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Staff Analytical Note/Note analytique du personnel—2024-3

Last updated: February 21, 2024

How changes in the share of constrained households affect the effectiveness of monetary policy

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DOI: https://doi.org/10.34989/san-2024-3 | **ISSN** 2369-9639

Key messages

- We quantify how a change in the share of constrained households—households with binding or close to binding credit constraints—affects the sensitivity of aggregate output to real interest rates changes.
- If the income of constrained households varies more than proportionately with changes in gross domestic product (GDP), an increase in the share of constrained households makes aggregate output more sensitive to real interest rates. This is the empirical case in Canada, where we estimate that a 1% decline in GDP leads to a decline of between 1.1% and 1.63% in the disposable income of constrained households.
- Monetary policy becomes more effective as the share of constrained households increases even though the direct effects of real interest rate changes are weakened. This is because stronger indirect effects, which operate through the income response of constrained households, more than compensate for the weaker direct effects.
- Our analysis shows that a lower share of constrained households in the first quarter of 2022 made monetary policy between 2% and 17% less effective than in the fourth quarter of 2019. This means that the effect of the same change in real interest rates on GDP was between 2% and 17% smaller in the first quarter of 2022 than it was in the fourth quarter of 2019.
- While the lower share of constrained households may have contributed to less effective monetary policy after the COVID-19 recession, effectiveness should revert to normal as the share of constrained households returns to pre-pandemic levels.
- Although we focus on the share of constrained household, other transmission channels are important for the evolution of monetary policy effectiveness over our period of analysis. These channels include the direct effects of real rates on mortgage and other debt payments.

Model

We use a stylized two-agent new Keynesian model based on Bilbiie (2008) to show how changing the share of constrained households affects the effectiveness of monetary policy.[1]

The economy has two types of households: constrained and unconstrained. Constrained households constitute a share $\lambda \in [0,1)$ of all households, while the unconstrained constitute the remaining share of $1 - \lambda$.

[1] While we use a two-agent new Keynesian model, the same forces operate in a richer heterogenous agent new Keynesian model. For an example, see Appendix E in Acharya and Dogra (2020).

The key difference between the two types of households is that unconstrained households can access credit markets to borrow and save, while constrained households cannot. Thus, changes in monetary policy directly affect unconstrained households but not constrained households.

To capture the idea that constrained and unconstrained households have different exposures to cyclical fluctuations in GDP, we assume that the sensitivity of income of constrained households to GDP is given by:

$$y_{h,t} = \chi y_t,$$

where lower case variables denote log deviations of the variable from its steady-state level. In the empirically relevant case with $\chi > 1$, a 1% decline in GDP results in a more than one-for-one decline in the income accruing to constrained households.

Monetary policy effectiveness as a function of the share of constrained households

In this section we provide a high-level description of the forces at work in the model. We relegate the formal analysis to the **Appendix**.

Suppose monetary policy raises real interest rates by dr. Because constrained households are unable to access credit markets, the higher real interest rate does not directly affect their consumption decisions. In contrast, unconstrained households respond to the higher interest rate by reducing their current consumption demand by $dc_{u,t} = -\gamma dr$, where γ denotes the intertemporal elasticity of substitution. Because unconstrained households account for $1 - \lambda$ of total consumption, total spending declines by $-(1 - \lambda)\gamma dr$. This is the first-round or direct effect, and the magnitude of this effect declines in λ. Increasing the share of constrained households means that fewer unconstrained households are directly affected by the higher real interest rates.

The initial decrease in spending reduces the income accruing to constrained households by $-\chi(1 - \lambda)\gamma dr$. Since constrained households spend all their income, this drop in income further reduces total spending by $-\lambda\chi(1 - \lambda)\gamma dr$. This second-round effect reduces the income of constrained households even further, causing them to reduce spending by another $-(\lambda\chi)^2(1 - \lambda)\gamma dr$. Each decline in spending reduces the income accruing to constrained households, which results in another round of declines. This is like the old Keynesian consumption multiplier. Summing up the effects of every round, the total decline in spending, and hence output, is given by:

$$dy = -\left\{\underbrace{(1-\lambda)\gamma dr}_{direct\ effect} + \underbrace{[\lambda\chi(1-\lambda)dr + (\lambda\chi)^2(1-\lambda)dr + \cdots]}_{indirect\ effect}\right\}$$
$$= \frac{-(1-\lambda)\gamma}{1-\lambda\chi}dr$$

