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Data Externalities, Market Power, and the Optimal Design of Central Bank Digital Currencies*

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Abstract

We study the optimal design of a central bank digital currency (CBDC) in an economy where private payment service providers (PSPs) collect and monetize transaction data and may have market power. Payments data create social benefits through law enforcement and monitoring but also impose privacy costs and negative externalities by enabling profiling and surplus extraction. In our model, the central bank chooses CBDC fees, transaction rewards, and data-collection intensity, taking into account their effects on private payment adoption. We show that a data-collecting CBDC can either raise or lower private payment adoption and aggregate data production relative to cash, depending on the balance between PSP market power and the social costs of privately monetized data. In a calibration to the U.S. economy, the introduction of CBDC raises aggregate data collection, private PSP market share, and PSP profits. But when PSP competition is stronger, data are more valuable, or data-processing costs are lower, the optimal CBDC policy reduces aggregate data production if negative data externalities are sufficiently strong.

Keywords: Central Bank Digital Currencies, Data Privacy, Data Externalities, Payment Service Providers, Market Power, Liquidity Constraints

JEL Classification: G2, L14

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1 Introduction

Digital payments have expanded rapidly, and the business models of private payment service providers (PSPs) increasingly rely on the collection and use of transaction data. At the same time, these providers may possess market power in payment services. These developments raise questions for public payments policy, particularly as many central banks consider whether to introduce a central bank digital currency (CBDC) and how to design it.

A key design choice concerns data. Unlike cash, a CBDC can be configured to collect, store, and transmit transaction information. Such information can be socially valuable for law enforcement and monitoring purposes, with fraud detection and AML/CFT compliance as leading examples. Yet data collection also entails privacy costs for users, and private monetization of payments data can generate additional social costs if it enables profiling or surplus extraction. In this environment, CBDC design is not only about how much privacy to offer relative to cash, but also about how the introduction of a CBDC reshapes private payment adoption, market power, and the aggregate amount of data produced by the payment system.

Our paper studies these interactions in a general equilibrium payments model with two means of payment: public money (cash or CBDC) issued by the central bank and private credit provided by payment service providers. The model incorporates three key elements emphasized in the CBDC literature: (i) a liquidity cost of holding payment balances when nominal interest rates are positive (e.g., Keister and Sanches, 2022), (ii) privacy concerns associated with transaction data (e.g., Garratt and van Oordt, 2021; Ahnert et al., 2025), and (iii) market structure and market power in payments (e.g., Andolfatto, 2020 and Chiu et al., 2023). These ingredients jointly determine households' adoption decisions across payment instruments, the pricing and data policies of private providers, and the welfare consequences of alternative CBDC designs.

The model is built around a standard monetary framework based on Lagos and Wright (2005). In each period, buyers decide whether to use public money or private credit for decentralized transactions. Public money is costly to hold when nominal interest rates are positive, capturing the liquidity cost of payment balances. Buyers also differ in an idiosyncratic cost or benefit of using private credit relative to public money, which generates an endogenous cutoff determining private adoption. A payment instrument is summarized by a scheme that includes a fixed account fee (or subsidy), a per-transaction fee (or reward), and a data-collection intensity. Private providers choose their scheme to maximize profits, taking household adoption and usage decisions into account. The central bank chooses an analogous scheme for a CBDC, including its data policy.

Data collection and privacy enter directly at the transaction level. When a transaction is conducted using private credit, the provider collects data proportional to transaction size, the buyer suffers a privacy loss, and the provider bears a convex cost of data collection. The provider can also monetize these data in an outside market, where we interpret the data price as reflecting the real downstream value created by uses such as targeting, screening, or customization. When a transaction is conducted using a CBDC, the central bank can also choose to collect transaction data; the buyer bears the same type of transaction-specific privacy loss, and the central bank bears the collection cost, but the public rail does not monetize data. Beyond these private costs and revenues, the model features data externalities. Recorded transactions create a positive externality through public monitoring and enforcement, while privately monetized data create a negative externality insofar as they enable profiling and surplus extraction. These forces create wedges between private incentives and the socially optimal amount and location of data generation.

Within this framework, the optimal CBDC payment policy chooses fees, rewards, and a data-collection rule to maximize social welfare, recognizing that private providers respond strategically and that households adjust their payment choices. A CBDC affects outcomes along two margins. First, it changes the relative attractiveness of public money and private credit, thereby shifting private adoption. Second, it changes the composition and intensity of data generation across payment rails, which matters because publicly used data and privately monetized data have different welfare implications.

We derive three main results. First, the optimal CBDC can and often should collect data. Collecting some data on CBDC transactions is desirable when the public value of recorded transactions is sufficiently large relative to the privacy loss and the resource cost of data handling. In that case, a CBDC can replicate part of the informational value of digital payments without introducing private monetization incentives.

Second, replacing cash with a data-collecting CBDC can either increase or decrease the market share of private credit and the aggregate amount of data produced in the payment system. When market power leads to inefficiently low private-rail adoption under cash, it can be optimal for the CBDC to tilt some users toward private credit, even if that raises the private rail's market share. Conversely, when the negative externality from privately monetized data is sufficiently large, it can be optimal for the CBDC to crowd out private-rail usage. Through this channel, a data-collecting CBDC can reduce aggregate data production even though cash itself collects no data, because transactions migrate away from a privately monetized data regime toward a public rail with lower and differently valued data intensity.

Third, market structure shapes these comparisons. As private payment markets become more competitive, the cash regime tends to generate higher private adoption and more aggregate data because private providers expand usage more aggressively. Under an optimally designed CBDC, by contrast, the central bank can keep adoption and data generation closer to policy targets. As a result, the comparison between cash and CBDC tilts toward cases in which the CBDC reduces private market share and aggregate data as private competition intensifies.

To keep the baseline framework parsimonious, we study regimes in which the public payment instrument is either cash or CBDC and summarize privacy concerns with a common buyer-side privacy cost. The appendix then considers two extensions. One allows cash and CBDC to coexist, so that CBDC enters as an intermediate public option between cash and private payments; this preserves the main comparative logic while enriching the institutional environment. The other allows for heterogeneity in buyers' privacy sensitivity, showing how CBDC can also improve allocations by screening users across rails and clarifying that the baseline efficient-quantity result need not survive under heterogeneous privacy costs.

We calibrate the model to the U.S. economy. Under the calibrated parameterization, relative to the cash regime, the CBDC regime generates more aggregate data collection, as well as a higher market share and profits for private rails. This suggests that, at the calibrated values, the market-power and adoption distortion dominate the negative externality from privately monetized data, so the optimal CBDC policy increases the usage of the private rail. However, if competition among private PSPs intensifies, payments data become more valuable, or data-processing costs fall, then the optimal payment policy shifts toward reducing aggregate data production when the negative data externality is sufficiently large.

Related Literature

Our work contributes to and expands upon the literature on CBDC. The three key frictions in our model are also highlighted in the existing literature, while our paper, as far as we know, is the first attempt to provide a unified framework to study how the interaction among these forces affects the optimal CBDC payment policy. The role of liquidity and the optimal interest rate for a CBDC have been the focus of many monetary models of CBDC (e.g., Bhattarai et al., 2024; Brunnermeier and Niepelt, 2019; Chiu and Davoodalhosseini, 2023;

Keister and Sanches, 2022, Williamson, 2022; and Whited et al., 2023). The inefficiency due to the market power of banks is a motivation for CBDC issuance in some macro models of CBDC (Andolfatto, 2020, Chiu et al., 2023). Furthermore, the issues of privacy concerns and data externalities have been underscored in recent studies on the implications of CBDC for consumers and firms (Garratt and van Oordt, 2021; Ahnert et al. 2025; Garratt and Lee, 2022; Cheng and Izumi, 2026; Tinn, 2025), highlighting the multifaceted challenges faced in the design and implementation of central bank digital currencies.[1]

Our paper also relates to the emerging literature on the role of payments data in credit provision and aggregate economic activity. Parlour et al. (2022) and Kang et al. (2025) study how FinTech competition in payment services and nonbank digital money disrupt the traditional banking model, in which payment flows provide information about borrowers credit risk, and its implications for financial inclusion, bank lending, credit allocation, and consumer welfare. He et al. (2023) analyze the open banking framework that allows FinTech entrants to access customer transaction data and improve borrower screening. Kang and Wang (2024) examine the welfare implications of using payment data to infer aggregate demand information. Chiu and Koeppl (2026) and Gomis-Porqueras and Wang (2023) discuss the incentives of big digital payment rails to monetize user data and provide payment services.

Finally, our paper analyzes the strategic interaction between private payment services and public payment policy. See Liu et al. (2025) and Zhu and Hendry (2019) for related work.

The rest of the paper proceeds as follows. Section 2 describes the environment. Section 3 characterizes the optimal allocation. Section 4 analyzes the market equilibrium under a given set of government policies. Section 5 studies the optimal CBDC design depending on the availability of the policy instruments. Section 7 calibrates the model to the US economy. Section 8 summarizes our results and points out directions for future work.

2 Environment

The model builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is discrete, $t = 0, 1, \dots$, and agents discount next-period payoffs at a rate of $\beta \in (0, 1)$. Each

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[1] See Acemoglu et al. (2022), Choi et al. (2019), Bergemann et al. (2022), and Ichihashi (2020) for modeling data externalities in broader contexts.

period has two stages: a decentralized market (DM), followed by a centralized market (CM). In the DM, a buyer meets a seller and trades a specialized good q. The buyer's utility from q is $u(q)$ with $u'(q) > 0$, $u''(q) < 0$, and $u'(0) = \infty$; the seller produces q units at a linear cost of q and derives no utility from consuming it. The terms of trade are determined by buyers making take-it-or-leave-it offers. In the CM, all agents settle positions and trade a general good x; the utility from x is $U(x)$ and producing x costs x.

Payment Rails

The DM trades are settled on one of two separate payment rails. The private PSP provides the credit rail (c) and extends credit to support payments. The public-money rail (m) is operated by the central bank in the form of cash or CBDC payments.[2] Each rail is characterized by a policy triple $\theta = (\Phi, \phi, \alpha)$: a fixed (membership) fee Φ levied in the CM; a proportional fee ϕ on the DM transaction value q (a reward if $\phi < 0$); and a data-collection intensity α, so that a payment of size q generates $d = \alpha q$ units of data. Recording data costs $C(d)$ with $C'(d) > 0$, $C''(d) > 0$, and $C'(0) = 0$.

Before choosing a rail, each buyer draws an idiosyncratic adoption term ϵ from a CDF $F(\epsilon)$ with support $[\epsilon_{\min}, \epsilon_{\max}]$, where $\epsilon_{\min} < 0 < \epsilon_{\max}$. The term ϵ captures, in reduced form, transaction-specific adoption factors that are not explicitly modeled in our framework. A positive $\epsilon > 0$ indicates that the private rail entails a net adoption *cost* (e.g., set-up/learning effort, weaker merchant acceptance in the buyers usual venues, or higher intrinsic privacy loss); a negative $\epsilon < 0$ indicates that the private rail delivers a net *benefit* (e.g., loyalty ecosystems or greater perceived convenience).

Table 1: Transaction options

	Private rail	Public rail (m)
Adoption cost/benefit	$\epsilon \sim F(\epsilon)$	0
Data intensity	α	α_m
Fixed fee	Φ	Φ_m
Proportional fee	ϕ	ϕ_m

Note: Cash corresponds to $(\Phi_m, \phi_m, \alpha_m) = (0, 0, 0)$.

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[2] The baseline analysis studies regimes in which the public rail is either cash or CBDC. Appendix **??** extends the framework to allow cash and CBDC to coexist as distinct public payment options.

Within each period, the DM opens first, where buyers and sellers match and trade q on the chosen rail. The CM then opens. Agents trade goods x and settle credit balances from the previous DM. The private payment rails post (Φ, ϕ, α) and the government sets the public-rail triple $(\Phi_m, \phi_m, \alpha_m)$. Buyers decide which rail to use for the upcoming DM and pay any fixed fees. Public-rail users also choose the money balance to carry into the next DM. Figure 1 summarizes the timing, where we use PSP to denote the private payment service provider.

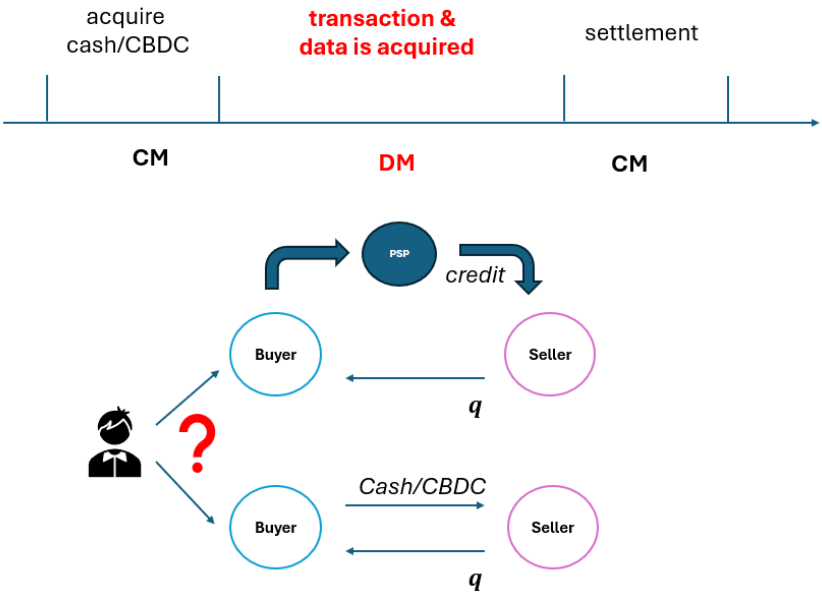

Figure 1: Timing and Payment Rail Choice

Payment Data and Welfare Channels

Data generated on the two rails affects welfare through three channels summarized in Figure 2. First, at the individual level, a transaction that generates d units of data incurs a privacy loss of $\ell_0 d$ with $\ell_0 > 0$ for the buyer.[3] Second, in the aggregate, the stock of transaction records,

$$D_T = D + D_m,$$

generates a *positive* externality $e_m D_T$. We interpret e_m as the reduced-form public benefits of recorded transactions for monitoring and enforcement purposes, with AML/CFT com-

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[3] The baseline model assumes a common marginal privacy-loss parameter ℓ_0 across buyers. Appendix **??** considers an extension with heterogeneous privacy sensitivity and shows how this modifies the intensive-margin efficiency result and creates a screening role for CBDC.

pliance and fraud detection as the leading examples. More broadly, e_m can also capture other beneficial public uses of payments data, such as supervisory monitoring or improved measurement of economic activity.

