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Modelling Canadian mortgage debt and payments in a semi-structural model

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Excerpt

We show how Canadian mortgage debt dynamics can be modelled in a semi-structural macroeconomic model, such as the Bank of Canada's LENS. The model we propose accounts for Canada's unique mortgage debt structure.

Topics: Economic models, Monetary policy transmission

JEL codes: E27, E43, E47, G51

Extrait

Nous montrons comment la dynamique de la dette hypothécaire canadienne peut être modélisée dans un modèle macroéconomique semi-structurel comme LENS utilisé par la Banque du Canada. Notre modèle reproduit la structure singulière de la dette hypothécaire canadienne.

Sujets : Modèles économiques, Transmission de la politique monétaire

Codes JEL : E27, E43, E47, G51

1. Introduction

Mortgage debt-servicing costs are a key channel in Canada's monetary policy transmission mechanism. Interest rate movements influence these costs and can significantly affect the discretionary income available for households to spend on goods and services. Yet, most structural macroeconomic models do not account for some of the intricacies of the mortgage market's structure.

This shortcoming is generally acceptable when monetary policy shocks are small or infrequent. However, over the past two years, the tightening of monetary policy has been very rapid, leading to a historic increase in debt-servicing costs (**Chart 1**) and meaningful changes to household preferences for various mortgage terms.

Looking ahead, the magnitude and timing of monetary policy pass-through will depend greatly on the structure of household debt. For example, if the proportion of households holding variable-payment mortgages increases, then monetary tightening will pass through to household finances more quickly. And if long-term fixed contracts grow as a share of outstanding mortgage debt, rate increases may take longer to have their full impact on consumer spending. Moreover, if some mortgage holders are able to lower their principal payments in the face of rising interest costs, they may not have to cut their discretionary spending. Thus, when assessing the transmission of monetary policy through this income channel, using a model capable of capturing these nuances is important.

Chart 1: Households' debt-servicing costs increased to a historic level in 2022

Mortgage payments as a ratio of household disposable income, quarterly

Source: Statistics Canada Last observation: 2023Q2

For such exercises, staff at the Bank of Canada rely primarily on microsimulations initialized using detailed microdata on individual mortgages.[1] Yet, most structural macroeconomic models do not include such details for computational reasons and

[1] See teNyenhuis and Su (2023).

instead use aggregated data to capture debt dynamics. In this note, we present a model of the Canadian mortgage debt landscape with two useful features:

- It is simple enough to use inside the Bank's Large Empirical and Semi-structural model (LENS) (Gervais and Gosselin 2014).
- It approximates reasonably well some of the properties of detailed microsimulations.

The model's purpose is to enrich LENS's properties, providing a more realistic accounting of the monetary policy transmission mechanism and how it interacts with household finances.

Our work is influenced by Bove, Dées and Thubin (2020), who expand the Bank of France's semi-structural model. Additionally, to model the interest rates on mortgage debt, we leverage detailed data from the publicly available A4 filings dataset, which is compiled by the Office of the Superintendent of Financial Institutions and aggregated by the Bank of Canada. The A4 data are divided by term and type of contract and thus capture the impact on rates from changes in the composition of mortgage debt.

Under a range of hypothetical policy rate scenarios, our model predicts that, even if rates begin to fall, the required payment rate on mortgage debt will continue to climb in the coming years. This higher rate will then weigh on the amount of income households have available for discretionary spending.

The note is organized as follows. In section 2, we describe the structure of the model and its methodology. In section 3, we highlight some basic model properties. In section 4, we discuss the implications for household finances predicted by our model under four alternative policy paths.