This shows that if monetary policy changes real interest rates by dr, the direct effect of this change, $(1-\lambda)\gamma dr$, declines when the share of constrained households increases. This is simply because a higher λ implies a greater share of constrained households that are not directly affected by the change in real interest rates. However, to obtain the net effect of a change in real interest rates on output, one must also incorporate the impact of constrained households through indirect second-order effects. If we take both direct and indirect effects into account, we find that a higher share of constrained households implies a larger change in output for the same increase in real interest rates in the case where $\chi \geq 1$.[2]

In other words, when the elasticity of constrained households' income is greater than 1, a greater share of these households makes monetary policy more effective. This is because stronger indirect effects, operating through the spending response of constrained households, more than compensate for the weaker direct effects from a higher share of constrained households. When the elasticity of the disposable income of constrained households equals 1, the stronger indirect effects exactly offset the weaker direct effects, and the share of constrained households has no impact on the effectiveness of monetary policy.

Measuring monetary policy effectiveness in Canada

We now use the model described above to evaluate how monetary policy effectiveness has changed in Canada since the COVID-19 recession. For this exercise, we need two inputs:

- the elasticity of constrained household income to total output (χ)
- the path for the share of constrained households for this period (λ_t)[3]

In what follows, we discuss how we measure each of these ingredients.

Measuring the income elasticity of constrained households in Canada

Ideally, our measure of income elasticity would be an estimate of the elasticity of the disposable income (post tax and transfers) of constrained households in Canada to movements in aggregate income.[4] However, since we do not have an off-the-shelf estimate of this elasticity, we use two approaches to calibrate χ.

Our first approach is to use available estimates for the US economy. Using data on earnings from the Longitudinal Employer-Household Dynamics, Patterson (2023) finds that the

[2] Notice that the derivative with respect to the share of constrained households is $\frac{d}{d\lambda}\left(\frac{dy_t}{-dr_t}\right) = \frac{\chi-1}{(1-\lambda\chi)^2}\gamma \geq 0$ for $\chi \geq 1$.

[3] While the intertemporal elasticity of substitution γ is important for determining the total effect of monetary policy, its value has no impact on the relative effectiveness of monetary policy, which is what we are interested in.

[4] While we abstract from tax and transfers in the model, our measurement focuses on households' disposable income as this is the income measure that better maps to constrained household consumption choices and thus, to the amplification mechanism described in the previous section.

elasticity of pre-tax-and-transfer earnings to GDP is 1.95 for demographic groups with a high marginal propensity to consume (MPC). To transform this estimate into the elasticity of post-tax-and-transfer income, we must account for the progressivity of the Canadian tax-transfer system. We do so by following the approach of Heathcote, Storesletten and Violante (2017) and model the relationship between pre- and post-government income as a log-affine function:

$$\log\tilde{Y}_{h,t} = \tau_t + (1-\tau_P)\log Y_{h,t},$$

where $Y_{h,t}$ denotes household income before taxes and transfers, $\tilde{Y}_{h,t}$ denotes household disposable income, τ_t denotes the level of taxation and τ_P measures the progressivity of the tax-transfer system. This specification implies a simple relationship between the pre- and post-tax income elasticity:

$$\chi_{\text{post-tax}} = \tau_P + (1-\tau_P)\chi_{\text{pre-tax}}.$$

Setting $\tau_P = 0.3$ to capture the progressivity of the Canadian tax-and-transfer system, we arrive at a post-tax-and-transfer elasticity of $\chi = 1.63$.[5]

Our second approach follows Bilbiie, Primiceri and Tambalotti (2022) and identifies χ from the cyclicality in measures of earnings inequality. This is because, empirically, constrained households are more likely to earn lower wages. Consequently, a higher cyclical exposure for constrained households translates into countercyclical movements in income inequality.[6] Using the p90–p10 earnings percentile ratio for Canada reported in the Global Repository of Income Dynamics dataset as our measure for income inequality, we obtain an estimate of $\chi = 1.15$.[7] Transforming this estimate into the elasticity of post-tax-and-transfer income, we get $\chi = 1.10$.