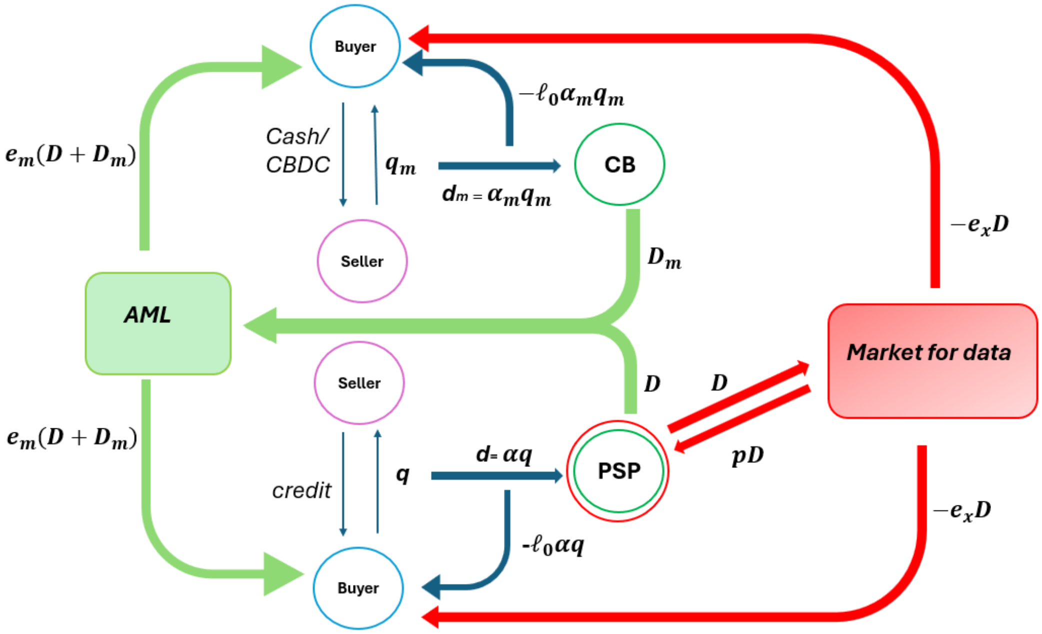

Figure 2: Data flow across rails and welfare channels

Note: The blue arrows capture the transaction-specific privacy loss of buyers; the green arrows capture the positive externality for data generated on both the private and public rails; the red arrows capture the data monetization and associated negative externality for data generated on the private rail. Note that the public rail does not monetize its data, so there are no $e_x D_m$ and pD_m terms in the red channel.

Third, the private rail monetizes its database, earning revenue from selling data at a price p per unit of data in the CM. We interpret p as the reduced-form marginal social value created by the downstream use of payment data, for example, through improved matching, screening, fraud detection, or product customization, as reflected in the willingness of downstream firms to pay for such information. During the monetization process, PSPs and downstream data purchasers ignore the resulting harm from downstream data use (e.g., group profiling and price discrimination of buyers in the CM). We capture this *negative* spillover by $e_x D_c$ with $e_x > 0$. A microfoundation for (p, e_x) is provided in Appendix C; in the main text, we treat (p, e_x) as reduced-form primitives.

We assume that the data collected on the public rail involve only the first two channels but not the third channel. This reflects the institutional distinction between private and public payment systems: a private PSP can treat transaction records as a commercial asset, whereas a central bank operates under a public mandate that restricts the use of payment data to policy, monitoring, and enforcement purposes.

Public Payment Policy

The government issues the public payment instrument (cash/CBDC) and chooses $(\Phi_m, \phi_m, \alpha_m)$. Monetary policy fixes the gross money-growth rate $\gamma > \beta$ and pays no interest on retail balances. Legacy cash corresponds to $(\Phi_m, \phi_m, \alpha_m) = (0, 0, 0)$, as there are no fees and no data collection. In contrast, the policy design $(\Phi_m, \phi_m, \alpha_m)$ for a CBDC allows nonzero entries because fees are feasible with a CBDC and basic metadata (e.g., time stamp, pseudonymous wallet, amount) can be recorded on the central bank's ledger. Finally, seigniorage and fee revenues (or rewards) are rebated via lump-sum transfers to balance the government's budget. In that sense, the fiscal and monetary authorities are consolidated.

3 Planner's Problem

We first characterize a planner's problem as a benchmark for welfare analysis to guide the optimal design of CBDC. The planner chooses the trade size and data intensity on each rail, as well as the rail for each household (characterized by ϵ). Payment data on the private rail generates positive down-stream value (p) and the associated negative externality (e_m). By contrast, data generated from CBDC transactions is not used to generate downstream value nor does it impose a negative externality. It is straightforward that the optimal solution is characterized by a participation threshold $\hat{\epsilon}$ under which households use the private rail and above which they use the public rail.

Specifically, the planner solves the problem below:

$$\begin{aligned}
\max_{q, q_m, \alpha, \alpha_m, \hat{\epsilon}} \mathcal{W} \;=\; & F(\hat{\epsilon})\big[u(q) - q - \ell_0 \alpha q - C(\alpha q)\big] \\
& +\; (1 - F(\hat{\epsilon}))\big[u(q_m) - q_m - \ell_0 \alpha_m q_m - C(\alpha_m q_m)\big] \\
& -\; \int_{-\infty}^{\hat{\epsilon}} \epsilon \, dF(\epsilon) \\
& +\; (p - e_x + e_m)\, F(\hat{\epsilon})\, \alpha q \;+\; e_m \, [1 - F(\hat{\epsilon})]\, \alpha_m q_m. \qquad (1)
\end{aligned}$$

The first two lines respectively capture buyers' utilities on the private and public rails, net of the data-collection costs. The third line is the participation cost incurred by agents with idiosyncratic shock $\epsilon \leq \hat{\epsilon}$ who adopt the private rail. The last line records the net social value of data: private data generate $(p-e_x+e_m)$ per unit, whereas public-rail data generate e_m only. The first-order conditions (FOCs) $\partial\mathcal{W}/\partial q = \partial\mathcal{W}/\partial\alpha = \partial\mathcal{W}/\partial q_m = \partial\mathcal{W}/\partial\alpha_m = \partial\mathcal{W}/\partial\hat{\epsilon} = 0$ yield the following result.

Proposition 3.1 *The planner's choice* $(\alpha^*, \alpha_m^*, q^*, q_m^*, \hat{\epsilon}^*)$ *satisfies*

$$
\begin{aligned}
q^* &: \quad u'(q^*) = 1; && (2)\\
q_m^* &: \quad u'(q_m^*) = 1; && (3)\\
\alpha^* &: \quad p + e_m = e_x + \ell_0 + C'(\alpha q^*); && (4)\\
\alpha_m^* &: \quad e_m = \ell_0 + C'(\alpha_m q_m^*); && (5)\\
\hat{\epsilon} &: \quad \hat{\epsilon}^* = \ell_0 \alpha_m^* q^* + C(\alpha_m^* q_m^*) - \ell_0 \alpha^* q^* - C(\alpha q^*) \\
&\qquad + (p - e_x + e_m)\, \alpha^* q^* - e_m \alpha_m^* q_m^*. && (6)
\end{aligned}
$$

The planner chooses trade sizes so that the marginal utility of good q equals its marginal production cost on each rail, i.e., $u'(q^*) = u'(q_m^*) = 1$, and sets data generation intensity so that its marginal social benefit equals its marginal social cost. The participation threshold is set such that the marginal participant, $\epsilon = \hat{\epsilon}^*$, obtains the same welfare on either rail.

The optimal payment policy has two main implications. First, whether the public rail should collect any data depends on the net social value of a public record, $e_m - \ell_0$. Second, given that decision, the downstream value of private data relative to the associated negative spillover, $p - e_x$, determines which rail should feature higher data intensity and the direction in which the planner should steer buyers. The implications are summarized in Proposition 3.2.

Proposition 3.2 *The planner's solution implies that*

1. *Data is collected on the public rail, or* $\alpha_m^* > 0$*, iff* $e_m > \ell_0$.
2. *The private rail should have a higher data intensity,* $\alpha^* > \alpha_m^*$*, iff* $p > e_x$.
3. *The participation cut-off favours the private rail,* $\hat{\epsilon}^* > 0$*, iff* $p > e_x$.

Finally, we summarize the amount of data generated on the two rails:

$$D = F(\hat{\epsilon}^*)\,\alpha^* q^*,$$
$$D_m = \big[1 - F(\hat{\epsilon}^*)\big]\alpha_m^* q_m^*.$$

The associated net contribution of data to welfare is

$$\mathcal{L}^* = (e_m - e_x)D + e_m D_m.$$

These objects will be useful below when we compare market outcomes with the planner benchmark. Note that the planner cares not only about the total amount of data, $D + D_m$, but also about its allocation across the private and public rails because the two types of records affect welfare differently.

4 Market Equilibrium

In this section, we characterize the decentralized (market) equilibrium for a given public payment policy $(\Phi_m, \phi_m, \alpha_m)$ and the nominal interest rate $\iota \equiv (\gamma - \beta)/\beta$, which determines the opportunity cost of holding public money balances. We solve the market equilibrium in three steps. First, we solve the buyers' problems regarding which rail to use (given their ϵ) and the trade size, q_m and q, on each rail, taking as given the public and private payment policies, $(\Phi_m, \phi_m, \alpha_m)$ and (Φ, ϕ, α). Second, taking the buyer's response as given, we analyze the private rail's profit-maximization problem and obtain its optimal policy triple. Third, we combine the two steps to define and characterize a market equilibrium.[4]

4.1 Buyers' Problem

Buyers take as given the policy triples on the public rail $\theta_m \equiv (\Phi_m, \phi_m, \alpha_m)$ and on the private rail $\theta \equiv (\Phi, \phi, \alpha)$, as well as the nominal opportunity cost ι.

Public money (m). As shown in Appendix B.1, the dynamic problem for a buyer who intends to pay with public money in the DM can be collapsed into the following static

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[4] The seller's choices are trivial: they do not hold money when $\gamma > \beta$ (equivalently, $\iota > 0$), and, being subject to neither fees nor transaction-specific privacy losses, accept both money and credit, taking the buyer's take-it-or-leave-it offer.

problem:

$$U_m(\theta_m) = \max_{q_m} \Big\{ u(q_m) - q_m - \ell_0 \alpha_m q_m - \Phi_m - \phi_m q_m - \iota q_m \Big\}. \tag{7}$$

The associated FOC is

$$u'(q_m) = 1 + \phi_m + \alpha_m \ell_0 + \iota. \tag{8}$$

The buyer equates the marginal utility from increasing the public-money transaction size to its marginal cost. Relative to the first-best condition $u'(q_m) = 1$, the public rail is distorted by three wedges: the proportional fee ϕ_m, the transaction-specific privacy cost $\alpha_m \ell_0$, and the liquidity cost ι from holding public money.

Private credit (c). When the buyer plans to use the private payment rail, money balances are unnecessary for $\iota > 0$. Again, as shown in Appendix B.1, we can collapse the buyer's dynamic problem into the following static version:

$$U(\theta) = \max_{q} \Big\{ u(q) - q - \ell_0 \alpha q - \Phi - \phi q \Big\}, \tag{9}$$

The FOC is

$$u'(q) = 1 + \phi + \alpha \ell_0. \tag{10}$$

The buyer equates the marginal utility from increasing the private-rail transaction size to its marginal cost. Relative to the first-best condition $u'(q) = 1$, the private rail is distorted by two wedges: the proportional fee ϕ and the transaction-specific privacy cost $\alpha \ell_0$. Unlike on the public rail, there is no liquidity wedge because private payments do not require holding money balances in advance.

Payment rail choice. The marginal adopter $\hat{\epsilon}$ is defined by indifference:

$$\hat{\epsilon} \; = \; U(\theta) - U_m(\theta_m). \tag{11}$$

All buyers with $\epsilon \leq \hat{\epsilon}$ choose private credit; the rest use public money.

The comparative statics for q_m, q, and $\hat{\epsilon}$ with respect to policy instruments are derived in Appendix B.2.

4.2 Private Payment Rail's Problem

Taking the public payment policy $\theta_m = (\Phi_m, \phi_m, \alpha_m)$ as given and considering the buyer's best response $q(\theta)$ and the adoption cutoff $\hat{\epsilon}(\theta, \theta_m)$ from Section 4.1, the private payment rail chooses its policy triple $\theta = (\Phi, \phi, \alpha)$ to maximize its profits:

$$\Pi(\theta; \theta_m) = \Big[\Phi + (\phi + \alpha p)\, q - C(\alpha q)\Big]\, F(\hat{\epsilon}), \tag{12}$$

where p is the (exogenous) price per unit of data sold and $C(\cdot)$ is the convex data-collection cost. As derived in Appendix B.3, the three FOCs with respect to $(\alpha_m, \phi_m, \Phi_m)$ can be simplified to:

$$p = \ell_0 + C'(\alpha q), \tag{13}$$

$$\phi = -\,\alpha \ell_0, \tag{14}$$

$$\Phi = \frac{F(\hat{\epsilon})}{f(\hat{\epsilon})} + C(\alpha q) - \alpha q\, C'(\alpha q), \tag{15}$$

where f is the density of ϵ.

