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Labor Market Shocks and Monetary Policy

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Abstract

We develop a heterogeneous-agent New Keynesian model featuring a frictional labor market with on-the-job search to quantitatively study the positive and normative implications of employer-to-employer (EE) transitions for inflation. We find that EE dynamics played an important role in shaping the differential inflation dynamics observed during the Great Recession and COVID-19 recoveries, with the former exhibiting subdued EE transitions and inflation despite both episodes sharing similar unemployment dynamics. The optimal monetary policy prescribes a strong positive response to EE fluctuations, implying that central banks should distinguish between recovery episodes with similar unemployment but different EE dynamics.

Topics: Business fluctuations and cycles; Inflation and prices; Labour markets; Monetary policy
JEL codes: E12, E24, E52, J31, J62, J64

Résumé

Nous mettons au point un modèle néokeynésien à agents hétérogènes intégrant un marché du travail frictionnel avec recherche d'emploi en cours d'emploi. Notre objectif est d'étudier de façon quantitative les implications positives et normatives des transitions d'employeur à employeur pour l'inflation. Nous constatons que ces transitions ont contribué de façon importante aux différentes dynamiques d'inflation observées durant les reprises qui ont suivi la Grande Récession et la pandémie de COVID-19. Bien que la dynamique du chômage ait été similaire dans les deux cas, les transitions d'employeur à employeur et l'inflation ont été moins marquées pendant la reprise post-pandémie. La politique monétaire optimale exige une forte réponse positive face aux fluctuations des transitions d'employeur à employeur. Par conséquent, les banques centrales devraient aborder différemment les reprises durant lesquelles la dynamique de ces transitions diffère malgré une dynamique de chômage similaire.

Sujets : Cycles et fluctuations économiques; Inflation et prix; Marchés du travail; Politique monétaire
Codes JEL : E12, E24, E52, J31, J62, J64

1 Introduction

The Federal Reserve has a dual mandate of fostering price stability and maximum employment. Its main tool in this endeavor is the federal funds rate, which it sets based on inflation and measures of economic slack. Slack measures regarding the labor market tend to focus on the *quantity* of employment (e.g. the unemployment rate) and underemphasize its *quality* dimension. There has been a growing interest in understanding the role of employer-to-employer (EE) transitions in determining the quality of employment and what this implies for inflation dynamics. EE flows affect production costs via firm competition for workers and the productive capacity of the economy by facilitating labor reallocation between firms. Job mobility is also a key determinant of income dynamics at the individual level and thus impacts aggregate demand—all important mechanisms for the determination of inflation.

In this paper, we quantitatively analyze the positive and normative implications of EE flows for inflation and monetary policy and ask two questions: First, how much and through which channels do EE flows affect macroeconomic outcomes, particularly inflation? Second, what is the optimal monetary policy within a class of Taylor rules, taking EE flows explicitly into account?

Our paper makes three contributions. First, we develop a model that combines the Heterogeneous Agent New Keynesian (HANK) framework with a frictional labor market and on-the-job search (OJS). Second, we use the model to quantify the impact of EE fluctuations over the business cycle on inflation. The quantitative framework lends itself to analyzing a broad set of scenarios. We first show that muted worker mobility caused around 0.25 percentage points lower inflation during the 2016–2019 recovery episode, which saw "missing inflation" despite a historically large decline in unemployment. We also find that the elevated EE rate during the "Great Resignation" of 2021–2022 generated around 0.60 percentage points additional inflation. Critically, our analysis provides a full-decomposition of the channels through which fluctuations in EE transitions affect inflation in the model not accounted for by standard demand and supply shocks. Third, we study optimal monetary policy within a class of Taylor rules under a dual-mandate central bank objective. The optimal policy prescribes a more aggressive response to the unemployment gap than what is prescribed by the commonly used coefficient on the unemployment gap in the literature, and a strong positive response to the EE gap. In practice, this allows the central bank to distinguish recovery episodes where job mobility is high from those where job mobility is low, even in the face of similar unemployment dynamics.

We start by developing a Two Agent New Keynesian (TANK) model with a frictional labor market to demonstrate our key insights vis-a-vis the interaction between EE flows and inflation, which serves as a segue to our full quantitative (HANK) model. In this model, a hand-to-mouth household (HtM) consumes its entire labor income, while a permanent-income household (PIH) has labor income, collects firm profits, and can save in a riskless asset. Unemployed and employed

members of these households search for jobs in a frictional labor market and work in firms that produce labor services. We assume that all firm-worker pairs are homogeneous in productivity and that workers hired out of unemployment receive a fraction of flow surplus as wage, while poached workers extract the entire surplus in the new match and thus always switch jobs when possible. The rest of the model follows the New Keynesian tradition. Monopolistically competitive intermediate firms buy labor services in a competitive market to produce differentiated goods, which are then sold to final-good producers. The monetary authority follows a Taylor rule to control the short-term nominal rate.

We use this model to study how job mobility dynamics affect inflation and other macroeconomic outcomes. In particular, starting from the steady state, we raise the EE rate by exogenously increasing employed workers' contact probability with external firms, modeled as a shock to the relative job search efficiency of employed workers, i.e., "OJS efficiency shock." Fixing the real interest rate, a higher EE rate increases output, the real marginal cost for intermediate firms, and inflation. This rise in output is driven by an increase in aggregate labor income due to wage increases from more-frequent job switches. The rise in the marginal cost (and thus inflation) is caused by a decline in the expected match value for labor services firms as match durations decrease.[1] Thus, intermediate firms must pay a higher price for labor services to convince the service firms to hire new workers, leading to a rise in their real marginal costs. Allowing for real rate adjustments, the rise in inflation induces a more than one-for-one increase in the nominal rate through the central bank's reaction. Despite the rise in labor incomes, the rise in the real rate together with lower firm profits lead to lower aggregate demand and output. Therefore, in equilibrium, the shift in EE behaves like a cost-push shock.

Importantly, we demonstrate how household heterogeneity changes the quantitative effects of an increase in EE rate on macroeconomic outcomes by comparing the TANK model with a nested Representative Agent New Keynesian (RANK) version. We uncover two key differences between these models. First, the presence of HtM households in the TANK model mitigates the decline in aggregate demand, as their demand is less elastic to the real interest rate. Second, the monetary policy response to a rise in inflation is more aggressive in the TANK model. However, despite a substantially higher increase in the real rate, inflation in the TANK model still experiences a larger increase. Hence, the RANK model not only underestimates the quantitative impact of EE fluctuations on inflation, but also induces a less-aggressive monetary policy response, as it overstates the decline in aggregate demand. Overall, real outcomes and policy implications differ significantly when the model incorporates household heterogeneity.

We continue by extending the TANK model into a quantitative HANK framework with a richer labor market that can be more tightly bridged to the data. We highlight three reasons

[1] In a separate exercise, we show that wage increases for employed workers with external offers have a limited role in explaining how worker mobility affects inflation.

for building our fully fledged model. First, to correctly quantify the macroeconomic effects of job mobility dynamics, it is crucial to capture the extent of wealth heterogeneity and allow for an interaction between heterogeneity in the marginal propensities to consume (MPC) and idiosyncratic income fluctuations, a relationship emphasized in the HANK literature. Second, capturing supply-side effects of job switches necessitates a richer labor market with productivity differences across matches in the labor services sector. Third, to discipline the wage dynamics of job switchers, job stayers, and job losers in the model with the data, we need to model the evolution of individual productivity both on and off the job.

For these reasons, our HANK model features the following additional elements relative to the TANK model. We allow individuals to save by investing in shares of a mutual fund to self insure against idiosyncratic income fluctuations due to unemployment risk, human capital depreciation when unemployed, and stochastic retirement. Worker-firm pairs in the labor services sector are heterogeneous in productivity due to differences in their match-specific productivity and workers' accumulated human capital. The wage is an endogenous piece-rate of the pair's output, which is determined through Bertrand competition. The heterogeneity in match-specific productivity together with this bargaining protocol allows the model to capture the productivity effects of job switches. Finally, an individual's human capital stochastically appreciates when employed and depreciates when unemployed. This feature allows the model to capture wage growth among job stayers and scarring effects of job loss. The economy is subject to aggregate shocks to demand, labor productivity, and the efficiency of OJS.

Estimating shocks and studying optimal monetary policy requires solving and simulating the model under aggregate shocks efficiently. We overcome the well-known challenges associated with solving HA models under aggregate uncertainty by implementing the sequence-space Jacobian (SSJ) method of Auclert, Bardóczy, Rognlie, and Straub (2021). One complication in adapting this method to our setting is that endogenous worker distributions *directly* enter into the equilibrium conditions and optimization problems. This is in contrast to more-familiar settings where only scalars enter equilibrium conditions in the sequence space. To this end, we also offer a methodological contribution by extending the SSJ method to accommodate a multi-stage model with search frictions, where one needs to keep track of discretized worker distributions.

We calibrate the steady state of the HANK model to match the fraction of hand-to-mouth individuals and relevant labor market moments of the U.S. economy prior to the Great Recession, as well as targets for disciplining the New Keynesian components, including the average level of markups, the slope of the Phillips curve, and the responsiveness of the nominal rate to inflation and unemployment gaps. We then jointly estimate processes for aggregate shocks to demand, labor productivity, and the efficiency of OJS by targeting empirical moments regarding the correlations of the unemployment rate, EE rate, and inflation with output, as well as their standard deviations. We show that shocks to OJS efficiency account for more than 40 percent

of fluctuations in inflation via a variance decomposition.

We then use the calibrated model to quantify (i) the drag on inflation due to suppressed EE flows during the expansion following the Great Recession and (ii) the increase in inflation from elevated worker mobility during the recovery from the COVID-19 recession. These exercises are motivated by our empirical observations on (i) a stable EE rate during the former recovery episode that is well below the level implied by its historical negative relationship with the unemployment rate and (ii) a sizable rise in the EE rate during the latter recovery episode. In simulating our model to emulate the former recovery episode, we first back out the sequence of positive demand and negative OJS efficiency shocks that replicate the declining path of unemployment and stable EE rates in this period. We then compare the outcomes to a counterfactual economy matching *only* the declining unemployment by estimating demand shocks alone. We find that the OJS efficiency shocks that our estimation infers lowers annual inflation by 0.23 percentage points at the peak. In a similar exercise, we simulate the recent recovery from the COVID-19 recession and show that the sizable increase in the EE rate during this period causes 0.56 percentage points higher inflation. The prediction of the model that higher worker mobility generates an increase in the real marginal cost (and inflation) is consistent with our empirical observation on the differential growth of unit labor costs (ULC) between the last two recovery episodes that feature similar unemployment but differential EE dynamics.

Next, we decompose the various channels through which an increase in job mobility results in higher inflation in our model. This exercise leverages the SSJ method and essentially applies the implicit function theorem on equilibrium conditions to express an outcome variable as a linear function of the shocks and other endogenous variables. An increase in OJS efficiency implies a lower expected match value for service firms as they face shorter match durations and more frequent wage rebargaining. This direct effect necessitates an increase in the price of their output (which is the real marginal cost for intermediate firms) to maintain the free-entry condition. Quantitatively, the direct effect explains 139 percent of the total increase in real marginal cost. However, a higher OJS efficiency also reduces tightness in equilibrium because of (i) an increased supply of labor services arising from improvements in the match-productivity distribution and (ii) lower demand due to an increase in the real rate and unemployment, both of which dictate a decline in tightness to clear the market for labor services. All else the same, when the labor market is more slack, firms find it easier to fill vacancies and workers find it more difficult to contact other firms, generating an increase in the expected match value of service firms. Therefore, the price of labor services declines to respect the free-entry condition. The decline in tightness in turn explains -42 percent of the increase in the marginal cost. Thus, labor market effects explain 97 percent of the total increase in the marginal cost. The remaining 3 percent is accounted for by changes in the real rate due to changes in inflation and unemployment.

Finally, we study the normative implications of job mobility for monetary policy. We con-

sider an augmented Taylor rule that responds to both deviations of unemployment and EE rates from their steady state values as well as the inflation gap. Under a dual-mandate central bank objective, we find that the optimal policy that minimizes the fluctuations in inflation and output gaps features a positive and large coefficient on the EE gap as well as a large negative coefficient on the unemployment rate. This allows the monetary authority to distinguish between identical recoveries with respect to unemployment through differences in job mobility dynamics. In practice, this optimal policy implies that during the last two recoveries with very similar declines in the unemployment rate, monetary policy response should have been more aggressive between 2021–2022 when job mobility was much higher. Importantly, the optimal policy yields a 12 percent lower central bank loss relative to a case where we ignore EE dynamics and optimize over only the unemployment gap coefficient. We conclude that explicitly accounting for worker transitions matters for the conduct of monetary policy.

Related literature. The HANK literature emphasizes empirically realistic distributions of wealth and MPCs to correctly quantify the transmission mechanisms of monetary policy (Kaplan, Moll, and Violante, 2018; Auclert, Rognlie, and Straub, 2020; Auclert, Rognlie, and Straub, 2023, among many others). This literature, however, often assumes a stylized labor supply block using an exogenous labor income process. We add to this literature by combining a HANK framework with a search model of the labor market featuring rich heterogeneity and OJS. This allows us to endogenize income risk and how it varies with aggregate fluctuations and correlates with MPCs, which are key elements of HANK models (Acharya and Dogra, 2020 and Patterson, 2022). We use the model to uncover positive and normative implications of worker mobility on aggregate dynamics. To this end, on the computational side, we extend the SSJ method of Auclert, Bardóczy, Rognlie, and Straub (2021) to handle discretized worker distributions that directly enter into optimization problems and equilibrium conditions.

Several papers bring together elements from search models with those from the New Keynesian literature. Ravn and Sterk (2016) develop a tractable New Keynesian model with uninsurable risk and characterize the interactions between unemployment risk, aggregate demand and monetary policy. Gornemann, Kuester, and Nakajima (2021) develop a fully stochastic New Keynesian model with uninsurable idiosyncratic risk and search frictions. Relative to these papers, our model features OJS and heterogeneity across jobs, which we use to study how worker mobility affects aggregate dynamics. Our paper also complements recent studies that employ labor search models to investigate how rigidity in existing workers' wages affect unemployment fluctuations through its impact on job-finding rates (Fukui, 2020) and job-separation rates (Blanco, Drenik, Moser, and Zaratiegui, 2023). We highlight how changes in existing workers' wages due to job switching and wage rebargaining affect aggregate dynamics.

Our work is most closely related to Moscarini and Postel-Vinay (2022), Faccini and Melosi (2023), and Alves (2020), who focus on the role of the job ladder in inflation dynamics. Moscarini

and Postel-Vinay (2022) are the first in this literature to introduce search frictions with OJS into a RANK framework. Their work establishes that the distribution of job quality is an important determinant of wage pressures on inflation and emphasizes the state-contingent effects of this distribution on inflation over the business cycle. Faccini and Melosi (2023) build on their work and highlight the role of worker mobility in explaining the missing inflation after the Great Recession and additional inflation after the COVID-19 recession, similar to our positive analysis. Relative to these papers, our HANK model features imperfect insurance against income fluctuations, and therefore changes in the job mobility are an important determinant of aggregate demand as well as supply. In particular, we show that the RANK model underestimates the quantitative effect of EE fluctuations on inflation and that real outcomes and monetary policy implications differ significantly when the model incorporates heterogeneity across households. In terms of mechanisms, we also highlight that even without wage re-bargaining upon an external offer, in periods of high worker mobility, inflation still increases because hiring a new worker becomes less valuable due to a shorter expected match duration. Finally, Alves (2020) embeds the key insights from Moscarini and Postel-Vinay (2022) in a HANK model and obtains sizable demand side effects from changes in job mobility. Our work differs from his in three important ways. First, our model features richer labor-market heterogeneity by allowing for differences in human capital as well as match productivity. Second, we not only quantify the total effect of job mobility on inflation but also decompose its underlying drivers. Third, we study the normative implications of job mobility for monetary policy and show that responding to fluctuations in the EE rate explicitly reduces welfare losses due to fluctuations in inflation and output.

Outline. Section 2 presents our TANK model and provides intuitions and insights for our main quantitative results. Section 3 presents our quantitative model combining the HANK framework with a search model of the labor market. Section 4 discusses the calibration of model's parameters and estimation of aggregate shocks. Section 5 explains how EE fluctuations affect macroeconomic outcomes and inflation, and Section 6 studies the normative implications of EE fluctuations for monetary policy. Section 7 concludes.

2 A TANK model with a frictional labor market and OJS

In this section, we illustrate the effects of job mobility dynamics on macroeconomic and labor market outcomes in a TANK framework with labor market frictions, where both the employed and unemployed search for jobs.[2] We use this model to explain the key mechanisms through which job mobility affects the economy, with a special focus on the role of household heterogeneity in inflation, which serves as a stepping stone to our fully fledged HANK model in Section 3.

[2]The model presented here is inspired by Masao Fukui's insightful discussion of our paper at the San Francisco Fed Macroeconomics and Monetary Policy Conference on March 2023.

2.1 Environment

Time is discrete and runs forever. The economy is populated by a unit mass of individuals that belong to one of two types of households, firms in three vertically integrated sectors (producing labor services, intermediate goods, and final goods), and a monetary authority.

Households. Individuals are infinitely lived and discount the future with factor $\beta \in (0,1)$. A fraction μ belongs to a hand-to-mouth (HtM) household and consumes their real *labor* income $c_t^{HtM} = W_t$. The remaining fraction $1-\mu$ belongs to a permanent-income household (PIH) that saves in a risk-free asset a at rate r_t, whose consumption c_t^{PIH} solves:

$$\max_{c_t^{PIH}} \sum_{t=0}^{\infty} \beta^t u(c_t^{PIH}) \quad \text{s.t.} \quad c_t^{PIH} + a_t = (1+r_t)a_{t-1} + Z_t,$$

where Z_t is the total real income of the PIH household consisting of labor income and firm profits, described below.[3] Assuming a constant relative risk aversion (CRRA) utility function with parameter $\sigma > 0$, c_t^{PIH} is given by[4]:

$$c_t^{PIH} = \frac{1}{\sum_{s=0}^{\infty} \beta^{s/\sigma} q_{t+s}^{1-1/\sigma}} \Big((1+r_t)a_{t-1} + \sum_{s=0}^{\infty} q_{t+s} Z_{t+s} \Big), \tag{1}$$

where $q_{t+s} = \frac{1}{1+r_{t+1}} \cdots \times \frac{1}{1+r_{t+s}}$ with $q_t = 1$ and we impose a no-Ponzi condition $\lim_{s\to\infty} q_{t+s} a_{t+s} = 0$. There is perfect risk-sharing within each household, and aggregate consumption is $C_t = (1-\mu)\, c_t^{PIH} + \mu c_t^{HtM}$.

Firms. Labor services firms hire workers in a frictional labor market to produce labor services, which are sold in a competitive market to intermediate goods firms producing differentiated varieties of intermediate inputs. These intermediate goods firms are monopolistically competitive, are subject to pricing frictions, and face a downward-sloping demand from final goods firms. Final goods firms produce the consumption good by combining intermediate inputs using a constant elasticity of substitution (CES) technology.

Labor market and wages. The labor market is frictional and features random search. Both unemployed and employed individuals search for jobs, and their probability of contacting a vacancy depends on their job search efficiency and the tightness, θ_t. In particular, a worker's contact rate per unit of search efficiency is given by $f(\theta_t)$ (defined below). The job search efficiency of the unemployed is normalized to 1, and the relative job search efficiency of the

[3] We assume that the PIH household owns the firms and collect their profits, while the HtM household does not hold assets. This assumption is motivated by the fact that some households do not possess any liquid assets, including shares of publicly traded firms. Thus, their demand is not directly impacted by changes in asset values.

[4] Appendix A.1 provides the derivation of Equation (1).

employed is denoted by ν_t. When workers contact a vacancy, we assume that all job offers are accepted and real wages are paid according to a predetermined rule described below. In the beginning of each period, matches dissolve at an exogenous rate δ. We assume that all matches are homogeneous in productivity and produce one unit of labor service each period.

On the other side of this labor market, labor services firms pay a fixed cost κ to post vacancies and sell their output to intermediate firms at nominal price P_t^l ($p_t^l = P_t^l/P_t$ in units of the final good).[5] As such, output of each worker-firm pair is valued at p_t^l in real terms. We assume that when individuals find a job out of unemployment, they receive a real wage equivalent to fraction $\alpha \in [0,1]$ of match revenue p_t^l. When they find a new job while employed, we assume that they extract the entire surplus in this new match and receive a real wage of p_t^l. Each period, there are two types of employed workers in the labor market: those who found a job out of unemployment and collect wages equal to αp_t^l and those who found a new job while employed and collect wages equal to p_t^l. We denote the masses of these worker groups by $e_t(\alpha)$ and $e_t(1)$, respectively, and the mass of unemployed workers is given by u_t. Aggregate labor income is then given by

$$W_t = \alpha p_t^l e_t(\alpha) + p_t^l e_t(1), \tag{2}$$

while aggregate profits in the economy are $\Gamma_t = \Gamma_t^I + \Gamma_t^S$, where Γ_t^I and Γ_t^S are the aggregate per-period real profits of the intermediate and labor services firms, which we describe in Section 2.2. Per-capita real income of the PIH household is then given by[6]:

$$Z_t = W_t + \Gamma_t/(1-\mu). \tag{3}$$

Finally, the laws of motion that govern workers' end-of-period labor market status are as follows:

$$\begin{aligned} u_t &= (1 - f(\theta_t))u_{t-1} + \delta(1 - f(\theta_t))(1 - u_{t-1}) \\ e_t(\alpha) &= f(\theta_t)u_{t-1} + (1-\delta)(1 - \nu_t f(\theta_t))e_{t-1}(\alpha) + \delta f(\theta_t)(1 - u_{t-1}) \\ e_t(1) &= (1-\delta)e_{t-1}(1) + (1-\delta)\nu_t f(\theta_t)e_{t-1}(\alpha), \end{aligned} \tag{4}$$

where we allow workers who lose their jobs to search for a new one within the same period.

Monetary authority. The monetary authority controls the short-term nominal interest rate i_t according to the following reaction function:

$$i_t = i^* + \Phi_\pi(\pi_t - \pi^*) + \Phi_u(u_t - u^*). \tag{5}$$

[5] Unless otherwise stated, we use uppercases for nominal variables and lowercases for their real counterparts.
[6] Since all profits accrue to the PIH household, total profits $\Gamma_t/(1-\mu)$ are scaled by their measure.

Here, i^* denotes the steady-state nominal interest rate, Φ_π governs the responsiveness of the central bank to deviations of inflation from its target π^*, and Φ_u controls how much the central bank responds to deviations of the unemployment rate from its steady state value u^*. Finally, the real interest rate r_t satisfies the Fisher equation:

$$1 + i_t = (1 + \pi_{t+1})(1 + r_{t+1}). \tag{6}$$

The timing convention for these variables is as follows: The nominal interest rate i_t is indexed to the period in which it is set and is the interest rate that applies between periods t and $t+1$. The inflation rate is denoted by the period in which it is measured; i.e., π_{t+1} is the realized inflation between periods t and $t+1$. The real rate has the same timing convention as inflation: r_{t+1} is the ex-post realized real interest rate from t to $t+1$.

Timing of events. At the start of each period, (unanticipated) aggregate and job destruction shocks realize (details in Section 2.4). Next, the job search stage opens: firms post vacancies, workers search for jobs, and new matches are formed. This is followed by the production stage where each worker-firm pair produces labor services. Intermediate firms produce differentiated goods using labor services and set their prices subject to nominal rigidities, while final goods firms produce the numeraire good using intermediate goods. Given the unemployment rate and inflation, the monetary authority sets the nominal interest rate. Next, intermediate and service firms realize their profits, and service firms pay wages to their workers. In the final stage of the period, households pool income across their members and decide on how much to consume.

2.2 Production

The economy has three vertically integrated sectors that we now describe in more detail.

Final goods. The final-good producer purchases differentiated intermediate goods $y_t(j)$ at relative price $p_t(j) = P_t(j)/P_t$ and produces the final consumption good Y_t using the technology:

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}},$$

where η is the elasticity of substitution between varieties and solves the following problem:

$$\max_{\{y_t(j)\}} \; Y_t - \int p_t(j) y_t(j) dj.$$

This problem determines the demand for each intermediate good, $y_t(j) = p_t(j)^{-\eta} Y_t$, as a function of the relative price of variety $p_t(j)$ and aggregate demand conditions Y_t, and implies an ideal price index satisfying $1 = \left(\int p_t(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$ that intermediate firms take as given.

