard friction is small, which happens when the benefit b from shirking is small, the trading desk can be incentivized to exert effort with little skin in the game, that is, with a small ϕ_i. The more beneficial it becomes to shirk, the higher ϕ_i must be.

When the asset market is perfectly competitive, as in He and Krishnamurthy (2018), there are two cases depending on how attractive it is for the trading desk to shirk. In the first case, shirking is attractive so that the constraint that incentivizes trading desk i to exert effort, $\phi_i\Delta \geq b$, binds. As a result, the intermediation frictions affect the asset price.

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preferences, the lack of a wealth effect implies that K_i plays no role in asset demand and equilibrium prices.

In the second case, the incentive constraint doesn't bind, and the first-best solution can be obtained through choosing the optimal contract ϕ_i.

When the asset market is imperfectly competitive, intermediation frictions always affect the asset price. Intuitively, this is because one instrument (per households-bank pair), ϕ_i, cannot correct two frictions: moral hazard and imperfect competition.

Corollary 4. *There is no contract $\phi_i = \phi \ \forall i$ that implements the price and allocation of a frictionless market, in which both banks and households compete in an auction that induces truthful bidding, that is, avoids bid shading.*

Model with capital constraints. So far, the intermediation friction came from moral hazard. Now we add capital constraints. Suppose that each trading desk purchases $a_i(p)$ of the asset if the asset market clears at price p, and makes loans L to an unmodeled sector of the economy, which we normalize to 0 w.l.o.g. The desk is subject to a Basel III-type capital constraint: $\kappa p a_i(p) \leq E$, where E denotes the total equity capital.

From He and Krishnamurthy (2018) we know that the capital constraint binds only if the moral hazard incentive constraint binds. Given this, it is not surprising that the auction clearing price is analogous to the price of Proposition 2, where dealers face a similar capital constraint.

Proposition 6. *In equilibrium $\phi_i = \frac{1}{1+m}$ for all i, and the market clears at*

$$P^* = \frac{1}{1 + \lambda\kappa}\left(\mu - \left(\frac{\rho_I\sigma^2}{1+m}\right)\left(\frac{N-1}{N-2}\right)\frac{A}{N}\right),$$

$$\text{with } \lambda = \begin{cases} 0 & \text{if } \frac{E}{\kappa} \geq B \\ \frac{B}{E} - \frac{1}{\kappa} > 0 & \text{if } \frac{E}{\kappa} < B \end{cases} \text{ with } B = \mu\frac{A}{N} - \left(\frac{\rho_I\sigma^2}{1+m}\right)\left(\frac{N-1}{N-2}\right)\frac{A^2}{N}. \quad (24)$$

Trading desk i buys amount $\frac{A}{N}$, its bank obtains $\phi\frac{A}{N}$, and each mass of households receives $(1-\phi)\frac{A}{N}$.

Does competition matter? We now analyze whether imperfect competition in the asset market matters for whether and how the asset price is affected by intermediary frictions.

To vary the degree of competition, we vary the number of banks (or trading desks) who compete for the asset. More bidders in an auction translates into greater competition.[17]

Corollary 5. *Define* $\bar{N} : \mu = \frac{(4+\bar{N}(2\bar{N}-5))\phi\rho_I\sigma^2 A}{(\bar{N}-2)^2\bar{N}}$ *and let* $\mu > 0$.

(i) *Intermediary financing frictions always affect the asset price.*

(ii) *Let the asset market become less competitive in that N decreases to N'. For $N' \geq \bar{N}$, the shadow cost of the capital constraint increases, so that the market price is more strongly affected by the capital constraint. For $N' < \bar{N}$, the shadow cost decreases and the market price is less strongly affected by the capital constraint.*

Competition matters in two ways. First, with imperfectly competitive asset markets, it is no longer the case that intermediation frictions—either moral hazard or capital constraints—can be corrected by choosing intermediary remuneration, ϕ_i, optimally. To overcome (or at least reduce) the extra friction which arises from the fact that the asset market isn't perfectly competitive, a more complex remuneration scheme would be necessary.

Second, when the market is less competitive as a result of fewer intermediaries competing for the asset, the shadow cost of the capital constraint changes. Intuitively, a positive shadow cost guarantees that the capital constraint binds: $\kappa P^* a_i^*(P^*) \leq E$. The shadow cost is higher; the larger $P^* a_i^*(P^*)$ would be relative to E in a setting without the constraint. Thus, to

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[17]Note that this is different from the main text, in which we measure competition by the extent to which the market price that arises in the market with market power differs to the price that would arise in a perfectly competitive market. Measuring this price wedge directly gives a more precise idea of the impact of market power on prices than counting the number of market participants. However, since this wedge is endogenous, it is less useful for analyzing how changes in market power affect prices. Crucially, both measures of competition are qualitatively identical in that the price wedge (and the price impact) decreases monotonically when the number of market participants increases.

understand how the shadow cost changes, we must think through how $P^* a_i^*(P^*)$ changes as the number of bidders N decreases.

There are two opposing effects. On the one hand, each bidder wins more: $a_i^*(P^*) = \frac{A}{N}$ increases. On the other hand, the less competitive auction clears at a lower price: P^* decreases. When the quantity effect dominates the price effect, the shadow cost increases as N decreases. Whether this is true or not depends on how competitive the market is, i.e., the number of bidders. If the degree of competition is sufficiently strong $(N' \geq \bar{N})$, the quantity effect dominates, otherwise $(N' < \bar{N})$ the price effect dominates.