Equation (13) pins down the optimal data intensity $\bar{\alpha}$ on the private rail. Intuitively, the private payment rail chooses data collection intensity to equate the marginal benefit of data collection, p, to the marginal cost, which consists of two parts: the marginal cost of data collection, $C'(d)$, and the transaction specific privacy cost of the buyer, ℓ_0 (the private rail internalizes its user's individual privacy cost). Note that the private rail's data policy does not respond to the public payment policy. Equation (14) states that the private rail fully compensates the buyer for the transaction-specific privacy loss. Equation (14) states that the private rail sets Φ to balance revenue from captured buyers and from the marginal buyer. Increasing Φ by 1 dollar increases the revenue from captured buyers by $F(\hat{\epsilon})$. However, the rail loses the marginal buyer with density $f(\hat{\epsilon})$, each bringing $\Phi + (\phi + \alpha p)q - C(\alpha q) = \Phi - C(\alpha q) + \alpha q C'(\alpha q)$ revenue to the rail.

Substituting (15) into (12), we can rewrite the private rail's equilibrium profit more compactly as:

$$\Pi(\hat{\epsilon}) = \frac{[F(\hat{\epsilon})]^2}{f(\hat{\epsilon})}.$$

4.3 Market Equilibrium: Definition and Characterization

We now combine the buyer's optimal choices (Section 4.1) and the private rail's optimal policy (Section 4.2) to define and characterize a stationary decentralized equilibrium for a given public-money policy $\theta_m = (\Phi_m, \phi_m, \alpha_m)$ and nominal interest rate ι.

Definition 4.1 (Market equilibrium) *Given θ_m and ι, a (stationary) market equilibrium is a tuple*

$$\left(q_m, q, \alpha, \Phi, \phi, \hat{\epsilon}\right)$$

such that:

(i) Buyers choose q_m and q according to (8) and (10), and select the private rail iff $\epsilon \leq \hat{\epsilon}$, where $\hat{\epsilon}$ is defined by (11).

(ii) The private rail chooses (α, Φ, ϕ) to maximize (12), satisfying the FOCs (13)-(15).

The next proposition gathers the equilibrium relationships in closed form.

Proposition 4.1 (Equilibrium) *In any market equilibrium, the following conditions hold:*

(a) Public rail. *Buyers who use public money choose q_m to satisfy*

$$u'(q_m) = 1 + \phi_m + \alpha_m \ell_0 + \iota. \tag{16}$$

(b) Private rail trade size. *Buyers on the private rail face*

$$u'(q) = 1. \tag{17}$$

Since u' is strictly decreasing, this equality uniquely pins down the first-best quantity $q = q^*$.

(c) Private data intensity. *The private rail's optimal data intensity solves*

$$p = \ell_0 + C'(\alpha q^*). \tag{18}$$

We denote the unique solution by $\alpha = \bar{\alpha}$.

(d) Proportional fee (rebate). *Given $\bar{\alpha}$, the proportional fee satisfies*

$$\phi = -\bar{\alpha}\ell_0. \tag{19}$$

Hence the private rail exactly offsets the marginal transaction-specific privacy loss.

(e) Fixed fee. *The fixed fee is*

$$\Phi = \frac{F(\hat{\epsilon})}{f(\hat{\epsilon})} + C(\bar{\alpha}q^*) - \bar{\alpha}q^*\, C'(\bar{\alpha}q^*). \tag{20}$$

(f) Adoption cutoff. *The marginal adopter $\hat{\epsilon}$ is given by*

$$\hat{\epsilon} = u(q^*) - q^* - \Phi - \big[u(q_m) - q_m - \ell_0\alpha_m q_m - \Phi_m - \phi_m q_m - \iota q_m\big]. \tag{21}$$

Items (e)-(f) jointly pin down $(\Phi, \hat{\epsilon})$.

(g) Private rail profits. *Equilibrium profits are*

$$\Pi(\hat{\epsilon}) = \frac{\big[F(\hat{\epsilon})\big]^2}{f(\hat{\epsilon})}. \tag{22}$$

The choice of q_m is intuitive: the buyer equates marginal utility to marginal cost, which consists of three parts: the price paid to the seller, the variable transaction fee paid to the PSP, the privacy cost, and the cost of holding real balances.

The above equilibrium conditions also reveal three key features of the private rail's behavior. First, the private rail restores the first-best trade size q^* by offering a proportional *rebate* that exactly offsets buyers' marginal privacy loss: equation (19) implies $\phi = -\bar{\alpha}\ell_0$, so the buyer's net marginal price is one and the quantity condition (17) implies $q = q^*$. Second, the PSP's choice of data intensity $\bar{\alpha}$ in (18) depends only on its own data revenue p, the buyer's private privacy cost ℓ_0, and the technological cost $C'(\cdot)$; it ignores the broader data externalities that enter the planner's objective, so data production on the private rail is generally inefficient. Third, the PSP monopolist adopts a classic two-part tariff pricing strategy, which sets the per-unit fee to the marginal cost (or the per-unit reward to the marginal benefit) and captures the consumer surplus through the fixed fee. In equilibrium, the PSP's profit can be expressed solely as a function of the adoption measure. We can interpret that the PSP provides a payment service (characterized by θ), and attracts users by offering them a certain level of surplus. In equation (22), $F(\hat{\epsilon})$ captures the units of payment services sold by the PSP and $F(\hat{\epsilon})/f(\hat{\epsilon})$ captures the unit consumer surplus extracted. The tradeoff is that to increase $F(\hat{\epsilon})$, the PSP must lower Φ, or equivalently, $F(\hat{\epsilon})/f(\hat{\epsilon})$.

4.4 Benchmark: Cash as the Public Instrument

In this section, we analyze the market equilibrium where the public rail features cash with the policy triple

$$(\Phi_m, \phi_m, \alpha_m) = (0, 0, 0),$$

Buyers using cash face only the liquidity cost ι and the money-rail FOC (16) for q_m is

$$u'(q_m) = 1 + \iota, \tag{23}$$

The associated payoff is

$$U_m^{\text{cash}} = u(q_m) - q_m - \iota q_m. \tag{24}$$

All private-rail conditions (17)-(22) remain unchanged. The marginal adopter has $\hat{\epsilon}_0$, which satisfies

$$\hat{\epsilon}_0 = \big[u(q^*) - q^* - \ell_0 \bar{\alpha} q^* - \Phi - \phi q^*\big] - \big[u(q_m) - q_m - \iota q_m\big]. \tag{25}$$

Government choice of ι. With cash, the only policy lever is the nominal opportunity cost ι (money growth or implicit interest on cash). The government chooses ι to maximize social welfare $\mathcal{W}$, taking the private rail's behavior as given. Totally differentiating,

$$\frac{\partial \mathcal{W}}{\partial \iota} = \frac{\partial \mathcal{W}}{\partial q_m}\frac{\partial q_m}{\partial \iota} + \frac{\partial \mathcal{W}}{\partial \hat{\epsilon}}\frac{\partial \hat{\epsilon}}{\partial \iota}. \tag{26}$$

Using (23), $\partial q_m/\partial \iota = 1/u''(q_m) < 0$. From Lemma B.4, $\partial \hat{\epsilon}/\partial \iota = q_m > 0$. The partials of welfare w.r.t. q_m and $\hat{\epsilon}$ (using (45) and the definition of $\mathcal{W}$ evaluated at $\alpha_m = 0$) are:

$$\frac{\partial \mathcal{W}}{\partial q_m} = (1 - F(\hat{\epsilon}_0))\,[u'(q_m) - 1], \tag{27}$$

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \hat{\epsilon}} = f(\hat{\epsilon}_0)\Big\{ &- \hat{\epsilon}_0 + \big[u(q^*) - q^* - \ell_0 \bar{\alpha} q^* - C(\bar{\alpha} q^*)\big] - \big[u(q_m) - q_m\big] \\ &+ (p - e_x + e_m)\bar{\alpha} q^*\Big\}.\end{aligned} \tag{28}$$

Evaluating at the Friedman rule $\iota = 0$ (so $u'(q_m) = 1$ and $q_m = q^*$), the first term in (26) vanishes. Condition (28) then determines the sign of $\partial \mathcal{W}/\partial \iota$ at zero inflation.

Proposition 4.2 (Friedman rule under cash) *With cash as the public payment instrument, the Friedman rule ($\iota = 0$) is optimal if and only if*

$$\big(p - e_x + e_m\big)\, \bar{\alpha} q^* - \ell_0 \bar{\alpha} q^* - C(\bar{\alpha} q^*) \ < \ \hat{\epsilon}_0, \tag{29}$$

where $\bar{\alpha}$ is given by (18) and $\hat{\epsilon}_0$ is the equilibrium cutoff under $(\Phi_m, \phi_m, \alpha_m) = (0, 0, 0)$ and $\iota = 0$.

Equation (29) illustrates that the Friedman rule is not optimal if the marginal social value of additional private rail data, $(p - e_x + e_m)\bar{\alpha} q^*$, dominates the marginal private privacy cost and production cost of that data, net of the private rail adoption cost threshold. In that case, the planner wants to push more users onto the private rail, which requires making cash relatively less attractive by deviating from the Friedman rule.

5 Optimal CBDC Design and Its Consequences

In this section, we first derive the optimal (second-best) CBDC policy that maximizes social welfare given the private rail's best response (Subsection 5.1), and analyze the behavior of data intensity and market share in the CBDC regime. We then compare the cash regime and the CBDC regime regarding market share and aggregate data production (Subsections 5.2-5.3).

5.1 Optimal CBDC Design

In this section we study the optimal design of a CBDC as a public payment instrument. Like cash, the CBDC is characterized by $\epsilon = 0$, but it is more flexible in that the policy triple $(\Phi_m, \phi_m, \alpha_m)$ can take nonzero values. We begin by analyzing the case where the government has full control over all three instruments $(\Phi_m, \phi_m, \alpha_m)$, and later restrict attention to settings where ϕ_m cannot be used.

Given the private rail's scheme $\theta = (\Phi, \phi, \alpha)$ and the government's policy $\theta_m = (\Phi_m, \phi_m, \alpha_m)$, buyers optimally choose (i) which rail to use, (ii) how much public money to carry into the DM, and (iii) their trade size in the DM. The private and public rails set their payment and data policies simultaneously. We consider a Nash equilibrium in which (1) the private rail chooses θ to maximize its profit, taking θ_m as given, and (2) the government designs θ_m to maximize social welfare $\mathcal{W}$, taking θ as given.

The government cannot directly dictate $(\hat{\epsilon}, q_m)$, but can influence them through the instruments $(\Phi_m, \phi_m, \alpha_m)$. Totally differentiating $\mathcal{W}$ with respect to each policy instrument, the first-order conditions (FOCs) are:

$$0 = \frac{\partial \mathcal{W}}{\partial \hat{\epsilon}} \frac{\partial \hat{\epsilon}}{\partial \alpha_m} + \frac{\partial \mathcal{W}}{\partial \alpha_m} + \frac{\partial \mathcal{W}}{\partial q_m} \frac{\partial q_m}{\partial \alpha_m}, \tag{30}$$

$$0 = \frac{\partial \mathcal{W}}{\partial \hat{\epsilon}} \frac{\partial \hat{\epsilon}}{\partial \phi_m} + \frac{\partial \mathcal{W}}{\partial q_m} \frac{\partial q_m}{\partial \phi_m}, \tag{31}$$

$$0 = \frac{\partial \mathcal{W}}{\partial \hat{\epsilon}} \frac{\partial \hat{\epsilon}}{\partial \Phi_m}. \tag{32}$$

Lemma B.4 gives $\partial \hat{\epsilon} / \partial \Phi_m = 1$, and equation (32) immediately implies $\frac{\partial \mathcal{W}}{\partial \hat{\epsilon}} = 0$, i.e. the planner's indifference condition between rails. Using this result, equation (31) reduces to $\frac{\partial \mathcal{W}}{\partial q_m} = 0$, which pins down the efficient public-rail trade size $q_m = q^*$. Finally, substituting these results into (30) yields $\frac{\partial \mathcal{W}}{\partial \alpha_m} = 0$, which determines the socially optimal public-rail data intensity α_m^* by:

$$C'(\alpha_m q^*) = e_m - \ell_0. \tag{33}$$

The marginal adopter under the second-best CBDC policy is then characterized by:

$$\hat{\epsilon}^{SB} = \left[\ell_0 \alpha_m^* q^* + C(\alpha_m^* q^*)\right] - \left[\ell_0 \bar{\alpha} q^* + C(\bar{\alpha} q^*)\right] + (p - e_x + e_m)\bar{\alpha} q^* - e_m \alpha_m^* q^*. \tag{34}$$

To implement $q_m = q^*$, the government sets the proportional fee (or reward) on the CBDC to offset privacy and liquidity wedges:

$$\phi_m = -\alpha_m^* \ell_0 - \iota.$$

Hence both rails trade at the first-best quantity, with $u'(q) = u'(q_m) = 1$ and $q = q_m = q^*$. The fixed fee Φ_m is then chosen to place the marginal adopter at $\hat{\epsilon}^{SB}$:

$$\Phi_m = \frac{F(\hat{\epsilon}^{SB})}{f(\hat{\epsilon}^{SB})} + C(\alpha_m^* q^*) - \alpha_m^* q^* C'(\alpha_m^* q^*) + (e_m - e_x)\bar{\alpha} q^*.$$

We collect these results in the following proposition.