Intermediate goods. Intermediate firms produce $y_t(j)$ using a linear technology with labor services as the only input: $y_t(j) = l_t(j)$, where $l_t(j)$ denotes the amount of labor services bought from labor services firms. They set the price for their differentiated good, taking into account the demand from the final-good producer and price adjustment costs à la Rotemberg (1982). Pricing frictions render last period's *relative* price $p_{t-1}(j)$ a state variable for intermediate goods firms. These firms solve the following profit maximization problem:

$$\Theta(p_{t-1}(j)) = \max_{p_t(j)} \; p_t(j)y_t(p_t(j)) - p_t^l y_t(p_t(j)) - Q(p_{t-1}(j), p_t(j))Y_t + \frac{1}{1+r_{t+1}}\Theta(p_t(j)), \quad (7)$$

where adjustment costs are given by $Q(p_{t-1}(j), p_t(j)) = \frac{\eta}{2\vartheta}\log\left(\frac{p_t(j)}{p_{t-1}(j)}(1+\pi_t) - \pi^*\right)^2$. Appendix A.2 shows that this problem yields the following New Keynesian Phillips curve (NKPC):

$$\frac{\log\left(1+\pi_t-\pi^*\right)(1+\pi_t)}{1+\pi_t-\pi^*} = \vartheta\left(p_t^l - \frac{\eta-1}{\eta}\right) + \frac{1}{1+r_{t+1}}\frac{\log\left(1+\pi_{t+1}-\pi^*\right)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*}\frac{Y_{t+1}}{Y_t}, \quad (8)$$

where $mc_t = p_t^l$ is the real marginal cost of production. Finally, the aggregate per-period real profits of intermediate firms are given by

$$\Gamma_t^I = \left(1 - p_t^l - \frac{\eta}{2\vartheta}\log(1+\pi_t-\pi^*)^2\right)Y_t, \quad (9)$$

where we use the fact that in equilibrium relative prices are always one, $p_t(j) = 1$, by symmetry.

Labor services. A continuum of labor services firms post vacancies, incurring cost κ per vacancy. Labor market tightness, θ_t, is defined as the ratio of vacancies v_t to the aggregate measure of job searchers, including both unemployed and employed workers weighted by their job search efficiencies, $S_t = u_{t-1} + \delta(1-u_{t-1}) + \nu_t(1-\delta)(1-u_{t-1})$. Let $M(v,S)$ be a constant-returns-to-scale (CRS) matching function that determines the number of matches as a function of vacancies and effective job seekers. We define $q(\theta) = \frac{M(v,S)}{v} = M\left(1, \frac{1}{\theta}\right)$ as the firm's contact rate and $f(\theta) = \frac{M(v,S)}{S} = M(\theta, 1)$ as the worker's contact rate per unit of search efficiency, where the CRS assumption implies that θ is sufficient to determine these rates.

We now turn to the problem of labor services firms. Consider a firm that employs a worker out of unemployment. The worker-firm pair produces one unit of labor services. The output is then sold to intermediate goods producers at real price p_t^l, and the worker is paid real wages equal to αp_t^l. The real value of this firm J_t is then given by

$$J_t = (1-\alpha)p_t^l + \frac{1}{1+r_{t+1}}(1-\delta)\Big(1 - \nu_{t+1}f(\theta_{t+1})\Big)J_{t+1}. \quad (10)$$

Note that the value to the firm of a match formed by poaching a worker is zero, as the worker extracts the entire match revenue as wage by assumption. Therefore, the per-period real profits

of service firms are given by

$$\Gamma_t^S = (1-\alpha)p_t^l e_t(\alpha). \tag{11}$$

The real value of a firm posting a vacancy is

$$V_t = -\kappa + q(\theta_t)\frac{u_{t-1}+\delta(1-u_{t-1})}{u_{t-1}+\delta(1-u_{t-1})+\nu_t(1-\delta)(1-u_{t-1})}J_t, \tag{12}$$

where we incorporate the fact that the value of poaching is zero. Further, a free-entry condition implies that expected profits are just enough to cover the cost of a vacancy, i.e, $V_t = 0$.

2.3 Equilibrium

Goods market clearing requires that output Y_t should be equal to aggregate demand C_t, and labor market clearing requires labor demand Y_t to be equal to employment $1 - u_t$[7]:

$$C_t = Y_t = (1-u_t). \tag{13}$$

Given the monetary policy rule in Equation (5) and a path of OJS efficiency ν_t, an equilibrium is a sequence of consumption levels c_t^{PIH} and c_t^{HtM}, real price of labor services p_t^l, market tightness θ_t, interest rates r_t and i_t, inflation π_t, and worker masses u_t, $e_t(\alpha)$, and $e_t(1)$ such that

- Household consumption decisions are given by Equation (1) and $c_t^{HtM} = W_t$, where the aggregate real labor income W_t and the per capita income of the PIH household Z_t are given by Equations (2) and (3), and firm profits satisfy Equations (9) and (11).
- Aggregate demand is $C_t = (1-\mu)\, c_t^{PIH} + \mu c_t^{HtM}$.
- Labor market tightness θ satisfies the law of motion for unemployment in Equation (4).
- Employed masses $e_t(\alpha)$ and $e_t(1)$ evolve according to laws of motion in Equation (4).
- The free-entry condition $V_t = 0$ in Equation (12) yields the real value of matched firm J_t. Equation (10) gives the real price of labor services p_t^l.
- The real interest rate satisfies the Fisher equation (6).
- Inflation π_t satisfies the NKPC in Equation (8).

The stationary equilibrium of the model is obtained by setting aggregate shocks to zero. We provide a solution algorithm for the steady state of this model in Appendix A.3. We solve the economy's transitional dynamics using the sequence-space Jacobian (SSJ) method developed by Auclert, Bardóczy, Rognlie, and Straub (2021). In doing so, we cast the model as a Directed Acyclic Graph (DAG), presented in Figure A.1. Further relevant computational details are provided in Section 4.2 in the context of our full HANK model and in Appendices A.4 and B.2.

[7]Here, we assume that vacancy creation cost κ does not exhaust real resources.

2.4 Results

In this section, we use our model to study how job mobility dynamics affect inflation and other macroeconomic outcomes. We use the same parameter values from the HANK model described in Section 3 and calibrated in Section 4.[8]

2.4.1 Understanding mechanisms: TANK vs RANK

Our main experiment involves introducing exogenous variation in job mobility dynamics. We implement this by simulating a series of shocks to the relative job search efficiency of the employed ν, which we label as OJS efficiency shocks. To investigate the role of household heterogeneity, we also study the same set of experiments in a nested RANK model, which we obtain by setting the fraction of HtM households to $\mu = 0$. We defer how we estimate OJS shocks and how we might microfound them to Section 4.3 to keep the discussion tight.

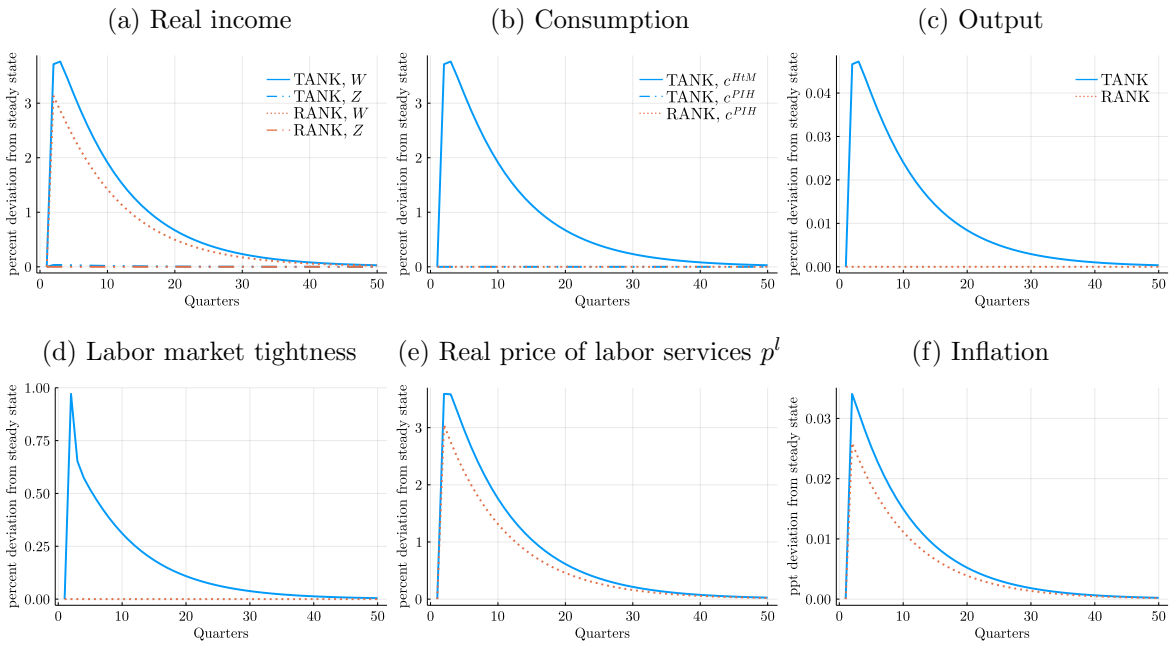
TANK vs RANK under constant real rate. We begin by investigating how OJS efficiency shocks affect economic outcomes under a constant real rate; i.e., the monetary authority adjusts the nominal rate one for one with inflation. Figure 1 presents impulse responses from a 1 percent increase in ν that gradually recovers.

We first focus on results from the RANK model. Aggregate real labor income W increases upon a rise in the OJS efficiency (Panel (a)). A higher OJS efficiency raises the probability that a worker with $\alpha < 1$ is poached by another firm and experiences a real wage increase as she extracts the full flow surplus in her next job. Higher wages paid by labor services firms lead to an increase in the real price of labor services p^l charged to intermediate firms. Because of nominal rigidities, intermediate firms cannot immediately adjust their prices, leading to a decline in their profits Γ^I. As shown in Figure A.2, total profits Γ decline. Because the PIH household owns these firms and collects their profits, the total income of the PIH household Z does not change, as the rise in labor income is offset by the decline in profits. Thus, aggregate demand c^{PIH} (Panel (b)), output (Panel (c)), and market tightness (Panel (d)) do not change in equilibrium.

However, more frequent poaching upon a rise in OJS efficiency reduces the expected duration of a match and leads to an increase in wages paid to workers. Both of these channels lower the

[8]There are two exceptions. First, while in the HANK model we set the relative job search efficiency of employed ν to match the average EE rate in the data, we set a different value for this parameter in the TANK model. This is because the TANK model does not feature any job ladder relative to productivity and workers with $\alpha < 1$ are forced to move to a new job upon an outside offer. This assumption implies that the average EE rate in the TANK model would be a function of the mass $e(\alpha)$, and that the EE rate is not very informative. We simply set $\nu = 0.5$ in the TANK model to capture empirical findings by Faberman, Mueller, Şahin, and Topa (2022). They show that hours spent searching and mean applications sent among employed job-searchers are around half of those among unemployed workers. Second, both in the TANK and HANK models, we set the cost of vacancy creation κ to match the average unemployment rate in the data, which naturally yields different values. In addition, there are two parameters specific to the TANK model: the share of HtM households in the economy μ and the piece rate for jobs formed out of unemployment α. We set $\mu = 0.3$ and $\alpha = 0.5$.

Figure 1: Impulse responses to a positive OJS efficiency shock: Constant real rate



Notes: This figure presents the dynamics of aggregate real labor income W, the per capital income of the PIH household Z, consumption, output, labor market tightness, the real price of labor services p^l, and inflation in an economy subject to a series of positive OJS efficiency shocks. Blue lines plot responses in the TANK model and red lines plot responses in the RANK model. In this analysis, we keep the real interest rate r constant.

expected value from a match $\mathbb{E}J$ for the labor services firm.[9] Given a fixed real interest rate and absent an equilibrium change in labor market tightness, the decline in $\mathbb{E}J$ necessitates an increase in p^l for equilibrium conditions to hold (Panel (e)). The NKPC in Equation (8) implies that inflation is driven entirely by the relative price of labor services p^l, which determines the real marginal cost for intermediate firms. As a result, inflation increases as well (Panel (f)).

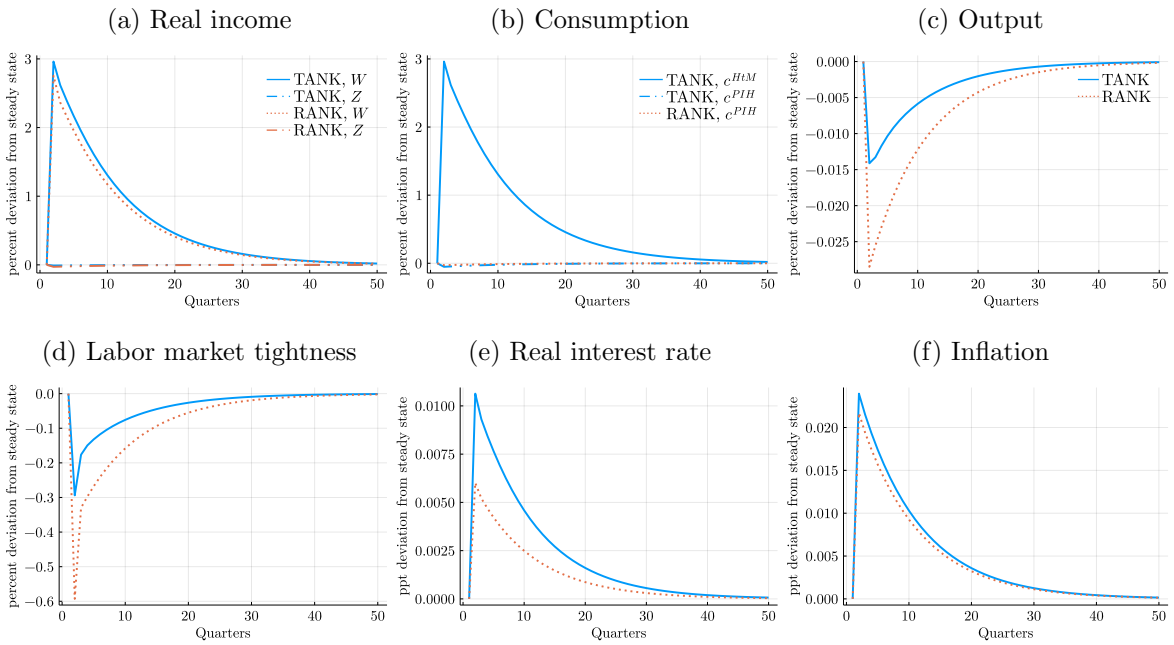
In the TANK model, aggregate labor income increases more than it does in RANK because a higher OJS efficiency now has positive real effects: aggregate demand and market tightness increase. Unlike in the RANK model, higher labor income arising from increased job-to-job moves directly translates into higher demand among HtM individuals (Panel (b)), given that they do not receive dividends. This increase in demand leads to an increase in output. Further, an increase in tightness amplifies the contact probability of employed workers, $\nu f(\theta)$.

In the TANK model, the real price of labor services p^l (and thus inflation) increases by more than it does in the RANK model. This is because the increase in $f(\theta)$ further amplifies the decline in the expected value from a match for the labor services firm $\mathbb{E}J$ due to the direct impact of ν and thus the rise in p^l.

Overall, according to the RANK model under a constant real rate, the rise in job mobility acts

[9] Here, $\mathbb{E}J = (1-\delta)\Big(1-\nu_{t+1}f(\theta_{t+1})\Big)J_{t+1}/\left[1+r_{t+1}\right]$.

Figure 2: Impulse responses to a positive OJS efficiency shock: Real rate response



Notes: This figure presents the dynamics of aggregate real labor income W, total income of PIH households, consumption, output, labor market tightness, real interest rate, and inflation in an economy subject to a series of positive OJS efficiency shocks. Blue lines plot responses in the TANK model and red lines plot responses in the RANK model.

like a pure cost-push shock for intermediate firms without any effect on demand or productivity, as we assumed all worker-firm matches are homogeneous in productivity. As a result, a positive OJS efficiency shock is completely absorbed by the rise in the marginal cost of production for intermediate firms and inflation. In the TANK model, the shock increases not only the marginal cost but also demand. Hence, without a monetary policy response, inflation in the TANK model rises much more than it does in RANK. However, as we discuss below, demand responses to OJS shocks can change if real rates adjust in equilibrium.

TANK vs RANK under real rate response. Next, we study the same experiment but allow the monetary authority to respond to the shock according to the reaction function in Equation (5). Figure 2 summarizes the results from this exercise.

We again start by focusing on the RANK case. Recall that a higher OJS efficiency only leads to an increase in inflation in the RANK model under a constant real rate. Here, through the response of the monetary authority, higher inflation induces an increase in the nominal interest rate i and therefore an increase in the real interest rate r (Panel (e)).[10] Despite the rise in real labor income (Panel (a)), the rise in the real rate together with the decline in total profits

[10] An increase in inflation induces a more than one-for-one increase in the nominal rate since $\Phi_\pi > 1$. This effect dominates the downward pressure on nominal rate due to the higher unemployment rate (Figure A.3).

(Figure A.3) lead to a slight decline in aggregate demand (Panel (b)). Lower aggregate demand implies lower output (Panel (c)) and labor market tightness (Panel (d)).

In equilibrium, the direct effect of a higher OJS efficiency on the expected value of a match $\mathbb{E}J$—again through shorter match durations and higher wages paid to workers—dominates the effects of general equilibrium changes in tightness, the unemployment rate, and the real rate on $\mathbb{E}J$.[11] As a result, $\mathbb{E}J$ declines and the price of labor services p^l and inflation rise. Because of the decline in demand, the increase in inflation is now much smaller (Figure 2 Panel (f)) than the increase under constant real rate (Figure 1 Panel (f)).

TANK model outcomes differ from those in the RANK model in two important ways. First, the decline in aggregate demand is smaller in the TANK model. This is because the demand of the HtM household increases as its real labor income increases. Thus, output and labor market tightness *decline less* in the TANK model. Second, the monetary authority has leeway to increase the nominal rate (as well as the real rate) by much more to curb inflation in the TANK model. Unlike the PIH household, the demand of the HtM household is impacted only indirectly by the real rate through its effect on market tightness and income. As a result, the demand of the HtM household is less elastic to changes in the real rate than that of the PIH household. This allows the monetary authority to implement a more aggressive rate increase to fight inflation, keeping the rise in inflation in the TANK model close to that in the RANK model. However, despite this more aggressive increase in the real rate, inflation in TANK is still slightly higher than it is in RANK, albeit the gap is substantially narrower compared to the constant real rate case.

Overall, while a positive OJS shock leads to a rise in the marginal cost of production (and inflation) and a decline in demand in both the RANK and TANK models, there are two key differences in the magnitude of their outcomes. First, the presence of HtM households in the TANK model mitigates the decline in aggregate demand and market tightness. Second, the monetary policy response to the shock is more aggressive in the TANK model than in the RANK model. These results indicate that real outcomes and the associated policy implications of shocks to the job ladder differ greatly when the model incorporates household heterogeneity.

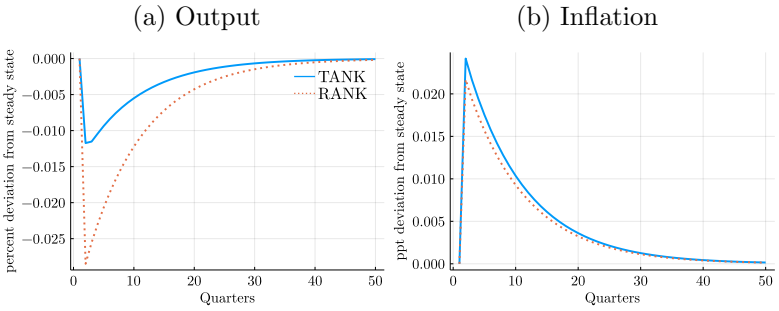
2.4.2 Understanding mechanisms: Wage increases upon external offers

The main reason behind the rise in p^l upon a positive OJS efficiency shock is the decline in the expected match value $\mathbb{E}J$, driven by *both* a shorter match duration and higher wages paid to poached workers. Would the positive relationship between the EE rate and inflation disappear in the absence of wage increases upon external offers? To answer this question, we set the piece rate of workers hired out of unemployment α close to 1 instead of to 0.5.[12] This way, external contacts have almost no effect on workers' wages as job switches do not entail wage gains.

[11] In Section 5.4, we use our HANK model and provide a full decomposition of inflation to these responses.

[12] Here, we also recalibrate the cost of vacancy creation κ to match the same steady-state unemployment rate as in the baseline model, keeping all other parameters the same.

Figure 3: Impulse responses to a positive OJS efficiency shock: Without wage increases



Notes: This figure presents the dynamics of output and inflation in an economy subject to a series of positive OJS efficiency shocks. Blue lines plot responses in the TANK model and red lines plot responses in the RANK model. Here, we set the piece rate of workers who are hired out of unemployment α to close to 1 instead of to 0.5 in the baseline model. This way, external contacts have almost no effect on currently employed workers' wages.

Figure 3 shows that a positive OJS efficiency shock still leads to a decline in demand and output and to a rise in inflation in both versions of the model.[13] Recall that the rise in the real marginal cost p^l and inflation is driven by the decline in the expected match value $\mathbb{E}J$. In the baseline model with wage increases, a rise in OJS efficiency reduces $\mathbb{E}J$ because of a decline in expected match duration and an increase in wages paid to workers. When we eliminate wage rebargaining here, only the match-duration channel is active. In this case, intermediate firms still need to pay a higher price for labor services, given that labor services firms expect to form shorter-lived matches. Overall, a similar inflation response in this exercise highlights that the effects of job mobility on inflation do not solely rest on wage increases upon an external offer.

In this stylized model, we assume that all outside offers are accepted and result in a wage increase. In the HANK model in Section 3, we introduce a wage bargaining protocol where the incumbent firm can make a counteroffer upon the worker receiving an outside offer. In that model, external offers might lead to wage increases not only when workers switch jobs but also when they remain with their current firms.[14] As we discuss later, the intuitions from the exercises in this section also carry over to the fully fledged HANK model.

2.4.3 Taking stock and motivations for a HANK framework

This section showed that a positive OJS efficiency shock leads to a decline in demand and tightness and to an increase in the real marginal cost and inflation in both the TANK and RANK models. We also find that the magnitudes of these changes differed significantly between the two models. This suggests that incorporating household heterogeneity is relevant for both quantifying how job mobility dynamics affect macroeconomic outcomes and the conduct of monetary policy.

[13] We note that the change in the value of α does not change the quantitative results in the RANK model given that the total income of the PIH household does not depend on how firm revenue is shared as profits vs wages.

[14] The full quantitative model features match heterogeneity and therefore job switches are endogenous.

While our TANK model is useful for understanding the critical channels through which job mobility dynamics affect the economy, we have three motivations for studying a quantitative HANK model with a richer labor market. First, while the TANK model highlights the role of self-insurance in determining the economic outcomes and the response of monetary policy, it is limited by the assumption that agents are either fully-insured or hand-to-mouth; i.e, self-insurance is independent of labor income fluctuations. To quantify the effects of job mobility on the economy and allow for an interaction between MPC heterogeneity and idiosyncratic income risk, we need to more realistically capture wealth heterogeneity across individuals. Second, the TANK model captures only the inflationary effects of job mobility and omits that most job switches are productivity enhancing. In order to capture the productivity effects of job mobility, the labor market must feature heterogeneity in match productivity. Third, this stylized model is not suitable for disciplining wage dynamics. Specifically, the model assumes that an employed worker is forced to switch jobs upon contact and the associated wage gains are substantial. While we find that this wage increase is not the main driver of inflation when job mobility increases, the magnitudes of the responses are still affected by how much a worker's wage increases upon job switch.[15] In addition, some external offers may not necessarily increase the worker's wage (say, for example, when the offer is much worse than the current terms). This model also assumes that job stayers—those who do not receive an external offer and do not experience a job loss—do not experience any wage gains, which is counterfactual. Finally, in terms of wage losses upon displacement, the model cannot account for the scarring effects of unemployment, given that it assumes that all workers hired from unemployment receive the same wage and that unemployment does not affect a worker's productivity or labor market outcomes beyond causing a temporary wage loss. For these reasons, we need to explicitly model changes in worker productivity off- and on-the-job together with an endogenous determination of wages to discipline income dynamics among job losers, switchers, and stayers with the data.

3 A HANK model with a frictional labor market and OJS

We now describe our quantitative model that combines a New Keynesian framework with heterogeneous agents and a frictional labor market, where both employed and unemployed workers search for jobs. In our presentation of this model, we focus components that are new relative to the TANK model described in the previous section.

3.1 Environment

The economy is populated by a measure one of ex-ante identical individuals, firms in three vertically integrated sectors, a mutual fund, a fiscal authority, and a monetary authority.

[15] For example, the decline in output in the TANK model without wage increases upon external offers (Panel (a) of Figure 3) is smaller than the decline in output in the baseline TANK model (Panel (c) of Figure 2).

Firms. While this model incorporates the same three vertically integrated firms as in the TANK model, there are two differences. First, worker-firm matches in the labor services sector are heterogeneous in productivity, the details of which are discussed below. Second, the production function of the intermediate goods firms depends on aggregate productivity z as well.

Individuals. An individual's life consists of a working stage and a retirement stage. At the working stage, individuals are heterogeneous in their holdings of mutual fund shares $s \geq 0$, employment status e (employed E or unemployed U), human capital (skill) $h \in \mathcal{H} = \{\underline{h}, \ldots, \overline{h}\}$,—among the employed—match-specific productivity $x \in \mathcal{X} \equiv \{\underline{x}, \ldots, \overline{x}\}$, and endogenous and history-dependent piece-rate $\alpha \in (0, 1]$ governing the share of output they receive as wages.