C   Return volatility

We illustrate why observing volatility, σ_t^2, facilitates identification by means of an example. For this, recall that we construct return volatility from the prices, p_{tij}^S, a dealer i charges when selling a to-be-issued bond to a trader j prior to the auction t, that is, before observing her value realization, $\tilde{v}_{ti}(\cdot)$, on auction day: $\sigma_t^2 = Var(p_{tij}^S)$. Further, refer to the distribution of $\tilde{v}_{ti}(\cdot)$, specified in equation (16), by F_t^v and the distribution of the shadow cost, which is a random variable given its dependence on θ_{ti}, by $F_t^{\lambda\kappa}$.

If we didn't observe σ_t^2, we would need to impose some structure on the data-generating process to identify our parameters of interest. Here we provide one example of this process, which is by no means exclusive. Assume that each dealer charges a common price to its clients, that may depend on the value distribution, F_t^v, and a trade-specific markup. This markup depends on the shadow cost of the capital constraint that the dealer faces at that moment. To formalize this idea, let dealer i observe a realization of the shadow cost, $\lambda\kappa_{tij}$, from the distribution $F_t^{\lambda\kappa}$ before selling to buyer j. The dealer charges a markup of $m_t(\lambda\kappa_{tij})$, where $m_t(\cdot)$ maps the shadow cost draw into $\mathbb{R}$. To identify our parameters of interest, we would need to replace σ_t^2 in equation (16) by $Var(m_t(\boldsymbol{\lambda\kappa_{tij}}))$. From here we see that identification becomes challenging. For instance, we would need to specify a functional form

for $m_t(\cdot)$, and estimate a system of equations, which includes equation (16), the distribution of shadow costs, and function $m_t(\cdot)$.

D   Robustness analysis

We conduct a series of robustness checks to validate our risk aversion and shadow cost estimates. All risk aversion estimates are presented in Appendix Table A2; Appendix Figure A2 shows the distribution of shadow cost estimates for all specifications.[18]

We start by analyzing the sensitivity of our parameter estimates to the number of steps included in the values functions. In our benchmark specification, we include all functions with at least two steps (which are essentially all functions) to avoid a potential bias coming from omitting functions. Given that we linearly interpolate between steps using our model, we might be concerned about doing this when there are few steps. Our results, however, are robust to using value functions with more steps—three to six, where we do not include robustness for seven steps since not all dealers use the maximum allowable number of steps in all auctions.

Next, we estimate equation (17) with quantities expressed in million C\$. In our benchmark specification, we normalize quantities by the auction supply to avoid our estimates being affected by the fact that the Bank of Canada issued larger amounts of debt during the exemption period than in regular times. Given that dealers have an obligation to actively participate in the auctions, the increased supply implies that dealers demanded larger amounts (see Appendix Figure A3). Further, since dealers are supposed to bid competitively, and are given a price range when bidding, increasing the total demand decreases the slope in the dealer's bidding function and willingness to pay during the exemption period (relative to the case in which we normalize demand by the supply). The model rationalizes smaller slopes by smaller risk aversion and shadow cost parameters.

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[18]A robustness analysis of the value estimates is provided in Allen et al. (2023).

Third, we verify robustness with respect to our measure of volatility. In our benchmark specification, we construct volatility using trades where we observe dealers selling the to-be-auctioned security in a five trading day window prior to auction. This is natural given that most trading prior to an auction occurs in the one week between the tender open call and the auction close. The more days we include, the larger the volatility. This effect is stronger during the exemption period than during regular times, so that the slope in the dealer's willingness to pay is steeper when assuming zero shadow costs. To rationalize that the observed slope is lower, the shadow costs are higher than in the benchmark specification. This effect goes in the opposite direction when including fewer days to construct the volatility index. Moreover, the fewer days we include, the less likely a security is traded, so that the volatility index is missing for the auction of that security. To avoid dropping these auctions entirely, we use the average volatility of same maturity-type auctions within the quarter—in our benchmark specification there is no need to do this.

In addition, we could estimate our model using different volatility indices. One alternative is to use the Implied Volatility Index for Canadian Treasuries, which measures the expected volatility in the Treasury market over the next 30 days (Chang and Feunou (2014)). Given that this volatility drops more strongly during the exemption period than our volatility index, shadow cost estimates are higher when relying on the implied volatility.

Another alternative is to construct return volatility using post-auction trades. We refrain from doing so, because dealers do not know what happens after the auction at the time they bid. Further, post-auction prices likely depend on the realization of the dealer's private information, and with that their willingness to pay, in the auction. This implies that the post-auction volatility—an independent variable in equation (17)—is a function of the dependent variable and would lead to a simultaneous equation bias.

E Proofs

We first present the proofs of all propositions, and then of all corollaries.

Proof of Proposition 1. We consider the case in which supply has bounded support, $(0, \overline{A}]$ with $\overline{A} < 0$, but the proof generalizes to the case of unbounded support. For ease of notation, we omit the type θ_i in this proof, and instead include an $i-$subscript, e.g., $p_i(a) = p_i(a, \theta_i)$.