Proposition 5.1 (Second-best CBDC equilibrium) *The government's optimal CBDC* *policy triple* $(\phi_m^{SB}, \Phi_m^{SB}, \alpha_m^{SB})$ *implements the following allocation:*

$$\begin{aligned} q_m &= q^*, \alpha_m = \alpha_m^*, \\ q &= q^*, \alpha = \bar{\alpha}, \qquad (35) \\ \hat{\epsilon}^{SB} &= \left[\ell_0 \alpha_m^* q^* + C(\alpha_m^* q^*)\right] - \left[\ell_0 \bar{\alpha} q^* + C(\bar{\alpha} q^*)\right] + (p - e_x + e_m)\bar{\alpha} q^* - e_m \alpha_m^* q^*. \end{aligned}$$

where * *denotes first-best (planner) quantities. The corresponding equilibrium fee structure is*

$$\phi = -\bar{\alpha}\ell_0, \tag{36}$$

$$\Phi = \frac{F(\hat{\epsilon}^{SB})}{f(\hat{\epsilon}^{SB})} + C(\bar{\alpha} q^*) - \bar{\alpha} q^* C'(\bar{\alpha} q^*), \tag{37}$$

$$\phi_m^{SB} = -\alpha_m^* \ell_0 - \iota, \tag{38}$$

$$\Phi_m^{SB} = \frac{F(\hat{\epsilon}^{SB})}{f(\hat{\epsilon}^{SB})} + C(\alpha_m^* q^*) - \alpha_m^* q^* C'(\alpha_m^* q^*) + (e_m - e_x)\bar{\alpha} q^*. \tag{39}$$

Equation (38) shows that the proportional fee (or reward) on CBDC is used to neutralize the individual privacy wedge and the liquidity wedge ι. Note that the public payment policy does not affect q or α, but it can use the fixed fee to place the marginal adopter exactly at the planner's indifference point (34) according to equation (39).

5.2 Private vs. Public Rails: Data Intensity and Market Shares

Next, we analyze the data production and market shares of the public and private payment rails under the second-best CBDC policy. We organize the analysis around two questions: (1) Which rail, private or CBDC, generates more transaction data? (2) How does the private rail's market share under the CBDC compare with the planner's benchmark?

First, consider data intensity. Though both rails face the same collection cost $C(\cdot)$ and the same buyer privacy loss $\ell_0 \alpha q$, their perceived benefits differ: the private rail values data solely at the monetization price p, while the CBDC values it at the AML benefit e_m.

Proposition 5.2 (Data intensity across rails) *In equilibrium, the private rail collects more data than the CBDC rail,* $\bar{\alpha} > \alpha_m^*$*, if and only if* $p > e_m$.

Second, compare the equilibrium market share to the planner's optimum. Note that the

CBDC replicates α_m^*, so any deviation from the first best stems from the private rail's choice of $\bar{\alpha}$. Given that the public payment policy cannot influence $\bar{\alpha}$, it leverages Φ_m to influence the market shares of the two payment rails.

Proposition 5.3 (Market share vs. planner) *When $e_x > e_m$, the private rail over-collects data (i.e., or $\bar{\alpha} > \alpha^*$). In that case, the optimal CBDC fee implies $\hat{\epsilon}^{SB} < \hat{\epsilon}^*$, diverting users to the public rail to pay with the CBDC.*

5.3 Cash vs. CBDC: Market Share and Aggregate Data

Another perspective is to compare the CBDC regime with the cash regime $(\Phi_m, \phi_m, \alpha_m) = (0, 0, 0)$. With a CBDC, the public payment policy gains additional levers $(\alpha_m, \phi_m, \Phi_m)$, which can alter both (i) the private rail's market share and (ii) the aggregate amount of data produced. One may think that replacing cash with a data-collecting CBDC should always raise total data and erode the private rail's share; both conjectures may be wrong.

Proposition 5.4 (Cash vs. CBDC) *Assume $\iota = 0$ (so the size of cash transactions is privately efficient) and $p > e_m$ (so $\bar{\alpha} > \alpha^*$). There exist thresholds $\underline{e} < \bar{e}$ such that, relative to the cash regime, the CBDC regime*

(i) lowers the private rail's market share if and only if $e_x > \underline{e}$;

(ii) lowers the aggregate volume of data if and only if $e_x > \bar{e}$.

The intuition for market share is as follows. The CBDC policy is set to solve two inefficiencies: market power (so the private PSP under-provides payment services) and externality (so the private PSP over-produces data). When e_x is smaller, market power is the dominant concern, so the optimal CBDC policy expands the PSP's market share. When e_x is large, the externality is the dominant concern, so the optimal policy shrink's the PSP's market share.

Regarding data production, if $e_m > \ell_0$, then $\alpha_m^* > 0$, and the CBDC regime would add to aggregate data collection for fixed adoption (or $\hat{\epsilon}$). However, the optimal CBDC policy sets Φ_m to steer adoption if desired. When $p > e_m$, we have $\bar{\alpha} > \alpha_m^*$, and if Φ_m^{SB} diverts some transactions away from the private rail, then the net effect on aggregate data collection is ambiguous. The Aggregate data fall if and only if two conditions hold simultaneously:

$$\hat{\epsilon}_0 > \hat{\epsilon}^{SB} \quad \text{and} \; \big[1 - F(\hat{\epsilon}^{SB})\big]\alpha_m^* q^* \; < \; \big[F(\hat{\epsilon}_0) - F(\hat{\epsilon}^{SB})\big]\bar{\alpha} q^*.$$

The first inequality states that, relative to the cash regime, the CBDC pulls users off the private rail; the second indicates that the volume of $\bar{\alpha}$*-intensity records* lost on the private rail exceeds the volume of α_m^**-intensity records* gained on the CBDC. This case arises only when e_x is large, prompting the government to set Φ_m^{SB} high enough to curb the negative externality by shifting a substantial mass of transactions to the CBDC rail.

5.4 Numerical Illustration

Figure 3 reports a numerical illustration of the model. The figure overlays three allocations: the planners benchmark (dotted black), the cash equilibrium (dashed blue), and the CBDC equilibrium (solid red). The four panels summarize, respectively, welfare W, aggregate data production D_T, PSP profit, and the private-rail market share S. We graph the variables against e_x, while fixing other model parameters (see the notes in Figure 3 for details about the functional forms and parameter values).

Panel (Welfare W). Welfare varies with the strength of the externality e_x. The planner is the highest by construction, the cash equilibrium is the lowest, and the CBDC regime typically lies in between. Relative to cash, the CBDC payment policy improves welfare by setting (Φ, ϕ, α) to steer transactions across the two rails.

Panel (Aggregate data D_T). The D_T panel is the key object for Proposition 5.4. Under the planner, D_T falls with e_x because a stronger negative externality increases the shadow cost of data, and efficiency calls for less data-intensive activity. Under cash, D_T is flat because the public payment policy is fixed, and the private incentives do not internalize e_x. Under CBDC, higher e_x induces stronger diversion away from the private rail, reducing private rail scale and thus data production. Beyond the cutoff $\bar{e}$, $D_T^{\text{CBDC}}(e_x) < D_T^{\text{cash}}(e_x)$.

Panel (Per-PSP profit). Per-PSP profit declines as e_x rises because the CBDC policy progressively compresses the private rails residual demand (lower volume, and thus lower rents). The gap relative to cash reflects how strongly the CBDC policy tilts usage away from the private rail at each e_x.

Panel (Private-rail market share S). The market-share panel shows the quantity reallocation.

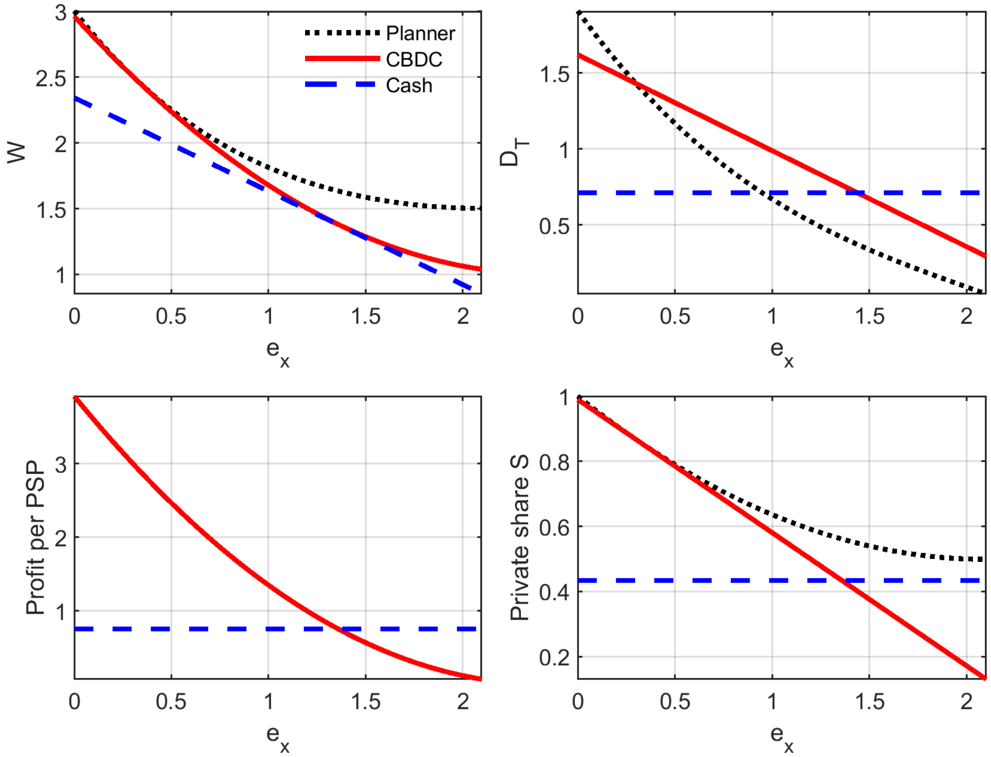

Figure 3: Optimal public payment policy Φ, ϕ, α

Notes. The figure shows a numerical example to illustrate how welfare W, aggregate data production D, private PSP's profit and the market share of private PSP change with e_x. We compare planner's allocation, the market equilibrium with cash as the public payment instrument, and the market equilibrium with CBDC as the public payment instrument. We assume utility function $u(q) = q^\eta/\eta$ with $\eta = 0.5$, and data processing cost function $C(d) = Ad^\psi/\psi$ with $A = 1.1$ and $\psi = 2$. The other parameter values are $\iota = 0, \epsilon \sim U[-2, 2], e_m = 0.3, \ell_0 = 0.2$ and $p = 2$.

Under the cash regime, the private share is flat against e_x. Under the CBDC regime, as e_x rises, the CBDC fee schedule makes public rail relatively more attractive, so $S^{\text{CBDC}}(e_x)$ falls. This decline in S is the mechanism behind the threshold in the D_T panel: sufficiently strong diversion eventually pushes aggregate data below the cash benchmark.

Taken together, the S and D_T panels provide a visual confirmation of Proposition 5.4: increasing e_x strengthens the planners incentive to use the CBDC fee to reallocate transactions away from the private rail; this reduces the private rail market share, and beyond a cutoff $\bar{e}$, it reduces aggregate data relative to cash. The reallocation moves welfare toward the planner benchmark by partially internalizing the externality.

6 Cournot Competition among Private PSPs

The baseline model considers a single, monopolistic PSP operating on the private rail. We now let *multiple* symmetric PSPs compete *à la* Cournot: each firm chooses a *quantity of users*, i.e., a market share $s_i \geq 0$, while data intensity and proportional fee (α_i, ϕ_i) are set as in the monopoly case, and a common fixed fee Φ adjusts to clear the market (one can interpret that the PSPs compete to provide a payment service bundle that guaranties users a certain level of surplus). This setup lets us gauge how competition alters equilibrium outcomes and provides the framework for our calibration in Section 7.

6.1 Market feasibility and inverse demand

There are $N \geq 1$ symmetric private PSPs, indexed by $i = 1, \ldots, N$. Each chooses a user share $s_i \geq 0$ and a policy pair (α_i, ϕ_i). Total private adoption is $S = \sum_{i=1}^{N} s_i$. Note that $S \in [0, 1]$ and $1 - S$ is the adoption of public rail.

Market feasibility. With multiple PSPs, we require a *single* fixed fee Φ that rationalizes the aggregate share S. Formally, for any active PSP i the marginal buyer with idiosyncratic cost $\hat{\epsilon}$ is indifferent between using the public rail and that PSP:

$$\widetilde{U}_i - \Phi - U_m = \hat{\epsilon}, \qquad F(\hat{\epsilon}) = S.$$

Here,

$$\widetilde{U}_i(\alpha_i, \phi_i, q) \;\equiv\; u(q) - q - \phi_i\, q - \ell_0\, \alpha_i\, q$$

denotes the buyer's utility on PSP i before paying Φ. Because ϵ is one-dimensional and PSPs are symmetric, the same $\hat{\epsilon}$ must apply for all i; otherwise, buyers would strictly prefer the PSP with the higher net utility. This differs from the single-rail benchmark, where Φ is that rail's own choice; here Φ is the market-clearing fee implied by S through the inverse demand

$$\Phi \;=\; \widetilde{U}_i(\alpha_i, \phi_i, q) - U_m - F^{-1}(S). \tag{40}$$

Given buyers' first-order condition $u'(q) = 1 + \phi_i + \ell_0\alpha_i$, PSP i's profit is

$$\pi_i \;=\; s_i\Big[\widetilde{U}_i - U_m - F^{-1}(S) + p\,\alpha_i q + \phi_i q - C(\alpha_i q)\Big].$$

The next result shows that, under Cournot competition on adoption shares, each PSP's *per-user* policy on data intensity and proportional fee replicates the benchmark from the main model.