Individuals are born with skill h drawn from distribution Γ^h. During their working lives, they experience stochastic appreciation or depreciation of these skills depending on their employment status, as in Ljungqvist and Sargent (1998). In particular, an employed individual's skill increases by Δh percent with probability π^E, while an unemployed individual's skill depreciates by Δh percent with probability π^U in each period. Formally,

$$h' = \begin{cases} h \times (1 + \Delta h) & \text{with probability } \pi^E \\ h & \text{with probability } 1 - \pi^E \end{cases}$$

when employed, and,

$$h' = \begin{cases} h \times (1 - \Delta h) & \text{with probability } \pi^U \\ h & \text{with probability } 1 - \pi^U \end{cases}$$

when unemployed. We model human capital dynamics to capture wage changes upon employment transitions in the data. For instance, human capital depreciation during unemployment allows the model to capture the scarring effects of unemployment, while human capital appreciation during employment helps the model generate wage growth among job stayers.

Individuals trade shares of the mutual fund and make consumption decisions (bought at price P_t) in the face of idiosyncratic income risk due to the stochastic evolution of their human capital and frictions in the labor market. Each period, working-age individuals retire with probability ψ^R. Retirees ($e = R$) finance consumption through their private savings and from pension income ϕ^R. They die with probability ψ^D, upon which they are replaced with unemployed individuals.[16]

Labor market. As in the TANK model, labor services firms operate in a labor market that features random search, where θ_t denotes tightness. Different from the TANK model, worker-firm matches are heterogeneous in match-specific productivity and worker skill. Upon contact,

[16]When an individual dies, she is replaced by an offspring who inherits her mutual fund holdings and enters the working stage as unemployed with the lowest skill level $\underline{h}$.

a worker-firm pair draws a match-specific productivity x from distribution Γ^x, which remains constant throughout the match. The match operates a production technology given by $F(h,x) = hx$. The worker is paid real wages according to a predetermined rule $w(h,x,\alpha)$ (detailed below) every period until the termination of the match. The match can dissolve because of an exogenous job separation, which occurs at rate δ, retirement, or an endogenous job-to-job transition by the worker. Unemployed individuals receive unemployment insurance (UI) benefits from the government according to the function $UI(h) = \phi^U F(h,\underline{x})$ (denoted in consumption units), where we assume that UI payments are designed as a replacement rate ϕ^U of output the worker would have produced working at a job with the lowest match productivity $\underline{x}$. On the other side of this labor market, labor services firms pay a per-period fixed cost κ to post vacancies and sell their output to intermediate firms at nominal price P_t^l ($p_t^l = P_t^l/P_t$ in units of the final good).

Wage determination. In each period, the wage paid to an employed worker is an endogenous piece-rate α of the flow output from the match. We follow a static bargaining protocol—a simplified version of the dynamic bargaining protocol in Postel-Vinay and Robin (2002)—for the determination of α, where firms Bertrand compete based on flow output (instead of present values).[17] Figure 4 summarizes this bargaining protocol.

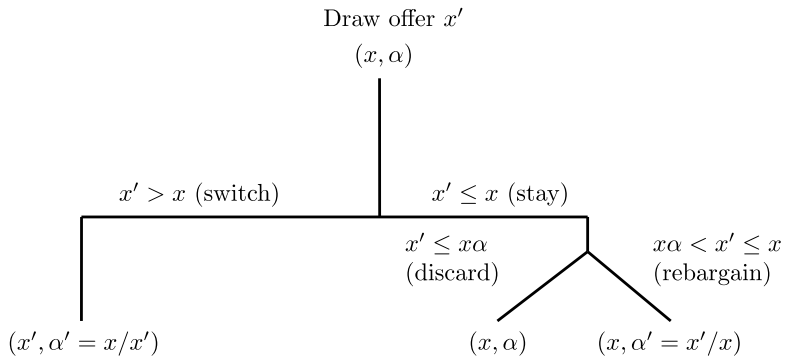
Consider a worker with human capital h employed in a match with productivity x and piece rate α, whose wage is given by $w(h,x,\alpha) = \alpha\phi^E F(h,x)$, where $\phi^E \in (0,1)$ represents the maximum share of output that a worker with maximum piece rate $\alpha = 1$ can capture as wage.[18] Suppose this worker meets with a new firm with a higher productivity $x' > x$, in which case she switches jobs. This is because the most the incumbent firm can offer to the worker is $w(h,x,1) = \phi^E F(h,x)$. We assume that the new firm is willing to match this wage, i.e., $w(h,x',\alpha') = w(h,x,1)$, which implies a new piece rate $\alpha' = x/x'$ for the worker. The worker is better off switching to the more-productive firm given that the piece rate can only become (weakly) larger in the future upon new contacts, as discussed below.

Now suppose the same worker receives an offer with a lower productivity $x' < x$, resulting in the worker staying with the incumbent firm. This case induces two scenarios. First, the new productivity x' could be so low that even the maximum potential wage from the new job cannot match the worker's current wage, i.e., $w(h,x',1) < w(h,x,\alpha)$, which happens when $x' < \alpha x$. In this case, the worker simply discards the offer and continues with the same piece rate. Second, x' could be sufficiently high to serve as a credible threat for the worker to bid up her wage with the incumbent firm. This happens when $w(h,x,1) > w(h,x',1) > w(h,x,\alpha)$, i.e., $x > x' > \alpha x$,

[17] While it is possible to allow bargaining over the entire present discounted value of the match surplus, this would introduce an additional loop into the solution, where for a given wage function, we would have to check whether it is consistent with the worker and firm value functions, as in Krusell et al. (2010). To avoid this added computational burden, we assume that bargaining takes place over the flow surplus.

[18] This assumption guarantees that whenever $\phi^E < p^l$, the firm's flow profit is greater than zero. As a result, there are no firms with negative surplus.

Figure 4: Wage determination



in which case the incumbent firm matches the maximum potential wage from the outside offer, $w(h, x, \alpha') = w(h, x', 1)$, implying an updated piece-rate $\alpha' = x'/x$.

The heterogeneity in match-specific productivity together with this bargaining protocol allows the model to capture the productivity effects of job switches, which we previously ignored in Section 2. In addition, workers are not forced to accept all external offers. Instead, they now decline offers from external contacts with lower match productivity; i.e., job switches are rational.

There are three sources of wage growth in this model. First, output increases as the worker's human capital appreciates over time, leading to higher wages given a piece rate. Second, poaching yields a wage increase as the worker extracts the entire surplus from the incumbent firm. Third, external offers not good enough to poach the worker but with sufficiently high match productivity lead to an increase in the piece rate via rebargaining with the incumbent firm. The last two channels capture the potential inflationary effects of EE flows that stem from wage increases in excess of productivity gains upon job switches and from counteroffers of current employers.[19]

The piece rate for a worker hired out of unemployment follows the same logic. We assume unemployment is akin to being employed at the lowest match productivity $\underline{x}$ and hence, for a new match with productivity x', the piece rate is given by $\alpha' = \underline{x}/x'$. We further assume that all offers out of unemployment are accepted.[20]

Mutual fund. A risk-neutral mutual fund owns all the firms in the economy, holds all nominal bonds B_t issued by the government, and sells shares in return. The fund pays a nominal dividend

[19]We explicitly model the last two sources of wage growth for three reasons. First, a higher EE rate is empirically associated with higher wage growth for both stayers and switchers (Moscarini and Postel-Vinay, 2016). Second, in the absence of wage increases due to external offers, wage growth among stayers would counterfactually be attributed to human capital appreciation alone. Third, while wage-inflation induced by EE is not indispensable for generating a positive relationship between the level of EE rate and inflation (see Section 2.4.2), it has effects on how EE dynamics shape demand through consumption responses triggered by future income changes.

[20]Under our baseline calibration, we verify that all new matches out of unemployment indeed have positive surplus in equilibrium. This is despite the opportunity cost of accepting an offer, since we ultimately estimate OJS to be less efficient than searching while unemployed. It turns out that the dynamic gains of being employed dominate the option value of waiting for another match with higher productivity.

D_t per share and can be traded by individuals at price P_t^s.

Fiscal and monetary authorities. The government implements a linear consumption tax τ_c and a progressive income tax. For any gross income level ω, net income is given by $\tau_t \omega^{1-\Upsilon}$, where τ_t captures the level of taxation and $\Upsilon \geq 0$ captures the rate of progressivity built into the tax system, as in Benabou (2002) and Heathcote, Storesletten, and Violante (2014).[21] Together with the tax revenue, the government issues nominal bonds B_t to finance UI benefits, retirement pensions, and an exogenous stream of nominal expenditures G_t. The central bank sets the short-term nominal interest rate using the reaction function given in Equation (5).

Timing of events. First, (unanticipated) aggregate shocks realize, the details of which are discussed in Section 4.3. Then, the government sets taxes and spending, and exogenous retirement, mortality and job destruction shocks are realized. Next, workers' skill evolve based on their beginning-of-period employment status, and new workers replenish the dead. This is followed by the job search stage where firms post vacancies and individuals search for jobs. Once new contacts are made, match productivities are observed, new matches are formed, and job switches occur. Then, in the production stage, each match produces labor services; intermediate firms produce differentiated goods and set their prices subject to nominal rigidities; and final goods are produced using intermediate goods. Given the unemployment rate and inflation, the monetary authority sets the nominal rate. Next, intermediate and service firms realize their profits; service firms pay wages to workers; the mutual fund pays out dividends; and the government collects taxes, issues new bonds, pays out UI and retirement benefits, and spends an exogenous amount. Finally, individuals decide how much to consume and how many shares to buy.

3.2 Individuals

Individuals decide on accepting a job offer while employed, as well as share holdings and consumption subject to a budget constraint and a short-selling constraint for shares. We cast the problems recursively, where time subscripts encode all the relevant aggregate state variables. Below, we present the problem of unemployed, employed, and retired individuals in turn. The value functions are expressed as of the end of a period at the consumption stage.

Unemployment. Let $V_t^U(s,h)$ denote the value of unemployed individuals with s shares of the mutual fund and skill h in period t. The problem of the unemployed worker is given by

$$\begin{aligned} V_t^U(s,h) = \max_{s' \geq 0,\, c} \; & u(c) + \beta(1-\psi^R)\mathbb{E}_{h'|h}\left[\Omega_{t+1}^U(s',h')\right] + \beta\psi^R V_{t+1}^R(s') \\ \text{s.t.} \quad & P_t c(1+\tau_c) + P_t^s s' = P_t \tau_t UI(h)^{1-\Upsilon} + (P_t^s + D_t)s, \end{aligned} \tag{14}$$

[21] Parameter τ is inversely related to the tax rate. Under a linear schedule with $\Upsilon = 0$, the tax rate is $1 - \tau$.

where we express the budget constraint in nominal terms. Here, $\Omega^U_{t+1}(s', h')$ is the value of the job search for unemployed workers at the beginning of the next period, which is described below. Unemployed workers receive dividends D_t from the mutual fund in proportion to their share holdings s, receive real after-tax UI benefits, and make a consumption-saving decision.

Employment. Let $V^E_t(s, h, x, \alpha)$ denote the value of employed individuals with s shares, skill h, match productivity x, and piece rate α. The employed individual's problem is given by

$$\begin{aligned} V^E_t(s,h,x,\alpha) = \max_{s'\geq 0,\, c} \; & u(c) + \beta(1-\psi^R)\mathbb{E}_{h'|h}\left\{(1-\delta)\Omega^E_{t+1}(s',h',x,\alpha) + \delta\Omega^U_{t+1}(s',h')\right\} + \beta\psi^R V^R_{t+1}(s') \\ \text{s.t.} \quad & P_t c(1+\tau_c) + P^s_t s' = P_t\tau_t w(h,x,\alpha)^{1-\Upsilon} + (P^s_t + D_t)s. \end{aligned} \tag{15}$$

Employed individuals also collect dividends, collect a real after-tax wage, and choose consumption and share holdings before entering the next period. At the beginning of the next period, the job might dissolve exogenously, in which case the worker becomes unemployed and searches for a new job. If not, the worker can engage in OJS, whose value is given by $\Omega^E_{t+1}(s', h', x, \alpha)$.

Retirement. Finally, the value of retirement is given by

$$\begin{aligned} V^R_t(s) = \max_{s'\geq 0,\, c} \; & u(c) + \beta(1-\psi^D)V^R_{t+1}(s') \\ \text{s.t.} \quad & P_t c(1+\tau_c) + P^s_t s' = P_t\tau_t(\phi^R)^{1-\Upsilon} + (P^s_t + D_t)s. \end{aligned} \tag{16}$$

Retirees only face mortality risk and make consumption-saving decisions given pension income.

Job search problems. Let $f(\theta_t)$ be the worker's contact rate per unit of search efficiency. The value of the job search for an unemployed worker is

$$\Omega^U_t(s,h) = f(\theta_t)\,\mathbb{E}_x V^E_t(s,h,x,\underline{x}/x) + (1-f(\theta_t))\,V^U_t(s,h), \tag{17}$$

where the job search efficiency of the unemployed is normalized to one. The value of OJS is

$$\begin{aligned} \Omega^E_t(s,h,x,\alpha) = {} & \nu_t f(\theta_t)\,\mathbb{E}_{\widetilde{x}}\left[\max\left\{V^E_t(s,h,\widetilde{x},x/\widetilde{x}), V^E_t(s,h,x,\max\{\alpha,\widetilde{x}/x\})\right\}\right] \\ & + (1-\nu_t f(\theta_t))\,V^E_t(s,h,x,\alpha), \end{aligned} \tag{18}$$

where ν_t is the search efficiency of the employed relative to the unemployed, as in Section 2. Upon contact, the worker-firm pair draws match productivity x and expectations are taken with respect to the sampling distribution Γ^x. The first term inside the expectation represents the worker's value when she switches to a new job with match productivity $\widetilde{x}$ and new piece rate $\alpha' = x/\widetilde{x}$. The second term represents the worker's value of staying with the incumbent firm, with either the current piece rate α (if $\widetilde{x} < \alpha x$) or a higher piece rate $\widetilde{x}/x$ (if $\widetilde{x} > \alpha x$).

3.3 Production

The final-good producer and intermediate firms follow the same equations described in Section 2 except that the production function of intermediate firms becomes $y_t(j) = z_t l_t(j)$, where z_t is aggregate productivity. The NKPC changes to

$$\frac{\log\left(1+\pi_t-\pi^*\right)\left(1+\pi_t\right)}{1+\pi_t-\pi^*} = \vartheta\left(\frac{p_t^l}{z_t}-\frac{\eta-1}{\eta}\right)+\frac{1}{1+r_{t+1}}\frac{\log\left(1+\pi_{t+1}-\pi^*\right)\left(1+\pi_{t+1}\right)}{1+\pi_{t+1}-\pi^*}\frac{Y_{t+1}}{Y_t}, \quad (19)$$

which is identical to Equation (8) except that the real marginal cost is $mc_t = p_t^l/z_t$.

Labor services. Tightness θ_t is the ratio of vacancies v_t to the aggregate measure of job search $S_t = \int d\mu_t^U(s,h) + \nu_t \int d\mu_t^E(s,h,x,\alpha)$, where μ^U and μ^E are the distributions of unemployed and employed workers over their relevant states at the search stage within the period.

Consider a firm that employs a worker with skill h and piece rate α in a match with productivity x. The pair produces $F(h,x)$ units of labor services, which is then sold to intermediate firms at real price p_t^l. The real value of this firm $J_t(h,x,\alpha)$ is given by

$$\begin{aligned} J_t(h,x,\alpha) = {} & p_t^l F(h,x) - w(h,x,\alpha) + \frac{1}{1+r_{t+1}}(1-\psi^R)(1-\delta) \\ & \times \mathbb{E}_{h'|h}\Big\{ (1-\nu_t f(\theta_{t+1}))\, J_{t+1}(h',x,\alpha) + \nu_t f(\theta_{t+1}) \int_{\underline{x}}^{x} J\left(h',x,\max\{\alpha,\widetilde{x}/x\}\right) d\Gamma^x(\widetilde{x})\Big\}, \end{aligned} \quad (20)$$

where the match survives if the worker does not retire, does not exogenously separate into unemployment, and does not find a new job through OJS. The worker accepts the new job offer if $\widetilde{x} > x$, in which case the firm's value is 0. If $\widetilde{x} < x$, then the firm keeps the worker either at a higher piece rate $\widetilde{x}/x$ (if $\widetilde{x} > \alpha x$) or at the current piece rate α (if $\widetilde{x} < \alpha x$). As firms are owned by the risk-neutral mutual fund, they discount the future at the real interest rate r_{t+1}.

The real value of posting a vacancy is

$$\begin{aligned} V_t = {} & -\kappa + q(\theta_t)\frac{1}{S_t}\left[\int_{s,h}\int_{\widetilde{x}} J_t\left(h,\widetilde{x},\underline{x}/\widetilde{x}\right) d\Gamma^x(\widetilde{x})\, d\mu_t^U(s,h)\right. \\ & \left. + \nu_t \int_{s,h,x,\alpha}\int_{x}^{\overline{x}} J_t\left(h,\widetilde{x},x/\widetilde{x}\right) d\Gamma^x(\widetilde{x})\, d\mu_t^E(s,h,x,\alpha)\right], \end{aligned} \quad (21)$$

where the first and second terms capture the value of filling a vacancy out of unemployment and employment, respectively. A free-entry condition implies that $V_t = 0$.

Mutual fund. The mutual fund issues shares at price P^s and owns all firms and government bonds. No arbitrage implies that the return on shares must equal the return on bonds:

$$\frac{P_{t+1}^s + D_{t+1}}{P_t^s} = 1 + i_t. \quad (22)$$

The mutual fund is not allowed to retain any funds. All balances (positive or negative) are distributed to share owners in the form of dividends given by

$$D_t = B_{t-1} - \frac{B_t}{1+i_t} + P_t\Gamma_t^I + P_t\Gamma_t^S, \tag{23}$$

where the aggregate real profits of intermediate and labor services firms are as follows[22]:

$$\Gamma_t^I = \left(1 - \frac{p_t^l}{z_t} - \frac{\eta}{2\vartheta}\log(1+\pi_t-\pi^*)^2\right) Y_t, \tag{24}$$

and

$$\Gamma_t^S = \int \left(p_t^l F(h,x) - w(h,x,\alpha)\right) d\lambda_t^E(s,h,x,\alpha), \tag{25}$$

where $\lambda_t^E(s,h,x,\alpha)$ is the distribution of employed workers at the end of the period. Equation (23) clarifies that the mutual fund collects payments for the existing debt obligations B_{t-1}, collects profits of intermediate and labor services firms Γ_t^I and Γ_t^S, and finances all new debt issuances B_t. The remaining balance accrues to the individuals as dividends.

Fiscal authority. The fiscal authority taxes individuals and issues bonds to finance expenditures G_t as well as UI and retirement benefits. The government budget constraint is

$$\begin{aligned} B_{t-1} + G_t + P_t\int UI(h)d\lambda_t^U(s,h) + P_t\int \phi^R d\lambda_t^R(s) = \frac{B_t}{1+i_t} + P_t\tau_c\int c(l,s,h,x,\alpha)d\lambda_t(e,s,h,x,\alpha) \\ + P_t\int \left(UI(h) - \tau_t UI(h)^{1-\Upsilon}\right) d\lambda_t^U(s,h) \\ + P_t\int \left(w(h,x,\alpha) - \tau_t w(h,x,\alpha)^{1-\Upsilon}\right) d\lambda_t^E(s,h,x,\alpha) \\ + P_t\int \left(\phi^R - \tau_t(\phi^R)^{1-\Upsilon}\right) d\lambda_t^R(s), \end{aligned} \tag{26}$$

where the left-hand side is total government expenses and the right-hand side is total government revenues generated from issuing bonds and consumption and income taxation, respectively. Here, $\lambda_t(\cdot)$, $\lambda_t^E(\cdot)$ and $\lambda_t^U(\cdot)$, $\lambda_t^R(\cdot)$ are the distributions of all, employed, unemployed, and retired individuals over relevant state variables at the end of the period, respectively.

Monetary authority. A monetary authority controls the short-term nominal interest rate i_t following the Taylor rule in Equation (5), and the real interest rate r_t satisfies the Fisher equation in Equation (6). Timing conventions of these variables are the same as in Section 2.

[22]We assume that vacancy cost κ is psychic in that it does not consume resources and does not affect profits.

3.4 Equilibrium

Market clearing requires that labor services demanded by intermediate firms Y_t/z_t is equal to the aggregate supply of labor services and that mutual fund shares demanded by all individuals aggregate to one. Formally, these conditions are given by:

$$Y_t/z_t = \int F(h,x)\, d\lambda_t^E(s,h,x,\alpha), \tag{27}$$

$$1 = \int g_t^{Us}(s,h) d\lambda_t^U(s,h) + \int g_t^{Es}(s,h,x,\alpha) d\lambda_t^E(s,h,x,\alpha) + \int g_t^{Rs}(s) d\lambda_t^R(s), \tag{28}$$

where g_t^{es} denotes the saving decision of workers with employment status $e \in \{E,U,R\}$.

Definition of equilibrium. Given fiscal policy instruments that determine UI replacement rate ϕ^U, retirement transfers ϕ^R, tax parameters $\{\tau_c, \tau_t, \Upsilon\}$, and government spending G_t, monetary policy, and paths of aggregate shocks (discussed in the next section), an equilibrium of the model is a sequence of individual decision rules for consumption $g_t^{Ec}, g_t^{Uc}, g_t^{Rc}$ and mutual fund share demand $g_t^{Es}, g_t^{Us}, g_t^{Rs}$, intermediate and labor services firm profits Γ_t^I and Γ_t^S, dividends D_t, real price of labor services p_t^l, share price P_t^s, labor market tightness θ_t, interest rates r_t, i_t, inflation π_t, bond holdings B_t, and worker distributions $\left\{\lambda_t^E, \lambda_t^U, \lambda_t^R\right\}$ such that

- Nominal and real interest rates satisfy the Taylor rule (5) and the Fisher equation (6).
- Intermediate and labor services firm profits satisfy Equations (24) and (25), respectively.
- Share prices satisfy Equation (22) and dividends are given by Equation (23).
- Bonds are such that the government budget constraint in Equation (26) holds every period.
- Individual decisions g_t^{Ec}, g_t^{Uc}, g_t^{Rc}, g_t^{Es}, g_t^{Us} and g_t^{Rs} are optimal.
- Labor market tightness θ_t and real price of labor services p_t^l are such that the value of posting a vacancy expressed in Equation (21) is zero and Equation (27) holds.
- Inflation π_t satisfies the NKPC in Equation (19).
- The shares market clears as specified in Equation (28).[23]
- The worker distribution evolves according to the laws of motion in Appendix B.1.1.

As indicated above, market tightness θ_t and real price of labor services p^l are jointly determined to satisfy the free-entry condition and the market-clearing condition for labor services. This is because the supply of labor services in Equation (27) is determined by the distribution of workers, which is governed by contact rate $f(\theta)$. Raising the supply of labor services requires an increase in market tightness, which is induced by a rise in the price of labor services through the free-entry condition in Equation (21).

[23] We do not check for goods market clearing due to Walras's Law.

Table 1: Externally calibrated parameters

Parameter	Explanation	Value	Reason
σ	Curvature in utility function	2	Standard
ψ^R	Retirement probability	0.00625	40 years of work stage
ψ^D	Death probability	0.0125	20 years of retirement stage
Δh	Skill appreciation/depreciation amount	0.275	Set
π^E	Skill appreciation probability	0.018	Wage growth for job stayers
ξ	Matching function elasticity	1.6	Set
η	Elasticity of substitution	6	20 percent markup
ϑ	Price adjustment cost parameter	0.021	Slope of Phillips curve, Galí and Gertler (1999)
x_G	Government spending/GDP ratio	0.19	Total net federal outlay/GDP
x_B	Debt/GDP ratio	2.43	Total public debt/GDP
τ_c	Consumption tax rate	0.0312	Sales tax receipt/consumption expenditure
Υ	Progressivity of income tax	0.151	Heathcote, Storesletten, and Violante (2014)
ρ_τ	Responsiveness of income tax parameter to debt level	0.10	Auclert, Rognlie, and Straub (2020)
π^*	Steady-state inflation rate	0.00496	2 percent annual inflation rate
Φ_π	Responsiveness of interest rate to deviations from inflation target	1.5	Taylor (1993) and Galí and Gertler (1999)
Φ_u	Responsiveness of interest rate to deviations from unemployment target	-0.25	Taylor (1993) and Galí and Gertler (1999)

Notes: This table summarizes externally calibrated parameters. See the main text for a detailed discussion.

The stationary equilibrium of the model is obtained by setting aggregate shocks to zero. In steady state, we assume that tax parameter τ^* clears the government budget constraint and that outstanding bonds and government expenditures are a fraction of steady-state level of output $B^* = x_B Y^*$ and $G^* = x_G Y^*$, respectively. We provide details on the solution of the steady state in Appendix B.1 and details on the computation of the economy's transitional dynamics in Section 4.2 and Appendix B.2.

4 Calibration and estimation

We now discuss how we discipline our model. First, we calibrate the steady-state of the model to match moments of the U.S. economy prior to the Great Recession—specifically, over the period 2004–2006. Second, we describe how we estimate shock processes for discount rate β, aggregate labor productivity z, and OJS efficiency ν using the model. A model period is set to a quarter.