2. Methodology and model description

We start with a basic identity, where the existing stock of mortgage debt ($\boldsymbol{D_t}$) rises with new mortgage issuances ($\boldsymbol{L_t}$) and declines with principal repayments ($\boldsymbol{P_t}$).

$$\boldsymbol{D_t = D_{t-1} + L_t - P_t} \tag{1}$$

The data for the stock of debt and principal payments come directly from Statistics Canada's National Balance Sheet Accounts and are available at a quarterly frequency. We can thus derive data on new issuances as the residual that makes the identity hold.[2]

2.1 New mortgage issuances

New mortgage issuances evolve according to an error-correction model. As shown in equation (2), the cointegrating vector that determines the long-run value of new issuances ($\hat{l}_t$) is estimated as a function of nominal residential investment (h_t) and the real effective interest rate on new mortgages (R_t^F).[3] A coefficient of one is imposed on residential investment to achieve a balanced steady-state growth path.

The value of new mortgage issuances (l_t) gradually adjusts toward this equilibrium level, governed by the dynamic equation (equation 3). Here again, the sum of coefficients on the independent and lagged dependent variables are constrained to sum to one, achieving a balanced growth path. Both equations are estimated from the first quarter of 1990 until the first quarter of 2023.

$$\hat{l}_t = \beta_0 + h_t + \beta_1 R_t^F \tag{2}$$

$$\Delta l_t = \alpha_0 \left(l_{t-1} - \hat{l}_{t-1} \right) + \sum_{i=1}^{3} \alpha_i (\Delta l_{t-i}) + (1 - \alpha_1 - \alpha_2 - \alpha_3)(\Delta h_t) + \epsilon_t \tag{3}$$

In **Chart 2**, we show the long-run values extracted from the cointegrating vector and compare them with the observed data.

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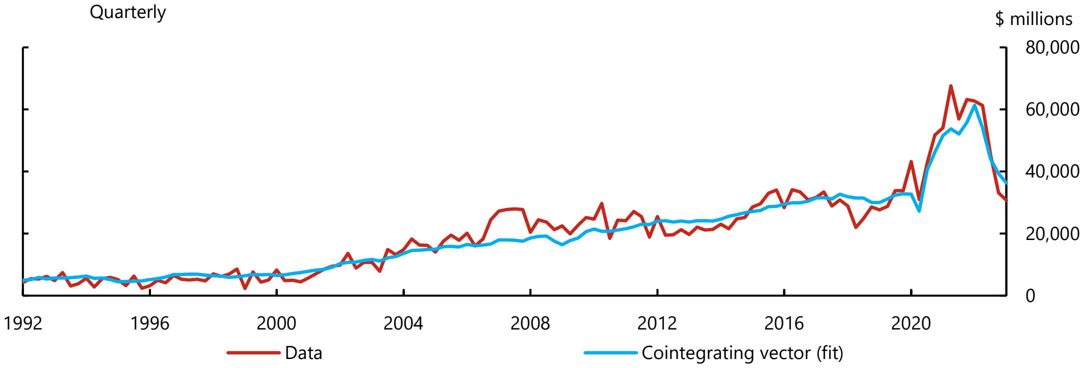
[2] Statistics Canada's principal payment data include only obligated principal payments.

[3] In this and all following equations, the lower-case letters indicate that the log of the variable is in use. In this equation, for instance, l = log(L) and h = log(H).

Chart 2: The cointegrating vector captures the key movements in mortgage credit
Quarterly



Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2023Q2

2.2 Principal and interest payments

Beginning with the accounting identities in equation (4) and equation (5), we model the behaviour of $\boldsymbol{R_t^S}$ and $\boldsymbol{\delta_t^S}$, which are the effective interest and principal payment rates on the stock of mortgage debt, respectively:[4]

$$\boldsymbol{I_t = R_t^S D_{t-1}} \tag{4}$$

$$\boldsymbol{P_t = \delta_t^S D_{t-1}} \tag{5}$$

2.2.1 Principal payments

We construct the principal payments rate ($\boldsymbol{\delta_t^S}$) following the methodology of Bove, Dées and Thubin (2020), with extensions to model certain features of the Canadian mortgage market.

