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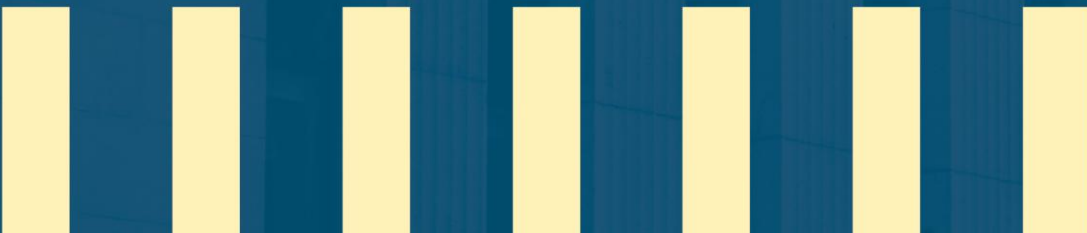
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Optimal Conventional and Unconventional Monetary Policy Mix*

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Abstract

This paper examines the optimal coordination of conventional and unconventional monetary policy tools in an economy with heterogeneous households and mortgage debt. We build a dynamic stochastic general equilibrium (DSGE) model featuring three household types—savers, borrowers, and renters—and include housing investment, long-term fixed-rate mortgages, and a housing production sector. The central bank controls both the short-term interest rate and the long-term rate by adjusting the maturity composition of government bonds. We show that household heterogeneity significantly alters the optimal policy response to macroeconomic shocks. Specifically, after a cost-push shock, optimal policy calls for increasing the short-term rate to contain inflation while simultaneously lowering the long-term rate to ease financial pressures on indebted households and renters. This combination speeds up the recovery of investment and output, stabilizes inflation, but exacerbates consumption inequality. By contrast, in a representative-agent model, the optimal response is to raise both rates. Our results underscore the need to consider distributional consequences in monetary policy design and indicate that yield curve control can serve as a valuable tool in heterogeneous economies.

Keywords: Monetary policy, household heterogeneity, yield curve control.

JEL Classification: E40, E43, E52.

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1 Introduction

The post-pandemic inflation surge in advanced economies reignited debates over the appropriate mix of conventional and unconventional monetary policy tools. In the United States and the Euro Area, central banks reacted by first ending asset purchases and then raising short-term policy rates. This sequence of actions led to a sharp increase in long-term interest and mortgage rates, amplifying the financial burden on indebted households. These developments raise a key question: should conventional and unconventional monetary policy instruments move in the same direction when responding to inflationary pressures, especially in economies where household debt is widespread?

This paper addresses this question by studying the optimal design of monetary policy in an economy with heterogeneous households and long-term mortgage debt. We develop a dynamic stochastic general equilibrium (DSGE) model featuring three types of households—patient savers, impatient borrowers, and hand-to-mouth renters—and incorporating housing investment, fixed-rate long-term mortgages, and a housing production sector. The central bank in our model controls both the short-term nominal interest rate and the long-term rate, through adjustments in the maturity composition of government bonds held by the private sector, summarized by the relative supply of long-term bonds. This setup enables us to study how standard interest rate policy and unconventional instruments—such as quantitative easing and yield curve control—jointly contribute to macroeconomic stabilization.

Our analysis reveals that household heterogeneity significantly alters the optimal response of monetary policy to macroeconomic shocks. Specifically, following an adverse cost-push shock, we find that it is optimal for the central bank to raise the short-term interest rate to contain inflationary pressures while at the same time lowering the long-term rate to ease the negative wealth effects on indebted households and reduce the rental burden for renters. This policy mix supports a faster recovery in housing investment and aggregate investment, but also widens the consumption inequality. In contrast, when household heterogeneity is removed from the model—by eliminating mortgage debt and assuming a representative household—the optimal policy response becomes more conventional: both short- and long-term rates are raised in response to inflationary shocks.

Methodologically, we derive a second-order approximation to household utility and construct a social welfare function using Pareto weights. We solve for the optimal policy under commitment, allowing the central bank to choose both the short-term rate and the relative supply of long-term bonds. Our results suggest that short-term tightening combined with long-term easing can better stabilize inflation while also limiting the contraction in housing markets and mitigating financial stress among vulnerable households.

The paper contributes to the optimal monetary policy literature in three main ways. First, it extends the standard New Keynesian framework by incorporating long-term fixed-rate mortgage contracts and a housing production sector, allowing for a richer analysis of monetary policy transmission through housing. Second, it introduces a welfare-based criterion that captures the distributional effects of monetary policy on heterogeneous households. Third, it offers new insights into the role of

unconventional monetary policy in highly indebted household environments.

This paper extends several strands of the literature. It is closely related to Iacoviello (2005), who introduces housing and collateral constraints into DSGE models, and Kydland et al. (2016), who stresses long-term mortgage contracts. We incorporate both fixed-rate mortgages and a housing production sector, which lets us distinguish between the flow and stock of household debt. Our model is also related to Chen et al. (2012) and Gertler and Karadi (2011, 2013), who study the macroeconomic effects of large-scale asset purchases, but we depart from them by explicitly analyzing the welfare implications of monetary policy in a heterogeneous-agent setting. In doing so, we contribute to the growing literature on the distributional effects of monetary policy (e.g., Cloyne et al., 2019) and on optimal policy under heterogeneity (see Bilbiie (2008) and Bilbiie (2025)). In these models, heterogeneity breaks the sufficiency of the aggregate output gap and raises the welfare cost of real activity fluctuations, thereby amplifying the weight placed on output stabilization relative to the representative-agent benchmark. In our framework, heterogeneous balance sheets and borrowing constraints further strengthen this mechanism, making coordination between short- and long-term interest rates crucial for welfare-improving stabilization.

More recently, Alexandrov and Brunnermeier (2026) show that while interest rate policy can achieve efficient aggregate outcomes near steady state, similar to our findings, active balance-sheet management becomes essential when shocks push the economy away from it because it prepares the economy’s risk exposure for future rate movements. Interest rates create bond-price fluctuations that steer consumption and wealth, while balance-sheet policy ensures that agents hold the appropriate level of risk to make these rate changes effective. Overall, the optimal mix implies that larger central-bank balance sheets require more aggressive future rate adjustments, and increases in idiosyncratic risk call for rate cuts and expanded bond holdings.

A related strand of the literature studies the optimal mix of monetary and fiscal policy under heterogeneity (see Bilbiie et al. (2024) and McKay and Wolf (2022)), showing that fiscal transfers can, alongside monetary policy, relax the trade-off between macroeconomic stabilization and inequality. In our framework, yield curve control also generates redistribution by lowering debt-service costs and relaxing borrowing constraints, thus shifting resources from savers to borrowers. However, long-term rate management operates through asset prices and borrowing conditions rather than directly through disposable income, as fiscal transfers do. Thus, while the additional monetary instrument improves aggregate stabilization, it does not eliminate distributional tensions, highlighting the second-best nature of unconventional monetary policy, especially when fiscal space is limited.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the monetary transmission with household heterogeneity. Section 4 derives optimal targeting rules and discusses the implications of heterogeneity for the design of monetary policy. Section 5 outlines the calibration strategy. Section 6 reports the quantitative results, including impulse responses under different policy regimes. Section 7 concludes with policy implications and directions for future research.

2 Model

The model is a closed-economy DSGE model with housing and household debt. The economy has three types of households: patient (savers), impatient (borrowers), and renter (hand-to-mouth agents) households, as in Cloyne et al. (2019). We allow for long-term fixed-rate mortgages and distinguish between the flow and stock of household debt, following Kydland et al. (2016), Garriga et al. (2017), and Alpanda and Zubairy (2017). The government issues short- and long-term bonds to finance deficits, and the long rate affects aggregate demand through its impact on borrowers' mortgage interest burden. The central bank jointly controls short and long rates: it sets the short rate via a Taylor rule and the long rate by adjusting the stock of long-term government bonds. Production is standard.

2.1 Households

The economy is populated by three types of infinitely lived agents $a \in \{P, I, R\}$, whose intertemporal preferences over consumption $c_{a,t}$, housing $h_{a,t}$, and labor $n_{a,t}$ are given by:

$$E_t \sum_{\tau=t}^{\infty} \beta_a^{\tau-t} e^{v_t} \left(\log c_{a,\tau} + \xi \log h_{a,\tau} - \frac{n_{a,\tau}^{1+\vartheta}}{1+\vartheta} \right), \quad (1)$$

where t indexes time, ξ is the weight on housing, ϑ is the inverse Frisch elasticity of labor supply, and β_a is the type-specific time-discount parameter with $\beta_I < \beta_P < 1$.¹ The preference shock v_t is common to all households and follows a stationary AR(1) process.

2.1.1 Patient households (savers)

Patient households (P) are the savers in the economy; they accumulate owner-occupied and rental housing, capital, short- and long-term government bonds, and extend mortgage loans to borrowers. Their period budget constraint is given by:

$$c_{P,t} + q_{h,t}(i_{hP,t} + i_{hR,t}) + q_{k,t}i_{k,t} + \frac{Q_{S,t}}{P_t}b_{S,t} + \frac{Q_{L,t}}{P_t} \left(b_{L,t} - \frac{\kappa}{\pi_t}b_{L,t-1} \right) + \frac{L_t}{P_t} \leq w_{P,t}n_{P,t} \quad (2)$$

$$+ r_{h,t}h_{R,t} + r_{k,t}k_{t-1} + \frac{P_{t-1}}{P_t}(b_{S,t-1} + b_{L,t-1}) + \left(R_{t-1}^d + \kappa_d \right) \frac{D_{t-1}}{P_t} + \frac{\Xi_t}{P_t} - tax_t - \Gamma_t \left(\frac{Q_{L,t}}{P_t}b_{L,t} + \frac{D_t}{P_t} \right)$$

where P_t denotes the aggregate price level, while Ξ_t refers to nominal profits of intermediate-goods producers and tax_t denotes real taxes paid to the government, both in lump-sum fashion. $w_{P,t}$ is the real wage on the labor services of patient households, $q_{h,t}$ and $q_{k,t}$ denote the (real) relative prices of housing and capital, respectively, while $i_{hP,t}$, $i_{hR,t}$ and $i_{k,t}$ are the patient households' investment purchases in owner-occupied housing, rental housing, and capital. The related laws of motion for

¹Since renter households are hand-to-mouth, β_R is irrelevant for the dynamics. We set $\beta_R = \beta_I$ in the welfare analysis.

the stock of these assets are given by:

$$h_{P,t} = (1 - \delta_h) h_{P,t-1} + i_{hP,t}, \quad (3)$$

$$h_{R,t} = (1 - \delta_h) h_{R,t-1} + i_{hR,t}, \quad (4)$$

$$k_t = (1 - \delta_k) k_{t-1} + i_{k,t}, \quad (5)$$

where δ_h and δ_k are depreciation rates of housing and capital. Similarly, $r_{h,t}$ and $r_{k,t}$ denote the rental income that patient households receive from their rental housing and capital holdings.

$Q_{S,t}$ and $Q_{L,t}$ denote the nominal prices of short- and long-term government bonds issued in period t , while $b_{S,t}$ and $b_{L,t}$ denote outstanding quantities of these bonds. A short bond issued in period $t - 1$ pays P_{t-1} in nominal terms in period t , while a long term bond issued in period $t - 1$ pays decaying coupon payments of $P_{t-1}, \kappa P_{t-1}, \kappa^2 P_{t-1}, \dots$ in periods $t, t + 1, t + 2, \dots$, respectively. Note that in period t , the *ex coupon* nominal price of a long-term bond issued in period $t - 1$ is given by $\frac{\kappa}{\Pi_t} Q_{L,t}$, where $\Pi_t = P_t/P_{t-1}$ is the inflation factor, which allows us to include long term bonds in the households' budget constraint above in recursive fashion.²

L_t is the amount of new lending extended to impatient households, while D_{t-1} denotes the stock of mortgage debt carried over from the previous period. On the latter, the patient households receive a fraction κ_d of principal payments and an average interest payment of R_{t-1}^d from borrowers. The laws of motion for the stock of debt and the average interest charged on the debt are respectively given by:

$$\frac{D_t}{P_t} = (1 - \kappa_d) \frac{D_{t-1}}{P_t} + \frac{L_t}{P_t}, \quad (6)$$

and

$$\frac{D_t}{P_t} R_t^d = (1 - \kappa_d) \frac{D_{t-1}}{P_t} R_{t-1}^d + \frac{L_t}{P_t} R_t^l. \quad (7)$$

where R_t^l is current nominal fixed rate on new mortgage loans. Note that R_t^d is a choice variable for the patient households (joint with their choice of newly extended loans, L_t), while they take as given the economy-wide current fixed mortgage rate, R_t^l .

Following Chen et al. (2012), agents pay a transaction cost Γ_t on their long-term debt holdings and this cost is given by:

$$\Gamma_t = \Gamma_1 \left(\frac{Q_{L,t} b_{L,t}}{Q_{S,t} b_{S,t}} \right)^{\Gamma_2} \exp(\tilde{\varepsilon}_{\Gamma,t}) - 1, \quad (8)$$

where Γ_1 and Γ_2 are the level and elasticity parameters, and $\tilde{\varepsilon}_{\Gamma,t}$ is an exogenous AR(1) process. The stock of mortgage debt faces the same transaction cost Γ_t as long-term government bonds, ensuring that changes in long-term rates affect the interest burden of borrower households and thus their aggregate demand.

Patient households maximize utility subject to their budget constraint and No-Ponzi conditions. The optimality conditions for labor supply, owner-occupied housing, rental housing, and capital are,

²Note that $\kappa = 0$ reduces the long-term bond to a short-term bond.

respectively, given by:

$$n_{P,t}^{\vartheta} = \lambda_{P,t} w_{P,t}, \quad (9)$$

$$q_{h,t} = \xi_h \frac{c_{P,t}}{h_{P,t}} + E_t \left[\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} (1 - \delta_h) q_{h,t+1} \right], \quad (10)$$

$$q_{h,t} = r_{h,t} + E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) (1 - \delta_h) q_{h,t+1} \right], \quad (11)$$

$$q_{k,t} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) [(1 - \delta_k) q_{k,t+1} + r_{k,t+1}] \right], \quad (12)$$

where $\lambda_{P,t} = 1/c_{P,t}$ denotes the Lagrange multiplier on the period budget constraint. Similarly, optimal holdings of short- and long-term government bonds satisfy:

$$q_{S,t} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \frac{1}{\Pi_{t+1}} \right], \quad (13)$$

$$(1 + \Gamma_t) q_{L,t} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \frac{1 + \kappa q_{L,t+1}}{\Pi_{t+1}} \right], \quad (14)$$

and the corresponding nominal yields on short- and long-term bonds are given by:

$$1 + R_t = \frac{1}{q_{S,t}}, \quad (15)$$

$$1 + R_{L,t} = \frac{1}{q_{L,t}} + \kappa. \quad (16)$$

Finally, the optimality conditions for the flow and stock of mortgage loans and the average interest on them are given, respectively, by:

$$1 + \Gamma_t = \Omega_{dP,t} + \Omega_{rP,t} R_t^l, \quad (17)$$

$$\Omega_{dP,t} + \Omega_{rP,t} R_t^d = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \frac{R_t^d + \kappa_d - (1 - \kappa_d) \Gamma_{t+1} + (1 - \kappa_d) [\Omega_{dP,t+1} + \Omega_{rP,t+1} R_t^d]}{\Pi_{t+1}} \right], \quad (18)$$

$$\Omega_{rP,t} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \frac{1 + (1 - \kappa_d) \Omega_{rP,t+1}}{\Pi_{t+1}} \right], \quad (19)$$

where $\Omega_{dP,t}$ and $\Omega_{rP,t}$ are the Lagrange multipliers on the laws of motion for the mortgage debt stock and the average interest on debt given in (6) and (7), respectively. Note that with full principal repayment for each period (i.e., $\kappa_d = 1$), we have $R_t^d = R_t^l$ for all t , and the above expressions would collapse to a familiar expression:

$$1 + \Gamma_t = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \frac{1 + R_t^l}{\Pi_{t+1}} \right], \quad (20)$$

with Γ_t capturing the spread on mortgages relative to the short rate. In our general case, this holds

only at the steady state with $R^d = R^l = R_L$ and:³

$$\frac{1 + R^d}{1 + R} = 1 + \Gamma. \quad (21)$$

2.1.2 Impatient households (borrowers)

Impatient households borrow from patient households to finance their investment in owner-occupied housing and hold no other assets. Their period budget constraint is:

$$c_{I,t} + q_{h,t} i_{hI,t} + \left(R_{t-1}^d + \kappa_d \right) \frac{D_{t-1}}{P_t} \leq w_{I,t} n_{I,t} + \frac{L_t}{P_t}, \quad (22)$$

where $w_{I,t}$ is their real wage and $i_{hI,t}$ is their residential investment purchases. Their housing stock evolves according to:

$$h_{I,t} = (1 - \delta_h) h_{I,t-1} + i_{hI,t}. \quad (23)$$

Each period, impatient households face a borrowing constraint on new loans:

$$\frac{L_t}{P_t} \leq \phi q_{h,t} i_{hI,t}, \quad (24)$$

where ϕ is the regulatory LTV ratio.

Their optimality conditions for labor supply and owner-occupied housing are:

$$n_{I,t}^\vartheta = \lambda_{I,t} w_{I,t}, \quad (25)$$

$$(1 - \phi \mu_t) q_{h,t} = \xi_h \frac{c_{I,t}}{h_{I,t}} + E_t \left[\left(\beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) (1 - \delta_h) (1 - \phi \mu_{t+1}) q_{h,t+1} \right], \quad (26)$$

where $\lambda_{I,t}$ and μ_t are the multipliers on the budget and borrowing constraints, respectively. The optimality conditions for the flow and stock of mortgage loans and their average interest rate are given by:

$$1 - \mu_t = \Omega_{dI,t} + \Omega_{rI,t} R_t^l, \quad (27)$$

$$\Omega_{dI,t} + \Omega_{rI,t} R_t^d = E_t \left[\left(\beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) \frac{R_t^d + \kappa_d + (1 - \kappa_d) [\Omega_{dI,t+1} + \Omega_{rI,t+1} R_t^d]}{\Pi_{t+1}} \right], \quad (28)$$

$$\Omega_{rI,t} = E_t \left[\left(\beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) \frac{1 + (1 - \kappa_d) \Omega_{rI,t+1}}{\Pi_{t+1}} \right], \quad (29)$$

where $\Omega_{dI,t}$ and $\Omega_{rI,t}$ are the Lagrange multipliers on the laws of motion for the mortgage debt stock and the average interest on debt given in (6) and (7), respectively. Note again that with full principal

³We provide a full derivation of the steady state in Appendix C.

repayment each period (i.e., $\kappa_d = 1$), the above expressions would collapse to the more familiar:

$$1 - \mu_t = E_t \left[\left(\beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) \frac{1 + R_t^l}{\Pi_{t+1}} \right]. \quad (30)$$

2.1.3 Renter households

Renter households are hand-to-mouth, and consume all their wage income on consumption goods and rental housing as:

$$c_{R,t} + r_{h,t}h_{R,t} = w_{R,t}n_{R,t}, \quad (31)$$

where $w_{R,t}$ is their real wage. Their optimality conditions imply the following static expressions:

$$n_{R,t}^\vartheta = \lambda_{R,t}w_{R,t}, \quad (32)$$

$$r_{h,t} = \xi_h \frac{c_{R,t}}{h_{R,t}}, \quad (33)$$

where $\lambda_{R,t} = 1/c_{R,t}$ denotes the Lagrange multiplier on the period budget constraint.

2.2 Production

The production side of the model is standard. A unit continuum of intermediate goods producers supplies slightly differentiated products and faces Rotemberg-type price adjustment costs. Perfectly competitive final goods producers aggregate these into a final good used for consumption, investment in capital and housing, and government spending. We also introduce adjustment costs for "investment goods producers" so that the relative price of investment goods may deviate from that of consumption goods.

2.2.1 Final goods producers

Final goods producers are perfectly competitive and aggregate the differentiated intermediate goods $y_t(j)$ for $j \in [0, 1]$ into a final good y_t using a standard Dixit-Stiglitz aggregator:

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\eta_{y,t}-1}{\eta_{y,t}}} dj \right)^{\frac{\eta_{y,t}}{\eta_{y,t}-1}}, \quad (34)$$

where $\eta_{y,t}$ is the elasticity of substitution between the differentiated goods that follows a stationary $AR(1)$ process. The resulting demand curve facing each intermediate goods firm is thus given by:

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\eta_{y,t}} y_t. \quad (35)$$

The final goods can then be used for consumption, investment in capital and housing, government expenditure, and resources spent on price adjustment costs by intermediate goods producers. Price

adjustment costs are described in more detail in the next subsection. Note that aggregate consumption and residential investment are the sum of the three types of households described previously as:

$$c_t = c_{P,t} + c_{I,t} + c_{R,t}, \quad (36)$$

$$i_{h,t} = i_{hP,t} + i_{hI,t} + i_{hR,t}. \quad (37)$$

2.2.2 Intermediate goods producers

There is a unit measure of monopolistically competitive intermediate goods producers indexed by j . Their technology is described by the following production function:

$$y_t(j) = z_t k_{t-1}(j)^\alpha \left[n_{P,t}(j)^{\theta_P} n_{I,t}(j)^{\theta_I} n_{R,t}(j)^{\theta_R} \right]^{1-\alpha} - f, \quad (38)$$

where f denotes the fixed cost, α is the share of capital in the overall production, and θ_P , θ_I , and θ_R denote the shares of patient, impatient, and renter households in the labor input, respectively, with $\theta_P + \theta_I + \theta_R = 1$. The aggregate productivity shock, z_t , follows an AR(1) process.

Firm j 's profits at period t are given by:

$$\begin{aligned} \frac{\Xi_t(j)}{P_t} = & \frac{P_t(j)}{P_t} y_t(j) - r_{k,t} k_{P,t-1}(j) - w_{P,t} n_{P,t}(j) \\ & - w_{I,t} n_{I,t}(j) - w_{R,t} n_{R,t}(j) - \frac{\kappa_p}{2} \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 y_t, \end{aligned} \quad (39)$$

where price stickiness is introduced through quadratic adjustment costs, as in Rotemberg (1982), with κ_p as the level parameter.