4.1 Calibration of parameters

Functional forms and externally calibrated parameters. Table 1 summarizes the externally calibrated parameters. The utility function over consumption is of the CRRA form with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. As is standard in the literature, we set the risk aversion parameter to $\sigma = 2$. As for the life cycle, workers spend on average 40 years in the labor force and 20 years in retirement, which require setting $\psi^R = 0.625$ percent and $\psi^D = 1.25$ percent on a quarterly basis.

Turning to the evolution of worker productivity, we use five equally-spaced (in logs) grid

points between the lowest value $\underline{h} = 1$ and the highest value $\overline{h} = 3$ for human capital. These choices imply that worker skills change by a proportion $\Delta h = (ln(3) - ln(1))/4 = 0.275$ between grid points when they appreciate while working or depreciate during unemployment. We discipline the probability of skill appreciation for the employed π^E by the annual wage growth of job stayers. Karahan, Ozkan, and Song (2022) document that this is around 2 percent for a large share of the U.S. population, which implies that expected quarterly wage growth of job stayers should be around 0.5 percent, which requires setting $\pi^E = 0.005/0.275 \approx 0.018$. We further assume that the match-specific productivity x is drawn from a log-normal distribution with standard deviation σ_x (to be discussed below). We discretize this process with 7 equally-spaced grid points (in logs) between the 1st and 99th percentiles of the log-normal distribution.

We pick a CES matching function so that the worker and firm contact rates are bounded between 0 and 1 and given by $f(\theta) = \theta(1+\theta^\xi)^{-1/\xi}$ and $q(\theta) = (1+\theta^\xi)^{-1/\xi}$, respectively. Here, ξ controls the elasticity of contact rates to tightness, and we set $\xi = 1.6$ following Schaal (2017).

The elasticity of substitution across intermediate goods varieties η controls the markup of prices over the marginal cost—and therefore the profit share—at the steady state. We set this parameter to 6 to obtain a profit share of $\eta/(\eta-1) = 20$ percent (Auclert, Bardóczy, Rognlie, and Straub, 2021; Faccini and Melosi, 2023). Without loss of generality, we normalize the productivity of the intermediate sector to $z = 1$ at the steady state. Finally, as Equation (19) shows, the price adjustment cost parameter ϑ directly dictates the slope of the Phillips curve. We set ϑ to 0.021 to match the slope of Phillips curve as estimated by Galí and Gertler (1999).

Given that Ricardian equivalence does not hold in our model, fiscal policy matters for how the economy responds to shocks. Over the period 2004-2006, the ratio of government spending to GDP was around 19 percent, so we set $x_G = 0.19$. We also calibrate the model to have a realistic amount of government debt. In the data, the ratio of debt stock to annual GDP averages to 60.8 percent over the same period. The quarterly frequency in the model dictates that we set this ratio to $x_B = 4 \times 0.608 = 2.43$. We set the consumption tax rate to $\tau_c = 3.02$ percent, which we obtain as the ratio of state and local sales tax receipts to personal consumption expenditures in the data for 2006. We set the parameter governing the progressivity of labor income taxes to $\Upsilon = 0.151$, as in Heathcote, Storesletten, and Violante (2014). The steady-state tax parameter τ^* clears the government budget constraint in Equation (26).

Turning to monetary policy, the central bank targets 2 percent annual inflation. Quarterly calibration requires us to set $\pi^* = 1.02^{1/4} - 1 \approx 0.496$ percent. In disciplining the Taylor rule, we follow Taylor (1993) and Galí (2015) and set the coefficient on inflation to $\Phi_\pi = 1.5$. Galí (2015) sets the the coefficient on the output gap to 0.125. To map this coefficient to the unemployment gap, we use an Okun's law coefficient of -2 (Okun, 1962) yielding $\Phi_u = -0.25$.

Internal calibration. The remaining nine parameters to be determined are the discount factor β, vacancy creation cost κ, job separation probability δ, job search efficiency of the employed ν,

Table 2: Internally calibrated parameters

Parameter	Explanation	Value	Target	Data	Model
β	Discount factor	0.981	Fraction of individuals with non-positive net liquid wealth	0.16	0.11
κ	Vacancy creation cost	0.670	Unemployment rate	0.051	0.052
δ	Job separation probability	0.091	EU rate	0.038	0.033
ν	Search efficiency of employed	0.108	EE rate	0.02	0.02
π^U	Skill depreciation probability	0.022	Earnings drop upon job loss	-0.35	-0.36
σ_x	Standard deviation parameter of match productivity distribution	0.063	Wage growth of job switchers	0.09	0.09
ϕ^E	Maximum share of output as wages	0.823	Labor share	0.67	0.74
ϕ^U	UI replacement rate	0.385	UI replacement rate	0.40	0.44
ϕ^R	Retirement benefit amount	0.473	Retirement income/labor income	0.34	0.41

Notes: This table summarizes internally calibrated parameters. See the main text for a detailed discussion.

skill depreciation probability when unemployed π^U, the standard deviation of the match specific productivity distribution σ_x, maximum share of output potentially paid to worker as wages ϕ^E, UI replacement rate ϕ^U, and retirement benefit amount ϕ^R. These parameters are calibrated internally by matching a set of data moments that we now describe. Table 2 summarizes the targeted moments and the calibrated parameter values. While all the parameters are jointly calibrated, Table 2 presents each parameter next to its most informative data moment.[24]

Given the recent work highlighting the role of asset distribution in the transmission of monetary policy, we target the fraction of HtM individuals in the labor force to discipline discount factor β. We define HtM individuals as those with non-positive net liquid wealth holdings. We use the 2004 panel of the Survey of Income and Program Participation (SIPP) and work with a sample of individuals aged 25–65, who do not own any business. 16 percent of our sample are HtM individuals according to our classification.

On the labor market side, we target a steady-state unemployment rate of 5.1 percent, as well as worker flows. We obtain the targets for the flow rates from various sources. Using data from the Current Population Survey (CPS), we compute the average monthly employment-to-unemployment separation rate over the period 2004-2006. We convert this monthly job loss rate to a quarterly frequency and obtain our target of 3.8 percent. To compute the job-to-job transition rate, we make use of quarterly data from the Longitudinal Employer-Household Dynamics (LEHD). We find that the job-to-job transition rate (or EE rate), measured as the job-switching rate of workers who do not have any intervening nonemployment spell, is around 2 percent over the same period. These three moments are informative about the vacancy creation cost κ, job separation rate δ, and employed search efficiency ν, respectively.

The probability of skill depreciation when unemployed π^U governs the magnitude of earnings

[24]We use the simulated method of moments, where we minimize the sum of squared percentage deviations of the model moments from their empirical counterparts.

drop upon job loss. Getting this moment right is important not only to discipline skill depreciation but also to get at the cost of job loss and the welfare effects of stabilization policy. A large literature has estimated the magnitude of earnings loss upon displacement using a variety of datasets and approaches (see, for example, Jacobson, LaLonde, and Sullivan, 1993; Stevens, 1997; Davis and von Wachter, 2011; Jarosch, 2021; Birinci, 2021, among others). Across these studies, the median estimate of earnings loss in the year of displacement is 35 percent. To this end, we simulate a panel of individuals, aggregate quarterly observations to an annual frequency, and run a distributed-lag regression on the model-generated data analogously with empirical studies.

Another aspect of the model is wage changes upon job switches, which determine the role of EE transitions in aggregate demand. Using the LEHD, we calculate the increase in earnings for continuously employed workers upon a job switch as around 9 percent. This moment informs the dispersion parameter for match productivity σ_x, which governs the increase in wages upon a job switch. Given σ_x, we set the location parameter of the match productivity distribution to $\mu_x = -\sigma_x^2/2$ so that its mean is normalized to one. Finally, we choose the maximum output share paid to workers ϕ^E to target an average labor income share of 0.67.

Turning to the fiscal transfers, we calibrate the UI replacement rate ϕ^U to match an average replacement rate of 40 percent. To discipline retirement pensions ϕ^R, we calculate the average retirement income to average labor income ratio in the SIPP. Specifically, we add up Social Security Income and pension incomes from federal, state, and local governments for each retiree in our sample and calculate the average per-person retirement income. We then divide this measure by the average labor income among non-retirees to obtain a target ratio of 0.34.

4.2 Solving for transitional dynamics

We now summarize how we solve the model with aggregate shocks, relegating further details to Appendix B.2. We use the SSJ method developed by Auclert, Bardóczy, Rognlie, and Straub (2021), which allows us to efficiently compute impulse responses. Importantly, we generalize the SSJ method for our context. Specifically, we allow model blocks to *directly* interact not only via aggregate variables but also through the discretized worker distributions. This modification is crucial because outcomes of the heterogeneous-agent (HA) block include distributions of employed and unemployed individuals, which are required inputs for labor service firms and other equilibrium conditions. First, the distribution of employed individuals across human capital and match productivity and the distribution of unemployed individuals across human capital at the job search stage, i.e., $\mu^E(h, x)$ and $\mu^U(h)$, respectively, affect the expected value of a match $\mathbb{E}J$ for firms deciding on vacancy creation. This is because (i) human capital affects the magnitude of output in a match and (ii) employed workers' match productivity with their current employer affects their job-acceptance decision and the piece rate that the poaching firm would offer to the worker (and thus their wage level) upon a new match. Second, the distribution of

employed workers across human capital, match productivity, and piece rate levels at the consumption/production stage in a period, $\lambda^E(h, x, \alpha)$, affects labor services firms' profits Γ^S by determining the output and wage levels in a match, which in turn affect dividends.[25]

To summarize, we generalize the SSJ method and incorporate discretized distributions across state variables as direct inputs and outputs along the DAG. In this sense, our approach combines the SSJ method of Auclert, Bardóczy, Rognlie, and Straub (2021) with Reiter (2009).

4.3 Estimation of aggregate shocks

We now use our calibrated model to estimate aggregate shock processes that we use in our positive and normative analysis in Sections 5 and 6.

We assume that the economy starts from steady state and is subject to demand, supply, and labor market shocks, which are modeled as innovations to the discount rate β, aggregate labor productivity z, and OJS efficiency ν. We assume AR(1) processes for β, z, and ν given by:

$$\beta_t = (1-\rho_\beta)\beta^* + \rho_\beta \beta_{t-1} + \sigma_\beta \epsilon_{\beta,t}, \quad z_t = (1-\rho_z)z^* + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}, \quad \nu_t = (1-\rho_\nu)\nu^* + \rho_\nu \nu_{t-1} + \sigma_\nu \epsilon_{\nu,t},$$

where ρ_j denotes the persistence of the AR(1) process, $\epsilon_j \sim N(0,1)$ is i.i.d. and $\sigma_j > 0$ denotes the standard deviation of innovations for $j \in \{\beta, z, \nu\}$.

Off the steady state, the government adjusts debt to satisfy its budget constraint. As a result, along a transition, debt can deviate from its state level $b^* = x_B Y^*$. In such cases, the fiscal authority follows an exogenous rule to adjust the tax parameter τ to eventually bring the real debt level back to steady state b^*.[26] This response function is given by

$$\tau_t = \tau^* - \rho_\tau \left(b_{t-1} - b^*\right) / Y^*. \tag{29}$$

The term ρ_τ controls the sensitivity of average taxes to debt.[27] Following Auclert, Rognlie, and Straub (2020), we set $\rho_\tau = 0.1$.

We estimate parameters of the AR(1) processes for β, z and ν (in total six parameters for the persistence and standard deviations for each) by targeting the autocorrelation of output, correlations of the unemployment rate, EE rate, and inflation with output, as well as the standard deviations of output, unemployment rate, EE rate, and inflation in the data.[28] This estimation

[25] If we were to assume directed search in the labor market, block-recursivity would allow us to drop $\mu^E(h, x)$ and $\mu^U(h)$. However, we would still need to keep track of $\lambda^E(h, x, \alpha)$ to calculate firm profits.

[26] Absent this countervailing force, the model would have a unit root resulting in debt exploding.

[27] As mentioned in Footnote 21, τ_t is inversely related to the average tax rate. The negative sign in front of ρ_τ means that whenever $b_{t-1} > b^*$, we have $\tau_t < \tau^*$, i.e., taxes increase.

[28] We obtain monthly data on the unemployment rate and core CPI from the BLS and take quarterly averages; quarterly data on real GDP from U.S. Bureau of Economic Analysis; monthly data on the EE rate from Fujita, Moscarini, and Postel-Vinay (2023), which we convert to a quarterly frequency by compounding $EE_t^{\text{qrt}} = 1-(1-EE_t)^3$; and monthly data on the number of vacancies from JOLTS and take quarterly averages and divide that by unemployment to obtain labor market tightness. All data cover the period between 1995:Q3 and 2008:Q4.

Table 3: Estimation of aggregate shocks: Data vs model moments

	Data			Model		
	Std. Dev	Autocorr.	Corr. w/ Y	Std. Dev	Autocorr.	Corr. w/ Y
Y	0.024	0.963	1	0.005	0.924	1
u	0.148	0.953	-0.882	0.092	0.859	-0.882
EE	0.090	0.907	0.147	0.068	0.765	0.145
θ	0.275	0.930	0.809	0.062	0.105	0.626
π	0.245	0.388	0.538	0.270	0.825	0.543

Notes: This table compares model outcomes with their empirical counterparts using the estimated AR(1) processes for the discount rate β, aggregate labor productivity z, and OJS efficiency ν. Y u, EE, θ, and π denote output, unemployment rate, EE rate, labor market tightness, and inflation, respectively.

yields $\rho_\beta = 0.909$, $\rho_z = 0.332$, $\rho_\nu = 0.936$ and $\sigma_\beta = 0.001$, $\sigma_z = 0.002$, and $\sigma_\nu = 0.003$.

Table 3 compares model moments with their data counterparts. The model generates nearly identical values for the correlations of unemployment rate, EE rate, and inflation with output, as well as the autocorrelation of output compared with their empirical counterparts. In terms of standard deviations, the model is less successful. A well-known shortcoming of labor search models is their inability to generate realistic unemployment volatility in response to productivity shocks (Shimer, 2005). Although unemployment volatility is still lower in our model than in the data, our model delivers a much higher volatility than the standard model, especially, because of shocks to the discount factor (see Table 4) and job ladder dynamics (Fujita and Ramey 2012). Our model also delivers inflation and EE rate volatilities close to their empirical values.

To gauge the contribution of each shock to the cyclicality of our target outcomes, we present a variance decomposition (Table 4). Shocks to β explain almost all fluctuations in output and 81.2 percent of fluctuations in the unemployment rate, while shocks to z and ν jointly account for the remaining 18.8 percent of fluctuations. We also find that shocks to ν are an important driver of EE movements and inflation; they account for 78.7 percent of fluctuations in the EE rate and 43.1 percent of fluctuations in inflation.

What are OJS efficiency shocks? We showed that fluctuations in OJS efficiency have sizable effects on the EE rate and inflation. A natural question to ask is: What are possible micro-foundations for these shocks? For instance, Bilal, Engbom, Mongey, and Violante (2022) show that worsening financial frictions lead to a decline in firm entry around the Great Recession. Importantly, they argue that the the job ladder collapse during this episode was due to the decline in vacancy creation among firms whose net poaching rate is typically high. Along these

To calculate moments, we take logs and detrend the time series of output (real GDP), unemployment rate, and tightness using the HP filter with a smoothing parameter of 10^5 and calculate correlations and standard deviations of their cyclical components. Because inflation is negative in some periods, we detrend the level of inflation. Finally, we calculate the percent deviation of EE rate from its sample average.

Table 4: Variance decomposition of moments

	Share of variance explained by		
	z	ν	β
Y	0.008	0.031	0.961
u	0.111	0.077	0.812
EE	0.070	0.787	0.143
θ	0.337	0.046	0.618
π	0.049	0.431	0.520

Notes: This table presents a variance decomposition of output, unemployment rate, EE rate, labor market tightness, and inflation, respectively. The columns represent the fraction of each moment's variance explained by shocks to aggregate productivity z, OJS efficiency ν, and discount factor β alone.

lines, to microfound OJS efficiency shocks, our framework can be extended to feature ex-ante labor-services firm heterogeneity, where the entry decision is subject to financial frictions. In this model, firms can choose to hire senior workers through poaching or junior workers out of unemployment who earn lower wages but are also less productive. Firms that are close to their borrowing constraints may opt to hire junior workers instead of poaching seniors. This would generate a depressed EE rate in a recovery despite a declining unemployment rate, a pattern consistent with the data during the post-Great Recession episode that we analyze in Section 5.1.

While such examples provide guidance on how to think about changes in the relative job-search efficiency of the employed, we do not attempt to microfound OJS shocks given the complexity of the framework. We simply treat the shocks as a reduced-form time-varying wedge between hiring out unemployment versus employment, estimate them using the model, and study their implications on aggregate dynamics. As Section 5 shows, an upward shift in OJS efficiency causes higher inflation and unemployment, implying that—unlike demand or supply shocks—they break down divine coincidence and lead to a trade-off for monetary policy.[29]

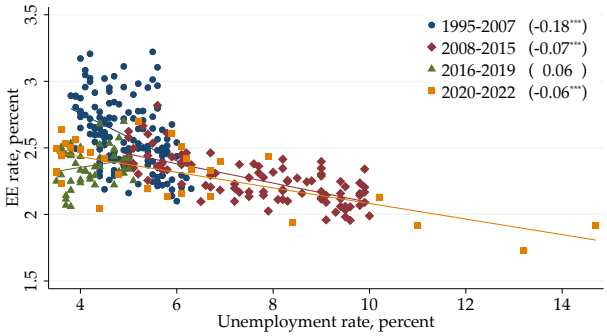
5 Positive implications of job mobility on inflation

In this section, we use the calibrated model to study the economy's response to exogenous shifts in worker mobility, which we capture by shocks to OJS efficiency ν. The shocks are disciplined by two historical episodes: (i) a stable EE rate despite a large decline in unemployment during the post-Great Recession expansion and (ii) a sizable rise in the EE rate post COVID-19 beyond what is implied by the historical correlation between unemployment and EE rates, which we document in Section 5.1. In Sections 5.2 and 5.3, we quantify (i) the drag on inflation due to

[29]Demand and productivity shocks typically move inflation and unemployment gaps in opposite directions—absent time varying real rigidities, which our model features—leading to divine coincidence. We show that OJS efficiency shocks move inflation and unemployment gaps in the same direction, breaking divine coincidence and introducing a trade-off for monetary policy. In this sense, OJS efficiency shocks act like cost-push shocks studied in the New Keynesian literature.

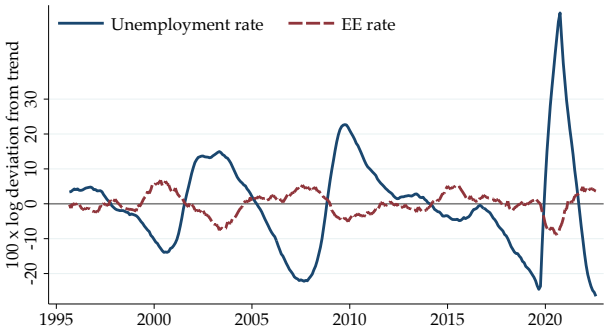
Figure 5: Unemployment rate, EE rate, and unit labor cost dynamics over time

(a) Unemployment vs EE rate

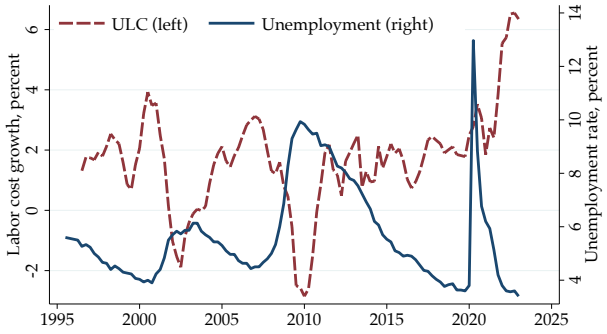


(b) Rolling correlation between unemployment and EE

(c) Cyclical components of unemployment and EE



(d) Unit labor cost vs unemployment rate



Notes: Panel (a) presents a scatter plot of monthly EE rate (Fujita, Moscarini, and Postel-Vinay, 2023) and unemployment rate across different time periods. Values in parenthesis report the coefficient from regressing the EE rate on the unemployment rate and *** denotes significance at the 1 percent level. Panel (b) presents the rolling correlation between the cyclical components of the logs of unemployment and EE rates using a five-year window. Panel (c) plots the time series of the cyclical components of log unemployment rate and EE rate. These series are detrended using the HP filter with a smoothing parameter of 10^5 and we take 12-month centered moving average for visual clarity. Panel (d) plots the four-quarter growth rate of the ULC index against the unemployment rate. We obtain the quarterly ULC index for nonfarm business sector from the BLS. For each quarter, we calculate the four-quarter growth rate and smooth the series by taking a four-quarter moving average.

muted worker mobility, relating this to the "missing inflation" puzzle during the former recovery episode, and (ii) the rise of inflation from elevated worker mobility during the latter recovery episode. Then, in Section 5.4, we fully decompose the channels through which OJS efficiency shocks affect inflation in the model.

5.1 Differential EE and labor cost dynamics in the last two recoveries

Panel (a) of Figure 5 presents a scatter plot of the monthly EE rate and the unemployment rate (i) prior to the Great Recession (1995-2007), (ii) during the Great Recession and the subsequent recovery (2008-2015), (iii) post-Great Recession (2016-2019), and (iv) the COVID-19 period (2020-2022). The correlation between the two series is negative and significant, except

during 2016-2019, when this correlation turned slightly positive (but insignificant). In other words, periods of tightening labor markets characterized by declining unemployment rates are typically also when the EE rate picks up, except for the 2016–2019 period. To present a more continuous view and to separate trend from the cycle, Panel (b) plots the rolling correlation between the cyclical components of log unemployment and EE rates over time using a five-year window. There is historically a strong negative comovement among the two series, which disappeared during 2016-2019. This breakdown of the correlation can be traced to a flat EE rate during this episode despite an around 25 percent decline in the unemployment rate from its trend to a historical low, as shown in Panel (c). This is in contrast to the recovery from the COVID-19 recession, when the unemployment rate declined by almost the same amount while the EE rate increased by around 8 percent above its trend.

Our focus in the quantitative exercises is to understand how the marginal cost of production p^l/z changes upon shocks to OJS efficiency. The closest data concept to p^l/z is the unit labor cost (ULC), which provides an index of labor costs adjusted for worker composition and productivity. Panel (d) presents a time-series of ULC growth. During the recovery from the COVID-19 recession, when the unemployment rate was below 4 percent, the growth of ULC reached around 6 percent. However, despite the unemployment rate similarly dropping below 4 percent between 2016 and 2019, ULC growth was only around 2 percent. As such, the differential rise in ULC growth during the last two recovery episodes cannot be accounted for by unemployment dynamics alone but can be attributed to their differential EE dynamics. In the next two sections, we show that this empirical observation is in line with our model's prediction that higher worker mobility generates an increase in the real marginal cost (and inflation).

5.2 Missing inflation due to muted worker mobility during 2016-2019

We now use our model to simulate the Great Recession recovery and quantify the size of the missing inflation due to muted worker mobility. To do so, we compare two model economies that mimic the path of the unemployment rate over the 2016-2019 period but *differ* in their equilibrium EE rate. The first economy features an endogenously rising EE rate due to a tightening labor market driven by positive demand shocks, consistent with its negative historical correlation with the unemployment rate but inconsistent with the observed pattern in this episode. The second economy, additionally subject to negative OJS shocks, not only features the same unemployment path as the first economy but also replicates the stable EE rate in the data.

Starting from the steady state described in Section 4, we allow for two shocks to hit the economy starting in 2016.[30] We model demand and OJS efficiency shocks as innovations to the

[30]That the economy is in steady state in 2016 is a plausible assumption from the labor market perspective. The unemployment rate in 2016 was around 5 percent, close to the estimates of the natural rate of unemployment.

discount factor β and ν, respectively, following AR(1) processes[31]:

$$\beta_t = (1-\rho_\beta)\beta^* + \rho_\beta \beta_{t-1} + \varepsilon_{\beta,t}, \qquad \nu_t = (1-\rho_\nu)\nu^* + \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t}. \tag{30}$$

The first economy features only demand shocks ($\varepsilon_{\nu,t} = 0$). We back out the path of $\varepsilon_{\beta,t}$ that generates a gradual decline in the unemployment rate by 15 percent relative to its steady state level of 5.2 percent.[32] To approximate the steady decline in the unemployment rate between 2016 and 2019, we assume the decline is linear and is completed within $\overline{T} = 16$ quarters from the onset of the first shock. Upon reaching its trough, the unemployment rate reverts back to steady state geometrically at rate ρ_u.[33] The second scenario also allows for shocks to OJS efficiency to mimic the actual expansionary episode during 2016-2019. We jointly estimate the path of the two shocks, $\varepsilon_{\beta,t}$ and $\varepsilon_{\nu,t}$, such that the economy generates the same unemployment rate path described in the first scenario and, in addition, has the EE rate unchanged throughout. [34]

The results are presented in Figure 6. Positive demand shocks alone in the first economy and the combination of positive demand and negative OJS shocks in the second economy generate identical paths for unemployment as intended (Panel (a)). The two economies differ in their EE rate by construction (Panel (b)): In the first economy, positive demand shocks increase vacancy creation and, consequently, generate an endogenous increase in the EE rate from its steady-state level of 2 percent to a peak of 2.06 percent. In the second economy, negative OJS efficiency shocks keep the EE rate suppressed as intended, in line with the data (Panel (c) of Figure 5).