These results imply that the estimate for the elasticity of post-tax-and-transfer income of Canadian constrained households is between 1.10 and 1.63. Since the range is greater than 1, our discussion in the previous section suggests that an increase in the share of constrained households tends to make monetary policy more effective.

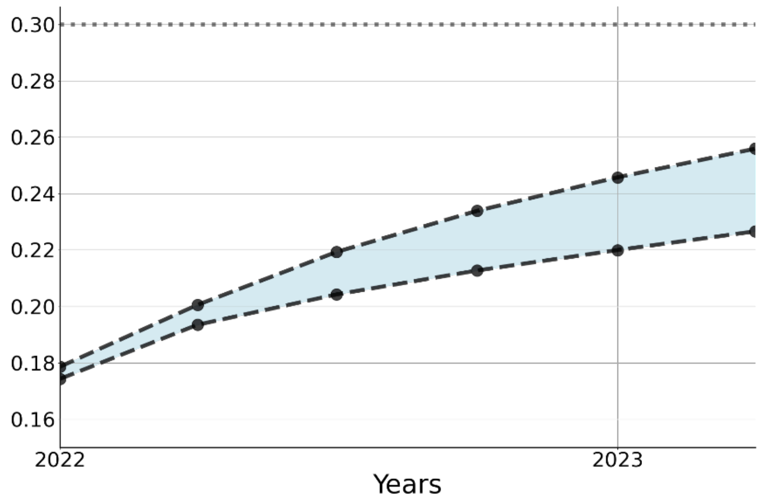
[5] The progressivity of 0.3 comes from Bank of Canada staff analysis on pre- and post-tax- and- transfers household income data from the Canadian public Census micro files. For a more comprehensive analysis of the progressivity of the Canadian income tax and transfer system, see Kurnaz and Yip (2022).

[6] In the model, the difference between the income of unconstrained and constrained households is given by $y_{u,t} - y_{h,t} = -\frac{(\chi-1)}{1-\lambda}y_t$. As a result, income inequality is countercyclical for $\chi > 1$.

[7] For more on Canadian data from the Global Repository of Income Dynamics, see Bowlus et al. (2022).

Chart 1: Share of constrained households

Projected paths for the share of constrained households



Note: The area between black dashed lines denotes the interval around our two scenarios for the evolution of real wages. The dotted grey line denotes the pre-pandemic share of constrained households of 0.30. Assumptions underlying each scenario are described in the text.

Source: Authors' calculations Last data plotted: 2023Q2

Share of constrained households

Now that we have an estimate for χ, we turn to our estimate for the time-series for the share of constrained households in Canada, λ_t. Since publicly available data are not yet available on households' balance sheets during and following the COVID-19 recession, we rely on a two-asset heterogenous agent new Keynesian model to project its time path. We base the model on Alves et. al. (2020) and calibrate it to the pre-pandemic Canadian economy of 2019.

We construct a time series of constrained households in two steps. First, we increase total household savings by 4.7% of total gross income. This reflects the effects on households' accumulated net savings between 2020 and 2022 from generous government income support policies and pandemic lockdowns.[8] We then use the model to project the evolution of the share of constrained households between the first quarter of 2022 and the second quarter of 2023. **Chart 1** shows the resulting share of constrained households λ_t under alternative scenarios for the evolution of real wages.[9]

Our results show the share of constrained households falls from a steady-state level of 30% in 2019 to less than 18% in the first quarter of 2022. This initial decline reflects our assumption

[8] We take this from the National Balance Sheet Accounts, which reports that households accumulated a total of Can$350 billion in excess savings from 2019Q4 to 2021Q4.

[9] In the first scenario, we assume that real wages hold constant at 2019 levels. In the second scenario, we depreciate real wages by accumulated inflation during 2021–22 and project that they return to 2019 levels by the end of the simulation.

that part of the 2020–21 accumulated savings went to households that, before the pandemic, held very little or no liquid savings. Over 2022 and later, households started to consume these savings as the economy reopened and income support policies were withdrawn. This caused the share of constrained households to go back to its pre-pandemic level.