The principal payments rate is modelled as a function with three terms:

- representing the principal payments rate on past mortgages
- computing the rate on new mortgages

[4] Throughout the note, the superscript “*S*” denotes an effective rate on the entire stock of debt, including existing and new mortgages. The superscript “*F*” denotes an effective rate on new debt flows only (new issuances).

- accounting for variable-rate mortgages with fixed payments, which exhibit unique properties

First, we model the principal rate on existing mortgages so that it increases concavely over time. This is because principal payments as a share of total payments rise as mortgages near maturity.

However, the principal rate on new mortgage originations ($\boldsymbol{\kappa_t}$) follows the identity below:

$$\boldsymbol{\kappa_t = \frac{R_t^F(1 + R_t^F)^{-M}}{1 - (1 + R_t^F)^{-M}}} \tag{6}$$

$\boldsymbol{M = average\ amortization\ period}$[5]

Finally, for the Canadian context, we draw a distinction between fixed-payment variable-rate mortgages and other mortgages. Fixed-payment variable-rate mortgages are unique because their principal payment rate is inversely related to the movements in interest rates. For example, when interest rates rise, the amount of principal paid down on existing loans decreases to keep total payments unchanged. Therefore, the principal rate for these mortgages can be modelled as follows:[6]

$$\boldsymbol{\phi_t = \delta_{t-1}^S - \Delta\left(R_t^{VRM,F} - \frac{1}{20}\sum_{i=0}^{20} R_{t-i}^{VRM,F}\right)} \tag{7}$$

To ensure a better fit to the data, we estimate empirically the share of new originations in the stock of mortgage debt ($\boldsymbol{\gamma}$). Altogether, the principal rate is modelled as follows:

[5] *M* is set to 100 quarters, representing a standard 25-year mortgage.

[6] We make a couple of simplifying assumptions to derive the equation. First, we assume a roughly equal distribution of issuances across time, placing equal weight on all past mortgage issuances. Second, we assume that variable-rate mortgages will be renewed every five years (20 quarters), which is the most common type of variable-rate mortgage.

$$\delta_t^S = (1-v)(1-\gamma)(\delta_{t-1}^S)^\alpha + \gamma\kappa_t + v(1-\gamma)\phi_t \tag{8}$$

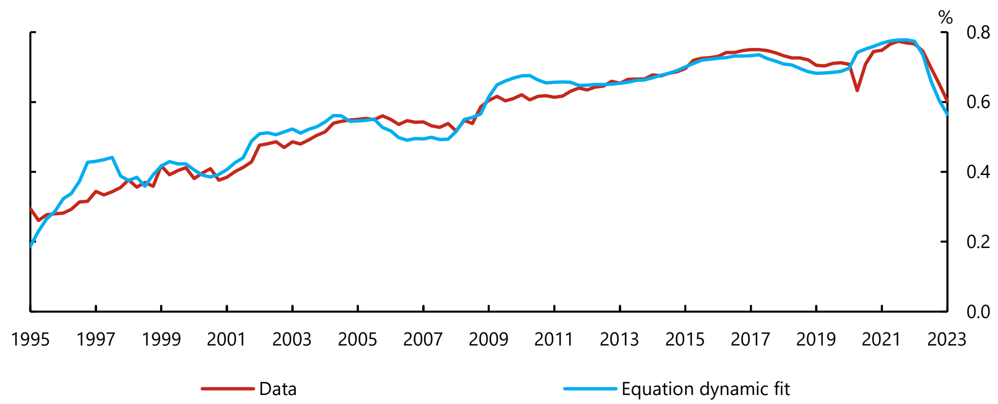
$$v = share\ of\ mortgages\ that\ have\ variable\ rates\ with\ fixed\ payments$$

$$0 < \alpha < 1 = curvature\ parameter^7$$

Chart 3 shows the in-sample dynamic fit of equation (8). The equation captures the key inflection points in the principal rate quite well, along with the general upward trend.