The first economy produces slightly more output (Panel (c)). The output difference is entirely attributable to the differences in average labor productivity (ALP) since the path of (un)employment is identical across the two economies. ALP initially decreases in both economies, as the increase in the job-finding rate means more unemployed join the ranks of the employed. Since the unemployed typically have lower human capital than the employed and accept offers with any match productivity, their entry to employment lowers ALP. ALP eventually increases in both economies because the higher job-finding rate results in a higher level of human capital and match productivity (Panel (d)). While the increase is slightly larger in the first economy

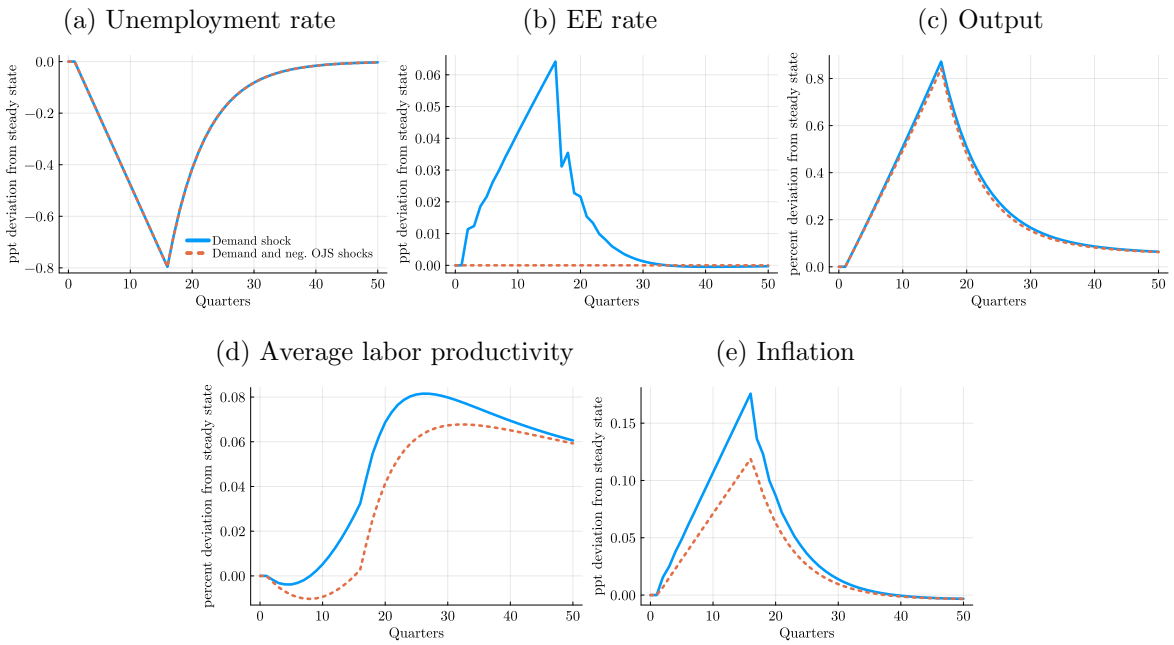
[31] Moscarini and Postel-Vinay (2022) show that discount factor shocks lead to a negative correlation between consumption and job creation in their model. This is because, in their RANK framework, firms are owned by the workers and they discount profits by the household's stochastic discount factor (SDF). When households become impatient and increase consumption, they also become impatient for firm profit streams, leading to a decline in firm value and vacancy creation. In our framework, it is not obvious whose SDF the firms should use given the rich household heterogeneity. We overcome this issue by introducing a risk-neutral mutual fund owning the firms, resulting in the firms discounting the future by the real interest rate.

[32] This is consistent with the decline in the unemployment rate attributable to an increase in the job-finding rate between 2016 and 2019. In particular, holding the separation rate fixed at its January 2016 level, the rise in the job-finding rate between 2016 and 2019 alone leads to a 15 percent decline in the unemployment rate.

[33] In Equation (30), we use the estimated (in Section 4.3) persistence parameters ρ_β and ρ_ν. For transparency, in this section, we abstract away from supply shocks.

[34] Figure B.4 (Panel (a)) plots the estimated paths of these shocks for both economies.

Figure 6: Effects of muted worker mobility on aggregate dynamics: Post-Great Recession exercise



Notes: This figure presents the dynamics of unemployment rate, EE rate, output, average labor productivity, and inflation in an economy subject to (1) only a series of positive demand shocks (solid-blue lines) and (2) series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). The shocks in the two economies are estimated to generate the same path of unemployment. The EE rate is untargeted in the first economy whereas the OJS efficiency shocks are such that the EE rate remains unchanged in the second economy, as in the data during the post-Great Recession episode.

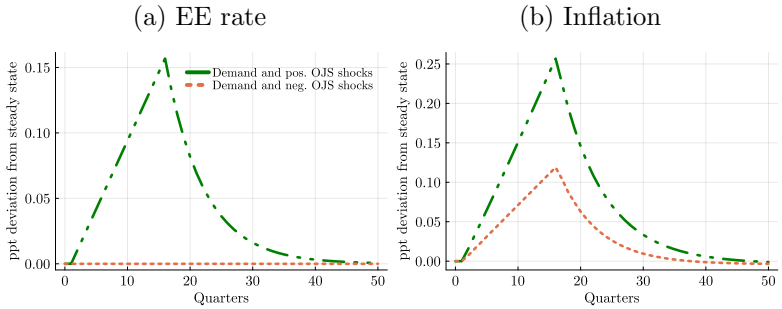
where the increase in EE transitions contributes to productivity-improving job switches, the gap is small because the match productivity distribution is a slow-moving object (Figure B.3).

If we were to solely focus on unemployment and output, and ignore job mobility, we would have inferred that the two economies were hit by very similar shocks. Looking at inflation changes this view, however. In particular, (annualized) inflation is 0.23 percentage points smaller in the economy with OJS efficiency shocks (Panel (e)).[35] This is not a small quantitative effect.[36] The 0.23 percentage points drag on inflation implies that inflation would have been around 2 percent in 2019 instead of 1.8 percent if the EE rate had increased during this recovery episode, in line

[35]To obtain this number, we calculate the annualized inflation rate in each economy and report their maximum difference, which materializes 16 quarters after the shock, when unemployment is at its lowest level.

[36]Recently, Bostanci, Koru, and Villalvazo (2022) and Pilossoph and Ryngaert (2022) document that higher inflation expectations lead to higher job-to-job transitions as workers try to escape inflation by moving to better-paying jobs. It is feasible to incorporate this channel into our framework by introducing (i) nominal wage rigidities, (ii) endogenous job search effort, and (iii) preventing rebargaining for a fraction of the external offers. This way, employed workers would be motivated to increase their search effort to increase the likelihood of an outside offer in episodes of high inflation given rigid nominal wages. In the event that the external offer is not matched, the worker would try to switch jobs. In this case, missing inflation explained by our model would be even larger because the increase in EE rate under the economy without OJS efficiency shocks would be larger. Therefore, we view our baseline estimate as a lower bound.

Figure 7: Effects of elevated worker mobility on aggregate dynamics: COVID-19 recovery exercise



Notes: This figure presents the dynamics of the EE rate and inflation in an economy subject to (1) a series of positive demand shocks and positive OJS efficiency shocks (dot-dashed-green lines) and (2) a series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). The shocks in the two economies are estimated to generate the same path of unemployment as in Figure 6 Panel (a). The EE rate in the first economy targets an increase by around 8 percent (or 0.16 ppts) as in the data during the recovery from the COVID-19 recession, while the second economy targets a constant EE rate.

with its historical negative co-movement with unemployment.[37]

5.3 Higher inflation due to strong worker mobility during 2021-2022

The recovery from the COVID-19 recession exhibited almost the same decline in the unemployment rate as in the post-Great Recession episode, but with a large increase in the EE rate. We now study the role of the high EE rate on the rise in inflation by comparing two economies that mimic the path of the unemployment rate during 2021–2022 but differ in their EE rates.

Starting from the same steady state, we simulate two different economies. The first economy features positive demand shocks and positive OJS efficiency shocks to emulate the path of unemployment and EE rates in this episode.[38] The second economy experiences positive demand shocks and negative OJS efficiency shocks such that the economy generates the same unemployment rate path as in the first economy but a flat EE rate. In other words, the second economy in this exercise is the same economy as the second economy (dashed-orange lines) represented in Figure 6.[39] The rest of the details are identical to our exercise in Section 5.2.[40]

Figure 7 presents the dynamics of the EE rate and inflation for these two economies.[41] Panel (a) shows that the first economy (dot-dashed green line) generates a 0.16 percentage points

[37]Inflation in 2019 was expected to be above 2 percent because the unemployment rate was deemed below its natural rate. Thus, the muted EE rate explains a sizable portion but not all of the missing inflation.

[38]Because the recovery of the unemployment rate in this episode is almost the same as that during the post-Great Recession episode, we also target a 15 percent (or 0.8 ppts) decline in the unemployment rate.

[39]We find that positive demand shocks alone generate a rise in the EE rate that is smaller than that was observed during the recovery from the COVID-19 recession. As Figure 6 (Panel (b)) shows, under positive demand shocks alone, the EE rate increases by 0.06 ppts, while the EE rate increased by 0.16 ppts in the data.

[40]Figure B.4 (Panel (b)) plots the estimated path of shocks for these two economies.

[41]In this section, we focus on discussing results on the EE rate and inflation for brevity as the intuitions of results are similar to those in our exercise in the previous section.

increase in the EE rate as in the data. Panel (b) shows that a higher EE rate leads to a larger increase in inflation. We find that annual inflation is 0.56 percentage points higher in the first economy. This implies that if the EE rate remained flat during the recovery following the COVID-19 recession (as it did during the post-Great Recession episode), the rise in inflation would have been 0.56 percentage points smaller.

Taking stock. Overall, our exercises in Sections 5.2 and 5.3 show that a recovery episode with an increasing EE rate—due to either the absence of negative OJS efficiency shocks (Figure 6) or presence of positive OJS efficiency shocks (Figure 7)—generates a larger increase in inflation when compared with a recovery episode with identical unemployment dynamics but a lower EE rate. This implies that the empirical Phillips curve would be flatter during a recovery with a muted EE rate—a result that we demonstrate in Figure B.5 using model simulations.

5.4 Decomposing the effect of OJS efficiency shocks on inflation

The preceding two case studies demonstrate that OJS shocks have sizable effects on inflation. We now quantify the channels through which a positive unit OJS shock translates to, on net, higher inflation by providing a decomposition of the impact response of inflation.[42] This decomposition leverages the DAG representation of the model in Figure B.1 and the system of Jacobians we compute to solve and simulate our model.

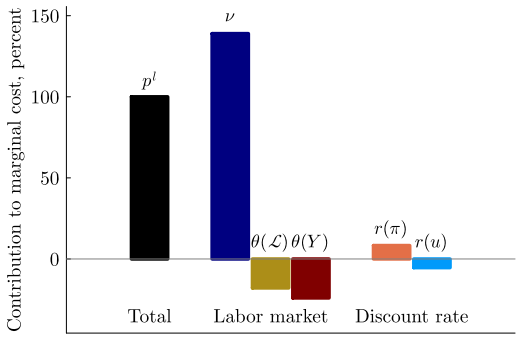
The NKPC in Equation (19) reveals that—to a first-order approximation—inflation is driven entirely by the relative price of labor services p^l, which, absent productivity shocks, alone determines the real marginal cost p^l/z for intermediate firms. Therefore, it is sufficient to provide a decomposition of shocks to OJS efficiency ν on p^l to fully understand the behavior of inflation.

The decomposition essentially applies the implicit function theorem to equilibrium conditions of choice in a particular order, to express one of the outcome variables as a (linear) function of the shocks and other endogenous variables. As discussed in Section 3.4, the free-entry condition is key in pinning down the price of labor services p^l. Hence, we can decompose equilibrium changes in p^l to contributions from various variables entering this condition. Namely, the input variables are labor market effects via OJS efficiency ν and market tightness θ, and discount rate effects via real rate r.[43] Because r is determined in the monetary policy block, taking π and u as inputs, the contribution of r to p^l can be further attributed to these variables, which we denote by $r(\pi)$ and $r(u)$. The contribution of θ too can be further dissected by recognizing that θ is pinned down in Equation (27). The variables that enter this condition and that have an effect on θ are aggregate output Y, which in turn affects the demand and supply of labor services. Therefore, the contribution of θ to p^l can be further broken down to contributions from output

[42] This exercise can be extended to the entire IRF period-by-period; for brevity we focus on the impact effect.

[43] We note that there are also composition effects via the search-stage worker distributions μ^E and μ^U. However, as we discuss below, these have relatively small quantitative effects on p^l.

Figure 8: Decomposition of channels that OJS efficiency shock affects marginal cost



Notes: This figure presents a decomposition of the overall impact increase in the marginal cost, i.e., the relative price of labor services p^l, explained by various channels in response to an increase in the OJS efficiency parameter ν—in particular, the fraction of the total change in p^l is accounted for by labor market effects and discount rate effects. ν refers to the direct effect of OJS efficiency on p^l; $\theta(\mathcal{L})$ refers to the effect of ν on market tightness θ through its effect on the total supply of labor services $\mathcal{L}$; $\theta(Y)$ denotes the effect of ν on θ through its effect on output Y; $r(\pi)$ denotes the effect of ν on real rate r through inflation π; and $r(u)$ refers to the effect of ν on real rate r through unemployment u.

$\theta(Y)$ and supply of labor services $\theta(\mathcal{L})$. Appendix B.3 provides further details, and the DAG presented in Figure B.2 illustrate these channels.

Figure 8 shows the percent contribution of each of these channels to the total increase in the relative price of labor services p^l in response to a positive unit shift in OJS efficiency ν.

Labor market effects. A shock to ν directly affects the expected match value for the labor services firm, $\mathbb{E}J$, by raising the worker's probability of receiving an outside offer. The higher frequency of such contacts raises the likelihood of wage re-bargaining or the worker quitting, both of which reduce the firm's value J.[44] Thus, an increase in ν leads to a decline in $\mathbb{E}J$.[45] All else the same, this decline in $\mathbb{E}J$ necessitates an increased price of labor services p^l for the free-entry condition to hold. Quantitatively, we find that the direct effect of ν on p^l, labeled as ν in Figure 8, explains 139 percent of the total (100 percent) increase in p^l upon impact.

There are further general equilibrium (GE) effects in addition to this direct effect. In the labor market, an increase in ν leads to a decline in tightness θ (Figure B.6 Panel (b)). How does this decline in θ affect p^l? According to the DAG in Figure B.2, θ enters the free-entry condition through its effects on the service firm.[46] For an unmatched service firm, a lower θ increases the

[44]Here, we note that the decline in the firm's value is especially driven by a shorter expected match duration, as we also showed in Section 2.4, and that wage re-bargaining has a relatively smal effect on $\mathbb{E}J$.

[45]A higher ν implies a higher weight for employed job-searchers in the aggregate measure of job searchers S, which has a distinct effect on $\mathbb{E}J$. We find this to be small due to offsetting forces. On the one hand, employed workers are typically more productive as they have higher skills than the unemployed. On the other hand, they are less likely to accept offers and more likely to dictate higher wages than new hires from unemployment.

[46]Market tightness θ also affects the distribution of employed workers over time in the heterogeneous agent (HA) block. However, this change does not affect the distribution of the employed at the search stage μ^E in the first period of a positive ν shock. Therefore, it has no effect on p^l upon impact.

probability of filling the vacancy $q(\theta)$. In addition, a lower θ reduces the worker's probability of contacting other firms in the future, implying less-frequent wage re-bargaining and longer match durations. Thus, a matched firm's value J and hence $\mathbb{E}J$ increase. All else constant, this higher expected match value requires a decline in p^l for the free-entry condition to hold. This GE effect of ν on p^l through θ overall accounts for -42 percent of the increase in p^l.

Because the GE effect through θ is large and of the opposite sign to the direct effect, we use the market-clearing condition for labor services in Equation (27) to further decompose the response of θ. From the DAG in Figure B.1, the direct effect of ν on θ is through the aggregate supply of labor services in the HA block, $\mathcal{L} = \int F(h,x)\, d\lambda_t^E(s,h,x,\alpha)$. All else the same, a higher ν implies a more-productive match distribution and, therefore, an increased supply of labor services. For the labor services market to clear, the productivity gains that raise $\mathcal{L}$ should be counteracted by a decline in θ. This effect of ν on θ through the supply of labor services, $\theta(\mathcal{L})$, explains -18 percent of the total increase in p^l.

In addition to $\mathcal{L}$, ν has a separate effect on θ through output Y, which declines in GE (Figure B.6 Panel (c)). The decline in output is driven by a lower aggregate demand (Panel (d)), which itself is a result of a higher unemployment rate (Panel (e)), a lower job finding rate, and a higher real rate. This lower output implies less demand for labor services $L = Y/z$. All else the same, a commensurate decline in θ is required for the labor services market to clear. This effect of ν on θ through output, $\theta(Y)$, accounts for -24 percent of the total increase in p^l. We conclude that the GE effects of ν on θ through $\mathcal{L}$ and Y mitigate a much larger direct effect of ν on p^l.

To summarize the labor market effects, a higher OJS efficiency ν implies lower expected match values as firms face more-frequent wage re-bargaining and shorter match durations. This direct effect entails a compensatory increase in p^l to maintain the free-entry condition. However, a higher ν increases supply of labor services and lowers aggregate demand, both of which require a decrease in labor market tightness θ to clear the market for labor services. This lower tightness translates to higher expected match values since firms fill vacancies faster and are less susceptible to quits. To satisfy the free-entry condition, this necessitates a decline in p^l to reduce firm entry, partially mitigating the rise in p^l due to the direct effect.

Discount rate effects. We now turn to the GE effects of ν on p^l through the real interest rate, which we label as the discount rate effects. These effects arise because of monetary authority's reaction to inflation and unemployment. In response to an increase in ν, the unemployment rate and inflation both increase (Figure B.6 Panels (e) and (f)). A higher inflation induces a more than one-for-one increase in the nominal rate i as we assume $\Phi_\pi > 1$ and therefore an increase in the real rate. This higher real rate reduces the valuation of service firms (see the third term on the right-hand side of Equation (20)), which in turn puts downward pressure on expected match value $\mathbb{E}J$. All else constant, this requires an increase in p^l for the free-entry condition to hold. Quantitatively, we find that the inflation channel $r(\pi)$ accounts for 8 percent of the total

increase in p^l. This is a much smaller effect compared to the labor market effects.

A similar reasoning implies that the increase in unemployment induces a decline in the real rate. This effect is also small; $r(u)$ explains -5 percent of the total increase in p^l.

Taking stock. While an increase in OJS efficiency ν increases p^l through its direct effect, GE forces through market tightness θ partially mitigate this increase. Overall, the labor market effects account for 97 percent of the total increase in p^l. The remaining 3 percent is accounted for by changes in the real rate due to the GE effects of ν on inflation and unemployment.

6 Monetary policy with labor market dynamics

Thus far, we have established that shifts in EE transitions relative to the unemployment rate can alter the relationship between unemployment and inflation as well as other macroeconomic outcomes. A natural question is whether our positive findings have normative implications. Specifically, does explicitly accounting for EE transitions as an additional proxy of economic slack matter for the conduct of monetary policy?

We now study the implications of considering job mobility dynamics when setting monetary policy. Specifically, we solve for the optimal monetary policy within a generalized Taylor rule—where the nominal interest rate reacts to inflation, unemployment, and the EE rate—under a dual-mandate central bank loss function. We then compare aggregate and worker level outcomes under both the optimal and baseline monetary policy, where the latter ignores EE dynamics and responds only to inflation and unemployment. This allows us to uncover the welfare effects of accounting for job mobility dynamics in monetary policy design. In all of these exercises, we assume the economy is subject to the three aggregate shocks estimated in Section 4.3.

Central bank loss function. We start by positing that the central bank sets monetary policy to minimize the following loss function:

$$\mathcal{W} = \text{var}(\pi_t - \pi^*) + \Psi \text{var}(Y_t - Y^*), \tag{31}$$

which penalizes the variance of quarterly inflation and output gaps. There are two motivations behind our objective function choice. First, our solution method relies on a first-order approximation, implying that it is not conducive to capturing the second-order effects of inflation on consumption and hence welfare. Therefore, we simply assume an objective function that explicitly involves inflation volatility. Second, this loss function approximates the dual mandate of the Federal Reserve Bank and is commonly used in the literature.

We choose the relative weight of the output gap as $\Psi = 0.25$, which is a conventional value in the literature (see, for example, Jensen 2002 and Walsh 2003).[47]

[47] According to Okun's law, changes in the output gap imply half the change in the unemployment gap. Using the relationship $u_t - u_t^* = \frac{Y_t - Y_t^*}{2}$, the loss function in Equation (31) reduces to $\mathcal{W} = \text{var}(\pi_t - \pi^*) + \text{var}(u_t - u^*)$.

Central bank reaction function. We study monetary policy within a class of Taylor-rule type monetary policy reaction functions. Specifically, we consider rules of the following form:

$$i_t = i^* + \Phi_\pi (\pi_t - \pi^*) + \Phi_u (u_t - u^*) + \Phi_{EE} (EE_t - EE^*), \tag{32}$$

where Φ_{EE} governs the responsiveness of the central bank to the EE rate. This is a generalized version of the baseline reaction function in Equation (5). We emphasize that this is a *feasible* policy, as the EE rate can be measured at the same frequency and using the same dataset as the unemployment rate, namely the CPS.

Optimal policy and its macroeconomic implications. The optimal monetary policy consists of a combination of coefficients of the generalized Taylor rule (32) that minimizes the loss function in Equation (31).[48] As our focus is on the response of monetary policy to the labor market, we keep the coefficient on inflation at its baseline value of $\Phi_\pi = 1.5$. We find that optimal policy prescribes $\Phi_u^* = -3.18$ and $\Phi_{EE}^* = 2.22$, implying a much more aggressive response to the unemployment gap than the calibrated value of $\Phi_u = -0.25$ (and $\Phi_{EE} = 0$), which is also oft-used in the literature. This strong response to the unemployment gap is then balanced by a large positive coefficient on the EE gap. In this case, the central bank loss shrinks by 78.7 percent relative to the baseline Taylor rule.

What do these coefficient values mean in practice? The optimal policy implies that the central bank should separate recovery episodes where job mobility is high from those where job mobility is low. For instance, despite the fact that the magnitude of the decline in the unemployment rate was similar between the last two recovery episodes, the rise in the nominal rate should have been more aggressive between 2021 and 2022 when job mobility was much higher.

What are the welfare consequences of ignoring EE dynamics in the Taylor rule? When we set $\Phi_{EE} = 0$ in Equation (32) and optimize over only the unemployment gap coefficient, we find $\Phi_u = -2.71$ (keeping $\Phi_{EE} = 0$). However, relative to the optimal monetary policy that also responds to job mobility dynamics ($\Phi_u^* = -3.18$ and $\Phi_{EE}^* = 2.22$), this policy yields 12 percent larger central bank loss. We conclude that explicitly accounting for worker transitions matters for the conduct of monetary policy.

Next, we compare the volatility of macroeconomic outcomes under the baseline and optimal monetary policies to understand how the optimal policy changes the extent of fluctuations in aggregate variables. Because the optimal policy minimizes fluctuations in inflation and output gaps, their volatilities are unsurprisingly smaller under the optimal policy, as shown in Table 5. As discussed above, the optimal policy features coefficients on inflation and unemployment gaps that are significantly larger in magnitude than those under the baseline policy. As a result, a

An equal weight on inflation and output is also consistent with descriptions of how the Federal Reserve trades off inflation and unemployment (Yellen, 2012; Debortoli et al., 2018).

[48] Appendix B.4 provides computational details on evaluating the objective function under alternative rules.

Table 5: Volatility of macroeconomic outcomes under baseline and optimal monetary policies

	π	Y	r	θ	u	C	p^l	p^s
Baseline Taylor rule	0.0013	0.0059	0.0019	0.0600	0.0047	0.0059	0.0203	0.1975
Optimal Taylor rule	0.0011	0.0020	0.0033	0.0175	0.0013	0.0020	0.0081	0.3051

Notes: This table presents standard deviations of variables under the baseline and optimal monetary policies. For each variable ω, we report the standard deviation of $\omega - \overline{\omega}$, where $\overline{\omega}$ denotes a steady-state value.

more-aggressive monetary policy response under the optimal policy leads to larger fluctuations in nominal and real rates. At the same time, this stronger response achieves lower fluctuations in labor market tightness, unemployment, consumption, and the real marginal cost p^l, but leads to larger fluctuations in real price of shares p^s.