Consider dealer i, and fix all other demand schedules at the equilibrium. To determine the best response, dealer i solves maximization problem (6). To simplify this problem, let $v_i(a) = \frac{\partial V_i(a)}{\partial a}$, denote $p_i'(a) = \frac{\partial p_i(a)}{\partial a}$, and abbreviate all functions, for instance, $p_i(\cdot)$ by p_i when useful. Further, let $\overline{a}_i^c$ be the largest amount that bidder i can win when submitting any demand function given others play an equilibrium function, and $\overline{a}_i^*$ be the largest amount the bidder wins when playing the equilibrium strategy. With this, and auxiliary distribution $G_i(a, p)$ which is defined in Proposition 1, the dealer's maximization problem becomes:

$$\max_{p_i \in \mathcal{B}} I(p_i) \text{ subject to } L(p_i) \geq 0, \text{ with} \tag{25}$$

$$I_i(p_i) = \int_0^{\overline{A}} F_i(p_i(a), p_i'(a), a) da \text{ with } F_i(p_i(a), p_i'(a), a) = [v_i(a) - p_i(a) - ap_i'(a)][1 - G_i(a, p_i(a))],$$

$$L_i(p_i) = E - \kappa\psi z - \int_0^{\overline{A}} H_i(p_i(a), p_i'(a), a) da \text{ with } H_i(p_i(a), p_i'(a), a) = \kappa[p_i(a) + p_i'(a)a][1 - G_i(a, p_i(a))].$$

Here we have integrated by parts to obtain $I(p_i)$ and $L(p_i)$. A function p_i^* is optimal if the following conditions are satisfied:

$$\frac{\partial(F_i + \lambda_i H_i)}{\partial p_i}(p_i^*(a), p_i^{*'}(a), a) - \frac{d}{da}\left(\frac{\partial(F_i + \lambda_i H_i)}{\partial p_i'}(p_i^*(a), p_i^{*'}(a), a)\right) = 0 \text{ for all } a \in [0, \overline{a}_i^*], \tag{26}$$

$$L_i(p_i^*) \geq 0 \text{ and } \lambda_i \geq 0, \tag{27}$$

$$\frac{\partial(F_i + \lambda_i H_i)}{\partial p_i'}(p_i^*(0), p_i^{*'}(0), 0) = \frac{\partial(F_i + \lambda_i H_i)}{\partial p_i'}(p_i^*(\overline{a}_i^*), p_i^{*'}(\overline{a}_i^*), \overline{a}_i^*) = 0. \tag{28}$$

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The last two conditions are the natural boundary conditions. They hold automatically given that $\frac{\partial(F_i+\lambda_i H_i)}{\partial p_i'}(p_i^*(a), p_i^{*'}(a), a) = -(1+\lambda_i\kappa)a[1 - G_i(a, p_i^*(a)]$, and $G_i(0, p_i^*(0)) = 0$, and $G_i(\bar{a}_i^*, p_i^*(\bar{a}_i^*)) = 1$ by definition of G_i.

Simplifying (26) gives: $-(1+\lambda_i\kappa)[1-G_i(a, p_i^*(a)]-[v_i(a)-(1+\lambda_i\kappa)(p_i^{*'}(a)a+p_i^*(a))]\frac{\partial G_i(a,p_i^*(a)}{\partial p_i} - \frac{d}{da}\big(-(1+\lambda_i\kappa)a[1-G_i(a, p_i^*(a)]\big) = 0$, where $\frac{d}{da}\big(a[1-G_i(a, p_i^*(a)]\big) = [1 - G(a, p_i^*(a))] - a\big[\frac{\partial G_i(a,p_i^*(a)}{\partial a} + \frac{\partial G_i(a,p_i(a)}{\partial p_i}p_i^{*'}(a)\big]$. This rearranges to condition (10). $\square$

Proof of Proposition 3. The proof of statement (i) is analogous to the proof of Proposition 1. There is only one difference, which comes from the fact that bidders pay the prices they bid for all units that they win instead of the market clearing price. This implies that $I_i(p_i)$ in maximization problem (25) is

$$I_i(p_i) = \int_0^{\overline{A}} F_i(p_i(a), a)da \text{ with } F_i(p_i(a), q) = [v_i(a) - p_i(a)][1 - G_i(a, p_i(a))]. \quad (29)$$

With slight abuse of notation, we are using the same labels as for the uniform price auction.

The proof of statement (ii) follows from Kastl (2012)'s original proof. The only difference is that the objective function is the Lagrangian, which is analogous to (25). $\square$

Proof of Proposition 4. When supply follows a Generalized Pareto distribution, we can solve for a function that fulfills condition (18) of Proposition 3. For this, we combine the insight that a dealer bids as if their true willingness to pay was $\frac{v(a)}{1+\lambda\kappa}$ for any given $\lambda \geq 0$, with a known result from the literature on equilibrium existence (e.g., Proposition 7 of Ausubel et al. (2014), Theorem 2 of Wittwer (2018))). In equilibrium, $\lambda > 0$ is pinned down by the capital constraint if the constraint binds, and is zero otherwise. $\square$

Proof of Proposition 5. To derive the equilibrium of the proposition, we guess and verify. We guess that there is a symmetric equilibrium in which all contracts are the same, $\phi_i = \phi$, and all trading desks i choose the same demand, $a(p) = \big(\frac{N-2}{N-1}\big)\frac{1}{\rho_I\sigma^2\phi}(\mu - p)$, for each p, and level of effort, $s_i = 0$. To verify that this equilibrium exists and derive the functional

form for ϕ, we begin in the auction stage. We let all trading desks other than i play the symmetric equilibrium and determine trading desk i's best response in the auction. Then we find contract ϕ_i that trading desk i's bank and households choose assuming that $\phi_j = \phi$ for all $j \neq i$. The proof is complete when we have shown that the best responses equal the guessed equilibrium.