Proposition 6.1 (PSP optimal data and proportional fee) *In the Cournot setting with inverse demand (40), each PSP's optimal policy on fees and data collection coincides with the benchmark from the main model:*

$$C'(\alpha_i q) = p - \ell_0, \tag{FOC$_\alpha$}$$

$$\phi_i = -\,\ell_0\, \alpha_i. \tag{FOC$_\phi$}$$

Consequently, the buyer's FOC implies $u'(q) = 1$ and the implemented trade size is the first best, $q = q^*$.

6.2 Symmetric equilibrium under cash

We now characterize the symmetric Cournot equilibrium when the public payment instrument is cash. Define the inverse hazard rate $H(\varepsilon) = F(\varepsilon)/f(\varepsilon)$. In a symmetric equilibrium each PSP chooses the same share s^*, so $S^* = Ns^*$ and $\hat{\epsilon}^* = F^{-1}(S^*)$. From PSP i's choice of share, the FOC $\partial\pi_i/\partial s_i = 0$ (holding rivals' shares fixed) yields

$$U_i - U_m - \hat{\epsilon} + p\alpha_i q + \phi_i q - C(\alpha_i q) - \frac{s_i}{f(\hat{\epsilon})} = 0,$$

where $\hat{\epsilon} = F^{-1}(S)$. Imposing symmetry ($s_i = s^*$ and $S^* = Ns^*$, hence $s^*/f(\hat{\epsilon}^*) = H(\hat{\epsilon}^*)/N$) and substituting (FOC$_\alpha$)–(FOC$_\phi$) (so $q = q^*$ and $\phi_i = -\ell_0 \alpha_i$) delivers

$$u(q^*) - q^* - (\ell_0 - p)\bar{\alpha} q^* - C(\bar{\alpha} q^*) - U_m - \hat{\epsilon}^* - \frac{H(\hat{\epsilon}^*)}{N} = 0. \tag{41}$$

Solving the implicit condition (41) for the cut-off $\hat{\epsilon}^*$, we then obtain the symmetric-equilibrium adoption share and profit:

$$s^*(N) \;=\; \frac{F\big(\hat{\epsilon}^*\big)}{N}, \qquad \pi(N) \;=\; \frac{F\big(\hat{\epsilon}^*\big)\, H\big(\hat{\epsilon}^*\big)}{N^2}. \tag{42}$$

Because $\phi_i = -\ell_0 \alpha_i$ in equilibrium (Proposition 6.1) the privacy cost and reward cancel in the buyer's net payoff, so the market-clearing fixed fee is simply

$$\Phi^*(N) \;=\; u(q^*) - q^* - U_m \;-\; F^{-1}\big(Ns^*(N)\big) \;=\; u(q^*) - q^* - U_m - \hat{\epsilon}^*. \tag{43}$$

To understand how competition shapes outcomes under cash, we differentiate the equilibrium condition (41) with respect to N. We assume a standard *increasing failure rate* (IFR) condition, i.e., $H'(\varepsilon) \geq 0$, which is satisfied by most standard families including uniform, exponential, logistic, and Weibull and Gamma with shape parameter greater than 1. Then, the next result shows the comparative statics implications for adoption, profits, and data volumes.

Proposition 6.2 (Comparative statics under cash) *Under IFR regularity condition, $H'(\varepsilon) \geq 0$, in the symmetric equilibrium characterized by (41):*

(i) total private adoption rises with competition, $\dfrac{dS^*}{dN} > 0$*;*

(ii) each PSP's share falls with competition, $\dfrac{ds^*}{dN} < 0$*;*

(iii) each PSP's profit falls with competition, $\dfrac{d\pi}{dN} < 0$*;*

(iv) aggregate private-rail data $D(N) = S^*(N)\,\bar{\alpha}\,q^*$ increases in N.

These conclusions formalize the intuition that, with cash as the public instrument, competition expands overall adoption but dilutes each provider's market share and profit; the increase in S^* also raises the aggregate stock of private-rail data.

Note that the Cournot formulation nests the baseline monopoly model as a special case. When $N = 1$, the Cournot first-order conditions for the per-user policy coincide with the monopoly PSPs conditions:

$$C'(\bar{\alpha}q^*) = p - \ell_0, \qquad \phi = -\ell_0\bar{\alpha}, \qquad u'(q) = 1 \;\Rightarrow\; q = q^*.$$

Moreover, the market-clearing fixed fee in Cournot,

$$\Phi = u(q^*) - q^* - U_m - \hat{\epsilon} \quad \text{with} \quad \hat{\epsilon} = F^{-1}(S),$$

together with the $N = 1$ adoption condition

$$Z - \hat{\epsilon} - \frac{F(\hat{\epsilon})}{f(\hat{\epsilon})} = 0 \qquad \Big(Z := u(q^*) - q^* + \bar{\alpha}q^*C'(\bar{\alpha}q^*) - C(\bar{\alpha}q^*) - U_m\Big),$$

is algebraically identical to the monopoly fixed-fee optimality condition

$$\Phi = \frac{F(\hat{\epsilon})}{f(\hat{\epsilon})} + C(\bar{\alpha}q^*) - \bar{\alpha}q^*C'(\bar{\alpha}q^*).$$

Hence, when $N = 1$, the Cournot equilibrium delivers exactly the same $(\bar{\alpha}, \phi, \Phi)$ and cutoff $\hat{\epsilon}$ as the benchmark cash economy.

6.3 Symmetric equilibrium under CBDC

We next turn to the Cournot environment under an optimally designed CBDC. Under the second-best CBDC policy (Subsection 5.1), the CBDC data intensity is fixed at α_m^*, and the proportional CBDC fee (or reward)

$$\phi_m^{SB} = -\alpha_m^*\ell_0 - \iota$$

offsets the privacy and liquidity wedges so that $q_m = q^*$. The government then uses the CBDC fixed fee Φ_m^{SB} to implement the target cutoff $\hat{\epsilon}^{SB}$. Hence, along the second-best CBDC policy, the aggregate market share of the private payment system is fixed at

$$S(N) = F(\hat{\epsilon}^{SB}),$$

independently of the number of private PSPs. Aggregate private-rail data are therefore also invariant to market structure:

$$D(N) = S(N)\,\bar{\alpha}q^* = F(\hat{\epsilon}^{SB})\,\bar{\alpha}q^*.$$

In the CBDC regime, although aggregate private adoption is fixed, entry still has competitive effects similar to the cash regime. Specifically, an increase in N reduces each PSPs market share, compresses the per-user profit margin (captured by a decline in Φ), and lowers total profits for each PSP, as shown below. In a symmetric equilibrium, each PSP serves

$$s^*(N) = \frac{F(\hat{\epsilon}^{SB})}{N},$$

so each firm's market share declines with N. The per-user profit margin is

$$m(N) \equiv \Phi^{SB}(N) + (\phi + \bar{\alpha}p)q^* - C(\bar{\alpha}q^*).$$

The symmetric Cournot first-order condition implies

$$m(N) = \frac{s^*(N)}{f(\hat{\epsilon}^{SB})} = \frac{F(\hat{\epsilon}^{SB})}{N\,f(\hat{\epsilon}^{SB})},$$

so the per-user margin falls with N. Since $\bar{\alpha}$ and q^* are fixed, the private fixed fee must also decline with competition:

$$\Phi^{SB}(N) = (\ell_0 - p)\bar{\alpha}q^* + C(\bar{\alpha}q^*) + \frac{F(\hat{\epsilon}^{SB})}{N\,f(\hat{\epsilon}^{SB})}.$$

Each PSP's total profit is the product of market share and per-user margin:

$$\pi(N) = s^*(N)\,m(N) = \frac{[F(\hat{\epsilon}^{SB})]^2}{N^2\,f(\hat{\epsilon}^{SB})},$$

which declines with competition at rate $1/N^2$.

Finally, because the private fixed fee $\Phi^{SB}(N)$ falls with N, the government must correspondingly adjust the CBDC fixed fee $\Phi_m^{SB}(N)$ to keep the equilibrium cutoff at $\hat{\epsilon}^{SB}$. In other words, under the second-best CBDC policy, the public fee schedule is adjusted with market structure so that the aggregate private share remains constant even though competition erodes the rents of individual PSPs.

Proposition 6.3 (Comparative statics under CBDC) *Along the second-best CBDC policy:*

(i) $S(N) \equiv F(\hat{\epsilon}^{SB})$ *is constant in* N*, while* $s^*(N) = F(\hat{\epsilon}^{SB})/N$ *is strictly decreasing in* N*;*

(ii) the private fixed fee $\Phi^{SB}(N)$ *is strictly decreasing in* N*;*

(iii) each PSP's per-user margin,

$$m(N) = \frac{F(\hat{\epsilon}^{SB})}{N\, f(\hat{\epsilon}^{SB})},$$

is strictly decreasing in N*;*

(iv) each PSP's profit,

$$\pi(N) = \frac{[F(\hat{\epsilon}^{SB})]^2}{N^2 f(\hat{\epsilon}^{SB})},$$

is strictly decreasing in N*;*

(v) aggregate private-rail data

$$D(N) = S(N)\, \bar{\alpha} q^*$$

are invariant to N*.*

In summary, Cournot competition reduces each individual PSP's market share, per-user margin, and total profit in both regimes. Under the second-best CBDC, however, the government adjusts the CBDC fixed fee so that the equilibrium cutoff $\hat{\epsilon}^{SB}$ remains unchanged. As a result, aggregate private adoption and aggregate private-rail data are held constant even though the rents of individual PSPs decline with entry. Under cash, by contrast, additional competition expands the total number of private-rail users and the stock of data they generate.

6.4 Private vs. public rails, cash vs. CBDC regimes

Having established these baseline effects of competition, we now ask whether our earlier findings regarding private vs. CBDC rails and cash vs. CBDC regimes still hold when the private side is populated by Cournot competitors.

Private versus CBDC rails. Every private PSP still solves the same per-user FOCs as in the monopoly baseline (Proposition 6.1). Consequently, the results regarding data generation intensity established earlier carry over verbatim: the private rail collects more data than the CBDC rail if and only if $p > e_m$, and the private rail collects more relative to the planner's target if and only if $e_x > e_m$.

Cash versus CBDC regimes. The comparison between the cash and CBDC regimes delivers the same qualitative insights as in the monopoly case, with one nuance that now depends on the number of entrants N: there exist two N-dependent thresholds

$$\underline{e}(N) \; < \; \bar{e}(N),$$

with $\underline{e}(N)$ and $\bar{e}(N)$ *decreasing* in N, such that:

(i) (*Market share*) the aggregate share of all private rails is lower under the CBDC than under cash iff $e_x > \underline{e}(N)$;

(ii) (*Aggregate data*) total data are lower under the CBDC than under cash iff $e_x > \bar{e}(N)$.

Recall from Section 6.2 that as N rises under cash, both the private-rail share, $S_{\text{cash}}(N)$, and the aggregate data, $D_{\text{cash}}(N) = S_{\text{cash}}(N)\,\bar{\alpha}\,q^*$, increase. By contrast, from Section 6.3, under the CBDC regime, both variables are policy-fixed (invariant in N), with $S_{\text{CBDC}} = F(\hat{\epsilon}^{SB})$ and $D_{\text{CBDC}} = S_{\text{CBDC}}\,\alpha_m^*\,q^*$. Consequently, the gaps between the two regimes,

$$\Delta S(N) \equiv S_{\text{cash}}(N) - S_{\text{CBDC}} \quad \text{and} \quad \Delta D(N) \equiv D_{\text{cash}}(N) - D_{\text{CBDC}},$$

grow with N, and both $\underline{e}(N)$ and $\bar{e}(N)$ decline with N, so the parameter interval in which a data-collecting CBDC reduces aggregate data becomes progressively wider as more private PSPs enter.

Intuitively, as discussed earlier in Section 5.3, the optimal CBDC policy addresses two inefficiencies by controlling the market share of the private rail: market power (under provision of private payment services) and data externality (over collection of payments data). It is optimal to reduce private market share (relative to the cash regime) when e_x is large enough. As N increases, the first inefficiency is alleviated, and the second inefficiency becomes more dominant. As a result, it is optimal to reduce the private market share and data collection for smaller threshold values of e_x.

7 Calibration

In this section, we calibrate the model with cash and use it to perform counterfactual analysis with CBDC.

7.1 Calibration Procedure and Results

We assume the following functional forms. The utility function in the DM is CRRA, or $u(q) = q^\eta/\eta$, with $\eta < 1$. The utility function in the CM is $U(x) = B \log x$. The cost of producing data is $C(d) = Ad^\psi/\psi$ with $\psi > 1$. The distribution of the cost to adopt the private rail (relative to the public rail), ϵ, follows a uniform distribution $\epsilon \sim U(\epsilon_{min}, \epsilon_{max})$.

We set $e_m = e_x = 0$, which is innocuous for calibration because these two parameters do not affect the decisions of private agents, and the public payment policy is passive with cash. There are 10 parameters in total: $(\iota, \eta, \epsilon_{min}, \epsilon_{max}, B, A, \ell_0, p, \psi, N)$. We normalize $\epsilon_{max} = 1$ and $\ell_0 = 1$. The nominal interest rate is calibrated externally as the average yield of AAA bonds from 1990 to 2024, $\iota = 5.61\%$. The remaining seven parameters are jointly calibrated internally to fit five point targets: cash volume share, cash value share, card reward rate, the ratio of data monetization revenue to total payment revenue, and the average money demand from 1990 to 2024, plus the fit of the money demand curve from 1990 to 2024.