Chart 3: The mortgage principal rate equation captures the main inflection points over history

Principal payments, as a share of mortgage debt, quarterly



Source: Statistics Canada

Last observation: 2023Q2

2.2.2 Interest payments

The last remaining piece of the model is calculating the effective interest rate on the stock of mortgages. To do this well, the model must account for the underlying structure of the debt Canadians hold. For this purpose, we employ the debt and rates data available in the A4 bank filings database. The A4 filings are a monthly database of new loan issuances, loans outstanding and their corresponding interest rates for each federally regulated financial institution in Canada, available from 2013 onward. The data can be divided by term and type of loan, painting a detailed picture of Canada's household debt structure.

[7] This parameter is calibrated to 0.995, which reasonably approximates the amortization curve of a typical 25-year mortgage.

With this new information, we can divide mortgage debt and rates into five subcategories:

- variable-rate mortgages
- bridge loans (less than one year amortization)
- one- or two-year fixed-rate mortgages
- three- or four-year fixed-rate mortgages
- five-year or longer fixed-rate mortgages

We can then estimate each subcomponent rate separately and use a weighted average to produce an effective interest rate:

$$\boldsymbol{R_t^S = \sum_j \omega_t^{S,j} R_t^{S,j}} \tag{9}$$

$$\boldsymbol{j} \in \{\boldsymbol{Variable}, \boldsymbol{Bridge}, \boldsymbol{1\text{-} \ or \ 2\text{-}year \ fixed}, \boldsymbol{3\text{-} \ or \ 4\text{-}year \ fixed}, \boldsymbol{5\text{-}year \ fixed}\}$$

To forecast the effective rate on outstanding mortgage debt, we must first forecast not only the subcomponent rates ($\boldsymbol{R_t^{S,j}}$) but also the share of each debt category out of the stock of mortgages ($\boldsymbol{\omega_t^{S,j}}$). The rates on the stock of each debt category are forecast empirically as a function of either the policy rate, the two-year risk-free rate or the five-year risk-free rate, depending on the term length of the rate in question:

$$\boldsymbol{R_t^{S,j} = \theta^j \varphi_t^j + (1 - \theta^j) R_{t-1}^{S,j} + \epsilon_{1,t}} \tag{10}$$

$$\boldsymbol{\varphi_t^j} = \begin{cases} \boldsymbol{Policy \ rate \ if \ j = variable} \\ \boldsymbol{2\text{-}year \ rate \ if \ j = bridge, 1\text{-} \ or \ 2\text{-}year \ fixed} \\ \boldsymbol{5\text{-}year \ rate \ if \ j = 3 \ or \ 4\text{-}year \ fixed, 5\text{-}year \ fixed} \end{cases}$$

The stock weights ($\boldsymbol{\omega_t^{S,j}}$) depend on their past value and also on the dynamics of the corresponding share of new mortgage issuances (the flow weight). The flow weights ($\boldsymbol{\omega_t^{F,j}}$) are forecast as autoregressive, mean-reverting processes:

$$\boldsymbol{\omega_t^{S,j} = \vartheta_0^j \omega_{t-1}^{S,j} + \vartheta_1^j \omega_t^{F,j} + \epsilon_{2,t}} \tag{11}$$

$$\boldsymbol{\omega_t^{F,j} = \eta_0^j \omega_{t-1}^{F,j} + (1 - \eta_0^j)(\bar{\omega}^{F,j}) + \epsilon_{3,t}} \tag{12}$$