A firm's objective is to choose the input quantities, output, and price each period to maximize the present value of profits (using the patient households' stochastic discount factor) subject to the demand from goods aggregators. The firm's optimality conditions with respect to its inputs are:

$$\Omega_t \alpha \frac{y_t + f}{k_{t-1}} = r_{k,t}, \quad (40)$$

$$\Omega_t (1 - \alpha) \theta_P \frac{y_t + f}{n_{P,t}} = w_{P,t}, \quad (41)$$

$$\Omega_t (1 - \alpha) \theta_I \frac{y_t + f}{n_{I,t}} = w_{I,t}, \quad (42)$$

$$\Omega_t (1 - \alpha) \theta_R \frac{y_t + f}{n_{R,t}} = w_{R,t}, \quad (43)$$

where Ω_t is the Lagrange multiplier on the demand function of final goods producers given in (35) and captures marginal cost of firms. The optimality with respect to pricing implies the following New Keynesian Phillips curve:

$$\left(\frac{\Pi_t}{\Pi} - 1\right) \frac{\Pi_t}{\Pi} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \frac{y_{t+1}}{y_t} \right] - \frac{\eta_{y,t} - 1}{\kappa_p} \left(1 - \frac{\eta_{y,t}}{\eta_{y,t} - 1} \Omega_t \right). \quad (44)$$

2.2.3 Capital and Housing producers

Investment goods producers are perfectly competitive, and they purchase $i_{k,t}$ and $i_{h,t}$ units of new investment goods from final goods firms at a relative price of 1, and turn these into effective units of installed capital and housing that can be purchased by end-users at relative prices of $q_{k,t}$ and $q_{h,t}$, respectively. The change in relative prices is due to adjustment costs in investment similar to Christiano et al. (2005) and Smets and Wouters (2007), which can potentially differ between capital and housing. The investment-goods producers' objective is thus to maximize:

$$E_t \sum_{\tau=t}^{\infty} \beta_P^{\tau-t} \frac{\lambda_{P,\tau}}{\lambda_{P,t}} \left\{ \left[1 - \frac{\kappa_{ik}}{2} \left(\frac{i_{k,\tau}}{i_{k,\tau-1}} - 1 \right)^2 \right] q_{k,\tau} i_{k,\tau} + \left[1 - \frac{\kappa_{ih}}{2} \left(\frac{i_{h,\tau}}{i_{h,\tau-1}} - 1 \right)^2 \right] q_{h,\tau} i_{h,\tau} - (i_{k,\tau} + i_{h,\tau}) \right\}, \quad (45)$$

where κ_{ik} and κ_{ih} are the investment adjustment cost parameters, and future profits are discounted using the patient households' stochastic discount factor. The first-order conditions for capital and residential investment yield the Tobin's marginal q expressions, which are summarized as follows:

$$i_{k,t} : \left(\frac{i_{k,t}}{i_{k,t-1}} - 1 \right) \frac{i_{k,t}}{i_{k,t-1}} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \left(\frac{i_{k,t+1}}{i_{k,t}} - 1 \right) \left(\frac{i_{k,t+1}}{i_{k,t}} \right)^2 \frac{q_{k,t+1}}{q_{k,t}} \right] + \frac{1}{\kappa_{ik}} \left(1 - \frac{1}{q_{k,t}} \right) - \frac{1}{2} \left(\frac{i_{k,t}}{i_{k,t-1}} - 1 \right)^2 \quad (46)$$

$$i_{h,t} : \left(\frac{i_{h,t}}{i_{h,t-1}} - 1 \right) \frac{i_{h,t}}{i_{h,t-1}} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \left(\frac{i_{h,t+1}}{i_{h,t}} - 1 \right) \left(\frac{i_{h,t+1}}{i_{h,t}} \right)^2 \frac{q_{h,t+1}}{q_{h,t}} \right] + \frac{1}{\kappa_{ih}} \left(1 - \frac{1}{q_{h,t}} \right) - \frac{1}{2} \left(\frac{i_{h,t}}{i_{h,t-1}} - 1 \right)^2 \quad (47)$$

2.3 Monetary and fiscal policies

The consolidated government budget constraint is given by:

$$g_t + \frac{P_{t-1}}{P_t} (b_{S,t-1} + b_{L,t-1}) = tax_t + \frac{Q_{S,t}}{P_t} b_{S,t} + \frac{Q_{L,t}}{P_t} \left(b_{L,t} - \frac{\kappa}{\Pi_t} b_{L,t-1} \right), \quad (48)$$

where the bond quantities above refer to privately-held bonds in circulation, and therefore exclude potential purchases by the central bank. The lump-sum taxes collected from patient households (relative to steady state output y) are assumed to respond positively to aggregate output and to the government's debt level so that the government cannot run a Ponzi scheme:

$$\frac{tax_t}{y} = \tilde{\tau} \left(\frac{y_t}{y} \right)^{\tau_y} \left(\frac{q_{S,t-1} b_{S,t-1} + q_{L,t-1} b_{L,t-1}}{q_S b_S + q_L b_L} \right)^{\tau_b}, \quad (49)$$

where $\tilde{\tau}$ is a level parameter, and τ_y and τ_b determine the elasticity of taxes to income and government debt, respectively. Government expenditure, g_t , follows an exogenous AR(1) process given by:

$$\log g_t = (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}. \quad (50)$$

The central bank targets the short-term interest rate and the slope of the yield curve. To achieve the latter, the central bank must adjust the maturity structure of government bonds, because the transaction cost term, Γ_t —which determines the spread—depends on households' holdings of long- relative to short-term government bonds. The target for the short-term nominal interest rate is determined using a standard Taylor rule:

$$1 + R_t = (1 + R_{t-1})^\rho \left[(1 + R) \left(\frac{\Pi_t}{\Pi} \right)^{a_\pi} \left(\frac{y_t}{y} \right)^{a_y} \right]^{1-\rho} \tilde{\varepsilon}_{R,t}, \quad (51)$$

where ρ determines the extent of interest rate smoothing, R is the steady-state value of the short rate, and a_π and a_y are the response coefficients for inflation and the output gap, respectively. $\tilde{\varepsilon}_{R,t}$ denotes the monetary policy shock, which follows an AR(1) process.

Finally, we define variable $\gamma_{b,t} = Q_{L,t} b_{L,t} / Q_{S,t} b_{S,t}$ as the supply of long-term bonds relative to short-term bonds, which follows a log-stationary AR(1) process:

$$\hat{\gamma}_{b,t} = \rho_{\gamma_b} \hat{\gamma}_{b,t-1} + \varepsilon_{\gamma_b,t} \quad (52)$$

where $\varepsilon_{\gamma_b,t}$ is an i.i.d. process. Similar to Chen et al. (2012), we treat large-scale asset purchase programs as shocks to the relative supply of long-term bonds, since by construction changes in $\gamma_{b,t}$ - whether expected or not - alter the composition of outstanding government liabilities.

The model's equilibrium is defined as prices and allocations such that households maximize the

discounted present value of their utility, all firms maximize the discounted present value of profits subject to their constraints, and all markets clear. The resource constraint of the economy reads:

$$c_t + i_{h,t} + i_{k,t} + g_t = y_t - \frac{\kappa_p}{2} \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma_p} \pi^{1-\varsigma_p}} - 1 \right)^2 y_t - \Gamma_t (q_{L,t} b_{L,t} + d_t). \quad (53)$$

3 Dissecting the monetary transmission mechanism

In this section, we examine the transmission channels of monetary policy. The short-term rate affects the consumption of patient households directly as they are the only holders of short-term debt, while its expected future path affects the long-term rate, $R_{L,t}$, and thereby the average mortgage rate, R_t^d , they charge on impatient households' stock of debt. Combining the first-order conditions for short-term bonds, $b_{S,t}$, long-term bonds, $b_{L,t}$, and log-linearizing, we obtain:

$$\hat{R}_{L,t} = \frac{1}{1 + \kappa_{QL}} \sum_{s=0}^{\infty} \left(\frac{\kappa_{QL}}{1 + \kappa_{QL}} \right)^s \left[\hat{R}_{t+s} + \frac{\hat{\Gamma}_{t+s}}{1 + \Gamma} \right] \quad (54)$$

where variables with a hat denote log deviations from the steady state. Equation (54) shows that the long-term rate depends on the current and expected future paths of the short-term policy rate, \hat{R}_t , and the relative supply of long-term bonds, $\hat{\Gamma}_t$. The latter is summarized by:

$$\hat{\Gamma}_t = \Gamma_2 \hat{\gamma}_{b,t} + \tilde{\varepsilon}_{\Gamma,1} \quad (55)$$

where $\hat{\gamma}_{b,t} = \widehat{q_L b_{L,t}} - \widehat{q_S b_{S,t}}$.⁴ A drop, for instance, in the relative supply of long-term bonds and a rebalancing towards short-term bonds lowers the long-term rate. As we show in the next section, the central bank can adjust this ratio to influence the yield curve and shield the economy from shocks. The resulting changes in the long-term rate alter the present value of patient households' wealth and thus their consumption, since they are the only holders of government bonds.

Impatient households instead borrow from patient households, so monetary policy indirectly affects their mortgage rates. Combining the patient households' first-order conditions for short- and long-term bonds, new mortgage issuance (l_t), and the average mortgage rate, and then log-linearizing, yields:

$$\hat{R}_t^l = \frac{\Gamma}{1 + \Gamma} \hat{R}_t + \frac{1}{1 + \Gamma} \hat{R}_{L,t} - \frac{\kappa_{QL}}{1 + \Gamma} \Delta \hat{R}_{L,t+1} \quad (56)$$

⁴When log-linearizing, we have considered $Q_{L,t} b_{L,t}$ and $Q_{S,t} b_{S,t}$ as one variable, respectively. This is in order to neutralize this variable from price effects, when designing the optimal monetary policy. In this case, a drop in the relative supply of long-term bonds would coincide with a fall in $\hat{\gamma}_{b,t}$ and thereby a drop in the long-term rate.

where we have imposed that mortgages are repaid in full within one period ($\kappa_d = 1$) for tractability and clearer intuition. Equation (56) shows how the central bank affects the mortgage rate, \hat{R}_t^l , through conventional and unconventional monetary policy. Under conventional policy, the short-term policy rate \hat{R}_t raises the mortgage rate, but steady-state transaction costs ($\Gamma > 0$) make this transmission incomplete. Conventional policy also affects the mortgage rate indirectly through its impact on the long-term rate $\hat{R}_{L,t}$. Unconventional policy works via changes in the relative supply of long-term bonds $\hat{\gamma}_{b,t}$, which influence $\hat{R}_{L,t}$ through equation (54). Again, steady-state transaction costs weaken this channel. Intuitively, when the central bank raises the short-term rate and, consequently, the long-term rate, patient households suffer a fall in the present discounted value of their wealth. Because they finance borrowers' mortgages, they pass part of the tighter monetary stance on impatient households, lifting thereby the mortgage rate, \hat{R}_t^l .

Finally, monetary policy affects renters' housing demand and thus the equilibrium rent, $\hat{r}_{h,t}$.⁵ Combining the first-order conditions of patient households for rental housing, $h_{R,t}$, and short-term bonds, $b_{S,t}$, with renters' first-order condition for housing and log-linearizing yields:

$$\hat{h}_{R,t} = -\frac{1}{r_h} \hat{q}_{h,t} + \hat{c}_{R,t} + \frac{\beta_P(1 - \delta_h)}{r_h} \left[-\left(\hat{R}_t - \pi_{t+1}\right) + \hat{q}_{h,t+1} \right] \quad (57)$$

This equation determines the equilibrium quantity of rental housing. A higher policy rate lowers real wages reducing renters' consumption, $\hat{c}_{R,t}$, and housing demand, as implied by (32) and (33). At the same time, the higher policy rate raises long-term and mortgage rates, causing impatient households to cut housing investment. These effects induce patient households to shift from housing to short- and long-term bonds, reducing the supply of rental housing. In equilibrium, this decline in supply accommodates the fall in renters' housing demand.

4 Optimal Monetary Policy

In this section, we analyze the optimal monetary policy from a utility-based welfare perspective. To keep the analysis tractable, we derive analytical results without capital accumulation, though our numerical exercises later allow for it.⁶ We focus on the optimal central bank response to inflationary shocks and therefore abstract from the zero lower bound.

We define social welfare V as a weighted average of the welfare of the three household types, choosing weights so that a constant consumption stream yields equal welfare across types:

$$V = (1 - \beta_P) V_P + (1 - \beta_I) V_I + (1 - \beta_R) V_R. \quad (58)$$

⁵Patient households have no market power over rents; rents are set by rental housing market demand and supply.

⁶The conclusions in this section are unchanged when capital accumulation is included. The analytical results are available upon request.

Here V_α , for $\alpha \in \{P, I, R\}$, denotes the welfare of patient, impatient, and renter households, respectively. The central bank maximizes V subject to the equilibrium conditions of the economy. Each V_α is derived from a second-order approximation to the corresponding utility function. The derivation of the aggregate welfare is summarized in the proposition that follows.

Proposition 1. *The discounted sum of the household utilities is given by:*

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t U_t = & \sum_{t=0}^{\infty} \beta^t \left\{ \omega_{c_P} \hat{c}_{P,t} - \frac{1}{2} \tilde{\omega}_{c_P} \hat{c}_{P,t}^2 + \omega_{c_I} \hat{c}_{I,t} - \frac{1}{2} \tilde{\omega}_{c_I} \hat{c}_{I,t}^2 + \omega_{c_R} \hat{c}_{R,t} - \frac{1}{2} \tilde{\omega}_{c_R} \hat{c}_{R,t}^2 \right. \\ & + \omega_{i_{h_P}} \hat{i}_{h_P,t} - \frac{1}{2} \tilde{\omega}_{i_{h_P}} \hat{i}_{h_P,t}^2 + \omega_{i_{h_I}} \hat{i}_{h_I,t} - \frac{1}{2} \tilde{\omega}_{i_{h_I}} \hat{i}_{h_I,t}^2 + \omega_{i_{h_R}} \hat{i}_{h_R,t} - \frac{1}{2} \tilde{\omega}_{i_{h_R}} \hat{i}_{h_R,t}^2 \\ & - \frac{1}{2} \omega_y \hat{y}_t^2 - \frac{1}{2} \omega_\pi \pi_t^2 - \omega_{\gamma_b} \left(\Gamma_2 \hat{\gamma}_{b,t} + \frac{\Gamma_2 (\Gamma_2 - 1)}{2\gamma_b} \hat{\gamma}_{b,t}^2 \right) \\ & \left. - \omega_{y_{c_P}} \hat{y}_t \hat{c}_{P,t} - \omega_{y_{c_I}} \hat{y}_t \hat{c}_{I,t} - \omega_{y_{c_R}} \hat{y}_t \hat{c}_{R,t} + \omega_{c_P c_I} \hat{c}_{P,t} \hat{c}_{I,t} + \omega_{c_P c_R} \hat{c}_{P,t} \hat{c}_{R,t} + \omega_{c_R c_I} \hat{c}_{R,t} \hat{c}_{I,t} \right\} \\ & + t.i.p. + O(\|\xi^3\|) \end{aligned}$$

Proof. In Appendix A.2 ■

Notice that in the welfare criterion derived above, linear terms also appear. As shown in Benigno and Woodford (2005), a properly defined welfare criterion features linear terms in addition to quadratic ones when the steady state is distorted. In such a case, the central bank must take into account the impact of stabilization policy on the average level of output or, in the context of the current setup, its components. In fact, in our framework the steady state is distorted not only because of monopolistic competition in the intermediate goods sector but also because of credit constraints faced by impatient households. Crucially, as shown in Proposition 1, given the heterogeneity in our setup, it is the welfare-relevant components of aggregate output—rather than output per se—that appear in the linear part of the welfare criterion. This reflects that the policymaker is concerned not only with minimizing fluctuations in output and its components, but also with, at least partly, correcting the heterogeneity in consumption and housing investment levels in the economy that arises from various shocks. As we show in Appendix C, given the distorted steady state, steady-state consumption levels across different household types are not equalized.⁷ Similarly, steady-state housing investment is heterogeneous as well, given that impatient households are always credit constrained. This implies that the policymaker will not aim to completely eliminate heterogeneity in consumption and/or housing investment. Rather, the policymaker seeks to limit any further widening of some or all of these forms of heterogeneity that emerge outside the steady state due to various shocks or the accompanying general equilibrium effects.

⁷The steady-state consumption of impatient households depends on their steady-state stock of housing. Moreover, since they are always credit constrained, their steady-state consumption also depends on the LTV ratio, ϕ . By contrast, the steady-state consumption of renters depends entirely on their steady-state labor income.

To gain better insight, consider an adverse supply shock. In this case, the monetary authority raises the short-term rate, which, via (54), leads to an increase in the long-term rate. This lowers the present discounted value of patient households' wealth and induces them to shift part of the burden on impatient households, thereby raising the average mortgage rate, R_t^d . This curbs the consumption of impatient households. Their housing investment also falls due to both the higher mortgage rates and the decline in real house prices, $q_{h,t}$, caused by the supply shock, which tightens their borrowing constraint through (24). At the same time, the contractionary nature of the shock reduces renters' demand for housing, pushing rents, $r_{h,t}$, downward. Patient households hence partly offset the negative impact of higher short- and long-term rates and lower rents on their own consumption by increasing the average mortgage rate they charge on impatient households. Beyond volatility, the direction of these effects now also matters for the policymaker—an outcome that would not arise under an efficient steady state. Therefore, heterogeneity shapes the optimal trade-offs faced by the policymaker. This is formalized in the proposition below.

Proposition 2. *In the absence of heterogeneity, with only patient households in the economy and no mortgage debt, the central bank faces the traditional inflation–output trade-off after a supply shock. With heterogeneity, however, following a supply shock the central bank can stabilize inflation by trading off aggregate output and/or individual consumption.*

Proof. In Appendix A.3 ■

As we show in the Appendix, with optimal monetary policy under commitment when only patient households reside in the economy (i.e. $\theta_P = 1$), the optimal trade-off receives the following form:

$$\omega_y (\hat{y}_t - \hat{y}_{t-1}) = -\frac{(\eta_y - 1) \omega_\pi \vartheta}{\kappa_p} \pi_t \quad (59)$$

where the terms ω_y and ω_π are nonlinear functions of the model structural parameters. Under heterogeneity instead, the optimal targeting criterion becomes:

$$\omega_y (\hat{y}_t - \hat{y}_{t-1}) + \underbrace{\omega_{y_{cP}} \Delta \hat{c}_{P,t} + \omega_{y_{cI}} \Delta \hat{c}_{I,t} + \omega_{y_{cR}} \Delta \hat{c}_{R,t}}_{\text{additional trade-off terms due to heterogeneity}} = -\frac{(\eta_y - 1) \omega_\pi \vartheta}{\kappa_p} \pi_t \quad (60)$$

where the coefficient in the consumption of each group is positive and $\Delta \hat{c}_{\alpha,t} = \hat{c}_{\alpha,t} - \hat{c}_{\alpha,t-1}$, for $\alpha \in \{P, I, R\}$. Under heterogeneity, the central bank is faced with additional trade-offs captured by the covariance between output and the consumption of each household type in the welfare criterion discussed in Proposition 1 above. The covariance terms vanish when considering the model without heterogeneity (i.e. $\theta_P = 1$). In this case thus $\omega_{y_{cP}} = \omega_{y_{cI}} = \omega_{y_{cR}} = 0$ in the welfare loss and the optimal criterion above collapses to that in the absence of heterogeneity, (59). From expression (60),

in the face of higher inflation, the central bank, *ceteris paribus*, has to lower aggregate output as well as the consumption of some or all population groups. Contrary to the textbook result where there is only one representative household in the economy, here the central bank is not constrained to trade all components of output off when faced with higher inflation. Even though the outcome is the same (i.e. lower output), not every household type bears the burden of stabilization to the same extent necessarily. This implies that inflation stabilization may come at the cost of rising consumption inequality.

4.1 Heterogeneity and the slope of the yield curve

In this section, we examine how heterogeneity interacts with the central bank’s implied optimal decisions on short- and long-term rates. From Proposition 1, we have shown that the relative long-term bond supply, $\hat{\gamma}_{b,t}$, enters the welfare criterion and is therefore one of the central bank’s stabilization objectives.⁸ Targeting this objective changes the supply of long-term bonds relative to short-term bonds, immediately affecting their price and thus long-term rates. Following demand or supply shocks, the central bank not only chooses optimally the trade-off between inflation and output (which affects the short-term rate through the Euler equation of patient households for short-term bonds), but also affects the long-term rate further via its optimal decision on the relative supply of long-term bonds.

The key question in the current setup is to what extent the optimal decision on the relative supply of long-term bonds depends *qualitatively* on heterogeneity, conditional on demand or supply shocks. From Proposition 2, it is clear that, qualitatively, the implied optimal decision about the short-term rate does not depend on heterogeneity. In the absence of heterogeneity, the central bank must lower inflation by inducing a recession following, for instance, an adverse supply shock. With heterogeneity, the central bank must still lower inflation, but now by trading off some or all components of aggregate output, thereby reducing it. In both cases, this is achieved by raising the short-term rate; heterogeneity only changes the magnitude of the required increase.

For the long-term rate, however, heterogeneity can have important *qualitative* implications for the optimal relative supply of long-term bonds. We first ask whether it is only inflation that affects the optimal decision for the relative supply of long-term bonds, or whether output (and some of its components) also matters. We show that under heterogeneity the optimal relative supply of long-term bonds depends not only on inflation but also on output and the consumption of each household type. When heterogeneity is turned off, the optimal relative supply depends solely on inflation. We formalize this result in Proposition 3 below.

⁸We abstract from central bank balance sheet considerations. An alternative would be to assume a central bank balance sheet where the central bank issues reserves (as in Sims and Wu, 2019) to finance its asset purchases, with a representative financial intermediary born in each period and exiting in the subsequent period (as opposed to Gertler and Karadi, 2011, 2013). Another alternative would be to assume that the central bank issues short-term bonds to finance its asset purchases (as in Kabaca et al., 2023) and faces an efficiency cost associated with asset purchases (as in Karadi and Nakov, 2021). In both cases, the central bank would transfer its profits/losses to the government in the form of remittances. The conclusions of this section would be unchanged under these assumptions.

Proposition 3. *In the absence of heterogeneity, the optimal relative supply of long-term bonds depends only on inflation. Under heterogeneity, the optimal relative supply of long-term bonds depends on inflation as well as on output and household-type consumption levels.*

Proof. In Appendix A.4 ■

Solving for the optimal relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, we obtain:

$$\begin{aligned} \omega_{\gamma_b} (1 - \Gamma_2) \hat{\gamma}_{b,t} &= \omega_{\gamma_b} \gamma_b - \frac{\omega_{c_P} - \omega_{c_I} + \omega_{c_R}}{R^l \Omega_{rP}} \\ &+ \frac{\tilde{\omega}_{c_P} + \omega_{c_P c_I} - \omega_{c_P c_R}}{R^l \Omega_{rP}} \hat{c}_{P,t} + \frac{\omega_{c_P c_I} + \omega_{c_R c_I} - \tilde{\omega}_{c_I}}{R^l \Omega_{rP}} \hat{c}_{I,t} + \frac{\tilde{\omega}_{c_R} + \omega_{c_R c_I} - \omega_{c_P c_R}}{R^l \Omega_{rP}} \hat{c}_{R,t} \\ &- \frac{(\eta_y - 1) \omega_\pi (\theta_R + \theta_I - \theta_P)}{\kappa_p R^l \Omega_{rP}} \pi_t + \frac{\omega_{y_{c_P}} + \omega_{y_{c_R}} - \omega_{y_{c_I}}}{R^l \Omega_{rP}} \hat{y}_t \end{aligned} \quad (61)$$

where using the definitions of weights in Appendix A.2 it is easy to show that $\omega_{y_{c_I}} + \omega_{y_{c_R}} - \omega_{y_{c_P}} > 0$.⁹ Note also that $\theta_R + \theta_I - \theta_P > 0$. Given that $0 < \Gamma_2 < 1$, expression (61) shows that when deciding upon the optimal relative supply of long-term bonds, the central bank takes into account not only inflation and output, but also the consumption of each group. That is, heterogeneity matters for the optimal setting of long-term bond supply and thereby for the long-term rate. In the Appendix, we show that in the absence of heterogeneity with only patient households residing in the economy and no mortgage debt (i.e. $\theta_P = 1$ and $\theta_I = \theta_R = 0$), the optimal decision about the relative long-term bond supply collapses to:

$$\omega_{\gamma_b} (1 - \Gamma_2) \hat{\gamma}_{b,t} = \omega_{\gamma_b} \gamma_b + \frac{(\eta_y - 1) \omega_\pi}{\kappa_p R^l \Omega_{rP}} \pi_t \quad (62)$$

In the absence of heterogeneity thus, the central bank sets the optimal relative supply of long-term bonds according to developments in inflation only. Notice also that the sign of the weight on inflation in the optimal relative long-term bond supply now changes. In (61), there is a negative relation between the inflation rate and the optimal relative long-term bond supply, given that $\theta_R + \theta_I - \theta_P > 0$ and $\eta_y > 1$.¹⁰ In the absence of heterogeneity in (62) instead, the sign on inflation is reversed implying a positive relation with the optimal relative long-term bond supply. This implies that heterogeneity alters the way the central bank sets the optimal relative supply of long-term bonds following supply shocks. The optimal criterion under heterogeneity, (61), prescribes that, ceteris paribus, the central bank has to lower the relative supply of long-term bonds after a supply shock that lifts inflation. In fact, since in this case inflation rises while output and the consumption of each group fall, the negative weight on inflation in (61) implies that the movements in inflation and output, though in opposite directions, have exactly the same (negative) effect on the optimal relative long-term

⁹Similarly, using the derivations in Appendix A.2, it is also easy to show that $\tilde{\omega}_{c_P} + \omega_{c_P c_I} - \omega_{c_P c_R} > 0$, $\omega_{c_P c_I} + \omega_{c_R c_I} - \tilde{\omega}_{c_I} > 0$ and that $\tilde{\omega}_{c_R} + \omega_{c_R c_I} - \omega_{c_P c_R} > 0$, so that the coefficient on each group's consumption is positive.