The share of cash transactions is derived from the Survey of Consumer Payment Choice (SDCPC) by the Federal Reserve Bank of Atlanta. The 2024 SDCPC Tables shows the number and average value of transactions with different payment methods for the years 2015-2024. We select the numbers for three payment methods: cash, debit, and credit. Let n_m, n_d, n_c be the number of transactions for cash, debit, and credit, respectively. Let v_m, v_d, v_d be the corresponding average transaction size. The volume share of cash transactions is calculated as $s_{vol} = n_m/(n_m + n_d + n_c)$, and the value share is calculated as $s_{val} = n_m \times v_m/(n_m \times v_m + n_d \times v_d + n_c \times v_c)$. We calculate the s_{vol} and s_{val} for each year and use the average as the targets. The target for cash volume share is 0.297, and the value share is 0.171.

We derive the card reward rate target from Drechsler et al. (2025) and SDCPC. Debit cards usually do not offer any rewards. Drechsler et al. (2025) suggests the average reward for the six largest card issuers is 1.57%. From SDCPC, the ratio of transaction value for credit cards over the sum of debit and credit cards is 0.519. We set the target for rewards to $0.519 \times 1.57\% = 0.815\%$.

Regarding data monetization revenue, the McKinsey report suggests that top-performing organizations attribute 11% of their revenue to data monetization. We set the target for data monetization revenue as a share of total payments revenue at 10%.[5]

In the model, money demand is defined as the ratio of cash balances to output. The data counterpart is the currency component of M1 over GDP ratio. The average money demand target, 0.06, is calculated as the average from 1990 to 2024.

We conduct a grid search over the seven parameters to minimize the squared percentage distance between the targets in the data and the model, applying equal weights to each point target. The MSE of the money demand curve (for the series 1990 to 2024) is given double the weights as it is implicitly used to pin down two model parameters.

Table 2 shows the calibrated parameter values. Table 3 shows the model implied point targets versus data. Figure 4 shows the model predicted money demand curve versus data. The model fits the data well.

Table 2: Parameters in the Calibrated Model

Parameter	Target	Value
Set externally		
ι nominal interest rate	AAA corporate bond yield	0.056
ϵ_{max}	normalized	1
ℓ_0	normalized	1
Jointly calibrated		
η	cash value share	0.920
B	money demand	0.800
ψ	money demand curve	1.150
A	reward	1.750
p	money demand curve	1.850
ϵ_{min}	cash volume share	-2.700
N	monetization/payment revenue	20

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[5] We also checked the annual reports for Visa and Mastercard. In recent years, the revenue from value-added services accounts for about 30 to 40% of total net revenues. This can be treated as an upper bound for data revenue. The target we use is roughly one third of the upper bound.

Table 3: Targets and Model Predicted Values

Target	Data	Model
Card rewards	0.815%	0.811%
Data monetization revenue/total payment revenue	0.110	0.110
Cash vol. share	0.297	0.294
Cash val. share	0.171	0.174
Average Money demand	0.060	0.058
MSE of money demand curve		0.007

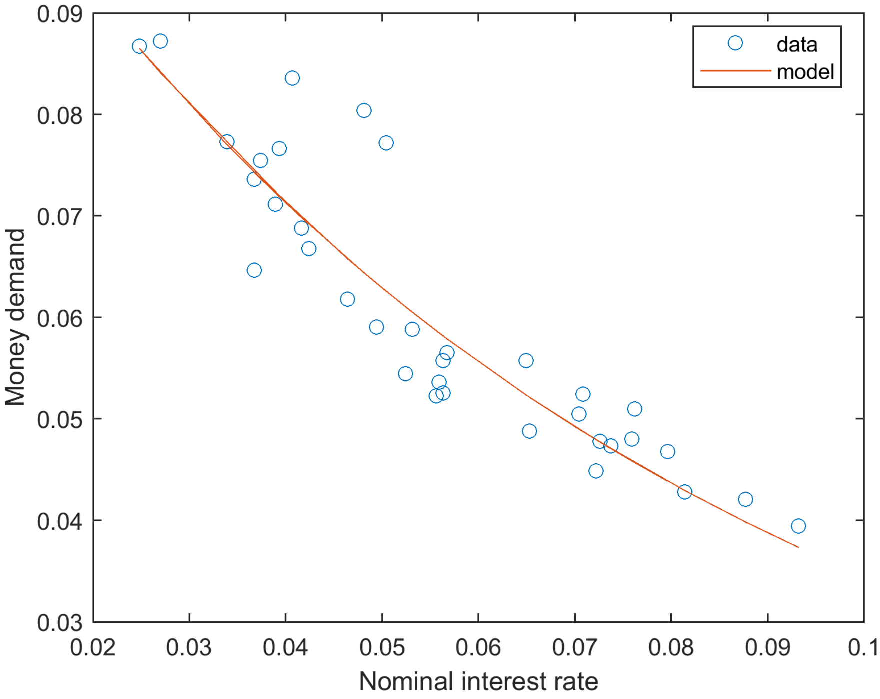

Figure 4: Money demand curve: data versus model

7.2 Counterfactual exercises

We use the calibrated model to run counterfactual policy experiments. Holding the model parameters fixed at the calibrated value, we *switch the payment regime* from cash to a CBDC (with optimal CBDC payment policies) and recompute the Cournot equilibrium. This exercise allows us to quantify the effects of introducing CBDC relative to the cash regime.

Figure 5 graphs welfare W, aggregate data D, per-PSP profit, and the private-rail market share S against the negative externality e_x in three cases: (i) the cash benchmark in dashed blue (the regime used for calibration); (ii) the CBDC equilibrium under the same calibrated parameters in solid red; and (iii) the planner benchmark in dotted black.

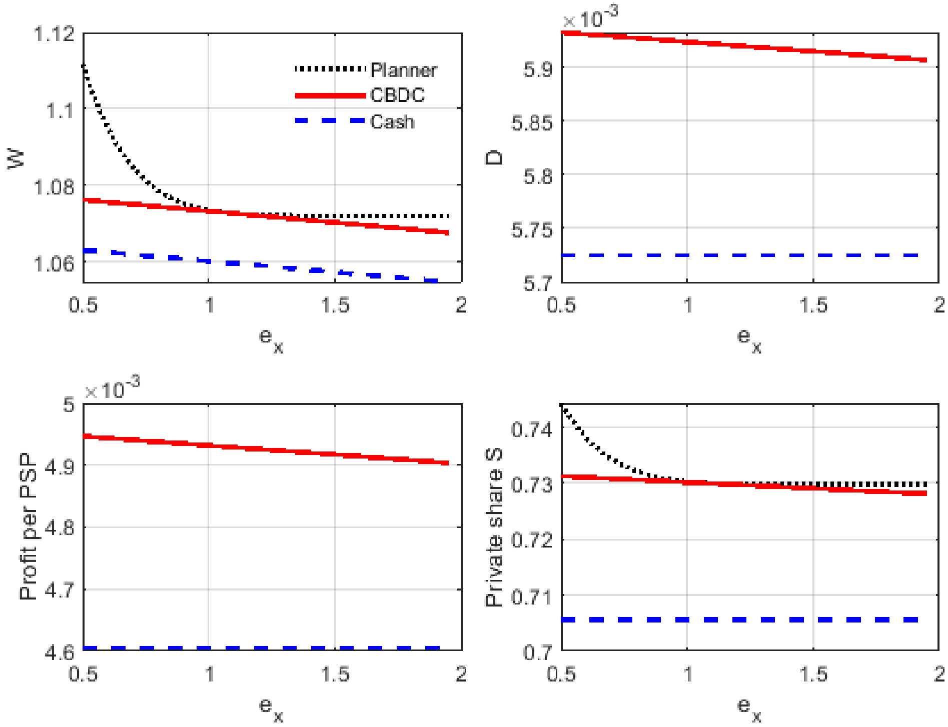

Figure 5: Counterfactual: cash versus CBDC

Notes. The economy is calibrated under the cash regime. Holding calibrated parameters fixed, we recompute equilibrium outcomes under the CBDC regime and under the planner as e_x varies (e_m is set to 1.1 in this exercise).

Figure 5 shows that welfare is higher in the regime with CBDC relative to the regime with cash. This is not surprising, given that the public CBDC payment policy can replicate the cash regime by setting $\alpha_m = \phi_m = \Phi_m = 0$. Under the calibrated parameters, relative to the cash regime, the CBDC regime generates more aggregate data collection, as well as a higher market share and profits for private rails. This suggests that market power is the dominant friction, and the public CBDC payment policy boosts the usage of the private rail. In the CBDC regime, all four variables decrease with e_x.

We carry out another counterfactual analysis: keeping everything the same as in Figure 5, except that we increase the number of private rails to $N = 60$. The result is shown in Figure 6. In this experiment, aggregate data production is lower with CBDC when e_x is high enough. A similar pattern applies to the market share and profits of private rails. When the market for payment services becomes more competitive (captured by a higher N), negative externality becomes a dominant concern when e_x is high. The figure is similar if we increase p (data becomes more valuable) or reduce A (the cost to process data decreases).

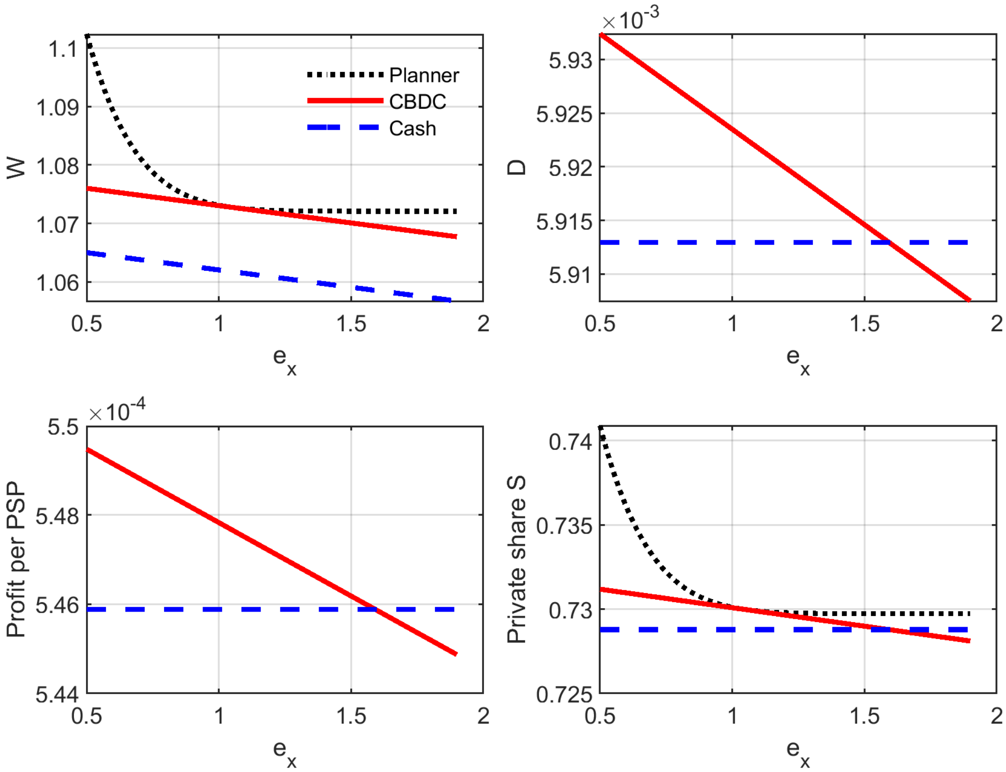

Figure 6: Counterfactual: cash versus CBDC

Notes. The parameters are the same as in Figure 5 except for $N = 60$.

8 Conclusion

This paper studies CBDC design in an economy where private PSPs provide payment services and monetize transaction data. We show that an optimal CBDC may collect some data when the public value of records exceeds privacy and processing costs. At the same time, private PSPs generally choose data collection to satisfy private incentives rather than social ones, so the CBDC can improve welfare by steering adoption across public and private rails. As a result, the introduction of a data-collecting CBDC can either increase or decrease private payment usage and aggregate data production relative to cash, depending on the balance between market power and the negative externalities from privately monetized data.

In our calibration to U.S. payment data, the optimal CBDC raises welfare and, under the baseline parameters, increases both private-rail market share and aggregate data collection; however, stronger competition, higher data value, or lower data-processing costs make it optimal to limit aggregate data production. Overall, the paper highlights that CBDC design cannot be separated from data policy and market structure.

While we develop a streamlined theoretical framework to study the optimal design of CBDC payment policy and calibrate it to the U.S. economy, several extensions would be valuable. One natural next step is to allow cash, CBDC, and private digital payment instruments such as cards to coexist, so that users can choose among payment methods with different privacy, convenience, and pricing attributes. Such an environment would permit a richer analysis of how payment design shapes data production, market power, and welfare. Another direction is to calibrate the model to countries with different payment mixes, privacy regimes, and market structures, to examine how the optimal CBDC policy varies across institutional settings. More broadly, the framework could also be used to study privacy regulation, including policies that restrict the secondary commercialization of payments data.

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A Proofs

Proof of Proposition 3.1. The FOCs are

$$\begin{aligned}
\partial \mathcal{W}/\partial q &= F(\hat{\epsilon})[u'(q) - 1 - \ell_0 \alpha + \alpha(p - e_x + e_m) - \alpha C'(\alpha q)] = 0, && (44)\\
\partial \mathcal{W}/\partial q_m &= [1 - F(\hat{\epsilon})][u'(q_m) - 1 - \ell_0 \alpha_m + \alpha_m e_m - \alpha_m C'(\alpha_m q_m)] = 0, && (45)\\
\partial \mathcal{W}/\partial \alpha &= F(\hat{\epsilon})\,[p + e_m - e_x - \ell_0 - C'(\alpha q)]\,q = 0, && (46)\\
\partial \mathcal{W}/\partial \alpha_m &= [1 - F(\hat{\epsilon})]\,[e_m - \ell_0 - C'(\alpha_m q_m)]\,q_m = 0, && (47)\\
\partial \mathcal{W}/\partial \hat{\epsilon} &= f(\hat{\epsilon})\,\{-\hat{\epsilon} + [u(q) - q - \ell_0 \alpha q - C(\alpha q)] - [u(q_m) - q_m - \ell_0 \alpha_m q_m - C(\alpha_m q_m)]\} = 0,\\
&+ \;\; f(\hat{\epsilon})\,((p - e_x + e_m)\,\alpha q - e_m \alpha_m q_m) = 0. && (48)
\end{aligned}$$

The results follow from rearrangement.