A trading desk with contract ϕ_i chooses her demand function $a_i(\cdot)$ and whether to exert effort or not, $s_i \in \{0, 1\}$, to maximize the expected utility she obtains from wealth

$$\omega_i(a_i(p), s_i) = \phi_i\{a_i(p)(\boldsymbol{R} - p) - s_i\Delta\} + s_i b \tag{30}$$

point-wise for each p and subject to market clearing, i.e., $\sum_i a_i(p) = A$. If the trading desk exerts effort, $s_i = 0$, her wealth in period 2 is ϕ_i of the return that the asset will generate, which is $a_i(p)(\boldsymbol{R} - p)$. If the desk shirks, $s_i = 1$, she obtains benefit b but suffers a loss which comes from the fact that the total generated wealth reduces by Δ. Given her contract, the trading desk's loss is ϕ_i of that. Thus, the trading desk exerts effort if the benefit of doing so is larger than the cost, which is the case when

$$\phi_i\Delta \geq b \Leftrightarrow \phi_i(1 + m) \geq 1 \text{ where } m = \frac{\Delta}{b} - 1. \tag{31}$$

Maximizing the objective function and imposing market clearance, we find that desk i chooses

$$a_i(p) = \left(\frac{\phi\rho_I\sigma^2}{N - 2} + \phi_i\rho_I\sigma^2\right)^{-1}(\mu - p) \tag{32}$$

in response to all other trading desks choosing the equilibrium guess. The auction clears at

$$P^c = \mu - \frac{(N - 1)\phi\phi_i\rho_I\sigma^2 A}{(N - 2)(\phi + (N - 1)\phi_i)}. \tag{33}$$

Anticipating how trading desks behave in the auction, households and the bank choose ϕ_i for trading desk i to maximize their joint expected utility from wealth. Given CARA utility,

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this is equivalent to

$$\max_{\phi_i} \text{Welfare}(\phi_i) = \sum_{j \in \{H,I\}} a_j(P^c)(\mu - P^c) - \frac{1}{2}a_j^2(P^c)\rho_j\sigma^2 \text{ subject to } \phi_i\Delta \geq b, \qquad (34)$$

where $a_H(p) = (1 - \phi_i)a_i(p), a_I(p) = \phi_i a_i(p)$, and $a_i(p)$, and P^c are given by (32) and (33), respectively. The solution to this problem pins down a mapping between ϕ_i and ϕ. In the symmetric equilibrium, ϕ_i must equal ϕ. Depending on the size of Δ and b, or equivalently m, there is a unique solution to this given $m \geq 0$, which is $\phi_i = \phi = \frac{\Delta}{b}$, or equivalently, $\phi_i = \phi = \frac{1}{1+m}$. Inserting this $\phi_i = \frac{1}{1+m}$ into the market clearing price completes the proof. $\qquad \square$

Proof of Proposition 6. As the moral hazard incentive constraint binds, so that $\phi_i = \frac{1}{1+m}$ when the capital constraint binds, the proof is analogous to the proof of Propositions 5. $\quad \square$

Proof of Corollary 1. (i) To show that there is no linear equilibrium, we take the perspective of dealer i and fix all other dealers' demand functions. Dealer i chooses an optimal quantity point a for each price p at which the market might clear. The point-wise first-order condition, which is the analogue to conditions (14) and (15), is

$$\frac{\mu - \rho\sigma[\sigma a + \iota\sigma_z z_i]}{1 + \lambda_i \kappa} = p + \mathbb{E}[\boldsymbol{\Lambda_i}|p, \lambda_i]a, \qquad (35)$$

where $\Lambda_i = \frac{\partial p}{\partial a}$ is dealer i's price impact, and λ_i denotes the Lagrange multiplier of the capital constraint. In equilibrium, dealer i's price impact equals the slope of the inverse residual supply curves, i.e., $\Lambda_i = -\left(\sum_{j \neq i} \frac{\partial a_j(\cdot)}{\partial p}\right)^{-1}$. From the first-order condition, we know that dealer i's best response is linear if and only if $\mathbb{E}[\boldsymbol{\Lambda_i}|p, \lambda_i] = \mathbb{E}[\boldsymbol{\Lambda_i}|\lambda_i]$, or equivalently, observing price realization p does not update the dealer's belief about other dealers' constraints. However, even if we assume that this holds for all dealers i, the equilibrium price is a function of λ_i of all i. Therefore, there cannot be a linear equilibrium.