¹⁰Parameter η_y is the steady state value of the elasticity of substitution between the differentiated goods which we set at 6, in line with Galí (2008).

bond supply. Thus, the central bank induces a decline in the long-term rate, thereby flattening the yield curve. In contrast, in the absence of heterogeneity, the central bank must increase the relative supply of long-term bonds (positive weight on inflation in (62)), thereby triggering an increase in the long-term rate.

Corollary. *Following an adverse supply shock without heterogeneity, the central bank raises both the short-term rate and the relative supply of long-term bonds (increasing the long-term rate). With heterogeneity, instead, the central bank raises the short-term rate but lowers the relative supply of long-term bonds.*

The intuition is as follows. Following an adverse supply shock, the central bank must raise the short-term rate.¹¹ This increases both the long-term and mortgage rates, further burdening impatient households that borrow to invest in housing. To dampen these effects, the central bank reduces the relative supply of long-term bonds, offsetting the upward impact of the policy rate on long-term and mortgage rates (see also (56)). This mitigates the fall in housing investment and speeds up the recovery, as shown in our quantitative exercise below. This mechanism is absent under homogeneity, where the central bank's main objective is simply to reduce inflation.

The next question that we ask is what heterogeneity implies for the optimal response following a positive demand shock. When it comes to the short-term policy rate, it is easy to show that the central bank needs to lift it.¹² When it comes to the relative supply of long-term bonds the conclusion is rather ambiguous. Contrary to the supply shock where inflation rises and output drops, observing again the optimal criterion (61) it is not obvious how the central bank has to move the optimal relative supply of long-term bonds following a demand shock, as output (and the consumption of each type) and inflation move in the same direction. We show that, in this case, the share of impatient households in the production function, θ_I , is crucial for the optimal decision of the central bank. We summarize the condition determining how the central bank should set the optimal relative supply of long-term bonds after a demand shock with heterogeneity in the following proposition.

Proposition 4. *Following a positive demand shock under heterogeneity, the central bank must lower the relative supply of long-term bonds if and only if:*

$$\theta_I > \left(\frac{1}{1 + \varsigma - \frac{2(1-\beta_I)}{c_I \zeta}} \right) (\theta_R(1 - \varsigma) + \theta_P(1 + \varsigma)), \quad \text{where} \quad \varsigma = \left(\frac{\eta_y - 1}{\kappa_p} \right)^2.$$

Otherwise, the central bank must increase the relative supply of long-term bonds.

Proof. In Appendix A.5 ■

¹¹In fact, plugging the inflation–output trade-off implied by (59) under homogeneity and by (60) under heterogeneity into the Euler equation for short-term bonds of patient households and solving for the short-term rate, \hat{R}_t , shows that higher inflation leads to a higher short-term rate.

¹²Again, this is shown by plugging the targeting criterion (60) in the Euler equation of the patient household and then solve for the policy rate, \hat{R}_t .

Table 1. Calibrated parameters

	Symbol	Value
Inflation target (gross, qtr.)	π	1.005
Time discount factor	$\beta_P, \beta_I = \beta_R$	0.9925, 0.9875
Inverse of the Frisch elasticity	ϑ	1
Level for housing in utility	ξ	1
LTV ratio on new regular mortgages	ϕ	0.8
Amortization rate on HH loans	κ_d	0.0175
Share of capital in production	α	0.25
Share of Patient agents and Renters in production function	θ_P, θ_R	0.30, 0.22
Share of Impatient agents in production function	θ_I	0.48
Depreciation rates	δ_h, δ_k	0.015, 0.03
Gross markup in price	θ_p	1.1
Fixed costs in production	f	0.11
Utilization cost level	κ_u	0.03
Tax level	Ξ	0.22
Taylor rule, smoothing	ρ	0.75
Taylor rule, inflation sensitivity	α_π	1.5
Taylor rule, output sensitivity	α_y	0.125

5 Calibration

Table 1 lists calibrated parameter values, and Table 2 reports the main ratios at the steady state of our model. The trend inflation factor, π , is set to 1.005, corresponding to 2% annual inflation. The time-discount factor of savers, β_P , is set to 0.9925 to match an annualized 3% real risk-free interest rate, while that of borrower and renters, $\beta_I = \beta_R$, is set to 0.9875, implying a 200 bps spread on the risk-free rate if borrowers were allowed to engage in non-mortgage borrowing. The level parameter for housing in the utility function, ξ , is set to 1 to ensure that the value of housing relative to annual GDP is 1.53, in line with the observed average over the last 30 years in the US Flow of Funds (FoF) data.

In the data, residential and non-residential investments are about 4.5% and 13% of output, respectively, while housing-to-GDP and capital-to-GDP ratios are 1.07 and 1.85 on an annualized basis.¹³ Based on these, we calibrate the quarterly depreciation rates for housing and capital stocks, δ_h and δ_k , to 1.5% and 3%, respectively. The share of capital in domestic production, α , is calibrated to 0.25 using the capital-output ratio and the model-implied rental rate of capital. The demand elasticity for differentiated intermediate goods at the steady state, η_y , is set to 6, implying a net markup in prices of 20%. The fixed cost of production, f , is set equal to 0.11 times the steady-state level of output to ensure that pure economic profits are zero at the steady state, thus eliminating the incentive for entry and exit in the long-run of the stochastic economy. We calibrate the coefficients,

¹³The capital stock reflects the tangible asset holdings of non-financial corporations, non-corporate businesses, and households minus the real estate and consumer durable holdings of households, using FOF data.

Table 2. Model steady-state target ratios

	Symbol	Model
Total consumption / GDP	c/y	0.625
share of patient households	c_P/c	0.2325
share of impatient households	c_I/c	0.5351
share of renter households	c_R/c	0.23
Residential investment / GDP	i_h/y	0.2173
Non-residential investment / GDP	i_k/y	0.1997
Government expenditure / GDP	g/y	0.20
Tax revenue / GDP	tax/y	0.22
Capital stock / GDP (qtr)	k/y	6.7
Housing stock / GDP (qtr)	h/y	3.62
share of patient households	h_P/h	0.48
share of impatient households	h_I/h	0.52
Mortgage debt / total housing value	d/h	0.37
average LTV on all outstanding loans	d/h_I	0.75
LTV on new regular loans	ϕ	0.75

Γ_1 and Γ_2 , in the transaction cost function Γ_t such that the steady state spread between the long-term rate and the short-term rate equals to 0.75 approximately. We consider this spread reasonable when looking at the spread between the 10-year treasury yield and the policy rate-sensitive 2-year yield in the U.S. historically. The implied values for Γ_1 and Γ_2 thus are 1.0025 and 0.0025, respectively, with a steady state ratio of long-term to short-term government debt, γ_b , equal to 2.

The LTV ratio on new mortgages, ϕ , is set to 0.75. Based on FOF data, the ratio of mortgage debt owed by all households relative to their real estate holdings, d/h , is around 0.28. Given that the LTV ratio is 0.75 for the average borrower, we can infer that borrower households own about 56% of the total housing stock. We therefore calibrate the wage share of patient households, θ_P , to 0.30, to hit this target. Steady-state government expenditure is calibrated to ensure that its share in output, g/y , is 20%. The level parameter for taxes, Ξ , is set to 0.22. We are interested in demand and supply shocks and set their persistence to 0.5.

6 Results

In this section, we analyze the effects of demand and supply shocks. The supply shock is captured by a markup shock, namely a shock to the elasticity of substitution across varieties $\eta_{y,t}$. The demand shock is captured by the preference shock, v_t , that enters the utility function of all household types symmetrically. We consider three cases: (i) monetary policy sets the short-term rate according to the interest rate rule in (51), while the relative supply of long-term bonds follows a stationary process (52); (ii) the central bank sets only the short-term rate optimally, while the relative supply of long-

term bonds continues to follow the stationary process in (52); and (iii) the central bank sets both the short-term rate and the relative supply of long-term bonds optimally.

The results of the previous section provide analytical insights about the trade-offs the central faces under the type of heterogeneity considered in the paper, as well as on the implications of the latter on the optimal decision about the relative long-term bond supply. However, they do not provide a complete picture of what the net effect will be on the long-term rate, and thereby on the mortgage rate. This section fulfills that purpose. We introduce shocks to the economy, and examine their effects on the interest rates and the aggregate economy, interpreting their dynamics using our analytical results.

Supply shock. We start with a stagflationary supply shock, modeled as a one-standard-deviation increase in the time-varying elasticity of substitution across varieties, $\eta_{y,t}$, in the optimal pricing equation (44) for intermediate goods producers. The impulse responses for the three cases we study appear in Figure 1.

In the benchmark case (black solid lines), the central bank follows the Taylor rule in (51), while the relative supply of long-term bonds follows the exogenous process in (52). The shock raises inflation and triggers monetary tightening, which deepens the recession caused by the supply disruption. As in Iacoviello (2005), the supply shock immediately lowers real house prices. The higher policy rate raises mortgage rates on new loans, and together with lower real house prices—making impatient households more credit-constrained—reduces their housing investment.

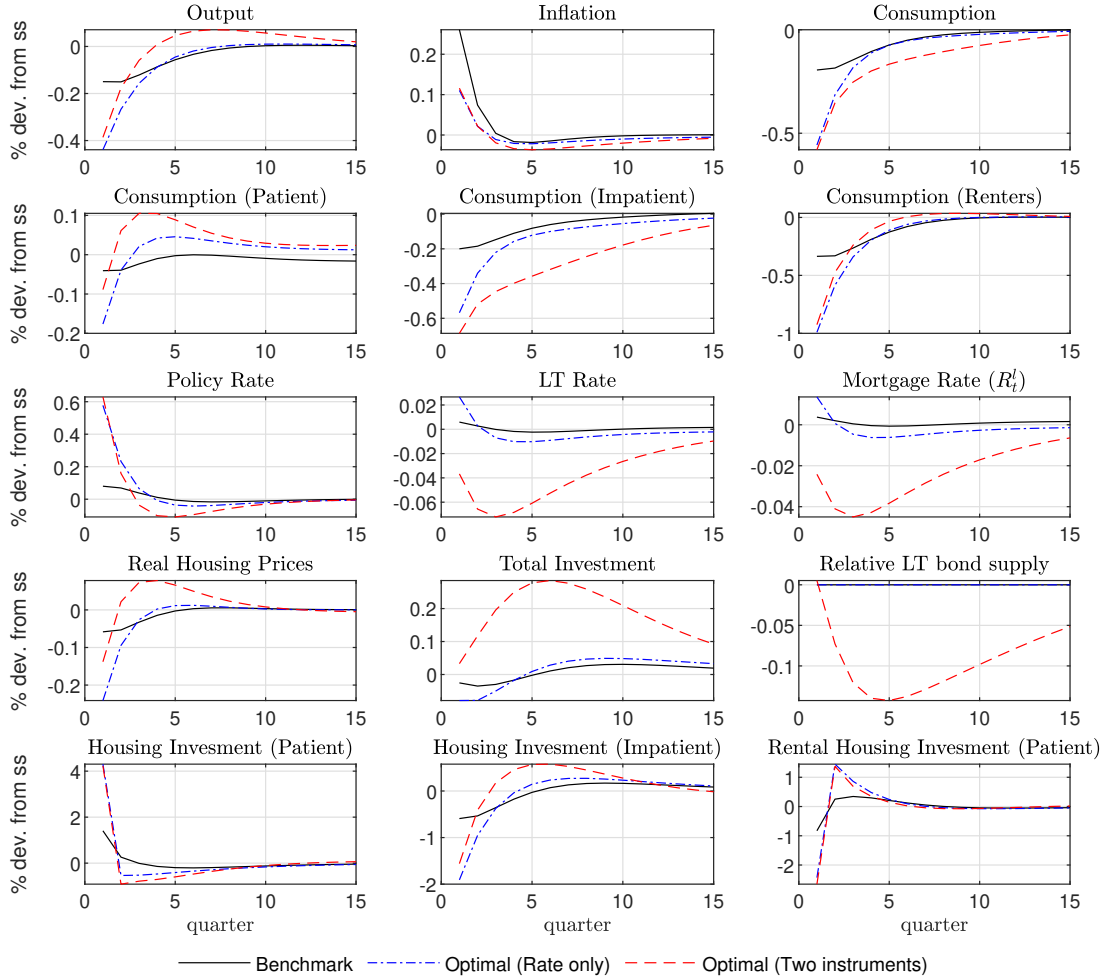
Patient households benefit from lower real house prices and expand their own housing stock. At the same time, the adverse supply shock reduces real wages (see top panel in Figure A.4 in Appendix A), which is particularly costly for renters who rely solely on labor income. They therefore reduce their housing demand, leading patient households to cut investment in rental housing. These developments lower consumption for impatient households and renters. Patient households' consumption also falls, driven by lower expected income from lending to impatient households, reduced wages, and a decline in the present discounted value of their wealth due to higher interest rates.

When the central bank optimally sets only the policy rate (blue dashed-dotted lines), it hikes more aggressively than in the benchmark case. The policy rate then slightly undershoots its steady state, because the central bank considers not only the jump in inflation but also that impatient households' and renters' consumption and housing demand depend on the long-term rate.¹⁴ It therefore finds it optimal to keep the policy rate lower than otherwise for several periods. Since, via (54), the long-term rate depends on the sequence of the short-term rate, it undershoots after its initial rise and remains persistently below the steady state. Through (56), this lowers mortgage rates in the medium run, allowing impatient households' housing investment to recover quickly and overshoot its benchmark path about a year after the shock.

Overall, this policy is effective in mitigating the impact of the supply shock on inflation, at the expense of a deeper recession in the first three quarters relative to the benchmark. This occurs

¹⁴Recall from section 3 that the long-term rate affects the mortgage rate paid by impatient households via (56) and renters' housing demand via (57).

FIGURE 1. Impulse responses following a supply shock.



Notes: Impulse response functions following a supply shock. The solid black lines show the responses from the benchmark model, in which the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted lines show the responses when the central bank optimally sets only the short-term (policy) rate, while the red dashed lines show the responses when the central bank optimally sets both the short-term (policy) rate and the relative supply of long-term bonds.

because the deeper recession lowers wages and further curbs household consumption, while the higher initial interest rates reduce capital and housing investment more than in the benchmark. Once investment and consumption begin to recover, output returns to steady state relatively quickly. The main reason for this rapid recovery is that the policy rate—and thus long-term and mortgage rates—is allowed to undershoot in the medium run despite the initial aggressive hike.

When the central bank optimally sets both the policy rate and the relative supply of long-term assets (red dashed lines), it raises the policy rate slightly more on impact compared to when it optimally sets only the policy rate, but then lets it fall faster and undershoot more. The optimal

relative supply of long-term bonds shrinks, as Proposition 3 and Corollary 1 suggest, causing a drop in both the long-term rate and the mortgage rate. The fall in the optimal relative supply of long-term bonds dominates the increase in the policy rate and thus leads to a decline in the long-term and mortgage rates even on impact. The decline in the mortgage rate supports a faster recovery, with an earlier overshoot in housing investment by impatient households, who benefit not only from lower borrowing costs but also from the rapid rise in real house prices that makes them less credit-constrained. The resulting increase in total investment mitigates downward pressures on output, generating a faster recovery than in the other two cases.

Note that the consumption of impatient households now falls more than in the other two cases. This is because they derive higher utility from housing due to lower mortgage rates, and therefore cut their goods consumption more. Specifically, lower mortgage rates together with the expected overshooting in real housing prices force them to reduce their current consumption more than in the other two cases, in order to increase housing investment, since they know they will be less credit-constrained in the future.

Regarding renters, their consumption now declines more relative to the benchmark, since the larger contraction in output initially depresses real wages more (see top panel in Figure A.4 in Appendix A). Compared to the case where only the policy rate is set optimally, their consumption recovers slightly faster due to the more rapid rebound in real wages driven by the steeper recovery in economic activity.

Overall, when the central bank optimally sets both the policy rate and the relative supply of long-term bonds, it succeeds in stabilizing inflation while also triggering a rapid recovery in economic activity with a persistent overshooting. However, this policy widens consumption inequality across households. Unlike impatient households and renters, the consumption of patient households falls by less than when only the policy rate is set optimally and later overshoots the steady state more persistently. This is driven by the increase in the present discounted value of their wealth due to the persistent undershooting of the policy rate and the decline in the long-term rate. In addition, the medium-run overshooting in housing investment by impatient households, combined with higher real housing prices that relax their credit constraints, generates extra income for patient households. Together, these effects more than offset the negative impact of the supply shock on their consumption.

Demand shock. We now consider a one-standard-deviation positive demand (preference) shock, with the impulse responses shown in Figure 2. Focusing first on the benchmark case (black solid lines), where the central bank sets the short-term rate according to the Taylor rule in (51) and the relative supply of long-term bonds follows the exogenous process in (52), the shock generates the standard effects. Consumption of all household types increases, boosting aggregate demand and raising inflation. In response, the central bank raises the short-term interest rate, which, via (54) and (56), leads to higher long-term and mortgage rates. The resulting increase in real interest rates dampens capital investment.

The expansion of economic activity pushes up wages, which increases renters' demand for consumption goods and housing. This, in turn, encourages patient households to invest more in rental

housing. Impatient households also raise their housing demand because of the lower real house prices. Although lower real house prices tighten their borrowing constraint, their housing first-order condition, equation (26), shows that housing demand depends on both current and expected future real house prices, $q_{h,t}$. Since future real house prices are expected to rise back toward their steady state, impatient households choose to invest in housing today, even though they will be less credit-constrained in the future when prices are higher.¹⁵ This effect, together with higher wages, more than offsets the increase in borrowing costs, raising their housing investment.

Patient households initially reduce investment in their own housing. They instead find it more profitable to invest in rental housing, given the stronger demand from renters, and to extend loans to impatient households. With fixed-rate mortgages, they benefit from higher borrowing costs by expanding loan supply. Only once impatient households' housing investment and rental housing investment fall below their steady-state levels do patient households start increasing investment in their own housing.¹⁶

Contrary to Iacoviello (2005), housing supply in our model is endogenous. The expansionary demand shock raises housing demand by impatient households and renters, but sharply reduces demand by patient households, so that total housing demand falls slightly (see Figure A.1 in Appendix A). This lowers real housing prices. The concurrent drop in housing investment, and thus housing supply, puts upward pressure on prices and partly offsets the demand effect. Overall, the demand effect dominates and real housing prices decline. With fixed housing supply, this price decline would be larger.

Let us now turn to the case where the central bank optimally sets only the short-term interest rate (blue dashed-dotted responses in Figure 2), while the relative long-term bond supply follows an exogenous $AR(1)$. In this case, the central bank raises the policy rate significantly more than under the Taylor rule benchmark. Via (54), this causes a larger immediate increase in the long-term rate, followed by a persistent medium-run undershooting; the mortgage rate follows a similar pattern. These sharper hikes in the policy, long-term, and mortgage rates more than offset the shock's positive effects on output, generating a mild recession. The consumption of impatient households is now much more subdued than in the benchmark, due to higher borrowing costs and a decline in their wages on impact caused by the recession.¹⁷ Higher interest rates also increase their cumulative debt repayment costs, further depressing their consumption.

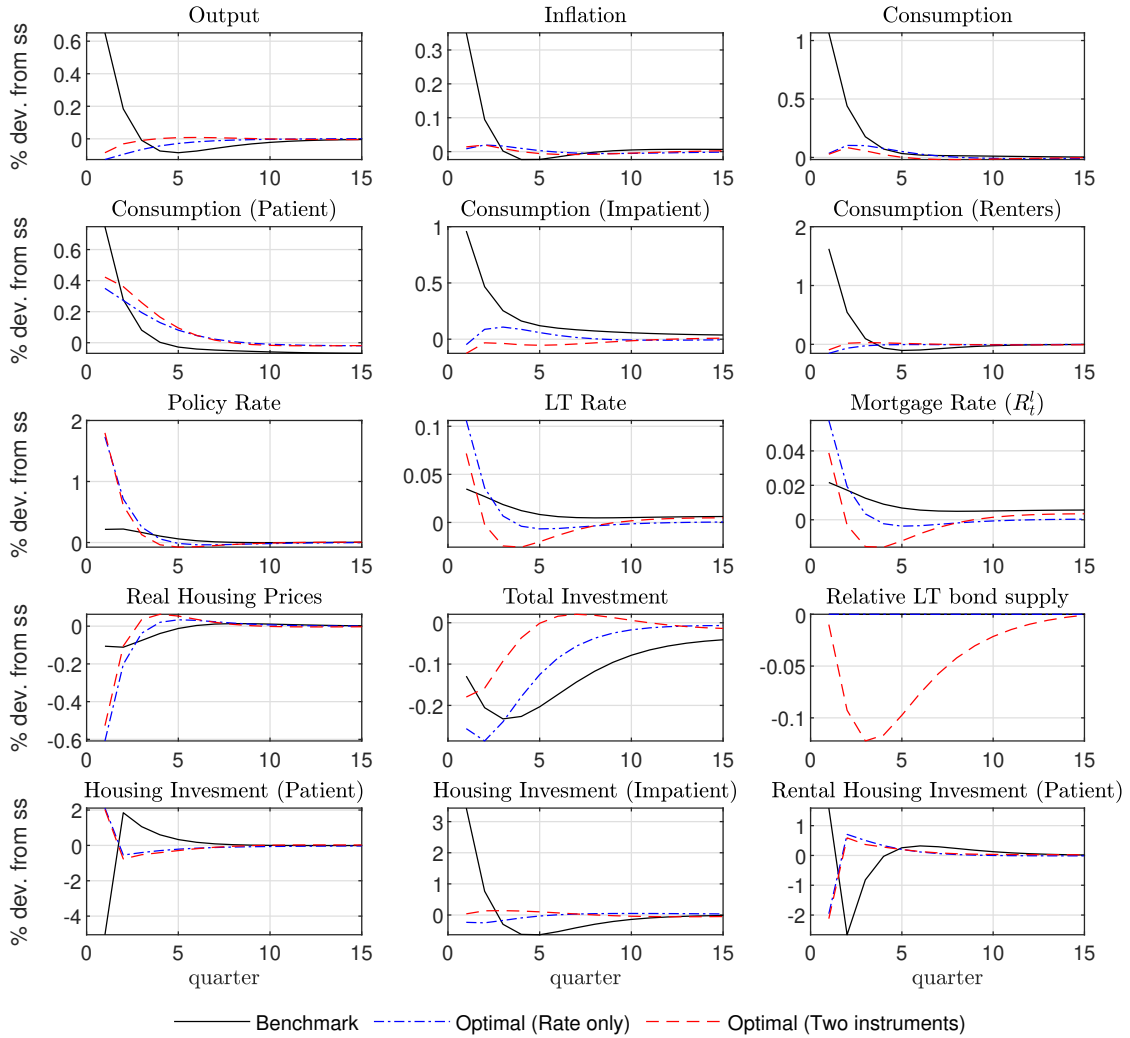
The decline in real housing prices is now larger in the first quarters following the positive demand shock. Impatient households' housing investment now falls, mainly because higher mortgage rates on impact tighten their credit constraints. At the same time, the recession lowers real wages (see top panel in Figure A.2 in Appendix A), reducing renters' housing demand (see Figure A.1 in Appendix

¹⁵Because the credit constraint is always binding, impatient households always borrow up to their limit. Being less credit-constrained in the future therefore means a higher debt repayment burden, which depresses future consumption. Hence, they find it optimal to invest more in housing today at lower real prices than in the future at higher prices.