Proof of Proposition 3.2. Define

$$G(\alpha) = \ell_0(\alpha_m^* - \alpha)q^* + C(\alpha_m^* q^*) - C(\alpha q^*) + (p - e_x + e_m)\,\alpha q^* + e_m \alpha_m^* q^*.$$

If $p = e_x$, then $\alpha^* = \alpha_m^*$ and $\hat{\epsilon}^* = G(\alpha_m^*) = 0$. If $p > e_x$, then $\alpha^* > \alpha_m^*$, and $G(\alpha_m^*) > 0$. In addition, for $\alpha \in (\alpha_m^*, \alpha^*]$, we have $G'(\alpha) = [p - e_x + e_m - \ell_0 - C'(\alpha q^*)]q^* = [C'(\alpha^* q^*) - C'(\alpha q^*)]q^* \geq 0$, with equality at $\alpha = \alpha^*$. It then follows $\hat{\epsilon}^* = G(\alpha^*) > 0$. If $p < e_x$, then $\alpha^* < \alpha_m^*$, and $G(\alpha_m^*) < 0$. In addition, for $\alpha \in [\alpha^*, \alpha_m^*)$, we have $G'(\alpha) = [p - e_x + e_m - \ell_0 - C'(\alpha q^*)]q^* = [C'(\alpha^* q^*) - C'(\alpha q^*)]q^* \leq 0$, with equality at $\alpha = \alpha^*$. It then follows $\hat{\epsilon}^* = G(\alpha^*) < 0$.

Proof of Proposition 4.2. At $\iota = 0$, $q_m = q^*$ so $\partial \mathcal{W}/\partial q_m = 0$ by (27). Hence $\partial \mathcal{W}/\partial \iota$ has the sign of $\partial \mathcal{W}/\partial \hat{\epsilon}$ (because $\partial \hat{\epsilon}/\partial \iota > 0$). Using (28) with $q_m = q^*$ and rearranging delivers condition (29). If the inequality holds, $\partial \mathcal{W}/\partial \iota|_{\iota=0} < 0$ and the Friedman rule is optimal. Otherwise, a positive ι (an inflation tax) can be welfare-improving by shifting adopters toward the private rail where data may have higher net social value.

Proof of Proposition 5.1. Step 1 (planner conditions). The first-order conditions for the government's problem with a full CBDC instrument set imply

$$\frac{\partial \mathcal{W}}{\partial \hat{\epsilon}} = 0, \qquad \frac{\partial \mathcal{W}}{\partial q_m} = 0, \qquad \frac{\partial \mathcal{W}}{\partial \alpha_m} = 0.$$

These three equalities are exactly the planner's indifference, quantity-efficiency, and data-intensity conditions. Hence the target allocation in part (i) reproduces the planner's $(q_m^*, \alpha_m^*, \hat{\epsilon}^{SB})$ while leaving the private rail at $(q^*, \bar{\alpha})$.

Step 2 (constructing a policy). Given $\alpha_m = \alpha_m^*$ from (33), set the proportional fee

$$\phi_m = -\alpha_m^* \ell_0 - \iota.$$

This removes both the privacy wedge $\alpha_m^* \ell_0$ and the liquidity wedge ι from the buyer's FOC, so $q_m = q^*$.

Step 3 (fixing the extensive margin). With $q_m = q^*$ and $\alpha_m = \alpha_m^*$ already in place, choose the fixed fee Φ_m as in (39). Substituting this expression into the definition of U_m and comparing with U yields $\hat{\epsilon} = \hat{\epsilon}^{SB}$ in (34). Hence the extensive margin is exactly the planner's cutoff.

Step 4 (private rail unaffected). None of the government choices in Steps 2-3 alter the private rail's profit-maximization problem, so the private rail remains at $(q^*, \bar{\alpha}, \phi, \Phi)$ from Proposition 4.1.

Steps 1-4 construct a feasible policy triple that achieves the allocation in part (i); therefore that allocation is attainable and second-best-optimal.

Proof of Proposition 5.2. The CBDC chooses α_m^* from $C'(\alpha_m^* q^*) = e_m - \ell_0$, whereas the private rail chooses $\bar{\alpha}$ from $C'(\bar{\alpha} q^*) = p - \ell_0$ (compare (4) and (5)). Hence $\bar{\alpha} > \alpha_m^*$ precisely when $p > e_m$.

Proof of Proposition 5.3. The planner sets α^* via $C'(\alpha^* q^*) = p - \ell_0 + e_m - e_x$, while the private rail uses $C'(\bar{\alpha} q^*) = p - \ell_0$. If $e_x > e_m$ then $\bar{\alpha} > \alpha^*$. Define $G(\alpha) = \ell_0(\alpha_m^* - \alpha)q^* + C(\alpha_m^* q^*) - C(\alpha q^*) + (p - e_x + e_m)\,\alpha q^* - e_m \alpha_m^* q^*$. Note $\hat{\epsilon}^{SB} = G(\bar{\alpha})$, $\hat{\epsilon}^{SB}(\alpha^*) = \hat{\epsilon}^*$, and $G'(\alpha) = [p - e_x + e_m - \ell_0 - C'(\alpha q^*)]q^* = [C'(\alpha^* q^*) - C'(\alpha q^*)]q^* < 0$ for $\alpha > \alpha^*$. It then follows $\hat{\epsilon}^{SB} < \hat{\epsilon}^*$.

Proof of Proposition 5.4. Let $\hat{\epsilon}_0$ be the adoption threshold with cash. Because $\hat{\epsilon}_0$ is independent of (e_x, e_m) whereas $\hat{\epsilon}^{SB}$ in (34) decreases in e_x, there is a unique $\underline{e}$ at which the two cut-offs coincide. For $e_x > \underline{e}$ we have $\hat{\epsilon}^{SB} < \hat{\epsilon}_0$, so the private rail's share falls.

Aggregate data equal $D_c + D_m = F(\hat{\epsilon})\bar{\alpha}q^* + [1 - F(\hat{\epsilon})]\alpha_m^* q^*$. Because cash generates no data, the change $\Delta D = (D_c + D_m)_{\text{CBDC}} - (D_c + D_m)_{\text{cash}}$ can be negative only if the mass diverted off the private rail is large enough. This tends to happen when e_x is large, or the government finds it optimal to divert transactions away from the private rail. In particular, we need $\hat{\epsilon}_0 > \hat{\epsilon}^{SB}$ and

$$(1 - F(\hat{\epsilon}^{SB}))\alpha_m^* q^* < (F(\hat{\epsilon}_0) - F(\hat{\epsilon}^{SB}))\bar{\alpha}q^*,$$

which yields the higher threshold $\bar{e}$ stated in the text.

Proof of Proposition 6.1. Fix (α_i, ϕ_i); the buyer's best response on PSP i satisfies

$$u'(q) = 1 + \phi_i + \ell_0\alpha_i, \qquad \frac{\partial q}{\partial \phi_i} = \frac{1}{u''(q)} < 0.$$

For a given S (hence $F^{-1}(S)$), PSP i's per-user margin is

$$\begin{aligned} B(\alpha_i, \phi_i, q, S) &= \big[u(q) - q - \phi_i q - \ell_0\alpha_i q\big] - U_m - F^{-1}(S) + p\,\alpha_i q + \phi_i q - C(\alpha_i q) \\ &= u(q) - q - \ell_0\alpha_i q - U_m - F^{-1}(S) + p\,\alpha_i q - C(\alpha_i q). \end{aligned} \tag{49}$$

Using the envelope theorem to account for the induced change in q,

$$\frac{\partial B}{\partial \phi_i} = \underbrace{\Big[u'(q) - 1 - \ell_0\alpha_i + p\,\alpha_i - \alpha_i C'(\alpha_i q)\Big]}_{\partial B/\partial q} \cdot \frac{1}{u''(q)}.$$

Substituting the buyer FOC $u'(q) = 1 + \phi_i + \ell_0\alpha_i$ yields

$$\frac{\partial B}{\partial \phi_i} = \big[\phi_i + p\,\alpha_i - \alpha_i C'(\alpha_i q)\big]\,\frac{1}{u''(q)}.$$

Hence the FOC w.r.t. ϕ_i is

$$\phi_i + p\,\alpha_i - \alpha_i C'(\alpha_i q) = 0. \tag{FOC$_\phi$}$$

Similarly, differentiating (49) with respect to α_i (again using the buyer FOC for the induced q) gives

$$\frac{\partial B}{\partial \alpha_i} = \big[p - \ell_0 - C'(\alpha_i q)\big]\Big(q + \frac{\partial q}{\partial \alpha_i}\Big),$$

so the FOC w.r.t. α_i is

$$C'(\alpha_i q) = p - \ell_0. \tag{FOC$_\alpha$}$$

Combining (FOC_ϕ) and (FOC_α) implies

$$\phi_i = -\,\ell_0\,\alpha_i.$$

Plugging this into the buyer FOC yields $u'(q) = 1$ so $q = q^*$. None of the conditions depend on s_i, S, or N, establishing the claim.

Proof of Proposition 6.2. Write the symmetric-equilibrium condition (41) as

$$Z - \hat{\epsilon}^* - \frac{H(\hat{\epsilon}^*)}{N} = 0, \qquad Z := u(q) - q - (\ell_0 - p)\,\bar{\alpha}\,q - C(\bar{\alpha}q) - U_m.$$

Implicit differentiation w.r.t. N yields

$$0 = -\,\frac{d\hat{\epsilon}^*}{dN} - \frac{H'(\hat{\epsilon}^*)}{N}\,\frac{d\hat{\epsilon}^*}{dN} + \frac{H(\hat{\epsilon}^*)}{N^2},$$

so

$$\frac{d\hat{\epsilon}^*}{dN} = \frac{H(\hat{\epsilon}^*)}{N^2\left(1 + \frac{H'(\hat{\epsilon}^*)}{N}\right)}. \tag{50}$$

If $H'(\varepsilon) \geq 0$ (IFR), the denominator of (50) is strictly positive. For any interior equilibrium ($0 < S^* < 1$ so $H(\hat{\epsilon}^*) > 0$), we have $d\hat{\epsilon}^*/dN > 0$. Hence

$$\frac{dS^*}{dN} = f(\hat{\epsilon}^*)\,\frac{d\hat{\epsilon}^*}{dN} > 0, \qquad \frac{ds^*}{dN} = \frac{1}{N}\frac{dS^*}{dN} - \frac{S^*}{N^2} < 0.$$

For profits, $\pi(N) = \frac{F(\hat{\epsilon}^*)\,H(\hat{\epsilon}^*)}{N^2}$, so

$$\frac{d\pi}{dN} = \frac{1}{N^2}\frac{d}{dN}\big[F(\hat{\epsilon}^*)H(\hat{\epsilon}^*)\big] - \frac{2F(\hat{\epsilon}^*)H(\hat{\epsilon}^*)}{N^3} = \frac{F(\hat{\epsilon}^*)H(\hat{\epsilon}^*)}{N^3}\left[\frac{H(\hat{\epsilon}^*)/N}{1 + H'(\hat{\epsilon}^*)/N} - 2\right] < 0,$$

where the inequality uses $H'(\hat{\epsilon}^*) \geq 0$ and $H(\hat{\epsilon}^*) > 0$. Finally, since $\bar{\alpha}$ and q^* are constant, aggregate private-rail data $D(N) = S^*(N)\,\bar{\alpha}\,q^*$ inherits the monotonicity of $S^*(N)$.

Proof of Proposition 6.3. Under the second-best CBDC policy, the equilibrium cutoff $\hat{\epsilon}^{SB}$ is held fixed, so

$$S(N) = F(\hat{\epsilon}^{SB})$$

and therefore

$$s^*(N) = \frac{F(\hat{\epsilon}^{SB})}{N}, \qquad D(N) = F(\hat{\epsilon}^{SB})\,\bar{\alpha}q^*.$$

The expressions for $\Phi^{SB}(N)$ and $\pi(N)$ given in Subsection 6.3 then imply immediately that $\Phi^{SB}(N)$ and $\pi(N)$ are strictly decreasing in N. The same is true for the per-user margin

$$m(N) = \frac{F(\hat{\epsilon}^{SB})}{N\, f(\hat{\epsilon}^{SB})}.$$

Hence market share per PSP, per-user margin, fixed fee, and profit per PSP all fall with entry, while aggregate private adoption and aggregate private-rail data remain constant.

B Derivations for Section 4

B.1 Buyer's Dynamic Problem and Reduction to Static Forms

This appendix shows how the CM/DM dynamic problems reduce to the static formulations $U_m(\theta_m)$ in (7) and $U(\theta)$ in (9).

Public-money rail. Let m be the buyer's real money balance entering the CM and b her debt position. The CM problem is

$$W_m(m,b) = \max_{x,y,m'} \left\{U(x) - y + \beta V_m(m')\right\} \quad \text{s.t.} \quad x + \gamma m' + b = y + m + T, \tag{51}$$

where V_m is the DM value if the buyer intends to use public money. In the DM,

$$V_m(m) = u(q(m)) - \ell_0 \alpha_m q(m) - \Phi_m - \phi_m q(m) - g(m) - \mathcal{L} + W_m(m,0), \tag{52}$$

with $g(m)$ the payment to the seller. For $\iota > 0$ the buyer brings just enough money, so $q(m) = g(m) = m$. Substituting (52) into (51), using linear CM preferences and eliminating m' yields

$$U_m(\theta_m) = \max_{q_m} \Big\{u(q_m) - q_m - \ell_0\alpha_m q_m - \Phi_m - \phi_m q_m - \iota q_m\Big\}, \tag{53}$$

which is (7) in the main text.