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(ii) Now let $N \to \infty$, so that each dealer's price impact converges to 0, and the market becomes perfectly competitive. Then, the following condition

$$\frac{\mu - \rho\sigma[\sigma a + \iota\sigma_z z_i]}{1 + \lambda_i \kappa} = p \Leftrightarrow \tilde{v}_i(a) = (1 + \lambda_i \kappa)^{-1}(\mu - \rho\sigma(\sigma a + \iota\sigma_z z_i)) \qquad (36)$$

characterizes the equilibrium demand of dealer i. This equilibrium exists if there are $\lambda_i \geq 0$ for all i such that the capital constraints are satisfied. The market clears at

$$P^\infty = \lim_{N \to \infty} \frac{\mu - \rho\sigma(\sigma\frac{A}{N} + \frac{1}{N}\sum_i \iota\sigma_z z_i)}{1 + \frac{1}{N}\sum_i \lambda_i \kappa} = \frac{\mu - \rho\iota\sigma\sigma_z \mathbb{E}[z_i]}{1 + \mathbb{E}[\lambda_i]\kappa} \qquad (37)$$

as $\lim_{N \to \infty} \frac{1}{N}\sum_i z_i = \mathbb{E}[z_i]$ and $\lim_{N \to \infty} \frac{1}{N}\sum_i \lambda_i = \mathbb{E}[\lambda_i]$ by the law of large numbers. $\qquad \square$

Proof of Corollaries 2 and 3. (i) When dealers only face aggregate uncertainty, equilibrium demand is given by Proposition 2. Further, we can infer the price impact $\Lambda = \frac{1}{N-2}\frac{\rho\sigma^2}{1+\lambda\kappa}$ and the market clearing price: $P^*(\Lambda) = \frac{1}{1+\lambda\kappa}\left(\frac{1}{N}\sum_i \mu_i - \frac{N-1}{N-2}\frac{A}{N}\rho\sigma^2\right)$, with $\mu_i = \mu - \rho\iota\sigma\sigma_z z_i$. In contrast, when bidders are price-takers and submit their true willingness to pay, the market clears at: $P^*(0) = \frac{1}{1+\lambda\kappa}\left(\frac{1}{N}\sum_i \mu_i - \frac{A}{N}\rho\sigma^2\right)$. Thus, $markup = P^*(0) - P^*(\Lambda) = \Lambda\frac{A}{N}$. From here it is easy to see that the slope of equilibrium demand (13), the market price $P^*(\Lambda)$, price impact, and the markup increase when $\lambda\kappa$ decreases. Further, we can compute the following elasticity:

$$\eta = \frac{\partial markup}{\partial\lambda\kappa}\frac{\lambda\kappa}{markup} = \frac{P^*(\Lambda)}{\partial\lambda\kappa}\frac{\lambda\kappa}{P^*(\Lambda)} = \frac{\partial\Lambda}{\partial\lambda\kappa}\frac{\lambda\kappa}{\Lambda} = \frac{1}{1+\lambda\kappa} - 1. \qquad (38)$$

(ii) Now consider the case in which dealers have private information, so that Proposition 1 applies. When the market is perfectly competitive and all dealers are price-takers, dealer demand is given by $\tilde{v}_i(a) = (1 + \lambda_i \kappa)^{-1}(\mu - \rho\sigma(\sigma a + \iota\sigma_z z_i))$ according to Corollary 1. The market clears at the price given in (37). When shadow costs decrease for all i, the demand curve becomes steeper and the market price increases. Further, we can compute elasticity η^∞ analogously to before. When at least some dealers share their bids, the effects depend

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on the way the shading factor changes relative to the true willingness to pay. This, in turn, depends on the distribution of signals, and supply, and the number of competing dealers. $\square$

Proof of Corollary 4. To show that there is no $\phi_i = \phi$ for all i that implements the price and allocation of a frictionless market, we compute the price and allocation of such a frictionless market and compare both to the analogue in our market setting.

In the frictionless market, agent of type $j \in \{H, I\}$ submits the following demand $\bar{a}_j(p) = \frac{1}{\rho_j \sigma^2}(\mu - p)$. Intuitively, each agent submits the marginal utility she achieves from winning amount $a_j(p)$, conditional on the auction clearing at p. The market would clear at price $\bar{P}^c = \mu - (\rho_H + \rho_I)\sigma^2 \frac{A}{N}$ at which $N\bar{a}_H(\bar{P}^c) + N\bar{a}_I(\bar{P}^c) = A$. Households obtain $\bar{a}_H(\bar{P}^c) = \frac{A}{N}\frac{\rho_H + \rho_I}{\rho_H}$, and the bank obtains $\bar{a}_I(\bar{P}^c) = \frac{A}{N}\frac{\rho_H + \rho_I}{\rho_I}$.

Comparing this price and the allocation to the one presented in Proposition 5, we see that it is not possible to obtain the frictionless price and frictionless allocation with the same ϕ. We can only obtain one of the two. $\square$

Proof of Corollary 5. Statement (i) follows from Proposition 5 and Corollary 4. To show statement (ii) we only need to determine how $\lambda > 0$ changes in N, since we already know that the market price increases when λ decreases.

$$\frac{\partial \lambda}{\partial N} = \frac{A(-\mu(N-2)^2 N + (4 + N(2N-5))\phi\rho_I\sigma^2 A)}{E(N-2)^2 N^3}$$

$$\frac{\partial \lambda}{\partial N} \begin{cases} < 0 & \text{if } \mu > c(N) = \frac{(4+N(2N-5))\phi\rho_I\sigma^2 A}{(N-2)^2 N} \\ > 0 & \text{otherwise} \end{cases}$$