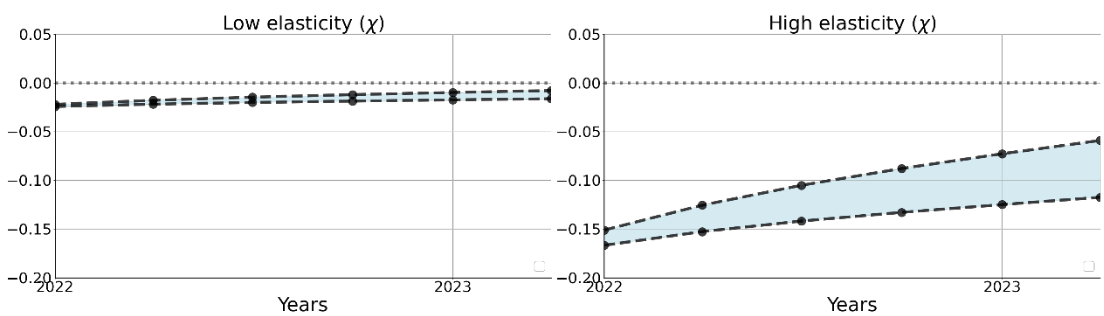
Time-varying effects of monetary policy

Given the elasticity of constrained income (χ) and time path for the share of constrained households (λ_t), we now measure the effectiveness of monetary policy over our projection horizon. **Chart 2** shows our results. The left panel is based on our lower estimate of the income elasticity of constrained households of 1.1, while the right panel uses our higher estimate of 1.63.

The smaller share of constrained households in 2022 led to a decline in monetary policy effectiveness, but the quantitative impact is relatively modest (**Chart 2**, both panels). For example, even though the share of constrained households in Canada falls from 30% to 18% in the first quarter of 2022 (a decline of 12 percentage points compared with pre-pandemic levels), the effectiveness of monetary policy declines by only 2% to 17%, depending on our estimate for the elasticity of disposable income for constrained households (χ). This means that the effect on GDP from the same change in real interest rates was between 83% and 98% as effective in the first quarter of 2022 than in the fourth quarter of 2019.

Chart 2: Effectiveness of monetary policy

Percent change of aggregate output to interest rate cut, relative to 2019Q4



Note: Numbers on the left axis in each panel denote the percent change in the response of aggregate output from a decrease in the interest rate relative to the value in the fourth quarter of 2019.

Source: Authors' calculations Last data plotted: 2023Q2

Decomposition into direct and indirect effects

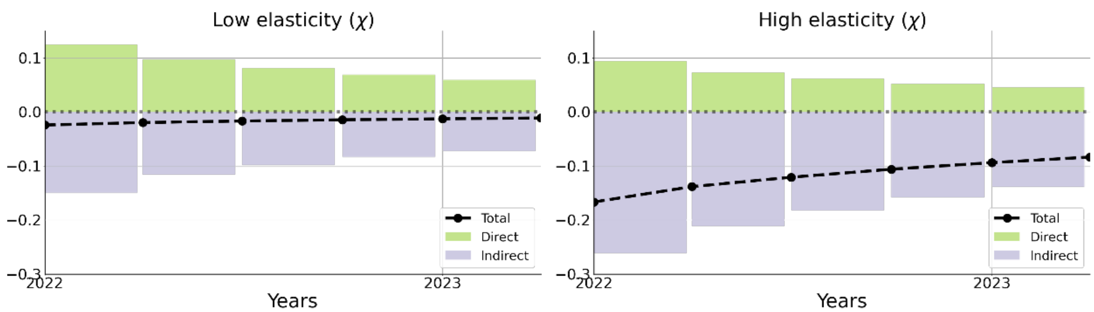
Our model also allows us to break down the contributions of direct and indirect effects to overall changes in monetary policy effectiveness. The lower share of constrained households relative to the fourth quarter of 2019 implies stronger direct (**Chart 3**, green bars) and weaker indirect effects (**Chart 3**, purple bars).[10]

[10] We report this breakdown only for one scenario, but results are similar under alternative paths.