Private-credit rail. Analogously, define $W(m,b)$ as the CM value if the buyer intends to use private credit. Since credit is available in the DM, the buyer sets $m' = 0$ for $\iota > 0$. The DM value is

$$V(q) = \Big[u(q) - \ell_0 \alpha q - \Phi - (1+\phi)q - \epsilon\Big] - \mathcal{L} + W(q,0), \tag{54}$$

leading to the static problem

$$U(\theta) = \max_q \Big\{ u(q) - q - \ell_0 \alpha q - \Phi - \phi q \Big\}, \tag{55}$$

i.e. (9).

Marginal adopter. Let $\hat{\epsilon}$ solve $U - \hat{\epsilon} = U_m$. Because F is the CDF of ϵ, buyers with $\epsilon \leq \hat{\epsilon}$ adopt private credit; others use public money, yielding (11).

Lemma B.1 (Reduction) *Under linear CM preferences and $\iota > 0$, the buyer's dynamic CM/DM problems on each rail reduce to the static programs* (7) *and* (9)*; the marginal adopter $\hat{\epsilon}$ satisfies* (11)*.*

Comparative statics used in the main text follow by differentiating these reduced problems.

B.2 Comparative Statics for q_m, q, and $\hat{\epsilon}$

Differentiating (8)-(10) and $\hat{\epsilon} = U(\theta) - U_m(\theta_m)$ yields:

$$q_{m,\alpha} \equiv \frac{\partial q_m}{\partial \alpha_m} = \frac{\ell_0}{u''(q_m)}, \tag{56}$$

$$q_{m,\phi} \equiv \frac{\partial q_m}{\partial \phi_m} = \frac{1}{u''(q_m)}, \tag{57}$$

$$q_{m,\iota} \equiv \frac{\partial q_m}{\partial \iota} = \frac{1}{u''(q_m)}. \tag{58}$$

Lemma B.2 *$q_{m,\alpha} < 0$, $q_{m,\phi} < 0$, and $q_{m,\iota} < 0$.*

Lemma B.3 *For the private rail,*

$$q_\alpha \equiv \frac{\partial q}{\partial \alpha} = \frac{\ell_0}{u''(q)} < 0, \tag{59}$$

$$q_\phi \equiv \frac{\partial q}{\partial \phi} = \frac{1}{u''(q)} < 0. \tag{60}$$

Lemma B.4 *The marginal adopter $\hat{\epsilon}$ responds to policy instruments as follows:*

$$\frac{\partial \hat{\epsilon}}{\partial \alpha} = \frac{\partial U}{\partial \alpha} = -q\,\ell_0, \tag{61}$$

$$\frac{\partial \hat{\epsilon}}{\partial \Phi} = \frac{\partial U}{\partial \Phi} = -1, \tag{62}$$

$$\frac{\partial \hat{\epsilon}}{\partial \phi} = \frac{\partial U}{\partial \phi} = -q, \tag{63}$$

$$\frac{\partial \hat{\epsilon}}{\partial \alpha_m} = -\frac{\partial U_m}{\partial \alpha_m} = q_m \ell_0, \tag{64}$$

$$\frac{\partial \hat{\epsilon}}{\partial \Phi_m} = -\frac{\partial U_m}{\partial \Phi_m} = 1, \tag{65}$$

$$\frac{\partial \hat{\epsilon}}{\partial \phi_m} = -\frac{\partial U_m}{\partial \phi_m} = q_m, \tag{66}$$

$$\frac{\partial \hat{\epsilon}}{\partial \iota} = -\frac{\partial U_m}{\partial \iota} = q_m. \tag{67}$$

Proofs. The signs follow from $u'' < 0$ and $\ell_0 > 0$.

B.3 Private rail FOCs and Derivations

This subsection derives the first-order conditions of the private rail's problem reported in Section 4.2. Recall the profit function

$$\Pi(\theta;\theta_m) = \Big[\Phi + (\phi + \alpha p)\,q - C(\alpha q)\Big]\,F(\hat{\epsilon}), \tag{68}$$

with $\theta = (\Phi, \phi, \alpha)$, $\theta_m = (\Phi_m, \phi_m, \alpha_m)$, and $q = q(\theta)$, $\hat{\epsilon} = \hat{\epsilon}(\theta, \theta_m)$ determined by buyers. We use: (i) $u'(q) = 1 + \phi + \alpha\ell_0$ (eq. (10)); (ii) Lemma B.3; (iii) Lemma B.4. Let f denote the density of ϵ.

FOC with respect to α.

$$\frac{\partial \Pi}{\partial \alpha} = \big[\Phi + (\phi + \alpha p)q - C(\alpha q)\big]\, f(\hat{\epsilon})\, \frac{\partial \hat{\epsilon}}{\partial \alpha} + F(\hat{\epsilon})\Big\{p\,q + (\phi + \alpha p)\,q_\alpha - \big[C'(\alpha q)\,q + \alpha C'(\alpha q)\,q_\alpha\big]\Big\}. \tag{69}$$

Using $\partial\hat{\epsilon}/\partial\alpha = -q\ell_0$ and $q_\alpha = \ell_0/u''(q)$, and combining with the other FOCs (below), we obtain

$$p = \ell_0 + C'(\alpha q). \tag{70}$$

FOC with respect to ϕ.

$$\frac{\partial \Pi}{\partial \phi} = \big[\Phi + (\phi + \alpha p)q - C(\alpha q)\big]\, f(\hat{\epsilon})\, \frac{\partial \hat{\epsilon}}{\partial \phi} + F(\hat{\epsilon})\Big\{q + (\phi + \alpha p)\, q_\phi - \alpha C'(\alpha q)\, q_\phi\Big\}. \tag{71}$$

With $\partial\hat{\epsilon}/\partial\phi = -q$ and $q_\phi = 1/u''(q)$, and using (70), we get

$$\phi = -\,\alpha \ell_0. \tag{72}$$

FOC with respect to Φ. Since $q_\Phi = 0$,

$$\frac{\partial \Pi}{\partial \Phi} = \big[\Phi + (\phi + \alpha p)q - C(\alpha q)\big]\, f(\hat{\epsilon})\, \frac{\partial \hat{\epsilon}}{\partial \Phi} + F(\hat{\epsilon}). \tag{73}$$

Using $\partial\hat{\epsilon}/\partial\Phi = -1$,

$$\Phi = \frac{F(\hat{\epsilon})}{f(\hat{\epsilon})} + C(\alpha q) - \alpha q\, C'(\alpha q). \tag{74}$$

Profits. Substituting (70)-(74) into (68) yields

$$\Pi(\hat{\epsilon}) = \frac{[F(\hat{\epsilon})]^2}{f(\hat{\epsilon})}.$$

Summary. The three FOCs reduce to

$$\begin{aligned} p &= \ell_0 + C'(\alpha q), \\ \phi &= -\,\alpha \ell_0, \\ \Phi &= \frac{F(\hat{\epsilon})}{f(\hat{\epsilon})} + C(\alpha q) - \alpha q\, C'(\alpha q), \end{aligned}$$

implying $q = q^*$ and $\alpha = \bar{\alpha}$ in equilibrium.

C A Microfoundation for p and e_x

Here, we base on Chiu and Koeppl (2023) to provide a micro-foundation for p and e_x.

Buyers in CM Suppose there is an additional CM where data is an input for facilitating transactions. There are two types of goods, customized goods, $q(i)$, and a generic good,

$\bar{q}$. A buyer j consumes only a specific type j customized good so that the marginal utility derived from $q(i)$ is $\bar{u}$ for $i = j$, and is zero for $i \neq j$. For a generic good, the marginal utility $\tilde{u} \in [0, \bar{u}]$ is randomly drawn from a distribution $H(u)$.

The buyer's utility is thus given by

$$U_j(q(i), \bar{q}) = \bar{u} \cdot q(i)\mathbf{1}_{i=j} + \tilde{u} \cdot \bar{q} \tag{75}$$

where $\mathbf{1}_{i=j}$ is an indicator function capturing whether the customized good matches the buyer's type. Finally, we assume that the preference for a particular customized good is private information for the buyer.

Sellers A seller can produce either a customized good or a generic good, but not both. The marginal cost of producing a customized good is 1, while for a generic good it is $1 + \bar{c}$. Hence, the customization of a product not only increases the utility from consuming it, but also reduces the cost of producing it.

Trading The seller makes a take-it-or-leave-it offer to the buyer for either producing the customized good or the generic good. The seller, however, cannot offer a customized good as the buyer's type is not known. Hence, the seller will offer a generic good at a price P that solves

$$\max_{P} P[1 - H(P)] - 1 - \bar{c}. \tag{76}$$

where $1 - H(P)$ is the probability of the buyer to accept the offer. The optimal price is given by the reciprocal of the hazard rate

$$P^* = \frac{1 - H(P^*)}{H'(P^*)} \tag{77}$$

We can normalize the sellers profits to zero at the optimal price P^* so that the expected surplus of the buyer from purchasing a generic good is given by

$$\mathcal{S}_b = \big(1 - H(P^*)\big)\big(\mathbb{E}[u|u \geq P^*] - P^*\big) \tag{78}$$

Information about Buyer's Preferences The seller can buy information from the private rail to learn the preference of the buyer for the customized good. With this information, the seller can offer a customized good and charge the price $P = \bar{u}$. The payoffs from trading can be summarized in the following table.

	Unknown (trade generic)	**Known (trade special)**
Buyer's surplus	$\mathcal{S}_b$	0
Seller's surplus	0	$\mathcal{S}_s = \bar{u} - 1$
Total surplus	$\mathcal{S}_b$	$\mathcal{S}_s = \bar{u} - 1$

When the preference of the buyer is known, selling a customized good increases the total surplus by

$$\mathcal{S} = \mathcal{S}_s - \mathcal{S}_b. \tag{79}$$

which is the social value of the information provided by the private rail. Sellers are willing to purchase the information from the private rail and offer a customized good up to a price that is equal to the additional surplus $\mathcal{S}_s$. This surplus is not only the additional social value $\mathcal{S}$ generated from the information but also the buyer's informational rent $\mathcal{S}_b$.

Suppose each unit of D from the DM can help the seller to customize x units of goods, then the price of data is

$$p = x\mathcal{S}_s,$$

and the data externality is

$$\mathcal{L}(D) = e_x D = x\mathcal{S}_b D.$$

D Calibration Notes

We calibrate the economy under the cash regime with Cournot competition in the private provision of payment services.

Functional forms:

$$u(q) = q^\eta/\eta,$$
$$C(d) = Ad^\psi/\psi,$$
$$U(x) = B\ln x.$$

The equilibrium conditions are:

$$u'(q) = q^{\eta-1} = 1 \Rightarrow q = q^* = 1$$

$$C'(\alpha q) = A(\alpha q)^{\psi-1} = p - \ell_0 \Rightarrow \alpha = \bar{\alpha} = \left(\frac{p-\ell_0}{A}\right)^{\frac{1}{\psi-1}} / q$$

$$\phi = -p\alpha + \alpha C'(\alpha q) = -\bar{\alpha}\ell_0$$

$$u'(q_m) = q_m^{\eta-1} = 1 + \iota \Rightarrow q_m = (1+\iota)^{\frac{1}{\eta-1}}$$

$$U_m = u(q_m) - q_m - \iota q_m$$

$$\hat{\epsilon} = \frac{N}{1+N}\Big[u(q) - q - (\ell_0 - p)\alpha q - C(\alpha q) - U_m\Big] + \frac{\epsilon_{min}}{1+N}$$

$$\Phi = u(q) - q - U_m - \hat{\epsilon}.$$

The targets are:

Table 4: Calibration Targets

Target variable	notation
Cash share of transactions in volume	s_v
Cash share of transactions in value	$s_\$$
Data monetization revenue/ payment revenue	DR
Average money demand	MD
Credit card rewards/transaction value	R
Money demand curve (fit 2 parameters)	

Formulas for targets:

1. Credit card rewards

$$R = -\phi = \alpha\ell_0$$

2. Data monetization revenue as a share of payment revenue

$$DR = \frac{\alpha p}{\frac{\Phi}{q} + \phi + \alpha p}.$$

3. Cash volume share

$$s_v = 1 - F(\hat{\epsilon}).$$

4. Cash value share

$$s_\$ = \frac{[1-F(\hat{\epsilon})]q_m}{F(\hat{\epsilon})q + [1-F(\hat{\epsilon})]q_m}.$$

5. Money demand

$$MD = M/GDP,$$

where GDP is the sum of (1) buyer's consumption in the DM, (2) buyer and seller's consumption in the CM, and (3) PSP's revenue.

$$GDP = [F(\hat{\epsilon})q + (1 - F(\hat{\epsilon}))q_m] + [\Phi + (\phi + \alpha p)\, q]\, F(\hat{\epsilon}) + 2B,$$

and the real money balance is:

$$M = [1 - F(\hat{\epsilon})]q_m.$$

6. Money demand curve

While deriving the money demand curve, we plug in the interest rate for the year t (while fixing other parameters) and calculate the model implied money demand for that year. We measure the distance between the model implied money demand curve and its empirical counterpart as:

$$MSE = \frac{1}{T}\sum_t \left(\frac{\frac{M_{t\text{model}}}{GDP_{t\text{model}}}}{\frac{M_t}{GDP_t}} - 1 \right)^2$$

where T is the number of data points on the money demand curve.