Note that cutoff $c(N)$ strictly decreases in N, and converges to 0 as $N \to \infty$. Therefore, given $\mu > 0$, there is some $\bar{N}$ at which $\mu = c(\bar{N})$ so that for $N > \bar{N}$, $\frac{\partial \lambda}{\partial N} < 0$ and for $N < \bar{N}$, $\frac{\partial \lambda}{\partial N} > 0$. If $\mu > \frac{7}{3}\rho_I\sigma^2\phi A$, $\frac{\partial \lambda}{\partial N} < 0$ for any N. $\square$

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Appendix Table A1: Bid functions are approximately linear

|	mean	median	sd
β_t	0.21	0.16	0.13
R_t^2	0.72	0.74	0.11
Adj. R_t^2	0.64	0.67	0.15
Within R_t^2	0.54	0.56	0.15

Appendix Table A1 shows the point estimate and R^2 from regressing bids on quantities and an auction-dealer fixed effect in each auction, $b_{tik} = \zeta_{ti} + \beta_t a_{tik} + \epsilon_{tik}$, using bidding functions with at least two steps. Bids are in bps of yields and quantities in percentage of supply.

Appendix Table A2: Robustness w.r.t. risk aversion

	(All)		(2020)		(2022)	
Main specification	0.0065	(0.0051)	0.0065	(0.0038)	0.0074	(0.0070)
Number of steps ≥ 3	0.0067	(0.0051)	0.0065	(0.0039)	0.0076	(0.0073)
Number of steps ≥ 4	0.0066	(0.0051)	0.0063	(0.0037)	0.0077	(0.0073)
Number of steps ≥ 5	0.0066	(0.0052)	0.0058	(0.0034)	0.0076	(0.0074)
Number of steps ≥ 6	0.0061	(0.0056)	0.0046	(0.0030)	0.0086	(0.0081)
Quantities in mil C\$	0.0002	(0.0001)	0.0001	(0.0001)	0.0002	(0.0002)
Volatility using 1 day	0.0063	(0.0058)	0.0059	(0.0044)	0.0067	(0.0070)
Volatility using 2 days	0.0056	(0.0054)	0.0046	(0.0037)	0.0067	(0.0069)
Volatility using 3 days	0.0061	(0.0057)	0.0056	(0.0042)	0.0066	(0.0069)
Volatility using 4 days	0.0066	(0.0053)	0.0064	(0.0041)	0.0073	(0.0071)
Volatility using 6 days	0.0066	(0.0052)	0.0066	(0.0041)	0.0074	(0.0065)
Volatility using 7 days	0.0067	(0.0052)	0.0067	(0.0047)	0.0071	(0.0060)
Volatility using 8 days	0.0069	(0.0049)	0.0067	(0.0046)	0.0069	(0.0059)
Volatility using 9 days	0.0068	(0.0048)	0.0067	(0.0046)	0.0068	(0.0058)
Volatility using 10 days	0.0071	(0.0049)	0.0071	(0.0049)	0.0067	(0.0055)

Appendix Table A2 presents the median of all risk-aversion estimates for all specifications in column (All), when the exemption started in column (2020), and when it ended in column (2022). The main specification uses functions with at least two steps, expresses quantities in percentage of supply, and relies on the volatility index that uses trades during five trading days before the day of the auction. The second to fifth rows show robustness with respect to the number of steps of the bidding/willingness to pay functions. The sixth row presents results when using quantities in million C\$. The remaining rows display the results for different volatility indices, constructed using N trading days prior to the auction, for N=1, ..., 10. Standard errors are presented in parentheses. They are stated in multiples of 100 to reduce the number of zeros.

Appendix Figure A1: Holders of Canadian government bonds
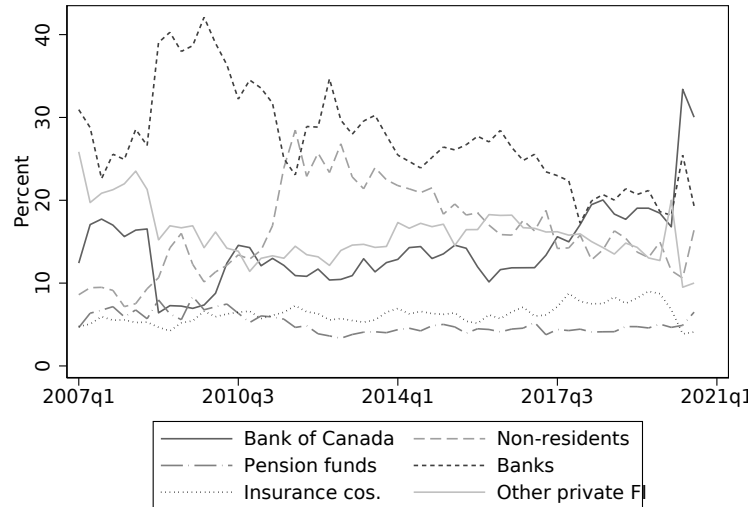
Appendix Figure A1 shows who holds Canadian government bonds and bills from 2007 until 2021 in percentage of par value outstanding: Bank of Canada, Non-residents, Canadian pension funds, Canadian banks, Canadian insurance companies, and other private firms. The bank category holdings are mostly driven by the eight banks we focus on, as they hold over 80% of the assets of all banks.
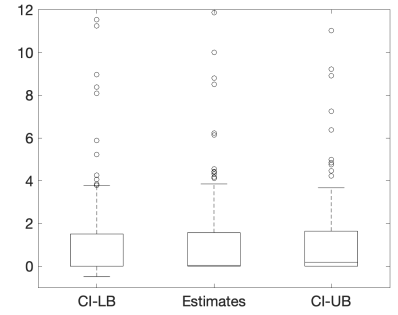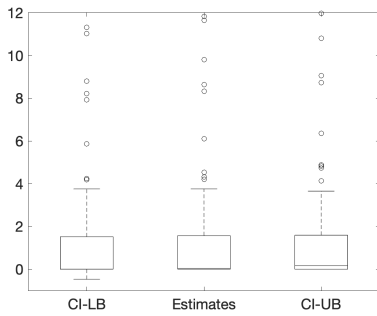
Appendix Figure A2: Robustness w.r.t. shadow costs