¹⁶These patterns also reveal a substitutability for patient households between investment in own housing and the provision of loans to the impatient or investing in rental housing.

¹⁷The real wage of impatient households falls on impact but soon starts rising and mildly overshoots for several quarters thereafter (see top panel in Figure A.2 in Appendix A). The results are available upon request.

FIGURE 2. Impulse responses following a positive demand shock.



Notes: Impulse response functions following a positive demand shock. The solid black lines show the responses from the benchmark model, in which the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted lines show the responses when the central bank optimally sets only the short-term (policy) rate, while the red dashed lines show the responses when the central bank optimally sets both the short-term (policy) rate and the relative supply of long-term bonds.

A) and thus rental housing investment by patient households.

Relative to the benchmark, patient households' consumption rises less because sharper increases in short- and long-term rates reduce the positive effect of the preference shock via a lower present discounted value of wealth. The increase in housing rents is also smaller (see bottom panel in Figure A.2 in Appendix A), as the negative rental housing supply effect on rents is mitigated by weaker rental housing demand. Consequently, patient households' rental income rises less, further dampening their

consumption response.

Overall, the central bank successfully curbs inflation relative to the benchmark but triggers a mild recession that fully offsets the shock’s positive output effects, as crowding out of investment dominates. Lower output reduces real wages, dampening impatient households’ consumption and explaining the modest drop in renters’ consumption. Stabilization therefore amplifies household asymmetries and raises consumption inequality—defined here as divergence in the direction of consumption responses. In particular, consumption rises for patient households but falls for impatient households and renters, who bear most of the adjustment.

When the central bank optimally sets both the policy rate and the relative supply of long-term bonds (red dashed lines in Figure 2), it controls inflation as effectively as when it optimally sets only the policy rate (blue dash-dotted lines), but the recession is now milder and less persistent. As before, both short- and long-term rates rise, but now the central bank offsets the upward pressure on the long rate by reducing the supply of long-term bonds. This partly reverses the sharp increase in the long rate caused by the higher short rate and, via (56), produces a smaller immediate rise in the mortgage rate. In subsequent quarters, the mortgage rate falls below its steady state. These dynamics allow housing investment by impatient households to rise slightly rather than fall. Their higher housing demand (see Figure A.1 in Appendix A) dampens the decline in overall housing demand, leading to a smaller initial drop and earlier overshooting of real house prices. By adjusting the supply of long-term bonds, the central bank also relaxes impatient households’ credit constraints. Total investment therefore falls by less and overshoots its steady-state level five quarters after the shock, mitigating the decline in output.

6.1 Optimal monetary policy with and without heterogeneity

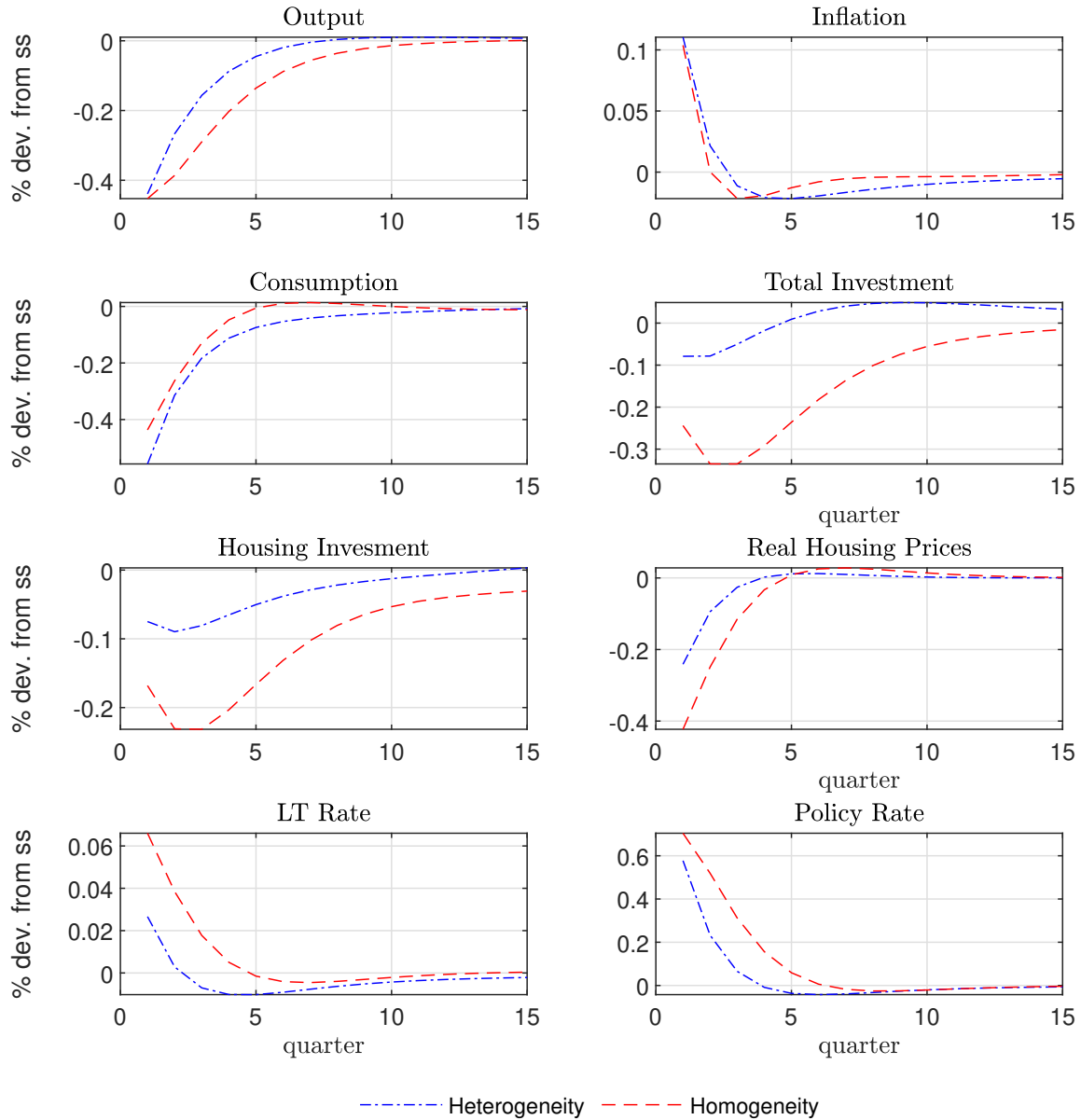
In this section, we compare our results to those from the version of the model with only patient households, namely a model without household heterogeneity and mortgage debt.¹⁸ We assume that the household sector consists solely of patient households who hold short- and long-term government bonds, invest in their own housing, and supply labor to intermediate goods firms. Since this is the only household type, the model excludes loan provision and investment in rental housing. The rest of the model is as in Section 2. We focus on the supply shock to save space.

We first consider optimal monetary policy conducted solely through the policy rate. The results are shown in Figure 3. Under homogeneity (red dashed lines), the central bank stabilizes inflation more effectively at the cost of a slightly more persistent output contraction. Policy is more aggressive in this case, as seen in the larger rate hike.

Under heterogeneity, the central bank instead tolerates more persistent deflation to limit the recession from the supply shock. This matches Proposition 2: with heterogeneous agents, the central bank faces a more complex inflation–output stabilization trade-off because it must account for how policy changes each type’s consumption and housing investment. Under homogeneity, patient

¹⁸In Appendix B, we also discuss the version of the model without renters and its implications for optimal policy.

FIGURE 3. Homogeneity vs Heterogeneity: Optimal Policy under One Instrument



Notes: Impulse response functions following a supply shock when optimal policy is conducted by setting the short-term (policy) rate only. The blue dashed-dotted lines show the responses corresponding to the original model taken from figure 1 while the red dashed lines show the responses in the absence of heterogeneity with patient households only.

households' diversified asset holdings (government bonds, physical capital, and housing) make them less exposed to the supply shock and to tighter policy. Higher interest rates reduce the present value of their wealth but are partly offset by lower real house prices, softening the hit to consumption and giving the central bank more room to stabilize inflation.

Under heterogeneity, the central bank recognizes that many households are indebted with no assets beyond housing (impatient) or rely solely on labor income (renters). As discussed above, impatient households' consumption is sensitive to borrowing costs, and renters' consumption is highly responsive to economic contractions and falling real wages. In aiming to curb inflation, the central bank must consider these channels, which leads it to raise the policy rate less than under homogeneity.

We now turn to the case where the central bank optimally sets both the policy rate and the relative supply of long-term bonds, and compare it to the benchmark in Figure 4. The key difference concerns the optimal relative supply of long-term bonds. Under homogeneity, as implied by Proposition 3 and the Corollary above, the optimal relative supply of long-term bonds (equation (62)) increases with inflation. Following a stagflationary supply shock, the central bank therefore raises the relative supply of long-term bonds, pushing up the long-term rate. In contrast, under heterogeneity, instead, the coefficient on inflation in the optimal rule for the relative supply of long-term bonds (equation (61)) is negative. The central bank must then reduce the relative supply of long-term bonds, causing the long-term rate to fall, as discussed in the previous section.

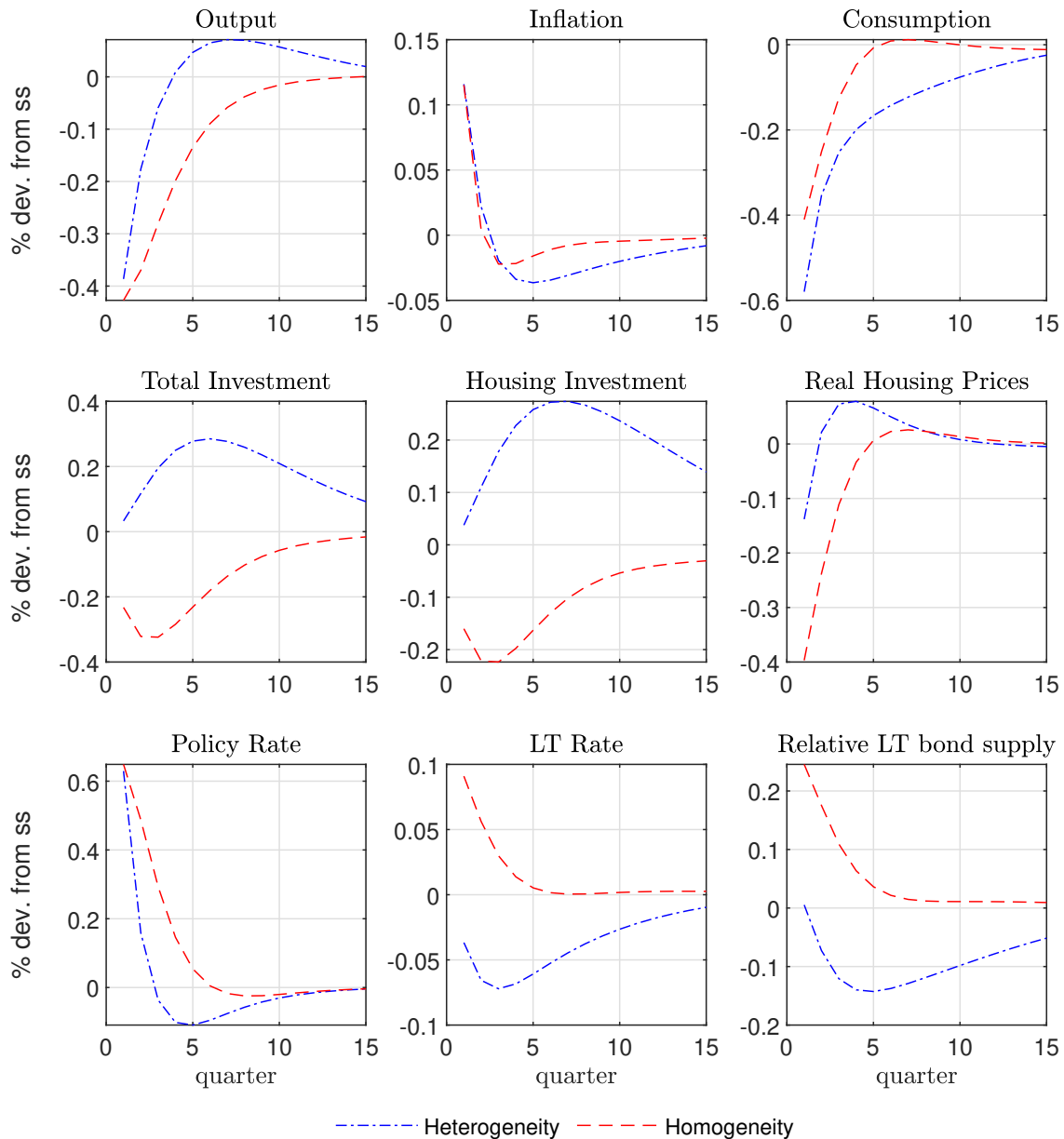
Additionally, we showed in Proposition 3 that under homogeneity the central bank does not place a weight on output fluctuations when choosing the relative supply of long-term bonds. Under heterogeneity, however (see (61)), it optimally sets the relative long-term bond supply by considering not only inflation but also output and household-specific consumption fluctuations. In particular, it accounts for the strong impact of long-term rate movements on impatient households and renters via the channels discussed in section 3 (see (56) and (57)). Consequently, following a supply shock, it optimally reduces the relative supply of long-term bonds to mitigate the adverse effects of the shock and the higher short-term rate on these groups.

The second key observation concerns real house prices. Under homogeneity, they fall more, making housing cheaper for patient households and softening their consumption decline. Under heterogeneity, the persistent fall in mortgage rates boosts housing demand, so real house prices decline less. This implies higher housing costs for patient households than otherwise but relaxes borrowing constraints for impatient households. Higher total investment in this case supports a faster recovery in economic activity. The policy rate rises in both cases but declines faster and undershoots more under heterogeneity. Hence, heterogeneity is crucial for optimal monetary policy design, particularly for decisions on the relative supply of long-term bonds and their impact on long-term rates, mortgage debt, and the broader macroeconomy.

7 Conclusion

This paper investigates the optimal mix of conventional and unconventional monetary policy in an economy with household heterogeneity and mortgage debt. Using a DSGE model with savers, borrowers, and renters, we show that the presence of heterogeneity fundamentally reshapes how monetary policy works and how it should be designed. After adverse supply shocks, the optimal response is a deliberately split strategy: raise the short-term rate to keep inflation in check, while

FIGURE 4. Homogeneity vs Heterogeneity: Optimal Policy under Two Instruments



Notes: Impulse response functions following a supply shock when optimal policy is conducted by setting the short-term (policy) rate as well as the relative supply of long-term bonds. The blue-circled are the responses corresponding to the original model taken from figure 1 while the red dashed are the responses in the absence of heterogeneity with patient households only.

simultaneously pushing down the long-term rate to ease the financial strain on indebted households and renters. This dual policy accelerates the rebound in investment and output, stabilizes inflation better, but comes at the cost of increased consumption inequality.

Our analysis underscores the need to consider the distributional effects of monetary policy, particularly in highly indebted economies. The results indicate that unconventional tools such as yield curve control and quantitative easing should be targeted to household balance sheet vulnerabilities. We also show that the optimal supply of long-term bonds depends on the type of shock and the degree of heterogeneity in the economy.

Future research could extend this framework by adding financial intermediaries, endogenous default risk, or central bank balance sheet constraints, and by examining fiscal–monetary policy interactions in heterogeneous-agent settings.

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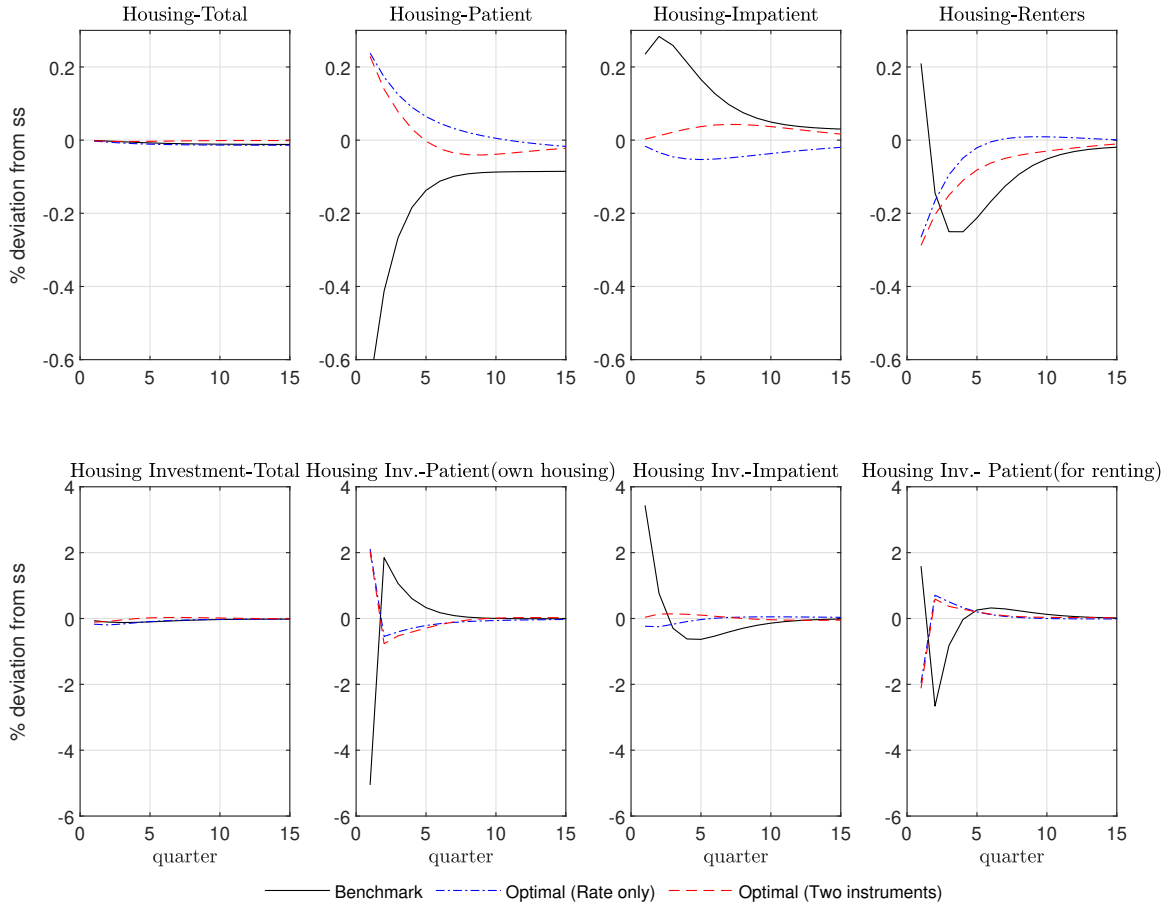
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Online Appendix

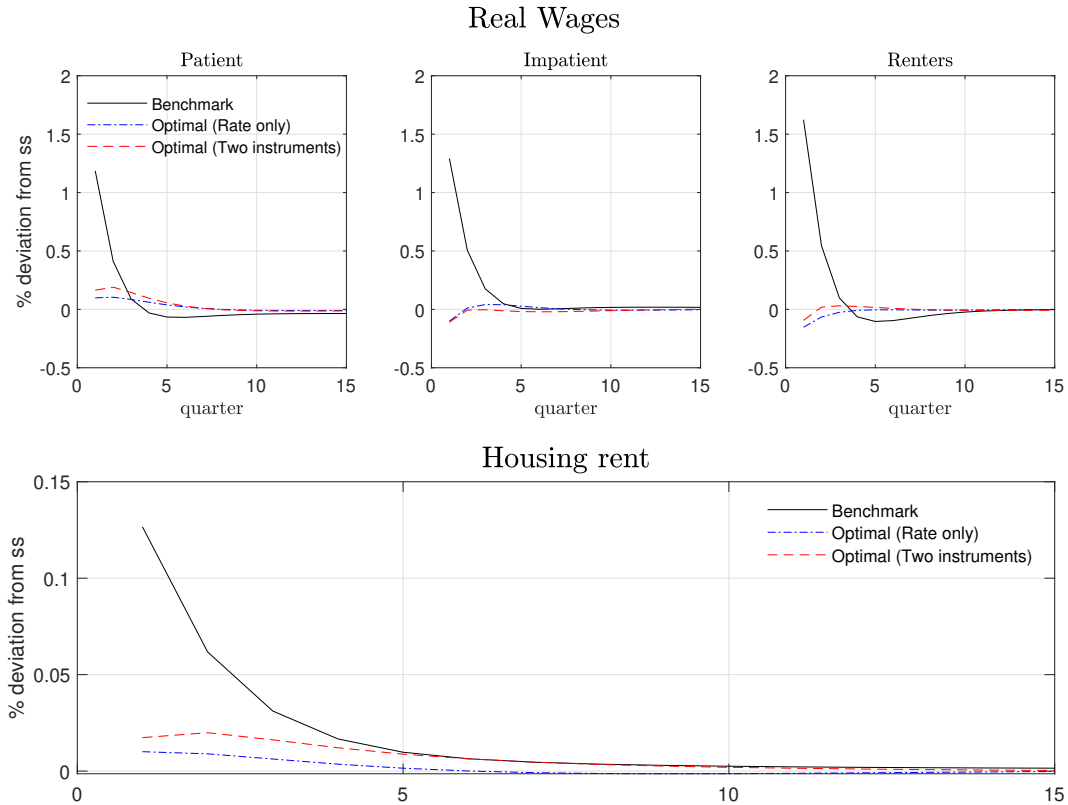
A Additional Figures

FIGURE A.1. Housing Demand and Housing Investment following a positive preference shock