Heterogeneous welfare effects of the optimal policy. We turn to studying consumption-equivalent welfare gains from the optimal policy according to the dual-mandate loss function in the aggregate and across heterogeneous groups.[49]

In the aggregate, we find that the optimal policy yields 0.16 percent additional lifetime consumption relative to the baseline policy. Importantly, the optimal policy generates heterogeneous welfare gains across subpopulations, as shown in Table 6. When we group individuals based on their position in the match quality distribution, we find that those in the bottom quintile have welfare gains of 0.24 percent and that those in the top quintile have welfare gains of 0.16 percent, while those who are in the middle (second, third, and fourth quintiles) have welfare gains of 0.13 percent. The optimal policy yields a smaller decline and faster recovery of the EE rate during economic fluctuations, relative to what would be experienced under the baseline policy. Thus, individuals at the bottom gain the most from the optimal policy because they benefit the most from climbing up the job ladder. Individuals at the top also experience substantial gains because their valuation of a smooth job ladder is high given that they have the most to lose in the case of a job loss. In terms of heterogeneous gains across the share distribution, smaller fluctuations in unemployment achieved by the optimal policy leads to larger welfare gains among wealth-poor individuals for whom unemployment risk is most costly. Larger fluctuations in the price of shares caused by the more-aggressive monetary policy response under the optimal policy is the main reason behind the smaller welfare gains in the top quintile of the share distribution. Finally, the unemployed experience larger welfare gains than the employed given that the former group benefits not only from a smoother job ladder while employed but also from speedier labor market recoveries that hasten job-finding and mitigate the productivity losses from unemployment.

[49] Appendix B.5 provides details on measuring these consumption-equivalent welfare changes.

Table 6: Heterogeneous welfare effects of optimal monetary policy

Match quality x			Share s			Human capital h			Employment e	
Bottom	Middle	Top	Bottom	Middle	Top	Bottom	Middle	Top	E	U
0.24	0.13	0.16	0.23	0.13	0.11	0.13	0.18	0.10	0.15	0.20

Notes: This table presents percent additional lifetime consumption gains from the optimal monetary policy relative to the baseline policy across different groups. We divide individuals based on their position in the distributions of match quality x, share holdings s, and human capital h, as well as their employment status e (employed E and unemployed U). Bottom and top refer to bottom and top quintiles of respective distributions and middle refers to second, third, and fourth quintiles of these distributions.

7 Conclusions

This paper quantitatively demonstrates that fluctuations in the EE rate have important macroeconomic and policy-relevant implications. On the positive side, we show that muted worker mobility during the recovery from the Great Recession caused around 0.25 percentage points lower inflation, while elevated worker mobility during the recovery from the COVID-19 recession generated around 0.60 percentage points additional inflation. Our analysis provides a full-decomposition of various channels through which fluctuations in the EE rate affect inflation in our model despite its complexity. On the normative side, we uncover that the optimal monetary policy that takes into account EE fluctuations prescribes a much more aggressive nominal rate response to unemployment than what is prescribed by the commonly used unemployment gap coefficient in the literature, and a strong positive response to the EE gap. This policy implies that the central bank should behave differently in recoveries when the EE rate increases than it does when the EE rate remains stable, even if the unemployment dynamics are identical.

Our model features a rich set of fiscal policy instruments, such as a consumption tax, progressive labor income tax, unemployment and retirement benefits, and government debt. Therefore, it provides a framework to quantitatively study fiscal and monetary policy interactions, accounting for rich labor market dynamics. In addition, it is straightforward to introduce other exogenous shocks (e.g., to monetary policy, markups, and other labor-market-related parameters) into our model. Given our solution method, it is feasible to estimate a richer set of such shocks jointly to evaluate the model's performance in matching time-series and cross-sectional empirical moments. We leave these considerations for future research.

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Online Appendix

A TANK model

This appendix provides derivations of some of the equations in the TANK model presented in Section 2, discusses its solution, and presents additional results.

A.1 Derivation of the consumption rule of PIH households

Here, we derive the consumption rule of PIH households. The intertemporal budget constraint (IBC) for PIH households from time t onward is:

$$\begin{aligned}
a_{t-1} &= \frac{1}{(1+r_t)}\Big(c_t^{PIH} - Z_t\Big) + \frac{1}{(1+r_t)}a_t \\
&= \frac{1}{(1+r_t)}\Big(c_t^{PIH} - Z_t\Big) + \frac{1}{(1+r_t)}\frac{1}{(1+r_{t+1})}\Big(c_{t+1}^{PIH} - Z_{t+1}\Big) + \frac{1}{(1+r_t)}\frac{1}{(1+r_{t+1})}a_{t+1} \\
&= \frac{1}{(1+r_t)}\Big(c_t^{PIH} - Z_t\Big) + \frac{1}{(1+r_t)}\frac{1}{(1+r_{t+1})}\Big(c_{t+1}^{PIH} - Z_{t+1}\Big) \\
&+ \frac{1}{(1+r_t)}\frac{1}{(1+r_{t+1})}\frac{1}{(1+r_{t+2})}\Big(c_{t+2}^{PIH} - Z_{t+2}\Big) + \frac{1}{(1+r_t)}\frac{1}{(1+r_{t+1})}\frac{1}{(1+r_{t+2})}a_{t+2} \\
&\vdots \\
a_{t-1} &= \frac{1}{(1+r_t)}\sum_{s=0}^{\infty} q_{t+s}(c_{t+s}^{PIH} - Z_{t+s}),
\end{aligned}$$

where we define $q_{t+s} = \frac{1}{1+r_{t+1}} \cdots \times \frac{1}{1+r_{t+s}}$ with $q_t = 1$ and impose a no-Ponzi condition $\lim_{s\to\infty} q_{t+s}a_{t+s} = 0$. We can equivalently express the IBC as follows:

$$\sum_{s=0}^{\infty} q_{t+s}c_{t+s}^{PIH} = (1+r_t)a_{t-1} + \sum_{s=0}^{\infty} q_{t+s}Z_{t+s}.$$

The Euler equation (assuming CRRA utility) is given by $(c_t^{PIH})^{-\sigma} = \beta(1+r_{t+1})(c_{t+1}^{PIH})^{-\sigma}$. We can express the Euler equation to tie current period to s-periods ahead consumption as:

$$\begin{aligned}
c_t^{PIH} &= \beta^{-1/\sigma}(1+r_{t+1})^{-1/\sigma}c_{t+1}^{PIH} \\
&= \beta^{-1/\sigma}(1+r_{t+1})^{-1/\sigma}\beta^{-1/\sigma}(1+r_{t+2})^{-1/\sigma}c_{t+2}^{PIH} \\
&= \beta^{-2/\sigma}(1+r_{t+1})^{-1/\sigma}(1+r_{t+2})^{-1/\sigma}c_{t+2}^{PIH} \\
&= \beta^{-2/\sigma}\left[(1+r_{t+1})(1+r_{t+2})\right]^{-1/\sigma}c_{t+2}^{PIH} \\
&\vdots \\
c_t^{PIH} &= \beta^{-s/\sigma}q_{t+s}^{1/\sigma}c_{t+s}^{PIH} \\
\Rightarrow c_{t+s}^{PIH} &= \beta^{s/\sigma}q_{t+s}^{-1/\sigma}c_t^{PIH}.
\end{aligned}$$

For $t = 0$, we have the following:

$$c_s^{PIH} = \beta^{s/\sigma} q_s^{-1/\sigma} c_0^{PIH}.$$

Plugging this expression into the IBC, we obtain the consumption decision rule of PIH households given by Equation (1) in the main text:

$$\begin{aligned}
\sum_{s=0}^{\infty} q_{t+s} c_{t+s}^{PIH} &= (1+r_t)a_{t-1} + \sum_{s=0}^{\infty} q_{t+s} Z_{t+s} \\
\sum_{s=0}^{\infty} q_{t+s} \beta^{s/\sigma} q_{t+s}^{-1/\sigma} c_t^{PIH} &= (1+r_t)a_{t-1} + \sum_{s=0}^{\infty} q_{t+s} Z_{t+s} \\
c_t^{PIH} &= \frac{1}{\sum_{s=0}^{\infty} \beta^{s/\sigma} q_{t+s}^{1-1/\sigma}} \Big((1+r_t)a_{t-1} + \sum_{s=0}^{\infty} q_{t+s} Z_{t+s} \Big).
\end{aligned}$$

This provides a closed-form solution for consumption, given the path of interest rate and income.

A.2 Solving the intermediate firm's problem

The problem of the intermediate firm can be solved analytically to obtain the NKPC. The pricing problem of an intermediate firm j with last period *relative* price $p_{t-1}(j)$ is given by

$$\Theta\left(p_{t-1}\left(j\right)\right) = \max_{p_t(j)} p_t\left(j\right) y_t\left(p_t\left(j\right)\right) - p_t^l y_t\left(p_t\left(j\right)\right) - \frac{\eta}{2\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right)^2 Y_t + \frac{1}{1+r_{t+1}} \Theta\left(p_t\left(j\right)\right).$$

Substituting in the CES demand for each variety, $y_t(j) = p_t(j)^{-\eta} Y_t$, the problem becomes

$$\Theta\left(p_{t-1}\left(j\right)\right) = \max_{p_t(j)} p_t\left(j\right)^{1-\eta} Y_t - p_t^l p_t\left(j\right)^{-\eta} Y_t - \frac{\eta}{2\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right)^2 Y_t + \frac{1}{1+r_{t+1}} \Theta\left(p_t\left(j\right)\right).$$

The first-order condition with respect to relative price $p_t(j)$ is given by

$$\begin{aligned}
0 = {} & \left(1-\eta\right) p_t\left(j\right)^{-\eta} Y_t + \eta p_t^l p_t\left(j\right)^{-\eta-1} Y_t \\
& - \frac{\eta}{\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right) \frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right) - \pi^*} \frac{1+\pi_t}{p_{t-1}\left(j\right)} Y_t + \frac{1}{1+r_{t+1}} \Theta'\left(p_t\left(j\right)\right),
\end{aligned}$$

and the envelope condition is

$$\Theta'\left(p_{t-1}\left(j\right)\right) = \frac{\eta}{\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right) \frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right) - \pi^*} \frac{p_t\left(j\right)\left(1+\pi_t\right)}{p_{t-1}\left(j\right)^2} Y_t.$$

Iterating the envelope condition forward by one period yields

$$\Theta'\left(p_t\left(j\right)\right)=\frac{\eta}{\vartheta}\log\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}\left(1+\pi_{t+1}\right)-\pi^*\right)\frac{1}{\frac{p_{t+1}(j)}{p_t(j)}\left(1+\pi_{t+1}\right)-\pi^*}\frac{p_{t+1}\left(j\right)\left(1+\pi_{t+1}\right)}{p_t\left(j\right)^2}Y_{t+1}.$$

Consolidating the envelope and the first-order conditions, we obtain:

$$0=\left(1-\eta\right)p_t\left(j\right)^{-\eta}Y_t+\eta p_t^l p_t\left(j\right)^{-\eta-1}Y_t-\frac{\eta}{\vartheta}\log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right)-\pi^*\right)\frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right)-\pi^*}\frac{1+\pi_t}{p_{t-1}\left(j\right)}Y_t$$
$$+\frac{1}{1+r_{t+1}}\underbrace{\frac{\eta}{\vartheta}\log\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}\left(1+\pi_{t+1}\right)-\pi^*\right)\frac{1}{\frac{p_{t+1}(j)}{p_t(j)}\left(1+\pi_{t+1}\right)-\pi^*}\frac{p_{t+1}\left(j\right)\left(1+\pi_{t+1}\right)}{p_t\left(j\right)^2}Y_{t+1}}_{\Theta'(p_t(j))}.$$

All firms set the same price due to symmetry, $p_t(j)=1\ \forall\, t,\, j$. Thus, the equation simplifies to

$$0=\left(1-\eta\right)Y_t+\eta p_t^l Y_t-\frac{\eta}{\vartheta}\frac{\log\left(1+\pi_t-\pi^*\right)\left(1+\pi_t\right)}{1+\pi_t-\pi^*}Y_t+\frac{1}{1+r_{t+1}}\frac{\eta}{\vartheta}\frac{\log\left(1+\pi_{t+1}-\pi^*\right)\left(1+\pi_{t+1}\right)}{1+\pi_{t+1}-\pi^*}Y_{t+1}.$$

Rearranging terms and using the definition of π_t, we obtain the NKPC in Equation (8):

$$\frac{\log\left(1+\pi_t-\pi^*\right)\left(1+\pi_t\right)}{1+\pi_t-\pi^*}=\vartheta\left(p_t^l-\frac{\eta-1}{\eta}\right)+\frac{1}{1+r_{t+1}}\frac{\log\left(1+\pi_{t+1}-\pi^*\right)\left(1+\pi_{t+1}\right)}{1+\pi_{t+1}-\pi^*}\frac{Y_{t+1}}{Y_t}.$$

A.3 Solving for a steady state

Nominal frictions are not relevant in the steady state, where prices rise by the rate of long-run inflation π^*; hence, intermediate firms do not incur price adjustment costs and the price level is indeterminate. Therefore, we solve for relative prices and allocations.

- Evaluating the NKPC in Equation (8) at the steady state yields the real marginal cost:

$$mc=p^l=\frac{\eta-1}{\eta}. \tag{A.1}$$

- Using Equation (9), per-period real profits of the intermediate firms are given by

$$\Gamma^I=(1-p^l)Y. \tag{A.2}$$

- Evaluating the law of motion for unemployment in Equation (4) at the steady state and solving for the worker's contact rate per unit of search efficiency $f\left(\theta\right)$, we obtain:

$$f\left(\theta\right)=\delta\left(1-u\right)/\left[u+\delta\left(1-u\right)\right]. \tag{A.3}$$

- Let $\widehat{u}$ and $\widehat{e}$ denote the masses of unemployed and employed job searchers at the search stage, respectively. They are given by:

$$\begin{aligned}\widehat{u} &= u + \delta\left(1-u\right),\\ \widehat{e} &= \left(1-\delta\right)\left(1-u\right).\end{aligned} \tag{A.4}$$

  We then back out the real value of a matched service firm J from Equation (12) imposing the free-entry condition $V = 0$ as follows:

$$J = \kappa/\left[q\left(\theta\right)\left(\widehat{u}/\left(\widehat{u}+\nu\widehat{e}\right)\right)\right]. \tag{A.5}$$

- Finally, solving for the relative price of labor services p^l in Equation (10) yields:

$$p^l = J\left[1-(1/(1+r))(1-\delta)(1-\nu f(\theta))\right]/(1-\alpha). \tag{A.6}$$

Next, we provide the algorithm to solve for the steady state of the TANK model.

1. Guess output Y and real interest rate r.
2. Solve for labor market outcomes:
   - Given Y, calculate the mass of unemployed at the steady state using the equilibrium condition $u = 1 - Y$.
   - Given u, use Equation (A.3) to obtain the worker's contact rate per unit of search efficiency $f\left(\theta\right)$. Given $f\left(\theta\right)$, calculate market tightness θ by inverting the CES matching function $\theta = \left(f^{\xi}/(1-f^{\xi})\right)^{1/\xi}$. Given $f\left(\theta\right)$ and θ, calculate the firm's contact rate $q\left(\theta\right) = f\left(\theta\right)/\theta$.
   - Given $f\left(\theta\right)$, $q\left(\theta\right)$, and definitions of $\widehat{u}$ and $\widehat{e}$ in Equation (A.4), calculate the value of a matched firm J using Equation (A.5). Then, use Equation (A.6) to obtain the relative price of labor services p^l.
   - Compute stationary worker distributions $e(\alpha)$ and $e(1)$ using Equation (4).
3. Solve for aggregate demand:
   - Given worker distributions, obtain aggregate real labor income W using Equation (2), aggregate real service firm profits Γ^S using Equation (11), and aggregate real intermediate firm profits using Equation (A.2).
   - Given aggregate profits $\Gamma = \Gamma^S + \Gamma^I$ and labor income, obtain the total income of PIH households $Z = W + \Gamma/(1-\mu)$.

- Given W and Z, solve for steady-state demand of the PIH household c^{PIH} using Equation (1) and of the HtM household using $c^{HtM} = W$. Then, obtain the aggregate demand using $C = (1-\mu)c^{PIH} + \mu c^{HtM}$.

4. Check whether goods market clearing $Y = C$ and the Phillips curve $p^l = (\eta - 1)/\eta$ hold. If not, return to Step 1 and update the guess of output Y and real rate r. Iterate until both of these equations hold. In practice, we use a non-linear solver over Y and r.

A.4 Solving for the transition path using DAG

We employ the SSJ method of Auclert, Bardóczy, Rognlie, and Straub (2021) to solve for the dynamics in our TANK model. In the following discussion, we broadly define the construction of the DAG for this model.

To solve our TANK model using the SSJ method, we first cast the model as a DAG, depicted in Figure A.1.[50] The leftmost red node contains exogenous variables that represent shocks the economy might be subject to, as well as endogenous variables (unknowns) whose dynamics we are interested in. The intermediate (green) nodes represent various model components and, importantly, demonstrate how each component relates to one another via their respective input and output variables. In this stylized TANK model—different from the HANK model—all of the intermediate nodes are "simple" blocks that relate various aggregate variables whose Jacobians can be computed using automatic differentiation. Finally, the rightmost red node represents the targets must equal zero in equilibrium (goods market clearing and the Phillips curve equation).[51] This final node might take inputs directly from the initial node with exogenous and endogenous variables, as well as outputs from the intermediate nodes.

As this model does not feature a heterogeneous-agent block, whose presence would have complicated the solution and for whom most of the value-added of the SSJ method is relevant to, we defer a detailed discussion of how we solve model dynamics until Section B.2.

A.5 Additional results

We present additional results from the TANK model that supplement the main results presented in Section 2. Figure A.2 plots the dynamics of total real profits ($\Gamma = \Gamma^S + \Gamma^I$), real service firm profits Γ^S, real intermediate firm profits Γ^I, and the unemployment rate following a positive OJS efficiency shock, as described in Section 2.4 under both RANK and TANK models under a constant real rate r. As we discussed in the main text, the rise in OJS efficiency leads to a decline in total real profits Γ in both models. Here, Panels (a) to (c) in Figure A.2 show

[50] For visual clarity, we consolidate the terminal "target" blocks that capture the two equilibrium conditions into a single node. One should think of the last node as consisting of two different ones representing each of the equilibrium conditions separately, with inputs from the relevant intermediate blocks.

[51] The number of unknown variables specified in the leftmost node must be equal to the number of target conditions in the rightmost node.

Figure A.1: DAG representation of the TANK model with a frictional labor market and on-the-job search

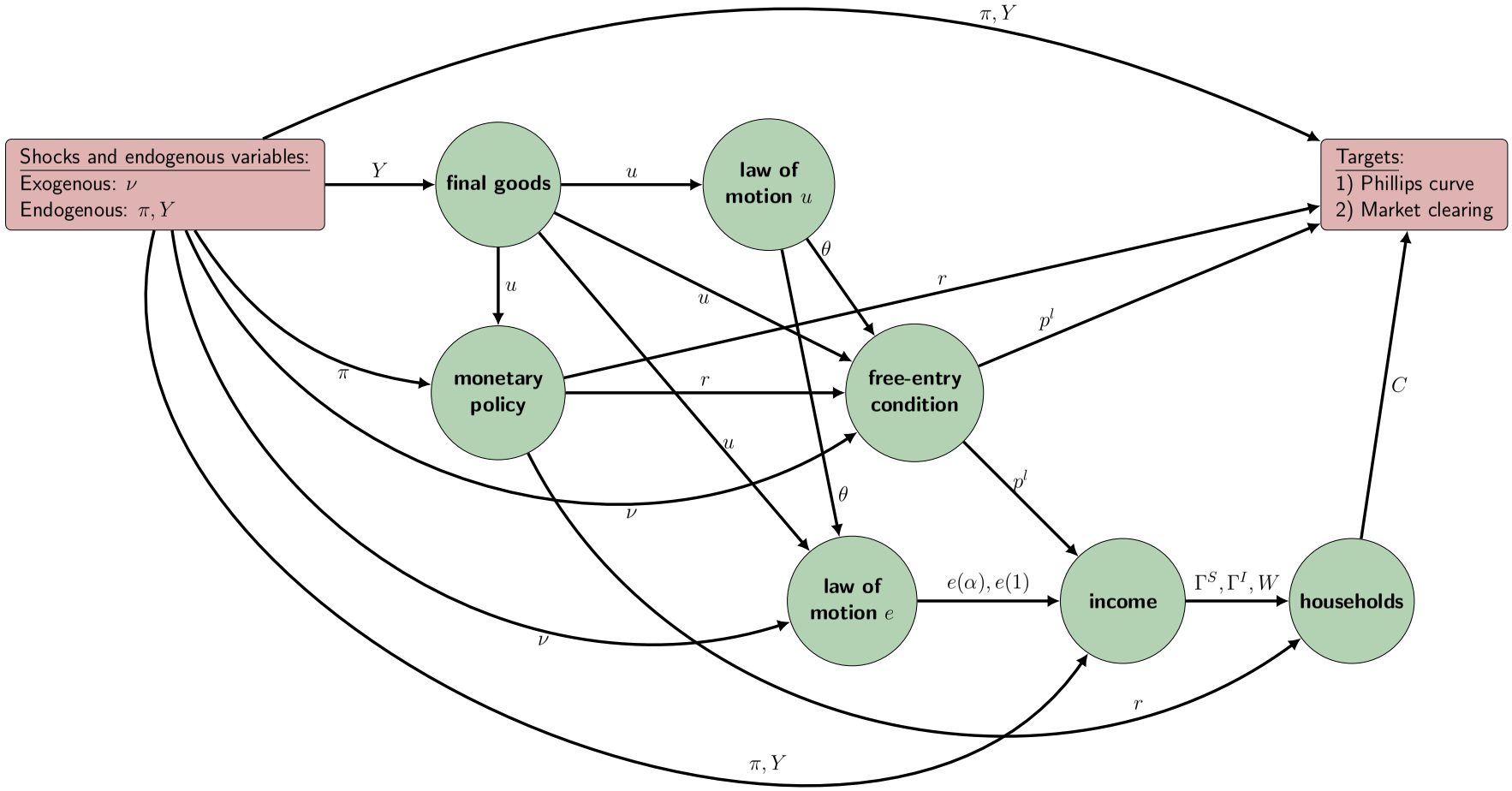
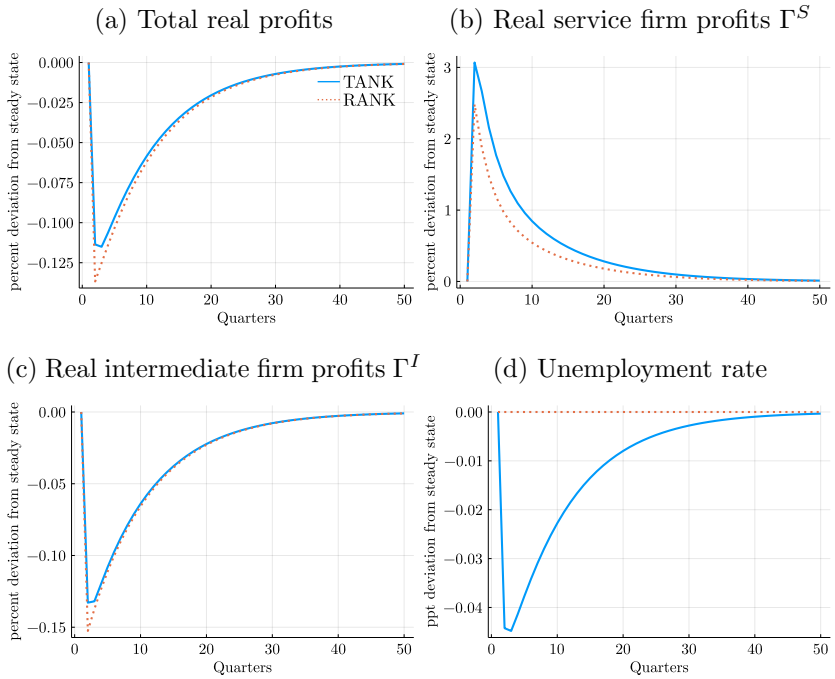


Figure A.2: Impulse responses to a positive OJS efficiency shock: Constant real rate

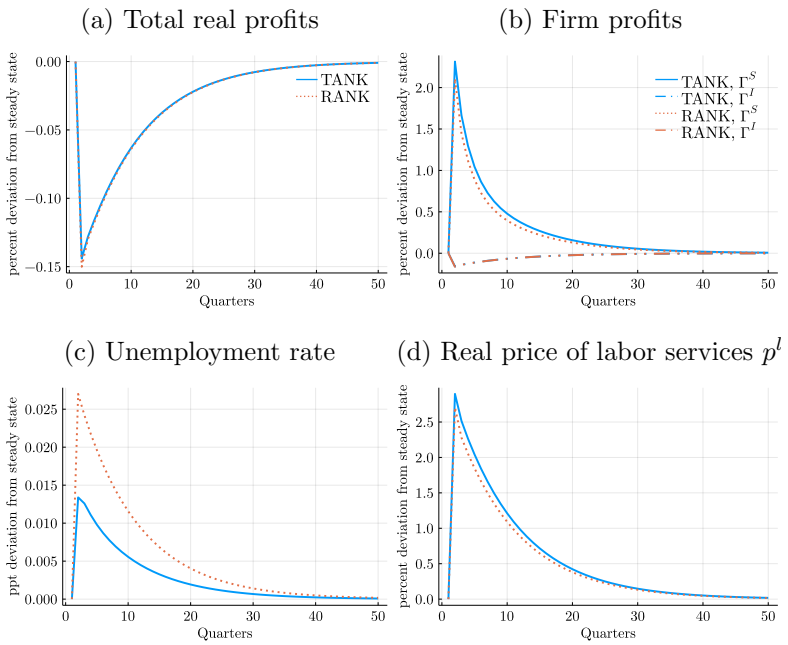


Notes: This figure presents the impulse responses of total real profits ($\Gamma = \Gamma^S + \Gamma^I$), real service firm profits Γ^S, real intermediate firm profits Γ^I, and unemployment rate in an economy subject to a positive OJS efficiency shock. Blue lines plot responses from the TANK model and red lines plot responses from the RANK model. In this analysis, we keep the real interest rate r constant.

that the decline in total profits is driven by the decline in the profits of intermediate firms, while the profits of service firms increase in both models. Because the OJS shock does not have any real effects in the RANK model under a constant real rate—it merely redistributes income from workers to firms, which were distributed back to workers as dividends—the unemployment rate does not change. However, the increase in demand following the shock in the TANK model leads to a rise in vacancy creation and a decline in the unemployment rate, as shown in Panel (d).