3. Model properties and impulse responses

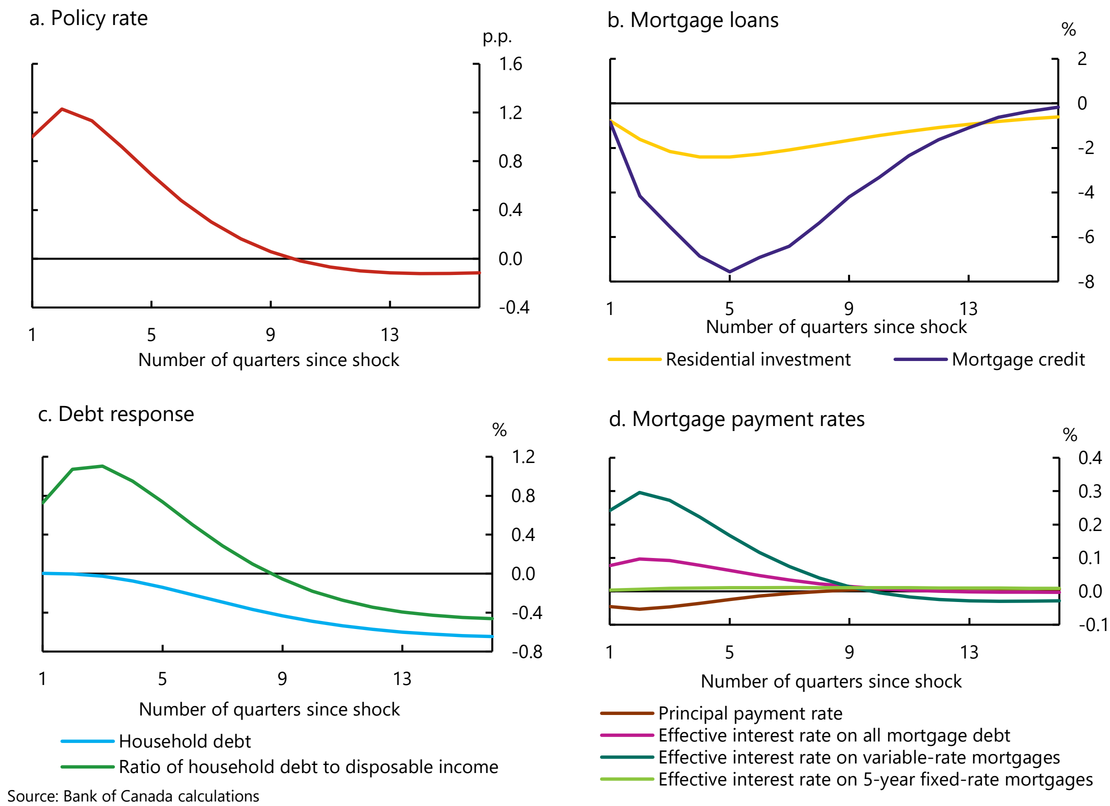
This section shows a collection of impulse responses derived from augmenting LENS with the system of equations described in section 2. To illustrate, we impose a temporary shock of 100 basis points to the policy rate (**Chart 4**, panel a).

This monetary policy tightening leads to a drop in both residential investment and demand for new loans (**Chart 4**, panel b). As a result, household debt also declines gradually. The household debt-to-income ratio initially rises as income falls. However, the ratio falls below the model's steady state after about eight quarters due to household deleveraging (**Chart 4**, panel c). This suggests that monetary policy tightening reduces household indebtedness in the long run.

Finally, the mortgage repayment rate falls in response to the policy rate shock. This is because interest costs represent a greater share of mortgage payments. When it comes to interest payments, the effective interest rate on variable-rate mortgages follows the policy rate closely. However, the effective rate on five-year fixed-rate mortgages has a muted and slow response (**Chart 4**, panel d). This is for two reasons:

- Given that this is a temporary rate hike, it has a relatively small impact on long-term borrowing costs.
- These mortgages have fixed rates, so the effective rate rises only when new mortgages or renewals are issued.