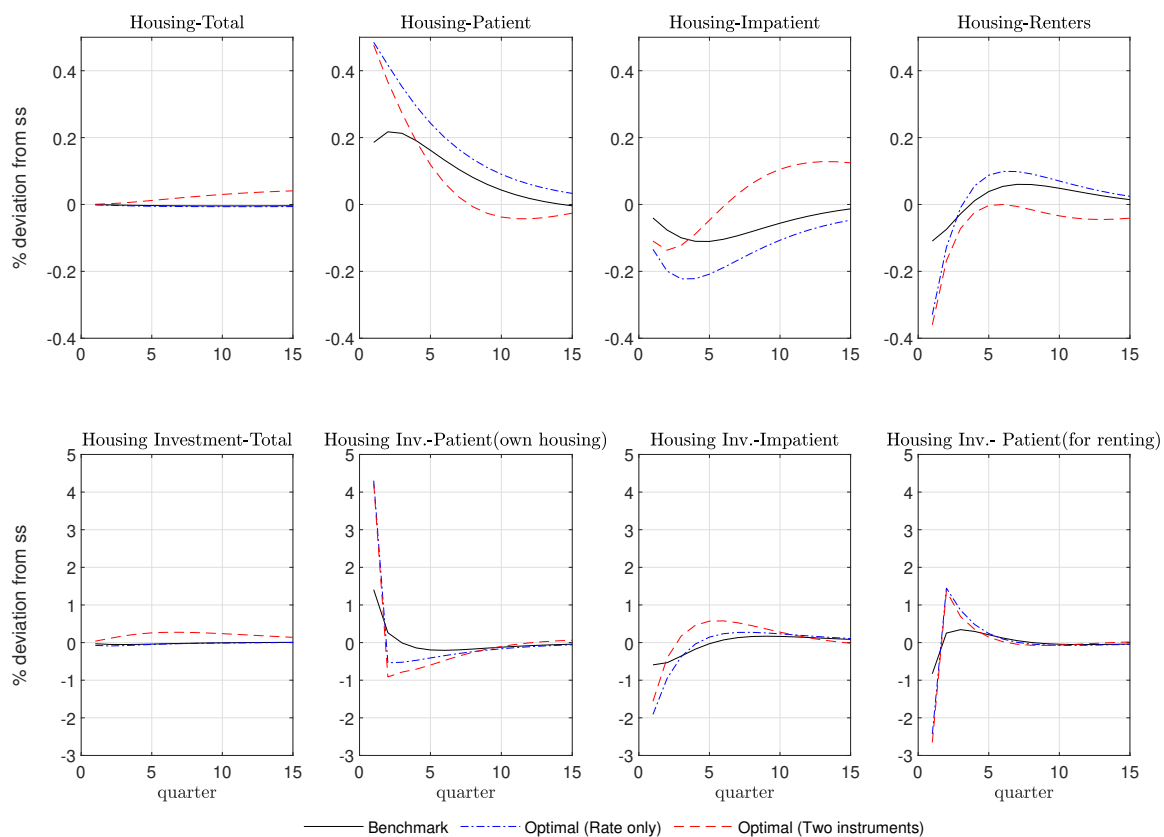
Notes: Impulse response functions following a positive demand shock, modeled as a positive preference shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

FIGURE A.2. Real Wages and Housing Rent following a positive preference shock



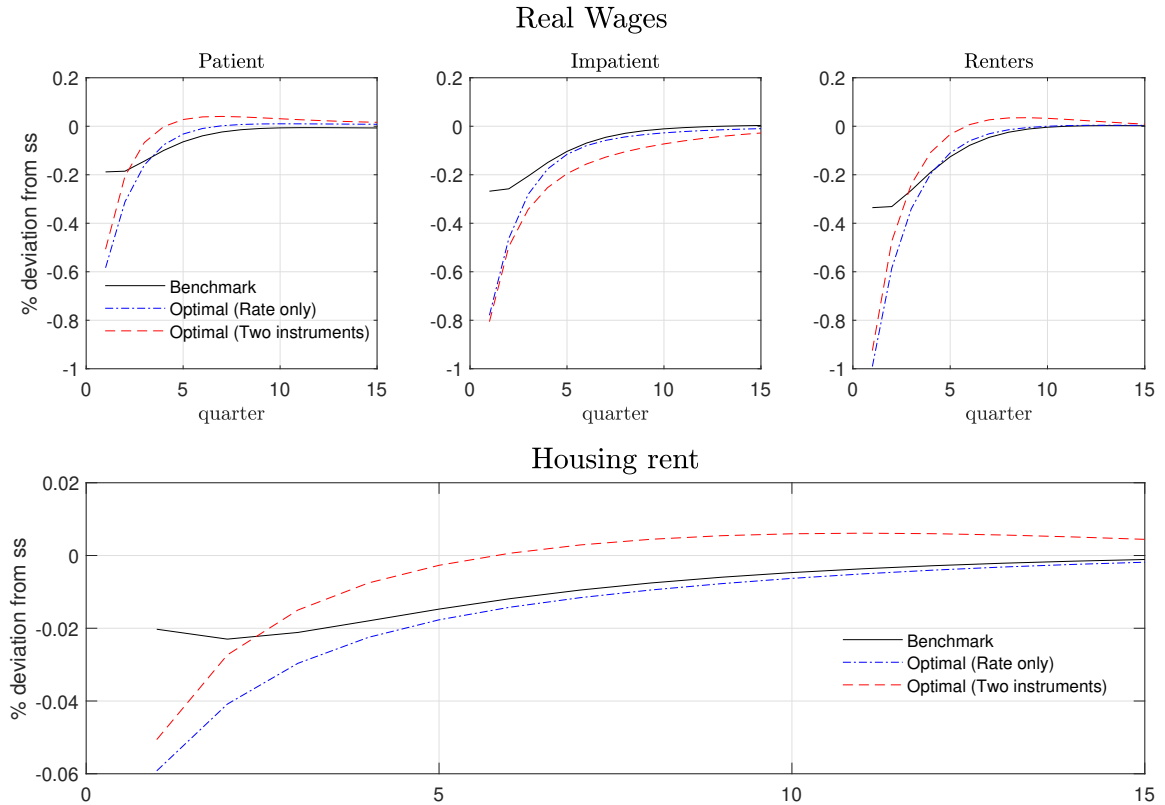
Notes: Impulse response functions following a positive demand shock, modeled as a positive preference shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted lines are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

FIGURE A.3. Housing Demand and Housing Investment following a supply shock



Notes: Impulse response functions following an adverse supply shock, modeled as a positive cost-push shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

FIGURE A.4. Real Wages and Housing Rent following a supply shock



Notes: Impulse response functions following an adverse supply shock, modeled as a positive cost-push shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted lines are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

A.1 Derivation of the resource constraint

Solving for the stock of mortgage debt in aggregate budget constraint of impatient households and substituting in the patient households' aggregate budget constraint, we obtain:

$$\begin{aligned}
& c_{P,t} + c_{I,t} + q_{h,t}(i_{hP,t} + i_{hI,t} + i_{hR,t}) + q_{k,t}i_{k,t} + q_{S,t}b_{S,t} + q_{L,t}b_{L,t} + \Gamma_t l_t \\
& = w_{P,t}n_{P,t} + w_{I,t}n_{I,t} + r_{h,t}h_{R,t} + r_{k,t}k_{t-1} + \frac{b_{S,t-1}}{\pi_t} + \frac{1 + \kappa_{QL,t}}{\pi_t}b_{L,t-1} - (1 - \kappa_d)\Gamma_t \frac{d_{t-1}}{\pi_t} + \frac{\Pi_t}{P_t} - tax_t
\end{aligned} \tag{A.1}$$

while solving for the rental costs in the renters' aggregate budget constraint and substituting in the expression above, we receive:

$$\begin{aligned}
& c_{P,t} + c_{I,t} + c_{R,t} + q_{h,t}(i_{hP,t} + i_{hI,t} + i_{hR,t}) + q_{k,t}i_{k,t} + q_{S,t}b_{S,t} + q_{L,t}b_{L,t} \\
& = w_{P,t}n_{P,t} + w_{I,t}n_{I,t} + w_{R,t}n_{R,t} + r_{k,t}k_{t-1} + \frac{b_{S,t-1}}{\pi_t} + \frac{1 + \kappa_{QL,t}}{\pi_t}b_{L,t-1} - \Gamma_t d_t + \frac{\Pi_t}{P_t} - tax_t
\end{aligned} \tag{A.2}$$

Note that real aggregate profits read as follows:

$$\frac{\Pi_t}{P_t} = \frac{1}{P_t} \int_0^1 P_t(i)y_t(i)di - w_{P,t}n_{P,t} - w_{I,t}n_{I,t} - w_{R,t}n_{R,t} - r_{k,t}k_{t-1} - \frac{\kappa_u}{1 + \varpi} [u_t^{1+\varpi} - 1] k_{t-1} - \frac{\kappa_p}{2} \left(\frac{\pi_t}{\pi_{t-1}^{\zeta_p} \pi^{1-\zeta_p}} - 1 \right)^2 y_t \tag{A.3}$$

Substituting (A.3) in (A.2), and abstracting from capital adjustment costs (i.e. $\kappa_u = 0$), we obtain:

$$\begin{aligned}
& c_{P,t} + c_{I,t} + c_{R,t} + i_{hP,t} + i_{hI,t} + i_{hR,t} + i_{k,t} + q_{S,t}b_{S,t} + q_{L,t}b_{L,t} \\
& = y_t + \frac{b_{S,t-1}}{\pi_t} + \frac{1 + \kappa_{QL,t}}{\pi_t}b_{L,t-1} - \Gamma_t d_t - \frac{\kappa_p}{2} \left(\frac{\pi_t}{\pi_{t-1}^{\zeta_p} \pi^{1-\zeta_p}} - 1 \right)^2 y_t - tax_t
\end{aligned} \tag{A.4}$$

The real government budget constraint is summarized by:

$$g_t + \frac{b_{S,t-1}}{\pi_t} + \frac{1 + \kappa_{QL,t}}{\pi_t}b_{L,t-1} = tax_t + q_{S,t}b_{S,t} + q_{L,t}b_{L,t} \tag{A.5}$$

Substituting out for newly issued and accumulated debt in (A.4) using the government budget constraint, we obtain the resource constraint of the economy:

$$c_t + i_{h,t} + i_{k,t} + g_t = y_t - \frac{\kappa_P}{2} \left(\frac{\pi_t}{\pi_{t-1}^{\zeta_P} \pi^{1-\zeta_P}} - 1 \right)^2 y_t - \Gamma_t (q_{L,t} b_{L,t} + d_t) \quad (\text{A.6})$$

where we assume that Γ_t has the following functional form:

$$\Gamma_t = \Gamma_1 \left(\frac{q_{L,t} b_{L,t}}{q_{S,t} b_{S,t}} \right)^{\Gamma_2} - 1 \quad (\text{A.7})$$

A.2 Proof of Proposition 1: Derivation of the welfare criterion

The derivation of the welfare criterion consists of taking the second order approximation to the utility of patient, impatient and renter households, respectively. For simplicity, we abstract from physical capital assuming labor as the only input in the production function.

Patient households

The second order approximation to the utility function of patient households reads as follows:

$$\begin{aligned} W_{P,t} = & U_P + U_{c_P} x_P \left(\hat{c}_{P,t} + \frac{1}{2} \left(1 + \frac{U_{c_P c_P} c_P}{U_{c_P}} \right) \hat{c}_{P,t}^2 \right) + U_{h_P} h_P \left(\hat{h}_{P,t} + \frac{1}{2} \left(1 + \frac{U_{h_P h_P} h_P}{U_{h_P}} \right) \hat{h}_{P,t}^2 \right) \\ & - U_{n_P} n_P \left(\hat{n}_{P,t} + \frac{1}{2} \left(1 + \frac{U_{n_P n_P} n_P}{U_{n_P}} \right) \hat{n}_{P,t}^2 \right) \end{aligned} \quad (\text{A.8})$$

where $U_{c_P}, U_{h_P}, U_{n_P}$ is the marginal utility of consumption, the marginal utility of housing and the marginal disutility of labor, respectively. Computing the marginal utility of consumption and housing, the marginal disutility of and the FOC w.r.t. labor at the steady state, we obtain:

$$W_{P,t} = U_P + U_{c_P} \left[\frac{c_P}{1 - \zeta} \hat{c}_{P,t} + \frac{U_{h_P} h_P}{U_{c_P}} \hat{h}_{P,t} - \frac{U_{n_P} n_P}{U_{c_P}} \left(\hat{n}_{P,t} + \frac{1}{2} (1 + \vartheta) \hat{n}_{P,t}^2 \right) \right] \quad (\text{A.9})$$

Impatient households

The second order approximation to the utility function of impatient households reads as follows:

$$\begin{aligned} W_{I,t} = & U_I + U_{c_I} x_I \left(\hat{c}_{I,t} + \frac{1}{2} \left(1 + \frac{U_{c_I c_I} c_I}{U_{c_I}} \right) \hat{c}_{I,t}^2 \right) + U_{h_I} h_I \left(\hat{h}_{I,t} + \frac{1}{2} \left(1 + \frac{U_{h_I h_I} h_I}{U_{h_I}} \right) \hat{h}_{I,t}^2 \right) \\ & - U_{n_I} n_I \left(\hat{n}_{I,t} + \frac{1}{2} \left(1 + \frac{U_{n_I n_I} n_I}{U_{n_I}} \right) \hat{n}_{I,t}^2 \right) \end{aligned} \quad (\text{A.10})$$

where $U_{c_I}, U_{h_I}, U_{n_I}$ is the marginal utility of consumption, the marginal utility of housing and the marginal disutility of labor, respectively. Computing the marginal utility of consumption and housing,

the marginal disutility of labor and the FOC w.r.t. labor at the steady state, we obtain:

$$W_{I,t} = U_I + U_{c_I} \left[c_I \hat{c}_{I,t} + \frac{U_{h_I}}{U_{c_I}} h_I \hat{h}_{I,t} - \frac{U_{n_I}}{U_{c_I}} n_I \left(\hat{n}_{I,t} + \frac{1}{2} (1 + \vartheta) \hat{n}_{I,t}^2 \right) \right] \quad (\text{A.11})$$

Renter households

The second order approximation to the utility function of renter households reads as follows:

$$\begin{aligned} W_{R,t} = & U_R + U_{c_R} c_R \left(\hat{c}_{R,t} + \frac{1}{2} \left(1 + \frac{U_{c_R c_R} c_R}{U_{c_R}} \right) \hat{c}_{R,t}^2 \right) + U_{h_R} h_R \left(\hat{h}_{R,t} + \frac{1}{2} \left(1 + \frac{U_{h_R h_R} h_R}{U_{h_R}} \right) \hat{h}_{R,t}^2 \right) \\ & - U_{n_R} n_R \left(\hat{n}_{R,t} + \frac{1}{2} \left(1 + \frac{U_{n_R n_R} n_R}{U_{n_R}} \right) \hat{n}_{R,t}^2 \right) \end{aligned} \quad (\text{A.12})$$

where $U_{c_R}, U_{h_R}, U_{n_R}$ is the marginal utility of consumption, the marginal utility of housing and the marginal disutility of labor, respectively. Computing the marginal utility of consumption and housing, the marginal disutility of and the FOC w.r.t. labor at the steady state, we obtain:

$$W_{R,t} = U_R + U_{c_R} \left[c_R \hat{c}_{R,t} + \frac{U_{h_R}}{U_{c_R}} h_R \hat{h}_{R,t} - \frac{U_{n_R}}{U_{c_R}} n_R \left(\hat{n}_{R,t} + \frac{1}{2} (1 + \vartheta) \hat{n}_{R,t}^2 \right) \right] \quad (\text{A.13})$$

The objective function of the policy maker is a weighted average of the three welfare measures:

$$W_t = (1 - \beta_P) W_{P,t} + (1 - \beta_I) W_{I,t} + (1 - \beta_R) W_{R,t} \quad (\text{A.14})$$

Note that the terms $\frac{U_{n_j}}{U_{c_j}} n_j$, for $j = P, I, R$, in the welfare of each group is equivalent to:

$$\frac{U_{n_j}}{U_{c_j}} n_j = w_j n_j = \theta_j y \quad (\text{A.15})$$

Using each group's corresponding second order approximation of the utility, we can write the the objective function as:

$$\begin{aligned} W_t = & (1 - \beta_P) \hat{c}_{P,t} + (1 - \beta_I) \hat{c}_{I,t} + (1 - \beta_R) \hat{c}_{R,t} + (1 - \beta_P) \xi_P \hat{h}_{P,t} + (1 - \beta_I) \xi_I \hat{h}_{I,t} + (1 - \beta_R) \xi_R \hat{h}_{R,t} \\ & - \frac{1 - \beta_P}{c_P} \theta_{P,y} \left(\hat{n}_{P,t} + \frac{1 + \vartheta}{2} \hat{n}_{P,t}^2 \right) - \frac{1 - \beta_I}{c_I} \theta_{I,y} \left(\hat{n}_{I,t} + \frac{1 + \vartheta}{2} \hat{n}_{I,t}^2 \right) - \frac{1 - \beta_R}{c_R} \theta_{R,y} \left(\hat{n}_{R,t} + \frac{1 + \vartheta}{2} \hat{n}_{R,t}^2 \right) \end{aligned} \quad (\text{A.16})$$

Combining the FOC of the firm's maximization problem w.r.t. to labor inputs, that define input demands, with the FOCs of each group w.r.t. labor supply and taking a first order approximation

of the resulting expression (and setting production costs f to zero), we obtain:

$$\hat{n}_{j,t} = \frac{1}{1 + \vartheta} \left[\hat{\Omega}_t + \hat{y}_t - \hat{c}_{j,t} \right] \quad (\text{A.17})$$

for $j = P, I, R$. Plugging the above expression in (A.16) and gathering terms:

$$\begin{aligned} W_t = & (1 - \beta_P) \left(1 + \frac{\theta_P y}{c_P (1 + \vartheta)} \right) \hat{c}_{P,t} + (1 - \beta_I) \left(1 + \frac{\theta_I y}{c_I (1 + \vartheta)} \right) \hat{c}_{I,t} + (1 - \beta_R) \left(1 + \frac{\theta_R y}{c_R (1 + \vartheta)} \right) \hat{c}_{R,t} \\ & + (1 - \beta_P) \xi_P \hat{h}_{P,t} + (1 - \beta_I) \xi_I \hat{h}_{I,t} + (1 - \beta_R) \xi_R \hat{h}_{R,t} \\ & - \left(\frac{(1 - \beta_P) \theta_P}{c_P} + \frac{(1 - \beta_I) \theta_I}{c_I} + \frac{(1 - \beta_R) \theta_R}{c_R} \right) \frac{y}{1 + \vartheta} \left(\hat{\Omega}_t + \frac{1}{2} \hat{\Omega}_t^2 + \hat{y}_t + \frac{1}{2} \hat{y}_t^2 + \hat{\Omega}_t \hat{y}_t \right) \\ & - \left(\frac{(1 - \beta_P) \theta_P y}{2 c_P (1 + \vartheta)} \right) \hat{c}_{P,t}^2 - \left(\frac{(1 - \beta_I) \theta_I y}{2 c_I (1 + \vartheta)} \right) \hat{c}_{I,t}^2 - \left(\frac{(1 - \beta_R) \theta_R y}{2 c_R (1 + \vartheta)} \right) \hat{c}_{R,t}^2 \\ & + \left(\hat{\Omega}_t + \hat{y}_t \right) \left[\frac{(1 - \beta_P) \theta_P y}{c_P (1 + \vartheta)} \hat{c}_{P,t} + \frac{(1 - \beta_I) \theta_I y}{c_I (1 + \vartheta)} \hat{c}_{I,t} + \frac{(1 - \beta_R) \theta_R y}{c_R (1 + \vartheta)} \hat{c}_{R,t} \right] \end{aligned} \quad (\text{A.18})$$

Let us now work with the marginal cost term, $\hat{\Omega}_t$. Log-linearization leads to:

$$\hat{\Omega}_t = \theta_P \hat{w}_{P,t} + \theta_I \hat{w}_{I,t} + \theta_R \hat{w}_{R,t} - \hat{z}_t \quad (\text{A.19})$$

Using the log-linearized FOCs of households w.r.t. to labor, we may rewrite the expression above as follows:

$$\hat{\Omega}_t = \vartheta \theta_P \hat{n}_{P,t} + \vartheta \theta_I \hat{n}_{I,t} + \vartheta \theta_R \hat{n}_{R,t} + \theta_P \hat{c}_{P,t} + \theta_I \hat{c}_{I,t} + \theta_R \hat{c}_{R,t} - \hat{z}_t \quad (\text{A.20})$$

and using the output equation we can simplify to:

$$\hat{\Omega}_t = \vartheta \hat{y}_t + \theta_P \hat{c}_{P,t} + \theta_I \hat{c}_{I,t} + \theta_R \hat{c}_{R,t} \quad (\text{A.21})$$

where *t.i.p.* denotes terms independent of policy (e.g. productivity shocks in our case) Plugging (A.21) in (A.18) and gathering terms, we obtain:

$$\begin{aligned}
W_t = & \left(\varpi_P - \zeta \frac{y\theta_P}{1+\vartheta} \right) \hat{c}_{P,t} + \left(\varpi_I - \zeta \frac{y\theta_I}{1+\vartheta} \right) \hat{c}_{I,t} + \left(\varpi_R - \zeta \frac{y\theta_R}{1+\vartheta} \right) \hat{c}_{R,t} \\
& + (1-\beta_P) \xi_P \hat{h}_{P,t} + (1-\beta_I) \xi_I \hat{h}_{I,t} + (1-\beta_R) \xi_R \hat{h}_{R,t} \\
& - \frac{1}{2} \tilde{\varpi}_P \hat{c}_{P,t}^2 - \frac{1}{2} \tilde{\varpi}_I \hat{c}_{I,t}^2 - \frac{1}{2} \tilde{\varpi}_R \hat{c}_{R,t}^2 \\
& + \vartheta \hat{y}_t \left[\left(\frac{(1-\beta_P)\theta_P y}{c_P} - \zeta y \theta_P \right) \hat{c}_{P,t} + \left(\frac{(1-\beta_I)\theta_I y}{c_I} - \zeta y \theta_I \right) \hat{c}_{I,t} + \left(\frac{(1-\beta_R)\theta_R y}{c_R} - \zeta y \theta_R \right) \hat{c}_{R,t} \right] \\
& + \left(\theta_P(\varpi_I - 1 + \beta_I) + \theta_I(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_I}{(1+\vartheta)} \right) \hat{c}_{P,t} \hat{c}_{I,t} \\
& + \left(\theta_P(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_R}{(1+\vartheta)} \right) \hat{c}_{P,t} \hat{c}_{R,t} \\
& + \left(\theta_I(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_I - 1 + \beta_I) - \frac{\zeta y \theta_R \theta_I}{(1+\vartheta)} \right) \hat{c}_{R,t} \hat{c}_{I,t} \\
& - \zeta y \left(\hat{y}_t + \frac{1}{2} \left(\vartheta + \frac{1}{1+\vartheta} \right) \hat{y}_t^2 \right) + t.i.p. + O(\|\xi^3\|) \tag{A.22}
\end{aligned}$$

where:

$$\varpi_j = (1-\beta_j) \left(1 + \frac{\theta_j y}{c_j (1+\vartheta)} \right) \quad \text{for } j = P, I, R$$

$$\zeta = \frac{(1-\beta_P)\theta_P}{c_P} + \frac{(1-\beta_I)\theta_I}{c_I} + \frac{(1-\beta_R)\theta_R}{c_R}$$

$$\tilde{\varpi}_j = \left[\frac{\theta_j^2 \zeta y}{1+\vartheta} + \frac{(1-\beta_j)\theta_j y}{c_j (1+\vartheta)} - \frac{2(1-\beta_j)\theta_j^2 y}{c_j (1+\vartheta)} \right] \quad \text{for } j = P, I, R$$