Figure A.3 presents model outcomes following the same shock, except the monetary authority reacts to the shock by following the baseline Taylor rule; i.e., the real interest rate is free to adjust according to the Taylor rule. Different from Figure A.2 under a constant real rate, the unemployment rate now increases in both models. This is driven by a decline in demand and vacancy creation. The smaller increase in the unemployment rate in the TANK model relative to RANK is explained by the smaller decline in demand and market tightness in the former. This is due to the HtM households in the TANK model mitigating fluctuations in demand and tightness as the demand of HtM households is less elastic to the real rate than that of PIH households.

Figure A.3: Impulse responses to a positive OJS efficiency shock: Real rate response



Notes: This figure presents the impulse responses of total real profits ($\Gamma = \Gamma^S + \Gamma^I$), real service firm profits Γ^S, real intermediate firm profits Γ^I, unemployment rate, and real price of labor services p^l in an economy subject to a positive OJS efficiency shock. Blue lines plot responses from the TANK model and red lines plot responses from the RANK model.

B HANK model

This appendix discusses the solution of the steady state and transition dynamics in the HANK model, provides details on the normative exercise, and presents additional results.

B.1 Solving for the steady state

B.1.1 Laws of motion for the worker distribution

We denote by λ_t the distribution of agents across individual states (i.e., share holdings s, human capital h, match productivity x, and piece rate α) at time t. As the population is normalized to one and the dead are replenished with unemployed workers, we have

$$\sum_{s,h,x,\alpha} \lambda_t^E(s,h,x,\alpha) + \sum_{s,h} \lambda_t^U(s,h) + \sum_{s} \lambda_t^R(s) = 1,$$

where $\lambda_t^E(\cdot)$, $\lambda_t^U(\cdot)$ and $\lambda_t^R(\cdot)$ denote the mass of employed, unemployed and retired workers by individual state variables, respectively, and we omit states that are not relevant for the agents. Also for reference below, let $\boldsymbol{S}_t^E(s';h,x,\alpha) = \left\{s \in \mathbf{S} :\ g_t^{Es}(s,h,x,\alpha) = s'\right\}$, $\boldsymbol{S}_t^U(s';h) = \left\{s \in \mathbf{S} :\ g_t^{Us}(s,h) = s'\right\}$, and $\boldsymbol{S}_t^R(s') = \left\{s \in \mathbf{S} :\ g_t^{Rs}(s) = s'\right\}$ denote the set of period t share

holdings s that map into a given level of share holdings s' in $t+1$ by employment status.

We now turn to explicitly writing down the system of equations that determine worker flows. To reduce notational clutter, we define $f_t = f(\theta_t)$ and suppress some of the function arguments.

Flows into employment. Conditional on not retiring, flows into employment include the following mutually exclusive events:

- Employed worker stays with the same employer; skill appreciates or does not appreciate.
  - The worker's piece rate can either (i) remain the same ($\alpha' = \alpha$) because either no meeting occurs or an offer is not met with a counteroffer or (ii) rise because of re-bargaining induced by an external offer. Considering inflows into a specific match productivity x' and piece rate α', it must be that the poaching firm's match productivity is $\widetilde{x} = x'\alpha'$ in the latter case. Further, it must be that the poaching firm's match productivity is higher than the current output share: $x\alpha < x'\alpha' = \widetilde{x}$.
- Employed worker accepts a new offer; skill appreciates or does not appreciate.
  - The worker's piece rate changes due to a job-to-job transition. Considering inflows into a specific match productivity x' and piece rate α', it must be that $\alpha' = \frac{x}{x'}$, where x is the productivity of the previous match. This implies that the previous match productivity must have been $x = \alpha' x'$.
- Employed worker loses job but finds a new job within the period; skill appreciates or does not appreciate.
  - Considering inflows into specific match productivity x' and piece rate α' from unemployment, it must be that $\alpha' = \frac{x}{x'}$. Here, it does not matter what the previous job's x or α was.
- Unemployed worker accepts a new offer; skill depreciates or does not depreciate.
  - The evolution of piece rate is similar to above.

We then have the following law of motion for the distribution of employed workers:

$$
\begin{aligned}
&\lambda_{t+1}^{E}\left(s',h',x',\alpha'\right)=\left(1-\psi^{R}\right)\times\\
&\Bigg(\sum_{s\in\boldsymbol{S}_{t}^{E}}\underbrace{\lambda_{t}^{E}\left(s,h'-1,x',\alpha'\right)}_{\substack{\text{no outside offer/discard offer;}\\ \alpha\text{ remains the same}}}\quad\pi^{E}\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\underbrace{\sum_{\widetilde{x}<x'\alpha'}}_{\text{discard offers}}\Gamma^{x}\left(\widetilde{x}\right)\right]\\
&+\underbrace{\sum_{\alpha}\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h'-1,x',\alpha\right)}_{\text{received offer from firm with productivity }\alpha'x'}\quad\pi^{E}\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\Gamma^{x}\left(x'\alpha'\right)\underbrace{\mathbf{1}_{x'\alpha'>x'\alpha}}_{\text{rebargain}}\right]\\
&+\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h',x',\alpha'\right)\left(1-\pi^{E}\right)\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\sum_{\widetilde{x}<x'\alpha'}\Gamma^{x}\left(\widetilde{x}\right)\right]\\
&+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h',x',\alpha\right)\left(1-\pi^{E}\right)\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\Gamma^{x}\left(x'\alpha'\right)\mathbf{1}_{x'\alpha'>x'\alpha}\right]\\
&+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h'-1,\underbrace{\alpha'x'}_{x},\alpha\right)\pi^{E}\left[\left(1-\delta\right)\nu\right]f_{t+1}\Gamma^{x}\left(x'\right)\qquad\text{(B.1)}\\
&+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h',x',\alpha\right)\left(1-\pi^{E}\right)\left[\left(1-\delta\right)\nu\right]f_{t+1}\Gamma^{x}\left(x'\right)\\
&+\sum_{\alpha}\sum_{x}\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h'-1,x,\alpha\right)\pi^{E}\delta\Gamma^{x}\left(x'\right)\quad\underbrace{\mathbf{1}_{\alpha'=\frac{x}{x'}}}_{\alpha'\text{ must be }\frac{x}{x'}}\\
&+\sum_{\alpha}\sum_{x}\sum_{s\in\boldsymbol{S}_{t}^{E}}\lambda_{t}^{E}\left(s,h',x,\alpha\right)\left(1-\pi^{E}\right)\delta\Gamma^{x}\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}\\
&+\sum_{s\in\boldsymbol{S}_{t}^{U}}\lambda_{t}^{U}\left(s,h'+1\right)\pi^{U}f_{t+1}\Gamma^{x}\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}\left(\cdot\right)+\sum_{s\in\boldsymbol{S}_{t}^{U}}\lambda_{t}^{U}\left(s,h'\right)\left(1-\pi^{U}\right)f_{t+1}\Gamma^{x}\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}\left(\cdot\right)\Bigg).
\end{aligned}
$$

Flows into unemployment. Conditional on not retiring, flows into unemployment include the following transitions:

- Employed worker loses job and does not find job; skill appreciates.
- Employed worker loses job and does not find job; skill does not appreciate.
- Unemployed worker does not find job; skill depreciates.
- Unemployed worker does not find job; skill does not depreciate.
- Dead retiree is reborn; inherits shares but draws new human capital.

Hence, we have the following law of motion for the distribution of unemployed workers:

$$\lambda_{t+1}^{U}(s',h') = (1-\psi^{R}) \times \Bigg(\sum_{\alpha} \sum_{x} \sum_{s \in \boldsymbol{S}_t^{E}} \lambda_t^{E}(s,h'-1,x,\alpha)\pi^{E}\delta(1-f_{t+1}) \\ + \sum_{\alpha} \sum_{x} \sum_{s \in \boldsymbol{S}_t^{E}} \lambda_t^{E}(s,h',x,\alpha)(1-\pi^{E})\delta(1-f_{t+1}) \\ + \sum_{s \in \boldsymbol{S}_t^{U}} \lambda_t^{U}(s,h'+1)\pi^{U}(1-f_{t+1}) + \sum_{s \in \boldsymbol{S}_t^{U}} \lambda_t^{U}(s,h')(1-\pi^{U})(1-f_{t+1}) \Bigg) \\ + \sum_{s \in \boldsymbol{S}_t^{R}} \lambda_t^{R}(s)\psi^{D}\Gamma^{h}(h'). \tag{B.2}$$

Flows into retirement. Flows into retirement include the following set of transitions:

- Employed worker retires.
- Unemployed worker retires.
- Retired worker does not die.

These inflows imply we have the following law of motion for the distribution of retirees:

$$\lambda_{t+1}^{R}(s') = \psi^{R} \sum_{s \in \boldsymbol{S}_t^{E},h,x,\alpha} \lambda_t^{E}(s,h,x,\alpha) + \psi^{R} \sum_{s \in \boldsymbol{S}_t^{U},h} \lambda_t^{U}(s,h) + \left(1-\psi^{D}\right) \sum_{s \in \boldsymbol{S}_t^{R}} \lambda_t^{R}(s). \tag{B.3}$$

B.1.2 Casting the model in relative prices and real variables

As in the TANK model, we start by deriving the equations governing relative prices, real dividends, and real profits of intermediate firms in steady state.

- Evaluating the NKPC in steady state, we obtain the real marginal cost $mc = p^{l}/z$:

$$mc = \frac{\eta - 1}{\eta}.$$

- The price of labor services is then given by

$$p^{l} = mc \times z = \frac{\eta - 1}{\eta} z. \tag{B.4}$$

- Per-period real profits of the intermediate firms are given by

$$\Gamma^{I} = (1 - mc)Y = \frac{Y}{\eta}. \tag{B.5}$$

- Real dividends are given by

$$\begin{aligned} d &= x_B Y - \frac{x_B Y(1+\pi^*)}{(1+i)} + \Gamma^I + \Gamma^S \\ &= x_B Y \frac{r}{1+r} + \Gamma^I + \Gamma^S. \end{aligned} \tag{B.6}$$

- Dividing the no-arbitrage condition by the aggregate price level P, we solve for the share price

$$\begin{aligned} \frac{(p^s + d)(1+\pi^*)}{p^s} &= 1 + i \\ p^s &= \frac{d}{r}. \end{aligned} \tag{B.7}$$

- Finally, we rewrite the government budget constraint in real terms as follows. Let $b_t = B_t/P_{t+1}$. Then, dividing both sides by P_t, multiplying the first term on the right-hand side by $\frac{P_{t+1}}{P_{t+1}}$, and recognizing that $1 + i_t = (1 + r_{t+1})(1 + \pi_{t+1})$, we get

$$\begin{aligned} b_{t-1} + g_t + \int UI\,(h)\, d\lambda_t^U\,(s,h) + \int \phi^R d\lambda_t^R(s) = \frac{b_t}{1+r_{t+1}} &+ \tau_c \int c\,(e,s,h,x,\alpha)\, d\lambda_t\,(e,s,h,x,\alpha) \\ &+ \int \left(UI\,(h) - \tau_t \left(UI\,(h)\right)^{1-\Upsilon} \right) d\lambda_t^U\,(s,h) \\ &+ \int \left(w(h,x,\alpha) - \tau_t w(h,x,\alpha)^{1-\Upsilon} \right) d\lambda_t^E\,(s,h,x,\alpha) \\ &+ \int \left(\phi^R - \tau_t \left(\phi^R\right)^{1-\Upsilon} \right) d\lambda_t^R(s). \end{aligned}$$

Here, the lower case variables b_{t-1} and g_t represent the real values of government debt and government spending, respectively. It is useful to define the real net revenue of government (tax proceeds minus outlays for pensions and unemployment insurance), $\mathcal{R}_t$, as

$$\begin{aligned} \mathcal{R}_t = &- \int UI\,(h)\, d\lambda_t^U\,(s,h) - \int \phi^R d\lambda_t^R\,(s) \\ &+ \tau_c \int c\,(e,s,h,x,\alpha)\, d\lambda_t\,(e,s,h,x,\alpha) + \int \left(UI\,(h) - \tau \left(UI\,(h)\right)^{1-\Upsilon} \right) d\lambda_t^U\,(s,h) \\ &+ \int \left(w(h,x,\alpha) - \tau w(h,x,\alpha)^{1-\Upsilon} \right) d\lambda_t^E\,(s,h,x,\alpha) + \int \left(\phi^R - \tau \left(\phi^R\right)^{1-\Upsilon} \right) d\lambda_t^R\,(s)\,. \end{aligned} \tag{B.8}$$

With these definitions, the government budget constraint in real terms is

$$\begin{aligned} b_{t-1} + g_t &= \frac{b_t}{1+r_{t+1}} + \mathcal{R}_t \\ \Rightarrow 0 &= (1+r_{t+1})(b_{t-1} + g_t - \mathcal{R}_t) - b_t. \end{aligned} \tag{B.9}$$

B.1.3 Solution algorithm for the steady state

We solve for the steady state using the following algorithm by bisecting over a nominal interest rate i that clears the share market given by Equation (28).

1. For a given nominal interest rate i and π^*, obtain r from the Fisher equation (6).
2. Outer loop: Guess a tax parameter τ, level of output Y, and service firm profits Γ^S.
   - Calculate the relative price of labor services using Equation (B.4), real bond holdings $b = x_B Y$, real government expenditures $g = x_G Y$, and real intermediate firm profits Γ^I using Equation (B.5).
   - Calculate real dividends d using Equation (B.6).
   - Calculate real share price p^s using Equation (B.7).
3. Inner loop: Guess a market tightness θ.
   - Calculate worker contact rate $f(\theta)$.
   - Solve the workers' problems given by Equations (14), (15), (16).
   - Compute the stationary worker distributions over state variables μ^E, μ^U, μ^R, λ, λ^E, λ^U, and λ^R.
   - Solve the matched firm problem in the labor services sector given by Equation (20).
   - Given the solution to the firm problem and worker distributions, calculate the implied market tightness $\widetilde{\theta}$ consistent with the free-entry condition $V = 0$, where V satisfies Equation (21).
   - Iterate over the inner loop until $\widetilde{\theta}$ agrees with the guessed market tightness θ.
4. Using the worker distributions, calculate the implied output $\widetilde{Y}$ using market clearing for labor services in Equation (27) and real service firm profits $\widetilde{\Gamma}^S$ in Equation (25).
5. Calculate the implied tax parameter $\widetilde{\tau}$ that clears the government budget constraint, which can be obtained from Equations (B.8) and (B.9) as:

$$\widetilde{\tau} = \frac{-\frac{r}{1+r} x_B Y - x_g Y + \tau_c \int c d\lambda + \int w d\lambda^E + \int UI d\lambda^U + \int \phi^R d\lambda^R}{\int w^{1-\Upsilon} d\lambda^E + \int UI^{1-\Upsilon} d\lambda^U + \int (\phi^R)^{1-\Upsilon} d\lambda^R}.$$

6. Iterate over the outer loop until $\widetilde{\tau}$, $\widetilde{Y}$, and $\widetilde{\Gamma}^S$ agree with guesses for τ, Y, and Γ^S.

B.2 Solving for the transition path using the SSJ method

In Section 4.2, we briefly discuss how we employ and expand the SSJ method of Auclert, Bardóczy, Rognlie, and Straub (2021) to solve for the transitional dynamics in our HANK model. In the following discussion, we provide additional details on this procedure.

Overview. We assume that the economy is in steady state at time $t = 0$. Entering period $t = 1$, there is an unexpected and transitory shock to the economy (e.g. productivity, discount rate, and labor market shocks). Because the shock is transitory, the economy eventually returns to the same real allocations but potentially at different nominal prices. We assume that this transition is completed by period $t = T$ for some large T.

We use the sequence-space Jacobian (SSJ) method developed by Auclert, Bardóczy, Rognlie, and Straub (2021), which allows us to efficiently compute impulse responses to aggregate shocks. To apply this method, we first rewrite model equations in terms of real variables and relative prices so that the initial and terminal steady states coincide.[52] We then cast the model as a DAG, presented in Figure B.1, which represents the model as various blocks and how they relate to one another. The nodes in the DAG can be classified into three groups: the initial node that contains exogenous shocks as well as endogenous variables to be solved for, the intermediate (green) nodes that represent blocks that contain the model's various components (such as the conduct of monetary policy via the Taylor rule, fiscal policy via the tax rule, or the heterogeneous agent household problem), and the terminal nodes that represent equilibrium conditions. The DAG relates each node by specifying variables used as inputs to and generated as outputs from these nodes. For each node, we compute the partial Jacobians of each output with respect to each input. We then forward accumulate these partial Jacobians along a topological sort of the DAG and use the implicit function theorem to obtain the general equilibrium Jacobians, which are used to compute the response of any endogenous variable to any exogenous shock. Importantly, using the equivalence of the impulse response function and the moving average representation of the process generating that variable, we can simulate time paths of aggregate variables and a large panel of individuals to obtain a rich set of aggregate and cross-sectional moments of the model under aggregate shocks.

Relative to a standard shooting algorithm to obtain general equilibrium impulse responses to a shock, the SSJ method provides major computational efficiency gains along two dimensions. The first improvement allows for the computation of policy function responses by a single backward value function iteration. The second improvement offers an efficient method of the forward iteration of equilibrium distributions in a model with rich heterogeneity. We closely follow Auclert, Bardóczy, Rognlie, and Straub (2021) to implement both of these improvements when solving for dynamics in our model.

Details. We first cast the model as a DAG, depicted in Figure B.1. Different from the DAG representation of the TANK model, the intermediate nodes in the DAG for the HANK model can be categorized into simple blocks and the heterogeneous agent block. An example of the former would be model components that relate various aggregate variables, such as the fiscal

[52] As we assume a trend inflation of 2 percent per year, the nominal variables in the initial and terminal steady states are not necessarily the same.

policy rule (Equation 29), the Taylor rule (Equation 6) or the expression for dividends and no-arbitrage that relate to the mutual fund (Equations 22 and 23). The latter is the most-complex model component wherein heterogeneous agents solve for decision rules that govern their consumption-saving choices and labor market outcomes, which play an important role in the dynamics of aggregates and distributions in the economy. Importantly, as we discuss in Section 4.2, model blocks *directly* interact not only via aggregate variables but also through the discretized worker distributions—the distribution of employed individuals across human capital and match productivity and the distribution of unemployed individuals across human capital at the job search stage, and the distribution of employed workers across human capital, match productivity, and piece rate levels at the consumption/production stage. We generalize the SSJ method to handle discretized worker distributions as direct inputs and outputs along the DAG.

In the HANK model, there are three exogenous variables that represent shocks to the economy, eight endogenous variables (unknowns) whose dynamics we are interested in, and thus eight target sequences that must equal zero in equilibrium (market clearing and consistency conditions) in the sequence space.

Formally, let $\zeta = \left(\{\pi_t, Y_t, p_t^l, b_t, u_t, \theta_t, \Gamma_t^S, e2e_t\}_{t=0}^{T-1}\right)$ represent the path of unknown endogenous variables and $\Theta = \left(\{z, \beta, \nu\}_{t=0}^{T-1}\right)$ represent the path of exogenous variables.[53] The system of equations, labeled as "targets" in the rightmost node, that govern the transition path is[54]:

$$H(\zeta;\Theta) = \begin{pmatrix} \frac{\log(1+\pi_t-\pi^*)(1+\pi_t)}{1+\pi_t-\pi^*} - \vartheta\left(\frac{p_t^l}{z_t} - \frac{\eta-1}{\eta}\right) - \frac{1}{1+r_{t+1}}\frac{\log(1+\pi_{t+1}-\pi^*)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*}\frac{Y_{t+1}}{Y_t} \\ \mathcal{L}_t - L_t \\ \mathcal{S}_t - 1 \\ (1+r_{t+1})(b_{t-1}+g_t-\mathcal{R}_t) - b_t \\ \mathcal{U}_t - u_t \\ \theta_t - q^{-1}\left(\kappa/\mathbb{E}J_t\right) \\ \Gamma_t^S - \Gamma_t^S \\ \mathcal{E}2\mathcal{E}_t - e2e_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{B.10}$$

The main purpose of setting up the model as a DAG is for the ability to systematically solve for Jacobians that summarize the partial equilibrium responses of each node's each output (including targets in the rightmost node) with respect to each direct input to that node. We are then able to forward accumulate—that is, apply the chain rule in a systematic fashion—

[53] Namely, the endogenous variables in the model are inflation, real output, real price of labor services, real debt, unemployment rate, labor market tightness, real profits of labor services firms and the mass of employer-to-employer transitions. The exogenous variables are the labor productivity in intermediate goods production, the discount factor and the on-the-job search efficiency.

[54] These equations in order capture the NKPC, market clearing for labor services, market clearing for mutual fund shares, the government budget balance, consistency of the unemployment rate, the free-entry condition, consistency of labor service profits and consistency of employer-to-employer transitions.

the partial Jacobians along a topological sort of the DAG to obtain the total Jacobians of any output (again including targets), with respect to changes in any exogenous variable or unknown endogenous variable. Simply put, a total Jacobian combines the direct and indirect responses of an output with respect to an input. For example, the response of the expected value of posting a vacancy $\mathbb{E}J$ (service firm block output) is affected directly by the real rate r through discounting in the firm's match value, but also indirectly through how the real rate affects demand and output, which ultimately affect market tightness, the unemployment rate, and the distribution of workers. These Jacobians are what we also use in our decomposition exercise.

Having obtained the total Jacobians of targets $H(\zeta;\Theta)$ with respect to endogenous unknowns ζ and to exogenous variables Θ, we can apply the implicit function theorem to compute the response of any endogenous unknown $d\zeta$ to a change in the exogenous variables $d\Theta$. Formally, let H_ζ and H_Θ denote the total Jacobians of the targets with respect to endogenous unknowns and exogenous variables; then, the impulse responses of unknowns is given by:

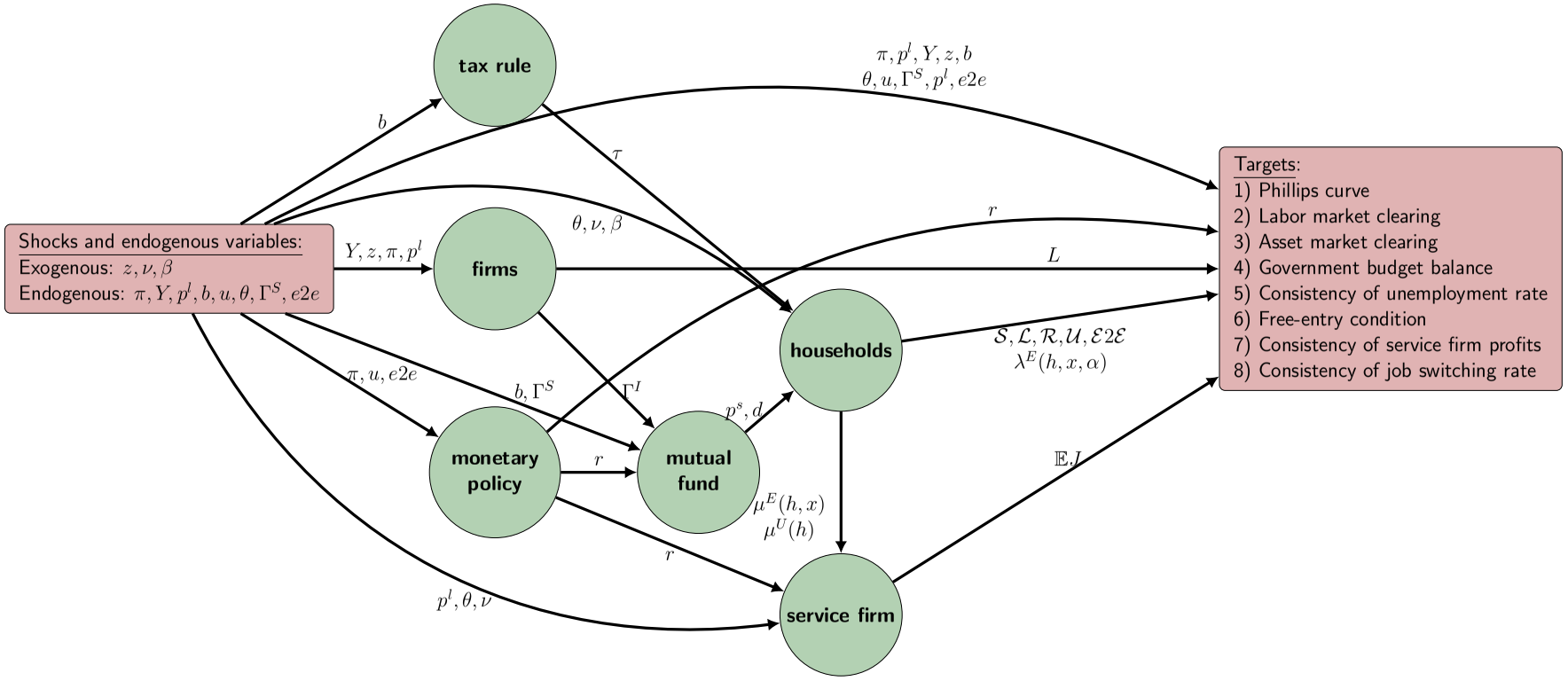
$$d\zeta = \underbrace{-H_\zeta^{-1}H_\Theta}_{G_\zeta} d\Theta,$$

where G_ζ denotes the GE Jacobians of the endogenous variables.