Chart 4: Responses to a rate hike of 100 basis point

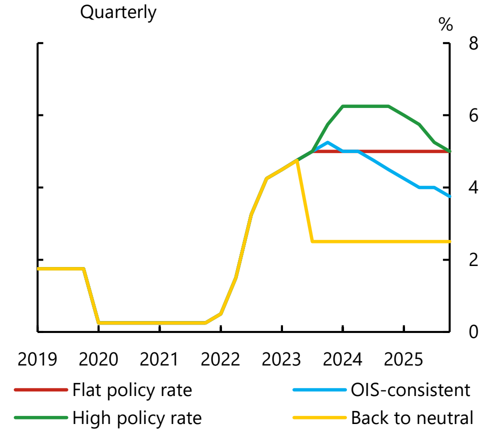


Source: Bank of Canada calculations

4. Scenario analysis

To show the implications of the model under various interest rate environments, we consider four alternative scenarios (**Chart 5**). In the first scenario, we assume the policy rate remains flat at 5% until the end of 2025. In the second, the policy rate evolves in line with implicit expectations in the overnight index swap market.[8] The third scenario has

Chart 5: Four scenarios for the overnight rate
Quarterly

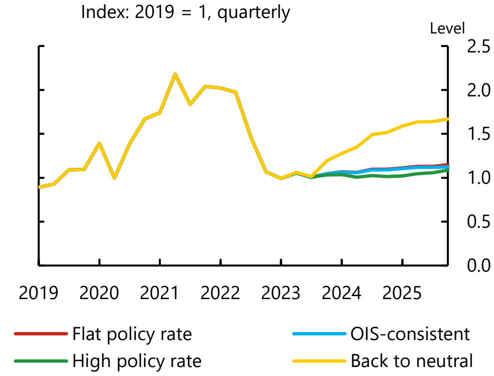


Source: Bank of Canada calculations Last Observation: 2023Q3

[8] The expectations were computed using data from August 9, 2023.

the policy rate jumping 100 basis points before gradually falling back to 5% by the end of 2025. Lastly, the fourth scenario assumes the policy rate is reset to its long-run neutral estimate and remains there over the forecast horizon. In all scenarios, agents anticipate the shocks, and we therefore assume they are aware of the full path of the policy rate. These scenarios are implemented via monetary policy shocks and are for illustrative purposes only. In practice, monetary policy responds internally to other shocks in the economy.

Chart 6: Mortgage issuances remain steady in most scenarios
Index: 2019 = 1, quarterly

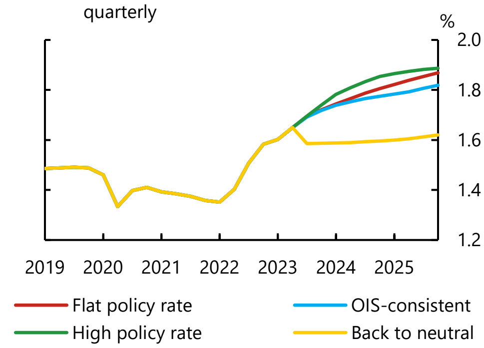


Sources: Statistics Canada and Bank of Canada calculations
Last observation: 2023Q2

In the first three scenarios, new mortgage issuances remain close to current levels until 2025, with the high-rate environment dampening demand (**Chart 6**). Naturally, the recovery in new debt acquisition is faster in the fourth scenario, as a sudden drop in interest rates generates significantly more demand for new loans.

Next, we compare the required mortgage payment rates in these four scenarios (**Chart 7**). Each rate is the sum of the required principal payment rate and the effective interest rate. In the first three scenarios, the required payment rates increase gradually over time as more borrowers renew their mortgages at higher rates. In the fourth scenario, the required payments drop considerably when the policy rate is set to neutral, largely due to a drop in required payments for the relatively small number of households with variable-rate and variable-payment mortgages. Beyond this initial drop, the required payment rate stabilizes as a balance emerges between households renewing at higher rates than their current term and those renewing at lower rates.

Chart 7: Required mortgage payment rates remain elevated in most scenarios
Payments, as a share of mortgage debt, quarterly



Sources: Statistics Canada and Bank of Canada calculations
Last observation: 2023Q2

Overall, the scenarios highlight an important lag in one of monetary policy's transmission channels. The impact of the tightening that began in early 2022 will continue to gradually materialize over the next few years. Therefore, barring a sudden drop in the policy rate, as shown in the fourth scenario, debt-servicing costs will likely continue to climb for many households, exerting a drag on discretionary spending.

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