(a) Main specification
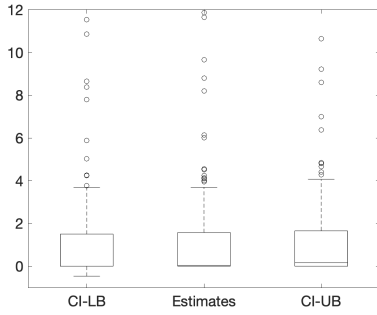
(b) With quantities
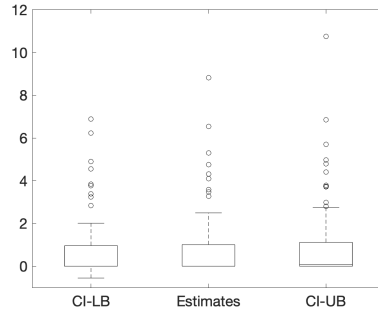
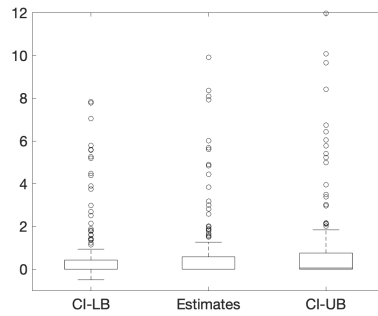(c) Number of steps ≥ 3

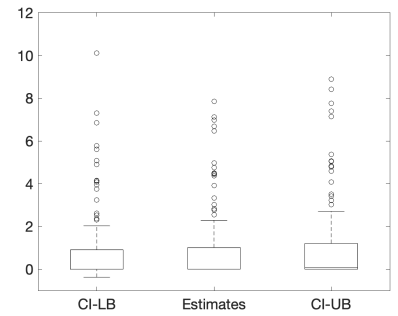(d) Number of steps ≥ 4

(e) Number of steps ≥ 5
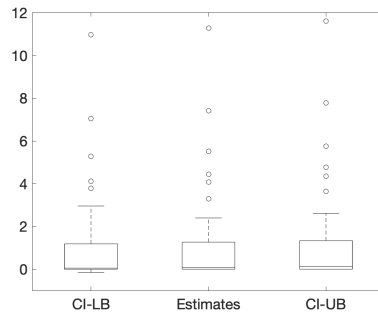
(f) Number of steps ≥ 6
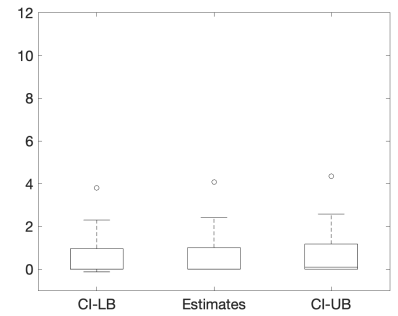
(g) Volatility (1 days)
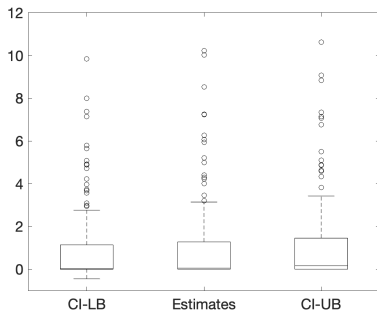
(h) Volatility (2 days)

(i) Volatility (3 days)