Equipped with the partial Jacobians of the intermediate variables and GE Jacobians of the unknown variables, we compute the GE Jacobians of the intermediate variables too—i.e., variables that appear on the arrows connecting intermediate nodes in the DAG—which allow us to compute their impulse responses with respect to exogenous variables as well.

Finally, we use the equivalence of the impulse response function (IRF) with the moving-average process representation of a time series. This allows us to flexibly simulate a time-path of aggregate variables and—given the path of these aggregate variables and policy responses to aggregate shocks—also simulate a large panel of workers. We in turn use this simulated worker panel to study a wide range of cross-sectional outcomes and evaluate the welfare effects of monetary policy.

Figure B.1: DAG representation of the HANK model with a frictional labor market and on-the-job search



B.3 Decomposing inflation effects of OJS shocks using the DAG

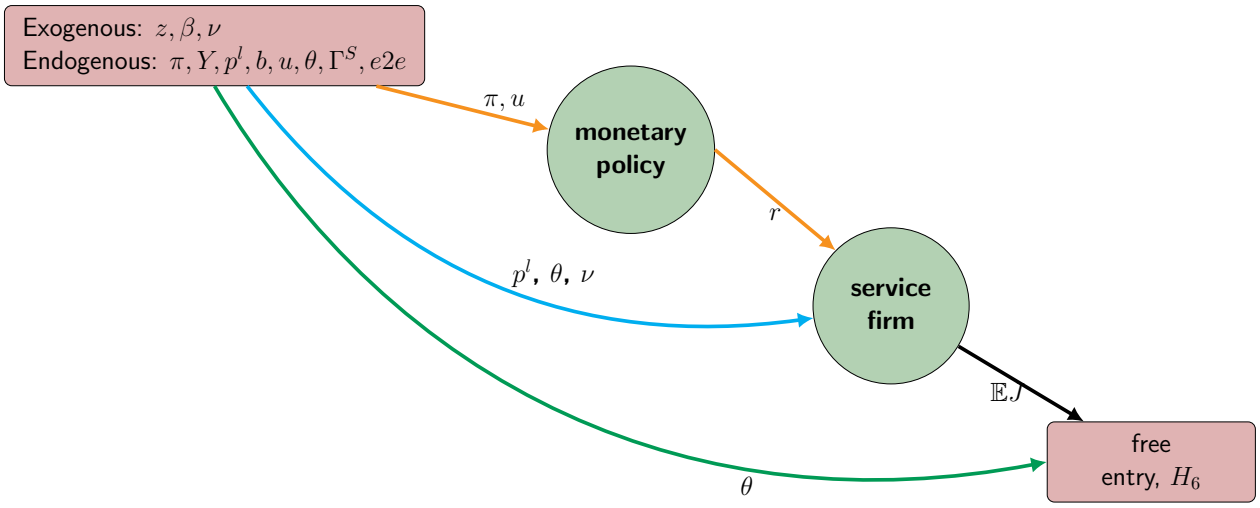
Some discussion is warranted to clarify how we operationalize the DAG and its associated input-output structure to decompose the various channels through which OJS shocks affect inflation in Section 5.4. We start from the total Jacobians of each block's outputs with respect to their inputs already computed when solving the model. We then use the implicit function theorem (IFT) for each block to compute the derivative of the output of interest with respect to all the endogenous and exogenous variables listed in the initial node in the DAG. In Section 5.4, where we use the free-entry condition for decomposing the changes in p^l, we obtain the derivative of p^l with respect to all variables by applying the IFT to this condition. We then multiply the total derivative of p^l with respect to these variables with the GE IRFs of these variables with respect to ν (also already computed while solving the model). As a result, we obtain the response of each component that makes up p^l with respect to the shock of interest ν. Specifically, we end up with (i) the effect of ν on p^l through the shock's direct effect on expected value from a match for labor services firms (light-blue line in Figure B.2), (ii) the GE effect of ν on p^l through the shock's indirect effect on equilibrium labor market tightness that dictates a change in p^l due to the change in the free-entry condition (green line in Figure B.2), and (iii) the GE effect of ν on p^l through the shock's indirect effects on equilibrium inflation and unemployment that necessitates a change in p^l due to the change in the real rate that affects the continuation value of labor services firms and thus the free-entry condition (orange lines in Figure B.2).

B.4 Evaluating alternative monetary policy rule coefficients

In this section, we provide computational details on how we evaluate the objective function in Equation (31) under alternative Taylor-rule coefficients as described in Section 6.

Naively, solving and simulating the model, and calculating the variances of inflation and unemployment to evaluate the objective function in Equation (31) under alternative Taylor-rule coefficients $\Phi_\pi, \Phi_u, \Phi_{EE}$ in Equation (32) requires the repeated computation of the entire SSJ system. As we discussed in Section 4.2, worker distributions directly enter the equilibrium conditions and agents' problems, as opposed to model blocks depending on one another only through aggregate variables. This added layer of complication in our context makes the computation of derivatives costly. To overcome this challenge, we use the approach described by McKay and Wolf (2022). The key insight is that firms and households do not care about the systematic component of monetary policy, and what matters for their decisions is the time path of the interest rate in response to the structural shocks in the economy. Based on this insight, one can compute model IRFs under alternative Taylor rule parameters without having to repeatedly compute the entire system of Jacobians, but only by solving a linear system of equations in structural shocks and a series of monetary *policy* shocks and by leveraging Jacobians computed once under the baseline parameterization.

Figure B.2: Decomposing the effect of OJS efficiency shocks on inflation



Notes: This figure presents model mechanisms through which an OJS efficiency shock ν affects the relative price of labor services p^l. Light-blue line refers to the effect of ν on p^l through the shock's direct effect on expected value from a match for labor services firms. Green line refers to the GE effect of ν on p^l through the shock's indirect effect on equilibrium labor market tightness that dictates a change in p^l due to change in free-entry condition. Finally, the orange line refers to the GE effect of ν on p^l through the shock's indirect effects on equilibrium inflation and unemployment that necessitates a change in p^l due to change real rate that affects the continuation value of labor services firms and thus the free-entry condition.

The key idea is to utilize *policy* shocks to monetary policy to compute IRFs to *non-policy shocks* under alternative Taylor rule coefficients, using *only* the Jacobian system computed once under the baseline monetary policy rule. The reason this approach works is that firms and households do not care about the systematic component of monetary policy—i.e., how aggressively the central bank reacts to inflation, unemployment and the EE rate separately—but they only care about the current and future path of interest rates.

Specifically, given the sequence-space truncation horizon T and alternative monetary policy coefficients $\widetilde{\Phi}_\pi, \widetilde{\Phi}_u, \widetilde{\Phi}_{EE}$, we solve the $T \times T$ linear system of equations for the path of policy news shocks $\boldsymbol{\varpi} = \{\varpi_t\}_{t=1}^T$ below:

$$\begin{aligned} \underbrace{\boldsymbol{i}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{i,\varpi}_{\Phi_\pi,\Phi_u}\boldsymbol{\varpi}}_{\text{IRF of } i \text{ under baseline}} &= \widetilde{\Phi}_\pi \underbrace{\left(\boldsymbol{\pi}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{\pi,\varpi}_{\Phi_\pi,\Phi_u}\boldsymbol{\varpi}\right)}_{\text{IRF of } \pi \text{ under baseline}} + \widetilde{\Phi}_u \underbrace{\left(\boldsymbol{u}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{u,\varpi}_{\Phi_\pi,\Phi_u}\boldsymbol{\varpi}\right)}_{\text{IRF of } u \text{ under baseline}} \\ &+ \widetilde{\Phi}_{EE} \underbrace{\left(\boldsymbol{EE}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{EE,\varpi}_{\Phi_\pi,\Phi_u}\boldsymbol{\varpi}\right)}_{\text{IRF of } EE \text{ under baseline}}, \end{aligned} \tag{B.11}$$

where $\boldsymbol{\Theta}^{Y,X}_{\Phi_\pi,\Phi_u}$ denotes the $T \times T$ Jacobian matrix of variable Y with respect to X for various X, Y combinations under the baseline monetary policy rule and ε is a non-policy/structural shock, i.e., shocks to supply, OJS efficiency, and demand as estimated in Section 4.3.

The left-hand side of Equation (B.11) is the combined IRF of the nominal interest rate to the structural shock ε and the sequence of policy news shocks $\boldsymbol{\varpi}$ under the baseline rule. The right-hand side scales each of the baseline IRFs of inflation, unemployment rate and EE rate to the structural and policy shocks by the alternative monetary policy coefficients to compute the IRF of the nominal interest rate under the alternative Taylor rule subject to the same shocks. The IRF to the structural shock ε under the *alternative* monetary policy rule is then equal to the IRF to ε and solved policy shocks $\{\varpi_t\}_{t=1}^{T}$ under the *baseline* rule.

Once we solve the system of equations in Equation (B.11), we can similarly compute the IRF of other model variables under the alternative Taylor rule that are relevant for the central bank's objective function, i.e., inflation and output volatilities. Using these IRFs, we can calculate variances and evaluate Equation (31) for any combination of $\widetilde{\Phi}_\pi, \widetilde{\Phi}_u, \widetilde{\Phi}_{EE}$.

B.5 Measuring welfare gains from the optimal policy

In this section, we discuss how we measure welfare gains under the optimal policy. To compute the aggregate consumption-equivalent welfare, we solve for $\overline{\chi}$—as in Lucas (1987)—that satisfies the following indifference condition:

$$\int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\left(1+\overline{\chi}\right) \overline{c}_t\left(e,s,h,x,\alpha\right)\right) \lambda\left(e,s,h,x,\alpha\right) = \int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\widetilde{c}_t\left(e,s,h,x,\alpha\right)\right) \lambda\left(e,s,h,x,\alpha\right),$$

where $\overline{c}_t\left(e,s,h,x,\alpha\right)$ and $\widetilde{c}_t\left(e,s,h,x,\alpha\right)$ denote the consumption of an individual with state (e,s,h,x,α) in date t under the baseline and optimal Taylor rules, respectively, while λ denotes the steady state distribution of agents. Here, $\overline{\chi}$ is the percent additional lifetime consumption that must be endowed at all future dates and states to all agents under the stationary distribution where the baseline Taylor rule is implemented so that the average welfare will be equal to that of an economy populated with the same agents but where the optimal policy is implemented.

Given the functional form of the utility function u in Section 4, $\overline{\chi}$ can be expressed as

$$\overline{\chi} = \left(\frac{\int \widetilde{V}^e\left(s,h,x,\alpha\right) \lambda\left(e,s,h,x,\alpha\right)}{\int \overline{V}^e\left(s,h,x,\alpha\right) \lambda\left(e,s,h,x,\alpha\right)} \right)^{\frac{1}{1-\sigma}} - 1,$$

where $\overline{V}$ and $\widetilde{V}$ denote value functions under the baseline and optimal Taylor rules respectively.

Finally, in order to obtain group-specific measures of welfare, as shown in Table 6, we divide the steady-state distribution into groups of interest. Let group $o \in \mathcal{O}$ be a subset of individual states within the set of all possible individual states $\mathcal{O}$. Then, group-specific welfare $\overline{\chi}_o$ solves:

$$\int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\left(1+\overline{\chi}_o\right) \overline{c}_t\left(e,s,h,x,\alpha\right)\right) \lambda^o\left(e,s,h,x,\alpha\right) = \int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\widetilde{c}_t\left(e,s,h,x,\alpha\right)\right) \lambda^o\left(e,s,h,x,\alpha\right),$$

where λ^o represents the steady state distribution of agents, conditional on being in group o.

B.6 Additional results

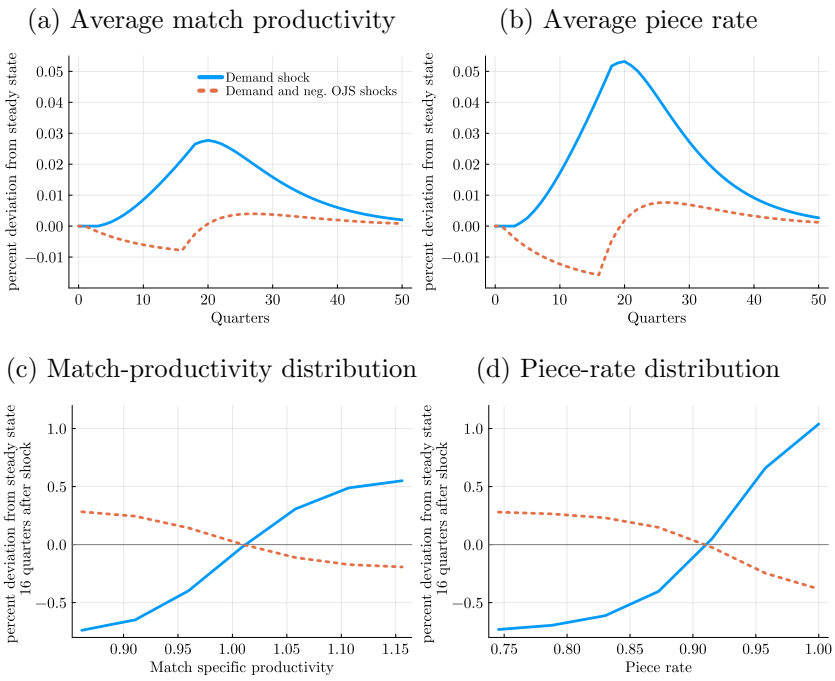
This appendix provides additional results to complement Section 5.

Effects of OJS efficiency shocks on ALP and piece rate. Section 5.2 studies the macroeconomic implications of negative OJS efficiency shocks by comparing outcomes between two different transitions starting from the same steady state. In doing so, we also present ALP dynamics across the two economies in Figure 6 to discuss their effects on output. In Figure B.3 Panel (a), we present the dynamics of average match-specific productivity between the two economies. We show that because negative OJS efficiency shocks limit employed workers' ability to change jobs, they temper the increase in average match productivity during the labor market recovery. In fact, as unemployed workers accept the first job offer, they initially lower the average match quality, which only recovers after 20 quarters as they climb up the ladder slowly in the midst of negative OJS efficiency shocks. Importantly, Panel (b) shows that the average piece rate increases slightly more when there are only positive demand shocks. This implies that wages in this economy are higher than wages in the economy with negative OJS efficiency shocks, despite similar ALP dynamics, as shown in Panel (d) of Figure 6. Therefore, inflationary wage pressures are stronger when we observe an EE rate rise coincidently with the unemployment decline. Quantitatively, this effect is small, as evidenced by a very small gap between the response of average piece rates across both economies. This is the reason the model still generates a rise in inflation even without wage re-bargaining for incumbent workers with external offers.

We also compare the evolution of match-specific productivity and piece rate in the cross-section between the two economies, as looking at averages alone may mask interesting results across heterogeneous agents. The next two panels plot *changes* in the distributions of match-specific productivity (Panel (c)) and of the piece rate (Panel (d)) 16 quarters after the shock relative to their respective steady states across the two economies. Under positive demand shocks alone, the match productivity and piece-rate distributions exhibit a rightward shift. In contrast, when there are negative OJS efficiency shocks as well, both distributions shift leftward as these shocks decelerate the job ladder.

Estimated shocks. In Sections 5.2 and 5.3, we implement two separate exercises to quantify the magnitude of missing inflation due to the low EE rate during the post-Great Recession episode and the additional rise in inflation due to a high EE rate during the recovery from the COVID-19 recession, respectively. In these exercises, we estimate demand and OJS efficiency shocks to match the unemployment rate and EE rate dynamics. Panel (a) and Panel (b) in Figure B.4 provide the estimated path of innovations to discount factor $\varepsilon_{\beta,t}$ and OJS efficiency $\varepsilon_{\nu,t}$ processes given in Equation (30) for these exercises.

Figure B.3: Effects of negative OJS efficiency shocks on productivity and piece rate distributions



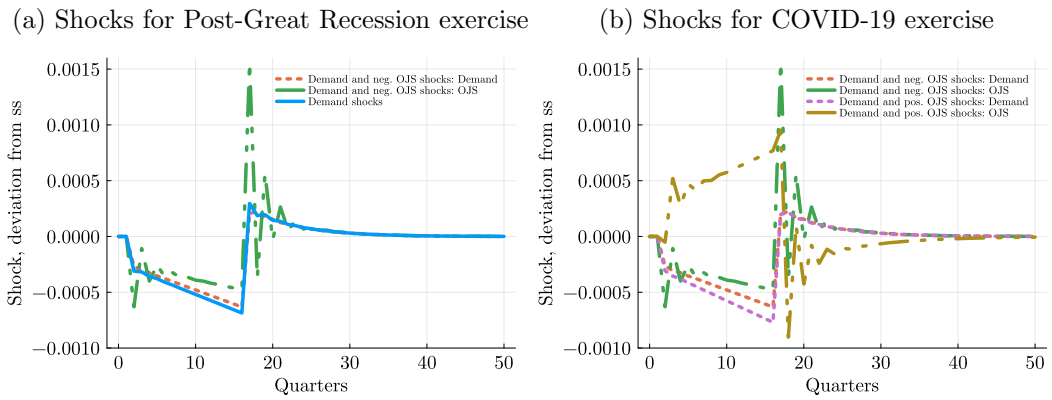
Notes: This figure presents dynamics of average match-specific productivity x (Panel (a)) over time, average piece rate over time (Panel (b)), and changes in the distribution of match-specific productivity (Panel (c)) and the distribution of piece rate (Panel (d)) 16 quarters after the shock relative to their respective steady state in an economy with (i) only a series of positive demand shocks (solid-blue lines) and (ii) series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). These shocks in the two economies are calibrated such that they lead to the same path of the unemployment rate. The additional negative OJS shocks in the second economy are estimated to keep the EE rate unchanged.

Implications on the slope of the observed Phillips Curve. Results in Sections 5.2 and 5.3 show that two recovery episodes with identical unemployment rates but different EE rate dynamics generate different inflation dynamics. As a result, the slope of the observed Phillips Curve would be different across these two recoveries. Figure B.5 shows this result using simulations from our exercises in Sections 5.2 (Great Recession recovery) and 5.3 (COVID-19 recovery).

In Figure B.5, the blue circles plot quarterly inflation against quarterly unemployment rate for the economy with positive demand shocks alone, while the orange diamonds represent the same for the economy with positive demand shocks and negative OJS efficiency shocks. Because the latter economy generates a smaller rise in inflation, Figure B.5 shows that the slope of the Phillips curve is flatter during the post-Great Recession episode.

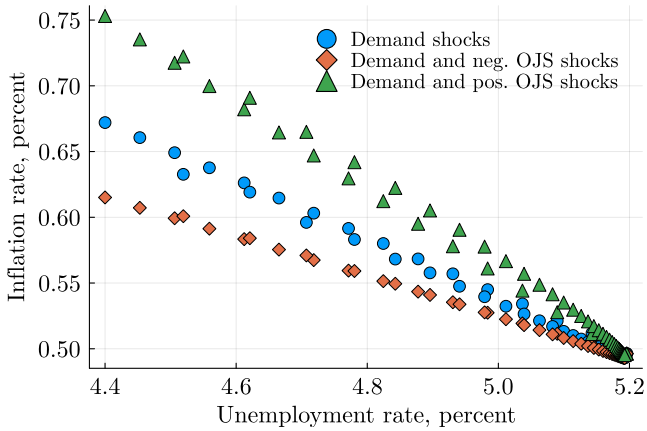
In Figure B.5, the green triangles plot quarterly inflation against quarterly unemployment rate for the post-COVID economy. Because this economy experiences a larger increase in inflation given the same unemployment rate path, Figure B.5 demonstrates that the slope of the Phillips Curve is steeper during the recovery from the COVID-19 recession.

Figure B.4: Estimated path of innovations to demand and OJS efficiency



Notes: This figure plots the estimated path of innovations to discount factor $\varepsilon_{\beta,t}$ and OJS efficiency $\varepsilon_{\nu,t}$ processes given in Equation (30) for the post-Great Recession exercise (Panel (a)) in Section 5.2 and for the COVID-19 recovery exercise (Panel (b)) in Section 5.3. In Panel (a), the solid blue line is the sequence of innovations to discount factor to match the path of unemployment in the post-Great Recession episode without targeting the path of the EE rate. The dashed-orange and solid-green lines are the paths of innovations to discount factor and OJS efficiency, respectively, to jointly match the same unemployment rate as in the first economy as well as the flat EE rate in the post-Great Recession period. In Panel (b), the dashed-orange and solid-green lines are the same as those in Panel (a) that generate the same unemployment rate and flat EE rate in the recovery episode from the COVID-19 recession, while dashed-purple and solid-dark-gold lines are the paths of innovations to discount factor and OJS efficiency, respectively to jointly match the same unemployment rate as well as the rise in EE rate in this episode.

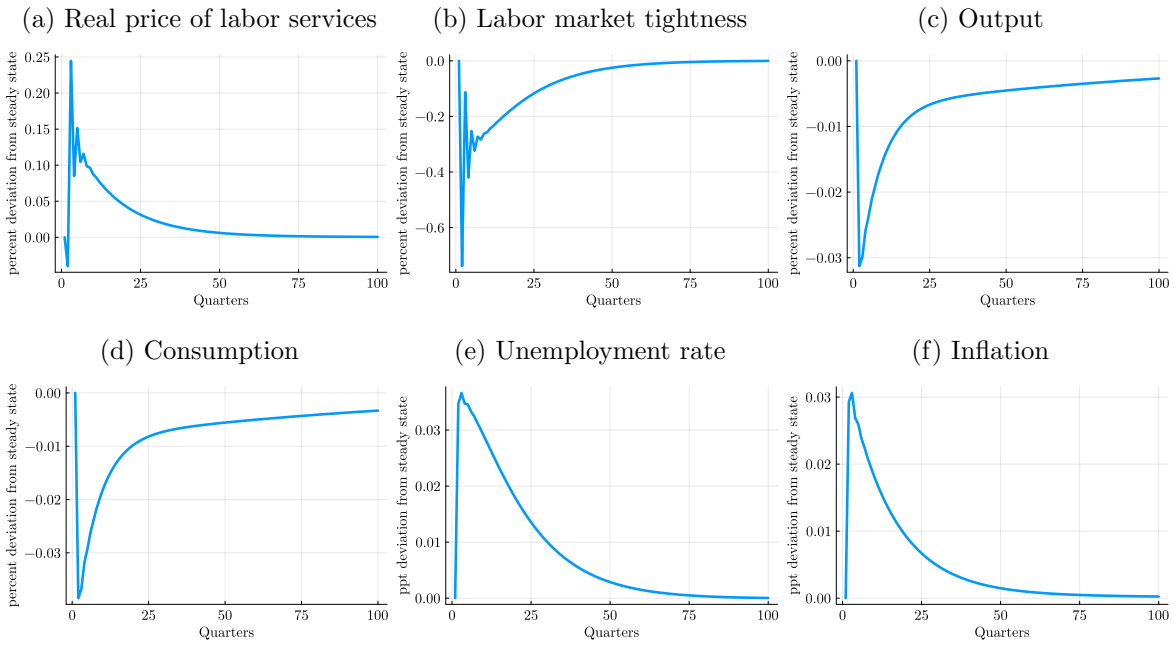
Figure B.5: Effects of OJS efficiency shocks on the slope of NKPC



Notes: This figure plots quarterly inflation against the quarterly unemployment rate for three different recovery episodes with the same unemployment dynamics: Blue circles represent a recovery episode with positive demand shocks alone; orange diamonds represent a recovery episode with positive demand shocks and negative OJS efficiency shocks generating a flat EE rate as in the recovery from the post-Great Recession; green triangles represent a recovery episode with positive demand shocks and positive OJS efficiency shocks generating a rise in the EE rate as in the recovery from the COVID-19 recession.

Impulse responses to an OJS efficiency shock. In Section 5.4, we refer to impulse responses of model outcomes to a positive unit shock to the OJS efficiency parameter ν. This reference supported our discussion of decomposing various channels through which an OJS shock affects inflation in our model. Here, we provide these impulse responses in Figure B.6. As discussed, an increase in ν leads to a decline in tightness, output, and consumption and to an increase in the real price of labor services, unemployment rate, and inflation.

Figure B.6: Impulse responses to a positive OJS efficiency shock



Notes: This figure presents impulse responses of various outcomes to a positive unit shock to the OJS efficiency parameter ν.