Taking a second order approximation of the resource constraint (A.6) under zero indexation:

$$\begin{aligned}
\hat{y}_t = & \frac{c_P}{y} \left(\hat{c}_{P,t} + \frac{1}{2} \hat{c}_{P,t}^2 \right) + \frac{c_I}{y} \left(\hat{c}_{I,t} + \frac{1}{2} \hat{c}_{I,t}^2 \right) + \frac{c_R}{y} \left(\hat{c}_{R,t} + \frac{1}{2} \hat{c}_{R,t}^2 \right) \\
& + \frac{i_P}{y} \left(\hat{i}_{P,t} + \frac{1}{2} \hat{i}_{P,t}^2 \right) + \frac{i_I}{y} \left(\hat{i}_{I,t} + \frac{1}{2} \hat{i}_{I,t}^2 \right) + \frac{i_R}{y} \left(\hat{i}_{R,t} + \frac{1}{2} \hat{i}_{R,t}^2 \right) \\
& + \frac{\kappa_P}{2} \pi_t^2 + \frac{(q_L b_L + d)}{y} \left(\hat{\Gamma}_t + \frac{1}{2} \hat{\Gamma}_t^2 \right) - \frac{1}{2} \hat{y}_t^2 + O(\|\xi^3\|) \tag{A.23}
\end{aligned}$$

Substituting the expression above in (A.22) and gathering terms, we get:

$$\begin{aligned}
W_t = & \left(\varpi_P - \zeta \frac{y\theta_P}{1+\vartheta} - \zeta c_P \right) \hat{c}_{P,t} + \left(\varpi_I - \zeta \frac{y\theta_I}{1+\vartheta} - \zeta c_I \right) \hat{c}_{I,t} + \left(\varpi_R - \zeta \frac{y\theta_R}{1+\vartheta} - \zeta c_R \right) \hat{c}_{R,t} \\
& + (1 - \beta_P) \xi_P \hat{h}_{P,t} + (1 - \beta_I) \xi_I \hat{h}_{I,t} + (1 - \beta_R) \xi_R \hat{h}_{R,t} \\
& - \left(\tilde{\varpi}_P + \frac{1}{2} \zeta c_P \right) \hat{c}_{P,t}^2 - \left(\tilde{\varpi}_I + \frac{1}{2} \zeta c_I \right) \hat{c}_{I,t}^2 - \left(\tilde{\varpi}_R + \frac{1}{2} \zeta c_R \right) \hat{c}_{R,t}^2 \\
& - \zeta i_P \left(\hat{i}_{P,t} + \frac{1}{2} \hat{i}_{P,t}^2 \right) - \zeta i_I \left(\hat{i}_{I,t} + \frac{1}{2} \hat{i}_{I,t}^2 \right) - \zeta i_R \left(\hat{i}_{R,t} + \frac{1}{2} \hat{i}_{R,t}^2 \right) \\
& + \vartheta \hat{y}_t \left[\left(\frac{(1 - \beta_P) \theta_{Py}}{c_P} - \zeta y \theta_P \right) \hat{c}_{P,t} + \left(\frac{(1 - \beta_I) \theta_{Iy}}{c_I} - \zeta y \theta_I \right) \hat{c}_{I,t} + \left(\frac{(1 - \beta_R) \theta_{Ry}}{c_R} - \zeta y \theta_R \right) \hat{c}_{R,t} \right] \\
& + \left(\theta_P (\varpi_I - 1 + \beta_I) + \theta_I (\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_I}{(1 + \vartheta)} \right) \hat{c}_{P,t} \hat{c}_{I,t} \\
& + \left(\theta_P (\varpi_R - 1 + \beta_R) + \theta_R (\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_R}{(1 + \vartheta)} \right) \hat{c}_{P,t} \hat{c}_{R,t} \\
& + \left(\theta_I (\varpi_R - 1 + \beta_R) + \theta_R (\varpi_I - 1 + \beta_I) - \frac{\zeta y \theta_R \theta_I}{(1 + \vartheta)} \right) \hat{c}_{R,t} \hat{c}_{I,t} \\
& - \frac{\zeta y \vartheta^2}{2(1 + \vartheta)} \hat{y}_t^2 - \frac{\zeta y \kappa_P}{2} \pi_t^2 - \zeta (q_L b_L + d) \left(\hat{\Gamma}_t + \hat{\Gamma}_t^2 \right) + t.i.p. + O(\|\xi^3\|) \tag{A.24}
\end{aligned}$$

Since we have abstracted from capital, $\hat{i}_{h,t}$ comprises total investment, \hat{i}_t . The second order approximation of the laws of motion of owner occupied, rental housing and the law of motion of housing of impatient households reads as follows:

$$\hat{h}_{j,t} = (1 - \delta_h) \hat{h}_{j,t-1} + \frac{i_{hj}}{h_j} \hat{i}_{hj,t} - \frac{1}{2} (1 - \delta_h)^2 \hat{h}_{j,t-1}^2 - \frac{1}{2} \frac{i_{hj}^2}{h_j^2} \hat{i}_{hj,t}^2 - \frac{1 - \delta_h}{h_j^2} (i_{hj} + 1 - \delta_h) \left(\hat{i}_{hj,t} \hat{h}_{j,t-1} \right) + \frac{1}{2} \hat{h}_{j,t}^2 \tag{A.25}$$

for $j = P, I, R$. Iterating (A.25) backwards, we obtain the following expression:

$$\hat{h}_{j,t} = (1 - \delta_h)^{t+1} \left(\hat{h}_{P,-1} - \frac{1}{2} \hat{h}_{P,-1}^2 \right) + \frac{i_{hj}}{h_P} \sum_{s=0}^t (1 - \delta_h)^{t-s} \hat{i}_{hj,s} - \frac{1}{2} \frac{i_{hj}^2}{h_P^2} \sum_{s=0}^t (1 - \delta_h)^{t-s} \hat{i}_{hj,s}^2 + O(\|\xi^3\|) \tag{A.26}$$

where $O(\|\xi^3\|)$ captures terms of order higher than two. Note that the first term in parenthesis on the RHS is independent of the policy that one chooses to apply in periods $t \geq 0$. Thus, if one takes the discounted value of these terms over all periods $t \geq 0$, one obtains:

$$\sum_{t=0}^{\infty} \beta^t \hat{h}_{j,t} = \left(\frac{1}{1 - (1 - \delta_h) \beta} \right) \frac{i_{hj}}{h_P} \sum_{t=0}^{\infty} \beta^t \hat{i}_{hj,t} - \left(\frac{1}{1 - (1 - \delta_h) \beta} \right) \frac{i_{hj}^2}{2h_P^2} \sum_{t=0}^{\infty} \beta^t \hat{i}_{hj,t}^2 + O(\|\xi^3\|) \tag{A.27}$$

We can now substitute (A.27) in the discounted sum of household's utility. Following Woodford

(2001), the discounted sum of utility of the representative household can be approximated by:

$$\sum_{t=0}^{\infty} \beta^t U_t = \sum_{t=0}^{\infty} \beta^t W_t + t.i.p. + O(\|\xi^3\|) \quad (\text{A.28})$$

or

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t U_t = & \sum_{t=0}^{\infty} \beta^t \left\{ \left(\varpi_P - \zeta \frac{y\theta_P}{1+\vartheta} - \zeta c_P \right) \hat{c}_{P,t} + \left(\varpi_I - \zeta \frac{y\theta_I}{1+\vartheta} - \zeta c_I \right) \hat{c}_{I,t} + \left(\varpi_R - \zeta \frac{y\theta_R}{1+\vartheta} - \zeta c_R \right) \hat{c}_{R,t} \right. \\ & + \left(\frac{1}{1-(1-\delta_h)\beta} \right) \left[(1-\beta_P) \xi_P \frac{i_{hP}}{h_P} \left(\hat{i}_{hP,t} - \frac{i_{hP}}{2h_P} \hat{i}_{hP,t}^2 \right) + (1-\beta_I) \xi_I \frac{i_{hI}}{h_I} \left(\hat{i}_{hI,t} - \frac{i_{hI}}{2h_P} \hat{i}_{hI,t}^2 \right) \right. \\ & \left. \left. + (1-\beta_R) \xi_R \frac{i_{hR}}{h_R} \left(\hat{i}_{hR,t} - \frac{i_{hR}}{2h_R} \hat{i}_{hR,t}^2 \right) \right] \right. \\ & - \zeta i_{hP} \left(\hat{i}_{hP,t} + \frac{1}{2} \hat{i}_{hP,t}^2 \right) - \zeta i_{hI} \left(\hat{i}_{hI,t} + \frac{1}{2} \hat{i}_{hI,t}^2 \right) - \zeta i_{hR} \left(\hat{i}_{hR,t} + \frac{1}{2} \hat{i}_{hR,t}^2 \right) \\ & - \frac{1}{2} (\tilde{\varpi}_P + \zeta c_P) \hat{c}_{P,t}^2 - \frac{1}{2} (\tilde{\varpi}_I + \zeta c_I) \hat{c}_{I,t}^2 - \frac{1}{2} (\tilde{\varpi}_R + \zeta c_R) \hat{c}_{R,t}^2 \\ & + \vartheta \hat{y}_t \left[\left(\frac{(1-\beta_P)\theta_P y}{c_P} - \zeta y \theta_P \right) \hat{c}_{P,t} + \left(\frac{(1-\beta_I)\theta_I y}{c_I} - \zeta y \theta_I \right) \hat{c}_{I,t} + \left(\frac{(1-\beta_R)\theta_R y}{c_R} - \zeta y \theta_R \right) \hat{c}_{R,t} \right] \\ & + \left(\theta_P(\varpi_I - 1 + \beta_I) + \theta_I(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_I}{(1+\vartheta)} \right) \hat{c}_{P,t} \hat{c}_{I,t} \\ & + \left(\theta_P(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_R}{(1+\vartheta)} \right) \hat{c}_{P,t} \hat{c}_{R,t} \\ & + \left(\theta_I(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_I - 1 + \beta_I) - \frac{\zeta y \theta_R \theta_I}{(1+\vartheta)} \right) \hat{c}_{R,t} \hat{c}_{I,t} \\ & \left. - \frac{\zeta y \vartheta^2}{2(1+\vartheta)} \hat{y}_t^2 - \frac{\zeta y \kappa_P}{2} \pi_t^2 - \zeta (q_L b_L + d) \left(\hat{\Gamma}_t + \frac{1}{2} \hat{\Gamma}_t^2 \right) \right\} + t.i.p. + O(\|\xi^3\|) \quad (\text{A.29}) \end{aligned}$$

Simplifying and gathering terms leads to:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t U_t = & \sum_{t=0}^{\infty} \beta^t \left\{ \omega_{c_P} \hat{c}_{P,t} - \frac{1}{2} \tilde{\omega}_{c_P} \hat{c}_{P,t}^2 + \omega_{c_I} \hat{c}_{I,t} - \frac{1}{2} \tilde{\omega}_{c_I} \hat{c}_{I,t}^2 + \omega_{c_R} \hat{c}_{R,t} - \frac{1}{2} \tilde{\omega}_{c_R} \hat{c}_{R,t}^2 \right. \\ & + \omega_{i_{hP}} \hat{i}_{hP,t} - \frac{1}{2} \tilde{\omega}_{i_{hP}} \hat{i}_{hP,t}^2 + \omega_{i_{hI}} \hat{i}_{hI,t} - \frac{1}{2} \tilde{\omega}_{i_{hI}} \hat{i}_{hI,t}^2 + \omega_{i_{hR}} \hat{i}_{hR,t} - \frac{1}{2} \tilde{\omega}_{i_{hR}} \hat{i}_{hR,t}^2 \\ & - \frac{1}{2} \omega_y \hat{y}_t^2 - \frac{1}{2} \omega_\pi \pi_t^2 - \omega_{\gamma_b} \left(\Gamma_2 \hat{\gamma}_{b,t} + \frac{\Gamma_2(\Gamma_2-1)}{2\gamma_b} \hat{\gamma}_{b,t}^2 \right) \\ & \left. - \omega_{y_{c_P}} \hat{y}_t \hat{c}_{P,t} - \omega_{y_{c_I}} \hat{y}_t \hat{c}_{I,t} - \omega_{y_{c_R}} \hat{y}_t \hat{c}_{R,t} + \omega_{c_P c_I} \hat{c}_{P,t} \hat{c}_{I,t} + \omega_{c_P c_R} \hat{c}_{P,t} \hat{c}_{R,t} + \omega_{c_R c_I} \hat{c}_{R,t} \hat{c}_{I,t} \right\} \\ & + t.i.p. + O(\|\xi^3\|) \quad (\text{A.30}) \end{aligned}$$

where:

$$\begin{aligned}
\omega_{c_P} &= \varpi_P - \zeta \frac{y\theta_P}{1+\vartheta} - \zeta c_P, & \omega_{c_I} &= \varpi_I - \zeta \frac{y\theta_I}{1+\vartheta} - \zeta c_I, & \omega_{c_R} &= \varpi_R - \zeta \frac{y\theta_R}{1+\vartheta} - \zeta c_R, \\
\tilde{\omega}_{c_P} &= \tilde{\varpi}_P + \zeta c_P - (1 - \beta_P), & \tilde{\omega}_{c_I} &= \tilde{\varpi}_I + \zeta c_I - (1 - \beta_I), & \tilde{\omega}_{c_R} &= \tilde{\varpi}_R + \zeta c_R - (1 - \beta_R), \\
\omega_{i_{h_P}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_P} \right) (1 - \beta_P) \xi_P \frac{i_{h_P}}{h_P} - \zeta i_{h_P}, & \omega_{i_{h_I}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_I} \right) (1 - \beta_I) \xi_I \frac{i_{h_I}}{h_I} - \zeta i_{h_I} \\
\omega_{i_{h_R}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_R} \right) (1 - \beta_R) \xi_R \frac{i_{h_R}}{h_R} - \zeta i_{h_R} \\
\tilde{\omega}_{i_{h_P}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_P} \right) (1 - \beta_P) \xi_P \frac{i_{h_P}^2}{h_P^2} + \zeta i_{h_P}, & \tilde{\omega}_{i_{h_I}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_I} \right) (1 - \beta_I) \xi_I \frac{i_{h_I}^2}{h_I^2} + \zeta i_{h_I} \\
\tilde{\omega}_{i_{h_R}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_R} \right) (1 - \beta_R) \xi_R \frac{i_{h_R}^2}{h_R^2} + \zeta i_{h_R} \\
\omega_y &= \frac{\zeta y \vartheta^2}{(1 + \vartheta)}, & \omega_\pi &= \zeta y \kappa_p, & \omega_{\gamma_b} &= \frac{\zeta (q_L b_L + d)}{\gamma_b} \\
\omega_{y_{c_P}} &= \vartheta \left(\zeta y \theta_P - \frac{(1 - \beta_P) \theta_P y}{c_P} \right), & \omega_{y_{c_I}} &= \vartheta \left(\zeta y \theta_I - \frac{(1 - \beta_I) \theta_I y}{c_I} \right), & \omega_{y_{c_R}} &= \vartheta \left(\zeta y \theta_R - \frac{(1 - \beta_R) \theta_R y}{c_R} \right) \\
\omega_{c_P c_I} &= \theta_P (\varpi_I - 1 + \beta_I) + \theta_I (\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_I}{(1 + \vartheta)}, & \omega_{c_P c_R} &= \theta_P (\varpi_R - 1 + \beta_R) + \theta_R (\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_R}{(1 + \vartheta)} \\
\omega_{c_R c_I} &= \theta_I (\varpi_R - 1 + \beta_R) + \theta_R (\varpi_I - 1 + \beta_I) - \frac{\zeta y \theta_R \theta_I}{(1 + \vartheta)} & & & & (A.31)
\end{aligned}$$

Note that in expression (A.30) we have used the second-order approximation of the transaction cost function, Γ_t . The latter reads as follows:

$$\hat{\Gamma}_t + \frac{1}{2} \hat{\Gamma}_t^2 = \frac{\Gamma_2}{\gamma_b} \hat{\gamma}_{b,t} + \frac{\Gamma_2 (\Gamma_2 - 1)}{2\gamma_b^2} \hat{\gamma}_{b,t}^2 \quad (A.32)$$

where, $\hat{\gamma}_{b,t}$, is the linearized version of $\gamma_{b,t}$. At this stage, it is convenient to linearize the equilibrium conditions that serve as constraints to the maximization problem of the central bank under commitment. Note that below, we consider the version of the model without capital. Taking a first-order Taylor approximation of the optimal pricing equation, we obtain the Phillips curve:

$$\pi_t = \beta_P E_t \pi_{t+1} + \frac{\eta_y - 1}{\kappa_p} \left(\hat{\Omega}_t + \hat{\Theta}_{p,t} \right) \quad (A.33)$$

where $\hat{\Theta}_{p,t}$:

$$\hat{\Theta}_{p,t} = -\frac{1}{\eta_y - 1} \hat{\eta}_{y,t} \quad (A.34)$$

Log-linearizing the equilibrium conditions from the optimization problem of patient households, we obtain:

(1) $h_{P,t}(i)$:

$$\hat{q}_{h,t} + \hat{\lambda}_{P,t} = -\frac{\xi_h}{q_h \lambda_P} \hat{h}_{P,t} + (1 - \delta_h) \beta_P E_t \left(\hat{\lambda}_{P,t+1} + E_t \hat{q}_{h,t+1} \right) \quad (A.35)$$

where $\hat{\lambda}_{P,t}$:

$$\hat{\lambda}_{P,t} = -\hat{c}_{P,t} \quad (A.36)$$

(2) $h_{R,t}(i)$:

$$\hat{q}_{h,t} + \left(1 - \frac{r_h}{q_h}\right) \hat{\lambda}_{P,t} = \frac{r_h}{q_h} \hat{r}_{h,t} + (1 - \delta_h) \beta_P E_t \left(\hat{\lambda}_{P,t+1} + E_t \hat{q}_{h,t+1} \right) \quad (\text{A.37})$$

(3) $k_{P,t}(i)$:

$$\hat{q}_{k,t} + \hat{\lambda}_{P,t} = E_t \left[\left(1 - \delta_k + \frac{r_k}{q_k}\right) \hat{\lambda}_{P,t+1} + \frac{r_k}{q_k} \hat{r}_{P,t+1} + (1 - \delta_k) \hat{q}_{h,t+1} \right] \quad (\text{A.38})$$

(4) $n_{P,t}(i)$:

$$\vartheta \hat{n}_{P,t} = \hat{\lambda}_{P,t} + \hat{w}_{P,t} \quad (\text{A.39})$$

(5) $b_{S,t}(i)$:

$$\hat{\lambda}_{P,t} = E_t \hat{\lambda}_{P,t+1} - E_t \pi_{t+1} - \hat{q}_{S,t} \quad (\text{A.40})$$

(6) $b_{L,t}(i)$:

$$\hat{\Gamma}_t + \hat{q}_{L,t} = E_t \hat{\lambda}_{P,t+1} - \hat{\lambda}_{P,t} - E_t \pi_{t+1} + \frac{\kappa_{qL}}{1 + \kappa_{qL}} E_t \hat{q}_{L,t+1} \quad (\text{A.41})$$

(7) $l_t(i)$:

$$\hat{\Gamma}_t = \Omega_{dP} \hat{\Omega}_{dP,t} + R^l \Omega_{rP} \left(\hat{\Omega}_{rP,t} + \hat{R}_t^l \right) \quad (\text{A.42})$$

(8) d_t :

$$\begin{aligned} & \frac{1}{\Omega_{dP} + \Omega_{rP} R^d} \left(\Omega_{dP} \hat{\Omega}_{dP,t} + \Omega_{rP} R^d \hat{\Omega}_{rP,t} + \Omega_{rP} R^d \hat{R}_t^d \right) = E_t \hat{\lambda}_{P,t+1} - \hat{\lambda}_{P,t} - E_t \pi_{t+1} \\ & + \frac{1 - \kappa_d}{R^d + \kappa_d - (1 - \kappa_d) \Gamma + (1 - \kappa_d) (\Omega_{dP} + \Omega_{rP} R^d)} \left[\left(\frac{1}{1 - \kappa_d} + \Omega_{rP} \right) R^d \hat{R}_t^d - E_t \hat{\Gamma}_{t+1} + \Omega_{dP} E_t \hat{\Omega}_{dP,t+1} + \Omega_{rP} R^d E_t \hat{\Omega}_{rP,t+1} \right] \end{aligned} \quad (\text{A.43})$$

(9) R_t^d :

$$\hat{\Omega}_{rP,t} = E_t \hat{\lambda}_{P,t+1} - \hat{\lambda}_{P,t} - E_t \pi_{t+1} + \frac{(1 - \kappa_d) \Omega_{rP}}{1 + (1 - \kappa_d) \Omega_{rP}} E_t \hat{\Omega}_{rP,t+1} \quad (\text{A.44})$$

Log-linearizing the FOCs of the impatient households' maximization, we obtain:

(10) $h_{I,t}(i)$:

$$\begin{aligned} & -\frac{\phi\mu}{1 - \phi\mu} \hat{\mu}_t + \hat{q}_{h,t} = \\ & \frac{\xi_h}{(1 - \phi\mu) q_h \lambda_I h_I} \left(\hat{\lambda}_{I,t} - \hat{h}_{I,t} \right) + \beta_I (1 - \delta_h) \left(E_t \hat{\lambda}_{I,t+1} - \hat{\lambda}_{I,t} + E_t \hat{q}_{h,t+1} - \frac{\phi\mu}{1 - \phi\mu} E_t \hat{\mu}_{t+1} \right) \end{aligned} \quad (\text{A.45})$$

where $\hat{\lambda}_{I,t}$:

$$\hat{\lambda}_{I,t} = -\hat{c}_{I,t} \quad (\text{A.46})$$

(11) $n_{I,t}(i)$:

$$\vartheta \hat{n}_{I,t} = \hat{\lambda}_{I,t} + \hat{w}_{I,t} \quad (\text{A.47})$$

$$(12) \ l_t(i) : \quad -\mu \hat{\mu}_t = \Omega_{dI} \hat{\Omega}_{dI,t} + R^l \Omega_{rI} \left(\hat{\Omega}_{rI,t} + \hat{R}_t^l \right) \quad (\text{A.48})$$

$$(13) \ d_t :$$

$$\frac{1}{\Omega_{dI} + \Omega_{rI} R^d} \left(\Omega_{dI} \hat{\Omega}_{dI,t} + \Omega_{rI} R^d \hat{\Omega}_{rI,t} + \Omega_{rI} R^d \hat{R}_t^d \right) = E_t \hat{\lambda}_{I,t+1} - \hat{\lambda}_{I,t} - E_t \pi_{t+1} + \frac{1 - \kappa_d}{R^d + \kappa_d + (1 - \kappa_d) (\Omega_{dI} + \Omega_{rI} R^d)} \left[\left(\frac{1}{1 - \kappa_d} + \Omega_{rI} \right) R^d \hat{R}_t^d + \Omega_{dI} E_t \hat{\Omega}_{dI,t+1} + \Omega_{rI} R^d E_t \hat{\Omega}_{rI,t+1} \right] \quad (\text{A.49})$$

$$(14) \ R_t^d :$$

$$\hat{\Omega}_{rI,t} = E_t \hat{\lambda}_{I,t+1} - \hat{\lambda}_{I,t} - E_t \pi_{t+1} + \frac{(1 - \kappa_d) \Omega_{rI}}{1 + (1 - \kappa_d) \Omega_{rI}} E_t \hat{\Omega}_{rI,t+1} \quad (\text{A.50})$$

$$(15) \ \text{Borrowing constraint}$$

$$\hat{l}_t = \phi q_h i_{hI} \left(\hat{q}_{h,t} + \hat{i}_{hI,t} \right) \quad (\text{A.51})$$

Log-linearizing the FOCs of renter households' maximization, we obtain:

$$(16) \ h_{R,t}^d :$$

$$\hat{r}_{h,t} = -\hat{\lambda}_{R,t} - \hat{h}_{R,t} \quad (\text{A.52})$$

where $\hat{\lambda}_{R,t}$:

$$\hat{\lambda}_{R,t} = -\hat{c}_{R,t} \quad (\text{A.53})$$

$$(17) \ n_{R,t}(i) :$$

$$\vartheta \hat{n}_{R,t} = \hat{\lambda}_{R,t} + \hat{w}_{R,t} \quad (\text{A.54})$$

Log-linearizing the resource constraint (excluding capital), we get:

$$\hat{y}_t = \frac{c_P}{y} \hat{c}_{P,t} + \frac{c_I}{y} \hat{c}_{I,t} + \frac{c_R}{y} \hat{c}_{R,t} + \frac{i_{hP}}{y} \hat{i}_{hP,t} + \frac{i_{hI}}{y} \hat{i}_{hI,t} + \frac{i_{hR}}{y} \hat{i}_{hR,t} \quad (\text{A.55})$$

where in the expression above the terms associated with Rotemberg price adjustment cost have dropped out, since we assume a zero steady-state inflation, $\Pi = 1$. As mentioned in the main body of the text, for the sake of simplicity and exposition, in the optimal monetary policy analysis we abstract from capital accumulation.