(j) Volatility (4 days)   (k) Volatility (6 days)   (l) Volatility (7 days)
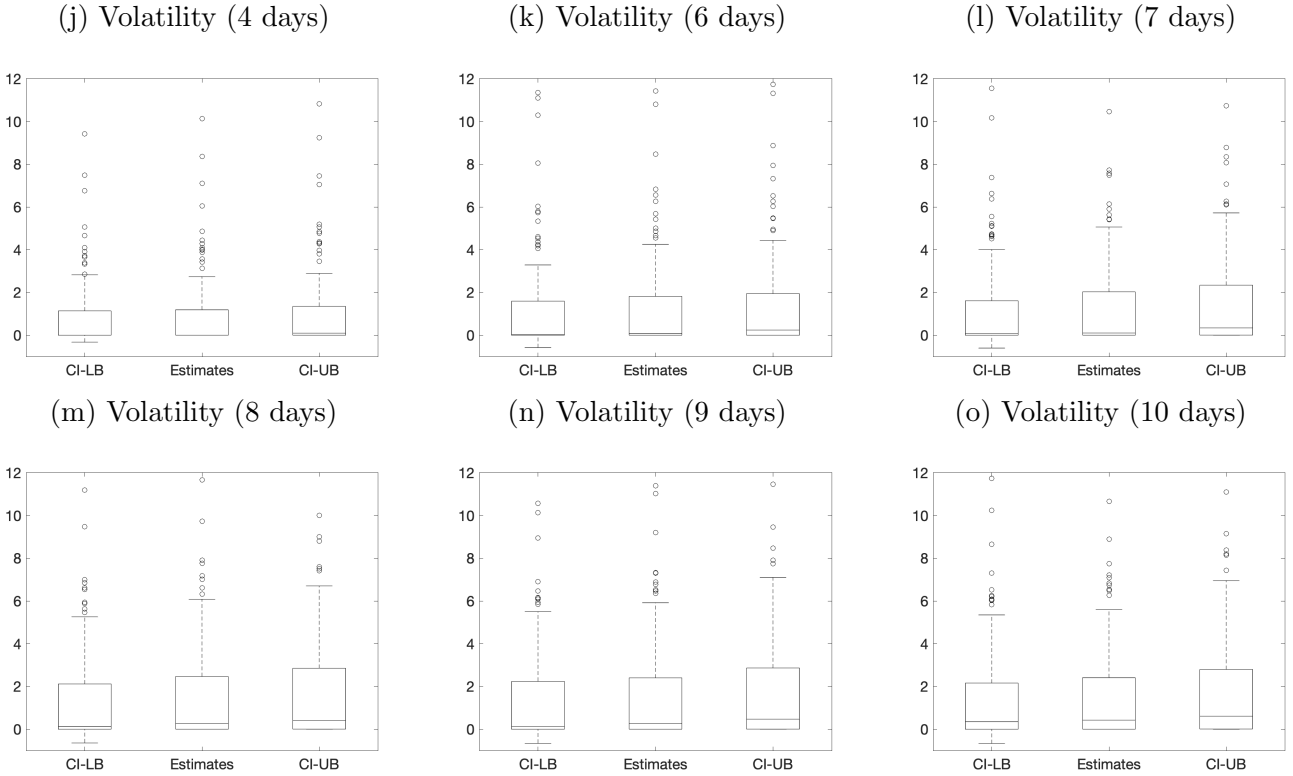
(m) Volatility (8 days)   (n) Volatility (9 days)   (o) Volatility (10 days)

Appendix Figure A2 shows the distribution of the lower and upper bounds of the 95% confidence intervals for each point estimate (CI-LB and CI-UB, respectively), in addition to the distribution of the point estimates (Estimates) pooled across all auctions, and excluding outliers for all model specifications. Panel (a) is identical to the RHS of Figure 7. In (b) we use quantities in million C$, in (c)-(f) we change the number of steps of value functions that are included in the estimation, and in (g)-(o) we use different volatility indices that are constructed pooling trades during 1-10 trading before the auction takes place.

Appendix Figure A3: Variation in quantities

(a) Supply and total demand

(b) Total demand in percentage of supply
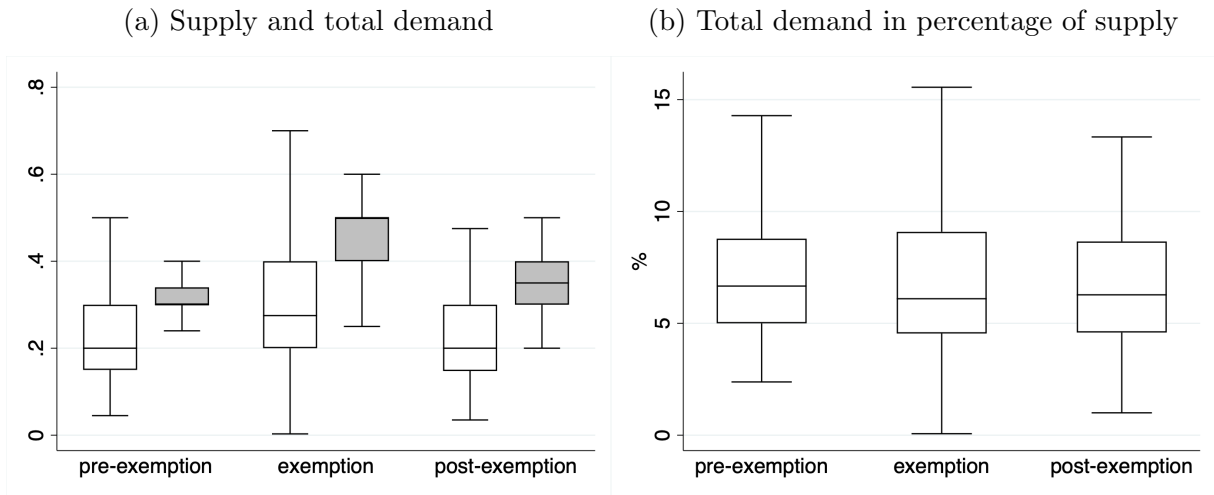
Appendix Figure A3a shows the distribution of the total amount a dealer demands in an auction before, during, and after the exemption period (in white) and the distribution of the supply (in gray). Demand is expressed in million C\$, and supply is in 10 million C\$ to make the two comparable. Appendix Figure A3b shows the distribution of the total amount demanded as percentage of supply across periods.