While overall changes in the effectiveness of monetary policy are modest, the impact on direct and indirect effects are substantial. The direct effects are strongest in the first quarter of 2022 when the share of constrained households is at its lowest value (**Chart 3**). For example, when the elasticity of the disposable income of constrained households equals 1.10, the 2% decrease in the effectiveness of monetary policy during the first quarter of 2022 is the net effect of a 15% decrease from indirect effects together with a 13% increase from direct effects.

Chart 3: Contributions of direct and indirect channels to monetary policy effectiveness

Percent change of aggregate output to interest rate cut, relative to 2019Q4



Note: Numbers on the left axis in each panel denote the percent change in the response of aggregate output from a decrease in the interest rate relative to the value in the fourth quarter of 2019. The dashed line denotes changes to total response, while bars show the contribution of direct (green) and indirect channels (purple).

Source: Authors' calculations | Last data plotted: 2023Q2

Conclusion

The quantitative importance of a change in the share of constrained households for monetary policy effectiveness depends on estimates for the elasticity of the disposable income of constrained households. Based on our range of estimates, we find that monetary policy was between 2% and 17% less effective in the first quarter of 2022 compared with 2019, which is a relatively modest reduction in effectiveness.

While we measure the effectiveness of monetary policy in terms of the sensitivity of output to a change in real interest rates, similar conclusions can be drawn about inflation. As long as the slope of the Phillips curve remained unchanged over this period, the lower share of constrained households should have also reduced the responsiveness of inflation to changes in real interest rates.

Appendix: Deriving the investment-saving equation

Unconstrained households

Unconstrained households have standard preferences, and their problem can be written as:

$$\max_{\{C_{u,t},A_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{u,t}^{1-1/\gamma}}{1-1/\gamma}$$

s.t.

$$P_t C_{u,t} + \tfrac{1}{1+i_t} A_t = P_t Y_{u,t} + A_{t-1},$$

where A_t denotes how much the unconstrained households choose to save in nominal terms at date t. $P_t Y_{u,t}$ denotes the nominal income accruing to an unconstrained household, and P_t denotes the aggregate price level.

Constrained households

Unlike the unconstrained, constrained households cannot access asset markets, and their problem can be written as:

$$\max_{\{C_{h,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{h,t}^{1-1/\gamma}}{1-1/\gamma}$$

s.t.

$$P_t C_{h,t} = P_t Y_{h,t}.$$

Thus, constrained households consume their entire income $C_{h,t} = Y_{h,t}$, where $P_t Y_{h,t}$ denotes the nominal income accruing to a constrained household.

Equilibrium and the aggregate Euler equation

In equilibrium, total consumption must equal GDP:

$$\lambda C_{h,t} + (1-\lambda) C_{u,t} = Y_t.$$

Since constrained households consume their entire income ($C_{h,t} = Y_{h,t}$) in the goods market clearing and rearranging, we get:

$$C_{u,t} = \tfrac{Y_t - \lambda Y_{h,t}}{1-\lambda}.$$

Taking a log-linear approximation of this condition around the steady-state, we can write:

$$c_{u,t} = \frac{1-\lambda\chi}{1-\lambda} y_t,$$

where we have substituted $y_{h,t}$ with χy_t following the assumption on heterogeneous exposures. This gives us an equation linking the log-deviations of consumption by the unconstrained household to deviations in aggregate income.

The consumption of the unconstrained households must also satisfy the Euler equation (already log-linearized):

$$c_{u,t} = -\gamma(i_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t c_{u,t+1}.$$

Substituting the expression for $c_{u,t}$ and rearranging:

$$y_t = -\frac{1-\lambda}{1-\lambda\chi}\gamma(i_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t y_{t+1}.$$

The term multiplying the real interest rate ($r_t = i_t - \mathbb{E}_t\pi_{t+1}$) captures the effectiveness of monetary policy, which depends on the share of constrained households (λ), the sensitivity of their income to GDP (χ) and the intertemporal elasticity of substitution of the household (γ). A one-time temporary unit increase in the real interest rate engineered by monetary policy lowers GDP by:

$$\frac{dy_t}{dr_t} = -\frac{1-\lambda}{1-\lambda\chi}\gamma,$$

which is the same as the expression in the main text.

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