A.3 Proof of Proposition 2

Let us work with the case of no heterogeneity first, namely $\theta_P = 1$ and $\theta_I = \theta_R = 0$. Assuming these values in the coefficients of the welfare criterion summarized in (A.31), it is easy to show that the welfare criterion collapses to:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t U_t = \sum_{t=0}^{\infty} \beta^t \left\{ \omega_{c_P} \hat{c}_{P,t} - \frac{1}{2} \tilde{\omega}_{c_P} \hat{c}_{P,t}^2 + \omega_{i_{h_P}} \hat{i}_{h_P,t} - \frac{1}{2} \tilde{\omega}_{i_{h_P}} \hat{i}_{h_P,t}^2 \right. \\ \left. - \frac{1}{2} \omega_y \hat{y}_t^2 - \frac{1}{2} \omega_\pi \pi_t^2 - \omega_{\gamma_b} \left(\Gamma_2 \hat{\gamma}_{b,t} + \frac{\Gamma_2 (\Gamma_2 - 1)}{2\gamma_b} \hat{\gamma}_{b,t}^2 \right) \right\} + t.i.p. + O(\|\xi^3\|) \end{aligned} \quad (\text{A.56})$$

where now the weights collapse to:

$$\begin{aligned} \omega_{c_P} &= \frac{(1 - \beta_P)y}{c_P(1 + \vartheta)}, \quad \tilde{\omega}_{c_P} = 1 - \beta_P \\ \omega_{i_{h_R}} &= \left(\frac{1}{1 - (1 - \delta_h) \beta_R} \right) (1 - \beta_R) \xi_R \frac{i_{h_R}}{h_R} - \frac{1 - \beta_P}{c_P} i_{h_R}, \quad \tilde{\omega}_{i_{h_P}} = \left(\frac{1}{1 - (1 - \delta_h) \beta_P} \right) (1 - \beta_P) \xi_P \frac{i_{h_P}^2}{h_P^2} \\ \omega_y &= \frac{1 - \beta_P}{c_P} y \vartheta^2, \quad \omega_\pi = \frac{1 - \beta_P}{c_P} y \kappa_p \end{aligned}$$

Setting $\theta_P = 1$ and $\theta_I = \theta_R = 0$, it is also easy to show that the weights on the covariance terms in (A.30) are zero. This implies that the central bank is not faced with additional trade-offs once heterogeneity is turned off. Notice also the linear terms in the welfare criterion (A.56). In this case, they arise because of the absence of a subsidy that would render the steady state efficient after removing the distortion due to monopolistic competition.

Updating and adjusting the equilibrium conditions (A.33) - (A.54) to account for $\theta_P = 1$ and $\theta_I = \theta_R = 0$ and taking the first order conditions of the central bank's maximization problem with respect to π_t and \hat{y}_t under commitment, we receive:

$$- \omega_\pi \pi_t - \lambda_{\pi,t} + \lambda_{\pi,t-1} = 0 \quad (\text{A.57})$$

$$- \omega_y \hat{y}_t + \frac{(\eta_y - 1)\vartheta}{\kappa_p} \lambda_{\pi,t} = 0 \quad (\text{A.58})$$

where $\lambda_{\pi,t}$ is the Lagrange multiplier on the Phillips curve in the central bank's maximization problem. Substituting out for $\lambda_{\pi,t}$ and $\lambda_{\pi,t-1}$ in (A.57) using (A.58), we arrive at the standard targeting rule under commitment:

$$\omega_y (\hat{y}_t - \hat{y}_{t-1}) = - \frac{(\eta_y - 1)\omega_\pi \vartheta}{\kappa_p} \pi_t$$

which is the standard trade-off under optimal monetary policy with commitment. Returning now to the case of heterogeneity and maximizing (A.30) subject to (A.33) - (A.54) with respect to π_t and y_t , under commitment, we receive:

$$- \omega_\pi \pi_t - \lambda_{\pi,t} + \lambda_{\pi,t-1} = 0 \quad (\text{A.59})$$

$$- \omega_y \hat{y}_t - \omega_{y_{c_P}} \hat{c}_{P,t} - \omega_{y_{c_I}} \hat{c}_{I,t} - \omega_{y_{c_R}} \hat{c}_{R,t} + \omega_{c_P} + \omega_{c_I} + \omega_{c_R} + \frac{(\eta_y - 1)\vartheta}{\kappa_p} \lambda_{\pi,t} = 0 \quad (\text{A.60})$$

Substituting out for \hat{y}_t using the log-linearized resource constraint (A.55) and gathering terms, we can write the FOC (A.60) as:

$$\begin{aligned}
& - \left(\frac{\omega_y c_P}{y} + \omega_{y c_P} \right) \hat{c}_{P,t} - \left(\frac{\omega_y c_I}{y} + \omega_{y c_I} \right) \hat{c}_{I,t} - \left(\frac{\omega_y c_R}{y} + \omega_{y c_R} \right) \hat{c}_{R,t} \\
& - \frac{\omega_y \hat{i}_{hP}}{y} \hat{i}_{hP,t} - \frac{\omega_y \hat{i}_{hI}}{y} \hat{i}_{hI,t} - \frac{\omega_y \hat{i}_{hR}}{y} \hat{i}_{hR,t} + \omega_{c_P} + \omega_{c_I} + \omega_{c_R} + \frac{(\eta_y - 1)\vartheta}{\kappa_p} \lambda_{\pi,t} = 0
\end{aligned} \tag{A.61}$$

Solving for $\lambda_{\pi,t}$ in (A.61) and substituting in (A.59), we receive the optimal targeting criterion of the central bank under heterogeneity:

$$\begin{aligned}
& \left(\frac{\omega_y c_P}{y} + \omega_{y c_P} \right) (\hat{c}_{P,t} - \hat{c}_{P,t-1}) + \left(\frac{\omega_y c_I}{y} + \omega_{y c_I} \right) (\hat{c}_{I,t} - \hat{c}_{I,t-1}) + \left(\frac{\omega_y c_R}{y} + \omega_{y c_R} \right) (\hat{c}_{R,t} - \hat{c}_{R,t-1}) \\
& + \frac{\omega_y \hat{i}_{hP}}{y} (\hat{i}_{hP,t} - \hat{i}_{hP,t-1}) + \frac{\omega_y \hat{i}_{hI}}{y} (\hat{i}_{hI,t} - \hat{i}_{hI,t-1}) + \frac{\omega_y \hat{i}_{hR}}{y} (\hat{i}_{hR,t} - \hat{i}_{hR,t-1}) \\
& = - \frac{(\eta_y - 1) \omega_\pi \vartheta}{\kappa_p} \pi_t
\end{aligned}$$

and using the resource constraint A.55 the above simplifies to:

$$\omega_y (\hat{y}_t - \hat{y}_{t-1}) + \omega_{y c_P} \Delta \hat{c}_{P,t} + \omega_{y c_I} \Delta \hat{c}_{I,t} + \omega_{y c_R} \Delta \hat{c}_{R,t} = - \frac{(\eta_y - 1) \omega_\pi \vartheta}{\kappa_p} \pi_t \tag{A.62}$$

A.4 Proof of Proposition 3

We derive the optimal relative supply for long-term bonds, $\hat{\gamma}_{b,t}$, by maximizing the welfare criterion (A.30) under commitment. To keep tractability and focus on the intuition of the result, we restrict ourselves to the case of full depreciation of housing ($\delta_h = 1$) and full mortgage debt repayment within one period ($\kappa_d = 1$). Doing so, we end up to the following optimal targeting criterion for the relative supply for long-term bonds:

$$\begin{aligned}
& \omega_{\gamma_b} (1 - \Gamma_2) \hat{\gamma}_{b,t} = \\
& \omega_{\gamma_b} \gamma_b - \frac{\omega_{c_P} - \omega_{c_I} + \omega_{c_R}}{R^l \Omega_{rP}} + \frac{\tilde{\omega}_{c_P} + \omega_{c_P c_I} - \omega_{c_P c_R}}{R^l \Omega_{rP}} \hat{c}_{P,t} + \frac{\omega_{c_P c_I} + \omega_{c_R c_I} - \tilde{\omega}_{c_I}}{R^l \Omega_{rP}} \hat{c}_{I,t} + \frac{\tilde{\omega}_{c_R} + \omega_{c_R c_I} - \omega_{c_P c_R}}{R^l \Omega_{rP}} \hat{c}_{R,t} \\
& - \frac{(\eta_y - 1) \omega_\pi (\theta_R + \theta_I - \theta_P)}{\kappa_p R^l \Omega_{rP}} \pi_t + \frac{\omega_{y c_P} + \omega_{y c_R} - \omega_{y c_I}}{R^l \Omega_{rP}} \hat{y}_t
\end{aligned} \tag{A.63}$$

where γ_b is the relative long-term to short-term steady state bond ratio. In the targeting criterion above, it is clear that when choosing the optimal relative supply of long-term bonds, the central bank considers not only inflation, π_t , but also output, \hat{y}_t , and each group's consumption. When we turn off heterogeneity instead (i.e. $\theta_P = 1$ and $\theta_I = \theta_R = 0$), the covariance terms drop out, meaning that $\omega_{c_P c_I} = \omega_{c_P c_R} = \omega_{c_R c_I} = 0$ and $\omega_{y c_I} = \omega_{y c_R} = \omega_{y c_P} = 0$, which we obtain after using the definition of these parameters in section A.2 above and the homogeneity assumption that $\theta_P = 1$ and $\theta_I = \theta_R = 0$. In this case, the optimal targeting criterion collapses thus to the following:

$$\omega_{\gamma_b} (1 - \Gamma_2) \hat{\gamma}_{b,t} = \omega_{\gamma_b} \gamma_b + \frac{(\eta_y - 1) \omega_\pi}{\kappa_p R^l \Omega_{rP}} \pi_t \tag{A.64}$$

Clearly, from the expression above and given that $\eta_y > 1$, under homogeneity the optimal relative supply of long-term bonds depends solely on the inflation rate.

A.5 Proof of Proposition 4

Plugging in the optimal targeting criterion for the relative long-term bond supply, (61), the log-linearized Phillips curve, (A.33), and gathering terms, we get that the composite coefficient on output, \hat{y}_t , in the resulting targeting criterion for the optimal relative supply of long-term bonds is:¹⁹

$$\frac{\zeta y}{R^l \Omega_{rP}} \left[-(\varsigma + 1)(\theta_I - \theta_P) + \theta_R(1 - \varsigma) + \frac{2(1 - \beta_I)\theta_I}{c_I \zeta} \right] \quad (\text{A.65})$$

where $\varsigma = \left(\frac{\eta_y - 1}{\kappa_p}\right)^2$. Note that since $\eta_y > 1$ and $\kappa_p > \eta_y$, as κ_p is the Rotemberg price adjustment cost parameter (and hence high enough), it follows that $\varsigma \in (0, 1)$. If the above composite coefficient is negative then the rise in output due to the positive demand shock will require a decline in the optimal relative supply of long-term bonds. If the composite coefficient is positive then, the optimal relative supply of long-term bonds must rise following a positive demand shock. We can now derive the share of impatient households that makes the composite coefficient on output negative. Considering thus the case where:

$$\left[-(\varsigma + 1)(\theta_I - \theta_P) + \theta_R(1 - \varsigma) + \frac{2(1 - \beta_I)\theta_I}{c_I \zeta} \right] < 0$$

and solving for θ_I , we receive:

$$\theta_I > \left(\frac{1}{1 + \varsigma - \frac{2(1 - \beta_I)}{c_I \zeta}} \right) (\theta_R(1 - \varsigma) + \theta_P(1 + \varsigma))$$

which guarantees that the optimal relative supply of long-term bonds declines after a positive demand shock. Otherwise, if:

$$\theta_I \leq \left(\frac{1}{1 + \varsigma - \frac{2(1 - \beta_I)}{c_I \zeta}} \right) (\theta_R(1 - \varsigma) + \theta_P(1 + \varsigma))$$

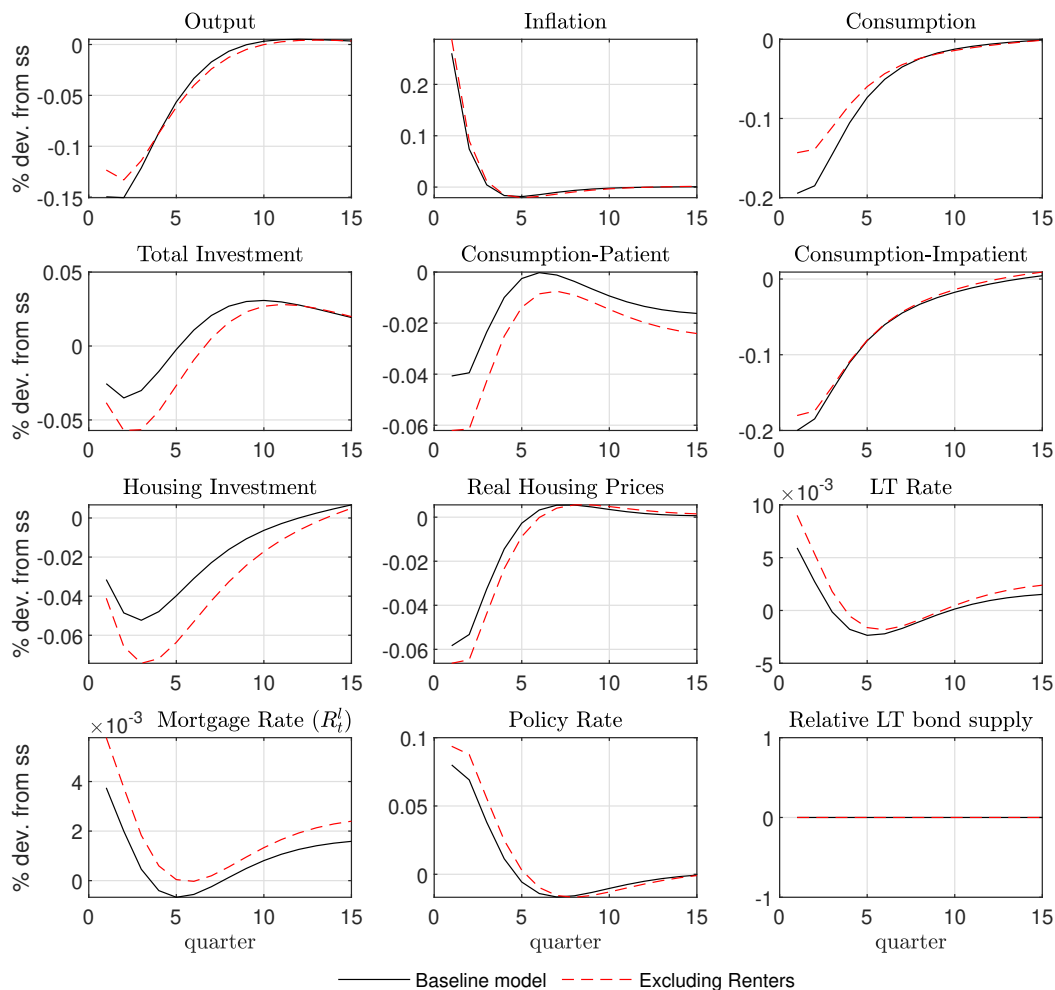
the optimal relative supply of long-term bonds must rise following a positive demand shock.

B The model without renters

In this section, we look at the importance of renters for the dynamics of the economy. For this reason, we set the share of renters in the production function (38) to zero, $\theta_R = 0$. At the same time, we keep the ratio of impatient to patient households the same as in the benchmark calibration. That is, we set $\theta_P = 0.3846$ and $\theta_I = 0.6154$. We consider all three cases as in the main text, namely, the case (Benchmark) where the central bank sets the policy rate following Taylor rule (51) and relative long-term bond supply, $\hat{\gamma}_{b,t}$, is exogenous as in (52), the case where the central bank sets optimally the policy rate while the relative long-term bond supply is exogenous (i.e. one instrument policy), and the case where the central bank sets optimally both the policy rate and the relative supply of long-term bonds. To save space, we restrict our focus to the supply shock and compare the results to those from the baseline calibration with three types of households, as presented in Figure 1. The results are displayed in Figures A.5, A.6 and A.7.

¹⁹Note that for simplicity we have set the inverse of the Firsch elasticity of labor supply, ϑ , to 1, as we also do in our baseline calibration presented at table 1 in the main text.

FIGURE A.5. Model without Renters vs Baseline model with three-types: Monetary Policy conducted via a Taylor Rule with exogenous relative Long-Term Bond supply



Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the baseline model (black solid lines from Figure 1) with three agents, where the central bank follows a Taylor rule and the relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, follows an $AR(1)$. The red-dashed lines are the corresponding impulse responses from the version of the model without renters.

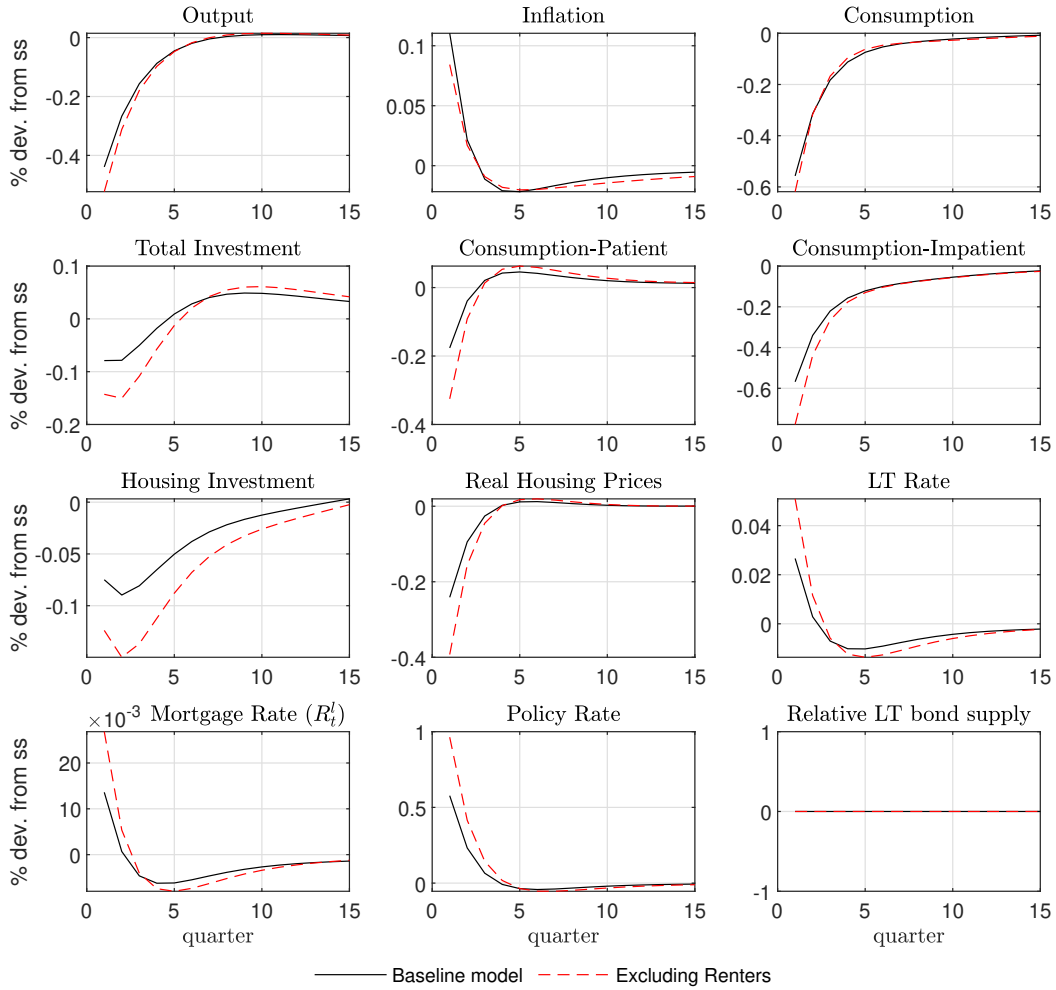
Looking at the responses in the benchmark case (Figure A.5) a few observations stand out. Excluding renters (red-dashed lines) leads to a slightly milder recession while inflation jumps more on impact. The milder recession is, obviously, partly due to the weaker decline in total consumption since the renters are now excluded. Other than that, impatient households (borrowers) are now more credit-constrained because real housing prices decline more when renters are excluded. In addition to being more credit-constrained, they also face higher borrowing costs that dampen the demand for housing and housing investment more compared to the baseline model. When renters are present, the downward pressures on private consumption following the supply shock are stronger which results in a milder jump in inflation and thereby in a milder policy rate hike. Subsequently, given equation (54), this implies that in the model with three-types, the long-term

rate and hence the mortgage rate rate increase less. Looking at the consumption of patient and impatient households, the latter type cuts more its consumption slightly less on impact compared to the baseline model, while patient households' consumption fluctuates at constantly lower levels in the absence of renters and this is due to the lower interest income from housing investment.

Turning now to Figure A.6 displaying the comparison under optimal policy with one instrument (i.e. only the policy rate set optimally), the key message is that in the absence of renters the central bank needs to raise the policy rate optimally more compared to the baseline model. This is because the absence of renters tends to make a given supply shock more inflationary and as such the central bank counteracts it by raising the policy rate more on impact. As a result, in the absence of renters, the long-term and mortgage rates rise higher than in the presence of renters. This explains why housing investment and real housing prices decline more than in the baseline model. Impatient households become thus more credit-constrained when renters are absent adding to the downward pressures in housing and thus total investment. The induced larger contraction of economic activity on impact reduces real wages more than in the baseline model, entailing a deeper dive in the consumption of impatient households in the first quarters after the shock. The inclusion of renters has thus visible implications for optimal monetary policy. Their presence implies a less contractionary monetary policy stance, so that the drop in housing investment is milder, while at the same time making impatient households less credit-constrained due to the milder drop in real housing prices.

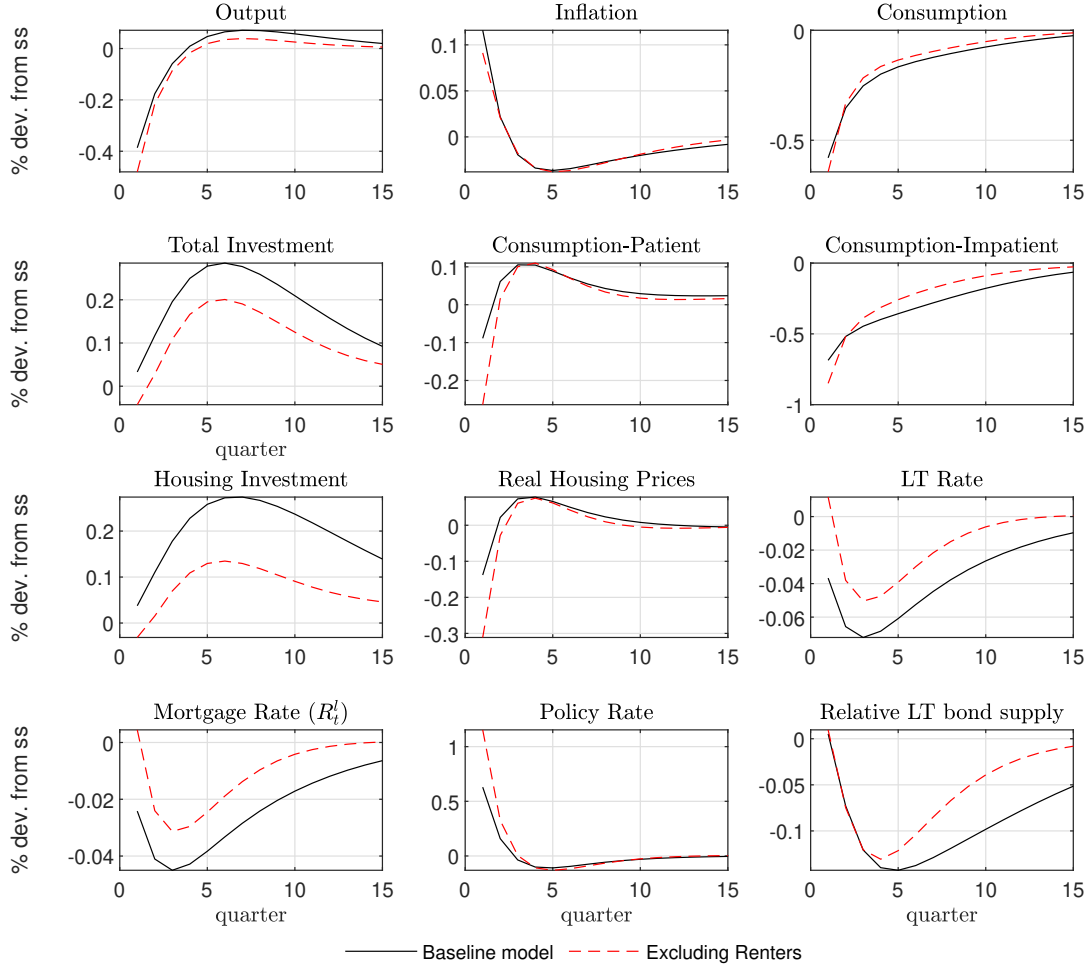
Finally, looking at the case of optimal policy with two instruments (i.e. policy rate and relative long-term bond supply), optimal monetary policy in the baseline model with three types implies a stronger flattening of the yield curve in the quarters following the shock. In fact, compared to the case where renters are absent, the central bank has to lift the policy rate substantially less (about 50bps less) and lower the long-term and the mortgage rate more by optimally decreasing the relative supply of long-term bonds more. Clearly, optimal policy in the presence of renters results in a widening of consumption inequality since the consumption of patient households drops substantially less on impact in the baseline model while the consumption of impatient stays persistently below its level in the absence of renters in the medium-run. The induced stronger flattening of the yield curve in the baseline model causes a boom in housing investment even on impact and to a quick rise in real housing prices. These imply higher wealth for patient households which explains why their consumption falls less. These effects are considerably weaker when renters are absent. The differences in the total consumption responses between the two models are marginal as the benefits from the improved response of patients' consumption are offset by the worsening of the response of impatient's consumption. When it comes to output, the amplified rise in housing and total investment in the baseline model due to the lower interest rates, compared to when renters are absent, seem to explain the milder contraction on impact and the subsequent amplified overshooting in the medium-term.

FIGURE A.6. Model without Renters vs Baseline model with three-types: Optimal Policy with One Instrument



Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the baseline model (black solid lines from Figure 1) with three agents, where optimal monetary policy is conducted by setting only the policy rate, while the relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, follows an $AR(1)$. The red-dashed lines are the corresponding impulse responses from the version of the model without renters.

FIGURE A.7. Model without Renters vs Baseline model with three-types: Optimal Policy under Two Instruments



Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the baseline model (black solid lines from Figure 1) with three agents, where optimal monetary policy is conducted by setting optimally both the policy rate and the relative supply of long-term bonds, $\hat{\gamma}_{b,t}$. The red-dashed lines are the corresponding impulse responses from the version of the model without renters.

C The steady state

C.1 List of steady-state conditions

$$\begin{aligned}
 \frac{1}{c_P} &= \lambda_P \\
 1 &= \xi_h \frac{c_P/y}{h_P/y} + \beta_P (1 - \delta_h) \\
 1 &= r_h + \beta_P (1 - \delta_h) \\
 1 &= \beta_P (1 - \delta_k + r_k) \\
 n_P^{1+\vartheta} &= \frac{w_P n_P/y}{z_P/y}
 \end{aligned}$$

$$\frac{1}{c_I} = \lambda_I$$

$$1 - \phi\mu = \xi_h \frac{c_I/y}{h_I/y} + \beta_I (1 - \delta_h) (1 - \phi\mu)$$

$$n_I^{1+\vartheta} = \frac{w_I l_I/y}{c_I/y}$$

$$\frac{1}{c_R} = \lambda_R$$

$$r_h = \xi_h \frac{c_R/y}{h_R/y}$$

$$n_R^{1+\vartheta} = \frac{w_R l_R/y}{c_R/y}$$

$$q_S = \frac{\beta_P}{\pi}$$

$$(1 + \Gamma) q_L = \frac{\beta_P}{\pi} (1 + \kappa q_L)$$

$$1 + R = \frac{1}{q_S}$$

$$1 + R_L = \frac{1}{q_L} + \kappa$$

$$1 + \Gamma = \Gamma_1 \left(\frac{q_L b_L/y}{q_S b_S/y} \right)^{\Gamma_2}$$

$$\frac{i_k}{y} = \delta_k \frac{k}{y}$$

$$\frac{i_{hP}}{y} = \delta_h \frac{h_P}{y}$$

$$\frac{i_{hI}}{y} = \delta_h \frac{h_I}{y}$$

$$\frac{i_{hR}}{y} = \delta_h \frac{h_R}{y}$$

$$\left(1 - \frac{1 - \kappa_d}{\pi} \right) \frac{d}{y} = \frac{l}{y}$$

$$\frac{l}{y} = \phi \frac{i_{hI}}{y}$$

$$\begin{aligned}
R^d &= R^l \\
1 + \Gamma &= \Omega_{dP} + \Omega_{rP}R^l \\
\Omega_{dP} + \Omega_{rP}R^d &= \beta_P \frac{R^d + \kappa_d - (1 - \kappa_d)\Gamma + (1 - \kappa_d)(\Omega_{dP} + \Omega_{rP}R_t^d)}{\pi_{t+1}} \\
\Omega_{rP} &= \beta_P \frac{1 + (1 - \kappa_d)\Omega_{rP}}{\pi}
\end{aligned}$$

$$\begin{aligned}
1 - \mu &= \Omega_{dI} + \Omega_{rI}R^l \\
\Omega_{dI} + \Omega_{rI}R^d &= \beta_I \frac{R^d + \kappa_d + (1 - \kappa_d)(\Omega_{dI} + \Omega_{rI}R^d)}{\pi} \\
\Omega_{rI} &= \beta_I \frac{1 + (1 - \kappa_d)\Omega_{rI}}{\pi}
\end{aligned}$$

$$\begin{aligned}
\frac{c_I}{y} + \frac{i_{hI}}{y} + \frac{R^d + \kappa_d}{\pi} \frac{d}{y} &= \frac{w_I n_I}{y} + \frac{l}{y} \\
\frac{c_R}{y} + r_h \frac{h_R}{y} &= \frac{w_R n_R}{y}
\end{aligned}$$

$$\begin{aligned}
q_k &= 1 \\
q_h &= 1
\end{aligned}$$

$$\begin{aligned}
\alpha &= r_k \frac{k}{y} \\
(1 - \alpha)\theta_P &= \frac{w_P n_P}{y} \\
(1 - \alpha)\theta_I &= \frac{w_I n_I}{y} \\
(1 - \alpha)(1 - \theta_P - \theta_I) &= \frac{w_R n_R}{y} \\
u = 1 \text{ and } \kappa_u &= \frac{\alpha}{k/y} \\
\Omega &= \frac{1}{\Theta_p} \\
y &= \frac{1}{\Theta_p} k^\alpha \left(n_P^{\theta_P} n_I^{\theta_I} n_R^{\theta_R} \right)^{1-\alpha} \\
\frac{c}{y} + \frac{i_k}{y} + \frac{i_h}{y} + \frac{g}{y} &= 1
\end{aligned}$$

$$\begin{aligned} \frac{g}{y} + \left(\frac{1+R}{\pi} - 1 \right) \frac{q_S b_S}{y} + \left(\frac{1+R_L}{\pi} - 1 \right) \frac{q_L b_L}{y} &= \frac{tax}{y} \\ \frac{q_L b_L}{q_S b_S} &= \gamma_b \\ \frac{tax}{y} &= \Xi \end{aligned}$$

$$\begin{aligned} \frac{h}{y} &= \frac{h_P}{y} + \frac{h_I}{y} + \frac{h_R}{y} \\ \frac{c}{y} &= \frac{c_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} \\ \frac{i}{y} &= \frac{i_k}{y} + \frac{i_h}{y} \\ \frac{i_h}{y} &= \frac{i_{hP}}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y} \end{aligned}$$

C.2 Solution algorithm to find the steady-state

$$q_k = q_h = 1 \tag{A.66}$$

$$q_S = \frac{\beta_P}{\pi} \tag{A.67}$$

$$\begin{aligned} 1 + R &= \frac{1}{q_S} \\ R &= \frac{1}{q_S} - 1 \end{aligned} \tag{A.68}$$

$$\begin{aligned} \frac{q_L b_L}{q_S b_S} &= \gamma_b \text{ and } 1 + \Gamma = \Gamma_1 \left(\frac{q_L b_L}{q_S b_S} \right)^{\Gamma_2} \\ \Gamma &= \Gamma_1 \gamma_b^{\Gamma_2} - 1 \end{aligned} \tag{A.69}$$

$$\begin{aligned} (1 + \Gamma) q_L &= \frac{\beta_P}{\pi} (1 + \kappa q_L) \\ q_L &= \frac{1}{(1 + \Gamma) \frac{\pi}{\beta_P} - \kappa} \end{aligned} \tag{A.70}$$

$$\begin{aligned}
1 + R_L &= \frac{1}{q_L} + \kappa \\
R_L &= \frac{1}{q_L} + \kappa - 1
\end{aligned} \tag{A.71}$$

Note that $\frac{1 + R_L}{1 + R} = 1 + \Gamma$

$$\begin{aligned}
\Omega_{rP} &= \beta_P \frac{1 + (1 - \kappa_d) \Omega_{rP}}{\pi} \\
\Omega_{rP} &= \frac{1}{\frac{\pi}{\beta_P} - (1 - \kappa_d)}
\end{aligned} \tag{A.72}$$

$$R^d = R^l \text{ and } 1 + \Gamma = \Omega_{dP} + \Omega_{rP} R^l$$

$$\text{and } \Omega_{dP} + \Omega_{rP} R^d = \beta_P \frac{R^d + \kappa_d - (1 - \kappa_d) \Gamma + (1 - \kappa_d) (\Omega_{dP} + \Omega_{rP} R_t^d)}{\pi_{t+1}}$$

$$1 + \Gamma = \beta_P \frac{R^d + 1}{\pi}$$

$$R^d = (1 + \Gamma) \frac{\pi}{\beta_P} - 1 \tag{A.73}$$

Note that $\frac{1 + R^d}{1 + R} = 1 + \Gamma = \frac{1 + R_L}{1 + R}$

$$R^l = R^d \tag{A.74}$$

$$1 + \Gamma = \Omega_{dP} + \Omega_{rP} R^l$$

$$\Omega_{dP} = 1 + \Gamma - \Omega_{rP} R^l \tag{A.75}$$

$$\Omega_{rI} = \beta_I \frac{1 + (1 - \kappa_d) \Omega_{rI}}{\pi}$$

$$\Omega_{rI} = \frac{1}{\frac{\pi}{\beta_I} - (1 - \kappa_d)} \tag{A.76}$$

$$\begin{aligned}
R^d &= R^l \text{ and } 1 - \mu = \Omega_{dI} + \Omega_{rI}R^l \\
\text{and } \Omega_{dI} + \Omega_{rI}R^d &= \beta_I \frac{R^d + \kappa_d + (1 - \kappa_d)(\Omega_{dI} + \Omega_{rI}R^d)}{\pi} \\
1 - \mu &= \beta_I \frac{1 + R^d - (1 - \kappa_d)\mu}{\pi} \\
\mu &= \frac{\frac{\pi}{\beta_I} - (1 + R^d)}{\frac{\pi}{\beta_I} - (1 - \kappa_d)} \tag{A.77}
\end{aligned}$$

$$\begin{aligned}
1 - \mu &= \Omega_{dI} + \Omega_{rI}R^l \\
\Omega_{dI} &= 1 - \mu - \Omega_{rI}R^l \tag{A.78}
\end{aligned}$$

$$\Omega = \frac{1}{\Theta_p} \tag{A.79}$$

$$\begin{aligned}
1 &= r_h + \beta_P(1 - \delta_h) \\
r_h &= 1 - \beta_P(1 - \delta_h) \tag{A.80}
\end{aligned}$$

$$\begin{aligned}
1 &= \beta_P(1 - \delta_k + r_k) \\
r_k &= \frac{1}{\beta_P} - 1 + \delta_k \tag{A.81}
\end{aligned}$$

$$\begin{aligned}
\alpha &= r_k \frac{k}{y} \\
\frac{k}{y} &= \frac{\alpha}{r_k} \tag{A.82}
\end{aligned}$$

$$\frac{i_k}{y} = \delta_k \frac{k}{y} \tag{A.83}$$

$$\frac{w_P n_P}{y} = (1 - \alpha)\theta_P \tag{A.84}$$

$$\frac{w_I n_I}{y} = (1 - \alpha)\theta_I \tag{A.85}$$

$$\frac{w_R n_R}{y} = (1 - \alpha)(1 - \theta_P - \theta_I) \tag{A.86}$$

$$\begin{aligned}
r_h &= \xi_h \frac{(1-\zeta) c_R/y}{h_R/y} \text{ and } \frac{c_R}{y} + r_h \frac{h_R}{y} = \frac{w_R n_R}{y} \\
[1 + \xi_h (1-\zeta)] \frac{c_R}{y} &= \frac{w_R n_R}{y} \\
\frac{c_R}{y} &= \frac{\frac{w_R n_R}{y}}{1 + \xi_h (1-\zeta)}
\end{aligned} \tag{A.87}$$

$$\begin{aligned}
r_h &= \xi_h \frac{(1-\zeta) c_R/y}{h_R/y} \\
\frac{h_R}{y} &= \xi_h \frac{(1-\zeta) c_R/y}{r_h}
\end{aligned} \tag{A.88}$$

$$\frac{i_{hR}}{y} = \delta_h \frac{h_R}{y} \tag{A.89}$$

$$\begin{aligned}
n_R^{1+\vartheta} &= \frac{w_R l_R/y}{(1-\zeta) c_R/y} \\
n_R &= \left(\frac{w_R l_R/y}{(1-\zeta) c_R/y} \right)^{\frac{1}{1+\vartheta}}
\end{aligned} \tag{A.90}$$

$$\begin{aligned}
1 - \phi\mu &= \xi_h \frac{(1-\zeta) c_I/y}{h_I/y} + \beta_I (1 - \delta_h) (1 - \phi\mu) \\
\frac{c_I}{y} &= \frac{[1 - \beta_I (1 - \delta_h)] (1 - \phi\mu) h_I}{\xi_h (1-\zeta) y}
\end{aligned}$$

$$\begin{aligned}
\frac{l}{y} &= \phi \frac{i_{hI}}{y} \text{ and } \frac{i_{hI}}{y} = \delta_h \frac{h_I}{y} \\
\frac{l}{y} &= \delta_h \phi \frac{h_I}{y}
\end{aligned}$$

$$\begin{aligned}
\frac{d}{y} &= \frac{\pi}{\pi - 1 + \kappa_d} \frac{l}{y} \\
\frac{d}{y} &= \frac{\pi \delta_h \phi}{\pi - 1 + \kappa_d} \frac{h_I}{y}
\end{aligned}$$

$$\begin{aligned} \frac{c_I}{y} + \frac{i_{hI}}{y} + \frac{R^d + \kappa_d}{\pi} \frac{d}{y} &= \frac{w_I n_I}{y} + \frac{l}{y} \\ \frac{[1 - \beta_I (1 - \delta_h)] (1 - \phi\mu)}{\xi_h (1 - \zeta)} \frac{h_I}{y} + \delta_h \frac{h_I}{y} + \frac{R^d + \kappa_d}{\pi - 1 + \kappa_d} \delta_h \phi \frac{h_I}{y} &= \frac{w_I n_I}{y} + \delta_h \phi \frac{h_I}{y} \\ \frac{h_I}{y} &= \frac{\frac{w_I n_I}{y}}{\frac{[1 - \beta_I (1 - \delta_h)] (1 - \phi\mu)}{\xi_h (1 - \zeta)} + \delta_h + \left(\frac{R^d + \kappa_d}{\pi - 1 + \kappa_d} - 1 \right) \delta_h \phi} \end{aligned} \quad (\text{A.91})$$

$$\frac{i_{hI}}{y} = \delta_h \frac{h_I}{y} \quad (\text{A.92})$$

$$\frac{l}{y} = \phi \frac{i_{hI}}{y} \quad (\text{A.93})$$

$$\frac{d}{y} = \frac{\pi}{\pi - 1 + \kappa_d} \frac{l}{y} \quad (\text{A.94})$$

$$\frac{c_I}{y} = \frac{[1 - \beta_I (1 - \delta_h)] (1 - \phi\mu)}{\xi_h (1 - \zeta)} \frac{h_I}{y} \quad (\text{A.95})$$

$$\begin{aligned} n_I^{1+\theta} &= \frac{w_I l_I / y}{(1 - \zeta) c_I / y} \\ n_I &= \left(\frac{w_I l_I / y}{(1 - \zeta) c_I / y} \right)^{\frac{1}{1+\theta}} \end{aligned} \quad (\text{A.96})$$

$$\begin{aligned} 1 &= \xi_h \frac{(1 - \zeta) c_P / y}{h_P / y} + \beta_P (1 - \delta_h) \\ \frac{c_P}{y} &= \frac{1 - \beta_P (1 - \delta_h)}{\xi_h (1 - \zeta)} \frac{h_P}{y} \end{aligned}$$

$$\begin{aligned} \frac{c}{y} + \frac{i_k}{y} + \frac{i_h}{y} + \frac{g}{y} &= 1 \text{ and } \frac{c}{y} = \frac{c_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} \text{ and } \frac{i_h}{y} = \frac{i_{hP}}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y} \\ \frac{1 - \beta_P (1 - \delta_h)}{\xi_h (1 - \zeta)} \frac{h_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} + \frac{i_k}{y} + \delta_h \frac{h_P}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y} + \frac{g}{y} &= 1 \\ \frac{h_P}{y} &= \frac{1 - \frac{c_I}{y} - \frac{c_R}{y} - \frac{i_k}{y} - \frac{i_{hI}}{y} - \frac{i_{hR}}{y} - \frac{g}{y}}{\frac{1 - \beta_P (1 - \delta_h)}{\xi_h (1 - \zeta)} + \delta_h} \end{aligned} \quad (\text{A.97})$$

$$\frac{i_{hP}}{y} = \delta_h \frac{h_P}{y} \quad (\text{A.98})$$

$$\frac{c_P}{y} = \frac{1 - \beta_P (1 - \delta_h)}{\xi_h (1 - \zeta)} \frac{h_P}{y} \quad (\text{A.99})$$

$$n_P^{1+\vartheta} = \frac{w_P n_P / y}{(1-\zeta) c_P / y}$$

$$n_P = \left(\frac{w_P n_P / y}{(1-\zeta) c_P / y} \right)^{\frac{1}{1+\vartheta}} \quad (\text{A.100})$$

$$\frac{i_h}{y} = \frac{i_{hP}}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y} \quad (\text{A.101})$$

$$\frac{i}{y} = \frac{i_k}{y} + \frac{i_h}{y} \quad (\text{A.102})$$

$$\frac{h}{y} = \frac{h_P}{y} + \frac{h_I}{y} + \frac{h_R}{y} \quad (\text{A.103})$$

$$\frac{c}{y} = \frac{c_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} \quad (\text{A.104})$$

$$\frac{tax}{y} = \Xi \quad (\text{A.105})$$

$$\frac{g}{y} + \left(\frac{1+R}{\pi} - 1 \right) \frac{q_S b_S}{y} + \left(\frac{1+R_L}{\pi} - 1 \right) \frac{q_L b_L}{y} = \frac{tax}{y} \text{ and } \frac{q_L b_L}{q_S b_S} = \gamma_b$$

$$\frac{g}{y} + \left[\frac{1+R}{\pi} - 1 + \left(\frac{1+R_L}{\pi} - 1 \right) \gamma_b \right] \frac{q_S b_S}{y} = \frac{tax}{y}$$

$$\frac{q_S b_S}{y} = \frac{\frac{tax}{y} - \frac{g}{y}}{\frac{1+R}{\pi} - 1 + \left(\frac{1+R_L}{\pi} - 1 \right) \gamma_b} \quad (\text{A.106})$$

$$\frac{q_L b_L}{y} = \gamma_b \frac{q_S b_S}{y} \quad (\text{A.107})$$

$$y = \frac{1}{\Theta_p} k^\alpha \left(n_P^{\theta_P} n_I^{\theta_I} n_R^{\theta_R} \right)^{1-\alpha}$$

$$y = \left(\frac{1}{\Theta_p} \right)^{\frac{1}{1-\alpha}} \left(\frac{k}{y} \right)^{\frac{\alpha}{1-\alpha}} n_P^{\theta_P} n_I^{\theta_I} n_R^{\theta_R} \quad (\text{A.108})$$

Now all variables in levels can be determined. And note that

$$\lambda_P = \frac{1}{c_P} \quad (\text{A.109})$$

$$\lambda_I = \frac{1}{c_I} \quad (\text{A.110})$$

$$\lambda_R = \frac{1}{c_R} \quad (\text{A.111})$$